Black Brane Solutions in Heterotic String Theory

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Based on [2412.02277]
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1.1. Introduction : Non-supersymmetric heterotic branes

Non-supersymmetric heterotic branes were proposed by

- Nonsupersymmetric Heterotic Branes [Kaidi,Ohmori,Tachikawa,Yonekura '23]
- On non-supersymmetric heterotic branes [Kaidi,Tachikawa,Yonekura '24]

They constructed world sheet CFTs describing the throat region of

- 0-brane
- 4-brane
- 6-brane
- 7-brane

We constructed black brane solutions.

Contents

1.2. Charge and branes

What is non-supersymmetric heterotic branes ?

$$\Downarrow$$

Cobordism conjecture predicts their existence.

Usually, we use

Completeness hypothesis(CH)

A gauge theory with gravity must contain all possible representations.

In a field theory consistent with quantum gravity,

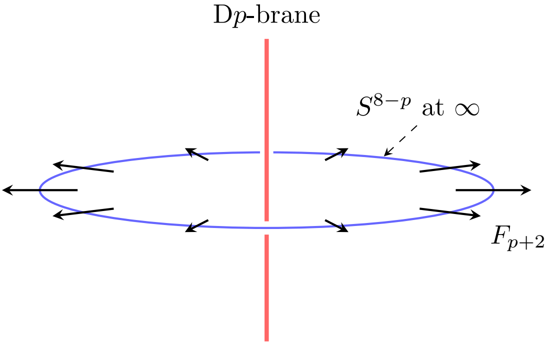
$$\text{charge} \Rightarrow \text{dynamical object}$$

1.2. D-branes

Example : D-branes

- D-branes carry RR-charges [Polchinski '95],
- but their existence was expected before its discovery.

$$S_{\mathsf{IIA}} \supset \int -\frac{1}{2}F_2 \wedge *F_2 - \frac{1}{2}F_4 \wedge *F_4$$



$$Q = \int_{S^{8-p}} *F_{p+2}$$

$\uparrow$generalize

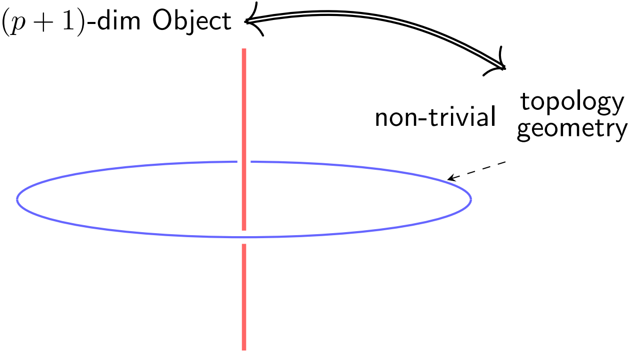
$$Q_e = \int_{S^2} \vec{E} \cdot \mathrm{d}^2\vec{S}$$
$$= \int_{D^3} \rho \mathrm{d}^3 V$$

1.3. Cobordism conjecture(CC)

Cobordism Conjecture(CC) [McNamara,Vafa '19]

All bordism classes in QG must vanish.

- Completeness hypothesis was refined.
  $\rightarrow$ It can now be applied to more subtle charges.

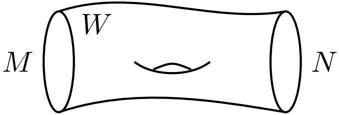


Cobordism Conjecture(CC) [McNamara,Vafa '19]

All bordism classes in QG must vanish.

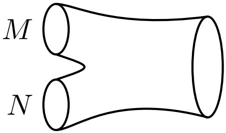
M and N are *bordant* if there is a $(n+1)$-dim mfd W s.t.

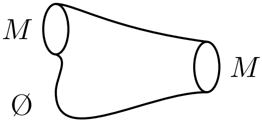
$$\partial W = M \sqcup (-N) \tag{1}$$

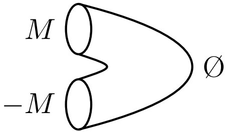


$\Longleftrightarrow M \overset{\text{bordant}}{\sim} N$

We can define group structure



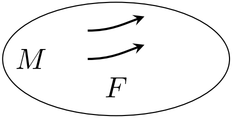




$[M] + [N] = [M \sqcup N]$ $\quad [M] + [\emptyset] = [M]$ $\quad [M] + [-M] = [\emptyset]$

1.3. CC $\to$ CH 1

M also can have additional structures such as flux F:



$$(M, n_M) \qquad n_M = \int_M F \in \mathbb{Z}$$

Assume that we don't have any charged object:

$$\mathrm{d}F = 0 \tag{2}$$

$$\boxed{\text{If } (M, n_M) \sim (N, n_N)\text{, then } 0 = \int_W \mathrm{d}F \overset{\text{Stokes}}{=} n_M - n_N \iff n_M = n_N.}$$

$$\Updownarrow$$

$$\boxed{\text{If } n_M \neq n_N\text{, then } (M, n_M) \not\sim (N, n_N).}$$

There are non-trivial classes for each $n_M \in \mathbb{Z}$.

