Worldsheet CFT_2 and Celestial CFT_2: An AdS_3/ CFT_2 perspective

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Introduction

Celestial conformal field theory

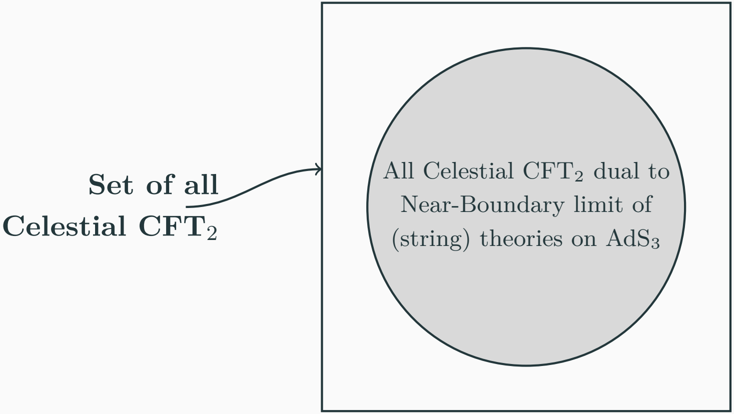
- Celestial holography: Quantum gravity in $(d+2)$ dimensional flat spacetime is dual to CFT on d-dimensional celestial sphere at the boundary null infinity.
- Symmetry group of Celestial CFT_d : Poincaré group $\text{ISO}(d+1,1)$ includes conformal group $\text{SO}(d+1,1)$ with translations realised as internal symmetry.
- Correlation functions compute the scattering amplitudes in $(d+2)$ flat space.
- Translations lead to distributional nature in correlation functions.
- Recent works consider CCFT_d with only Lorentz invariance $(\text{SO}(d+1,1))$. [Melton, Sharma, Strominger]

Celestial conformal field theory

- In these CCFT_d, correlation functions have the standard CFT form $\cdots$ Reconstruct Poincaré invariant S-matrix elements from these correlation functions.
- Examples include Liouville theory coupled to conformal matter fields $\cdots$ Celestial amplitudes for gluons and gravitons are holographically derived from correlation functions of Liouville vertex operators. [Melton-Sharma-Strominger-Wang, Stieberger-Taylor-Zhu, Donnay-Giribet-Valsesia]
- For this talk, Celestial CFT_d is any QFT with $\mathrm{ISO}(d+1,1)$ invariance on which $\mathrm{SO}(d+1,1)$ act as conformal group.

In this talk

- A method to construct Celestial CFT$_d$ from AdS$_{d+1}$/CFT$_d$ duality $\cdots$ Scaling limit where we zoom in the boundary of AdS.



Outline

- Introduction
- Zooming near the boundary of (EA)dS_3
- Our Proposal
- Near Boundary limit of CFT two-point function
- Near boundary limit of massive field theory on EAdS_3
- Near boundary limit of String theory
- Near boundary limit of Einstein Gravity
- Concluding Remarks

Zooming near the boundary of (EA)dS$_3$

Conformal Killing Vector fields of (EA)dS_3

- Consider CFT on (EA)dS_3 background

$$ds^2 = \frac{d\eta^2 + dz\, d\bar{z}}{\eta^2}, \qquad \text{Boundary at } \eta = 0.$$

- Conformal Killing Vector fields $\boxed{\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = \frac{2}{3}\nabla_\rho \xi^\rho\, g_{\mu\nu}}$ 10 parameters.

$$\mathcal{L}_m = -z^{m+1}\,\partial_z + \frac{1}{2}m(m+1)\,\eta^2\,\partial_{\bar{z}} - \frac{1}{2}(m+1)\,z^m\,\eta\,\partial_\eta\,, \qquad m \in \{0, \pm 1\}$$

$$\bar{\mathcal{L}}_m = -\bar{z}^{m+1}\,\partial_{\bar{z}} + \frac{1}{2}m(m+1)\,\eta^2\,\partial_z - \frac{1}{2}(m+1)\,\bar{z}^m\,\eta\,\partial_\eta$$

$$\mathcal{P}_{r,s} = -z^{r+\frac{1}{2}}\,\bar{z}^{s+\frac{1}{2}}\,\partial_\eta + 2\,\eta\left(s+\frac{1}{2}\right)z^{r+s}\partial_z + 2\,\eta\left(r+\frac{1}{2}\right)\bar{z}^{r+s}\partial_{\bar{z}}$$
$$+ (r+\frac{1}{2})(s+\frac{1}{2})\,\eta^2\,\partial_\eta\,, \qquad (r,s) \in \{\pm\frac{1}{2}\}$$

Conformal Killing Vector fields of (EA)dS$_3$

- The vector fields obey $\mathfrak{so}(4,1)$ algebra

$$[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n}\,, \qquad \left[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n\right] = (m-n)\bar{\mathcal{L}}_{m+n}$$

$$[\mathcal{L}_m, \mathcal{P}_{r,s}] = \frac{1}{2}(m-2r)\,\mathcal{P}_{m+r,s}\,, \quad \left[\bar{\mathcal{L}}_m, \mathcal{P}_{r,s}\right] = \frac{1}{2}(m-2s)\,\mathcal{P}_{r,m+s}$$

$$\left[\mathcal{P}_{r,s}, \mathcal{P}_{r',s'}\right] = 2\left(\epsilon_{rr'}\bar{\mathcal{L}}_{s+s'} + \epsilon_{ss'}\,\mathcal{L}_{r+r'}\right).$$

