Bridging two semiclassical confinement mechanisms: monopole and center vortex

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Based on:

PRL **133**, 171902 (2024) [arXiv:2405.12402 [hep-th]] with Yuya Tanizaki (YITP)

\+ [arXiv:2410.21392 [hep-th]] with Tatsuhiro Misumi (Kindai U.) and Yuya Tanizaki (YITP)
(special thanks to Mithat Ünsal(NCSU))

Confinement mechanisms

Two scenarios for quark confinement: monopole and center vortex

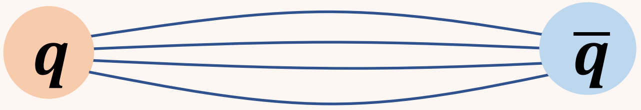
Dual superconductor picture (monopole condensation)

[Nambu ’74, ‘t Hooft ‘75, Mandelstam ‘76,…]

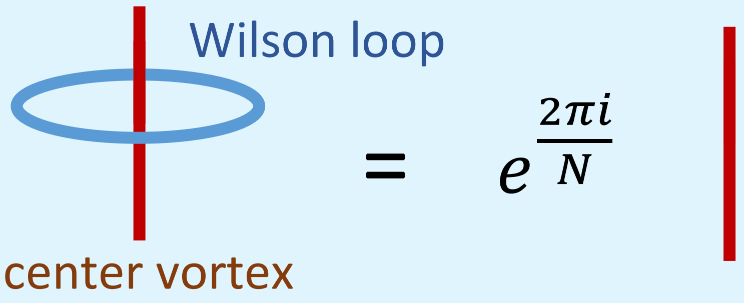
monopole condensation

⇒ dual Meissner effect

⇒ linear $q\bar{q}$-potential



Center-vortex proliferation [‘t Hooft ‘78, …]



Center vortex: rotating Wilson loop by $e^{\frac{2\pi i}{N}}$.

Proliferation ⇒ $\langle W(C)\rangle \sim e^{-\sigma\,(\mathrm{Area})}$

cf.) restoration of $\mathbb{Z}_N^{[1]}$: proliferation of co-dim-2 defects

Connection between them? [Ambjørn-Giedt-Greensite ‘99, Engelhardt-Reinhardt ‘99, Cornwall ’99, …]

“monopole as junction of center vortices”

Summary

Quark confiners: monopole and center vortex

Weak-coupling semiclassical realizations:

3d monopole semiclassics

[Ünsal ’07, Ünsal-Yaffe ’08,…]

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S^1}$ with “center-stabilizing deformation”

⇒ **confinement by 3d monopole gas**

2d center-vortex semiclassics

[Tanizaki-Ünsal ’22, …]

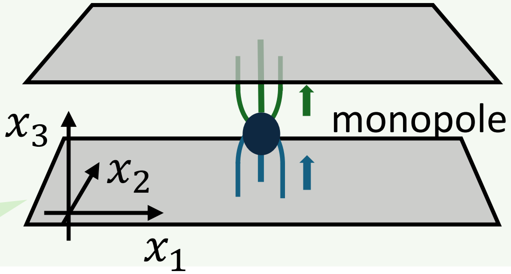
SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T^2}$ with ‘t Hooft flux

⇒ **confinement by 2d center-vortex gas**

This work: Consider an interpolating setup on $(\mathbb{R}^2 \times \boldsymbol{S^1}) \times \boldsymbol{S^1}$

Monopole in $\mathbb{R}^2 \times \boldsymbol{S^1}$

“monopole as junction of center vortices”



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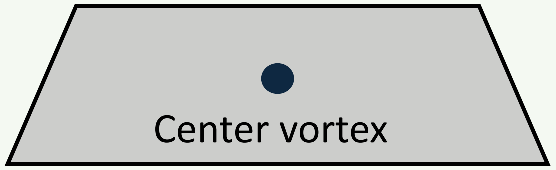
Center vortex in 2d



Outline

1. Introduction (2 slides)
2. Monopole semiclassics and center-vortex semiclassics (9 slides)
3. Monopole-vortex continuity (9 slides)
4. Summary

Semiclassical approaches to confinement

Motto: deforming SU(N) YM to **weakly-coupled** theory with **keeping confinement**.

compactification

center-stabilizing deformation
(to avoid deconfinement transition)

3d monopole semiclassics

[Ünsal ’07, Ünsal-Yaffe ’08,…]

SU(N) Yang-Mills on $\mathbb{R}^3 \times S^1$ with “center-stabilizing deformation”

⇒ **confinement by 3d monopole gas**

2d center-vortex semiclassics

[Tanizaki-Ünsal ’22, …]

SU(N) Yang-Mills on $\mathbb{R}^2 \times T^2$ with ‘t Hooft flux ($\mathbb{Z}_N^{[1]}$ background)

⇒ **confinement by 2d center-vortex gas**

Ansatz: adiabatic continuity conjecture

size of compactified S^1 or T^2

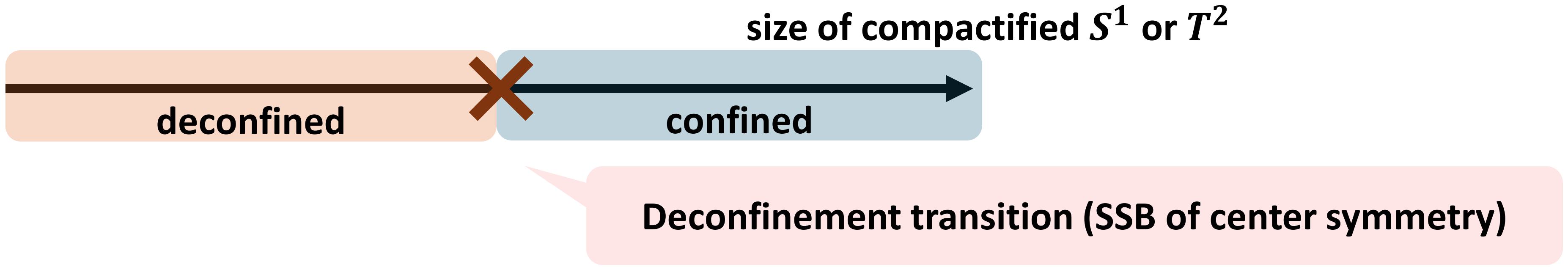
weak coupling

want to know

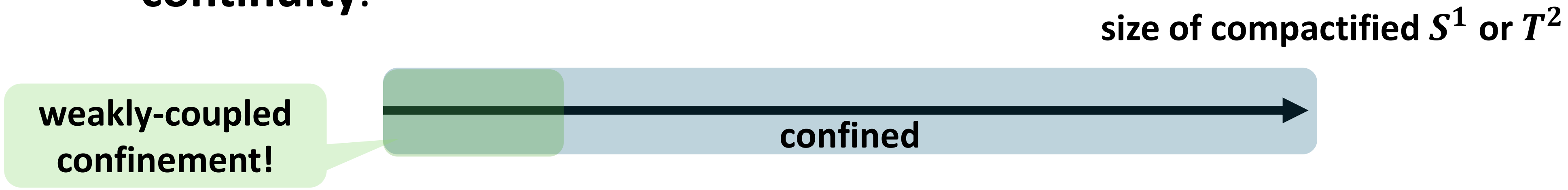
“adiabatic continuity” (confinement phase, w/o transition)

Adiabatic continuity

- With the naïve compactification, there is a deconfinement transition somewhere



- By adding "**center-stabilizing deformation**" (adding Polyakov-loop potential in 3d semiclassics; inserting 't Hooft flux in 2d semiclassics), we expect the **adiabatic continuity**.



