Breakdown of Hawking Evaporation opens new Mass Window for Primordial Black Holes as Dark Matter Candidate

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Abstract

- The Memory Burden (MB) effect completely change the mass ranges for PBHs to be dark matter (10^5g-10^{10}g)

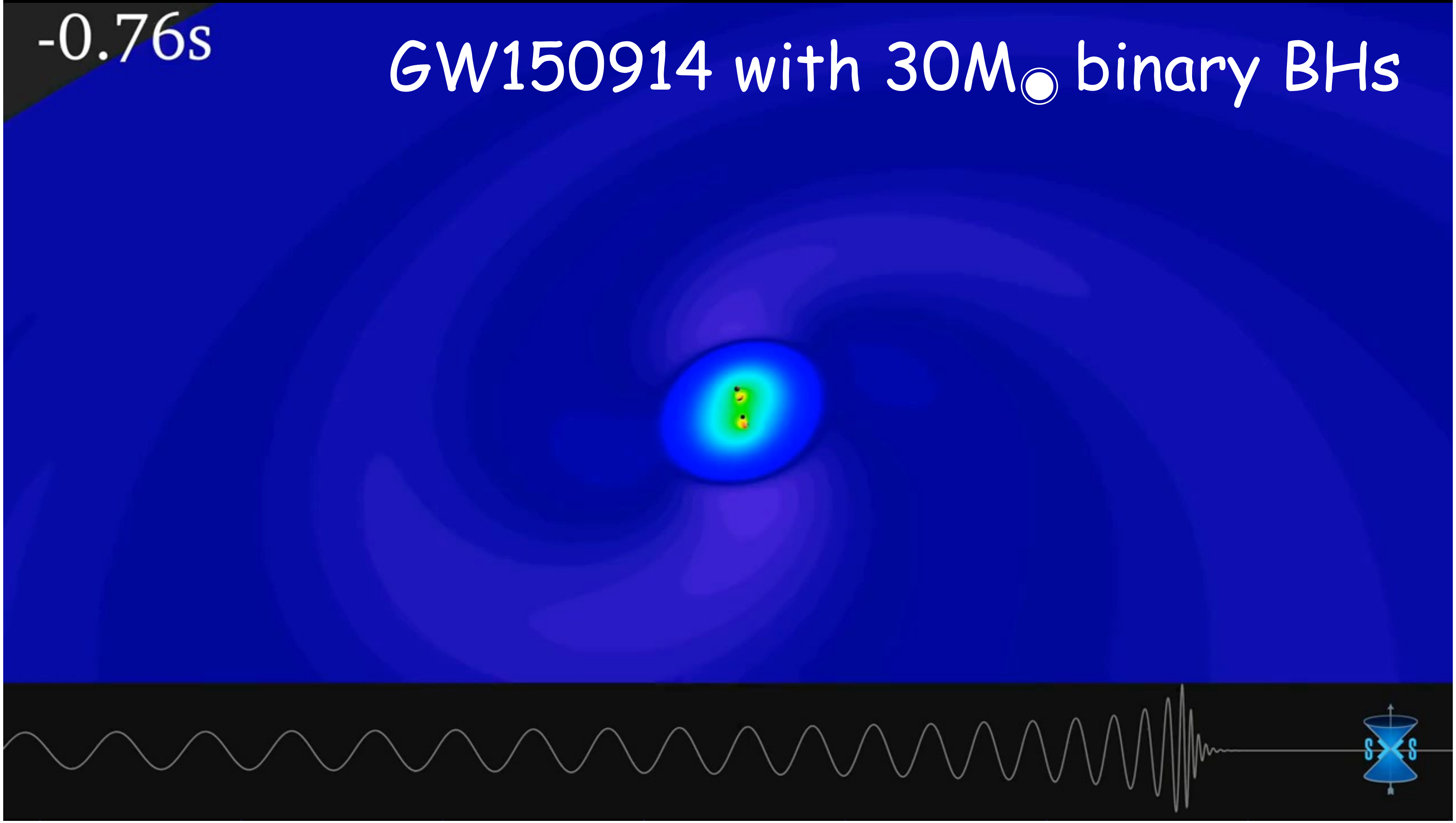
- The search for high-frequency GWs is a new direction for investigating phenomena in the early Universe.
- The targets are so many:
  1. Induced GW to produce dark matter PBHs with MB
  2. GWs from merging binary PBHs with subsolar mass
  3. Thermal/nonthermal graviton produced just after inflation
  4. 1^{st}-order phase transition at E >> weak scale
  5. …
- We can test high-frequency GWs by observing the electromagnetic wave converted from the GWs

Review of Primordial Black Holes

Detections of GWs from binary PBHs collide?

https://www.youtube.com/watch?v=1agm33iEAuo

GW150914 with $30M_{\odot}$ binary BHs

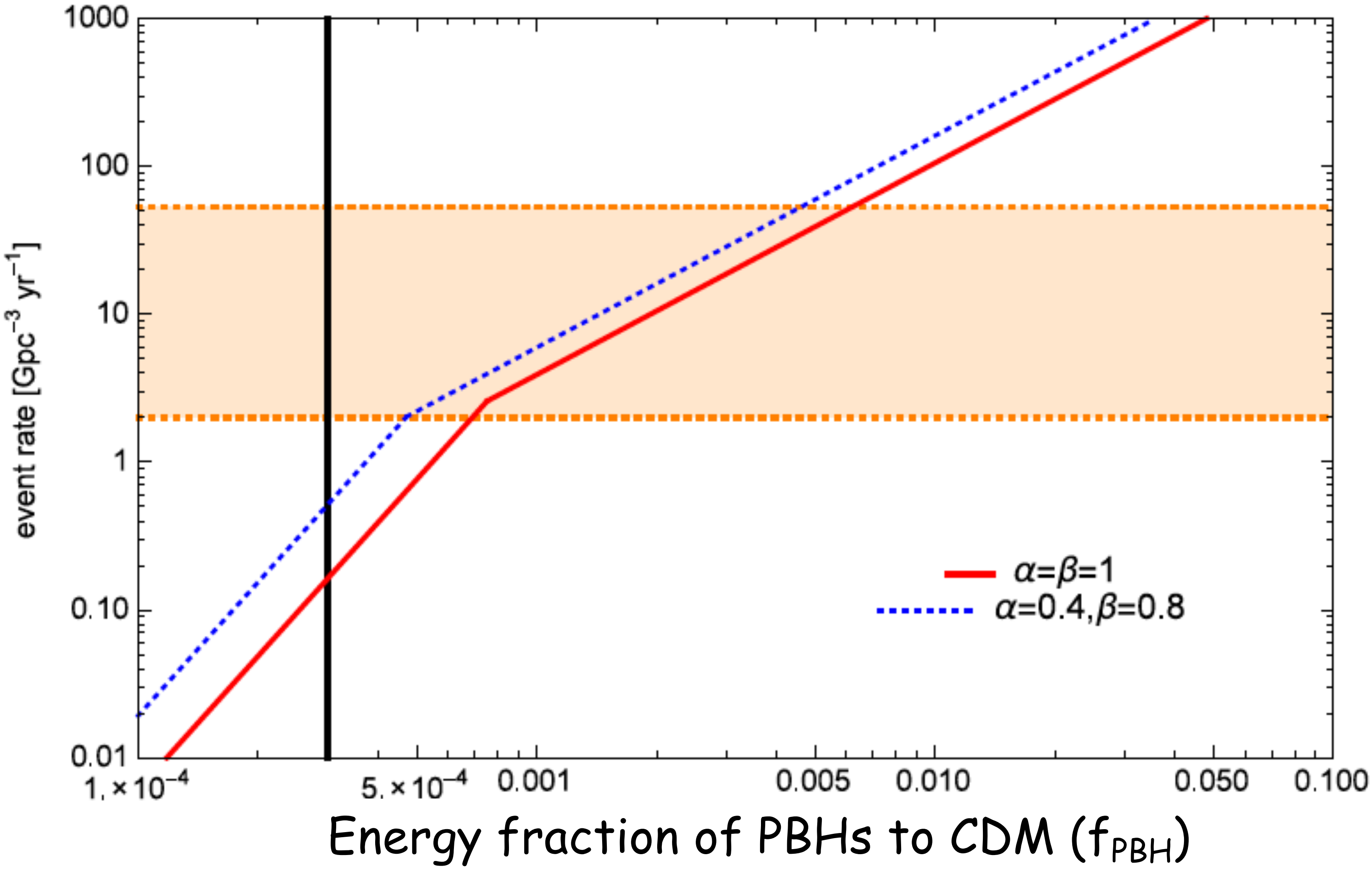


GW150914 and its merger rates for 30 M_{solar} masses BBH

Event rates of Binary BH mergers

M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama (2016).

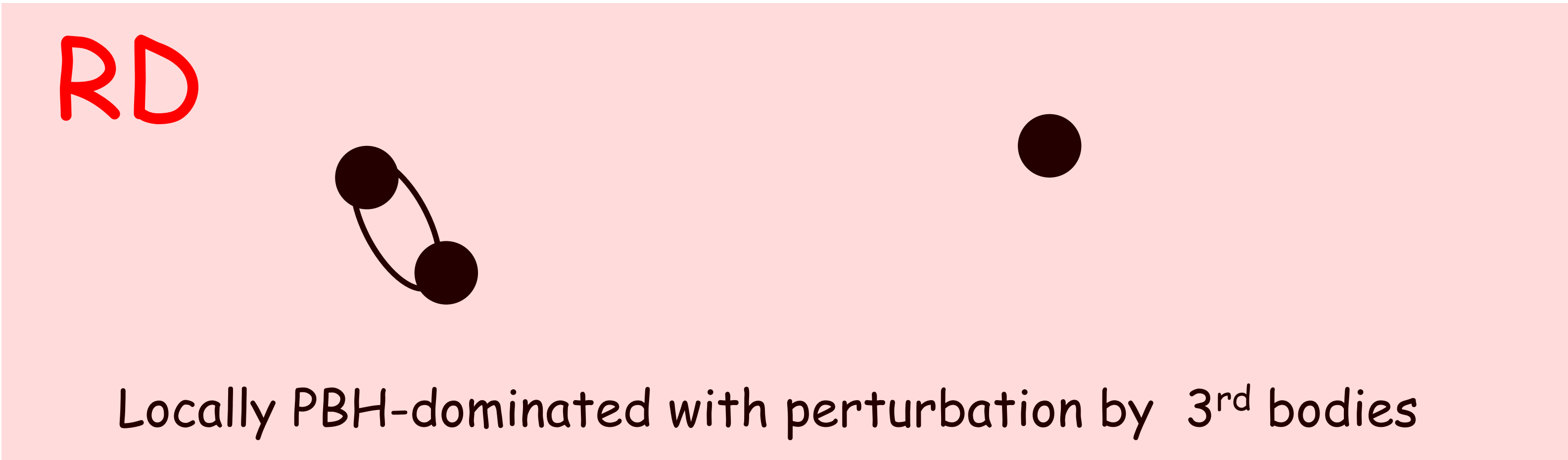
A 3-body effect is important for the BBH formations



Binary formations of PBHs in the radiation dominated epoch

M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama (2016).

- Three body effects are important



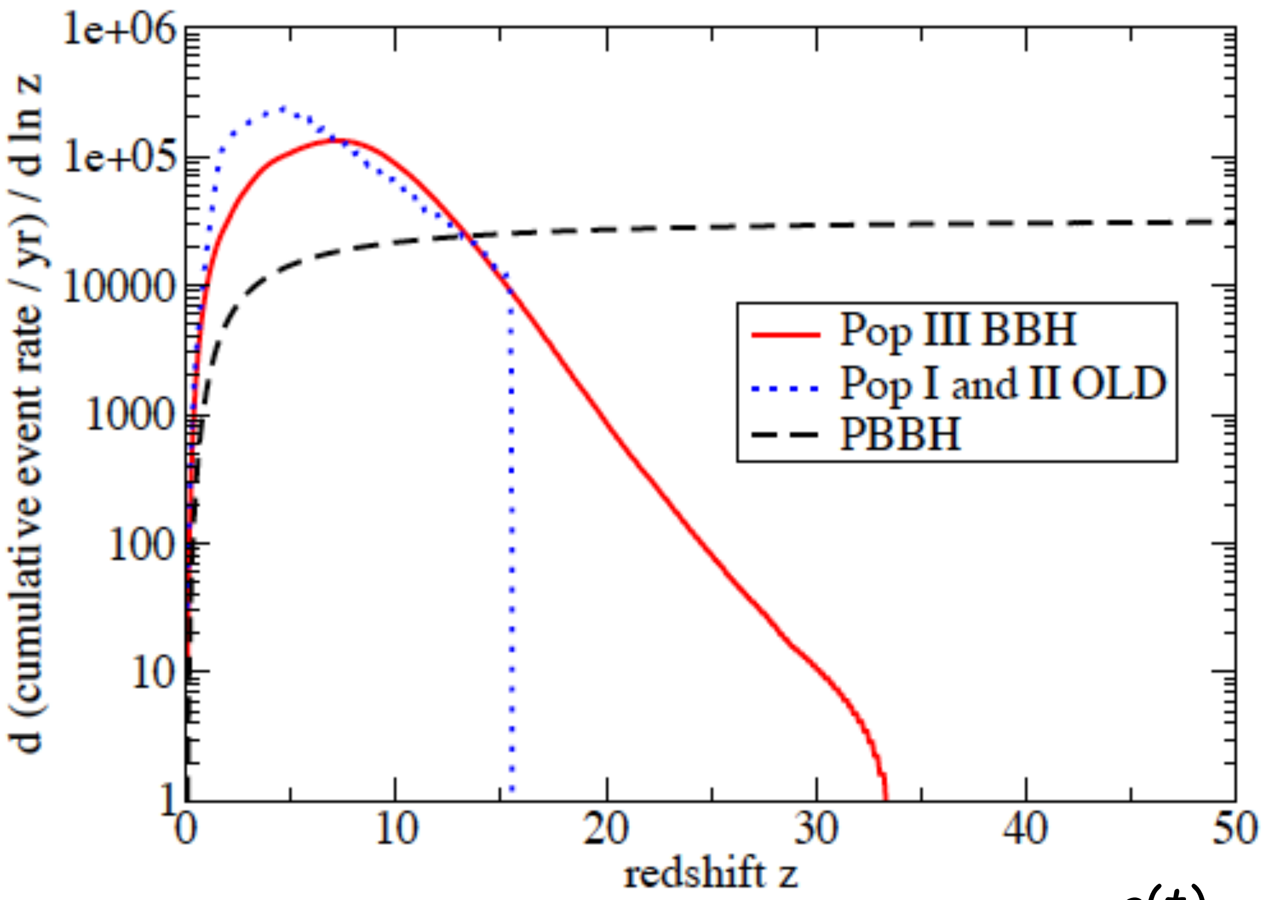
- Formation rate

$$\mathcal{R}_{\rm PBH}(z) = A_{\rm PBH}\left(\frac{t(z)}{\tau}\right)^{-\frac{34}{37}}$$

Z.-C. Chen and Q.-G. Huang, Astrophys. J. 864, 61 (2018), 1801.10327

DECIGO discriminates BPBHs from the normal BBHs

Takashi Nakamura et al, arXiv:1607.00897 [astro-ph.HE]



