Nonlinear Supersymmetry Without GSO Projection and Unstable D9-brane

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Based on hep-th/0010173 with T.Yoneya

1. Introduction

String Theory + GSO projections

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Supersymmetric string theory

# of Bosons = # of fermions

The systems of unstable D-branes

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various open string theories

with mixed or without GSO projection

Recently Unstable objects (Non-BPS D-brane, etc) are studied by many people, corresponding to

- Tachyon condensation,
- The topological classification of stable and unstable D-branes on the basis of K theory

etc....

$\Downarrow$

It is very important to study the system without GSO projections.

Yoneya's argument (Nucl.Phys. B576(2000) 219)

- Supersymmetry is hidden by a spontaneous breaking in tachyonic vacuum without GSO projection.
- How to realize the hidden SUSY.

He has investigated how $N = 2$ SUSY is buried in the property of spectra, vertex operators, boundary conditions, etc.

- Existence of would-be Goldstone fermion for spontaneously broken $N = 2$ SUSY in open string theory.

$$\|$$

massless fermions in the R-sector

- Existence of conserved supercurrent. And supercharges are not conserved,

$$\frac{dQ_\alpha}{d\tau} = -S_\alpha e^{-\phi}(\tau - i\pi) + S_\alpha e^{-\phi}(\tau + i\pi) = \begin{cases} 0 \\ -2S_\alpha e^{-\phi}(\tau - i\pi) \end{cases}$$

which is proportional to the fermion vertex operator at the string end ($\sigma = \pi$).

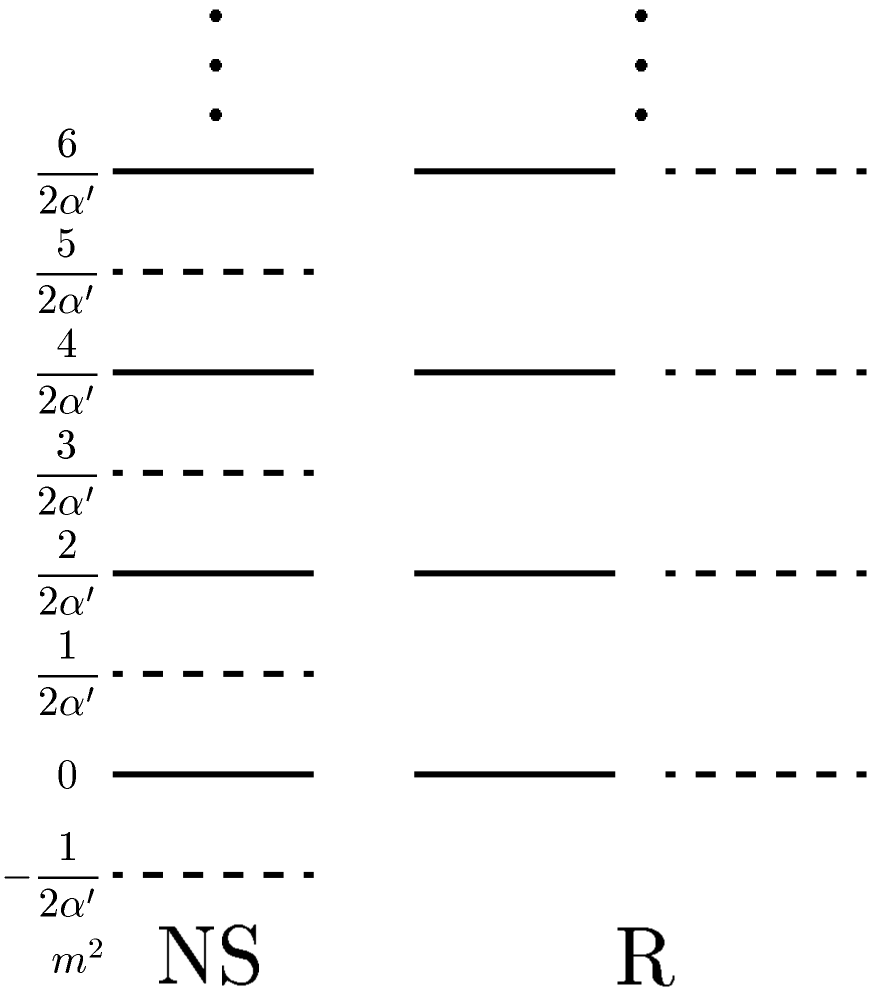
$$\Downarrow$$

massless fermions = Goldstone fermions

- Existence of a "trace formula" at tree level

$$\mathrm{Tr}[(-1)^F M^{2n}] = 0 \quad (n \neq 4).$$

  – This strongly suggests the existence of supersymmetry in a spontaneously broken phase.
  – All massive levels participate.



The effective theory is constructed
using only the massless modes
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SUSY should be realized nonlinearly to all orders
in the derivative (or α'−)expansion

Purpose of this work

To present further evidence for the hidden nonlinear SUSY without GSO projection.

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- The low-energy effective action for fermion scattering is constructed on the basis of Sen's proposal for the world-volume effective action.
- Study the low-energy theorem for the scattering of would-be Goldstone fermion, involving both the GSO and opposite-GSO projected sectors.
- And compared to the zero-slope limit of the 4-point amplitudes of the Goldstone fermions.

Plan of Talk

1. Introduction
2. Low-energy effective action
3. 4-fermion scattering without the GSO projection
4. Conclusions and discussions

2.Low-energy effective action

10D open string theory action without SUSY
in the low-energy limit
||
The world-volume action of unstable D9-brane.

We start with Sen's non-BPS Dp-brane action.

$$S_p = -\tau_p \int d^{p+1}\sigma \sqrt{-\det(\mathcal{G}_{\mu\nu} + \mathcal{F}_{\mu\nu})},$$

where

$$\mathcal{G}_{\mu\nu} = \eta_{\mu\nu}\Pi^m_\mu \Pi^n_\nu, \qquad \Pi^m_\mu = \partial_\mu X^m - i\bar{\theta}\Gamma^m \partial_\mu \theta,$$

$$\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\left[\bar{\theta}\Gamma_{11}\Gamma_m \partial_\mu \theta \left(\partial_\nu X^m - \frac{1}{2}\bar{\theta}\Gamma^m \partial_\nu \theta\right) - (\mu \leftrightarrow \nu)\right].$$

$\mu, \nu, \cdots$: indices of the world volume
$m, n, \cdots$: indices of the space-time

θ : Majorana Spinor in 10D with 32 components
$\theta_\pm$: the Weyl components: $\Gamma_{11}\theta_\pm = \mp\theta_\pm, \quad \bar{\theta}_\pm \Gamma_{11} = \pm\bar{\theta}_\pm.$

The action has the $N = 2$ supersymmetry:

$$\delta_\epsilon \theta = \epsilon, \qquad \delta_\epsilon X^m = \bar{\epsilon}\Gamma^m \theta,$$

$$\delta_\epsilon A_\mu = \bar{\epsilon}\Gamma_{11}\Gamma_m \theta \partial_\mu X^m - \frac{1}{6}(\bar{\epsilon}\Gamma_{11}\Gamma_m \theta \bar{\theta}\Gamma^m \partial_\mu \theta + \bar{\epsilon}\Gamma_m \theta \bar{\theta}\Gamma_{11}\Gamma^m \partial_\mu \theta).$$

Note: $\mathcal{G}_{\mu\nu}$, $\mathcal{F}_{\mu\nu}$ are separately invariant under SUSY.

c.f. BPS D-brane action constructed by Schwartz et. al:

$$S = S_{\mathrm{DBI}} + S_{\mathrm{WZ}}.$$

This action has κ-symmetry, then by the κ-gauge fixing, dof of bosons and fermions becomes equal.