Lattice works

- deformed YM on $\mathbb{R}^3 \times S^1$ [Bonati-Cardinali-D'Elia-Mazziotti '19] [Athenodorou-Cardinali-D'Elia '21]
- YM on $\mathbb{R}^3 \times T^2$ w/ 't Hooft twist [Bergner – González-Arroyo – Soler '25]

3d monopole semiclassics

[Ünsal ’07, Ünsal-Yaffe ’08,…] (cf. [Davies-Hollowood-Khoze-Mattis ’99,…] for SYM)

- **SU(N) Yang-Mills on $\mathbb{R}^3 \times S^1$ with “center-stabilizing deformation”** [Ünsal-Yaffe ’08]:

$$S = S_{YM} + \int d^3x \sum_{n=1}^{[N/2]} a_n \,|\mathrm{tr}\,(P^n)|^2$$

Add a potential for Polyakov loop (by hand) to keep center symmetry

⇒ Center symmetry is kept for **small S^1** (, realizing weak-coupling confinement)

- **3d effective theory on $\mathbb{R}^3$**

The Polyakov loop behaves as an adjoint scalar field.

At the center symmetric vacuum, “$\langle P \rangle \sim C$" (up to gauge)

⇒ adjoint higgsing $SU(N) \to U(1)^{N-1}$

e.g.) clock matrix for $N = 3$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$

3d effective theory = 3d $U(1)^{N-1}$ gauge theory + monopoles

- Polyakov confinement by dilute gas of monopoles (in 3d Abelian gauge theory) [Polyakov ’77]

Magnetic Debye screening ⇒ area law $\langle W(C) \rangle \sim e^{-\sigma\,(\mathrm{Area})}$

3d monopole semiclassics (some details)

"compactness of adjoint higgs"
[Kraan-van Baal '98] [Lee-Lu '98][Lee-Yi '97]

- **N kinds of monopoles**: $Q_{top} = 1/N$ fractional instantons

(N-1) BPS monopoles + KK monopole

Magnetic charge: $\vec{\alpha}_1$ $\vec{\alpha}_2$ .... $\vec{\alpha}_{N-1}$ $\vec{\alpha}_N (= -\vec{\alpha}_1 - \cdots - \vec{\alpha}_{N-1})$

$\vec{\alpha}_1, \cdots, \vec{\alpha}_{N-1}$: simple roots
$\vec{\alpha}_N$ $(= -\vec{\alpha}_1 - \cdots - \vec{\alpha}_{N-1})$: affine root

- **3d effective theory**

3d abelian duality: $U(1)^{N-1}$ gauge field→ $U(1)^{N-1}$-valued compact boson $\vec{\sigma}$ $(d\vec{\sigma} = * \vec{f})$

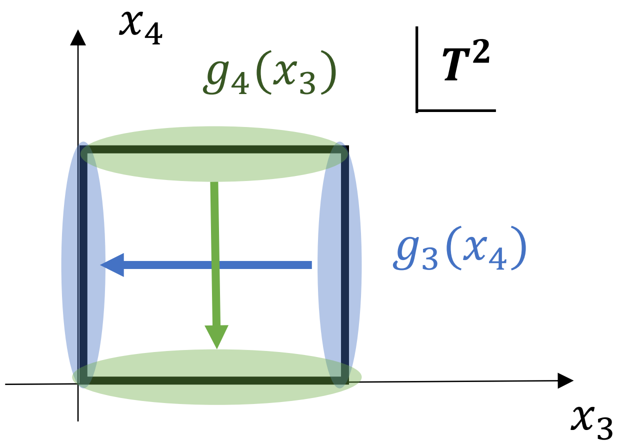
In terms of $\vec{\sigma}$ (dual photon/magnetic potential), the 3d effective theory is,

$$S = \int d^3x \left[\frac{\# g^2}{L} |d\vec{\sigma}|^2 - \# \, e^{-\frac{8\pi^2}{Ng^2}} \sum_{i=1,\cdots,N} \cos(\vec{\alpha}_i \cdot \vec{\sigma} + \theta/N) \right]$$

Monopole amplitude
$$[\mathcal{M}_i] \sim e^{-\frac{8\pi^2}{Ng^2}} \mathrm{e}^{i\vec{\alpha}_i \cdot \vec{\sigma} + i\,\theta/N}$$

2d center-vortex semiclassics

[Tanizaki-Ünsal ’22, ……] (cf. [Yamazaki-Yonekura ‘17])



Setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times T^2$ with ‘t Hooft flux

- **‘t Hooft flux for T^2 (or $\mathbb{Z}_N^{[1]}$ background)**

A unit ‘t Hooft flux ⇔ choose $g_3(0) g_4(L) g_3^\dagger(L) g_4^\dagger(0) = e^{\frac{2\pi \mathrm{i}}{N}}$.

$(g_3(x_4), g_4(x_3)$: transition functions on T^2)

Up to gauge, we can take $g_3 = S,\ g_4 = C$ (shift and clock matrices of $SU(N)$).

$$\begin{cases} a(\vec{x}, x_3 + L, x_4) = g_3^\dagger a g_3 - \mathrm{i}\, g_3^\dagger \mathrm{d} g_3 \\ a(\vec{x}, x_3, x_4 + L) = g_4^\dagger a g_4 - \mathrm{i}\, g_4^\dagger \mathrm{d} g_4 \end{cases}$$

e.g.) $N = 3$

$$S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},\ C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$

- **Consequences from ‘t Hooft-twisted compactification**
  - ✓**Center symmetry is kept at small T^2**

    Classically, $P_3 = S$ and $P_4 = C \Rightarrow \langle \mathrm{tr}\, P_3 \rangle = \langle \mathrm{tr}\, P_4 \rangle = 0$.
  - ✓**Perturbatively gapped gluons: O(1/NL) KK mass**
  - ✓**Numerical evidence for center vortex/fractional instantons** (as a local solution)

    [Gonzalez-Arroyo–Montero ’98, Montero ’99, ……].

    .

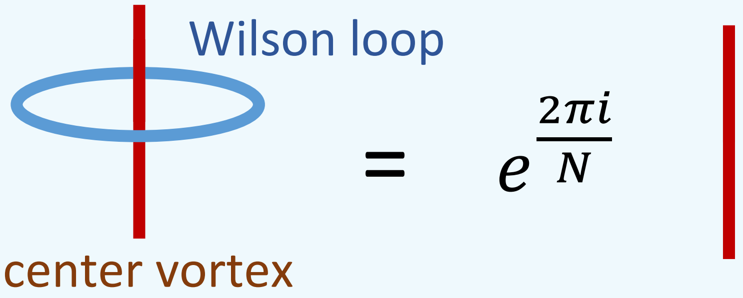
exists locally,
(not globally if `regularity’ at infinity is imposed)

Remark: center vortex?

Center vortex (in general context)

[‘t Hooft ‘78, …]

Co-dim-2 object carrying “magnetic flux of center element”:



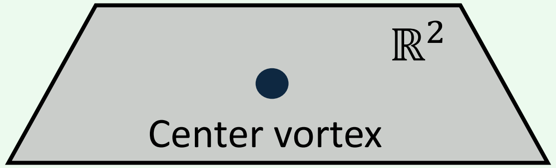
$$= e^{\frac{2\pi i}{N}}$$

(expected to play an important role in quark confinement)

