Top Mode Standard Model with Extra Dimensions

At Kyoto University

(March 7, 2001)

1. Introduction $\sim$ Brief History of TMSM $\sim$
2. ACDH model
3. An upper bound of the bulk QCD coupling and Naive Dimensional Analysis (NDA)
4. Analysis for the SD equation
5. Anomalous dimension of $\bar{\psi}\psi$
6. Summary and discussions

Michio Hashimoto (Tohoku Univ.)

This seminar is based on

hep-ph/0010260 .

1. Introduction

~ Brief History of TMSM ~

☆ Why is the top-quark mass so heavy?

It is nearly equal to the EWSB scale!

$$m_t \simeq 175 \text{ GeV}$$

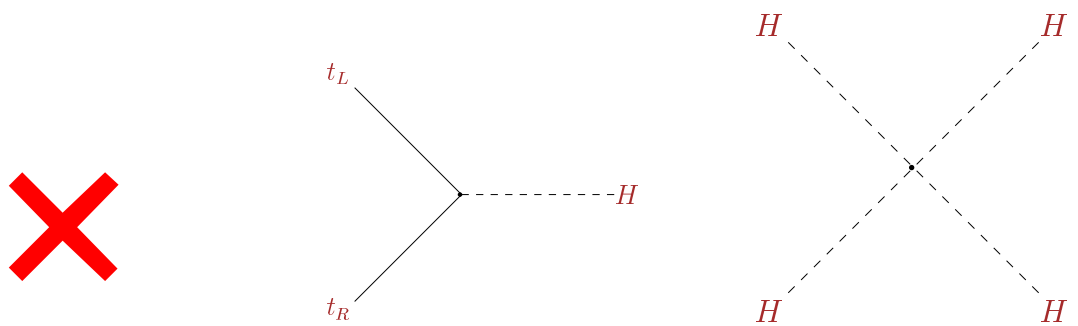
$$v \simeq 250 \text{ GeV}$$

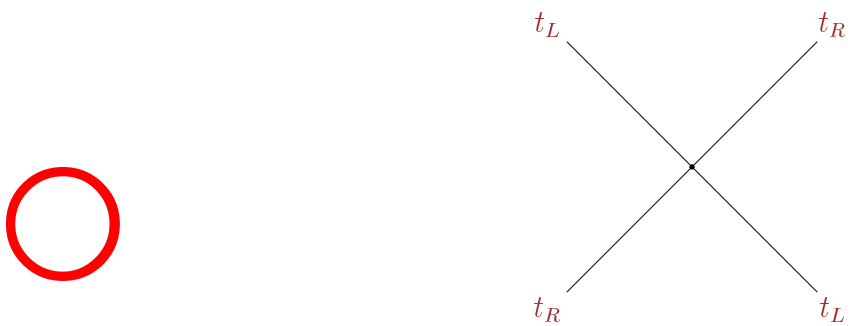
$\Rightarrow$ (Origin of EWSB) $\stackrel{??}{=}$ (The Top-Quark Condensation)

V. A. Miransky, M. Tanabashi, K. Yamawaki, PLB221('89)177; MPLA4('89)1043.
Y. Nambu, Fermi Inst. Rep. No. 89-08.

Top Mode Standard Model (TMSM)

$$\mathcal{L}_{\text{TMSM}} = \mathcal{L}_{\text{SM}} - \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{4-fermi}}$$





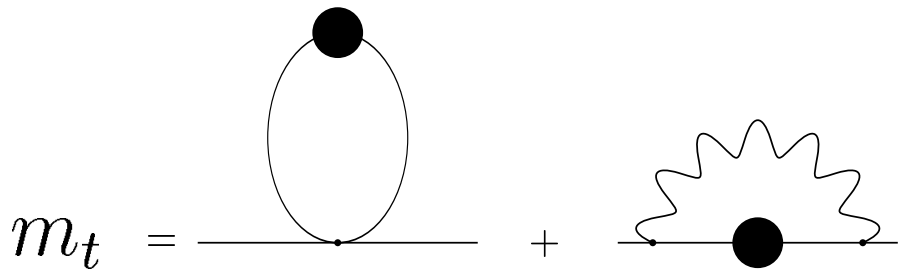
• (Schwinger–Dyson Equation) + (Pagels–Stokar Formula)

↑ The gap equation for the top-quark mass

↑ The formula for $f_\pi(v)$ in which only the mass function is needed

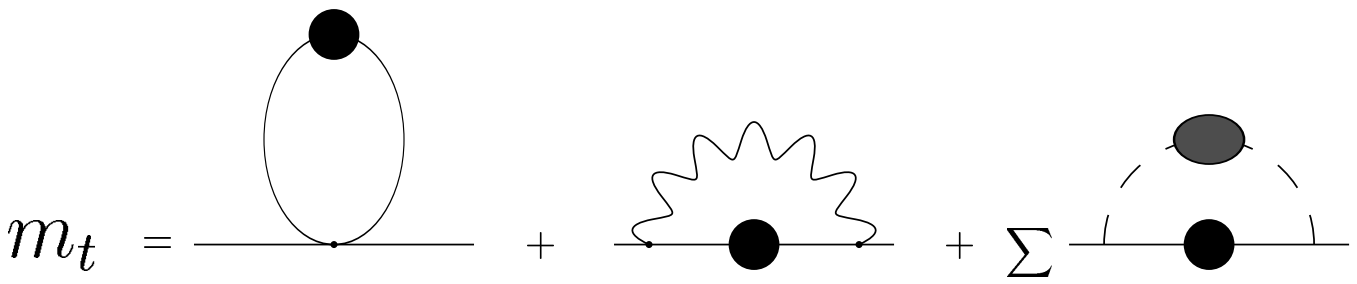
○ 4-fermi + QCD → $m_t = 250$ GeV

V. A. Miransky, M. Tanabashi, K. Yamawaki, PLB221('89)177; MPLA4('89)1043.



○ 4-fermi + QCD + composite Higgs loop → $m_t = 180$ GeV

M. Hashimoto, Prog. Theor. Phys. **100** (1998) 781.



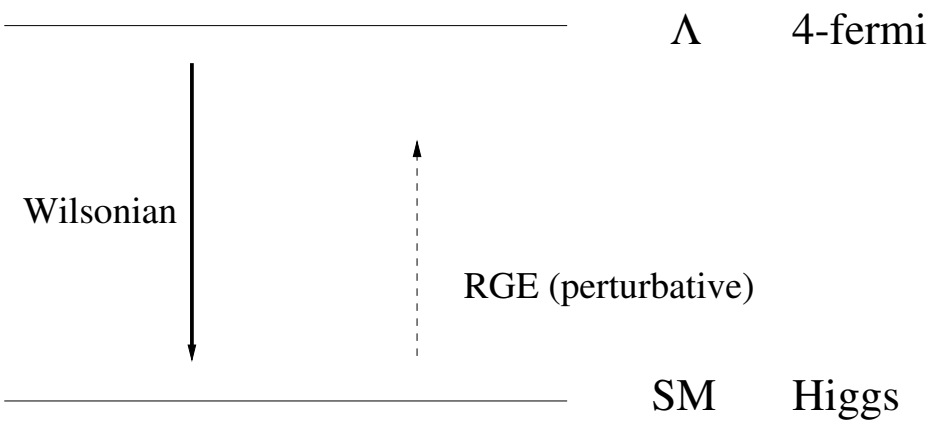
$\mathcal{O}(N_c^0)$ $\mathcal{O}(1/N_c)$

$(\Lambda = 10^{19}$ GeV)

• (RGE) + (compositeness condition)

↑

The boundary condition for 4-fermi interaction



compositeness condition

$$\frac{1}{y_t^2(\Lambda)} = 0 \quad \longleftarrow \text{for vanishing } Z_H(\Lambda)\partial_\mu H_0^\dagger \partial^\mu H_0$$

$$\frac{\lambda(\Lambda)}{y_t^4(\Lambda)} = 0 \quad \longleftarrow \text{for vanishing } (H_0^\dagger H_0)^2 \text{ at the cutoff scale } \Lambda$$