<u>In the case of D9-brane in Type IIA</u>

In the low-energy limit, we can set the static gauge,

$$\sigma^\mu \to x^\mu = X^m \quad (\mu = m, \cdots).$$

by assuming that configurations with various type of folding do not contribute.

In this gauge, we get

$$\mathcal{G}_{\mu\nu} = \eta_{\mu\nu} - i\bar{\theta}(\Gamma_\mu\partial_\nu + \Gamma_\mu\partial_\mu)\theta - \bar{\theta}\Gamma^\alpha\partial_\mu\theta\bar{\theta}\Gamma_\alpha\partial_\nu\theta,$$
$$\mathcal{F}_{\mu\nu} = F_{\mu\nu} - i\left[\bar{\theta}\Gamma_{11}\Gamma_\alpha\partial_\mu\theta\left(\delta^\alpha_\nu - \frac{i}{2}\bar{\theta}\Gamma^\alpha\partial_\nu\theta\right) - (\mu \leftrightarrow \nu)\right].$$

Here we change the normalization of the fields such that the final action has the standard normalization of field theory:

$$A_\mu \to \lambda A_\mu,$$
$$\theta = \frac{\lambda}{\sqrt{2}}\psi = \frac{\lambda}{\sqrt{2}}(\psi_+ + \psi_-),$$
$$\epsilon \to \frac{1}{\sqrt{2}}\epsilon,$$
$$\lambda \equiv \frac{1}{\tau_9} = [g_s(2\pi)^9\alpha'^5]^{1/2}$$

: Yang-Mills coupling constant in 10D.

The effective action:

$$S_{eff} = -\frac{1}{\lambda^2}\int d^{10}x \ \sqrt{-\det G_{\mu\nu}},$$

where

$$G_{\mu\nu} \equiv \mathcal{G}_{\mu\nu} + \mathcal{F}_{\mu\nu} = \eta_{\mu\nu} + \lambda F_{\mu\nu} + \lambda^2 S^{(2)}_{\mu\nu} + \lambda^4 S^{(4)}_{\mu\nu},$$

$$S^{(2)}_{\mu\nu} = -i\overline{\psi}_+\Gamma_\nu\partial_\mu\psi_+ - i\overline{\psi}_-\Gamma_\mu\partial_\nu\psi_-,$$

$$S^{(4)}_{\mu\nu} = -\frac{1}{4}(\overline{\psi}_+\Gamma^\alpha\partial_\mu\psi_+\overline{\psi}\Gamma_\alpha\partial_\nu\psi + \overline{\psi}_-\Gamma^\alpha\partial_\nu\psi_-\overline{\psi}\Gamma_\alpha\partial_\mu\psi).$$

We can regard this action as the low-energy effective action for spontaneously broken $N = 2$ SUSY.

$$\Downarrow$$

It should describe the scattering amplitude of massless Goldstone fermion mode at least in the order $O(\lambda^2)$. (It does!!)

Expanding the action up to λ^2:

$$\begin{aligned}
S_{eff} &+ \frac{1}{\lambda^2}\int d^{10}x \\
&= \int d^{10}x\Big(-\frac{1}{4}F^2_{\mu\nu} + \frac{i}{2}(\overline{\psi}_+\Gamma^\mu\partial_\mu\psi_+ + \overline{\psi}_-\Gamma^\mu\partial_\mu\psi_-) - \frac{\lambda^2}{32}(F^2_{\mu\nu})^2 + \frac{\lambda^2}{8}F^{\mu\nu}F_{\nu\alpha}F^{\alpha\beta}F_{\beta\mu} \\
&\qquad - i\frac{\lambda}{2}(\overline{\psi}_+\Gamma_\nu\partial_\mu\psi_+ + \overline{\psi}_-\Gamma_\mu\partial_\nu\psi_-)F^{\nu\mu} + i\frac{\lambda^2}{8}\overline{\psi}\Gamma^\mu\partial_\mu\psi F^2 \\
&\qquad + i\frac{\lambda^2}{2}(\overline{\psi}_+\Gamma_\mu\partial_\nu\psi_+ + \overline{\psi}_-\Gamma_\nu\partial_\mu\psi_-)F^{\mu\alpha}F_\alpha^{\ \nu} \\
&\qquad + \frac{\lambda^2}{8}(\overline{\psi}_+\Gamma^\mu\partial_\mu\psi_+ + \overline{\psi}_-\Gamma^\mu\partial_\mu\psi_-)^2 \\
&\qquad + \frac{\lambda^2}{8}(\overline{\psi}_+\Gamma_\mu\partial_\nu\psi_+\overline{\psi}_+\Gamma^\mu\partial^\nu\psi_+ + \overline{\psi}_-\Gamma_\mu\partial_\nu\psi_-\overline{\psi}_-\Gamma^\mu\partial^\nu\psi_-) \\
&\qquad - \frac{\lambda^2}{4}(\overline{\psi}_+\Gamma_\mu\partial_\nu\psi_+\overline{\psi}_+\Gamma^\nu\partial^\mu\psi_+ + \overline{\psi}_-\Gamma_\mu\partial_\nu\psi_-\overline{\psi}_-\Gamma^\nu\partial^\mu\psi_-) \\
&\qquad - \frac{\lambda^2}{4}\overline{\psi}_+\Gamma_\mu\partial_\nu\psi_+\overline{\psi}_-\Gamma^\mu\partial^\nu\psi_- + \text{higher orders}\Big).
\end{aligned}$$

The supertransformation law:

$$\delta\psi = \frac{1}{\lambda}\epsilon - \underbrace{i\frac{\lambda}{2}(\bar{\epsilon}\Gamma^{\mu}\psi)\partial_{\mu}\psi}_{*}, \quad \text{(c.f. Volkov-Akulov Action)}$$

$$\delta A_{\mu} = \frac{i}{2}\bar{\epsilon}\Gamma_{11}\Gamma_{\mu}\psi + \frac{\lambda^2}{24}(\bar{\epsilon}\Gamma_{11}\Gamma_{\nu}\psi\bar{\psi}\Gamma^{\nu}\partial_{\mu}\psi + \bar{\epsilon}\Gamma_{\nu}\psi\bar{\psi}\Gamma_{11}\Gamma^{\nu}\partial_{\mu}\psi) - \underbrace{i\frac{\lambda}{2}(\bar{\epsilon}\Gamma^{\nu}\psi)\partial_{\nu}A_{\mu}}_{*}.$$

But the supertransformation law does not look standard.