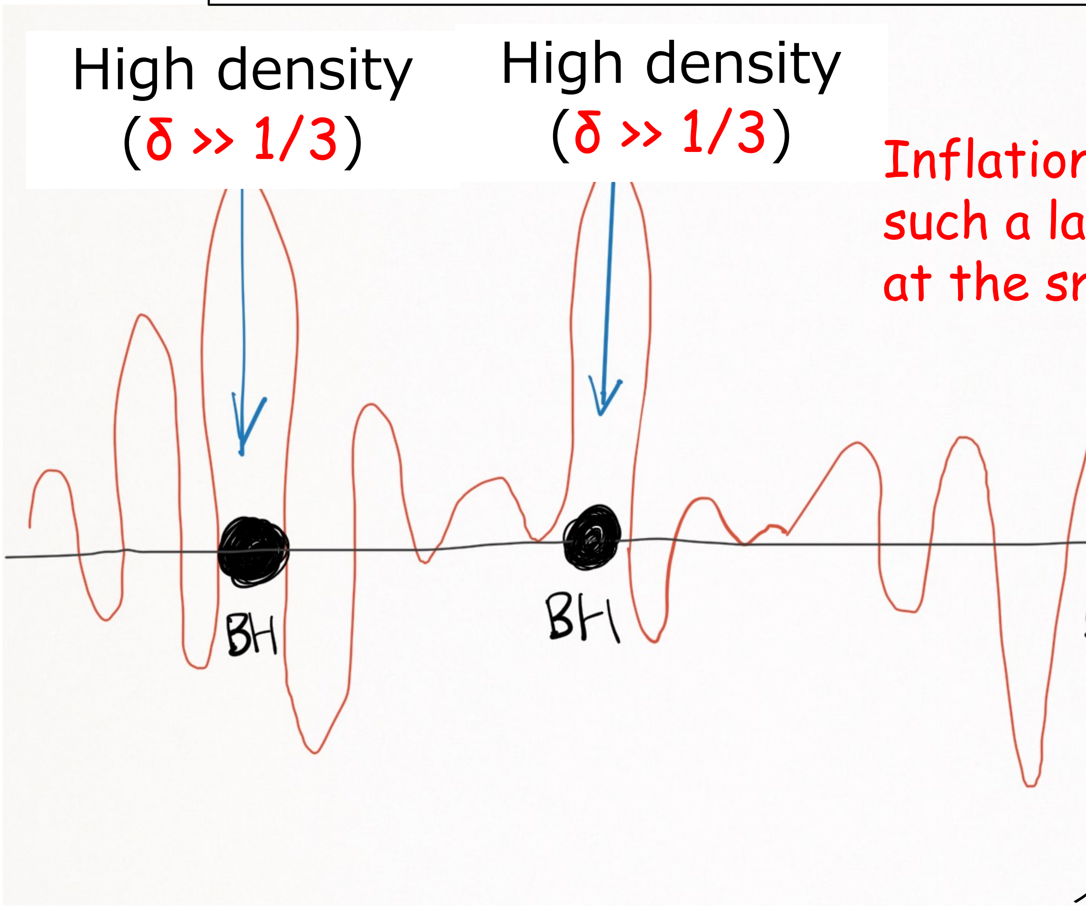
$$1/z \sim \frac{a(t)}{a(t_0)} \sim \left(t/10Gyr\right)^{2/3}$$

Primordial Black Holes

Bernard J. Carr, Astrophys. J. 201 (1975) 1

- High density perturbation ($\delta >> 1/3$) collapsed to PBHs

$$\delta > \delta_c \sim p / \rho \sim c_s^2 = w = 1/3$$



Inflation could have produced such a large perturbation at the small scales

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This is a cartoon

P_ζ (k) and PBH abundance β(M)

- Fraction of PBH to the total with Press Schechter formalism

For Peak Statistics,
e.g., see Yoo, Harada, KK et al (2018)(2020)

$$\beta(M) \equiv \frac{\rho_{\rm PBH}(M)}{\rho_{\rm tot}} = 2\int_{\delta_{\rm th}}^{\infty} d\delta \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) = {\rm erfc}\left(\frac{\delta_{\rm th}}{\sqrt{2}\sigma}\right)$$

$\sim 1/3 - 0.5$

For analytical derivations, see Harada, Yoo, KK (2013)

$\sigma \sim \overline{\delta\rho/\rho}$

- Relation between β and fluctuation σ (or β and Ω)

$$\beta(M) \sim {\rm erfc}\left(\frac{\delta_{\rm th}}{\sqrt{2}\sigma}\right) \simeq \sqrt{\frac{2}{\pi}}\frac{\sigma}{\delta_{\rm th}} \exp\left(-\frac{\delta_{\rm th}^2}{2\sigma^2}\right)$$

$\sim P_\zeta$

$$= 1.5 \times 10^{-18} \left(\frac{m_{\rm PBH}}{10^{15} g}\right)^{1/2} \left(\frac{\Omega_{\rm PBH} h^2}{0.1}\right)$$

Typical quantities of PBHs in RD

- Mass (horizon mass $=\rho(t_{form})\, H(t_{form})^{-3}$)

$$M_{PBH} \sim \rho(H^{-1}_{form})^3 \sim M^2_{pl} t_{from} \sim \frac{M^3_{pl}}{T^2_{form}} \sim 10^{15} g \left(\frac{T_{form}}{3\times 10^8 GeV} \right)^{-2} \sim 30 M_{\odot} \left(\frac{T_{form}}{40 MeV} \right)^{-2}$$

- Lifetime

$$\tau_{PBH} \sim \frac{M^3_{PBH}}{M^4_{pl}} \sim 4\times 10^{17} sec \left(\frac{M_{PBH}}{10^{15} g} \right)^3 \sim 3\times 10^{68} yrs \left(\frac{M_{PBH}}{30 M_{\odot}} \right)^3$$

- Hawking Temperature

$$T_{\rm PBH} \sim \frac{M^2_{pl}}{M_{\rm PBH}} \sim 10{\rm MeV} \left(\frac{M_{\rm PBH}}{10^{15} g} \right)^{-1} \sim 1\times 10^{-9}\ {\rm K} \left(\frac{M_{\rm PBH}}{30 M_{\odot}} \right)^{-1}$$

- Wave number *k* of horizon length

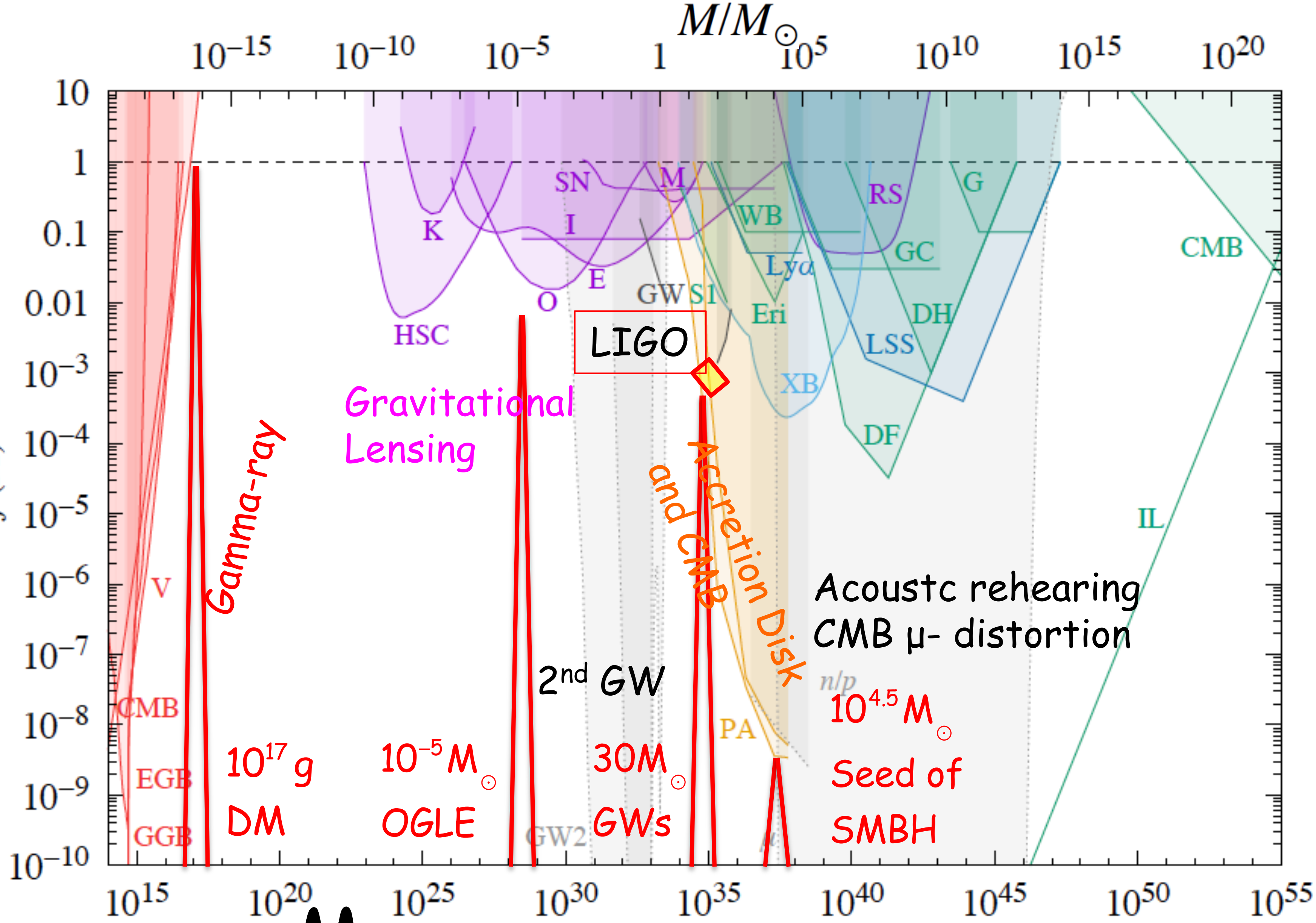
$$k = aH \sim 10^5 Mpc^{-1} \left(\frac{M_{PBH}}{10^4 M_{\odot}} \right)^{-1/2} \sim 10^5 Mpc^{-1} \left(\frac{T_{form}}{MeV} \right)^{+1}$$

- Fraction to CDM

$$f_{\rm fraction} \equiv \frac{\Omega_{PBH}}{\Omega_{CDM}} \sim 10^8 \left(\frac{M_{PBH}}{30 M_{\odot}} \right)^{-1/2} \sqrt{P_\delta} \exp\left[-\frac{1}{18 P_\delta} \right]$$

Upper bounds on the fraction to CDM

Carr, Kohri, Sendouda, J.Yokoyama (2009)(2022)



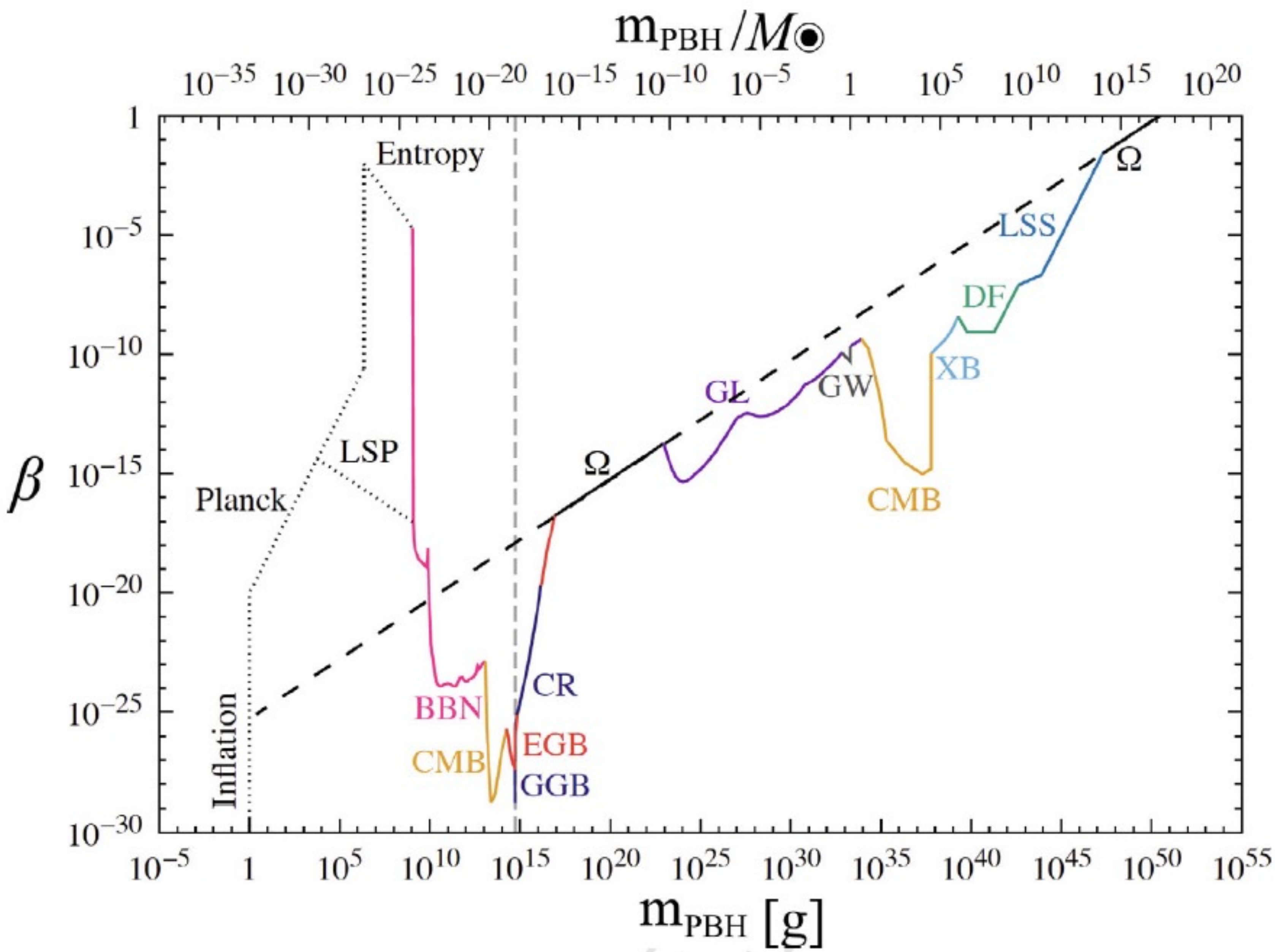
$f_{PBH}=\Omega_{PBH}/\Omega_{CDM}$

M_{PBH}

Mass in gram

Upper bounds on the fraction to CDM

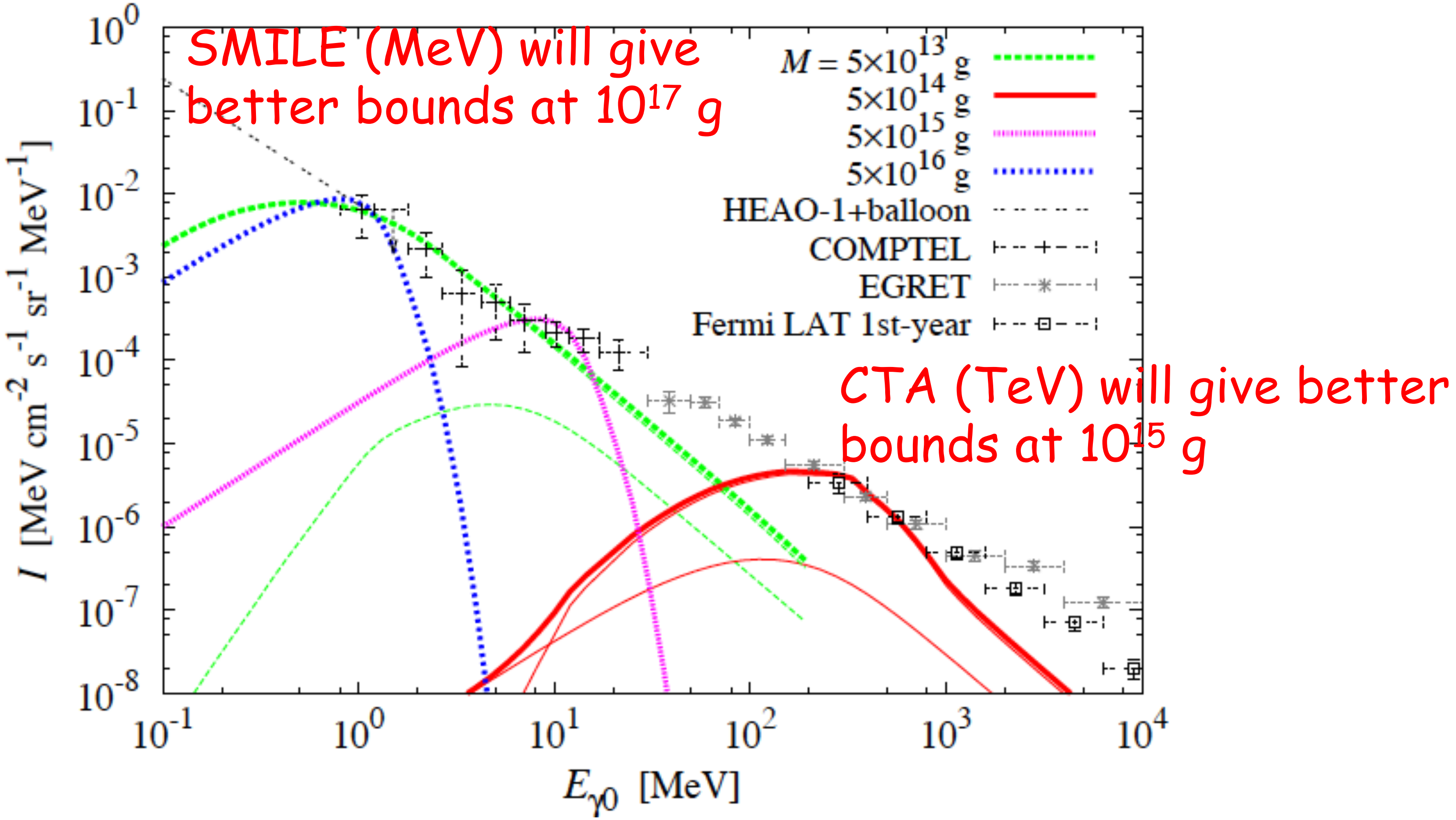
Carr, Kohri, Sendouda, J.Yokoyama (2009)(2022)