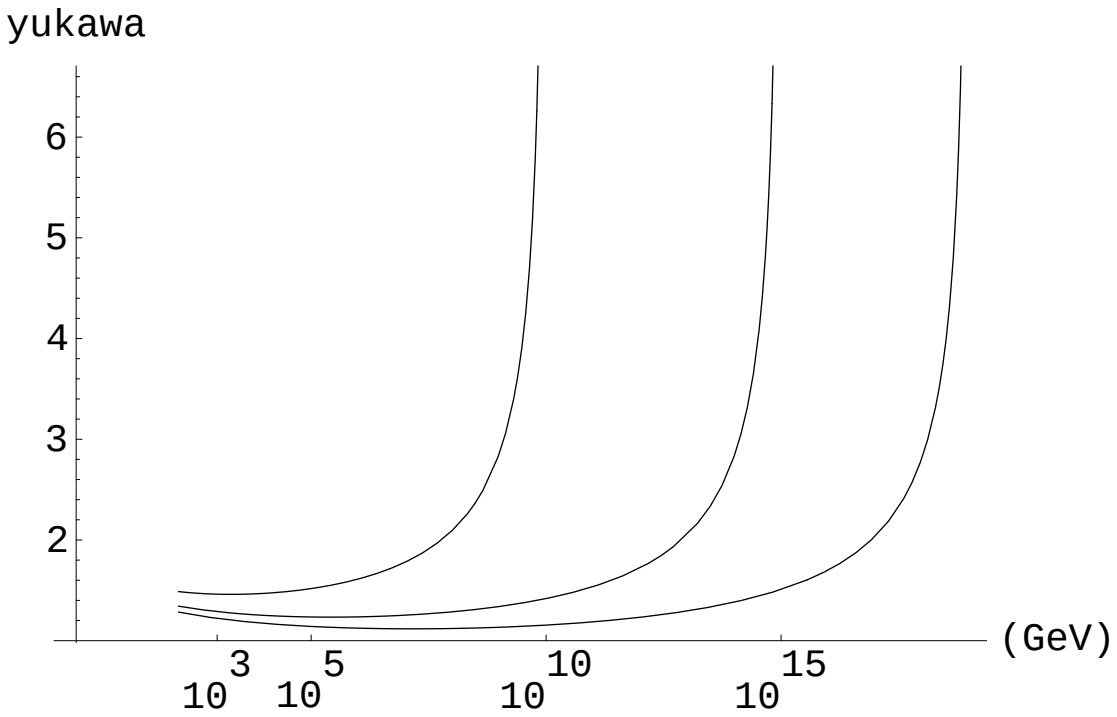
RGE for the SM

$$16\pi^2 \mu \frac{dy_t}{d\mu} = y_t \left(\left(N_c + \frac{3}{2} \right) y_t^2 - 3\frac{N_c^2 - 1}{N_c} g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_Y^2 \right)$$

$$16\pi^2 \mu \frac{d\lambda}{d\mu} = 4N_c \lambda y_t^2 - 4N_c y_t^4$$

$$+ 12\lambda^2 - 3\lambda(3g_2^2 + g_Y^2) + \frac{3}{4}\left(4g_2^4 + 2g_2^2 g_Y^2 + g_Y^4\right)$$

quasi IR-fixed point



The behavior of y_t for various cutoffs

$\Rightarrow$ full SM analysis $\cdots\cdots$ $m_t = 220$ GeV $\quad (\Lambda = 10^{19}$ GeV)

W. A. Bardeen, C. T. Hill, M. Lindner, PRD41('90)1647.

Problems

1. The original version of the TMSM predicted slightly larger top-quark mass. ($m_t > 200$ GeV)
2. What's the origin of 4-fermion interaction?
3. There is a serious fine-tuning problem.

$\longrightarrow$ ◎ Models in four dimensions

★ Top Color Model $SU(3)_1 \times SU(3)_2 \to SU(3)_{\rm QCD}$

C.T.Hill, PLB266('91)419; PLB345('95)483.

★ Top Seesaw Model

R.S.Chivukula, B.A.Dobrescu, H.Georgi, C.T.Hill, PRD59('99)075003.

$\longrightarrow$ ◎ Models with extra dimensions

★ Higgs particle and SM gauge bosons live in the bulk.

A. B. Kobakhidze, hep-ph/9904203.

- $m_t \simeq 300$ GeV, ($D = 6$, $\Lambda = 10$ TeV, $M_c = 1$ TeV)

★ The third generation and SM gauge bosons live in even dimensions. ($D = 6, 8, \cdots$) (ACDH Model)

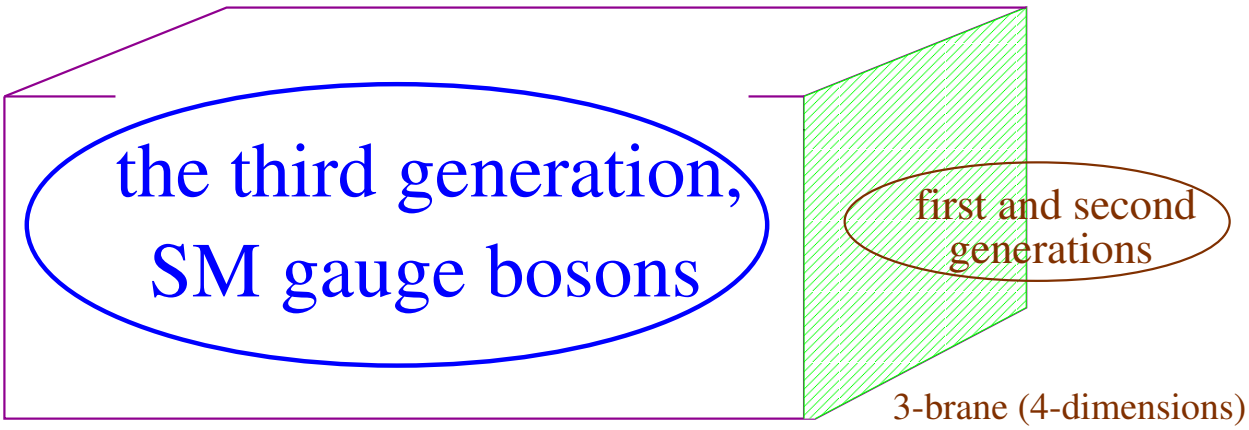
N. Arkani-Hamed, H. Cheng, B. A. Dobrescu, and L. J. Hall, Phys. Rev. **D62**, 096006 (2000).

- The bulk fermion dynamically condenses due to the bulk QCD.

2. ACDH model

- Arkani-Hamed, Cheng, Dobrescu, Hall (ACDH) proposed a following model:

$(t,b)_L$	t_R	b_R		1st generation
$(\nu_\tau,\tau)_L$	τ_R			2nd generation
gluon	W	Z	γ	



Basic Ideas

The bulk gauge theories rapidly become strongly coupled.

$$\hat{g}^2 \equiv g_D^2 \mu^\delta, \quad (\delta \equiv D-4)$$

$$\Downarrow$$

The condensation of the bulk fermion

$$\Downarrow$$

DEWSB !!!

(Dynamical chiral symmetry breaking(DχSB))

The Standard Model in six dimensions

- γ matrices in six dimensions ($2^3 \times 2^3$ matrix)

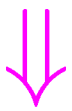
$$\Gamma^\mu = \begin{pmatrix} -\gamma^\mu & 0 \\ 0 & \gamma^\mu \end{pmatrix}, \Gamma^5 = \begin{pmatrix} 0 & iI \\ iI & 0 \end{pmatrix}, \Gamma^6 = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$$

- The chiral operator in six dimensions

$$\Gamma_\chi \equiv i^{(3-1)}\Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^5\Gamma^6 = \begin{pmatrix} -\gamma_5 & 0 \\ 0 & \gamma_5 \end{pmatrix}$$

- The chiral fermion in six dimensions

$$\Psi_\pm(x^\mu, y^1, y^2) \equiv \frac{1 \pm \Gamma_\chi}{2}\Psi(x^\mu, y^1, y^2)$$



The Lagrangian density in six dimensions

$$\mathcal{L}_{6\mathrm{D}} = \bar{\Psi}_\pm i\Gamma^M D_M \Psi_\pm - \frac{1}{2g_D^2}\mathrm{tr}(\mathcal{F}^{MN}\mathcal{F}_{MN})$$

- Our notation for field variables

6 dim.	$\mathcal{Q}_+$	$\mathcal{U}_-$	$\mathcal{D}_-$	$\mathcal{L}_+$	$\mathcal{E}_-$
4 dim.	$(t,b)_L$	t_R	b_R	$(\nu_\tau,\tau)_L$	τ_R