- $\delta\psi_{\pm}$ include no A_{μ}.

  e.g.) the linear case: $\delta\psi \sim \Gamma^{\mu\nu}\epsilon F_{\mu\nu}$

- Difficult to find the relation between this law and the standard linear transformation law.

and it is hard to compare this action with the scattering amplitudes.

$$\Downarrow$$

In fact, these forms of SUSY law and action are not unique because there is an ambiguity of field redefinition.

If we redefine fields as follows,

$$\begin{aligned}
\psi_{\pm} \to \psi_{\pm} &\mp \frac{1}{4}h_1\lambda\Gamma_{\mu\nu}\psi_{\pm}F^{\mu\nu} + \frac{1}{32}\lambda^2\Gamma_{\mu\nu\rho\sigma}\psi_{\pm}F^{\mu\nu}F^{\rho\sigma} \\
&- \frac{3}{16}\lambda^2\psi_{\pm}F^2 - \frac{1}{48}i\lambda^2(\bar{\psi}_{\pm}\Gamma^{\mu\nu\rho}\psi_{\pm})\Gamma_{\mu\nu}\partial_{\rho}\psi_{\pm} \\
&+ \frac{7}{24}i\lambda^2(\bar{\psi}_{\pm}\Gamma^{\mu}\partial_{\mu}\psi_{\pm})\psi_{\pm} - \frac{1}{12}i\lambda^2(\bar{\psi}_{\pm}\Gamma^{\mu}\partial^{\nu}\psi_{\pm})\Gamma_{\mu\nu}\psi_{\pm},
\end{aligned}$$

The supertransformation is changed as following:

$$
\begin{aligned}
\delta\psi_\pm =& \frac{1}{\lambda}\epsilon_\pm \pm \frac{1}{4}\Gamma^{\mu\nu}\epsilon_\pm F_{\mu\nu} \\
& + \lambda\Big[-\frac{i}{2}(\bar\epsilon\Gamma_\mu\psi)\partial_\mu\psi_\pm + \frac{1}{16}\Gamma^{\mu\nu}\Gamma^{\rho\sigma}\epsilon_\pm F_{\mu\nu}F_{\rho\sigma} \\
& \qquad - \frac{1}{32}\Gamma^{\mu\nu\rho\sigma}\epsilon_\pm F_{\mu\nu}F_{\rho\sigma} + \frac{3}{16}\epsilon_\pm F^2 \\
& \qquad \pm \frac{i}{4}\Gamma^{\mu\nu}\psi_\pm(\bar\epsilon\Gamma_{11}\Gamma_{[\nu}\partial_{\mu]}\psi) + \frac{i}{24}\Gamma_{\mu\nu}\partial_\rho\psi_\pm(\bar\psi_\pm\Gamma^{\mu\nu\rho}\epsilon_\pm) \\
& \qquad - \frac{7i}{24}\psi_\pm(\bar\epsilon_\pm\Gamma^\mu\partial_\mu\psi_\pm) - \frac{7i}{24}\epsilon_\pm(\bar\psi_\pm\Gamma^\mu\partial_\mu\psi_\pm) \\
& \qquad + \frac{i}{12}\Gamma^{\mu\nu}\psi_\pm(\bar\epsilon_\pm\Gamma_\mu\partial_\nu\psi_\pm) + \frac{i}{12}\Gamma^{\mu\nu}\epsilon_\pm(\bar\psi_\pm\Gamma_\mu\partial_\nu\psi_\pm)\Big], \\
\delta A_\mu =& \frac{i}{2}\bar\epsilon\Gamma_{11}\Gamma_\mu\psi + \lambda\left[-\frac{i}{8}\bar\epsilon\Gamma_\mu\Gamma^{\rho\sigma}\psi F_{\rho\sigma} - \frac{i}{2}(\bar\epsilon\Gamma^\nu\psi)\partial_\nu A_\mu\right].
\end{aligned}
$$

Restrict single chiral sector($\psi_- = \epsilon_- = 0$)
$\Rightarrow$the free linear transformation in $\lambda \to 0$ limit
up to $O(1/\lambda)$ of ψ_+ (trivial shift).

Supercurrents:

$$
S^\mu_\pm = \frac{1}{\lambda}\Gamma^\mu\psi_\pm \mp \frac{1}{4}\Gamma^{\alpha\beta}\Gamma^\mu\psi_\pm F_{\alpha\beta} + O(\lambda),
$$

Second term with only one chirality
= the one of the free linear theory.

And the effective action becomes

$$
\begin{aligned}
&S_{eff} + \frac{1}{\lambda^2}\int d^{10}x \\
&= \int d^{10}x \Big(-\frac{1}{4}F_{\mu\nu}^2 + \frac{i}{2}(\overline{\psi}_+\Gamma^\mu\partial_\mu\psi_+ + \overline{\psi}_-\Gamma^\mu\partial_\mu\psi_-) \\
&\qquad - \frac{\lambda^2}{32}(F_{\mu\nu}^2)^2 + \frac{\lambda^2}{8}F^{\mu\nu}F_{\nu\alpha}F^{\alpha\beta}F_{\beta\mu} \\
&\qquad + i\frac{\lambda^2}{4}(\overline{\psi}_+\Gamma_\mu\partial_\nu\psi_+ + \overline{\psi}_-\Gamma_\nu\partial_\mu\psi_-)F^{\mu\alpha}F_\alpha^{\ \nu} \\
&\qquad + \frac{i\lambda^2}{16}(\overline{\psi}_+\Gamma^{\mu\nu\rho}\psi_+ + \overline{\psi}_-\Gamma^{\mu\nu\rho}\psi_-)F_{\mu\sigma}\partial^\sigma F_{\nu\sigma} \\
&\qquad + \frac{\lambda^2}{4}\overline{\psi}_+\Gamma^\mu\partial_\mu\psi_+\overline{\psi}_-\Gamma^\nu\partial_\nu\psi_- \\
&\qquad - \frac{\lambda^2}{24}(\overline{\psi}_+\Gamma_\mu\partial_\nu\psi_+\overline{\psi}_+\Gamma^\mu\partial^\nu\psi_+ + \overline{\psi}_-\Gamma_\mu\partial_\nu\psi_-\overline{\psi}_-\Gamma^\mu\partial^\nu\psi_-) \\
&\qquad - \frac{\lambda^2}{4}\overline{\psi}_+\Gamma_\mu\partial_\nu\psi_+\overline{\psi}_-\Gamma^\mu\partial^\nu\psi_- + \text{higher orders}\Big).
\end{aligned}
$$

Even if we see only the usual projected sector, the SUSY transformation is not linear for $\alpha' \neq 0, \lambda \neq 0$.

$$\Uparrow$$

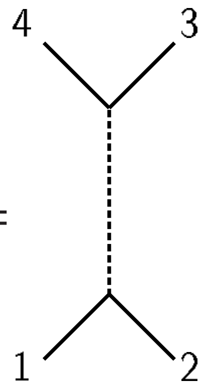
SUSY governing the Acton is in fact the nonlinear $N = 2$ SUSY rather than $N = 1$.