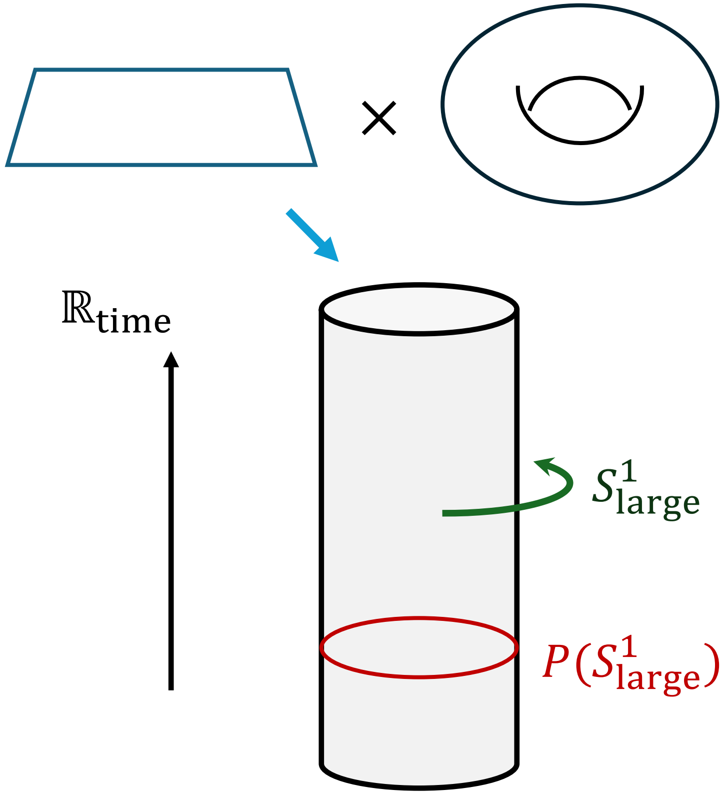
Center vortex (we consider here)

+ 1/N fractional instanton satisfying the BPS bound

$$S = \frac{8\pi^2}{Ng^2}, \qquad Q_{top} = 1/N$$



Fractional instanton (のきもち)



- <u>Reduction to QM</u> (cf. [Yamazaki-Yonekura '17])

Further spatial compactification:

$\mathbb{R}^2 \times T^2 \to \mathbb{R}_{\rm time} \times S^1_{\rm large} \times T^2$

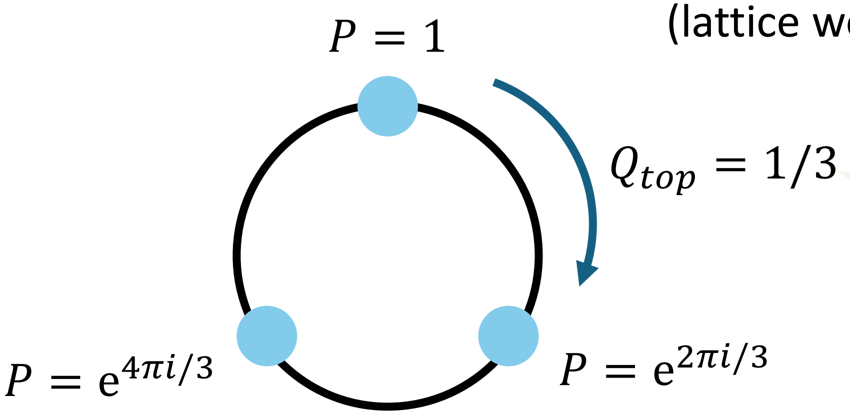
with $\mathrm{Size}\big(T^2\big) \ll \mathrm{Size}\big(S^1_{\rm large}\big) \ll \Lambda^{-1}$

2d EFT: $(SU(N) \to)\ \mathbb{Z}_N$ gauge

$\Rightarrow$ **N classical vacua** with $P(S^1_{\rm large}) = \mathrm{e}^{2\pi k i/N}\mathbf{1}$ $(k = 0, \cdots, N-1)$

- "Fractional instanton = tunneling event"

(lattice work: [Garcia Perez–Gonzalez-Arroyo–Soderberg '90; Itou '18])



- does not globally exist if the periodic BC is imposed
- can exist globally under the twisted BC

2d center-vortex semiclassics [Tanizaki-Ünsal '22]

- **Dilute gas of center vortices**

The center-vortex and anti-center-vortex vertices are:

$$[\mathcal{V}] = Ke^{-\frac{8\pi^2}{Ng^2}+i\frac{\theta}{N}}, \qquad [\bar{\mathcal{V}}] = Ke^{-\frac{8\pi^2}{Ng^2}-i\,\theta/N}$$

with a dimensionful constant K.

For calculating partition function, we compactify $\mathbb{R}^2$ without 't Hooft flux.
⇒ total topological charge is constrained $Q_{top} \in \mathbb{Z}$

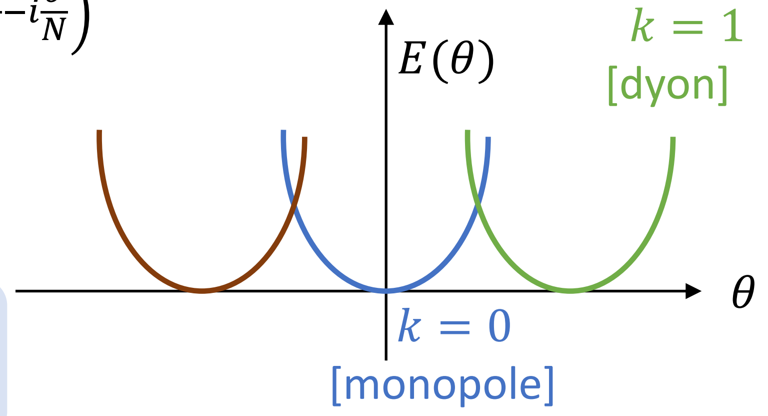
Then, the dilute gas approximation yields, (only configurations with $Q_{top} \in \mathbb{Z}$ are admitted)

$$Z_{2d} = \sum_{n,\bar{n}\geq 0} \frac{1}{n!\,\bar{n}!} \delta_{n-\bar{n}\in N\mathbb{Z}} \left(VKe^{-\frac{8\pi^2}{Ng^2}+i\frac{\theta}{N}}\right)^{n} \left(VKe^{-\frac{8\pi^2}{Ng^2}-i\frac{\theta}{N}}\right)^{\bar{n}}$$

$$= \sum_{k\in\mathbb{Z}_N} \exp\left[-V\left(-2Ke^{-\frac{8\pi^2}{Ng^2}}\cos\left(\frac{\theta-2\pi k}{N}\right)\right)\right]$$

N semiclassical vacua

Energy density of k-th vacuum
→multibranch structure!



✅ One can also derive area-law falloff of the Wilson loop from the dilute gas of center vortices.

Summary of Backgrounds / Question

Motto: deforming SU(N) YM to **weakly-coupled** one with **keeping confinement**.

<u>**3d monopole semiclassics**</u>
[Ünsal ’07, Ünsal-Yaffe ’08,…]
SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S}^1$ with "center-stabilizing deformation"
⇒ **3d** $U(1)^{N-1}$ **gauge theory**
+ monopole gas

⟷

<u>**2d center vortex semiclassics**</u>
[Tanizaki-Ünsal ’22, …]
SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T}^2$ with ‘t Hooft flux
⇒ **confinement by 2d center-vortex gas**

Question: Relation between them?
How monopole transmutes to center vortex?