- Zooming in near the boundary $\boxed{\eta = \epsilon\,\tilde{\eta}, \quad \epsilon \to 0 \quad \text{and} \quad \tilde{\eta} \quad \text{fixed}}$
- In terms of $\tilde{\eta}$ the CKVs become

$$\mathcal{L}_m = -z^{m+1}\,\partial_z - \frac{\tilde{\eta}}{2}(m+1)\,z^m\,\partial_{\tilde{\eta}} + \mathcal{O}(\epsilon^2)$$

$$\bar{\mathcal{L}}_m = -\bar{z}^{m+1}\,\partial_{\bar{z}} - \frac{\tilde{\eta}}{2}(m+1)\,\bar{z}^m\,\partial_{\tilde{\eta}} + \mathcal{O}(\epsilon^2)$$

$$\mathcal{P}_{r,s} = -z^{r+\frac{1}{2}}\,\bar{z}^{s+\frac{1}{2}}\,\frac{\partial_{\tilde{\eta}}}{\epsilon} + \mathcal{O}(\epsilon).$$

Group contraction of SO$(4,1)$ to ISO$(3,1)$

- In the limit $\epsilon \to 0$, $\mathcal{P}_{r,s}$ diverges $\cdots$ Define $P_{r,s} = \epsilon\, \mathcal{P}_{r,s}$ and also denote

$$\mathcal{L}_m \xrightarrow{\epsilon \to 0} L_m\,, \quad \bar{\mathcal{L}}_m \xrightarrow{\epsilon \to 0} \bar{L}_m$$

- Resultant algebra becomes Poincare ($\mathfrak{iso}(1,3)$),

$$[L_m, L_n] = (m-n)L_{m+n}\,, \qquad \left[\bar{L}_m, \bar{L}_n\right] = (m-n)\bar{L}_{m+n}$$
$$[L_m, P_{r,s}] = \frac{1}{2}(m-2r)\; P_{m+r,s}\,, \quad \left[\bar{L}_m, P_{r,s}\right] = \frac{1}{2}(m-2s)\; P_{r,m+s}$$
$$\left[P_{r,s}, P_{r',s'}\right] = 0.$$

- Near Boundary Limit: SO$(4,1)$ contracts to ISO$(3,1)$ with $P_{r,s}$ as translations $\cdots$ Subgroup SO$(3,1)$ (isometry of EAdS$_3$) remains unchanged.

Symmetries of manifold obtained after near boundary limit

- Lorentz group SO(3, 1) acts on the coordinates $(\tilde{\eta}, z, \bar{z})$ as

$$z \to \frac{az+b}{cz+d}, \ \bar{z} \to \frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}}, \ \tilde{\eta} \to \frac{\tilde{\eta}}{|cz+d|^2}.$$

and is the isometry group of the degenerate metric that we get after the near boundary limit $\boxed{ds^2_{\text{NB}} = \frac{dzd\bar{z}}{\tilde{\eta}^2}}$

- The translation $P_{r,s}$ acts as

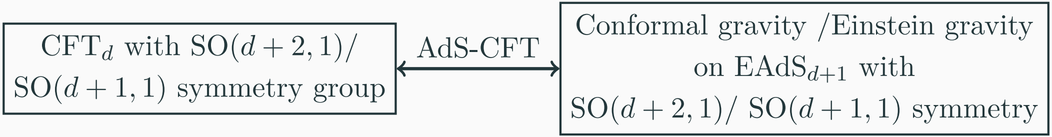
$$z \to z, \quad \bar{z} \to \bar{z}, \quad \tilde{\eta} \to \tilde{\eta} \ + \ a_{r,s}\, z^{r+\frac{1}{2}}\, \bar{z}^{s+\frac{1}{2}}, \qquad (r,s) \in \{\pm\frac{1}{2}\}.$$

and act as (Weyl) conformal transformations.

Our Proposal

Diagrammatic representation of the proposal

- In near boundary limit $\mathrm{SO}(d+2,1)$ contracts to $\mathrm{ISO}(d+1,1) \cdots$ Subgroup $\mathrm{SO}(d+1,1)$ remain unchanged.