3. 4-Fermion scattering without the GSO projection

We investigate the low-energy behavior of string scattering for the would-be Goldstone fermions in open string theory without GSO projection (i.e. including both chiral sectors.)

The formula for (s,t) amplitude with 4 massless fermion lines: (by J.H.Schwarz and C.C.Wu(1973))

$A_{1,2,3,4}(s,t) =$



$$= \int_0^1 dx\, x^{-\alpha' s-3/2}(1-x)^{-\alpha' t-3/2} \sum_{\ell=0}^{10} T^{(\ell)}(1,2)T_{(\ell)}(3,4)f_\ell(x).$$

where

- $T^{(\ell)}(1,2) = \overline{u}_1\Gamma^{(\ell)}u_2,\ \ T_{(\ell)}(3,4) = \overline{u}_3\Gamma_{(\ell)}u_4.$
- $\Gamma^{(\ell)}$: normalized antisymmetric product of ℓ Γ-matrices,

  $$\Gamma^{(\ell)}_{\mu_1,\cdots,\mu_\ell} = \frac{1}{\sqrt{\ell!}}\Gamma_{[\mu_1}\cdots\Gamma_{\mu_\ell]}, \qquad \Gamma^{\mu_1,\cdots,\mu_\ell}_{(\ell)} = \frac{1}{\sqrt{\ell!}}\Gamma^{[\mu_\ell}\cdots\Gamma^{\mu_1]},$$

  such that $\quad \mathrm{tr}(\Gamma^{(\ell)}_{\mu_1,\cdots,\mu_\ell}\Gamma^{\nu_1,\cdots,\nu_\ell}_{(\ell)}) = 32\delta^{\mu_1}_{[\nu_1}\delta^{\mu_2}_{\nu_2}\cdots\delta^{\mu_\ell}_{\nu_\ell]}.$

- $f_\ell(x) = \dfrac{1}{32}x^{5-\frac{\ell}{2}}(1-\sqrt{1-x})^{\ell-5} = \dfrac{1}{32}x^{\frac{\ell}{2}}(1+\sqrt{1-x})^{5-\ell}.$
- $s = -(k_1+k_2)^2 = -2k_1k_2, \quad t = -2k_2k_3, \quad u = -2k_1k_3.$

1 + 2 + 3 + 4+ case

$$A_{1+,2+,3+,4+}(s,t) = \int_0^1 dx\, x^{-\alpha' s-3/2}(1-x)^{-\alpha' t-3/2}\times$$
$$[T^{(1)}(1_+,2_+)T_{(1)}(3_+,4_+)(f_1(x)-f_9(x))$$
$$+T^{(3)}(1_+,2_+)T_{(3)}(3_+,4_+)(f_3(x)-f_7(x))]$$
$$= \frac{1}{2}\frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)}$$
$$\times\Big(-\alpha' t T^{(1)}(1_+,2_+)T_{(1)}(4_+,3_+)+\alpha' s T^{(1)}(1_+,4_+)T_{(1)}(2_+,3_+)\Big).$$

This has the total antisymmetry in external lines.

$\Rightarrow s-t$ duality: $\boxed{A_{1+,2+,3+,4+}(s,t) = -A_{4+,1+,2+,3+}(t,s).}$

This duality relation can be expressed by the matrix identity. We define the matrices F, and M as follows ($y = 1-x$):

$$\begin{pmatrix} T^{(1)}(1_+,2_+)T_{(1)}(3_+,4_+) \\ T^{(3)}(1_+,2_+)T_{(3)}(3_+,4_+) \end{pmatrix} = F \begin{pmatrix} T^{(1)}(1_+,4_+)T_{(1)}(3_+,2_+) \\ T^{(3)}(1_+,4_+)T_{(3)}(3_+,2_+) \end{pmatrix},$$
$$\begin{pmatrix} f_1(x)-f_9(x) \\ f_3(x)-f_7(x) \end{pmatrix} = M \begin{pmatrix} f_1(y)-f_9(y) \\ f_3(y)-f_7(y) \end{pmatrix}.$$

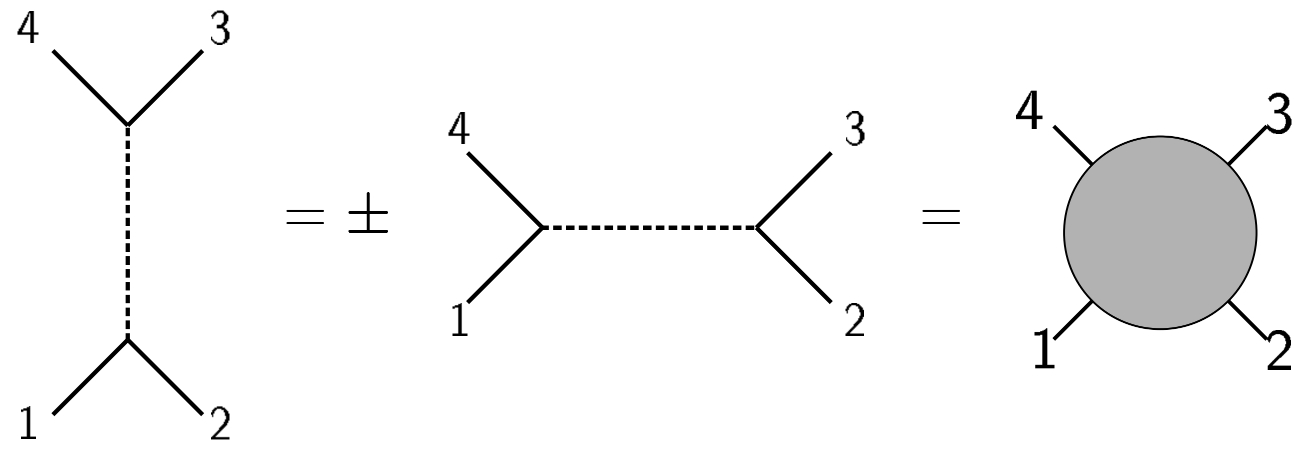
By using the Fierz arrangements, we get

$$F = \begin{pmatrix} -1/2 & -1/4 \\ -3 & 1/2 \end{pmatrix},\quad M = \begin{pmatrix} 1/2 & 3 \\ 1/4 & -1/2, \end{pmatrix},$$