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Interpolating setup [YH, Tanizaki '24] (cf. [Güvendik-Schäfer-Ünsal '24])

Interpolating setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times \overbrace{(\boldsymbol{S^1})_3 \times \underbrace{(\boldsymbol{S^1})_4}_{\text{center-stabilizing deformation}}}^{\text{'t Hooft flux}}$

(L_4: always small)

$L_3 \to \infty$

<u>3d monopole semiclassics</u>

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S^1}$ with center-stabilizing deformation

$L_3 \to L_4$

<u>2d center vortex semiclassics</u>

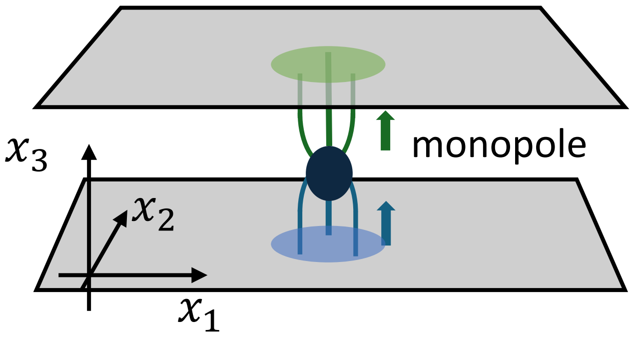
SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T^2}$ with 't Hooft flux

What we will see:

setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times \overbrace{(\boldsymbol{S^1})_3 \times \underbrace{(\boldsymbol{S^1})_4}_{\textbf{center-stabilizing deformation}}}^{\textbf{'t Hooft flux}}$

1. 3d effective theory on $\mathbb{R}^2 \times (S^1)_3 \Rightarrow$ 2d center-vortex gas on $\mathbb{R}^2$

2. BPS/KK monopole in $\mathbb{R}^2 \times (S^1)_3$ (3d monopole-instanton)
$\Rightarrow$ center vortex on $\mathbb{R}^2$ (2d center-vortex-instanton)



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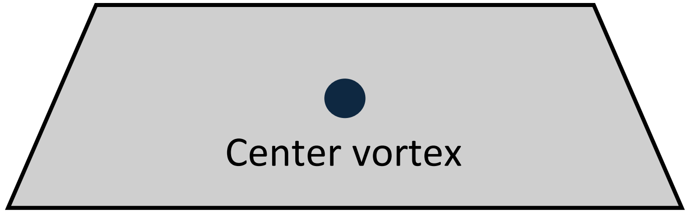


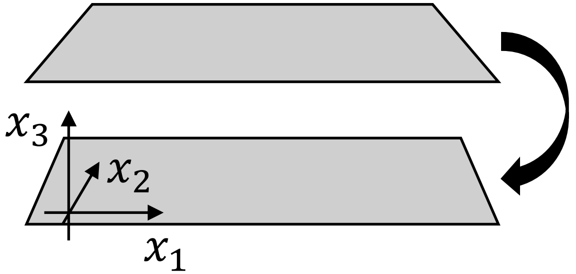
3d effective theory on $\mathbb{R}^2 \times \left(\boldsymbol{S^1}\right)_{\boldsymbol{3}}$

Interpolating setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times \overbrace{\left(\boldsymbol{S^1}\right)_{\boldsymbol{3}} \times \underbrace{\left(\boldsymbol{S^1}\right)_{\boldsymbol{4}}}_{\text{center-stabilizing deformation}}}^{\text{'t Hooft flux}}$

($\boldsymbol{L_4}$**: always small)**

small $\boldsymbol{L_4}$**, adjoint higgsing by** $\boldsymbol{P_4}$

3d $U(1)^{N-1}$ **gauge theory + monopoles on** $\mathbb{R}^2 \times \left(\boldsymbol{S^1}\right)_{\boldsymbol{3}}$
with "shift-twisted" boundary conditions



$\vec{\sigma}\,(\boldsymbol{x}, x_3 + L_3) = S^{-1}\vec{\sigma}(\boldsymbol{x}, x_3)$

In the gauge $\langle P_4 \rangle = C$ (clock matrix), the transition function for $\left(\boldsymbol{S^1}\right)_{\boldsymbol{3}}$ is $g_3 = S$ (shift matrix).

or, 't Hooft flux ⇒ $\left(\mathbb{Z}_N^{[0]}\right)_{3\mathrm{d}}$-twisted boundary condition ($\sim$ **Weyl permutation** for dual photon $\vec{\sigma}(\boldsymbol{x}, x_3)$)

Example: SU(2) case

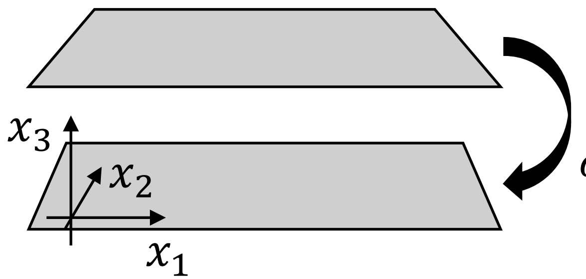
- Adjoint higgsing by $P_4(\propto \sigma_3$, up to gauge$)$: $SU(2) \to U(1)$ $\Rightarrow$ one compact scalar $\sigma \sim \sigma + 2\pi$

$$S_{3d}[\sigma] = \int d^3x \left[\frac{\# g^2}{L_4} |d\sigma|^2 - \#\, e^{-\frac{8\pi^2}{Ng^2}} \left(\cos\left(\sigma + \frac{\theta}{2}\right) + \cos\left(-\sigma + \frac{\theta}{2}\right) \right) \right]$$

- the transition function for $\left(\boldsymbol{S^1}\right)_{\mathbf{3}}$ is $g_3 \propto \sigma_1$(shift matrix):

$\Rightarrow$ flipping the basis $P_4 \mapsto -P_4$, equivalent to $\sigma\,(\boldsymbol{x}, x_3 + L_3) = -\sigma\,(\boldsymbol{x}, x_3)$

3d $U(1)$ gauge theory + monopoles on $\mathbb{R}^2 \times \left(\boldsymbol{S^1}\right)_{\mathbf{3}}$
with "shift-twisted" boundary conditions



$\boldsymbol{L_3 \ll \Lambda^{-1}}$**: restricted to "zeromode":** $\boldsymbol{\sigma = -\sigma}$

$\Rightarrow$ **2 vacua:** $\sigma = 0, \pi$

identical to extrema of the monopole potential
(3d-2d adiabatic continuity)

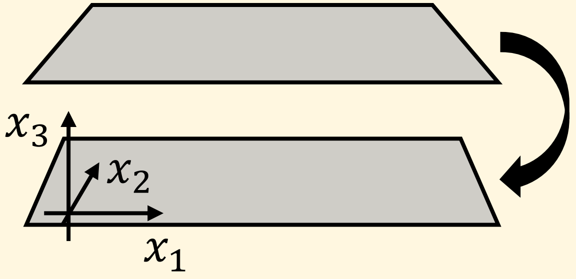
2d center-vortex gas

$$Z_{\mathbb{R}^2 \times \left(\boldsymbol{S^1}\right)_{\mathbf{3}}} \approx \sum_{k \in \mathbb{Z}_2} \mathrm{e}^{\#\, V_{2d} \mathrm{e}^{-\frac{8\pi^2}{2g^2}} \cos\left(\frac{\theta + 2\pi k}{2}\right)} = Z_{\text{2d vortex gas}}$$

From 3d monopole gas to 2d center-vortex gas

3d $U(1)^{N-1}$ gauge theory + monopoles on $\mathbb{R}^2 \times \left(\boldsymbol{S}^1\right)_3$
with "shift-twisted" boundary conditions

$$S_{3d}[\vec{\sigma}] = \int d^3x \left[\frac{\# g^2}{L_4} |d\vec{\sigma}|^2 - \#\, e^{-\frac{8\pi^2}{Ng^2}} \sum_{i=1,\cdots,N} \cos(\vec{\alpha}_i \cdot \vec{\sigma} + \theta/N) \right]$$



$\vec{\sigma}\,(\boldsymbol{x}, x_3 + L_3)$
$= S^{-1}\vec{\sigma}(\boldsymbol{x}, x_3)$

$L_3 \ll \Lambda^{-1}$: restricted to $\vec{\sigma} \; = S^{-1}\vec{\sigma}$

N vacua: $\vec{\sigma} = \vec{\sigma}_k = \frac{2\pi k}{N}(\vec{\mu}_1 + \cdots + \vec{\mu}_{N-1})$

$(k = 0, \cdots, N-1)$

identical to extrema of the monopole potential (**3d-2d adiabatic continuity**)