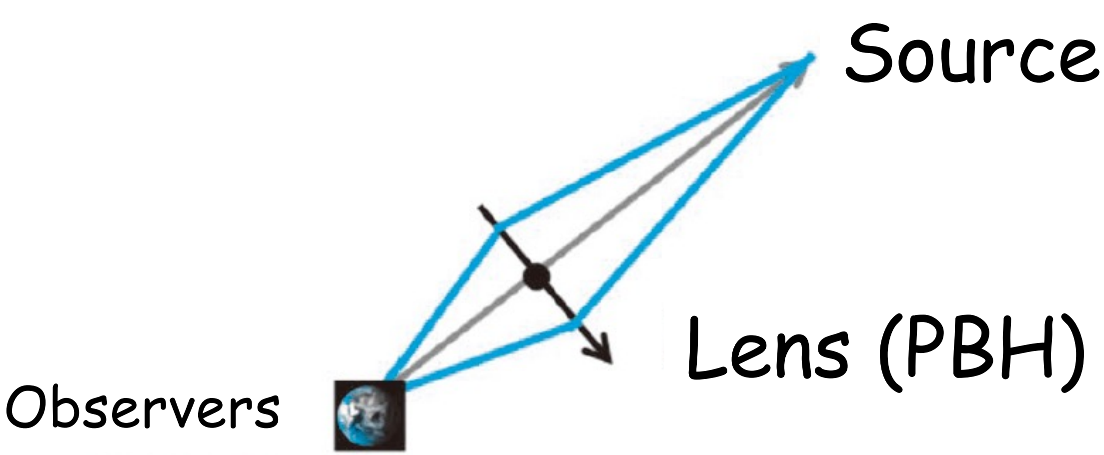
Evaporating PBHs through Hawking Process

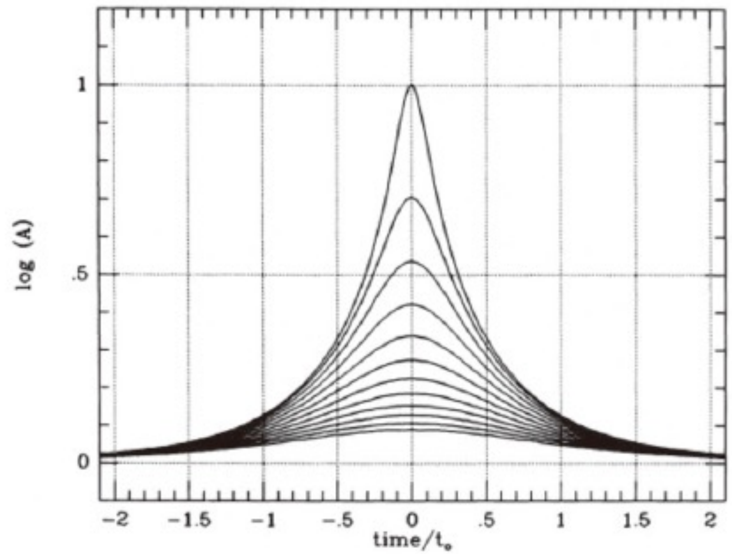
Carr, Kohri, Sendouda and Yokoyama (2010)

$$\mathrm{d}\dot{N}_s = \frac{\mathrm{d}E}{2\pi}\,\frac{\Gamma_s}{e^{E/T_{\rm BH}} - (-1)^{2s}}$$



Gravitational Lensing





Hiroko Niikura, https://stg.asj.or.jp/jp/activities/geppou/item/113-1_6.pdf

M31 lensing on PBHs modified by size-distribution and finite-size effects on bright star sources

Nolan Smyth, Stefano Profumo, Samuel English, Tesla Jeltema, Kevin McKinnon, Puragra Guhathakurta, arXiv:1910.01285 [astro-ph.CO]

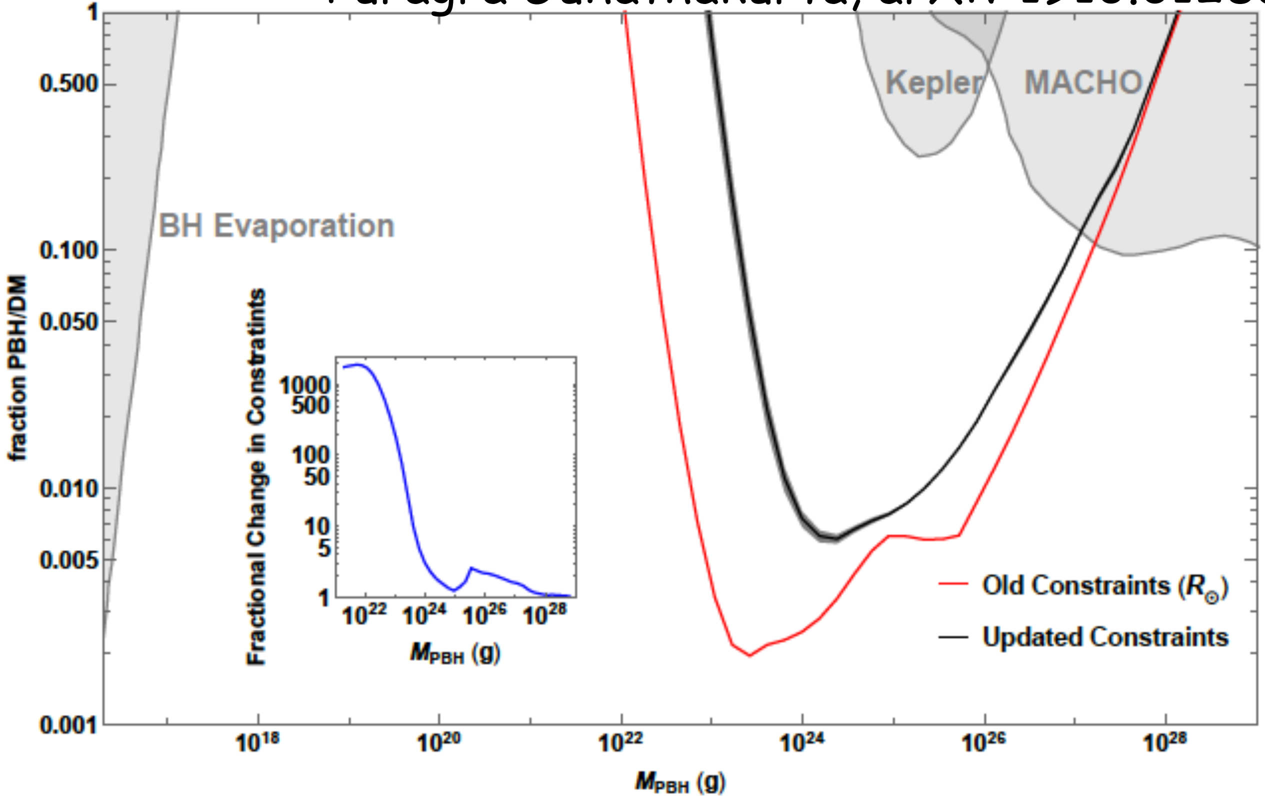
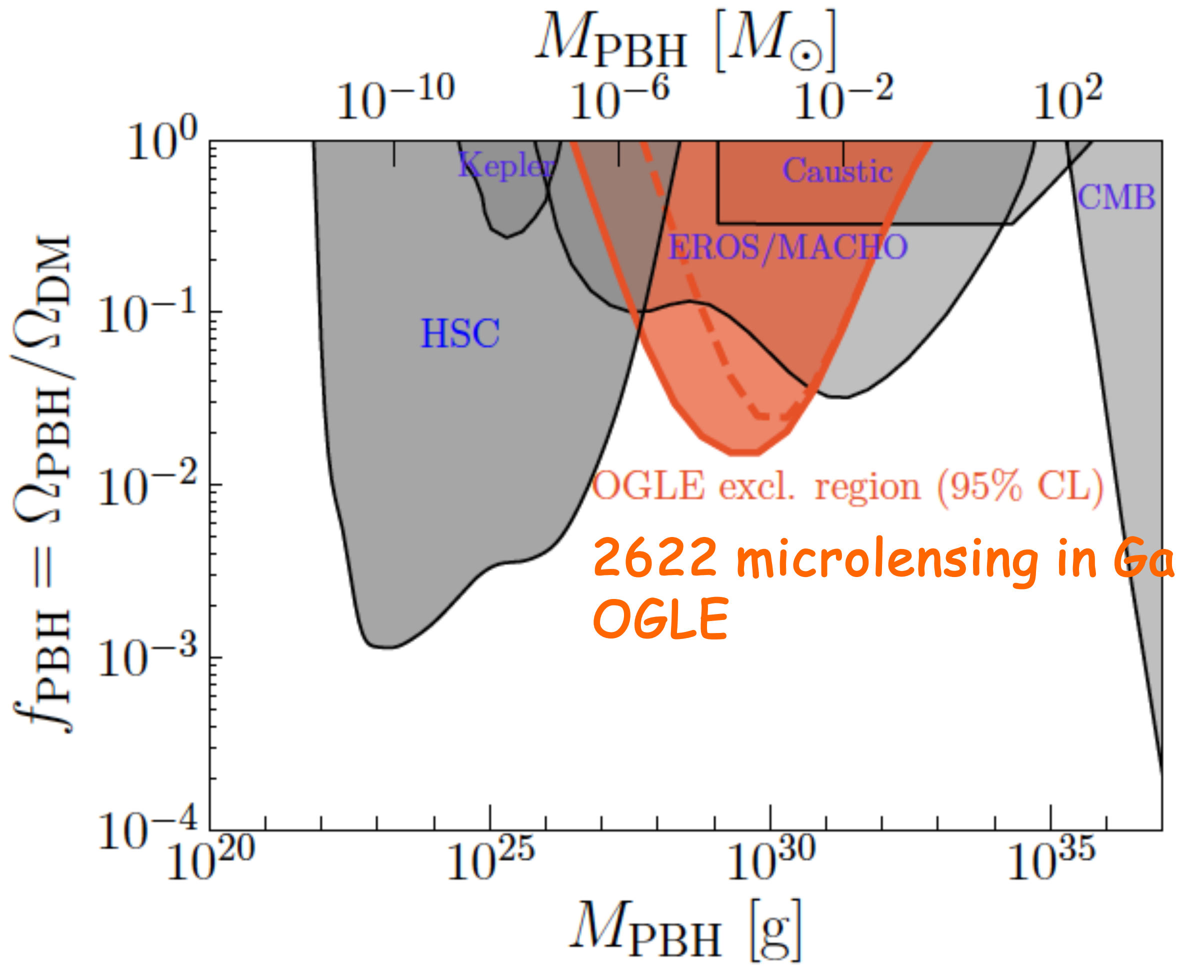


Figure 2. The constraints on primordial black holes as dark matter. The black line is the benchmark constraint and the primary result of this paper. The gray shading comes from the uncertainty in determining the stellar size distribution. The red line is

Gravitational lensing constrains on PBHs

Hiroko Niikura, Masahiro Takada, Shuichiro Yokoyama, Takahiro Sumi, Shogo Masaki,
arXiv:1901.07120 [astro-ph.CO]

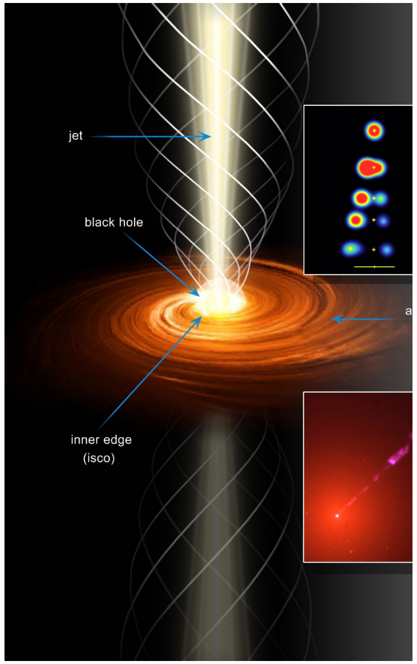


2622 microlensing in Galactic bulge by OGLE

CMB bound on PBHs by disk-accretion in the late MD epoch

Poulin, Serpico, Calore, Clesse, KK (2017)

- A non-spherical accretion disk (ADAF(slim) + Standard disk) around a PBH caused by an angular momentum emits radiation



$$\dot{M}_{\rm HB} \equiv 4\pi\lambda\,\rho_\infty v_{\rm eff} r_{\rm HB}^2 \equiv 4\pi\lambda\,\rho_\infty \frac{(GM)^2}{v_{\rm eff}^3}$$

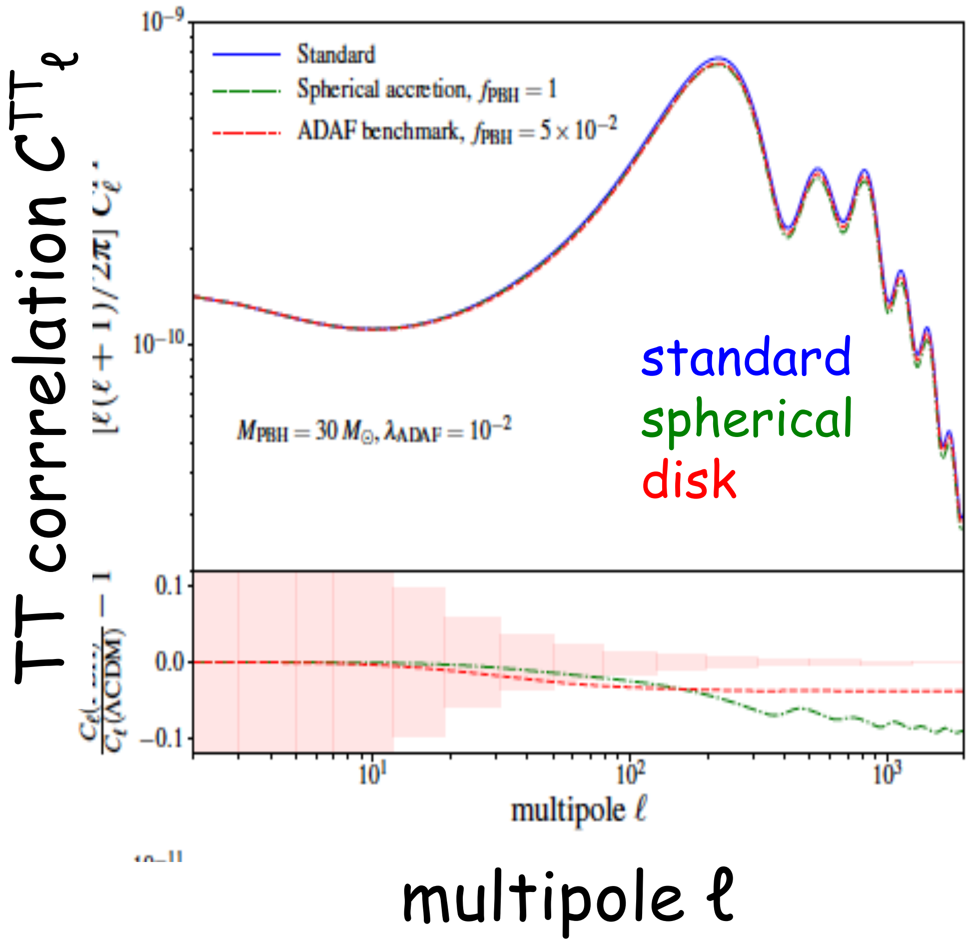
$$l \simeq \omega\, r_{\rm HB}^2 \simeq \left(\frac{\delta\rho}{\rho} + \frac{\delta v}{v_{\rm eff}}\right) v_{\rm eff} r_{\rm HB}$$