1.3. CC $\rightarrow$ CH 2

<u>One of the solution</u> : introduce an object (source) with charge n_M

$$\mathrm{d}F = j_m, \qquad \int_W j_m = n_M \tag{3}$$

If $(M, n_M) \sim (N, n_N)$, then

$$\begin{aligned} n_M = \int_W j_m &= \int \mathrm{d}F \\ &= n_M - n_N \Longleftrightarrow n_N = 0 \end{aligned} \tag{4}$$

$$\Downarrow$$

(M, n_M) can belong to trivial class.

To break the non-trivial classes, we need to introduce charged objects for any possible charges.

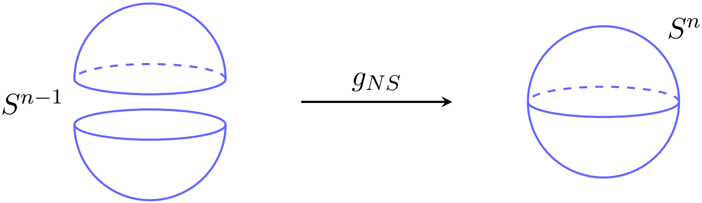
1.4. Charges in heterotic string

We focus on homotopy groups of gauge group in heterotic theory.

$$\pi_3((E_8 \times E_8) \rtimes \mathbb{Z}_2) \cong \mathbb{Z} \times \mathbb{Z}, \qquad \pi_7(\mathrm{Spin}(32)/\mathbb{Z}_2) \cong \mathbb{Z} \tag{5}$$

$\pi_{n-1}(G)$ classifies maps $S^{n-1} \to G$.

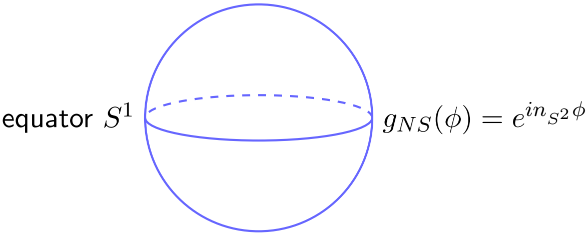
$\pi_{n-1}(G)$ means that non-trivial gauge field can be put on S^n



1.4. Dirac monopole

Example : Dirac Monopole $\Longleftrightarrow \pi_1(U(1)) \cong \mathbb{Z} = \left\{ n_{S^2} = \int_{S^2} \frac{iF}{2\pi} \right\}$

$$A_{\mathsf{north}} = \frac{i n_{S^2}}{2}(1-\cos\theta)\mathrm{d}\phi$$



$$A_{\mathsf{south}} = -\frac{i n_{S^2}}{2}(1+\cos\theta)\mathrm{d}\phi$$

$$A_{\mathsf{north}} = A_{\mathsf{south}} + g_{NS}^{-1}\mathrm{d}g_{NS} \tag{6}$$

$\pi_{n-1}(G)$ is a generalization of this situation

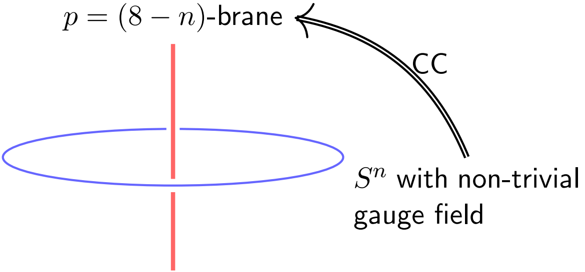
1.4. Charges in heterotic string 2

- These charges can be observed by following integrations:

$$\pi_3 \to \qquad (\nu_1, \nu_2) = \left(\int_{S^4} \frac{1}{2!} \mathrm{tr} \left(\frac{iF_1}{2\pi} \right)^2, \int_{S^4} \frac{1}{2!} \mathrm{tr} \left(\frac{iF_2}{2\pi} \right)^2 \right) \tag{7}$$

$$\pi_7 \to \qquad n_{S^8} = \int_{S^8} \frac{1}{4!} \mathrm{tr} \left(\frac{iF}{2\pi} \right)^4 \tag{8}$$

- According to CC, there must exists the objects which carry the topological charges.



1.5. Early works

- Non-SUSY heterotic branes were proposed by [Kaidi,Ohmori,Tachikawa,Yonekura '23] and [Kaidi,Tachikawa,Yonekura '24].
- The world sheet CFTs are constructed by them.