• T^2/Z_2 compactification

$$X(x^\mu, y^1 + 2\pi R, y^2) = X(x^\mu, y^1, y^2 + 2\pi R) = X(x^\mu, y^1, y^2),$$

$$\Psi_\pm(x^\mu, -y^1, -y^2) = -i\Gamma^5\Gamma^6\Psi_\pm(x^\mu, y^1, y^2),$$

$$= \begin{pmatrix} +\frac{1\mp\gamma_5}{2} & 0 \\ 0 & -\frac{1\pm\gamma_5}{2} \end{pmatrix} \Psi(x^\mu, y^1, y^2),$$

$$\mathcal{A}_\mu(x^\mu, -y^1, -y^2) = +\mathcal{A}_\mu(x^\mu, y^1, y^2),$$

$$\mathcal{A}_{5,6}(x^\mu, -y^1, -y^2) = -\mathcal{A}_{5,6}(x^\mu, y^1, y^2)$$

• The Fourier expansion of field variables to $\vec{y}(= (y^1, y^2))$

$$\Psi_+(x^\mu, y^1, y^2) = \frac{1}{\pi R}\begin{pmatrix} \frac{1}{\sqrt{2}}\psi_L^{(0)}(x^\mu) + \psi_L^{(\vec{n})}(x^\mu)\cos\left(\frac{\vec{n}\cdot\vec{y}}{R}\right) \\ \psi_R^{(\vec{n})}(x^\mu)\sin\left(\frac{\vec{n}\cdot\vec{y}}{R}\right) \end{pmatrix},$$

$$\mathcal{A}_\mu(x^\mu, y^1, y^2) = \frac{1}{\sqrt{2}\pi R}A_\mu^{(0)}(x^\mu) + \frac{1}{\pi R}A_\mu^{(\vec{n})}(x^\mu)\cos\left(\frac{\vec{n}\cdot\vec{y}}{R}\right),$$

$$\mathcal{A}_{5,6}(x^\mu, y^1, y^2) = \frac{1}{\pi R}A_{5,6}^{(\vec{n})}(x^\mu)\sin\left(\frac{\vec{n}\cdot\vec{y}}{R}\right)$$

$\Rightarrow$ Only SM particles remain as zero modes!

$$\Downarrow$$

Integrating y^1, y^2, the four dimensional action is obtained.

$$S = \frac{1}{2}\int d^4x \int_{-\pi R}^{\pi R}\int_{-\pi R}^{\pi R} dy^1 dy^2 \; \mathcal{L}_{\text{6D}}$$
$$\equiv \int d^4x \; \mathcal{L}_{\text{4D}}$$

- The kinetic term of the bulk gauge field

$$\frac{1}{g^2} = \frac{1}{n}\frac{(2\pi R)^\delta}{g_D^2}, \qquad (T^\delta/Z_n,\ \delta \equiv D-4)$$

- Masses of KK modes

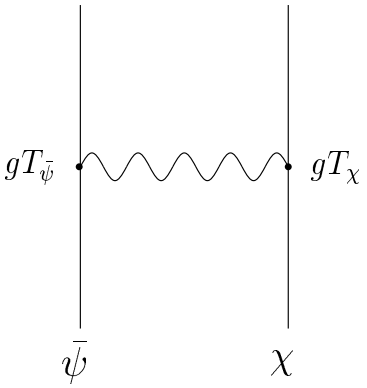
$$M_{\text{KK}}^2 = \frac{|\vec{n}|^2}{R^2}$$

- Couplings to KK modes

$$\mathcal{L}_{\text{int}} = g\bar{\psi}_{L,R}^{(\vec{n})}\gamma^\mu\psi_{L,R}^{(\vec{n})}A_\mu^{(0)} - ig\bar{\psi}_L^{(0)}\psi_R^{(\vec{n})}A_5^{(\vec{n})} + g\bar{\psi}_L^{(0)}\psi_R^{(\vec{n})}A_6^{(\vec{n})} + \text{(h.c.)},$$
$$\mathcal{L}_{\text{tri}} = g\partial^\mu A^{(0)\,\nu}[A_\mu^{(\vec{n})}, A_\nu^{(\vec{n})}] + g\partial^\mu A^{(\vec{n})\,\nu}[A_\mu^{(0)}, A_\nu^{(\vec{n})}] + g\partial^\mu A^{(\vec{n})\,\nu}[A_\mu^{(\vec{n})}, A_\nu^{(0)}]$$
$$- g\partial^\mu A^{(\vec{n})\,5,6}[A_{5,6}^{(\vec{n})}, A_\mu^{(0)}] + gA^{(\vec{n})\,\mu}[A_{5,6}^{(\vec{n})}, A_\mu^{(0)}]$$

$A_{5,6}^{(\vec{n})}(x^\mu)$: adjoint scalar fields

The Most Attractive Channel (MAC) Hypothesis



$$V(r) \sim \frac{g^2}{r} T_{\bar{\psi}} \cdot T_{\chi}$$
$$= -\frac{1}{2}\frac{g^2}{r}\left[C_2(\bar{\psi}) + C_2(\chi) - C_2(\bar{\psi}\chi)\right]$$

composite scalar	$SU(3) \times SU(2) \times U(1)$ representation	binding strength	relative binding for $\hat{g}_1 = \hat{g}_2 = \hat{g}_3$
$\overline{\mathcal{Q}}_+ \mathcal{U}_-$	$(\mathbf{1}, \mathbf{2}, +1/2)$	$\frac{4}{3}\hat{g}_3^2 + \frac{1}{15}\hat{g}_1^2$	1
$\overline{\mathcal{Q}}_+ \mathcal{D}_-$	$(\mathbf{1}, \mathbf{2}, -1/2)$	$\frac{4}{3}\hat{g}_3^2 - \frac{1}{30}\hat{g}_1^2$	0.93
$\overline{\mathcal{Q}}_+ \mathcal{D}^c_-$	$(\mathbf{3}, \mathbf{2}, +1/6)$	$\frac{2}{3}\hat{g}_3^2 + \frac{1}{30}\hat{g}_1^2$	0.5
$\overline{\mathcal{Q}}_+ \mathcal{U}^c_-$	$(\mathbf{3}, \mathbf{2}, -5/6)$	$\frac{2}{3}\hat{g}_3^2 - \frac{1}{15}\hat{g}_1^2$	0.43
$\overline{\mathcal{L}}_+ \mathcal{E}_-$	$(\mathbf{1}, \mathbf{2}, -1/2)$	$\frac{3}{10}\hat{g}_1^2$	0.21
$\overline{\mathcal{L}}^c_+ \mathcal{U}_-$	$(\mathbf{3}, \mathbf{2}, +1/6)$	$\frac{1}{5}\hat{g}_1^2$	0.14
$\overline{\mathcal{L}}_+ \mathcal{D}_-$	$(\mathbf{3}, \mathbf{2}, +1/6)$	$\frac{1}{10}\hat{g}_1^2$	0.07
$\overline{\mathcal{Q}}^c_+ \mathcal{E}_-$	$(\mathbf{3}, \mathbf{2}, -5/6)$	$\frac{1}{10}\hat{g}_1^2$	0.07

Table 1. $D = 6$

composite scalar	$SU(3) \times SU(2) \times U(1)$ representation	binding strength	relative binding for $\hat{g}_1 = \hat{g}_2 = \hat{g}_3$
$\overline{\mathcal{Q}}_+ \mathcal{U}_-$	$(\mathbf{1}, \mathbf{2}, +1/2)$	$\frac{4}{3}\hat{g}_3^2 + \frac{1}{15}\hat{g}_1^2$	1
$\overline{\mathcal{Q}}_+ \mathcal{D}_-$	$(\mathbf{1}, \mathbf{2}, -1/2)$	$\frac{4}{3}\hat{g}_3^2 - \frac{1}{30}\hat{g}_1^2$	0.93
$\overline{\mathcal{Q}}_+ \mathcal{Q}^c_-$	$(\mathbf{3}, \mathbf{1}, -1/3)$	$\frac{2}{3}\hat{g}_3^2 + \frac{3}{4}\hat{g}_2^2 - \frac{1}{60}\hat{g}_1^2$	$1 - \epsilon$
$\overline{\mathcal{U}}_- \mathcal{D}^c_+$	$(\mathbf{3}, \mathbf{1}, -1/3)$	$\frac{2}{3}\hat{g}_3^2 + \frac{2}{15}\hat{g}_1^2$	0.57
$\overline{\mathcal{Q}}^c_- \mathcal{L}_+$	$(\mathbf{3}, \mathbf{1}, -1/3)$	$\frac{3}{4}\hat{g}_2^2 + \frac{1}{20}\hat{g}_1^2$	0.57
$\overline{\mathcal{U}}^c_+ \mathcal{E}_-$	$(\mathbf{3}, \mathbf{1}, -1/3)$	$\frac{2}{5}\hat{g}_1^2$	0.29
$\overline{\mathcal{L}}_+ \mathcal{E}_-$	$(\mathbf{1}, \mathbf{2}, -1/2)$	$\frac{3}{10}\hat{g}_1^2$	0.21
$\overline{\mathcal{L}}_+ \mathcal{D}_-$	$(\mathbf{3}, \mathbf{2}, +1/6)$	$\frac{1}{10}\hat{g}_1^2$	0.07

Table 2. $D = 8$

RGE

- We take only Kaluza-Klein (KK) modes below the renormalization point μ into account.