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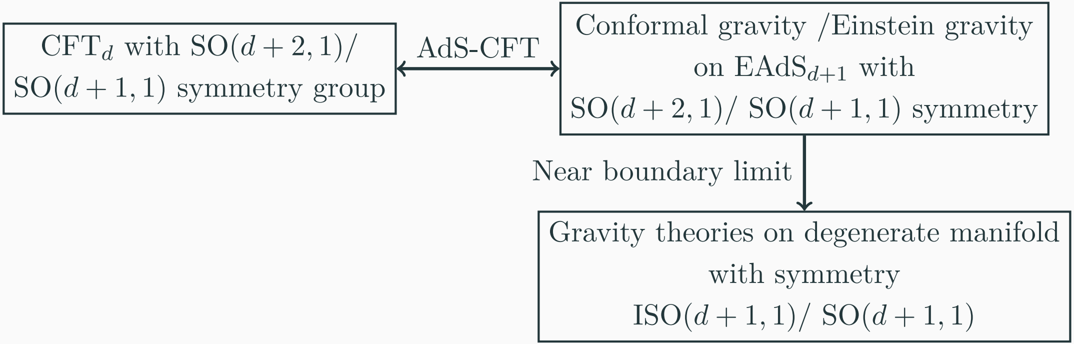
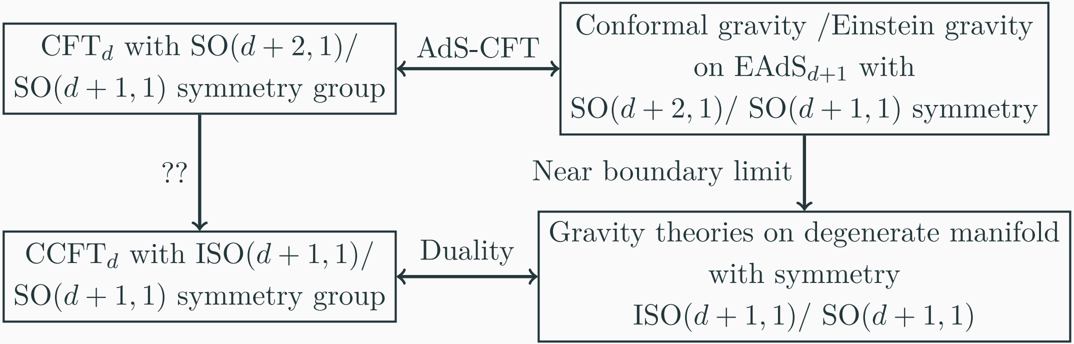


- In near boundary limit $SO(d+2,1)$ contracts to $ISO(d+1,1)$ $\cdots$ Subgroup $SO(d+1,1)$ remain unchanged.



Proposal continued $\cdots$

- Argument extends to conformal and Einstein gravity on Lorentzian AdS_{d+1} $\cdots$ Celestial CFT_d has symmetry group $\mathrm{ISO}(d,2)$ and $\mathrm{SO}(d,2)$.

- Scaling limit does not change the bulk isometry algebra $\mathrm{SO}(d+1,1)$ $\cdots$ Acts non-trivially on the correlation functions of bulk fields and therefore the bulk theory changes.

- Identification is based on only symmetries. Correlation functions of CCFT_d obtained this way may not necessarily compute scattering amplitudes in asymptotically flat $(d+2)$ dimensional spacetime.

Near Boundary limit of CFT two-point function

Near Boundary limit of CFT two-point function

- Two-point function on EAdS$_{d+1}$ conformally related to two-point function on flat space

$$\langle \Phi_\Delta(\eta_1, z_1, \bar{z}_1)\, \Phi_\Delta(\eta_2, z_2, \bar{z}_2) \rangle_{AdS_{d+1}} = \frac{\eta_1^\Delta\, \eta_2^\Delta}{\left((\eta_{12})^2 + \vec{x}_{12}^2 \right)^\Delta}, \quad \vec{x} \in \mathbb{R}^d$$

- In the near boundary limit

$$\langle \Phi_\Delta(\epsilon\, \tilde{\eta}_1, \vec{x}_1)\, \Phi_\Delta(\epsilon\, \tilde{\eta}_2, \vec{x}_2) \rangle_{AdS_{d+1}} = \frac{\epsilon^{2\Delta} \tilde{\eta}_1^\Delta\, \tilde{\eta}_2^\Delta}{(\epsilon^2 \tilde{\eta}_{12}^2 + \vec{x}_{12}^2)^\Delta}$$

$$\to \boxed{\epsilon^d \left(\pi^{\frac{d}{2}} \frac{\Gamma\left(\Delta - \frac{d}{2}\right)}{\Gamma(\Delta)} \frac{\tilde{\eta}_1^\Delta \tilde{\eta}_2^\Delta}{|\tilde{\eta}_{12}|^{2\Delta - d}} \delta^d(\vec{x}_1 - \vec{x}_2) \right) + \epsilon^{2\Delta} \left(\frac{\tilde{\eta}_1^\Delta \tilde{\eta}_2^\Delta}{|\vec{x}_1 - \vec{x}_2|^{2\Delta}} \right)}$$

- **Electric sector:** $\Delta > \frac{d}{2}$, **Magnetic sector:** $\frac{d-1}{2} \leq \Delta \leq \frac{d}{2}$