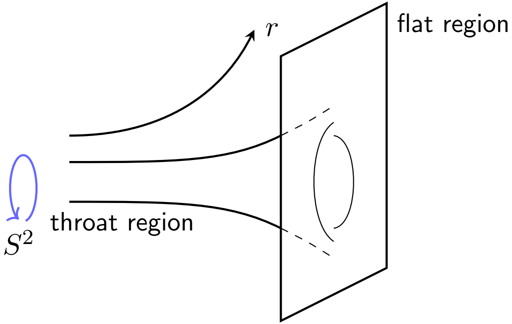
Homotopy group	p-brane	black brane
$\pi_7(\mathrm{Spin}(32)/\mathbb{Z}_2)$	0-brane*	??
$\pi_3((E_8 \times E_8) \rtimes \mathbb{Z}_2)$	4-brane*	partially studied
$\pi_1(\mathrm{Spin}(32)/\mathbb{Z}_2)$	6-brane	◯
$\pi_0((E_8 \times E_8) \rtimes \mathbb{Z}_2)$	7-brane	??

* Early works : [Polchinski '05],[Bergshoeff,Gibbons,Townsend '06]

* Related works : [Álvarez-García, Kneißl, Leedom, Righi '24]
$\to$ (-1)-brane in $\mathrm{Spin}(32)/\mathbb{Z}_2$
[Dierigl, Heckman, Montero, Torres '22]
$\to$ R7-brane in IIB

1.5. Black 6-brane solution

6-brane solution was given by the ref. [KOTY '23] and [KTY '24]

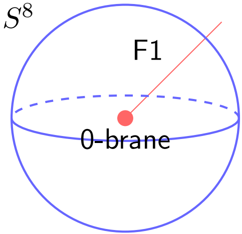


$$\mathrm{d}s^2 = \frac{\mathrm{d}R^2}{\left(1 - \frac{l_0}{R}\right)^2} + R^2 \mathrm{d}\Omega_2^2 + \mathrm{d}x_\mu \mathrm{d}x^\mu$$

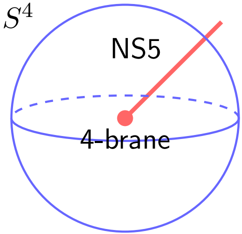
$$\frac{F}{2\pi} = \bigoplus_{i=1}^{16} \begin{pmatrix} 0 & q_i \\ -q_i & 0 \end{pmatrix} \frac{\epsilon}{4\pi} \qquad e^{-2\Phi} = g_s^{-2}\left(1 - \frac{l_0}{R}\right)$$

1.5. Early works 2

- [Polchinski '05]



- [Bergshoeff,Gibbons,Townsend '06]

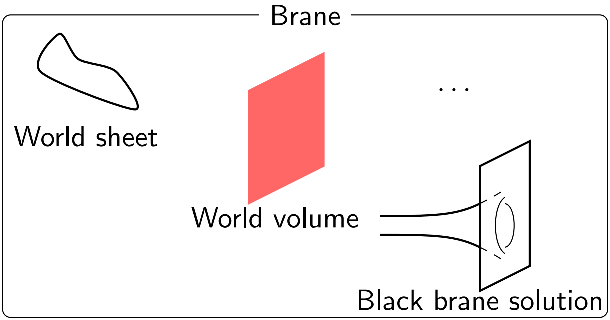


Black brane solutions of the other branes has not been constructed.

- Black 0-brane solution was not considered.
- Only asymptotic region of the 4-brane was considered.

1.5. Purpose

We need various expression.



- Purpose of our research is to construct black brane solutions for 0- and 4-brane and provide further evidence for the existence of these branes.

2.1. Action

Effective action for heterotic string

$$S \propto \int \mathrm{d}^{10}x \, \sqrt{-G} e^{-2\Phi} \left(\mathcal{R} + 4(\partial\Phi)^2 + \frac{\alpha'}{2} \mathrm{tr}'(F_{\mu\nu}F^{\mu\nu}) + \cdots \right) \tag{9}$$

- $\mathcal{R}$: Ricci scalar, Φ : Dilaton, F : YM field
- Fermionic part and B are omitted.
- tr' is normalized as

$$\mathrm{tr}'(X) = \frac{1}{2}\mathrm{tr}_{\mathsf{fund}}(X) \qquad X \in \mathfrak{so}(32) \tag{10}$$

$$\mathrm{tr}'(X) = \frac{1}{60}\mathrm{tr}_{\mathsf{adj}}(X) \qquad X \in \mathfrak{e}_8 \times \mathfrak{e}_8 \tag{11}$$