$$16\pi^2 \frac{d g_i}{d \ln \mu} = N_{\rm KK}(\mu)\, b_i' g_i^3 ,$$

$$16\pi^2 \frac{d y_t}{d \ln \mu} = N_{\rm KK}(\mu)\, y_t \left\{ \left(2^{\frac{\delta}{2}} N_c + \frac{3}{2} \right) y_t^2 - (6+\delta) \frac{N_c^2 - 1}{2N_c} g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_Y^2 \right\} ,$$

$$16\pi^2 \frac{d \lambda}{d \ln \mu} = N_{\rm KK}(\mu) \left\{ 2^{2+\frac{\delta}{2}} N_c \lambda y_t^2 - 2^{2+\frac{\delta}{2}} N_c y_t^4 \right.$$
$$\left. + 12\lambda^2 - 3\lambda \left(3g_2^2 + g_Y^2 \right) + \frac{3+\delta}{4} \left[2g_2^4 + \left(g_2^2 + g_Y^2 \right)^2 \right] \right\},$$

$$b_3' = -11 + \frac{2^{2+\frac{\delta}{2}}}{3} n_g + \frac{1}{2}\delta,$$

$$b_2' = -\frac{22}{3} + \frac{2^{2+\frac{\delta}{2}}}{3} n_g + \frac{1}{3}\delta + \frac{1}{6},$$

$$b_1' = \frac{2^{2+\frac{\delta}{2}}}{3} n_g + \frac{1}{10}$$

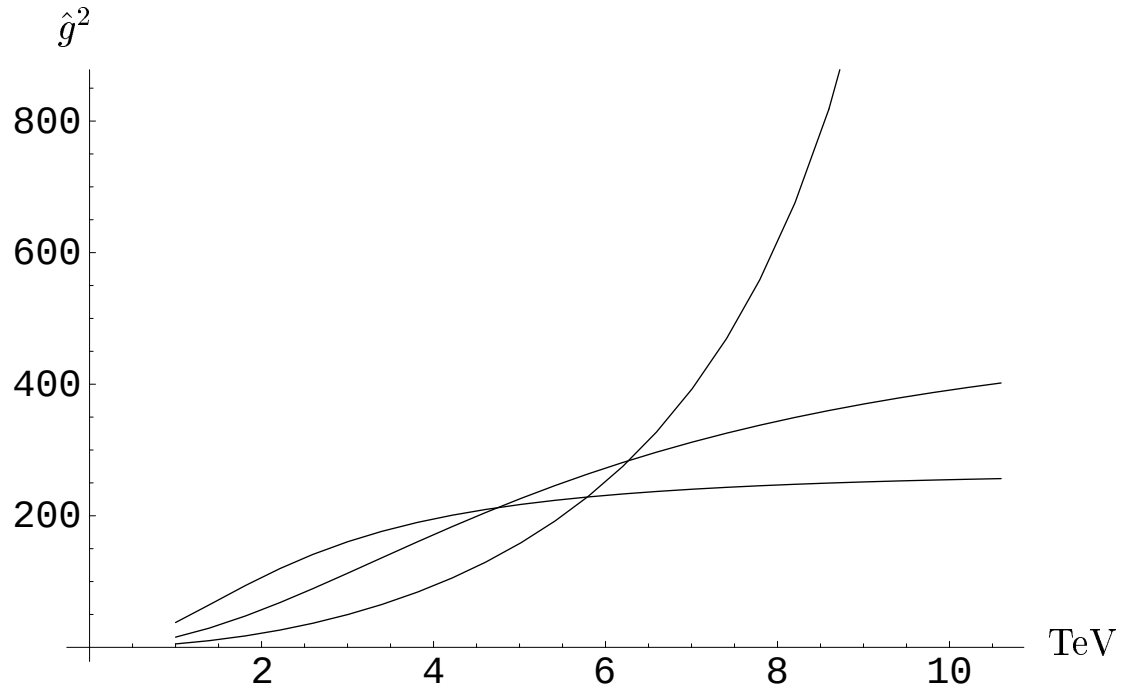
$N_{\rm KK}(\mu) \equiv$ # KK modes below μ

$$\simeq \frac{1}{n} \frac{\pi^{\frac{\delta}{2}}}{\Gamma(1+\delta/2)} (\mu R)^\delta, \quad (\mu \gg 1/R)$$

$$N_{\rm KK} \equiv N_{\rm KK}(\Lambda)$$

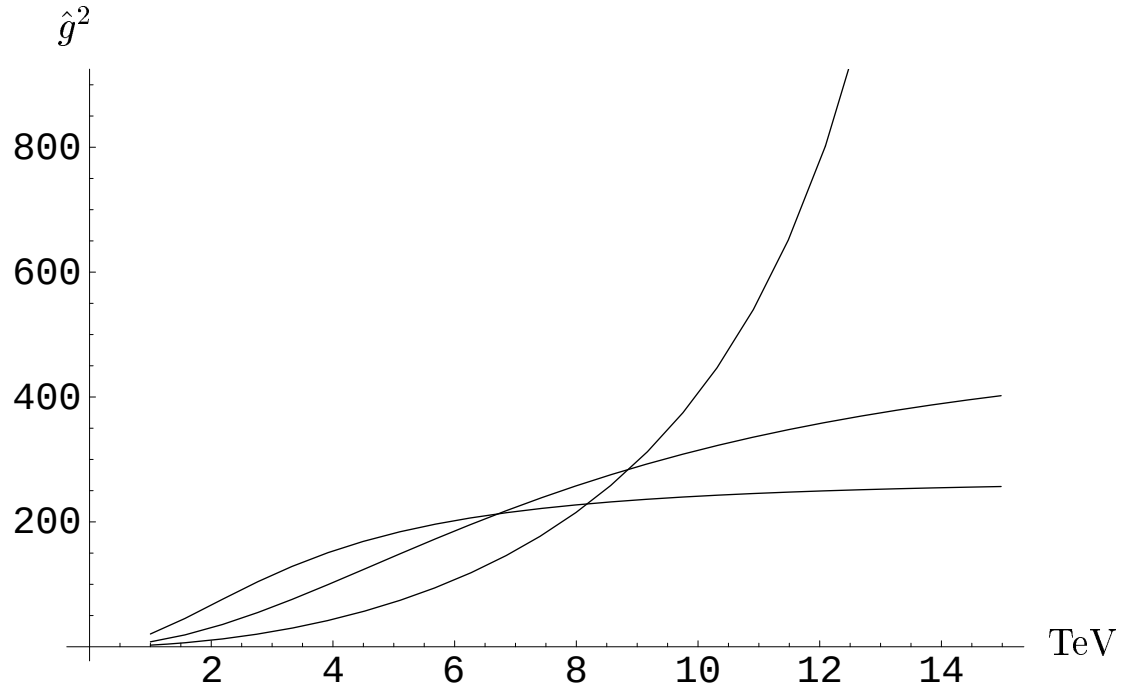
The running of the dimensionless gauge coupling in the bulk

$(D = 6)$



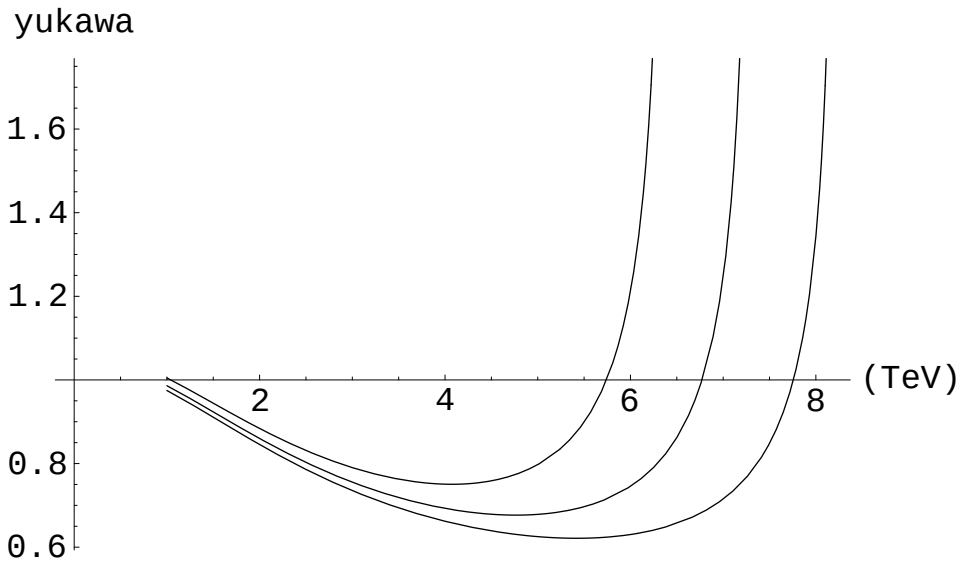
The running of gauge couplings of $SU(3)$, $SU(2)_L$, $U(1)_Y$ from bottom to up

$(M_c(\equiv 1/R) = 1$ TeV)