Electric sector

- Defining rescaled field $\boxed{\Phi_\Delta(\epsilon\,\tilde{\eta},\vec{x}) = \epsilon^{\frac{d}{2}}\,\tilde{\eta}^{\Delta}\,\tilde{\Phi}_\Delta(\tilde{\eta},\vec{x})}$
- In terms of $\tilde{\Phi}$, the two point function becomes

$$\langle\tilde{\Phi}_\Delta(\tilde{\eta}_1,\vec{x}_1)\,\tilde{\Phi}_\Delta(\tilde{\eta}_2,\vec{x}_2)\rangle_{\mathrm{NB}} = \pi^{\frac{d}{2}}\frac{\Gamma\left(\Delta-\frac{d}{2}\right)}{\Gamma(\Delta)}\frac{\delta^d(\vec{x}_1-\vec{x}_2)}{|\tilde{\eta}_{12}|^{2\Delta-d}},\;\Delta>\frac{d}{2}$$

- Two-point function of (d+1) Carrollian conformal primaries $\tilde{\Phi}_\Delta(\tilde{\eta},\vec{x})$ living on the degenerate metric

$$ds^2 = 0.\,d\tilde{\eta}^2 + d\vec{x}^2,\qquad \vec{x}\in\mathbb{R}^d$$

- Not surprising since $\mathrm{ISO}(d+1,1)$ is isomorphic to the conformal Carroll group in $(d+1)$ dimensions.

$\Delta > \frac{d}{2}$ as unitary bound in the electric sector

- As $\Delta \to \frac{d}{2}+$, finite part of the correlation function can be written as

$$\langle \tilde{\Phi}_\Delta(\tilde{\eta}_1, \vec{x}_1)\, \tilde{\Phi}_\Delta(\tilde{\eta}_2, \vec{x}_2) \rangle = -\frac{\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})} \ln |\tilde{\eta}_1 - \tilde{\eta}_2|^2 \; \delta^d(\vec{x}_1 - \vec{x}_2).$$

- In electric sector of a Carrollian CFT$_{d+1}$, the scalar operator with dimension $\Delta = \frac{d}{2}$ is not a conformal primary $\cdots$ derivative of scalar field is Carrollian conformal primary $\cdots$ similar to free scalar field with dimension zero.

- In a unitary Carrollian CFT$_{d+1}$ the dimension Δ of a scalar primary in the electric sector is bounded from below by $\frac{d}{2}$, i.e, $\Delta > \frac{d}{2}$.

- In a relativistic CFT$_{d+1}$ the corresponding bound $\Delta \geq \frac{d-1}{2}$ is less stringent.

Magnetic sector

- For $\Delta < \frac{d}{2}$, the magnetic sector has a dominant contribution.

- In terms of the rescaled field $\boxed{\Phi_\Delta(\epsilon\tilde{\eta}, \vec{x}) = \epsilon^\Delta\, \tilde{\eta}^\Delta \tilde{\Phi}_\Delta(\tilde{\eta}, \vec{x})}$ the two-point function becomes

$$\langle \tilde{\Phi}_\Delta(\tilde{\eta}_1, \vec{x}_1)\, \tilde{\Phi}_\Delta(\tilde{\eta}_2, \vec{x}_2)\rangle_{\text{NB}} = \frac{1}{|\vec{x}_1 - \vec{x}_2|^{2\Delta}}, \quad \frac{d-1}{2} \leq \Delta \leq \frac{d}{2}$$

- The lower bound on Δ is the unitarity bound of the relativistic CFT_{d+1} $\cdots$ Δ of a scalar primary in the magnetic sector is both bounded from below and above by $\frac{d-1}{2} \leq \Delta \leq \frac{d}{2}$.

- In the near-boundary limit, conformal primary becomes electric Carrollian primary if $\Delta > \frac{d}{2}$ and magnetic Carrollian primary if $\frac{d-1}{2} \leq \Delta \leq \frac{d}{2}$.

Near boundary limit of massive field theory on EAdS_3

Massive KG field action

- Consider scalar field of mass m on EAdS_3 with the following line element

$$ds^2 = d\phi^2 + e^{2\phi} d\gamma d\bar{\gamma}$$

where $-\infty < \phi < \infty$ and γ and $\bar{\gamma}$ are complex conjugates $\cdots$ boundary is at $\phi = \infty$.
- Relation with the old coordinates $\phi = -\ln \eta,\ \gamma = z,\ \bar{\gamma} = \bar{z}$
- The action of the scalar field is given by

$$S = \int d\phi\, d\gamma\, d\bar{\gamma}\, e^{2\phi} \Big((\partial_\phi \Psi)^2 + e^{-2\phi}\, \partial_i \Psi\, \partial_i \Psi + m^2\, \Psi^2 \Big)$$

- In terms of ϕ the near boundary limit becomes $\boxed{\phi = -\ln \epsilon + \tilde{\phi},\ \epsilon \to 0}$ with $\tilde{\phi}$ held fixed.