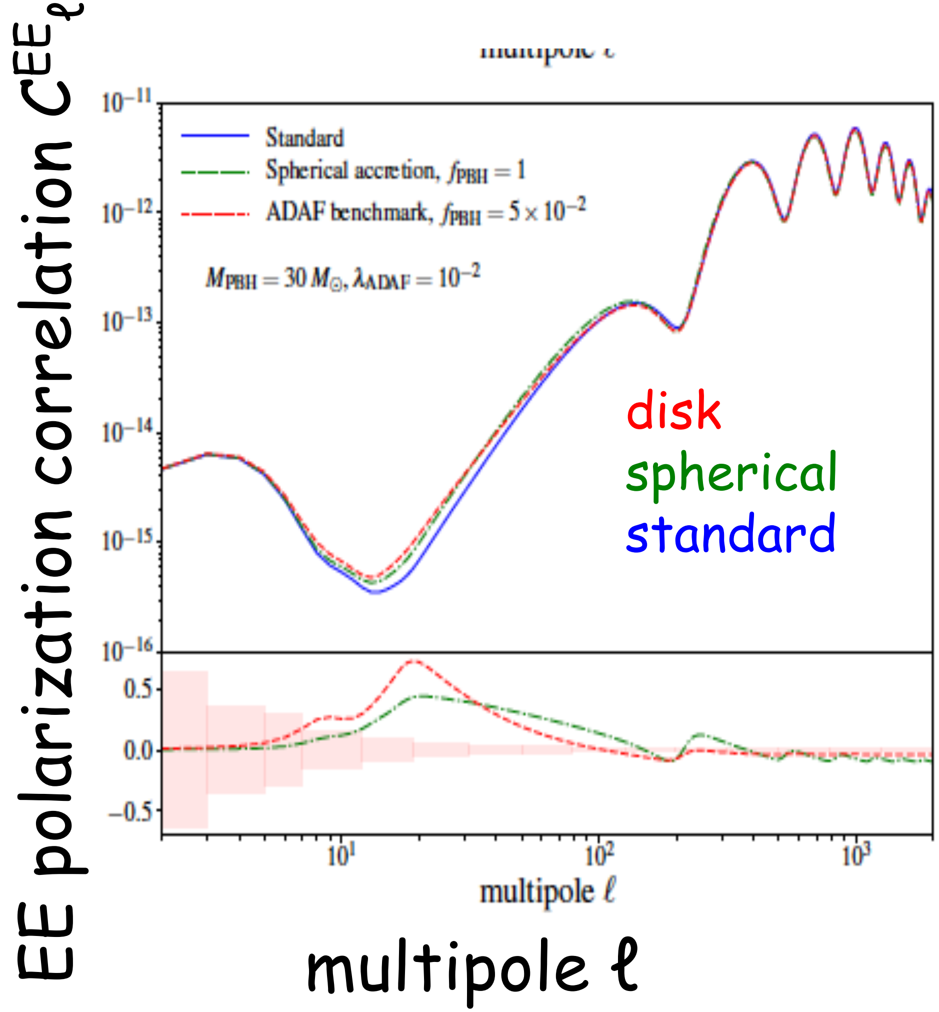
- CMB polarizations are affected

- From observations, we can constrain the number density of PBHs

Modified CMB anisotropy

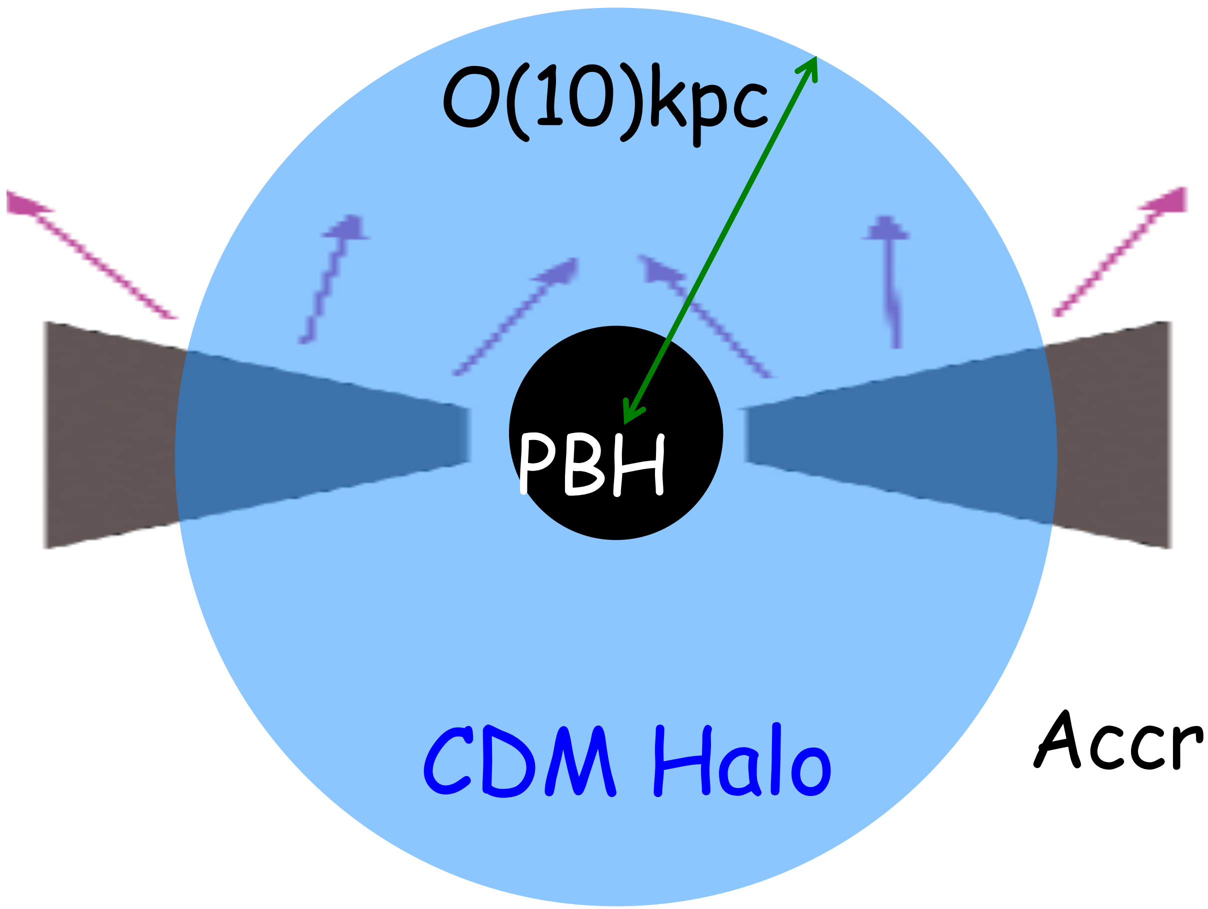
Poulin, Serpico, Calore, Clesse, Kohri (2017)





Cosmological baryon accretion onto the PBH + CDM halo system

Poulin, Serpico, Inman, Kohri (2020)



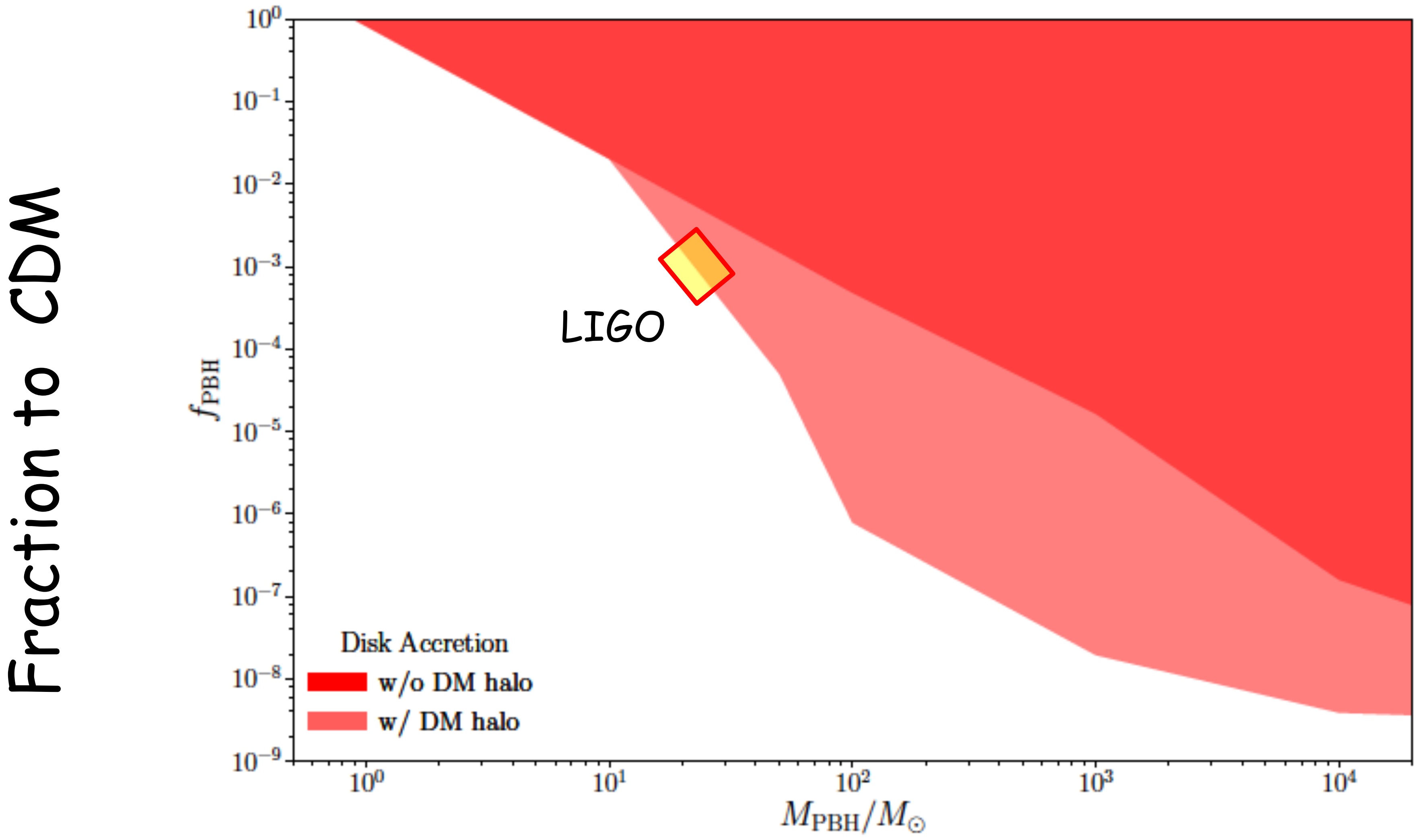
$\rho \propto r^{-2.25}$

Accretion onto CDM + PBH

z ~30 -- 100

CMB bound by disk-accretion in the MD epoch

Serpico, Poulin, Calore, Clesse, Kohri (2017)

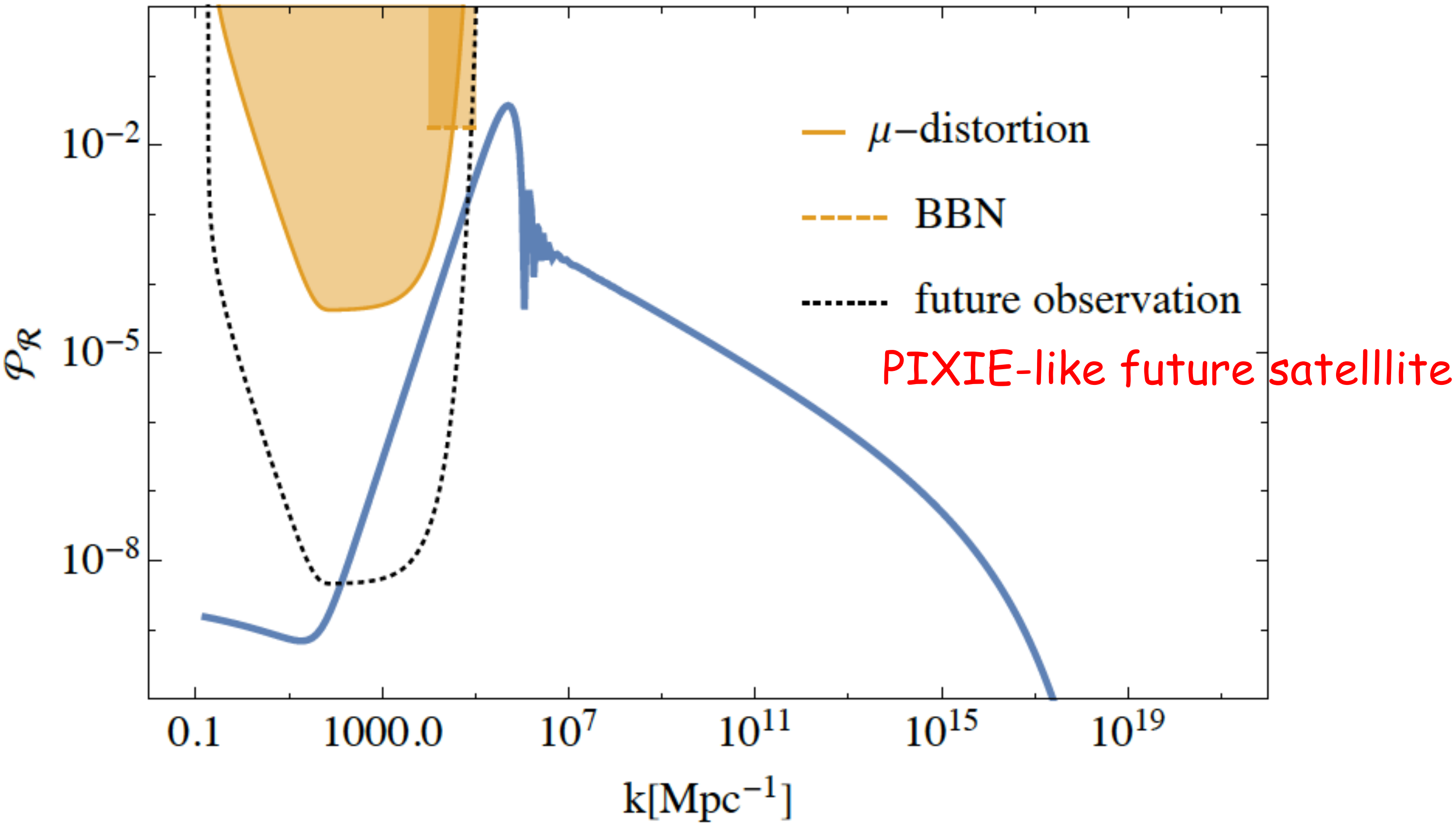


μ-distortion and acoustic reheating

~~e+γ+γ→e+γ : double Compton~~

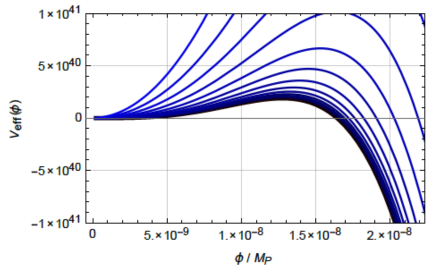
Kohri, Nakama, Suyama (2014)

e+γ→e+γ: Compton



Inomata, Kawasaki, Mukaida, Tada, Yanagida (2017)

Higgs stabilization due to evaporating PBHs?