2d center-vortex gas

$$Z_{\mathbb{R}^2 \times (S^1)_3} = \int_{\substack{\vec{\sigma}\,(\boldsymbol{x}, x_3+L_3) \\ = S^{-1}\vec{\sigma}(\boldsymbol{x}, x_3)}} \mathcal{D}\vec{\sigma}\; \mathrm{e}^{-S_{3d}[\vec{\sigma}]} \approx \sum_{\substack{\vec{\sigma}=\vec{\sigma}_k \\ k\in\mathbb{Z}_N}} \mathrm{e}^{-S_{3d}[\vec{\sigma}]} = \sum_{k\in\mathbb{Z}_N} \mathrm{e}^{\# V_{2d} \mathrm{e}^{-\frac{8\pi^2}{Ng^2}} \cos\left(\frac{\theta + 2\pi k}{N}\right)} = Z_{2d}$$

How monopole looks like in $\mathbb{R}^2 \times (S^1)_3$

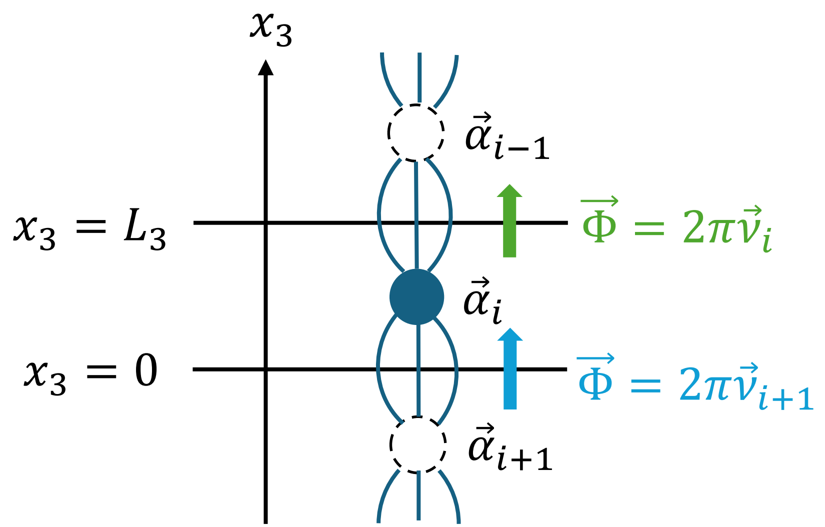
- BPS/KK monopole in 3d effective theory:

magnetic charge $\vec{\alpha}_i \Rightarrow \nabla^2 \vec{\sigma} \sim 2\pi\vec{\alpha}_i\, \delta^{(3)}(x - x_*)$

boundary condition: $\vec{\sigma}\,(\boldsymbol{x}, x_3 + L_3) = S^{-1}\vec{\sigma}(\boldsymbol{x}, x_3)$

$\Rightarrow$ "mirror image": infinite chain of BPS/KK monopoles

$$\vec{\sigma} \sim \text{"}\sum_{n\in\mathbb{Z}} \frac{\vec{\alpha}_{i-n\ (\mathrm{mod\ N})}}{|x - x_* - nL_3\, \hat{x}_3\,|}\text{"}$$



- A proper expression (with good convergence):

$$\vec{\sigma} \sim \sum_{k\in\mathbb{Z}} \left[\sum_{\ell\in\mathbb{Z}_N} \vec{v}_{i-\ell\ (\mathrm{mod\ N})} \left\{ \frac{1}{|x - x_* - (Nk+\ell)L_3\, \hat{x}_3|} - \frac{1}{|x - x_* - (Nk+\ell+1)L_3\, \hat{x}_3|} \right\} \right]$$

$\vec{v}_i$: weight vector of defining representation

$\vec{\alpha}_i = \vec{v}_i - \vec{v}_{i+1}$

outgoing magnetic flux

$\vec{\Phi} = 2\pi\vec{v}_i$

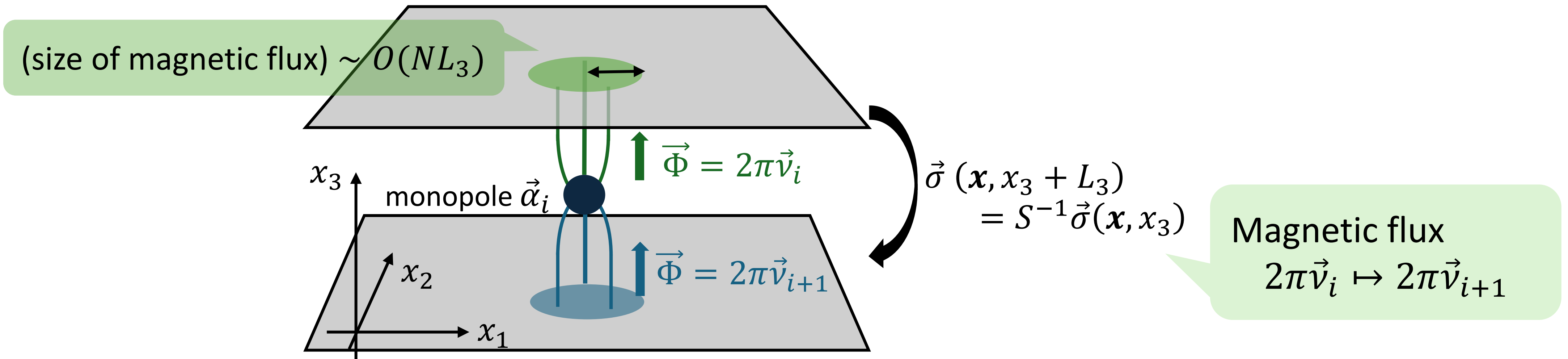
incoming magnetic flux

$\vec{\Phi} = 2\pi\vec{v}_{i+1}$

How monopole looks like in $\mathbb{R}^2 \times (S^1)_3$

This solution explains:

The $\vec{\alpha}_i$-monopole emits magnetic flux $2\pi\vec{\alpha}_i = 2\pi\vec{\nu}_i - 2\pi\vec{\nu}_{i+1}$



Suppose that the outgoing magnetic flux $2\pi\vec{\nu}_i$ goes upward

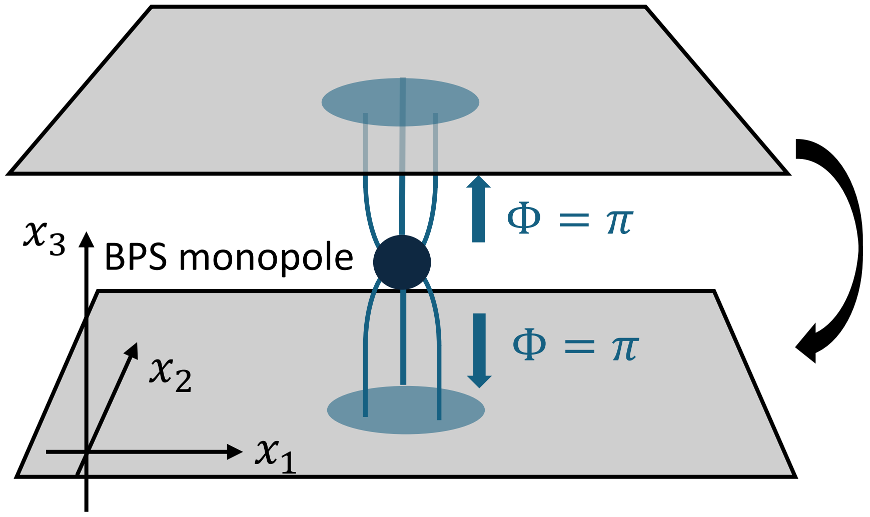
⇒ Shift-twisted boundary condition: $2\pi\vec{\nu}_i \mapsto 2\pi\vec{\nu}_{i+1}$ ⇒ The incoming flux $2\pi\vec{\nu}_{i+1}$ comes from bottom

The magnetic flux is localized in 2d ($\sim O(NL_3)$) .