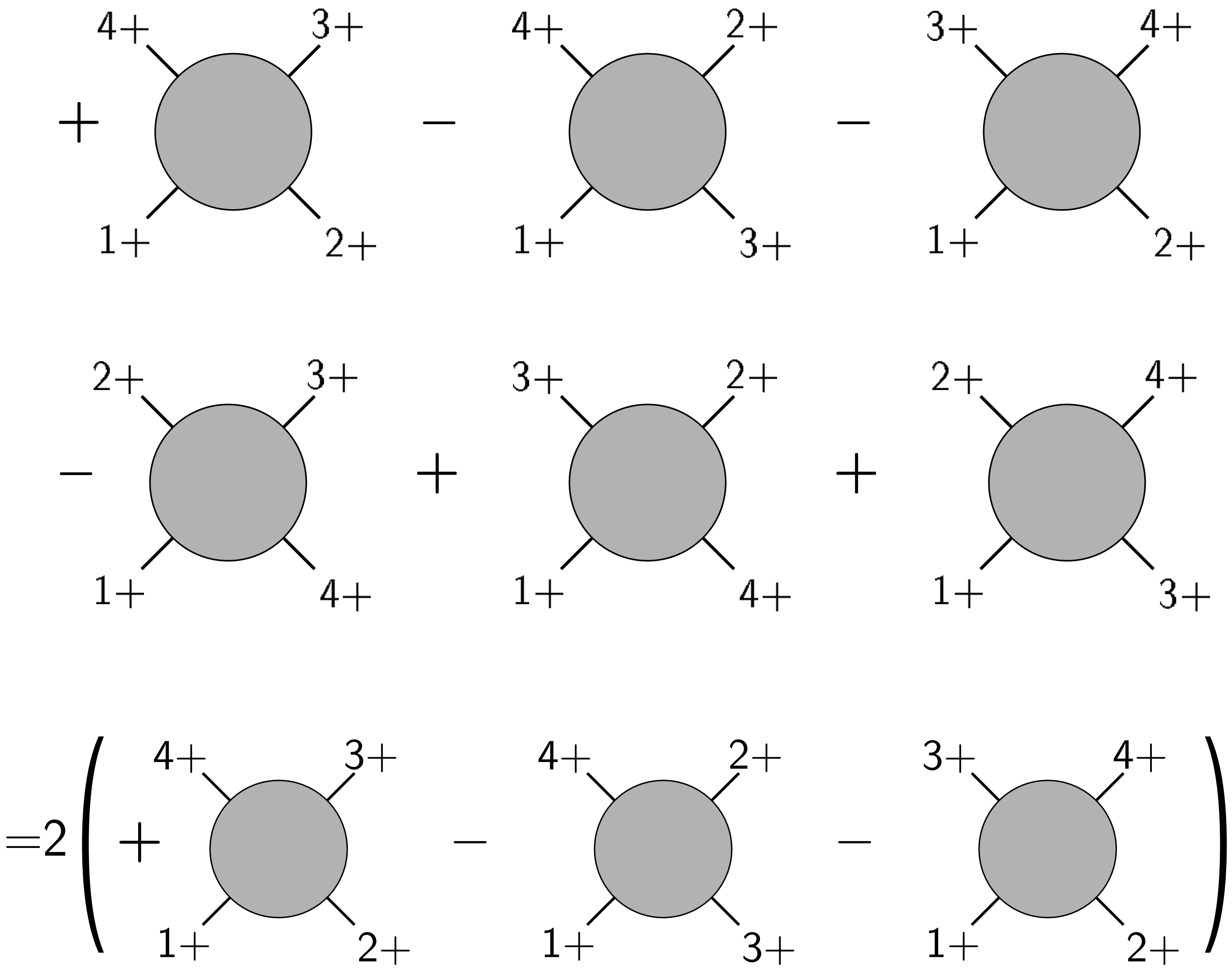
and they satisfy $\boxed{F^T M = -1}$, i.e.

$$\boxed{A_{1+,2+,3+,4+}(s,t) = -A_{1+,4+,3+,2+}(t,s).}$$

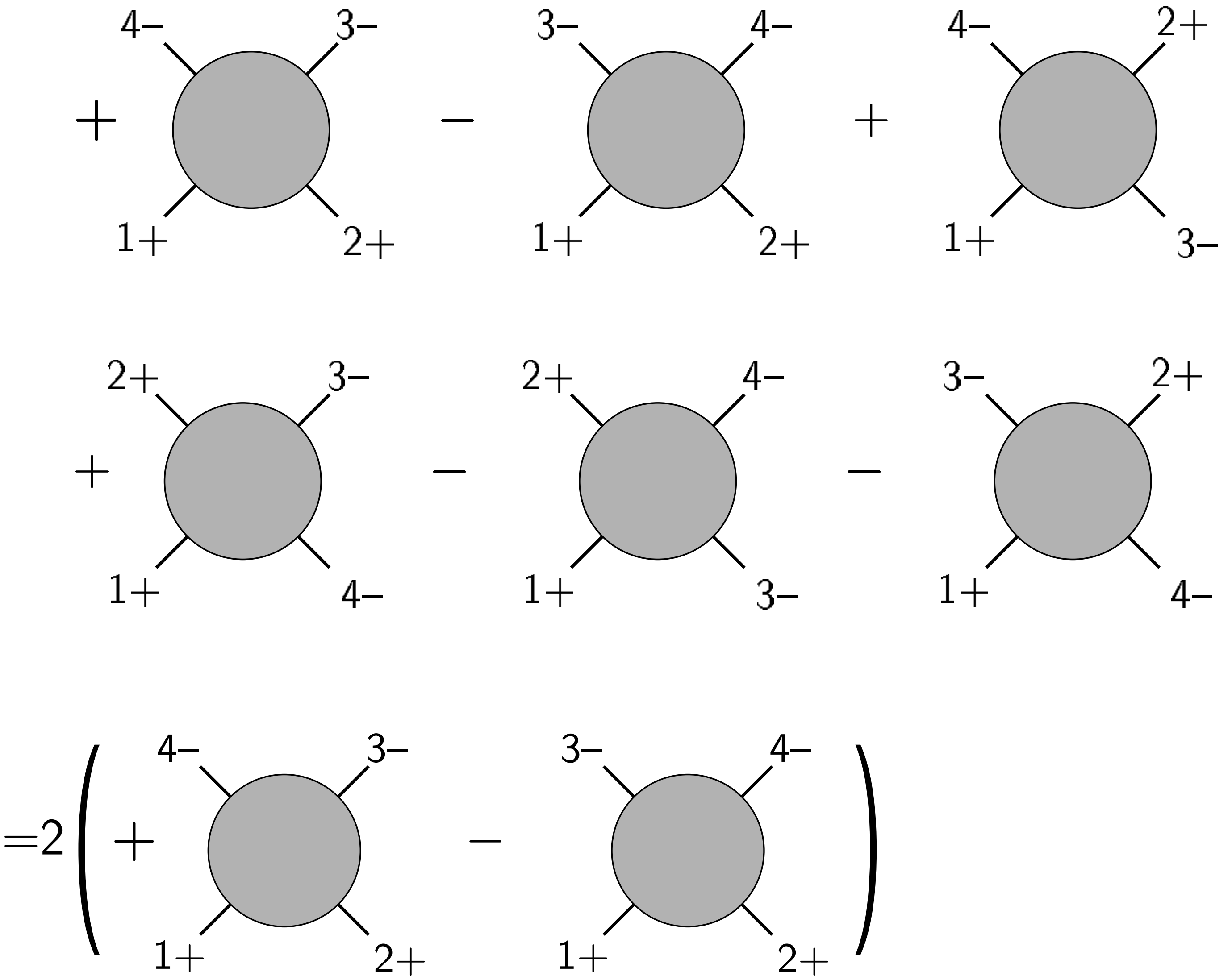
$s - t$ duality:



Total amplitude in the case of $(1+2+3+4+)$.