Near boundary limit of massive KG action

- In the near boundary limit, the action becomes

$$S_{\text{NB}} = \frac{1}{2}\int d\tilde{\phi}\, d\gamma\, d\bar{\gamma}\, e^{2\tilde{\phi}} \left((\partial_{\tilde{\phi}}\tilde{\Psi})^2 + m^2\, \tilde{\Psi}^2\right)$$

- The action is invariant under $\text{SO}(3,1)$ which acts as follows

$$\tilde{\Psi}'(\tilde{\phi}', \gamma', \bar{\gamma}') = \tilde{\Psi}(\tilde{\phi}, \gamma, \bar{\gamma})$$

$$\tilde{\phi}' = \tilde{\phi} + \ln|c\gamma + d|^2,\ \gamma' = \frac{a\gamma + b}{c\gamma + d},\ \bar{\gamma}' = \frac{\bar{a}\bar{\gamma} + \bar{b}}{\bar{c}\bar{\gamma} + \bar{d}},\ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{C})$$

- These coordinate transformations are the isometry of $\boxed{ds^2 = e^{2\tilde{\phi}} d\gamma d\bar{\gamma}}$ on which the scalar field $\tilde{\Psi}(\tilde{\phi}, \gamma, \bar{\gamma})$ lives.

- **Infinite Symmetries**: The SO(3,1) symmetries of the metric and the action can be enhanced to infinite-dimensional local symmetry of arbitrary holomorphic reparameterization

$$\tilde{\Psi}'(\tilde{\phi}', \gamma', \bar{\gamma}') = \tilde{\Psi}(\tilde{\phi}, \gamma, \bar{\gamma})$$
$$\tilde{\phi}' = \tilde{\phi} - \frac{1}{2} \ln \left| \frac{d\gamma'}{d\gamma} \right|^2, \ \gamma' = \gamma'(\gamma), \ \bar{\gamma}' = \bar{\gamma}'(\bar{\gamma})$$

- **Boundary Correlation function**: The boundary two-point correlation function for S_{NB} is computed using the bulk Green's function defined as

$$\left(-e^{-2\tilde{\phi}} \frac{\partial}{\partial \tilde{\phi}} \left(e^{2\tilde{\phi}} \frac{\partial}{\partial \tilde{\phi}} \right) + m^2 \right) G(\tilde{\phi}, \gamma, \bar{\gamma}; \tilde{\phi}', \gamma', \bar{\gamma}') = \frac{\delta(\tilde{\phi} - \tilde{\phi}') \delta^2(\gamma - \gamma')}{e^{2\tilde{\phi}}}$$

Green's function for S_{NB}

- The Green's function G is

$$G(\tilde{\phi},\gamma,\bar{\gamma};\tilde{\phi}',\gamma',\bar{\gamma}') = \frac{e^{-\Delta_+\tilde{\phi}}e^{-\Delta_-\tilde{\phi}'}}{2\sqrt{1+m^2}}\theta(\tilde{\phi}-\tilde{\phi}')\delta^2(\gamma-\gamma') + \frac{e^{-\Delta_-\tilde{\phi}}e^{-\Delta_+\tilde{\phi}'}}{2\sqrt{1+m^2}}\theta(\tilde{\phi}'-\tilde{\phi})\delta^2(\gamma-\gamma')$$

where $\boxed{\Delta_\pm = 1 \pm \sqrt{1+m^2}}$

- In terms of coordinate $\tilde{\eta} = e^{-\tilde{\phi}}$

$$G(\tilde{\eta},\gamma,\bar{\gamma};\tilde{\eta}',\gamma',\bar{\gamma}') = \frac{\tilde{\eta}^{\Delta_+}\tilde{\eta}'^{\Delta_-}}{2\sqrt{1+m^2}}\theta(\tilde{\eta}'-\tilde{\eta})\delta^2(\gamma-\gamma') + \frac{\tilde{\eta}^{\Delta_-}\tilde{\eta}'^{\Delta_+}}{2\sqrt{1+m^2}}\theta(\tilde{\eta}-\tilde{\eta}')\delta^2(\gamma-\gamma')$$

Boundary Correlation functions

- Taking $(\tilde{\eta}, \tilde{\eta}')$ to zero and using the extrapolate dictionary, the boundary correlators are

$$\langle O_{\Delta_+}(\gamma,\bar{\gamma})O_{\Delta_-}(\gamma',\bar{\gamma}')\rangle = \delta^2(\gamma-\gamma'),\ \Delta_+ + \Delta_- = 2$$
$$\langle O_{\Delta_+}(\gamma,\bar{\gamma})O_{\Delta_+}(\gamma',\bar{\gamma}')\rangle = \langle O_{\Delta_-}(\gamma,\bar{\gamma})O_{\Delta_-}(\gamma',\bar{\gamma}')\rangle = 0$$

- The boundary two-point function is a pure contact term because the theory is ultralocal in the boundary direction.