N Species of monopole (BPS/KK) can be included in extended moduli $x_3 \in [0, NL_3)$.

Example: SU(2) case

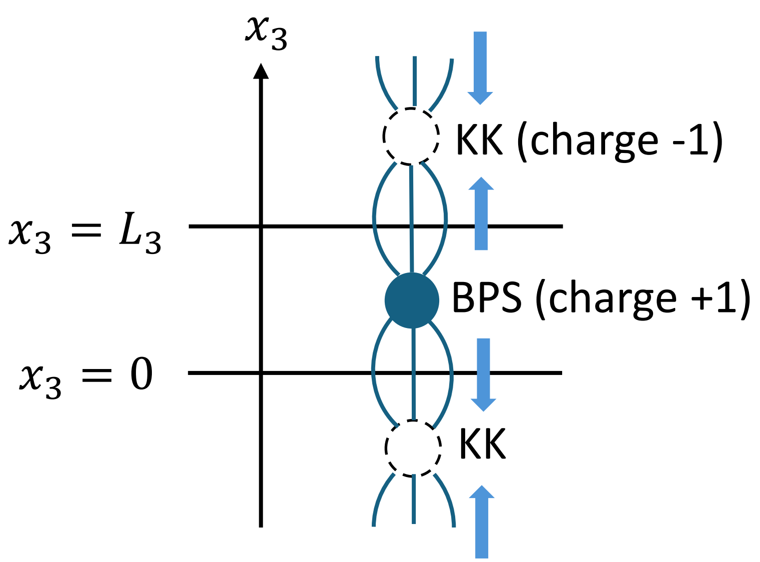
- One compact scalar $\sigma \sim \sigma + 2\pi$
- BPS monopole: magnetic charge +1, KK monopole: magnetic charge -1
- boundary condition: $\sigma\,(\boldsymbol{x}, x_3 + L_3) = -\sigma(\boldsymbol{x}, x_3)$



charge-conjugation

$$\sigma\,(\boldsymbol{x}, x_3 + L_3) = -\sigma\,(\boldsymbol{x}, x_3)$$

<u>“mirror image” solution:</u>



“Flux Fractionalization”:

1/N fractional magnetic flux, rotating the Wilson loop by a center element (-1)

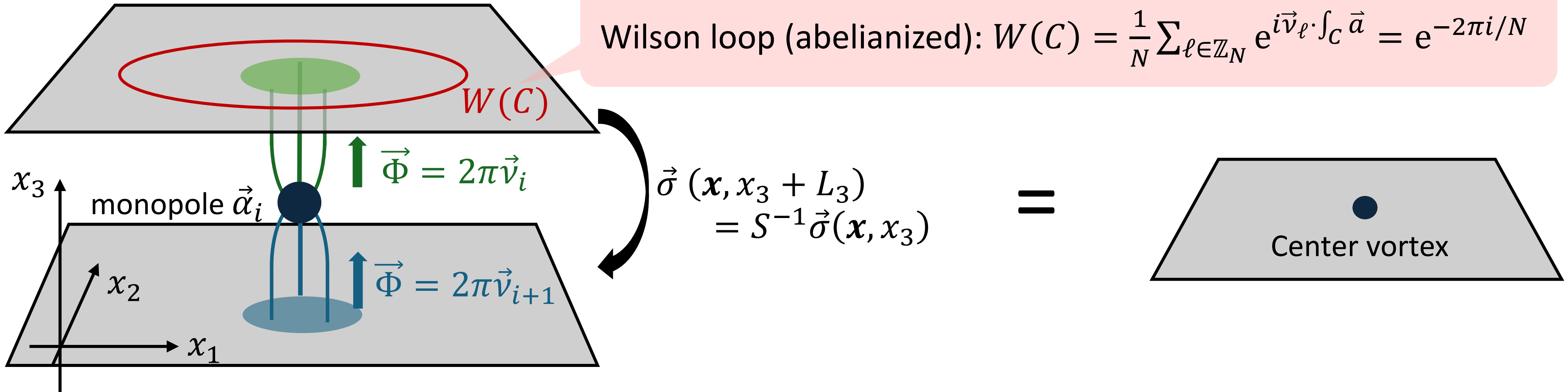
3d BPS/KK monopoles become 2d center vortex

- The magnetic flux (of size $O(NL_3)$) is indeed center vortex:
  Wilson loop acquires $\mathrm{e}^{-2\pi i/N}$ phase.
- **3d BPS/KK monopole-instanton = 2d center-vortex-instanton**:

The 3d/2d semiclassical confinement mechanisms are essentially same!

- **“monopole as junction of center vortex” (realizing the old expectation!)**

Wilson loop (abelianized): $W(C) = \frac{1}{N}\sum_{\ell\in\mathbb{Z}_N} \mathrm{e}^{i\vec{\nu}_\ell\cdot\int_C \vec{a}} = \mathrm{e}^{-2\pi i/N}$



Outline

1. Introduction (2 slides)
2. Monopole semiclassics and center-vortex semiclassics (9 slides)
3. Monopole-vortex continuity (9 slides)
4. Summary

Summary

Quark confiners: monopole and center vortex

Weak-coupling semiclassical realizations:

3d monopole semiclassics

[Ünsal ’07, Ünsal-Yaffe ’08,…]

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S}^1$ with “center-stabilizing deformation”

⇒ **confinement by 3d monopole gas**

2d center vortex semiclassics

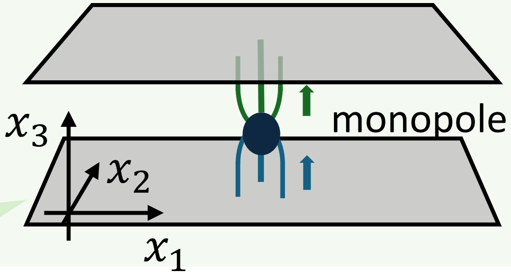
[Tanizaki-Ünsal ’22, …]

SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T}^2$ with ‘t Hooft flux

⇒ **confinement by 2d center-vortex gas**

This work: Consider an interpolating setup on $(\mathbb{R}^2 \times \boldsymbol{S}^1) \times \boldsymbol{S}^1$

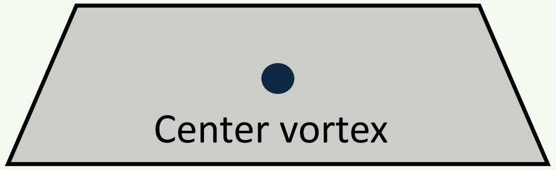
Monopole in $\mathbb{R}^2 \times \boldsymbol{S}^1$



“monopole as junction of center vortices”

=

Center vortex in 2d



Further developments / future directions

- Interplay between 3d/2d semiclassics
  - $\mathcal{N} = 1$ SYM, QCD(adj) [YH-Misumi-Tanizaki '24]:
    3d semiclassics is well developed, but 2d semiclassics was somewhat puzzling
    Perimeter-law in 2d ($\Leftarrow$ 3d double string picture), fate of bion mechanism…
  - QCD(F) (2d semiclassics unexpectedly works well, why?)
  - Other gauge groups…
  - …
- Monopole-vortex complex as soliton (in Higgs phase)
  In Higgs phase, monopole is confined in vortex [in progress, with Misumi-Nitta-Ohashi-Tanizaki]
- Center-vortex semiclassics with a non-minimal twist
  What if 't Hooft twist is not minimal? (related to large-N stuff (center stability, twisted Eguchi-Kawai…)) [in preparation, with Tanizaki-Ünsal]

おまけ: 3d/2d continuity for SYM

[YH, Misumi, Tanizaki ‘24]

$\mathcal{N} = 1$ super-Yang-Mills theory

$\mathcal{N} = 1$ SU(N) SYM $=$ **One-flavor massless adjoint QCD**

Field contents: SU(N) gluon a_μ+ adjoint Weyl fermion λ (gluino)

- **Why $\mathcal{N} = 1$ SYM ?:**

Some quantities (e.g., Witten index/partition function) are exactly kept under the spatial compactification (without deformation):

adiabatic continuity is "exact"

⇒ nice playground for examining semiclassics!

spatial compactification:
periodic boundary condition for fermion

IR scenario

$U(1)_{\text{chiral}} : \lambda \to e^{i\alpha}\lambda, \quad \bar{\lambda} \to e^{-i\alpha}\bar{\lambda}$
Only $(\mathbb{Z}_{2N})_{\text{chiral}}$ is non-anomalous.