Total amplitude in the case of $(1+2+3-4-)$.



$1+2+3-4-$ case

$$A_{1+,2+,3-,4-}(s,t) = \int_0^1 dx\, x^{-\alpha' s-3/2}(1-x)^{-\alpha' t-3/2}\times$$
$$[T^{(1)}(1_+,2_+)T_{(1)}(3_-,4_-)(f_1(x)+f_9(x))$$
$$+T^{(3)}(1_+,2_+)T_{(3)}(3_-,4_-)(f_3(x)+f_7(x))$$
$$+T^{(5)}(1_+,2+)T_{(5)}(3_-,4_-)f_5(x)],$$

$$A_{1+,4-,3-,2+}(t,s) = \int_0^1 dx\, x^{-\alpha' t-3/2}(1-x)^{-\alpha' s-3/2}\times$$
$$[T^{(0)}(1_+,4_-)T_{(0)}(3_-,2_+)(f_0(x)-f_{10}(x))$$
$$+T^{(2)}(1_+,4_-)T_{(2)}(3_-,2_+)(f_2(x)-f_8(x))$$
$$+T^{(4)}(1_+,4_-)T_{(4)}(3_-,2_+)(f_4(x)-f_6(x))].$$

In fact,

$$\boxed{A_{1+,2+,3-,4-}(s,t) = A_{1+,4-,3-,2+}(t,s).}$$

because we define F and M:

$$\begin{pmatrix} T^{(1)}(1_+,2_+)T_{(1)}(3_-,4_-) \\ T^{(3)}(1_+,2_+)T_{(3)}(3_-,4_-) \\ T^{(5)}(1_+,2_+)T_{(5)}(3_-,4_-) \end{pmatrix} = F \begin{pmatrix} T^{(0)}(1_+,4_-)T_{(0)}(3_-,2_+) \\ T^{(2)}(1_+,4_-)T_{(2)}(3_-,2_+) \\ T^{(4)}(1_+,4_-)T_{(4)}(3_-,2_+) \end{pmatrix},$$
$$\begin{pmatrix} f_1(x)+f_9(x) \\ f_3(x)+f_7(x) \\ f_5(x) \end{pmatrix} = M \begin{pmatrix} f_0(y)-f_{10}(y) \\ f_2(y)-f_8(y) \\ f_4(y)-f_6(y) \end{pmatrix},$$

then we get

$$F = \frac{1}{16}\begin{pmatrix} 10 & 6 & 2 \\ 120 & 8 & -8 \\ 252 & -28 & 12 \end{pmatrix},\ M = \frac{1}{32}\begin{pmatrix} 2 & 54 & 84 \\ 2 & 6 & -28 \\ 1 & -5 & 10 \end{pmatrix},$$

and

$$\boxed{F^T M = +1}.$$

$1+3-2+4-$ case

$$A_{1+,3-,2+,4-}(u,t) = \int_0^1 dx\, x^{-\alpha' u - 3/2}(1-x)^{-\alpha' t - 3/2} \times$$
$$[T^{(0)}(1_+,3_-)T_{(0)}(2_+,4_-)(f_0(x)+f_{10}(x))$$
$$+ T^{(2)}(1_+,3_-)T_{(2)}(2_+,4_-)(f_2(x)+f_8(x))$$
$$+ T^{(4)}(1_+,3_-)T_{(4)}(2_+,4_-)(f_4(x)+f_6(x))],$$

$$A_{1+,4-,2+,3-}(t,u) = \int_0^1 dx\, x^{-\alpha' t - 3/2}(1-x)^{-\alpha' u - 3/2} \times$$
$$[T^{(0)}(1_+,4_-)T_{(0)}(2_+,3_-)(f_0(x)+f_{10}(x))$$
$$+ T^{(2)}(1_+,4_-)T_{(2)}(2_+,3_-)(f_2(x)+f_8(x))$$
$$+ T^{(4)}(1_+,4_-)T_{(4)}(2_+,3_-)(f_4(x)+f_6(x))].$$

In this case, duality relation is as follows:

$$\boxed{A_{1+,3-,2+,4-}(u,t) = A_{1+,4-,2+,3-}(t,u).}$$

$$\begin{pmatrix} T^{(0)}(1_+,3_-)T_{(0)}(2_+,4_-) \\ T^{(2)}(1_+,3_-)T_{(2)}(2_+,4_-) \\ T^{(4)}(1_+,3_-)T_{(4)}(2_+,4_-) \end{pmatrix} = F \begin{pmatrix} T^{(0)}(1_+,4_-)T_{(0)}(2_+,3_-) \\ T^{(2)}(1_+,4_-)T_{(2)}(2_+,3_-) \\ T^{(4)}(1_+,4_-)T_{(4)}(2_+,3_-) \end{pmatrix},$$
$$\begin{pmatrix} f_0(x)+f_{10}(x) \\ f_2(x)+f_8(x) \\ f_4(x)+f_6(x) \end{pmatrix} = M \begin{pmatrix} f_0(y)+f_{10}(y) \\ f_2(y)+f_8(y) \\ f_4(y)+f_6(y) \end{pmatrix},$$

then we get

$$F = \frac{1}{16}\begin{pmatrix} 1 & 1 & 1 \\ 45 & 13 & -3 \\ 210 & -214 & 2 \end{pmatrix},\; M = \frac{1}{16}\begin{pmatrix} 1 & 45 & 210 \\ 1 & 13 & -14 \\ 1 & -3 & 2 \end{pmatrix},$$

and we get

$\boxed{F^T M = +1}$ $\Rightarrow$ contradict to the fermi antisymmetry.

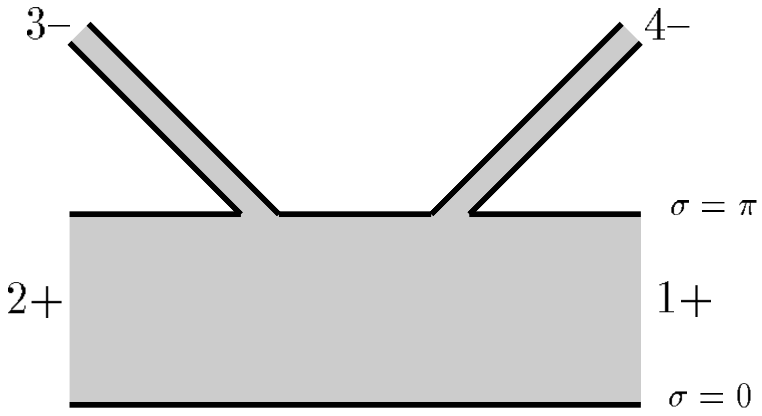
$\Rightarrow$ not contribute to the total amplitude.