- In the near boundary limit in AdS_{d+1} a scalar field of mass m corresponds to two boundary operators of dimensions

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2}.$$

Near boundary limit of String theory

Worldsheet theory for string propagation on (EA)dS$_3$ $\times \mathcal{N}$

- Conventional string theories do not have target space conformal invariance $\cdots$ The low energy effective theory is Einstein gravity.
- In the near boundary limit, string theories on AdS should be dual to Celestial CFTs with only conformal (Lorentz invariance) and not Poincare invariance.
- Bosonic string theory on EAdS$_3$ $\times \mathcal{N}$ in the presence of Neveu-Schwarz $B_{\mu\nu}$ field with EAdS$_3$ metric $\boxed{ds^2 = l^2\left(d\phi^2 + e^{2\phi}d\gamma d\bar{\gamma}\right)}$

Worldsheet theory continued ⋯

- For specific form of $B_{\mu\nu}$, the worldsheet Lagrangian on AdS_3 is given by

$$S = \frac{2l^2}{l_s^2} \int d^2w \left(\partial\phi\bar{\partial}\phi + e^{2\phi}\bar{\partial}\gamma\partial\bar{\gamma} \right) + S_{\rm int}$$

- Bosonic part of Lagrangian is given by WZW model for coset SL(2, C)/SU(2) $\cdots$ $S_{\rm int}$ describes string propagation on compact internal space. [Giveon-Kutasov-Seiberg, de Boer-Ooguri-Robins-Tannenhauser, Maldacena-Ooguri]

- The action can be written as

$$S = \int d^2w \left(\partial\phi\bar{\partial}\phi - \frac{2}{\alpha_+}\sqrt{g}R\phi + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} - \beta\bar{\beta}e^{-\frac{2}{\alpha_+}\phi} \right) + S_{\text{int}}$$

- (β,γ) holomorphic fields with weights $(1,0)$ and $(0,0)$ and $(\bar{\beta},\bar{\gamma})$ corresponding anti-holomorphic fields. $\cdots$ $\boxed{\alpha_+^2 = 2k - 4 = 2\frac{l^2}{l_s^2} - 4}$

  [Giveon, Kutasov, Sieberg]

- In the near boundary limit, neglecting the last term, one gets the free worldsheet action,

$$S_{\text{NB}} = \int d^2w \left(\partial\tilde{\phi}\bar{\partial}\tilde{\phi} - \frac{2}{\alpha_+}\sqrt{g}R\tilde{\phi} + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} \right) + S_{\text{int}}$$

Long strings

- Equations of motion of $\beta(\bar{\beta})$ are $\boxed{\bar{\partial}\gamma = 0, \ \partial\bar{\gamma} = 0.}$
- γ $(\bar{\gamma})$ is a holomorphic (anti-holomorphic) map from the worldsheet to the boundary sphere (S^2) in the target space. These wrapped strings are known as the "long-strings".
- $\mathrm{S_{NB}}$ has an affine $\mathrm{SL_2} \times \mathrm{SL_2}$ symmetry at level k generated by worldsheet currents $J^a(z)$ and $\bar{J}^a(\bar{z})$ with the central charge $c = \frac{3k}{k-2}$.
- The symmetries of the worldsheet action can be lifted to the space-time and become the Virasoro algebra of the dual $\mathrm{CFT_2}$. [Giveon, Kutasov, Sieberg]

Properties of long strings

- This spacetime dual CFT describe long strings with central charge $c_{\text{spacetime}} = 6kp$. p is number of times worldsheet wraps the boundary sphere. [Giveon, Kutasov, Sieberg]
- Seiberg and Witten argued that the space-time CFT_2 describing long strings contains Liouville field $\tilde{\phi}$ and modes of the fields describing location of the string in $\mathcal{N}$ with central charge c_{int}.
- The Liouville field $\tilde{\phi}$ has improvement term Q (for $p = 1$) and central charge c_L and modes in compact space has central charge c_{int}.
- In order to find the improvement term we use $\boxed{c_{\text{L}} + c_{\text{int}} = 6k}$

Properties of long strings continued $\cdots$

- Using the expression $\boxed{c_{\mathrm{L}} = 1 + 3Q^2}$ and $\boxed{c_{\mathrm{int}} = 26 - \frac{3k}{k-2}}$ one obtain

$$c_{\mathrm{L}} = 1 + 3Q^2 = 1 + 6\frac{(k-3)^2}{k-2} \implies Q = (k-3)\sqrt{\frac{2}{k-2}}.$$

- The Liouville field describes the radial ($\tilde{\phi}$) fluctuations of the long-string worldsheet.

- The space-time CFT_2 dual to the string theory described by the worldsheet action S_{NB} is an example of a Celestial CFT_2 (with only Lorentz invariance).

Near boundary limit of Einstein Gravity

Einstein gravity on locally (EA)dS$_3$

- Consider the metric of locally EAdS$_3$ in Fefferman-Graham gauge with the following line element

$$ds^2 = d\phi^2 + e^{2\phi}\, h_{ij}(\phi, x^i)\, dx^i dx^j \,, \quad x^i = \{\gamma, \bar{\gamma}\}.$$

- On a constant ϕ hypersurface, the two dimensional induced boundary metric $\boxed{a_{ij} = e^{2\phi} h_{ij}}$.
- The Einstein-Hilbert action for EAdS$_3$ gravity becomes,

$$S_{\text{EH}} = \int d\phi\, d^2\gamma\, e^{2\phi}\, \sqrt{h}\left(K^2 - K_{ij}K^{ij} + 2\right) + \int d\phi\, d^2\gamma \sqrt{h}\, \mathcal{R}^{(2)}$$

- $\mathcal{R}^{(2)}$ is the Ricci scalar of metric h_{ij} and extrinsic curvature K_{ij} is extrinsic curvature $\boxed{K_{ij} = \frac{1}{2}\partial_\phi a_{ij}, \quad K = a^{ij}\, K_{ij}}$.