Kohri and Matsui (2017)

- Potential with finite-temperature corrections

$$V_{\rm eff}(\phi) \simeq \frac{1}{2}\left(\lambda_{\rm eff} T_{\rm H}^2 + \kappa^2 T_{\rm H}^2\right)\phi^2 + \frac{\lambda_{\rm eff}}{4}\phi^4$$

$$\phi_{\rm max}^2 / T_{\rm H}^2 \approx \mathcal{O}(10)$$

- Probability to get over the potential

$$P(\phi > \phi_{\rm max}) \simeq \frac{\sqrt{2\langle\delta\phi^2\rangle_{\rm ren}}}{\pi\phi_{\rm max}} \exp\left(-\frac{\phi_{\rm max}^2}{2\langle\delta\phi^2\rangle_{\rm ren}}\right)$$

$$\langle\delta\phi^2\rangle_{\rm ren} / T_{\rm H}^2 \simeq \mathcal{O}(0.1)$$

- This gives,

$$\phi_{\rm max}^2 / \langle\delta\phi^2\rangle_{\rm rem} \sim 10^2$$

$$\mathcal{N}_{\rm PBH} \cdot P(\phi > \phi_{\rm max}) \lesssim 1$$

or

$$\beta \lesssim \mathcal{O}\left(10^{-21}\right)\left(\frac{m_{\rm PBH}}{10^9 {\rm g}}\right)^{3/2}$$

Secondary gravitational wave induced from large curvature perturbation (P_ζ >> r) at small scales

K. N. Ananda, C. Clarkson, and D. Wands, 2006
D.Baumann, P.J.Steinhardt, K.Takahashi and K.Ichiki,2007
R.Saito and J.Yokoyama, 2008
KK and T.Terada, 2018
R.-G. Cai, S. Pi, and M. Sasaki, 2019

- Power spectrum of the tensor mode

$$\langle h^r_{\boldsymbol{k}}(\eta) h^s_{\boldsymbol{k}'}(\eta)\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_h(k,\eta)\delta(\boldsymbol{k}+\boldsymbol{k}')\delta^{rs}, \qquad h_{ij}(\boldsymbol{x},\eta) = \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \left[h^+_{\boldsymbol{k}}(\eta)\mathrm{e}^+_{ij}(\boldsymbol{k}) + h^\times_{\boldsymbol{k}}(\eta)\mathrm{e}^\times_{ij}(\boldsymbol{k})\right]$$

- Omega parameter well inside the horizon

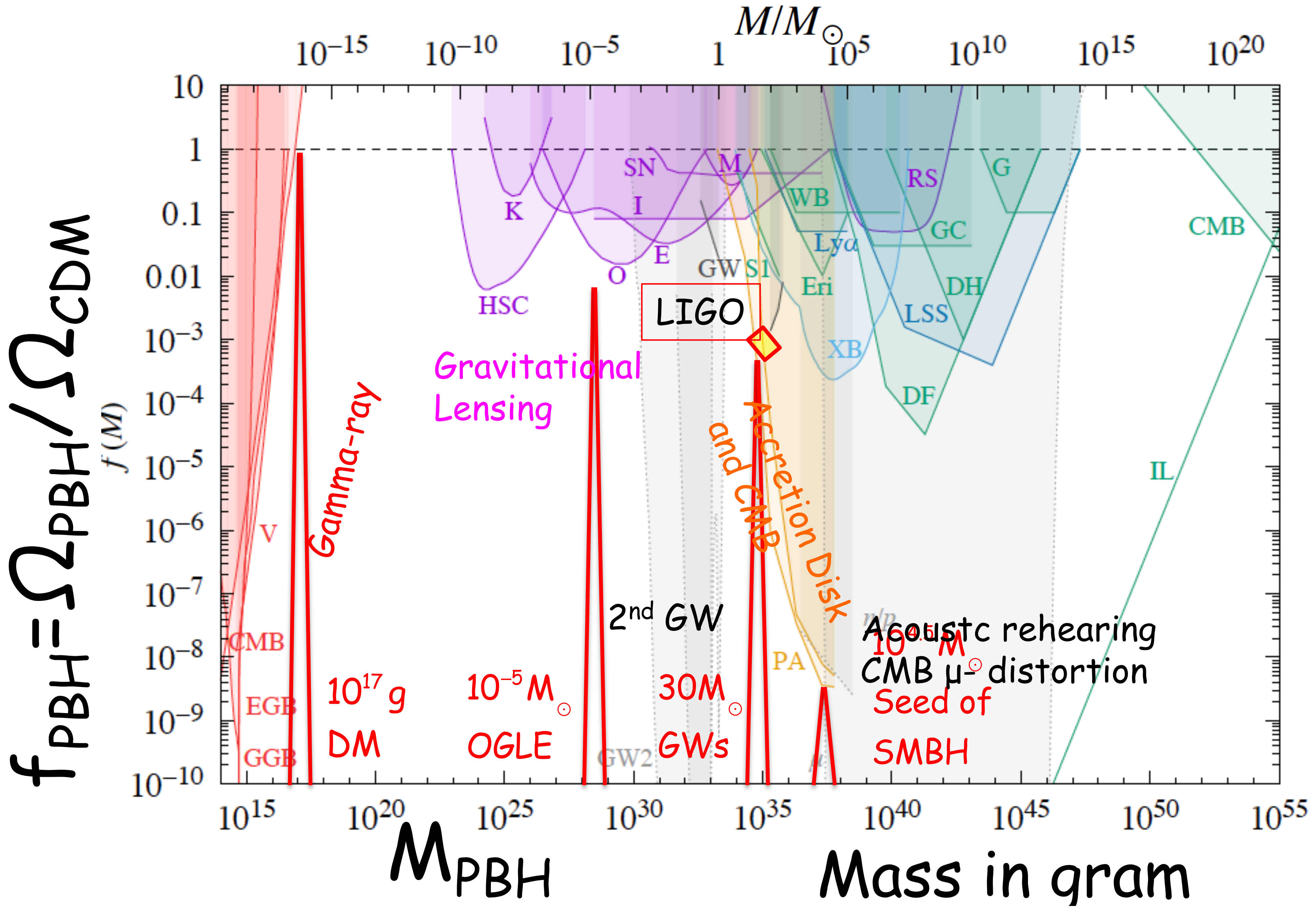
$$\Omega_{\mathrm{GW}}(k,\eta) = \frac{1}{3}\left(\frac{k}{\mathcal{H}}\right)^2 \mathcal{P}_h(k,\eta).$$

- Substituting the solution into this

$$\Omega_{\mathrm{GW,c}}(f) = \frac{1}{12}\left(\frac{f}{2\pi a H}\right)^2 \int_0^\infty \mathrm{d}t \int_{-1}^1 \mathrm{d}s \left[\frac{t(t+2)(s^2-1)}{(t+s+1)(t-s+1)}\right]^2 \times \overline{I^2(t,s,k\eta_c)}\,\mathcal{P}_\zeta\left(\frac{(t+s+1)f}{4\pi}\right)\mathcal{P}_\zeta\left(\frac{(t-s+1)f}{4\pi}\right)$$

Upper bounds on the fraction to CDM

Carr, Kohri, Sendouda, J.Yokoyama (2009-2022)



How to test PBHs?

- LIGO events ($\sim$30 $M_\odot$)
  - Strong lensing of FRBs
  - Anisotropies of GWs from PBHs
- DM (10^{17}g – 10^{23}g)
  - Induced GWs
  - MeV Gamma-ray
- Seeds of SMBHs ($\sim 10^4$ $M_\odot$)
  - Cosmological 21cm at $z > \sim O(10)$
  - CMB μ-distortion

Mechanisms to produce PBHs

- Chaotic-New inflation: J. Yokoyama, 1998), Multi-field inflation (Kawasaki, Sugiyama, Yanagida, 1998, …
- At the end of inflation: Lyth, Malik, Sasaki, Zabarra (2006), Preheating: Green and Malik (1999), Taruya (1998) …
- Blue-tilted spectrum (perturbative) Leach Grivell and Liddle, 2001, Kohri, Lyth and Melchiorri, 2007, …
- Ultra-slowroll? see Kristiano and J.Yokoyama, 2023, A. Riotto, 2023, …
- Tachyonic instability : Dhong Yeon Cheong, Kazunori Kohri, Seong Chan Park, arXiv:2205.14813
- Curvaton: Kawasaki, Kitajima, Yanagida (2012), Kohri, Lin, Matsuda (2012), …
- 1st-order Phase transition (+ pre-existing large curvature perturbation A_s)
  Byrnes, Hindmarsh, Young, Hawkins, 2018, Abe, Tada, Ueda, 2020, Franciolini, Musco, Pani, Urbano, 2022, Hashino, Kanemura, Tomo Takahashi, and M. Tanaka, 2022, …
- Collapse of Q-balls or topological defects (monopole, cosmic string, domain wall):
  Cotner, Kusenko, Sasaki, Takhistov, 2019, Hasegawa and Kawasaki, 2018, …
- Extra attractive forces (Yukawa interaction, …) : Kawana and Xie, 2021, Lu, Kawana, Kusenko, 2023, …
- …

Type-III Hilltop inflation models

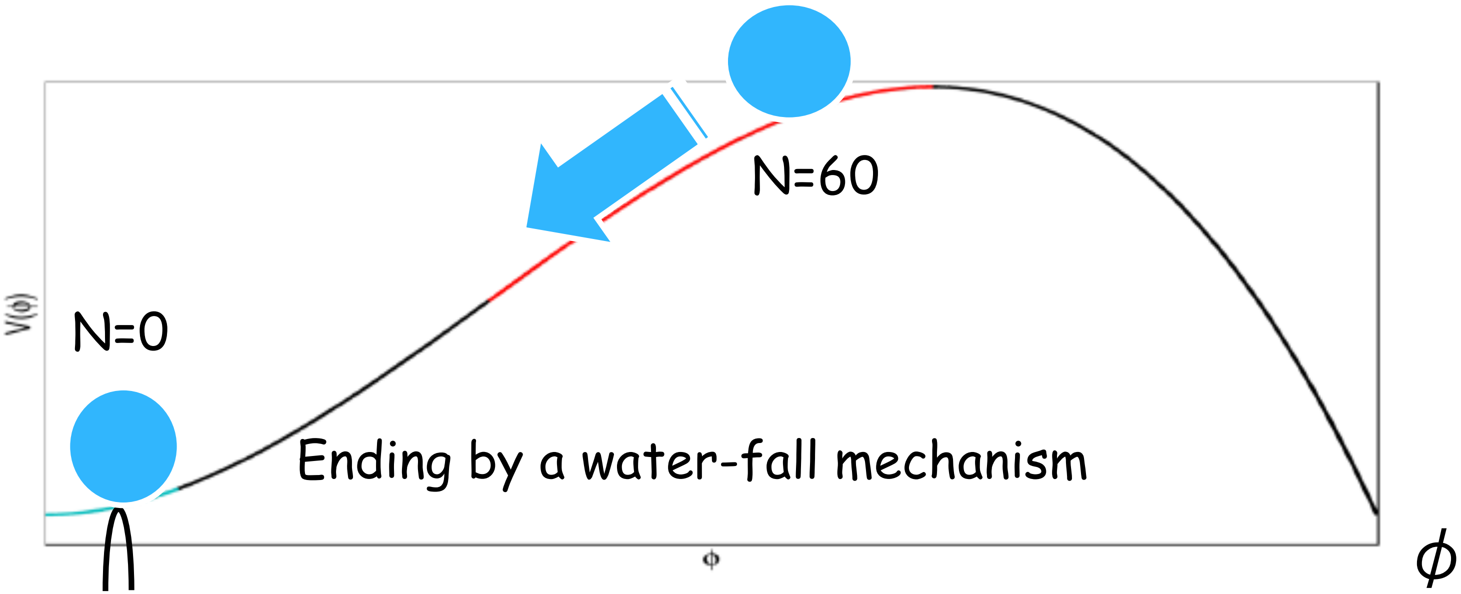
German, Ross, Sarkar (01)
KK, Lin and Lyth (07)

- Potential in supergravity, e.g.,

$$V(\phi) \;=\; V_0 + \frac{1}{2}m^2\phi^2 - \lambda\frac{\phi^p}{M_P^{p-4}} + \cdots$$

$$W = C\frac{\phi^\rho}{M_{pl}^{p-3}}, \quad \lambda \sim C m_{3/2}/M_{pl}$$ in SUGRA

Allahverdi, Kusenko and Mazumdar (06)



Large running spectral index

KK, Lin and Lyth (07)

- Spectrum

$$P_\zeta \sim \frac{V}{m_{\rm pl}^4 \varepsilon}$$

- Enhanced curvature perturbation at small scales due to a large running of running

$$\varepsilon \equiv \frac{1}{2}\left(m_{\rm pl}\frac{V'}{V}\right)^2 \to 0 \text{ for } \phi \downarrow$$

$$\beta_s = \frac{d^3 P_\zeta}{d(\ln k)^3} = 192\epsilon^3 + 192\epsilon^2\eta - 32\epsilon\eta^2 + (-24\epsilon + 2\eta)\xi^{(2)} + 2\sigma^{(3)}$$

Could be large!