Interpretation from the property of supercharges

Violation of the conservation of supercharges is proportional to the vertex operators at one end of an open string.

$\Downarrow$

In order to compensate the symmetry, the necessity of inserting the Goldstone fermion occurs at only one end of it.



The external lines of the same chiralities must always be paired as two adjacent lines.

The low-energy expansion of the amplitudes

- the case of single chirality

$$2A_{1+,2+,3+,4+}(s,t) - 2A_{1_+,2+,4+,3+}(s,u) - 2A_{3+,2+,4+,1+}(t,u)$$
$$= \frac{\pi^2}{2}\Big(\alpha' t T^{(1)}(1_+,2_+)T_{(1)}(4_+,3_+) - \alpha' s T^{(1)}(1_+,4_+)T_{(1)}(2_+,3_+)\Big),$$

- the case of mixed chirality

$$2A_{1+,2+,3-,4-}(s,t) - 2A_{1+,2+,4-,3-}(s,u)$$
$$= \frac{\pi^2}{2}\alpha'(t-u)T^{(1)}(1_+,2_+)T_{(1)}(3_-,4_-).$$

In both cases, massless poles corresponding to the gauge boson exchange are canceled,

$$\sim \frac{1}{-\alpha' s}T^{(1)}(1_+,2_+)T_{(1)}(3_-,4_-) - \frac{1}{-\alpha' s}T^{(1)}(1_+,2_+)T_{(1)}(4_-,3_-) = 0,$$

by the fermi antisymmetrization.

For $(1+,3-,2+,4-)$,

$$A_{1+,3-,2+,4-}(u,t) = A_{1+,4_-,2+,3-}(t,u)$$
$$= \Big[-\frac{3\alpha'\pi}{2}s - \frac{5\pi}{16}(1-\alpha' sK)\Big]T^{(0)}(1_+,3_-)T_{(0)}(2_+,4_-)$$
$$+\Big[-\frac{\alpha'\pi}{2}s + \frac{3\pi}{16}(1-\alpha' sK)\Big]T^{(2)}(1_+,3_-)T_{(2)}(2_+,4_-)$$
$$-\frac{\pi}{16}(1-\alpha' sK)T^{(4)}(1_+,3_-)T_{(4)}(2_+,4_-),$$

where $K = 2(1-\log 2)$.

This remains the order $(\alpha')^0$ terms. $(\Leftarrow (\bar{\psi}\Gamma^{(\ell)}\psi)^2 \quad (\ell = 0,2,4))$

$\Rightarrow$ contradict the nonlinear SUSY,
but not contribute to the total amplitudes.

Amplitude $A_{1+,2+,3-,4-}(s,t) - A_{1+,2+,4-,3-}(s,u)$ has Yukawa interactions $\overline{\psi}\psi\phi$ of fermions and tachyons ϕ.

The contribution of tachyon exchanges:

$$\frac{T^{(0)}(1_+,4_-)T_{(0)}(2_+,3_-)}{\alpha' t + \frac{1}{2}} - \frac{T^{(0)}(1_+,3_-)T_{(0)}(2_+,4_-)}{\alpha' u + \frac{1}{2}},$$

$$A_{1+,2+,3-,4-}(s,t) = A_{1+,4-,3-,2+}(t,s)$$
$$= \int_0^1 dx\, x^{-\alpha' t - 3/2}(1-x)^{-\alpha' s - 3/2}\left[T^{(0)}(1_+,4_-)T_{(0)}(3_-,2_+)(f_0(x) - f_{10}(x)) + \cdots\right].$$

This gives the order $(\alpha')^0$ terms, but the exact amplitude again does not have the order $(\alpha')^0$ contribution.

$$\Downarrow$$

The infinite tower of massive modes of the oppositely GSO-projected states is responsible for the cancellation of $O(\alpha'^0)$.

c.f. $\quad \mathrm{Tr}[(-1)^F M^{2n}] = 0 \quad (n \neq 4).$

Consistency between action and amplitude

Terms in the action which contribute to 4-point on-shell S-matrix elements, and amplitudes.

- single chiralities:
  - Action:
$$-\frac{\lambda^2}{4\cdot 3!}(\bar{\psi}_+\Gamma_\mu\partial_\nu\psi_+\bar{\psi}_+\Gamma^\mu\partial^\nu\psi_+ + \bar{\psi}_-\Gamma_\mu\partial_\nu\psi_-\bar{\psi}_-\Gamma^\mu\partial^\nu\psi_-).$$
  - Amplitude:
$$2A_{1+,2+,3+,4+}(s,t) - 2A_{1_+,2+,4+,3+}(s,u) - 2A_{3+,2+,4+,1+}(t,u)$$
$$= \frac{\pi^2}{2}\Big(\alpha' t T^{(1)}(1_+,2_+)T_{(1)}(4_+,3_+) - \alpha' s T^{(1)}(1_+,4_+)T_{(1)}(2_+,3_+)\Big).$$
- mixed chiralities:
  - Action:
$$-\frac{\lambda^2}{4}\bar{\psi}_+\Gamma_\mu\partial_\nu\psi_+\bar{\psi}_-\Gamma^\mu\partial^\nu\psi_-.$$
    (Another mixed term does not contribute to on-shell amplitude in this order.)
  - Amplitude:
$$2A_{1+,2+,3-,4-}(s,t) - 2A_{1+,2+,4-,3-}(s,u)$$
$$= \frac{\pi^2}{2}\alpha'(t-u)T^{(1)}(1_+,2_+)T_{(1)}(3_-,4_-).$$

Identifying $\dfrac{g^2\alpha'\pi^2}{2}$ to $\dfrac{\lambda^2}{4}$

$\Downarrow$

The open string amplitudes with the both GSO sectors satisfy the low-energy theorem of the nonlinear $N = 2$ SUSY.

Other terms of the action are the same as in the case with the usual GSO projection.

4. Conclusions and discussions

We showed the low-energy behavior of the 4-point amplitudes of would-be Goldstone fermions is perfectly consistent with the hidden N =2 SUSY.

Remained questions:

1. Extension to D9-$\overline{\text{D9}}$ system of Type IIB case and to general non-Abelian cases.
2. Generalization to higher orders in λ-expansion and to higher derivatives.($\Leftarrow$ a systematic geometric manner)
3. Including all supergravity background field.
   In this work, $\ell_P \ll \ell_{\text{SUSY}} \Rightarrow$ justified only in $g_s \ll 1$.