Near boundary limit of Einstien gravity

- In the near boundary limit, the action is

$$S_{\rm NB} = \int d\tilde{\phi}\, d^2\gamma\, e^{2\tilde{\phi}}\sqrt{h}\left(\tilde{K}^2 - \tilde{K}_{ij}\tilde{K}^{ij} + 2\right),\; -\infty < \tilde{\phi} < \infty$$

with $\tilde{K}_{ij} = \frac{1}{2}\partial_{\tilde{\phi}}\left(e^{2\tilde{\phi}}\, h_{ij}(\tilde{\phi}, x^i)\right)$.

- The theory in the near boundary limit of Einstein gravity is defined on the degenerate manifold with line element

$$ds^2 = 0.d\tilde{\phi}^2 + e^{2\tilde{\phi}}\, h_{ij}(\tilde{\phi}, x^i)\, dx^i dx^j .$$

- $S_{\rm NB}$ matches with the Carrollian limit of the Einstein-Hilbert action. [Hansen, Obers, Oling and Søgaard]

Properties of near boundary limit of Einstein gravity

- The solution to equations of motion obtained from $S_{\rm NB}$ are exact

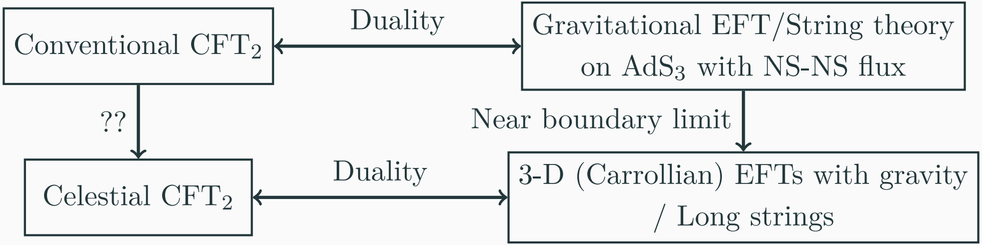
$$h_{ij}(\phi, x^i) = h^{(0)}_{ij}(x^i) + e^{-2\phi} h^{(1)}_{ij}(x^i) + e^{-4\phi} h^{(2)}_{ij}(x^i)\,, \quad h^{(2)}_{ij} = \frac{1}{4} h^{(1)}_{ik} h^{kl}_{(0)} h^{(1)}_{lj}\,.$$

$$\boxed{h^{ij}_{(0)}\, h^{(1)}_{ij} = 0, \qquad D^k\, h^{(1)}_{ki} = 0}.$$

- The tracelessness of $h^{(1)}_{ij}$ implies zero central charge.
- Conservation of $h^{(1)}_{ij}$ implies the boundary stress tensor leads to two commuting copies of Virasoro algebra.

Concluding Remarks

Conclusions



- We focused on theories with Lorentz invariance $\mathrm{SO}(1,3)$. These results can be extended to conformal field theories and conformal gravity on (EA)dS$_3$ (work in progress)
- String theory with target space conformal invariance?

Conclusions continued

- The existence of the Liouville sector in the long-string CFT_2 suggests that the Liouville sector could be a universal feature of all Celestial CFT_2.
- Our observation resonates with the fact that in the high-energy limit in asymptotically flat space-time, one can identify the string worldsheet with the celestial sphere. [Stieberger, Taylor]

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Thank You

Appendix A

- For the quantities defined as follows

$$\zeta_{ij} = \frac{1}{2}\partial_\phi h_{ij}, \quad \Theta = h^{ij}\zeta_{ij}$$

- The EE in the near boundary limit becomes

$$\boxed{\Theta - \frac{1}{2}\zeta_{ij}\zeta^{ij} + \frac{1}{2}\Theta^2 = 0}$$

$$\boxed{h^{kl}\nabla_k\zeta_{il} - h^{kl}\nabla_i\zeta_{kl} = 0}$$

$$\boxed{\left(\partial_{\tilde{\phi}}\Theta\, h_{ij} - \partial_{\tilde{\phi}}\zeta_{ij} + \zeta_{kl}\zeta^{kl}\, h_{ij} + \zeta_i^k\zeta_{kj} + 2\Theta h_{ij} - 2\zeta_{ij}\right) = 0}$$

All indices are raised and lowered by h_{ij}.

Digression: Carrollian CFT primer

- Carrollian holography is another candidate of flat space holography.
- The dual to quantum gravity on $(d+2)$ dimensional asymptotically flat spacetime is $(d+1)$ dimensional Carrollian conformal field theory defined on boundary null infinity.
- Null infinity is an example of a Carrollian manifold, which is endowed with a degenerate metric and non-vanishing vector field. For example, in Bondi gauge, at null infinity, the boundary metric of $(d+2)$-dimensional asmptotically flat spacetime is

$$ds^2 = 0.du^2 + dx^i\, dx^i, \qquad \vec{x} \in \mathbb{R}^d$$

- The conformal group of Carrollian manifold is isomorphic to the Poincaré group $\mathrm{ISO}(d+1,1)$.