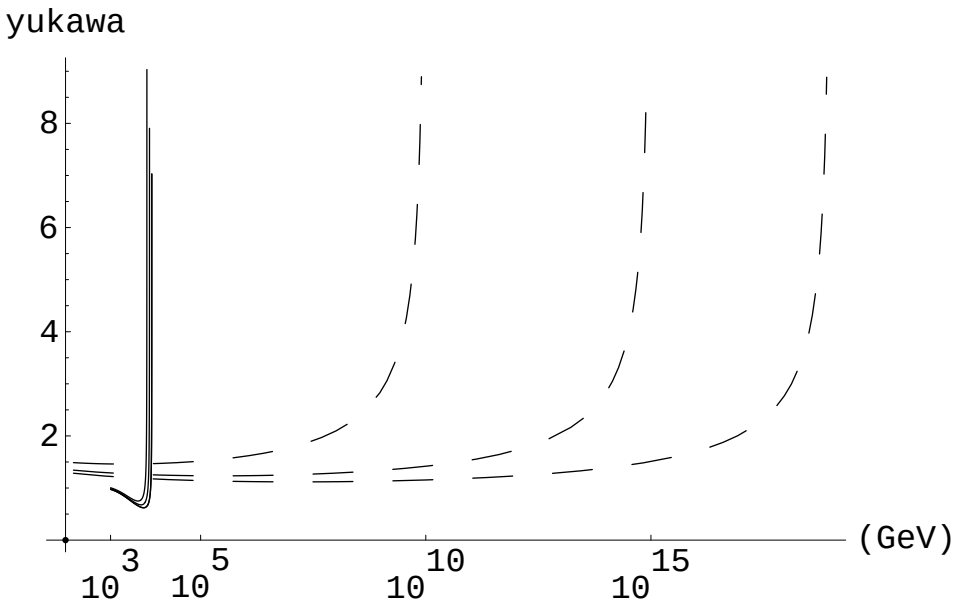


In the case composing Z_2 symmetry

The RGE for y_t $(D = 6)$



(Linear Scale)

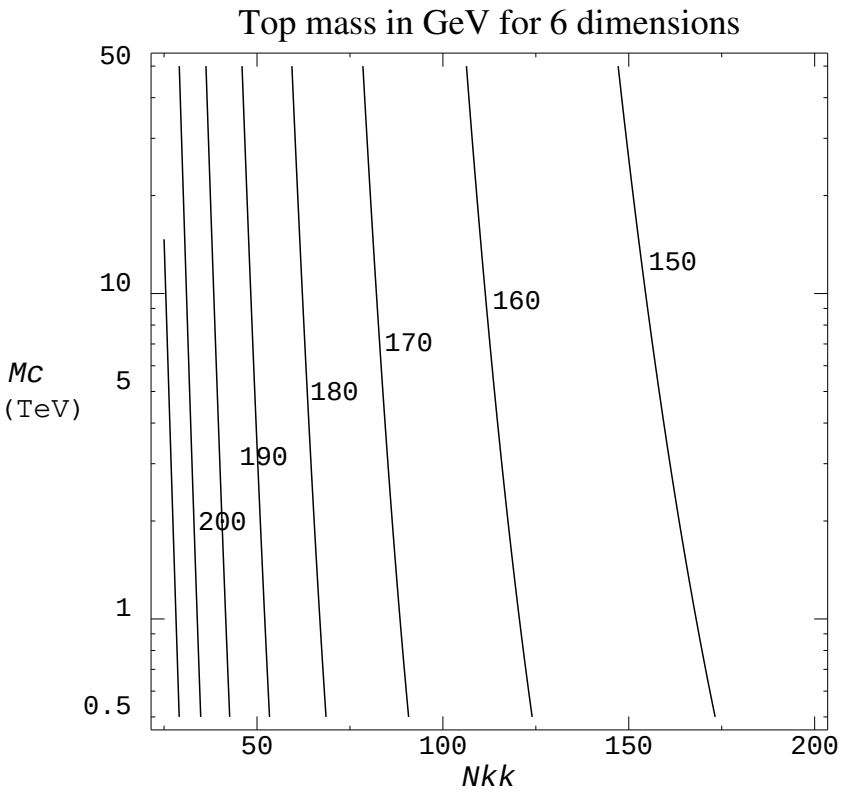


(Log Scale)

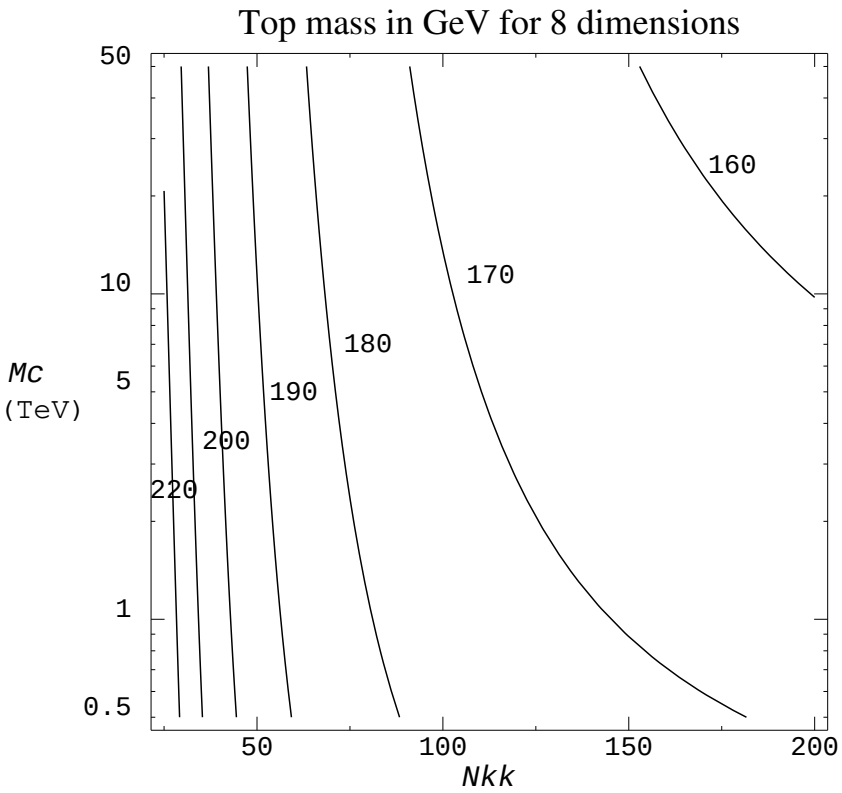
The solid line and the dashed line represent ACDH model and the SM, respectively.
We used $\Lambda = 10^{19}, 10^{15}, 10^{10}$ for SM and the same cutoff Λ for the ACDH model