2.2. Ansatz

For simplicity

- spherical symmetry
- static
- flatness along the brane

In terms of the fields,

$$\mathrm{d}s^2 = \mathrm{d}x^\mu \mathrm{d}x_\mu + \mathrm{d}r^2 + R(r)^2 \mathrm{d}\Omega_{8-p}^2 \tag{12}$$

$$A = A_{\theta^i}(\theta)\mathrm{d}\theta^i \tag{13}$$

$$\Phi = \Phi(r) \tag{14}$$

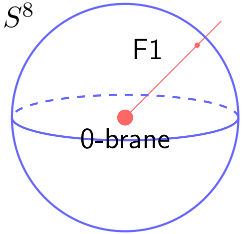
- $\mu = 0, 1, \ldots, p,$ $\quad \mathrm{d}\Omega_{8-p}^2$: unit S^{8-p} sphere metric
- WS theory : $\mathbb{R}^{p,1} \times \mathbb{R}_\Phi \times (S^{8-p}$part$)$
- θ^i : S^{8-p} direction

2.3. How to set $B = 0$: 0-brane case

- Equation of motion for B field:

$$\mathrm{d} * H = \frac{1}{4!}\mathrm{tr}\left(\frac{iF}{2\pi}\right)^4 + J_{F1} \tag{15}$$

In order to set $B = 0$, F1s are needed because $\mathrm{tr}(F^4)$ is nontrivial,



but they don't affect the SUGRA solution.

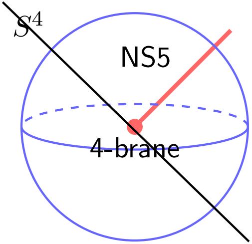
$$\rho_{F1} \sim g_s^0 \ll \rho_{\text{gauge}} \sim g_s^{-2} \tag{16}$$

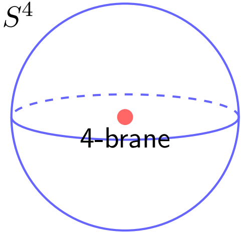
2.3. How to set $B = 0$: 4-brane case

- Bianchi identity for B field:

$$\mathrm{d}H = \frac{1}{2!}\mathrm{tr}'\left(\frac{iF_1}{2\pi}\right)^2 + \frac{1}{2!}\mathrm{tr}'\left(\frac{iF_2}{2\pi}\right)^2 + J_{NS5} \tag{17}$$

In order to set $B = 0$, the instanton # should be $(\nu, -\nu)$.





If there are NS5-branes, they affect the SUGRA solution.

$$\rho_{NS5}, \rho_{\text{gauge}} \sim g_s^{-2} \tag{18}$$

2.4. YM kinetic term

From spherical symmetry,

$$\mathrm{tr}'(F_{\mu\nu}F^{\mu\nu}) = -\frac{\mathsf{C}}{R^4} \tag{19}$$

- C is a constant depending on the charge $\pi_{n-1}(G)$
- R is the radius of S^n

Define the typical length scale :

$$l_0 = \sqrt{\frac{\alpha'\mathsf{C}}{n(n-1)}} \tag{20}$$

2.5. Extremal Case : Equations of Motion

Then, the EoM become

θ-direction

$$0 = \mathrm{d} * F + A \wedge *F, \tag{21}$$

r-direction

$$0 = \sigma'' - 2\left(\Phi - \frac{n}{2}\sigma\right)' \sigma' - \frac{n-1}{l_0^2}\left(e^{-2\sigma} - e^{-4\sigma}\right), \tag{22}$$

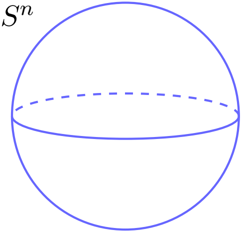
$$0 = \Phi'' - 2\left(\Phi - \frac{n}{2}\sigma\right)' \Phi' + \frac{n(n-1)}{4l_0^2} e^{-4\sigma} \tag{23}$$

where

$$\underbrace{l_0 = \sqrt{\frac{\alpha' \mathsf{C}}{n(n-1)}}}_{\text{typical length scale}}, \quad R = l_0 e^{\sigma}. \tag{24}$$

2.6. Example : Gauge field configuration

We want a non-trivial solution of YM eq with charge $\pi_{n-1}(G)$ on S^n.



- metric :

$$\mathrm{d}\Omega_n^2 = h_{ij}\mathrm{d}\theta^i\mathrm{d}\theta^j$$

- structure group of TS^n :