Transformation of Φ_Δ under SO(4, 1)

- Transformation of 3d conformal scalar primary under SO(4, 1)

$$\delta\Phi_\Delta = -\xi^\mu\,\partial_\mu\Phi_\Delta - \frac{\Delta}{3}\,\nabla_\mu\xi^\mu\,\Phi_\Delta$$

- The vector field $\xi = a_m\,\mathcal{L}_m + \bar{a}_m\,\bar{\mathcal{L}}_m + p_{r,s}\,\mathcal{P}_{r,s}$
- Generator Q_ξ of the conformal symmetry $[Q_\xi, \Phi_\Delta(\eta, z, \bar{z})] = \delta\Phi_\Delta$

Transformation of Φ_Δ under SO$(4,1)$

- The commutation relation between the generators SO(4,1) and $\Phi_\Delta(\eta, z, \bar{z})$

$$[\mathcal{L}_m, \Phi_\Delta] = z^{m+1}\,\partial_z\Phi_\Delta + \frac{1}{2}(m+1)\,z^m\,\eta\,\partial_\eta\Phi_\Delta - \frac{1}{2}\,m(m+1)\,\eta^2\,\partial_{\bar{z}}\Phi_\Delta$$

$$\left[\bar{\mathcal{L}}_m, \Phi_\Delta\right] = \bar{z}^{m+1}\,\partial_{\bar{z}}\Phi_\Delta + \frac{1}{2}(m+1)\,\bar{z}^m\,\eta\,\partial_\eta\Phi_\Delta - \frac{1}{2}\,m(m+1)\,\eta^2\,\partial_z\Phi_\Delta$$

$$[\mathcal{P}_{r,s}, \Phi_\Delta] = z^{r+\frac{1}{2}}\,\bar{z}^{s+\frac{1}{2}}\,\left(\partial_\eta\Phi_\Delta - \frac{\Delta\Phi_\Delta}{\eta}\right) - \left(2\,\eta\,\left(s+\frac{1}{2}\right)\,z^{r+s}\partial_z\Phi_\Delta\right.$$

$$\left. + 2\,\eta\,\left(r+\frac{1}{2}\right)\,\bar{z}^{r+s}\partial_{\bar{z}}\Phi_\Delta + \left((r+\frac{1}{2})(s+\frac{1}{2})\right)\,\left(\eta^2\partial_\eta\Phi_\Delta + \Delta\,\eta\,\Phi_\Delta\right)\right)$$

- $m \in \{0, \pm 1\}$ and $(r, s) \in \{\pm\frac{1}{2}\}$

- Rescaling $\eta = \epsilon\,\tilde{\eta}$ and defining Carrollian conformal primary field $\tilde{\Phi}_\Delta(\tilde{\eta}, z, \bar{z})$ as

$$\Phi_\Delta(\epsilon\,\tilde{\eta}, z, \bar{z}) = \tilde{\eta}^\Delta\,\epsilon\,\tilde{\Phi}_\Delta(\tilde{\eta}, z, \bar{z}), \qquad \epsilon \to 0$$

Transformation of $\tilde{\Phi}_\Delta$ under ISO$(3,1)$

- In the limit $\epsilon \to 0$, the commutator becomes

$$\left[L_m, \tilde{\Phi}_\Delta(\tilde{\eta}, z, \bar{z})\right] = z^{m+1}\, \partial_z \tilde{\Phi}_\Delta(\tilde{\eta}, z, \bar{z}) + (m+1)\, z^m \left(\frac{\tilde{\eta}}{2}\, \partial_{\tilde{\eta}} + \frac{\Delta}{2}\right) \tilde{\Phi}_\Delta(\tilde{\eta}, z, \bar{z}) + \mathcal{O}(\epsilon^2)$$

$$\left[\bar{L}_m, \tilde{\Phi}_\Delta(\tilde{\eta}, z, \bar{z})\right] = \bar{z}^{m+1}\, \partial_{\bar{z}} \tilde{\Phi}_\Delta(\tilde{\eta}, z, \bar{z}) + (m+1)\, \bar{z}^m \left(\frac{\tilde{\eta}}{2}\, \partial_{\tilde{\eta}} + \frac{\Delta}{2}\right) \tilde{\Phi}_\Delta(\tilde{\eta}, z, \bar{z}) + \mathcal{O}(\epsilon^2)$$

$$\left[P_{r,s}, \tilde{\Phi}_\Delta(\tilde{\eta}, z, \bar{z})\right] = z^{r+\frac{1}{2}}\, \bar{z}^{s+\frac{1}{2}}\, \partial_{\tilde{\eta}}\, \tilde{\Phi}_\Delta(\tilde{\eta}, z, \bar{z}) + \mathcal{O}(\epsilon^2)$$

- This is transformation law of a scalar primary field of dimension Δ in a Carrollian CFT_3 which lives on the manifold with degenerate metric

$$ds^2 = 0.d\tilde{\eta}^2 + dz\, d\bar{z}.$$