   The Goldstone fermions are expected to be absorbed by gravitino through the super Higgs effect.
4. The role of $N = 2$ SUSY in the dynamics of tachyon condensation.
   If $N = 2$ SUSY fully is restored,

   $$\langle \delta\psi \rangle = 0$$
   $$\Rightarrow \langle i\overline{\psi}_{\pm\beta}\partial_\mu\psi_{\pm\alpha}\rangle = -\frac{1}{5\lambda^2}\left(\Gamma_\mu\frac{1 \pm \Gamma_{11}}{2}\right)_{\alpha\beta}$$

   in terms of the fields before redefinition.

   Need to investigate the relation between vacuum expectation value and tachyon condensation.

The completeness relation:

$$T^{(0)}(1,4)T_{(0)}(3,2) = \frac{1}{32}\sum_{\ell=0}^{10} T^{(\ell)}(1,2)T_{(\ell)}(3,4).$$

Formulae used in calculation:

$$T^{(\ell)}(1,2) = -(-1)^{\frac{1}{2}\ell(\ell+1)}T^{(\ell)}(2,1),$$

$$T^{(\ell)}(1,2)T_{(\ell)}(3,4) = \pm(-1)^{\ell}T^{(10-\ell)}(1,2)T_{(10-\ell)}(3,4).$$

Field Redefinition Procedure

(originally by R.R.Metsaev et. al (1987))

A general form to redefine the fields (which preserve chirality):

$$
\begin{aligned}
\psi \to \psi &+ h_1\lambda\Gamma_{\mu\nu}\psi F^{\mu\nu} + h_2\lambda^2\Gamma_{\mu\nu\rho\sigma}\psi F^{\mu\nu}F^{\rho\sigma} \\
&+ h_3\lambda^2\psi F^2 + h_4 i\lambda^2(\overline{\psi}\Gamma^{\mu\nu\rho}\psi)\Gamma_{\mu\nu}\partial_\rho\psi \\
&+ h_5 i\lambda^2(\overline{\psi}\Gamma^\mu\partial_\mu\psi)\psi + h_6 i\lambda^2(\overline{\psi}\Gamma^\mu\partial^\nu\psi)\Gamma_{\mu\nu}\psi \\
&+ h_7 i\lambda^2(\overline{\psi}\Gamma^{\mu\nu\rho}\partial_\rho\psi)\Gamma_{\mu\nu}\psi + h_8 i\lambda^2(\overline{\psi}\Gamma^{\mu\nu\rho\sigma\tau}\partial_\tau\psi)\Gamma_{\mu\nu\sigma\rho}\psi,
\end{aligned}
$$

$$
A_\mu \to A_\mu + h_9 i\lambda^2(\overline{\psi}\Gamma_{\mu\nu\rho}\psi)F^{\nu\rho}.
$$

A general form of action:(without subscript $\pm$):

$$
\begin{aligned}
\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}^2 + \frac{i}{2}\overline{\psi}\Gamma^\mu\partial_\mu\psi + a_1\lambda^2(F^2)^2 + a_2\lambda^2(F)^4 \\
&+ b_0 i\lambda\overline{\psi}\Gamma_\mu\partial_\nu\psi F^{\mu\nu} + b_1 i\lambda^2\overline{\psi}\Gamma_{\mu\nu\rho}\psi\partial_\sigma F^{\sigma\nu}F^{\mu\rho} \\
&+ b_2 i\lambda^2\overline{\psi}\Gamma_{\mu\nu\rho}\psi F^{\mu\sigma}\partial_\sigma F^{\nu\rho} + b_3 i\lambda^2\overline{\psi}\Gamma_{\mu\nu\rho\sigma\tau}\partial_\tau\psi\partial_\sigma F^{\mu\nu}F^{\rho\sigma} \\
&+ b_4 i\lambda^2\overline{\psi}\Gamma_\mu\partial_\nu\psi F^{\mu\sigma}F_\sigma{}^\nu + b_5 i\lambda^2\overline{\psi}\Gamma^\mu\partial_\mu\psi F^2 \\
&+ c_1\lambda^2(\overline{\psi}\Gamma^\mu\partial_\mu\psi)^2 + c_2\lambda^2\,\overline{\psi}\Gamma^\mu\partial^\nu\psi\overline{\psi}\Gamma_\mu\partial_\nu\psi + c_3\overline{\psi}\Gamma^\mu\partial^\nu\psi\overline{\psi}\Gamma_\nu\partial_\mu\psi \\
&+ c_4\lambda^2\overline{\psi}\Gamma^{\mu\nu\rho}\partial_\rho\psi\overline{\psi}\Gamma_\mu\partial_\nu\psi + c_5\lambda^2\overline{\psi}\Gamma^{\mu\nu\rho}\partial_\rho\psi\overline{\psi}\Gamma_{\mu\nu\tau}\partial^\tau\psi.
\end{aligned}
$$

The transformations of coefficients under the redefinition:

$$
\begin{aligned}
&b_0 \to b_0 - 2h_1, \qquad b_1 \to b_1 - 2h_2 - h_9 + h_1^2, \quad b_2 \to b_2 - 2h_2 + b_0h_1, \\
&b_3 \to b_3 - \frac{1}{2}h_1^2 + h_2, \quad b_4 \to b_4 + 4h_1^2 - 4b_0h_1, \qquad b_5 \to b_5 + h_1^2 + h_3,
\end{aligned}
$$

$$
\begin{aligned}
&c_1 \to c_1 - 8h_4 - h_5 + 312h_8, \quad c_2 \to c_2 + 4h_4 + h_6 - 72h_8, \\
&c_3 \to c_3 - 8h_4 - h_6 + 120h_8, \quad c_4 \to c_4 - 4h_4 + h_6 + 2h_7 - 24h_8, \\
&c_5 \to c_5 + h_7 - 24h_8.
\end{aligned}
$$

Invariant combinations of the coefficient:

$$
\begin{aligned}
I_1 &= 4(b_2 + b_3) + {b_0}^2, \\
I_2 &= b_4 - {b_0}^2, \\
I_3 &= c_2 + \frac{2}{3}c_3 - \frac{1}{3}c_4 + \frac{2}{3}c_5.
\end{aligned}
$$

As long as these invariants are preserved, we can freely choose 8 coefficients out of 11 coefficients in the general form of the action.

Coefficients of our original action:

$$
a_1 = -\frac{1}{32}, \quad a_2 = \frac{1}{8}, \quad b_0^\pm = \mp\frac{1}{2}, \quad b_4^\pm = \frac{1}{2}, \quad b_5^\pm = \frac{1}{8},
$$
$$
c_1^\pm = c_2^\pm = \frac{1}{8}, \quad c_3^\pm = -\frac{1}{4}.
$$

For c_i, we choose,

$$
c_1 = c_3 = c_4 = c_5 = 0 \quad \Rightarrow \quad c_2 = -\frac{1}{24}
$$

For b_i, we choose

$$
b_0 = b_1 = 0 \quad \Rightarrow \quad b_4 = \frac{1}{4}, \quad b_3 = b_5 = 0.
$$

such that the coefficients of $O(\lambda)$ term are 0.

If we restrict to a single chiral sector, this choice is the same as Metsaev et.al.