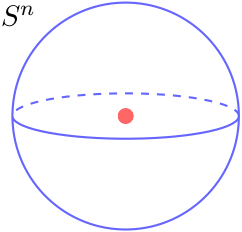
$$SO(n)$$

We can take a $\mathfrak{so}(n)$ gauge field configuration such that the connection is the same as Levi-Civita of the S^n:

$$F_{ijkl} = R_{ijkl} = h_{ik}h_{jl} - h_{il}h_{jk}, \tag{25}$$

2.6. Gauge field configuration

brane	gauge group	sphere	how to embed
4	$(E_8 \times E_8) \rtimes \mathbb{Z}_2$	S^4	$\mathfrak{so}(4) \simeq \mathfrak{su}(2) \times \mathfrak{su}(2) \subset \mathfrak{e}_8 \times \mathfrak{e}_8$
0	$\mathrm{Spin}(32)/\mathbb{Z}_2$	S^8	$\mathfrak{so}(8) \subset \mathfrak{so}(32)$



$$F_{ijkl} = R_{ijkl} = h_{ik}h_{jl} - h_{il}h_{jk},$$

Then we get

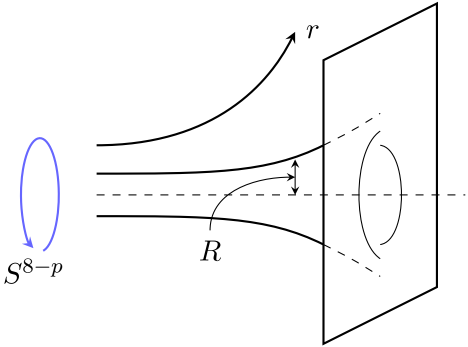
$$\mathrm{tr}'(|F|^2) = \sum_{ijkl} R_{ijkl}R_{ijkl} = -\frac{n(n-1)}{2R^4}. \tag{26}$$

We want black brane solutions like the 6-brane solution.

- throat region

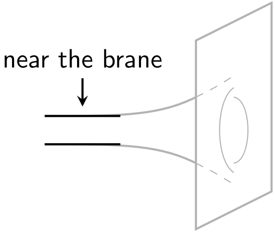
$$\mathbb{R}^{p,1} \times \mathbb{R}_{\text{linear dilaton}} \times (S^{8-p}\text{part}), \tag{27}$$

- asymptotically flat region



2.8. Throat region

EoM have a throat solution.



$$0 = \sigma'' - 2\left(\Phi - \frac{n}{2}\sigma\right)'\sigma' - \frac{n-1}{l_0^2}\left(e^{-2\sigma} - e^{-4\sigma}\right),$$

$$0 = \Phi'' - 2\left(\Phi - \frac{n}{2}\sigma\right)'\Phi' + \frac{n(n-1)}{4l_0^2}e^{-4\sigma}$$

$$\Phi = (\text{const}) - \sqrt{\frac{n(n-1)}{8}}\frac{r}{l_0} \tag{28}$$

$$R = l_0 \tag{29}$$

2.8. Flat region

Also, if we neglect $\mathrm{tr}'(|F|^2) \propto R^{-4}$.

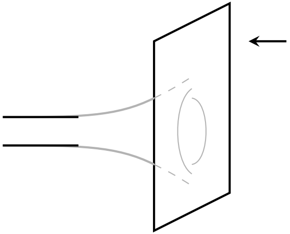
$$R = l_0 e^{\sigma}$$

r-direction

$$0 = \sigma'' - 2\left(\Phi - \frac{n}{2}\sigma\right)' \sigma' - \frac{n-1}{l_0^2}\left(e^{-2\sigma} - \cancel{e^{-4\sigma}}\right), \quad (30)$$

$$0 = \Phi'' - 2\left(\Phi - \frac{n}{2}\sigma\right)' \Phi' + \cancel{\frac{n(n-1)}{4l_0^2} e^{-4\sigma}} \quad (31)$$

we obtain flat spacetime



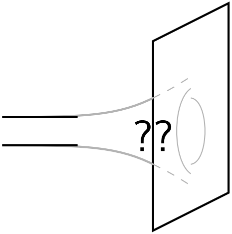
$$R = r + (\text{const}) \quad (32)$$

$$\Phi = (\text{const}) \quad (33)$$

2.9. Numerical calculations

These discussions are based on asymptotic analysis.

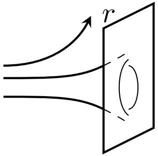
- We do not know whether the throat and flat regions are connected.



- Unfortunately, we could not find the analytical solution except for $n = 2, 9$ ($n = 2$ case $\rightarrow$ 6-brane).
  $\longrightarrow$ numerical calculation
- We need the initial conditions for numerical calculations.

2.9. Initial conditions

Consider a small perturbation to the throat



$$\Phi = (\text{const}) - \sqrt{\frac{n(n-1)}{8}}\frac{r}{l_0} + \delta\Phi. \tag{34}$$

$$R = l_0\left(1 + \delta R\right), \tag{35}$$

and linearize the EOM (and gauge fixing constraint).