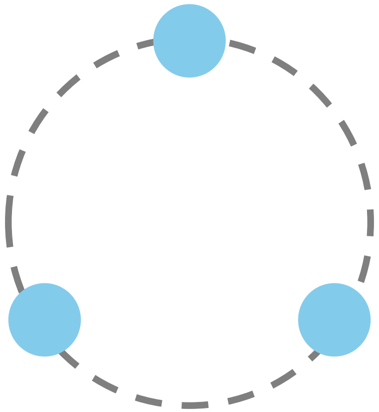
- **Symmetry:** $(\mathbb{Z}_{2N})_{\text{chiral}} \times \mathbb{Z}_N^{[1]}$

There is mixed anomaly.

- assume confinement ($\mathbb{Z}_N^{[1]}$ is unbroken)
  ⇒ **(Natural) IR scenario:** $(\mathbb{Z}_{2N})_{\textbf{chiral}} \to \mathbb{Z}_2$ **SSB**

N vacua $\{\ |k\rangle\ \}_{k=0,\cdots,N-1}$ with chiral condensate $\langle k|\text{tr}(\lambda\lambda)|k\rangle \sim e^{2\pi i k/N}$

consistent with Witten index [Witten'82]



Outline of this talk

- Two semiclassics:
  - SYM on $\mathbb{R}^3 \times \boldsymbol{S^1}$ with periodic BC [Davies-Hollowood-Khoze-Mattis '99,…][Ünsal '07]

  At small S^1, the vacuum structure can be understood through monopole/bion gas
  - SYM on $\mathbb{R}^2 \times \boldsymbol{T^2}$ with 't Hooft flux [Tanizaki-Ünsal '22]

  The vacuum structure is partially understood through center-vortex gas, but **some puzzling issues remain**.
- Aim of this talk: **understand/resolve (one of) these issues by the 3d/2d continuity.**

SYM on $\mathbb{R}^3 \times \boldsymbol{S^1}$ (with periodic BC)
⇒ 3d $U(1)^{N-1}$ gauge theory + monopoles

➡

SYM on $\mathbb{R}^2 \times \boldsymbol{T^2}$ with 't Hooft flux

SU(N) SYM on $\mathbb{R}^2 \times (S^1)_3 \times (S^1)_4$ with 't Hooft flux (small L_4, varying L_3)
⇒ 3d EFT on $\mathbb{R}^2 \times (S^1)_3$ with $\left(\mathbb{Z}_N^{[0]}\right)_{3d}$-twisted boundary condition.

3d monopole-bion semiclassics

[Davies-Hollowood-Khoze-Mattis ’99,… for SYM] [Ünsal ’07, … for QCD(adj)]

- SYM on $\mathbb{R}^3 \times \boldsymbol{S}^1$ (with periodic BC)
  3d EFT: two compact scalars (holonomy $\vec{\phi}$, dual photon $\vec{\sigma}$) & Cartan fermion $\vec{\lambda}$

SYM on $\mathbb{R}^3 \times \boldsymbol{S}^1$ $\Rightarrow$ 3d gauge field + fermion + holonomy (adjoint scalar)
(adjoint higgsing: $SU(N) \to U(1)^{N-1}$ & 3d abelian duality)
$\Rightarrow$ (holonomy $\vec{\phi}$, dual photon $\vec{\sigma}$) + Cartan fermion $\vec{\lambda}$ **(+ BPS/KK monopole-instantons)**

Monopole amplitude

$$[\mathcal{M}_i] \sim e^{-\frac{8\pi^2}{Ng^2}} \, \mathrm{e}^{i\vec{\alpha}_i\cdot\vec{z}+i\frac{\theta}{N}} \left(\vec{\alpha}_i \cdot \vec{\lambda}\right)^2$$

- **BPS/KK Monopole carries two fermionic zeromodes**

$\Rightarrow$ leading to gluino condensate $\langle \mathrm{tr}(\lambda\lambda)\rangle \neq 0$; but no bosonic potential

- **Bion (monopole-antimonopole pair) induces bosonic potential (for $(\vec{\phi}, \vec{\sigma})$)**

In terms of superpotential, the bion-induced potential is automatically included

2d center-vortex semiclassics / Questions

- One can feel "N vacua" in 2d semiclassics (by considering $\mathbb{R} \times S^1 \times T^2$ setup [Tanizaki-Unsal '22])
- **A few unsatisfactory points:**
  - Everything becomes heavy due to 't Hooft-twisted compactification, so the 2d dilute-gas effective theory is not straightforward.

  Since the center-vortex carries two zeromodes, we'd like to write $[\mathcal{V}] \sim e^{-\frac{8\pi^2}{Ng^2}+i\frac{\theta}{N}}(\lambda\lambda)$ But λ is heavy…

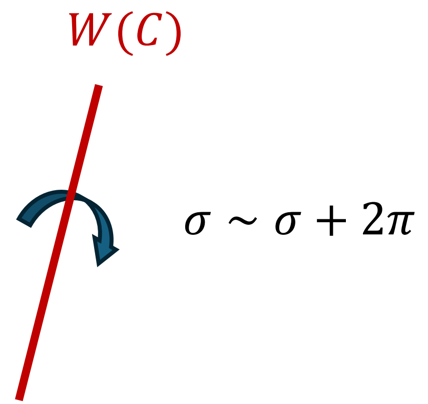
  - **Perimeter law of 2d Wilson loop? deconfinement? (counter-intuitive…)** [(From mixed anomaly,) Wilson loop should behave as a domain wall of discrete chiral symmetry in the 2d semiclassics.]

this talk

  - Role of magnetic bion in 2d? (magnetic bion causes confinement in 3d)

Let's observe the reduction from 3d to 2d!

Wilson loop in 3d semiclassics



- Let us consider SU(2) $\mathcal{N} = 1$ SYM, for simplicity:

The dual photon is a compact boson $\sigma \sim \sigma + 2\pi$

- **Wilson loop: a defect operator with nontrivial monodoromy $\boldsymbol{\sigma \sim \sigma + 2\pi}$**

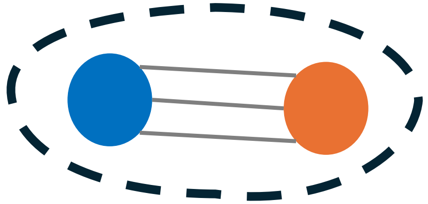
($\because$ Electromagnetic duality: 3d gauge field $\leftrightarrow$ compact scalar $\sigma \sim \sigma + 2\pi$)

- Bions give the bosonic potential:

Magnetic bion induces a potential $\sim \cos(2\sigma)$: **two minima**

Magnetic bion carries **no fermionic zeromodes**

but has **magnetic charge 2**

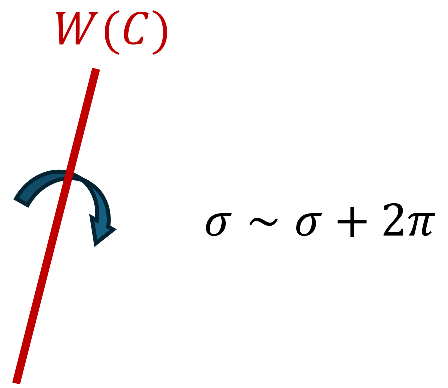


Magnetic bion
(molecule of BPS-$\overline{\mathrm{KK}}$ monopole)