Simple parameterization of running of spectral indexes of curvature perturbation

- Curvature perturbation

$$P_\zeta(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{\alpha_s}{2}\ln\left(\frac{k}{k_*}\right) + \frac{\beta_s}{6}\left(\ln\left(\frac{k}{k_*}\right)\right)^2}$$

$$A_s \equiv P_\zeta|_* \sim \left.\frac{V}{m_{\rm pl}^4 \varepsilon}\right|_* \sim (\delta T/T)^2$$

$$\varepsilon \equiv \frac{1}{2}\left(m_{\rm pl}\frac{V'}{V}\right)^2$$

- spectral index

$$n_s - 1 = dP_\zeta / d\ln k = 2\eta - 6\varepsilon$$

$$\eta \equiv m_{\rm pl}^2 \frac{V''}{V}$$

- running of n_s

$$\alpha_s = dn_s/d\ln k = -24\varepsilon^2 + 16\varepsilon\eta - \xi^{(2)}$$

$$\xi^{(2)} \equiv m_{\rm pl}^4 \frac{V'V'''}{V^2}$$

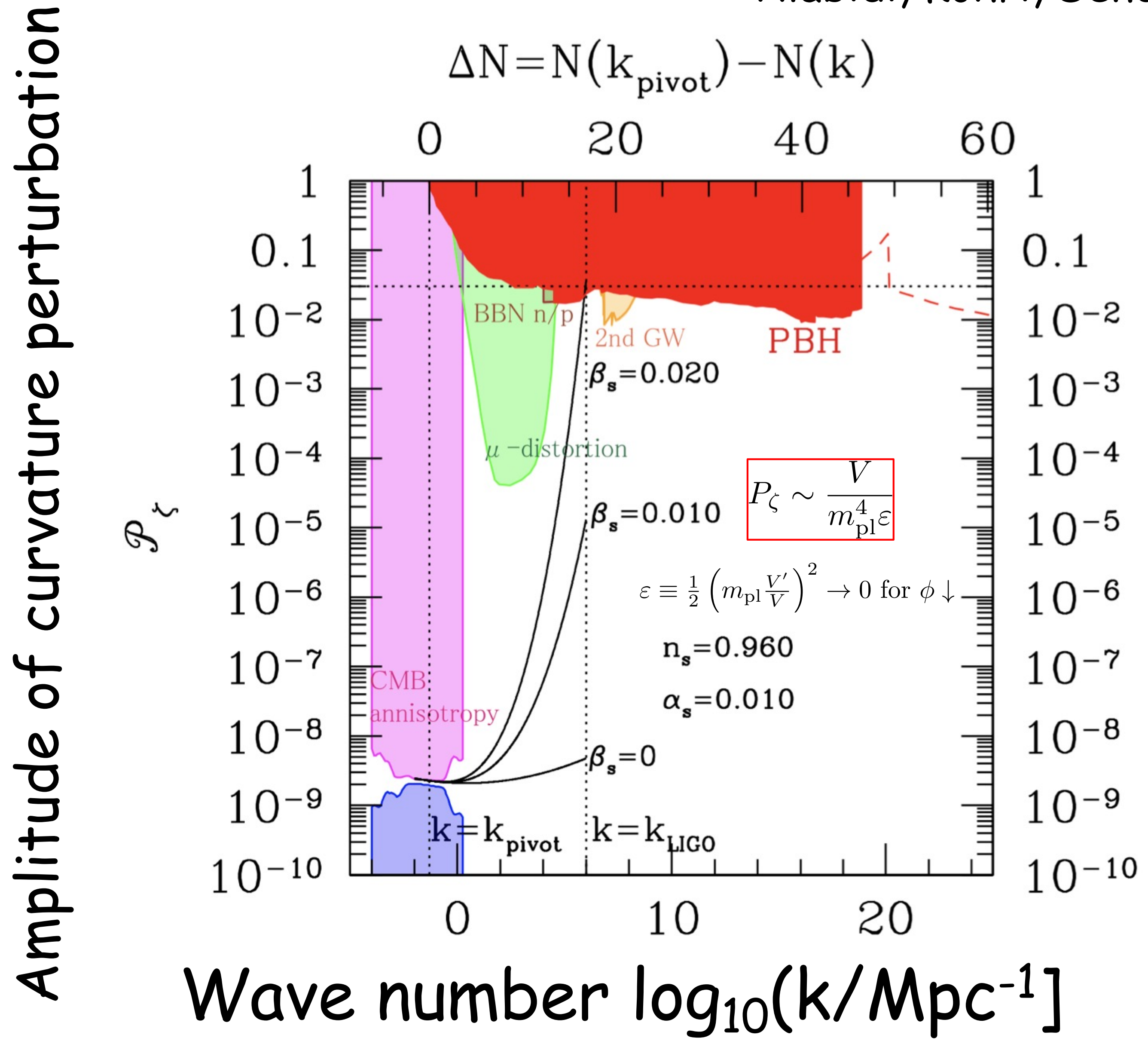
- running of running of n_s

$$\sigma^{(3)} \equiv m_{\rm pl}^6 \frac{(V')^2 V''''}{V^3}$$

$$\beta_s = d\alpha_s/d\ln k = 192\varepsilon^3 + 192\varepsilon^2\eta - 32\varepsilon\eta^2 + (-24\varepsilon + 2\eta)\xi^{(2)} + 2\sigma^{(2)}$$

Curvature perturbation $P_\zeta(k)$

Kohri and T.Terada, 2018
Alabidi, Kohri, Sendouda, Sasaki, 2013



Planck (2018)

$n_s = 0.9586 \pm 0.0056,$

$\alpha_s = 0.009 \pm 0.010,$

$\beta_s = 0.025 \pm 0.013.$

at 68% C.L.

For inflation models with a big running, see Kohri, Lin Lyth (2008)

k = p x a

Higgs-R^2 Inflation

Dhong Yeon Cheong, Kazunori Kohri, Seong Chan Park, arXiv:2205.14813 [hep-ph]

- Action of Higgs and R^2

$$S_J = \int d^4x\sqrt{-g_J}\left[\frac{M_P^2}{2}\left(R_J + \frac{\xi h^2}{M_P^2}R_J + \frac{R_J^2}{6M^2}\right) - \frac{1}{2}g^{\mu\nu}\nabla_\mu h\nabla_\nu h - \frac{\lambda(\mu)}{4}h^4\right]$$

- Conformal transformation

$$\alpha = M_P^2/12M^2$$

$$\sqrt{\frac{2}{3}}\frac{s}{M_P} = \ln\left(1 + \frac{\xi h^2}{M_P^2} + \frac{R_J}{3M^2}\right) \equiv \Omega(s).$$

- Action of scalaron (s) and Higgs (h)

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}G_{ab}g^{\mu\nu}\nabla_\mu\phi^a\nabla_\nu\phi^b - U(\phi^a)\right]$$

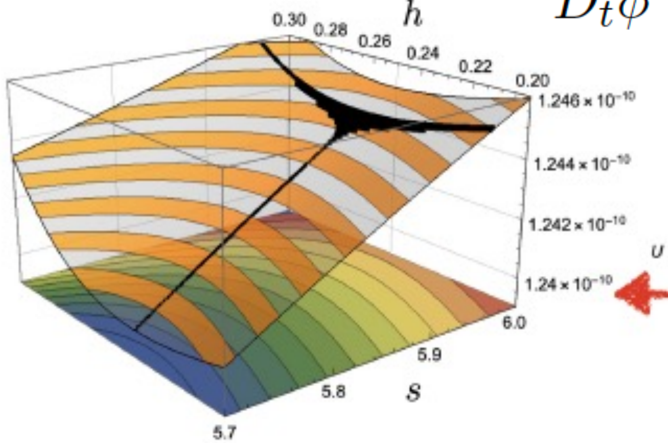
$$U(\phi^a) \equiv e^{-2\Omega(s)}\left\{\frac{3}{4}M_P^2M^2\left(e^{\Omega(s)} - 1 - \frac{\xi h^2}{M_P^2}\right)^2 + \frac{\lambda(\mu)}{4}h^4\right\}$$

$$g_{\mu\nu} = e^{\Omega(s)}g_{\mu\nu}^J \qquad G_{ab} = \begin{pmatrix}1 & 0\\ 0 & e^{-\Omega(s)}\end{pmatrix}$$

Motions on the potential of the Higgs-scalaron (s) system

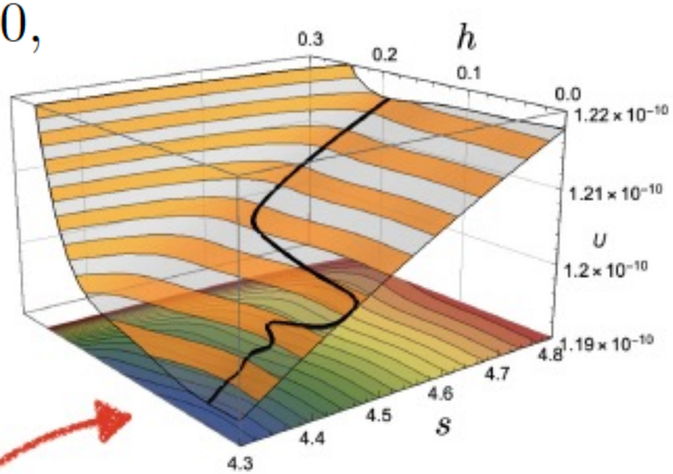
Dhong Yeon Cheong, Kazunori Kohri, Seong Chan Park, arXiv:2205.14813 [hep-ph]

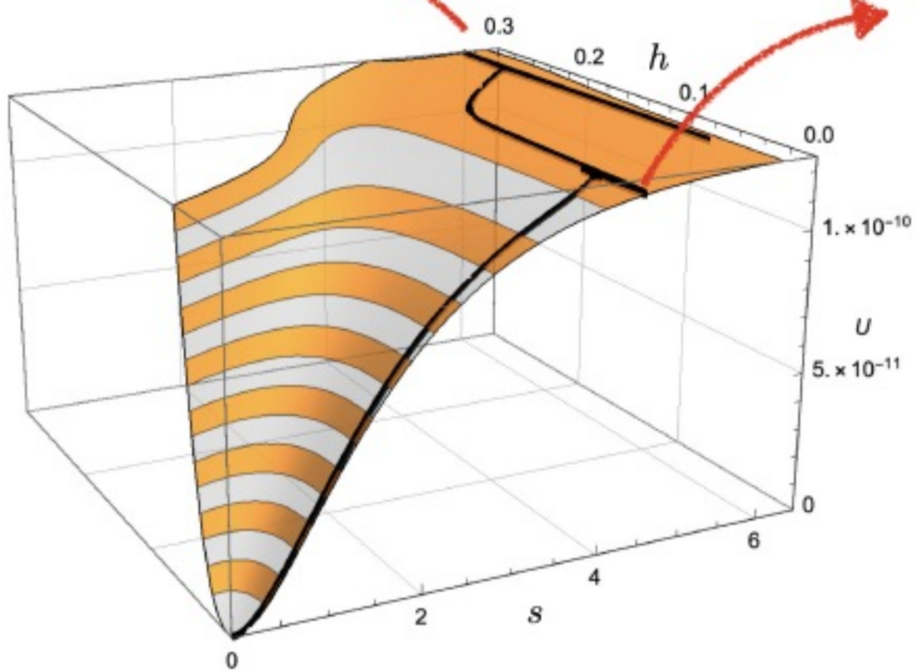
$$D_t\dot{\phi}^a + 3H\dot{\phi}^a + G^{ab}D_bU = 0,$$



$\phi^s = s$

$\phi^h = h$





Tachyonic Instability induced in Higgs-R^2 Inflation

Dhong Yeon Cheong, Kazunori Kohri, Seong Chan Park, arXiv:2205.14813 [hep-ph]

$$\ddot{Q}_N + 3H\dot{Q}_N + \left(\frac{k^2}{a^2} + M_{\rm eff}^2\right) Q_N = 2\dot{\phi}_0 \eta_\perp \dot{\mathcal{R}}$$

$$M_{\rm eff}^2 = U_{NN} + H^2 \epsilon \mathbb{R} - \dot{\theta}^2 \qquad \boxed{U_{NN} \ < \ 0,}$$

$$M_{\rm eff}^2 \simeq \frac{1}{\dot{s}^2 + e^{-\sqrt{\frac{2}{3}}s}\dot{h}^2}\left(e^{\sqrt{\frac{2}{3}}s}\dot{s}^2 \frac{\partial^2 U}{\partial h^2}\right) \simeq -3M^2\xi\left(1 - e^{-\sqrt{\frac{2}{3}}s}\right).$$

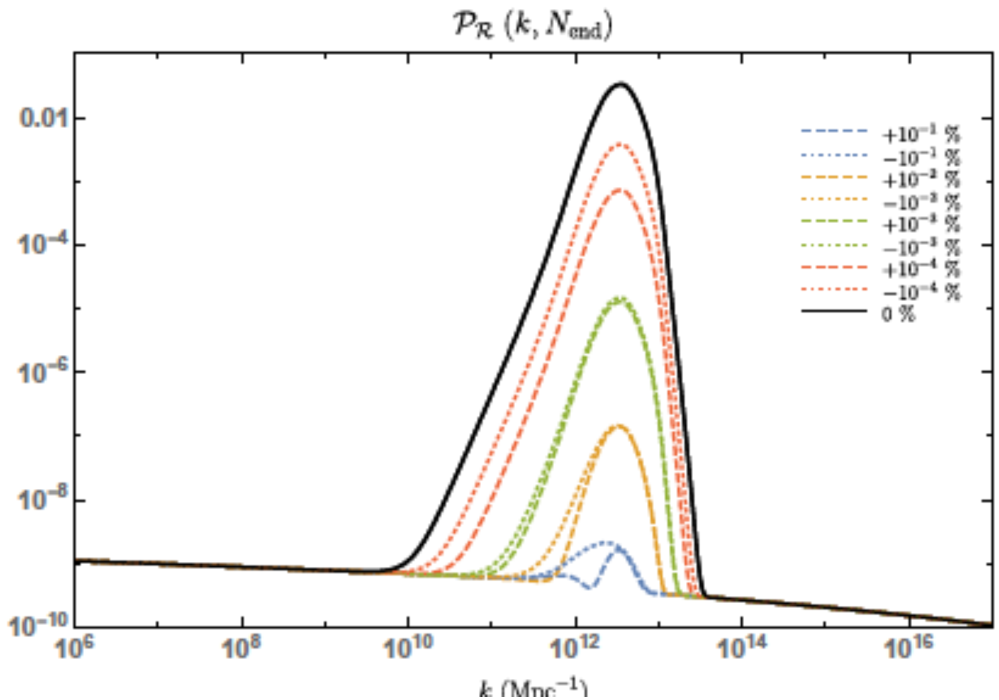
Hence Q_N can exhibit an *exponential* growth due to the tachyonic mass. This growth can be more rapid than cases implementing a USR phase.

$$Q_{N,k}(N_e) = e^{-\frac{3}{2}N_e}\left[d_3\, e^{-\frac{N_e}{2}\sqrt{9-4\frac{M_{\rm eff}^2}{H^2}-4\epsilon_k^2}} + d_4\, e^{\frac{N_e}{2}\sqrt{9-4\frac{M_{\rm eff}^2}{H^2}-4\epsilon_k^2}}\right]$$

$$\xrightarrow[|M_{\rm eff}^2| \gg H^2]{\epsilon_k^2 \ll 1} d_4\, e^{\left(\frac{|M_{\rm eff}|}{H} - \frac{3}{2}\right)N_e}$$

Primordial Black Holes and Second Order Gravitational Waves from Tachyonic Instability induced in Higgs-R^2 Inflation

Dhong Yeon Cheong, Kazunori Kohri, Seong Chan Park, arXiv:2205.14813 [hep-ph]



$$\delta\lambda_m/\lambda_m$$

$$\delta\lambda_m/\lambda_m \equiv (\lambda_m^{dev} - \lambda_m)/\lambda_m \sim 10^{-4}\,\%$$

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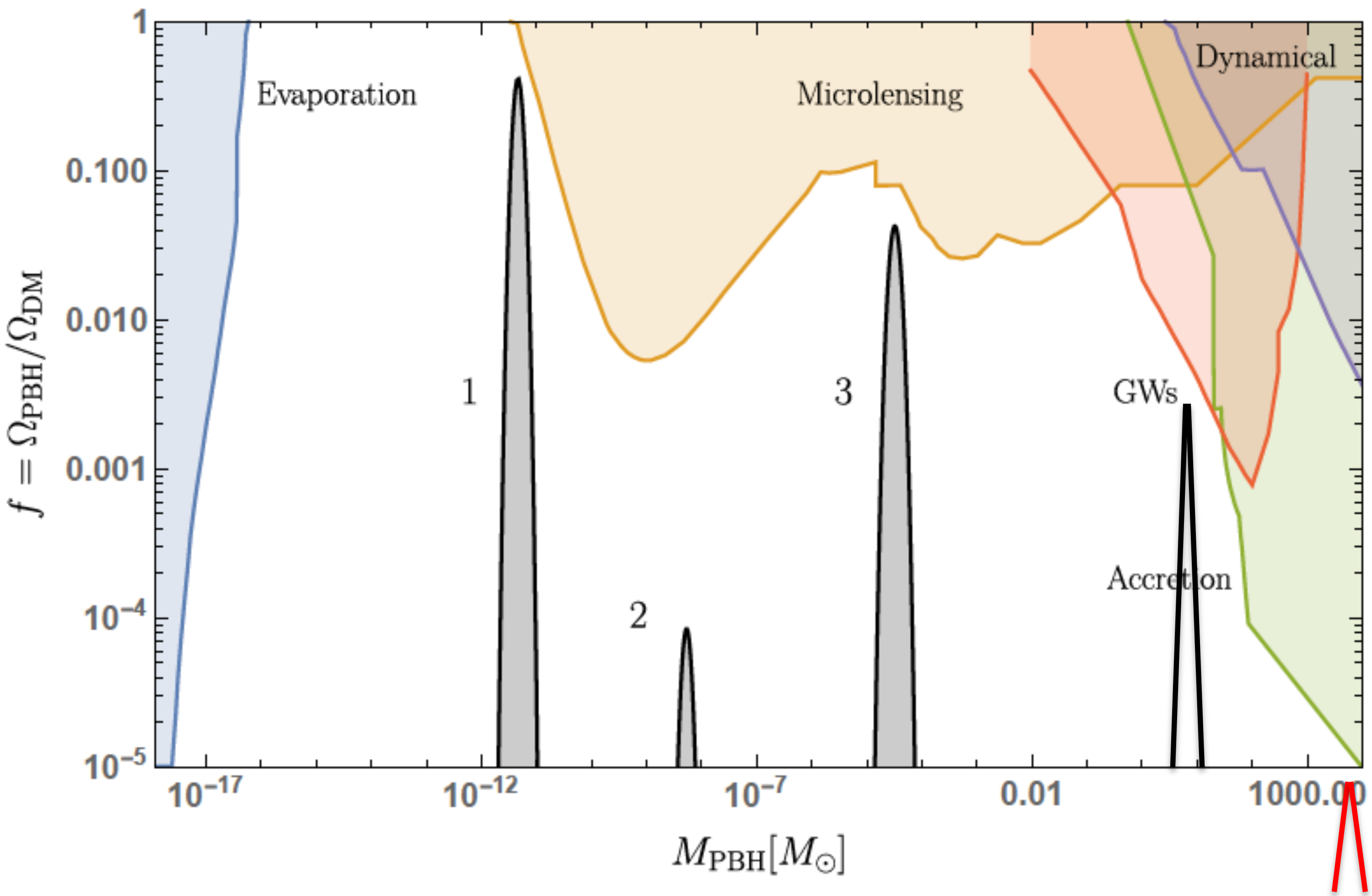