The mass of the top quark

$M_c \equiv 1/R$

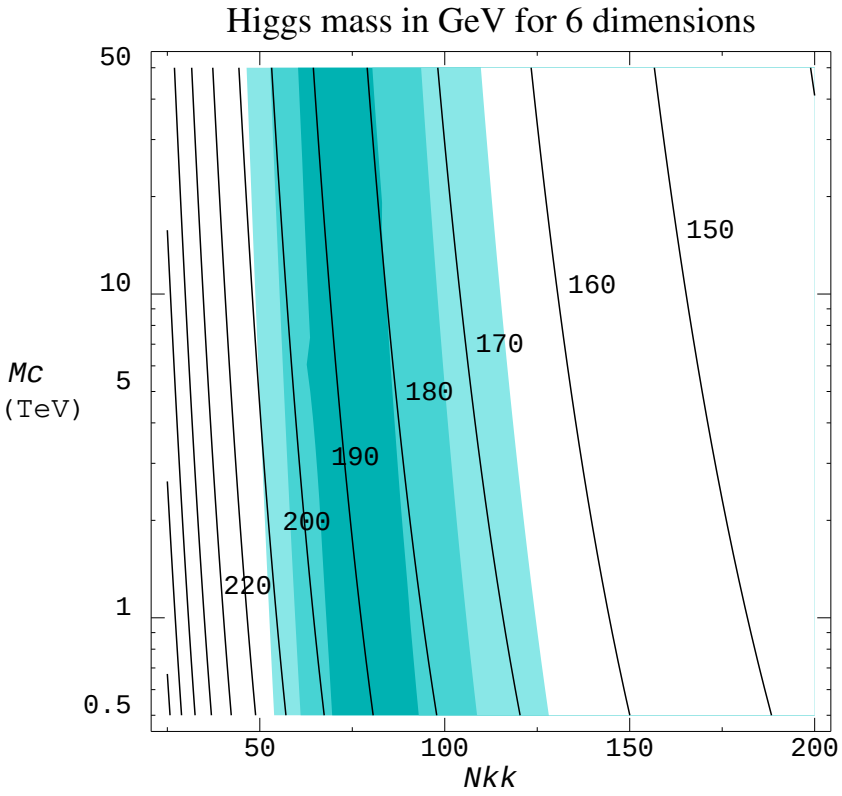


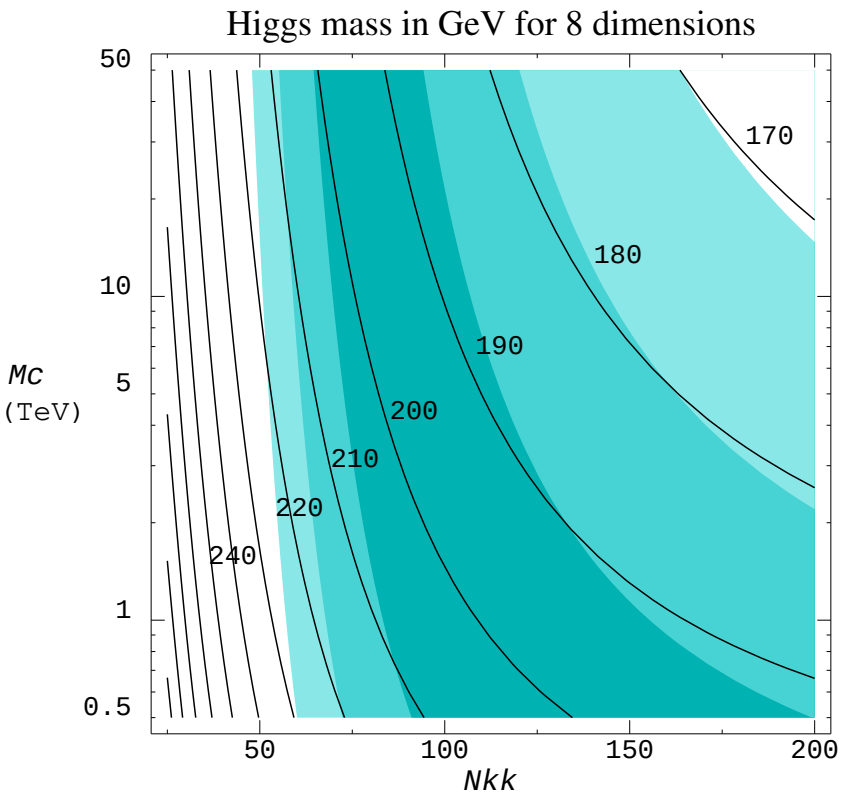
$\Lambda/M_c \simeq 4.9$ for $N_{\rm KK} = 75$



$\Lambda/M_c \simeq 2.1$ for $N_{\rm KK} = 100$

The prediction for the Higgs particle





The shaded regions correspond to the top mass lying within 1–3 σ (dark to light) of the experimental value ($m_t = 174.3 \pm 5.1$ GeV).

A problem of the ACDH model

- The gauge coupling of the bulk QCD does not grow arbitrarily large. Actually, we can find an upper bound.

- It is a quite non-trivial problem whether the top quark condenses or not.

- Let us investigate the condition for the DχSB.
  We study the dynamics of the bulk QCD by using the Schwinger-Dyson equation.

3. An upper bound of the bulk QCD coupling and Naive Dimensional Analysis (NDA)

- RGE for the gauge coupling in four dimensions

$$(4\pi)^2 \mu \frac{d}{d\mu} g = N_{\rm KK}(\mu) b' g^3,$$

$$N_{KK}(\mu) : \#\ \text{KK modes below}\ \mu$$

$$N_{\rm KK} = \frac{1}{n} \frac{\pi^{\delta/2}}{\Gamma(1+\delta/2)} (\mu R)^\delta,$$

$$b' = -\frac{26-D}{6} C_G + \frac{2^{D/2}}{3} T_R N_f,$$

T^δ/Z_n compactification, $\quad \delta \equiv D-4$

- The dimensionless gauge coupling in the bulk

$$\frac{1}{g^2} = \frac{(2\pi R)^\delta}{n} \frac{1}{g_D^2}, \quad \hat{g}^2 \equiv g_D^2 \mu^\delta,$$

$$\Longrightarrow \qquad \hat{g}^2 = \frac{(2\pi R\mu)^\delta}{n} g^2.$$

- RGE for $\hat{g}$ is given by

$$\mu \frac{d}{d\mu} \hat{g} = \frac{\delta}{2} \hat{g} + (1 + \delta/2) \Omega_{\rm NDA} b' \hat{g}^3,$$

$$\Omega_{\rm NDA} \equiv \frac{1}{(4\pi)^{D/2} \Gamma(D/2)}.$$

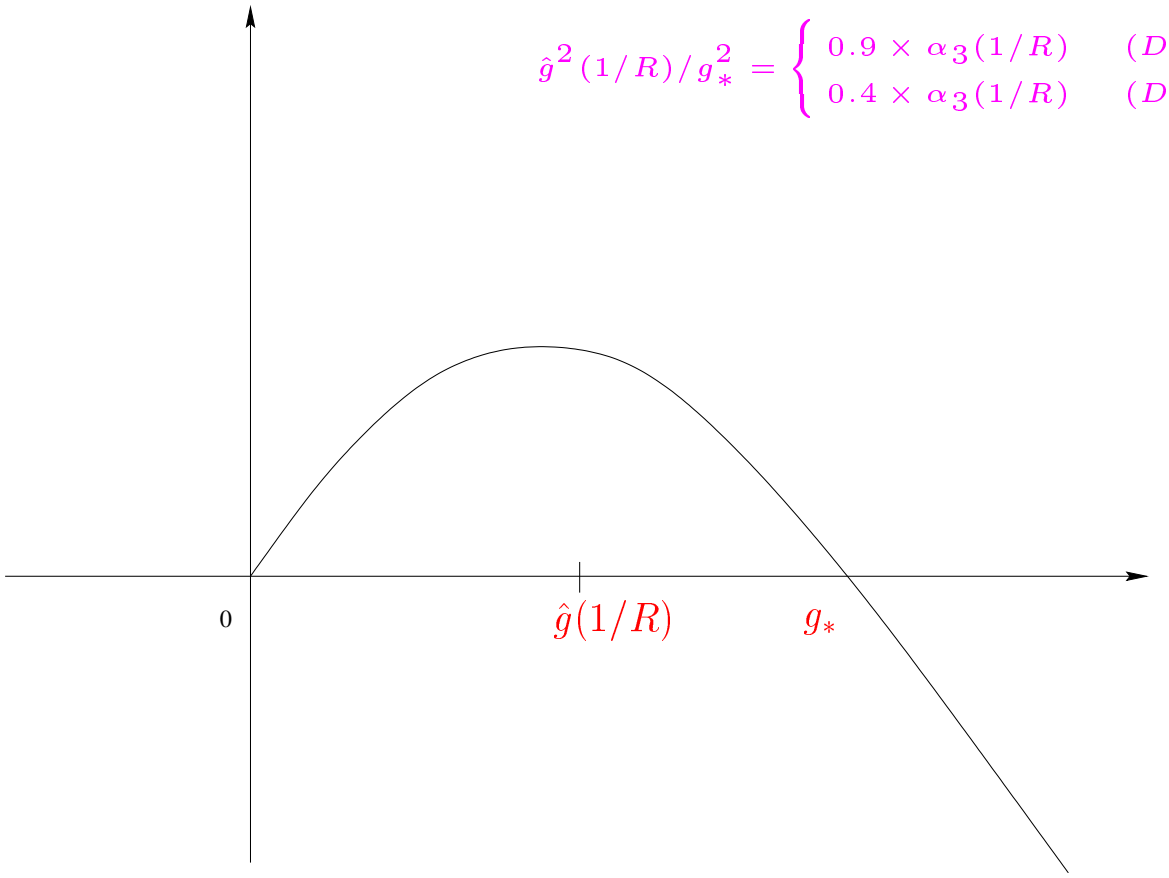
(the loop factor in D dimensions)

- Asymptotic free theory $\Rightarrow$ The ultra-violet fixed point

$(b' < 0)$ (g_*)

$$g_*^2 \Omega_{\rm NDA} = \frac{1}{-\left(\frac{2}{\delta} + 1\right) b'}.$$

$$\hat{g}^2(1/R)/g_*^2 = \begin{cases} 0.9 \times \alpha_3(1/R) & (D = 6) \\ 0.4 \times \alpha_3(1/R) & (D = 8) \end{cases}$$



The UV fixed point $=$ An upper bound for the bulk QCD coupling

The condition for the DχSB in the Naive Dimension Analysis

(NDA)

$$C_F \hat{g}^2 \Omega_{\rm NDA} > 1$$

(cf. NJL model)

On the other hand,

(The ACDH model)

$$C_F \hat{g}^2 \Omega_{\rm NDA} < C_F g_*^2 \Omega_{\rm NDA} \equiv \kappa_D \simeq \begin{cases} 0.09 & \text{for } D = 6 \\ 0.24 & \text{for } D = 8 \end{cases}$$

$\Rightarrow$ Does the DχSB really take place in the ACDH model?

$\longrightarrow$ Let us study this point furthermore by using the Schwinger-Dyson (SD) equation.