- **Double string picture**:

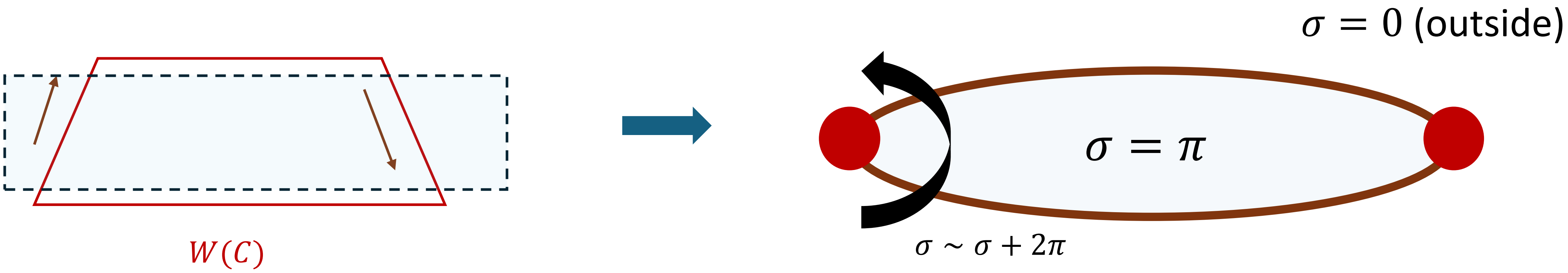
The Wilson loop (defect) emits two kinks ($\Delta\sigma = \pi$) [Anber-Poppitz-Sulejmanpasic '15]

Wilson loop in 3d semiclassics



- Let us consider SU(2) $\mathcal{N} = 1$ SYM, for simplicity.
- **Wilson loop: a defect operator with nontrivial monodoromy $\boldsymbol{\sigma \sim \sigma + 2\pi}$**
- **Magnetic bion potential:** $\sim \cos(2\sigma)$
- **Double string picture**:

The Wilson loop emits two kinks [Anber-Poppitz-Sulejmanpasic '15]



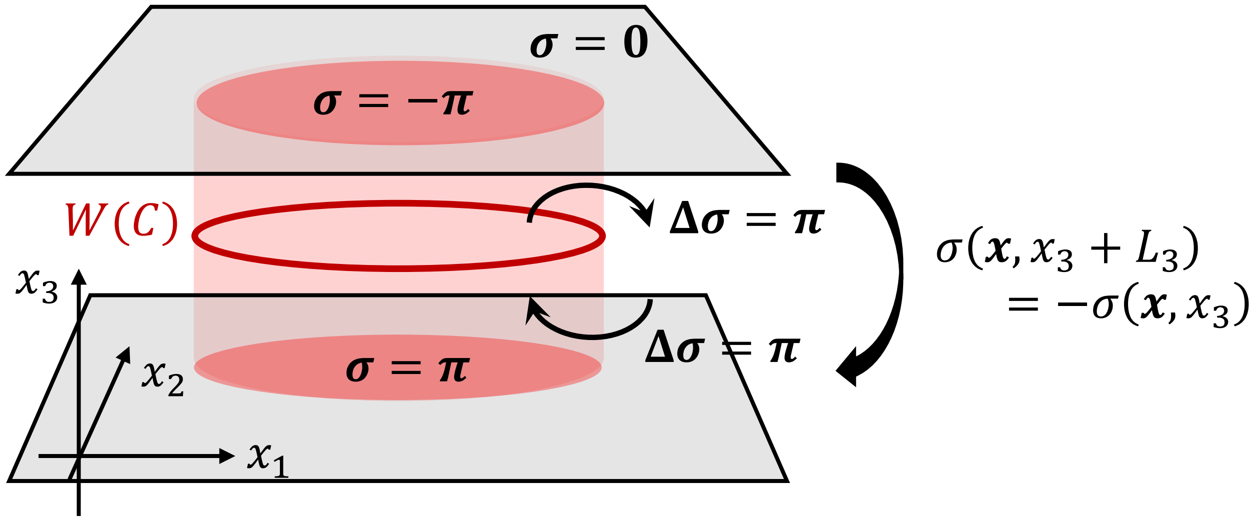
confining string = pair of two kinks

Wilson loop transmutes to domain wall

- Now, we look at the reduction from 3d semiclassics to 2d semiclassics:
  SU(2) $\mathcal{N}=1$ SYM on $\mathbb{R}^2\times(\boldsymbol{S}^1)_3\times(\boldsymbol{S}^1)_4$ (with small L_4, large-but-finite L_3).
  $\Rightarrow$ 3d EFT on $\mathbb{R}^2\times(\boldsymbol{S}^1)_3$ with $\left(\mathbb{Z}_N^{[0]}\right)_{3\mathrm{d}}$-twisted boundary condition
  (= "charge-conjugation-twisted" BC for $N=2$)
- Consider a large Wilson loop $|C|\gg L_3$:

This is domain wall of $(\mathbb{Z}_{2N})_{\mathbf{chiral}}$!

(Area) = L_3 × (Perimeter)



Confining string extends along the compactified direction

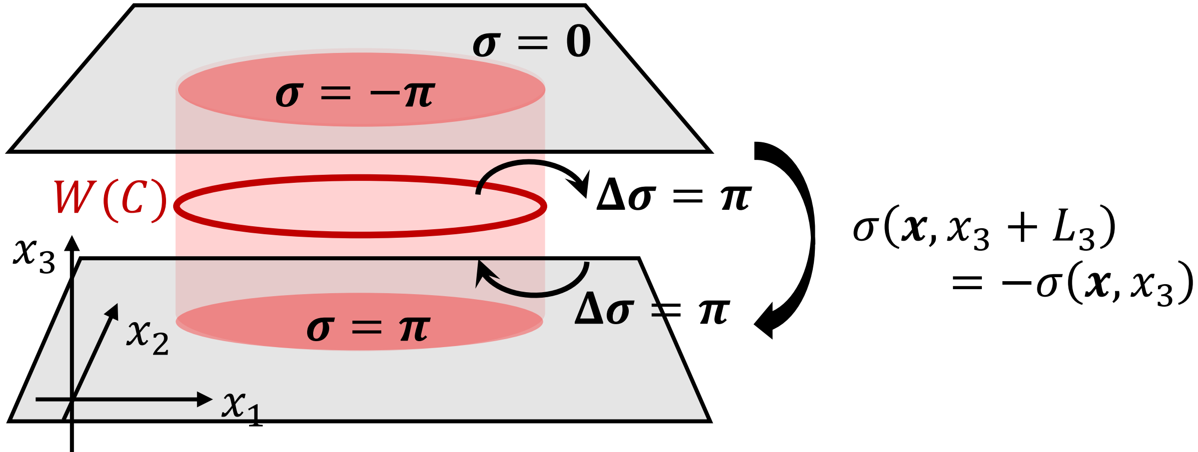
(Technical) Summary

- One of unclear points of 2d center-vortex semiclassics in $\mathcal{N}=1$ SYM:

 2d Wilson loop follows the perimeter law. deconfinement? What happens?

- The 3d-2d continuity gives an explanation:
  The 3d double-string picture explains that **the Wilson loop becomes** $(\mathbb{Z}_{2N})_{\text{chiral}}$ **domain wall** in the 2d perspective.

3d area law / 2d perimeter law
(Area) = L_3 × (Perimeter)



Generalization to SU(N)/ QCD(adj) is easy