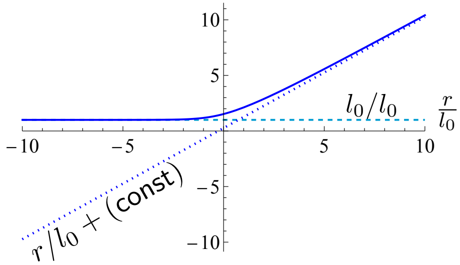
$$\delta\Phi' - \frac{n}{2}\delta R' = 0 \tag{36}$$

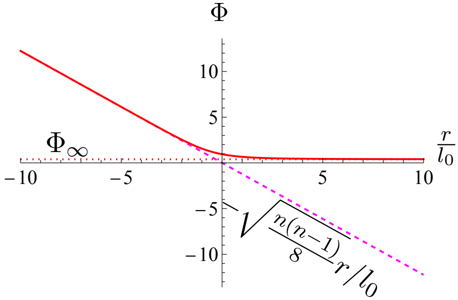
$$\delta R'' - 2\sqrt{\frac{n(n-1)}{8}}\delta R' - 2(n-1)\delta R = 0 \tag{37}$$

- Linearized equations are solvable. $\delta R = Ae^{\lambda r}, \quad \delta\Phi = \frac{n}{2}\delta R$
- We use the solution to give initial conditions.

2.10. Results

R and Φ for 4-brane are





- There are throat region and flat region.
- Two regions are smoothly connected.
- The results are same for $p = 0$.

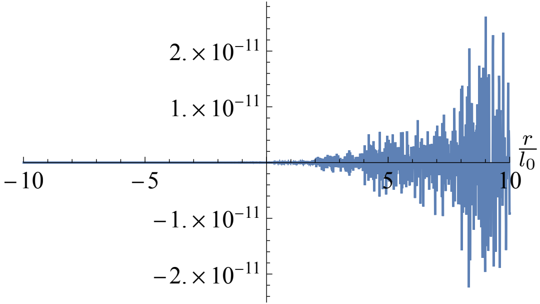
2.11. Check(Hamiltonian constraint)

We performed two additional calculations.

- Gauge fixing constraint from $r \to r'(r)$
- 6-brane analytical solution

Our numerical solutions satisfy gauge fixing constraint.

$$0 = \frac{n}{4}\sigma'^2 - \left(\Phi - \frac{n}{2}\sigma\right)'^2 + \frac{n(n-1)}{8l_0^2}(2e^{-2\sigma} - e^{-4\sigma}) \tag{38}$$



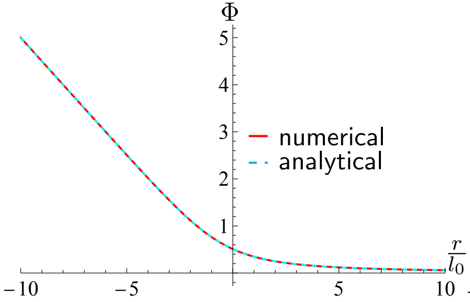
2.11. Check(6-brane)

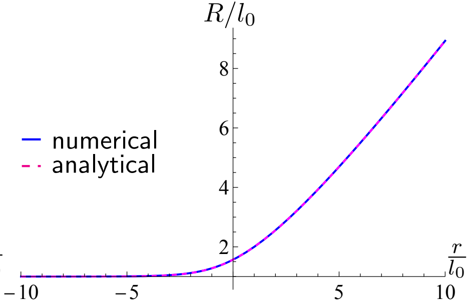
Analytical solution for 6-brane [KOTY'23,KTY'24].

$$\mathrm{d}s^2 = \mathrm{d}x_\mu \mathrm{d}x^\mu + \frac{\mathrm{d}R^2}{(1-\frac{l_0}{R})^2} + R^2 \mathrm{d}\Omega_2^2 \tag{39}$$

$$e^{-2\Phi} = g_s^{-2}\left(1 - \frac{l_0}{y}\right) \tag{40}$$

Compare analytical and our numerical calculations





2.12. Reliablity of the SUGRA 1

If the curvature is small, we can trust SUGRA.
$\rightarrow$ We need a large charge C(or a large $l_0 = \sqrt{\alpha' \mathsf{C}/n(n-1)}$).

- 4-brane
  We found $\nu = 1240$ configuration.

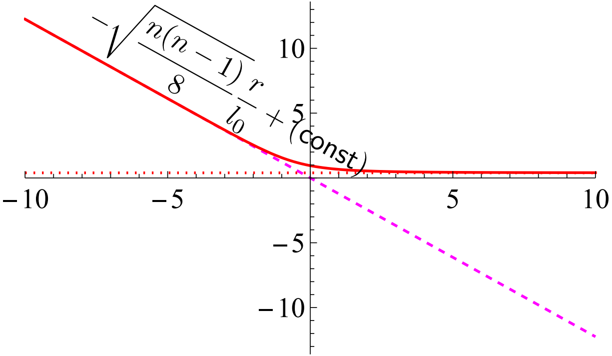
$$l_0 = \sqrt{\frac{\alpha' \mathsf{C}}{4(4-1)}} = \sqrt{\frac{\alpha'|\nu|}{2}} \sim 25\sqrt{\alpha'} \tag{41}$$

  where the instanton number of $E_8 \times E_8$ is $(\nu, -\nu)$.