4. Analysis for the SD equation

- The ladder Schwinger-Dyson (SD) equation

- The propagator of the bulk fermion

$$iS^{-1}(p) = A(-p^2)\not{p} - B(-p^2)$$

- Running effects of the gauge coupling

$\Rightarrow$ The Higashijima–Miransky approximation

After angular integration, the gauge coupling is replaced to the running one. (the improved ladder approximation)

$$g_D^2 \to g_D^2(p_E, q_E) = \frac{\hat{g}^2(|p_E|)}{(p_E^2)^{\delta/2}}\theta(|p_E| - |q_E|) + (p_E \leftrightarrow q_E)$$

Landau gauge gives $A(-p^2) = 1$.

★ We consider the necessary condition for the DχSB .

$\to$ We use the upper bound of the gauge coupling:

$$\hat{g} \to g_*$$

• The improved ladder SD equation

$$B(x) = (D-1)\kappa_D \int_{M_0^2}^{\Lambda^2} dy y^{D/2-1} \frac{B(y)}{y + B^2(y)} K_B^{\rm imp}(x,y),$$

$$K_B^{\rm imp}(x,y) = \frac{1}{x^2}\left(1 - \frac{y}{3x}\right)\theta(x-y) + (x \leftrightarrow y), \qquad (D=6)$$

$$K_B^{\rm imp}(x,y) = \frac{1}{x^3}\left(1 - \frac{y}{2x} + \frac{y^2}{10x^2}\right)\theta(x-y) + (x \leftrightarrow y), \quad (D=8)$$

$$\kappa_D \equiv C_F g_*^2 \Omega_{\rm NDA}$$

M_0 : The IR-cutoff arising from the compactification

Λ : The UV-cutoff

• Numerical analysis $(D=6)$

$$B_i = 5\kappa_6 \left[\sum_{j=1}^{i} \frac{x_j B_j}{x_j + B_j^2} \frac{x_j^2}{x_i^2}\left(1 - \frac{x_j}{3x_i}\right) + \sum_{j=i+1}^{i_\Lambda} \frac{x_j B_j}{x_j + B_j^2}\left(1 - \frac{x_i}{3x_j}\right)\right],$$

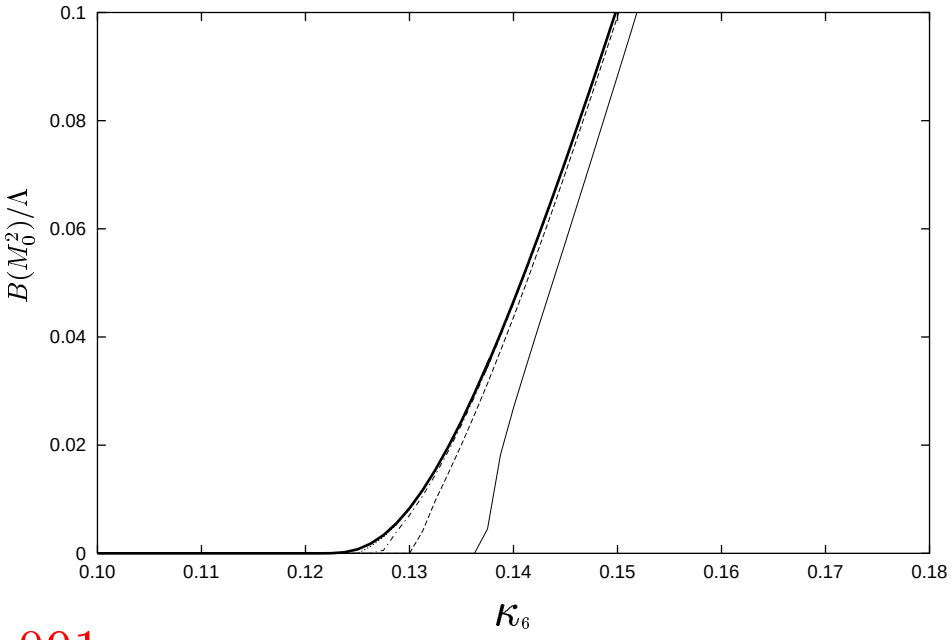
$$B_i^{(n+1)} \equiv 5\kappa_6 \left[\sum_{j=1}^{i} \frac{x_j B_j^{(n)}}{x_j + (B_j^{(n)})^2} \frac{x_j^2}{x_i^2}\left(1 - \frac{x_j}{3x_i}\right) + \sum_{j=i+1}^{i_\Lambda} \frac{x_j B_j^{(n)}}{x_j + (B_j^{(n)})^2}\left(1 - \frac{x_i}{3x_j}\right)\right],$$

$$B_j^{(n=0)} = M_0, \qquad \text{for } j = 1, 2, \cdots, i_\Lambda,$$

$$x_j \equiv M_0^2 \exp\left[\frac{j-1}{i_\Lambda - 1} \ln \frac{\Lambda^2}{M_0^2}\right], \qquad B_j \equiv B(x_j).$$

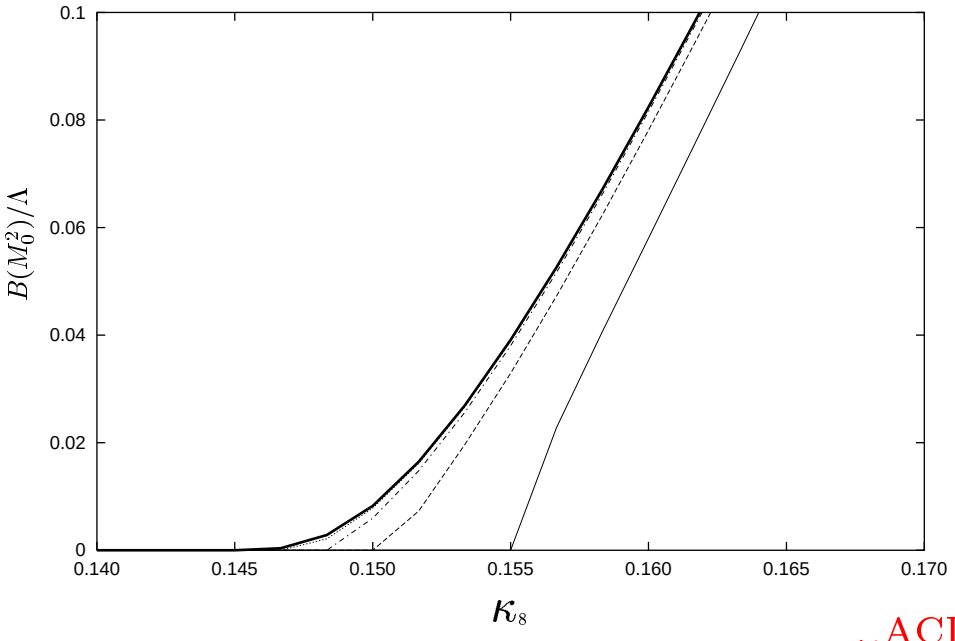
- Results of the numerical analysis:

$(D = 6)$



$\kappa_6^{\mathrm{ACDH}} = 0.091$

$(D = 8)$



$\kappa_8^{\mathrm{ACDH}} = 0.242$

In both graphs, lines from right to left represent graphs for $\Lambda^2/M_0^2 = 10^3, 10^4, 10^5, 10^6, 10^{10}$, respectively.

5. Anomalous dimension of $\bar{\psi}\psi$

- Large anomalous dimension of $\bar{\psi}\psi$ in the walking-Technicolor

  $\sim$ suppression of the FCNC problem

The asymptotic behavior of the mass function and the anomalous dimension

- Operator Product Expansion

$$-i\int d^D x e^{iq\cdot x} T[\psi(x)\bar{\psi}(0)] = c_1^M(q, g_D; \mu)\Gamma_M + c_{\bar{\psi}\psi}(q, g_D; \mu)(\bar{\psi}\psi) + \cdots ,$$

- Wilson coefficient $c_{\bar{\psi}\psi}$ in the tree approximation

$$c_{\bar{\psi}\psi}(q, \hat{g}; \mu) = \frac{(D-1)}{2^{\frac{\delta}{2}}} \frac{C_F}{N} \frac{\hat{g}^2}{\mu^\delta} \frac{1}{q^4}$$

- The propagator of the fermion field

$$-iS(p) = \frac{1}{\not{p} - B(-p^2)} \simeq \frac{\not{p}}{p^2} + \frac{B(-p^2)}{p^2} + \cdots ,$$