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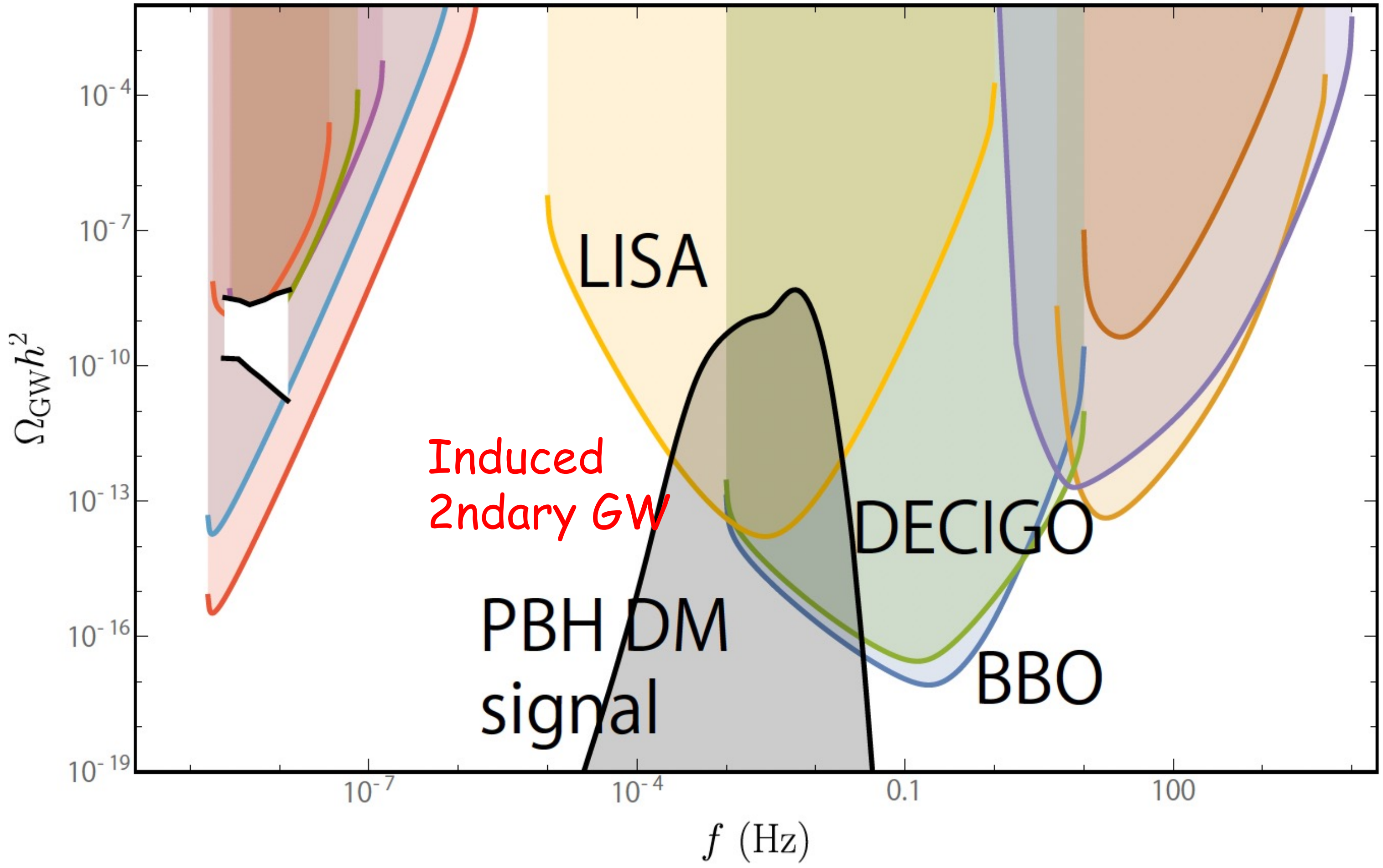
Dhong Yeon Cheong, Kazunori Kohri, Seong Chan Park, arXiv:2205.14813 [hep-ph]

See also, K. Kohri and T. Terada, arXiv:2009.11853

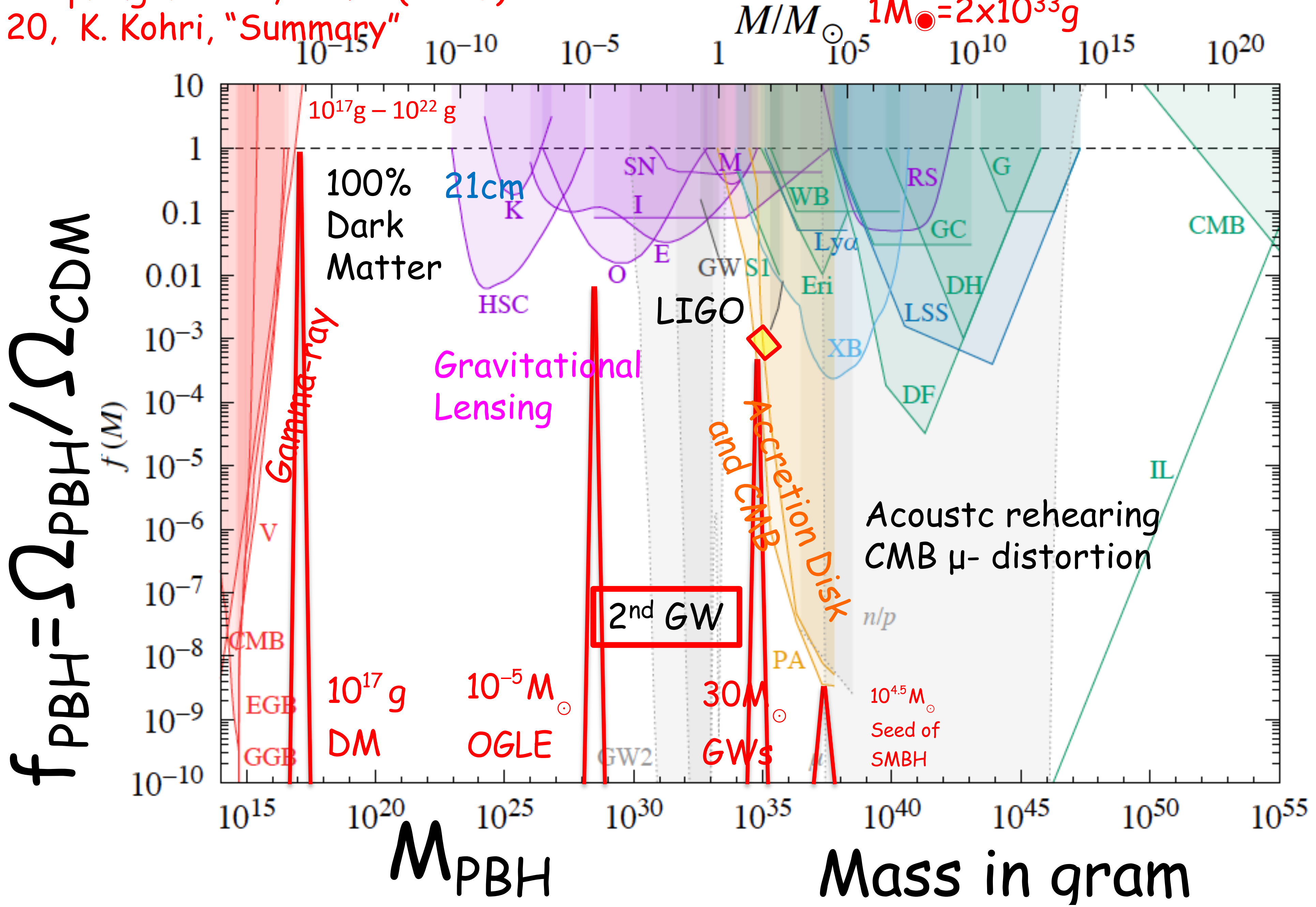


Pulsar Timing Array
and
Gravitational Wave Backgound

Upper bounds on the fraction to CDM

See Springer Book, "PBH" (2025)
§ 20, K. Kohri, "Summary"

Carr, Kohri, Sendouda, J.Yokoyama (2009)(2020)

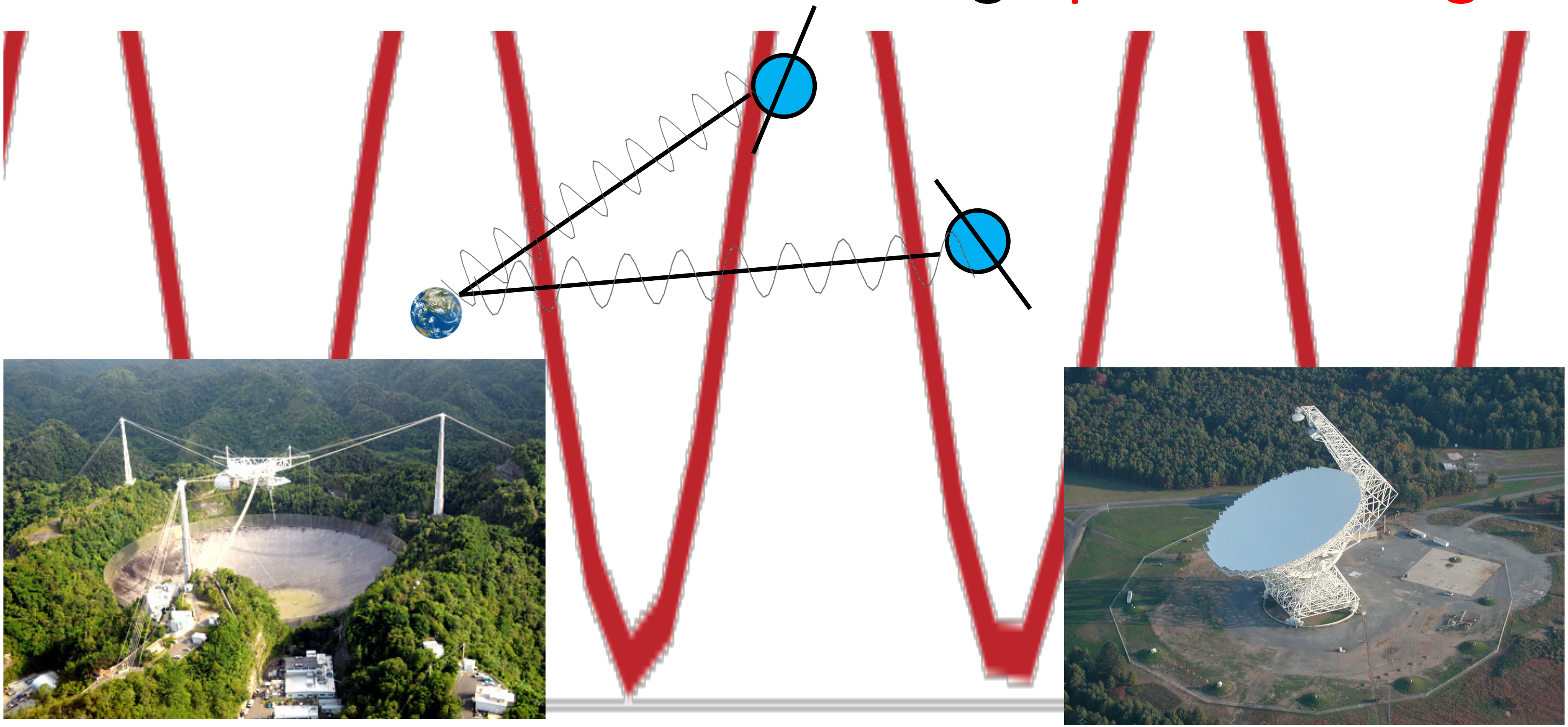


Gabriella Agazie, et al, The NANOGrav15yr collaboration, arXiv:2306.16213 [astro-ph.HE]

NANOGrav 15yr

(North American Nanohertz Observatory for Gravitational Waves)

found stochastic GWs through pulsar timing

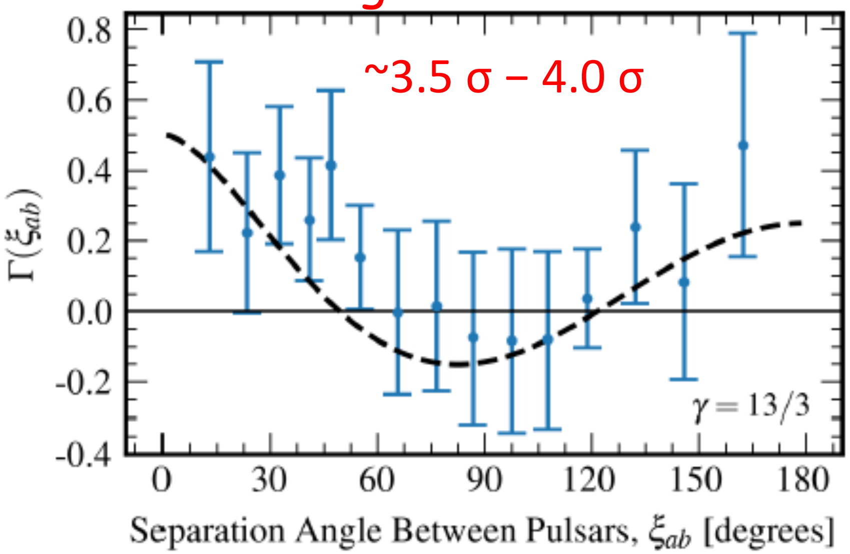


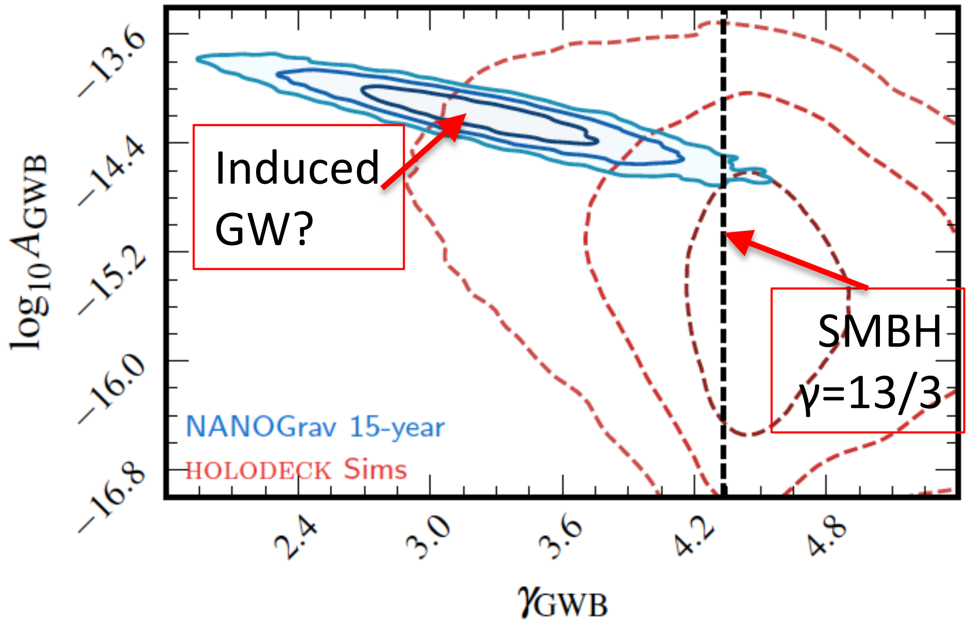
The 305-meter dish of the William E. Gordon Telescope, The Arecibo Obs.