- 0-brane
  We don't know how to get large charge configurations...

2.12. Reliablity of the SUGRA 2

If the string coupling is small, we can trust SUGRA, but



$$g_s \to \infty \text{ as } r \to -\infty \ldots$$

We can not trust SUGRA deep in the throat.

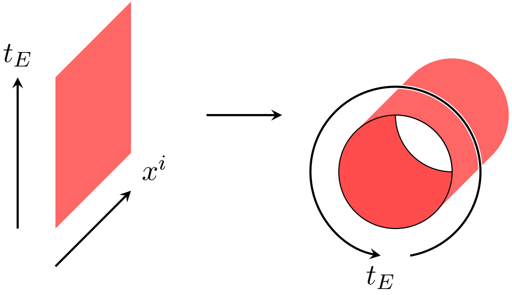
Non-extremal case

3.1. Non-extremal case

In the non-extremal case, there is a horizon where S^1 shrinks to zero.
$\rightarrow$ we can hide the subtle region behind horizon.

「臭いものには蓋をしろ」

In order to study finite temperature system, We go to Euclidian signature and compactify the Euclidian time direction on S^1.



3.1. Ansatz

Again, we assume

- spherical symmetry
- static
- flatness along the brane

$$\mathrm{d}s^2 = \mathrm{d}x_i \mathrm{d}x^i + M(r)^2 \mathrm{d}t_E^2 + \mathrm{d}r^2 + R(r)^2 \mathrm{d}\Omega_{8-p}^2 \tag{42}$$

where

$$i = 1, 2, \ldots, p, \quad t_E \sim t_E + 2\pi$$

Other fields are same as Extremal case.

Equations of motion :

$$0 = \Sigma'' - 2\left(\Phi - \frac{n}{2}\sigma - \frac{1}{2}\Sigma\right)'\Sigma' \quad \overset{\text{integrate}}{\rightarrow} \quad \Sigma' = De^{2(\Phi - \frac{n}{2}\sigma - \frac{1}{2}\Sigma)} \tag{43}$$

$$0 = \sigma'' - 2\left(\Phi - \frac{n}{2}\sigma - \frac{1}{2}\Sigma\right)'\sigma' - \frac{n-1}{l_0^2}\left(e^{-2\sigma} - e^{-4\sigma}\right) \tag{44}$$

$$0 = \Phi'' - 2\left(\Phi - \frac{n}{2}\sigma - \frac{1}{2}\Sigma\right)'\Phi' + \frac{n(n-1)}{4l_0^2}e^{-4\sigma} \tag{45}$$

- $l_0 = \sqrt{\frac{\alpha' \mathsf{C}}{n(n-1)}}, \qquad R = l_0 e^{\sigma}, \qquad M = l_0 e^{\Sigma}$
- D is a constant.

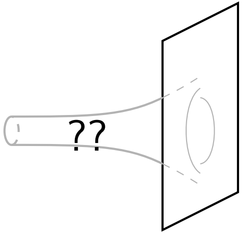
3.2. Asymptotic behavior

This system can have a flat region.

$$\Sigma' = De^{2(\Phi - \frac{n}{2}\sigma - \frac{1}{2}\Sigma)} \tag{46}$$

$$0 = \sigma'' - 2\left(\Phi - \frac{n}{2}\sigma - \frac{1}{2}\Sigma\right)'\sigma' - \frac{n-1}{l_0^2}\left(e^{-2\sigma} - \cancel{e^{-4\sigma}}\right) \tag{47}$$

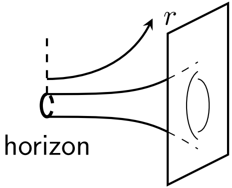
$$0 = \Phi'' - 2\left(\Phi - \frac{n}{2}\sigma - \frac{1}{2}\Sigma\right)'\Phi' + \cancel{\frac{n(n-1)}{4l_0^2}e^{-4\sigma}} \tag{48}$$



Again, we could not solve these equations analytically...

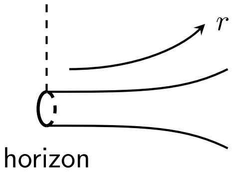
3.3. Initial conditions 1

We want to solve the EoM starting from the horizon.



However, this attempt fails. . .

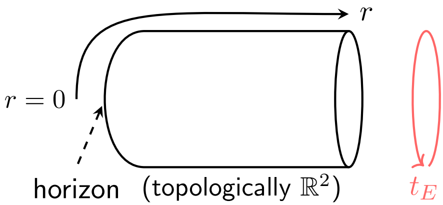
We solved the EOM starting near the horizon.



3.3. Initial conditions 2

We need an approximate solution near the horizon.