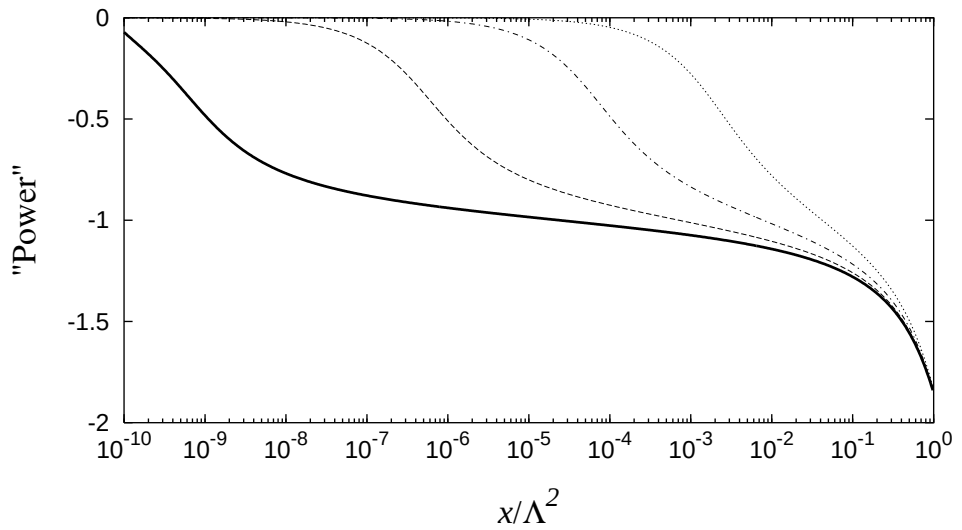
$$B(-p^2) \propto (-p^2)^{(\gamma_m^* + 2 - D)/2}$$

γ_m^* : The anomalous dimension on the fixed point

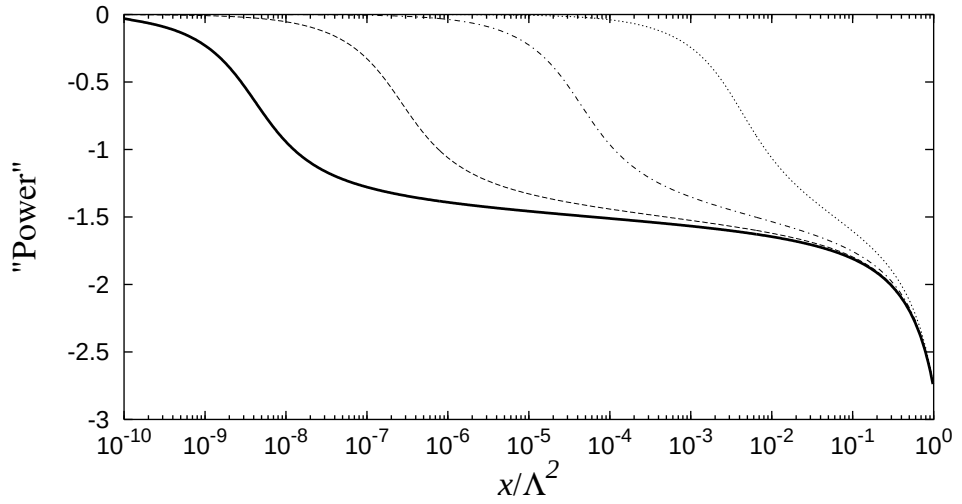
$$\text{“power”} \equiv \frac{d(\log B(x))}{d(\log x)} \quad \rightarrow \quad \gamma_m^* = 2\,\text{“power”} + (D-2)$$

- The asymptotic bahavior of the mass function:

$(D = 6)$



$(D = 8)$



The bold line represents the case of $\kappa_D \simeq \kappa_D^{\rm crit}$.

$\gamma_m^* = 2$ (for $D = 6$), $\gamma_m^* = 3$ (for $D = 8$)

- Analytical solution by using the following approximation:

$$K_B^{\rm imp}(x,y) \sim \frac{1}{x^{D/2-1}}\theta(x-y) + (x \leftrightarrow y),$$

$$\frac{B(x)}{M} = \frac{\sqrt{\tilde{\omega}^2+1}}{\tilde{\omega}} \left(\frac{x}{M^2}\right)^{-\frac{\nu}{2}} \sin\left(\frac{\nu\tilde{\omega}}{2}\ln\frac{x}{M^2} + \theta\right)$$

$$\tilde{\omega} \equiv \sqrt{\kappa_D/\kappa_D^{\rm crit} - 1}, \quad \nu \equiv \frac{D}{2} - 1, \quad \tan\theta \equiv \tilde{\omega}$$

- The critical point $\kappa_D^{\rm crit}$ in this approach:

$$\kappa_D^{\rm crit} = \frac{1}{8}\frac{D-2}{D-1},$$

$$\kappa_6^{\rm crit} = 0.10, \quad \kappa_8^{\rm crit} \simeq 0.11$$

$$\kappa_6^{\rm crit}({\rm num.}) \simeq 0.12, \quad \kappa_8^{\rm crit}({\rm num.}) \simeq 0.15$$

- The essential singularity type scaling

$$M = \Lambda \exp\left[\frac{-\pi}{\nu\sqrt{\kappa_D/\kappa_D^{\rm crit} - 1}}\right]$$

$\Longrightarrow$ suppression of the fine tuning problem

- The anomalous dimension from the analytical solution

$$\gamma_m^* = \frac{D}{2} - 1$$

- Finiteness of the decay constant

$$\begin{aligned}\left(F_\pi^{(D)}\right)^2 &= \frac{2T_R}{(2\pi)^{D/2}\Gamma(D/2)} \int_0^{\Lambda^2} dx x^{D/2-1} \frac{\Sigma^2(x) - \frac{x}{D}\frac{d\Sigma^2(x)}{dx}}{(x+\Sigma^2(x))^2}, \\ &\simeq \int_0^{\Lambda^2} dx x^{D/2-3}\Sigma^2(x), \\ &= M^D \int_M^{\Lambda} dp_E \frac{1}{p_E^3} \quad \rightarrow \quad \text{finiteness of } F_\pi^{(D)}\end{aligned}$$

- Relation between the decay constant in four and D dimensions

$$F_\pi^2 = \frac{(2\pi R)^\delta}{n}\left(F_\pi^{(D)}\right)^2,$$

6. Summary and discussions

- If the following condition satisfies, the DχSB does not take place due to the bulk QCD:

$$\kappa_6 < \kappa_6^{\rm crit} = 0.122, \quad \kappa_8 < \kappa_8^{\rm crit} = 0.146.$$

The ACDH model

× $\kappa_6^{\rm ACDH} = 0.091$, ○ $\kappa_8^{\rm ACDH} = 0.242$.

Simple solutions

- We may consider only the case of $D \geq 8$.
- The model where the second generation of quarks and leptons live in the bulk

↓

We should consider the mechanism of the suppression to the charm condensation.

- ⋮

★ Large anomalous dimension

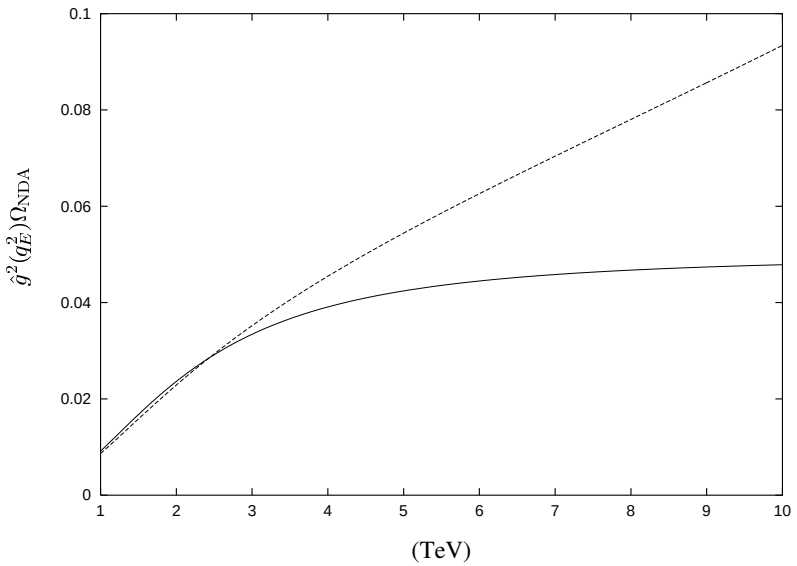
$$\boxed{\gamma_m^* = \frac{D}{2} - 1}$$

★ The finiteness of the decay constant

$$\boxed{(F_\pi^{(D)})^2 \sim M^D \int dp_E \frac{1}{p_E^3}}$$

Outlook

- Non-decoupling effects arising from many KK modes (summing up finite corrections to the vacuum polarization)
  The upper bound of the dimensionless gauge coupling in the bulk tends to increase.



- We should calculate Λ/M_c for the realistic value of m_t in our approach.
- The model building and restrictions for the cutoff
  - Landau pole of $U(1)_Y$ is not so far from the compactified scale. $\rightarrow$ Λ must be small.
  - S, T–parameters

$$S, T \propto \sum_{\vec{k}} \frac{1}{M_{\vec{k}}^2} \simeq \begin{cases} \log \Lambda R & (D = 6) \\ (\Lambda R)^{\delta - 2} & (D > 6,\ \delta = D - 4) \end{cases}$$

  - Generally speaking, a large cutoff is needed to resolve the FCNC problem.