- Set $r = 0$ at horizon using r shift.
- Expand the fields in powers of r, and determine the coefficients for each order.



Metric which is smooth at the horizon must be

$$\begin{aligned} \mathrm{d}s^2 &= M^2 \mathrm{d}t_E^2 + \mathrm{d}r^2 + \cdots \\ &\simeq \underbrace{r^2 \mathrm{d}t_E^2 + \mathrm{d}r^2}_{\mathbb{R}^2 \text{ polar coordinate}} + \cdots \end{aligned} \tag{49}$$

$\Rightarrow D$ is determined.

3.3. Initial conditions 3

Results of the expansion:

$$M = r\left(1 + \frac{n(n-1)}{24}\left(\frac{2}{\tilde{R}_0^2} - \frac{1}{\tilde{R}_0^4}\right)(r/l_0)^2 + \cdots\right) \tag{50}$$

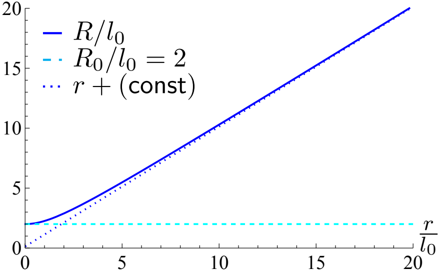
$$R = R_0\left(1 + \frac{n-1}{4}\left(\frac{1}{\tilde{R}_0^2} - \frac{1}{\tilde{R}_0^4}\right)(r/l_0)^2 + \cdots\right) \tag{51}$$

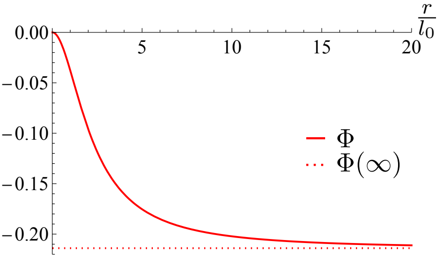
$$\Phi = \Phi_0 - \frac{n(n-1)}{16\tilde{R}_0^4}(r/l_0)^2 + \cdots \tag{52}$$

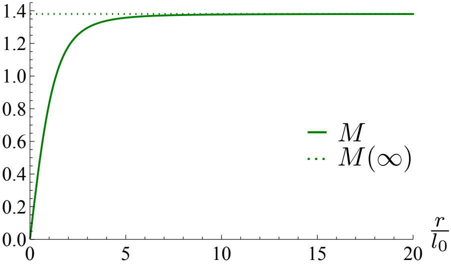
- R_0 and Φ_0 are field values at the horizon.
- $\tilde{R}_0 = R_0/l_0$

* The radius of convergence is too small to represent the entire brane.

3.4. Results







- $R_0/l_0 = 2, \Phi_0 = 0$
- These are the $p = 4$ solution.
- These are same for $p = 0$.

3.5. Check

We performed three additional calculations.

- Gauge fixing constraint
- 6-brane analytical solution
- Near extremal and near horizon limit : $R = l_0$

Near extremal and near horizon solution :

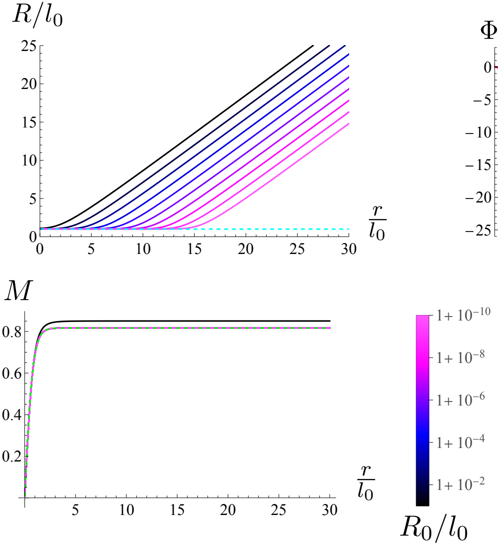
$$M/l_0 = \sqrt{\frac{8}{n(n-1)}} \tanh\left(\sqrt{\frac{n(n-1)}{8}} \frac{r}{l_0}\right) \tag{53}$$

$$e^{-2\Phi} = e^{-2\Phi_0} \cosh^2\left(\sqrt{\frac{n(n-1)}{8}} \frac{r}{l_0}\right) \tag{54}$$

$$R = l_0 \tag{55}$$

3.5. Near extremal and near horizon limit

limit of $R_0 \to l_0$



- Dahsed lines are the near extremal solution.
- As $R_0 \to l_0$, the solution approaches extremal.

4. Summary

- Purpose of our research is to construct black brane solutions for 0- and 4-brane and provide further evidence for the existence of these branes.

- We have constructed numerical solutions for extremal and non-extremal case.

- We checked consistency from some points of view.