The 100-meter Green Bank Telescope

The NANOGrav 15-year Data Set: Evidence for a Gravitational-Wave Background

Gabriella Agazie, et al, The NANOGrav15yr collaboration, arXiv:2306.16213 [astro-ph.HE]





$$h_c(f) = A_{\rm GWB}\left(\frac{f}{f_{\rm yr}}\right)^{\alpha}$$

$$S_{ab}(f) = \Gamma_{ab}\,\frac{A_{\rm GWB}{}^2}{12\pi^2}\left(\frac{f}{f_{\rm yr}}\right)^{-\gamma} f_{\rm yr}^{-3}$$

$$\Omega(f) = \frac{2\pi}{3H_0^2} f^2 h_c(f)^2 = \Omega_{yr}\left(\frac{f}{f_{\rm yr}}\right)^{\beta}$$

γ = 3 – 2α = 5 – β

β = 5 – γ (∼2)

Secondary gravitational wave induced from large curvature perturbation ($P_\zeta >> r$) at small scales

K. N. Ananda, C. Clarkson, and D. Wands, 2006
D. Baumann, P. J. Steinhardt, K. Takahashi and K. Ichiki,2007
R.Saito and J. Yokoyama, 2008
K. Kohri and T. Terada, 2018
R.-G. Cai, S. Pi, and M. Sasaki, 2019

- Power spectrum of the tensor mode

$$\langle h^r_{\boldsymbol{k}}(\eta) h^s_{\boldsymbol{k}'}(\eta)\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_h(k,\eta)\delta(\boldsymbol{k}+\boldsymbol{k}')\delta^{rs}, \qquad h_{ij}(\boldsymbol{x},\eta) = \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \left[h^+_{\boldsymbol{k}}(\eta)\mathrm{e}^+_{ij}(\boldsymbol{k}) + h^\times_{\boldsymbol{k}}(\eta)\mathrm{e}^\times_{ij}(\boldsymbol{k})\right]$$

- Omega parameter well inside the horizon

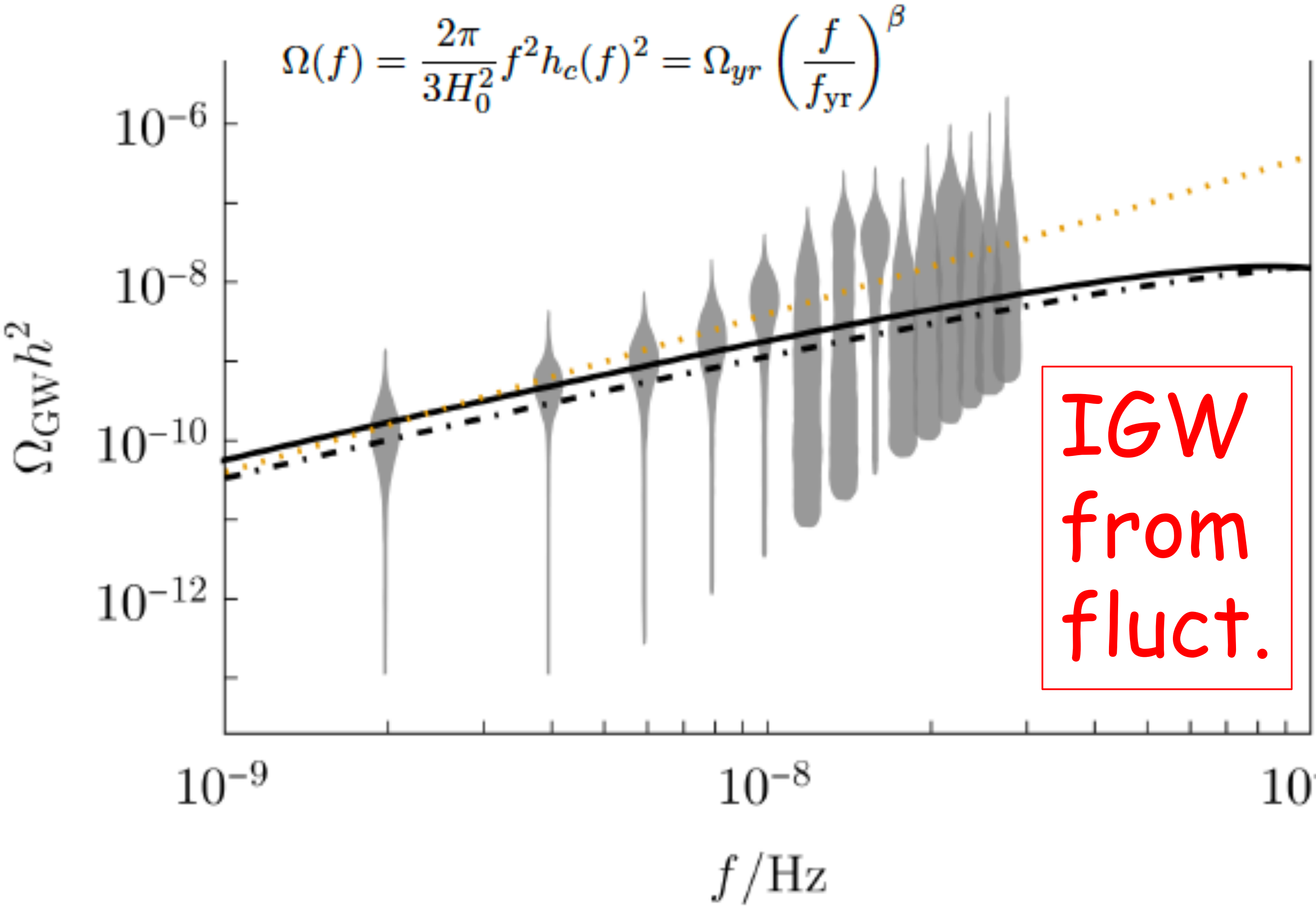
$$\Omega_{\rm GW}(k,\eta) = \frac{1}{3}\left(\frac{k}{\mathcal{H}}\right)^2 \mathcal{P}_h(k,\eta).$$

- Substituting the solution into this

$$\Omega_{\rm GW,c}(f) = \frac{1}{12}\left(\frac{f}{2\pi aH}\right)^2 \int_0^\infty \mathrm{d}t \int_{-1}^1 \mathrm{d}s \left[\frac{t(t+2)(s^2-1)}{(t+s+1)(t-s+1)}\right]^2 \times \overline{I^2(t,s,k\eta_c)}\,\mathcal{P}_\zeta\left(\frac{(t+s+1)f}{4\pi}\right)\mathcal{P}_\zeta\left(\frac{(t-s+1)f}{4\pi}\right)$$

NANOGrav15yr by Induced GW and sub-solar PBHs

Keisuke Inomata, Kazunori Kohri, Takahiro Terada, arXiv:2306.17834 [astro-ph.CO]



β=2 (γ=3) for

P_ζ~δ-func (→β~2)

A braod P_ζ (→β~3).

Gaussian ζ

$\delta_c = 0.45$

$W(x) = e^{-x^2/2}$

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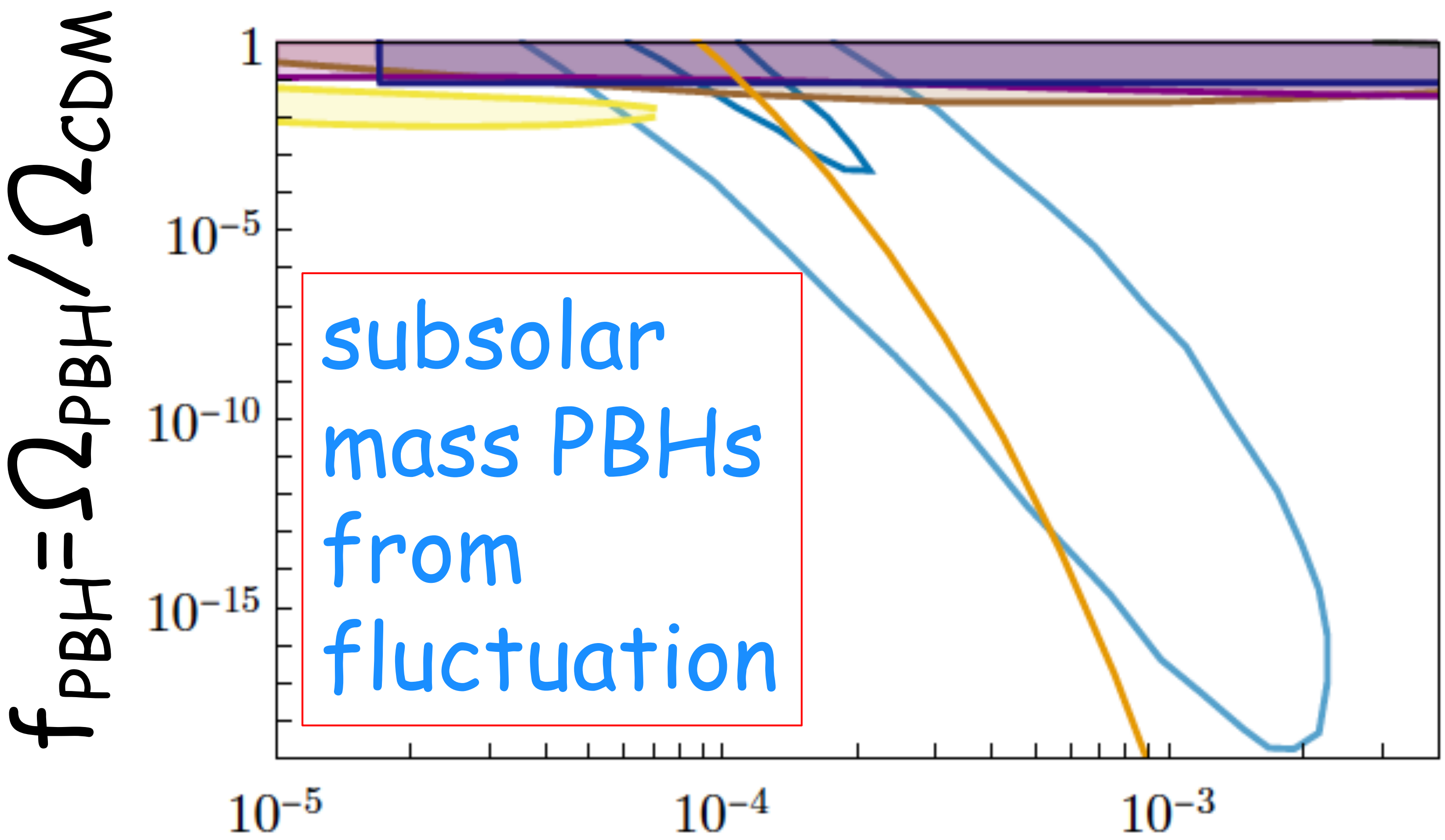
$f_{PBH} = \Omega_{PBH}/\Omega_{CDM} \sim O(0.01)$ -- $O(0.1)$



$M_{PBH} \sim O(10^{-4})\ M_\odot$

$1M_\odot = 2\times10^{33}g$

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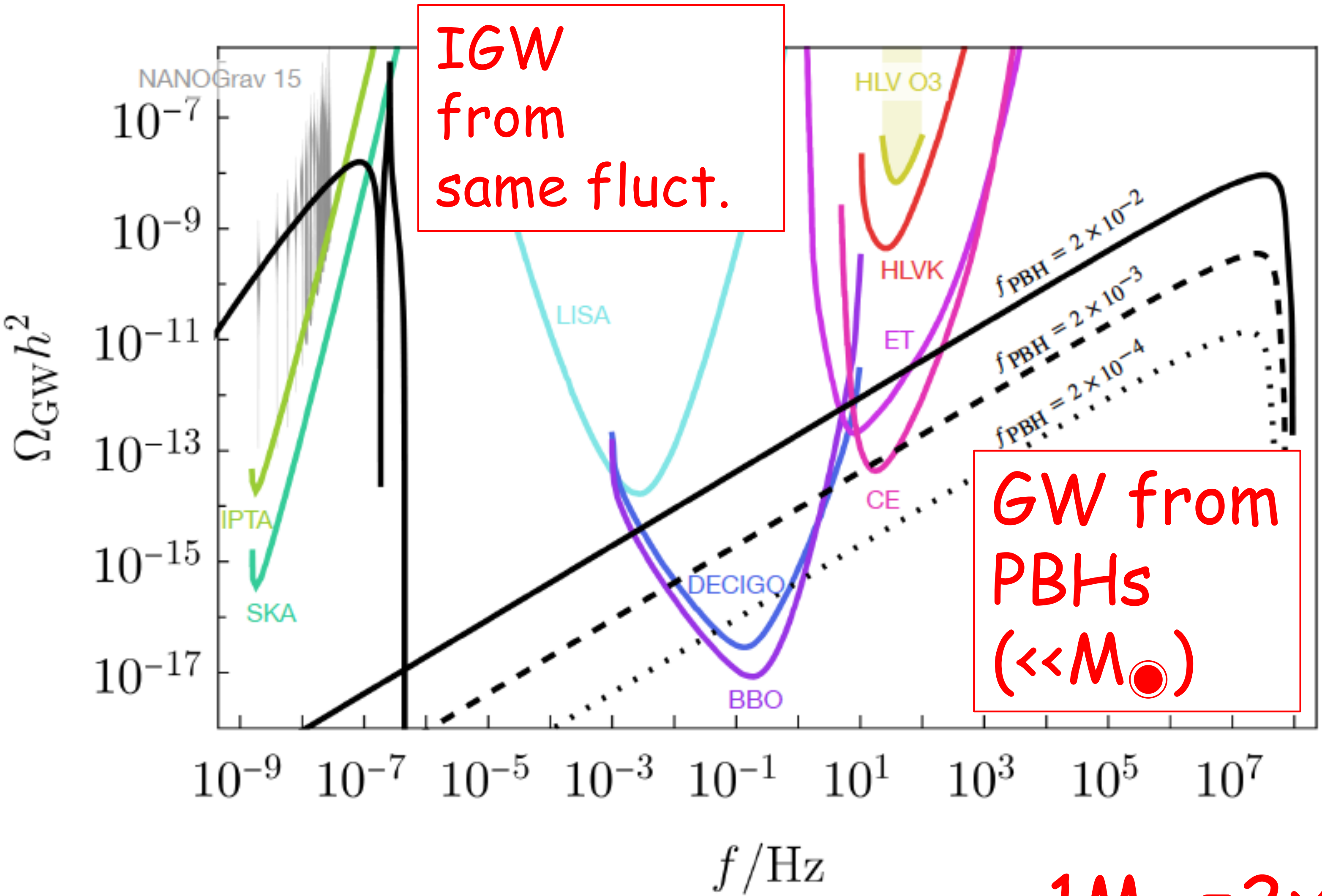
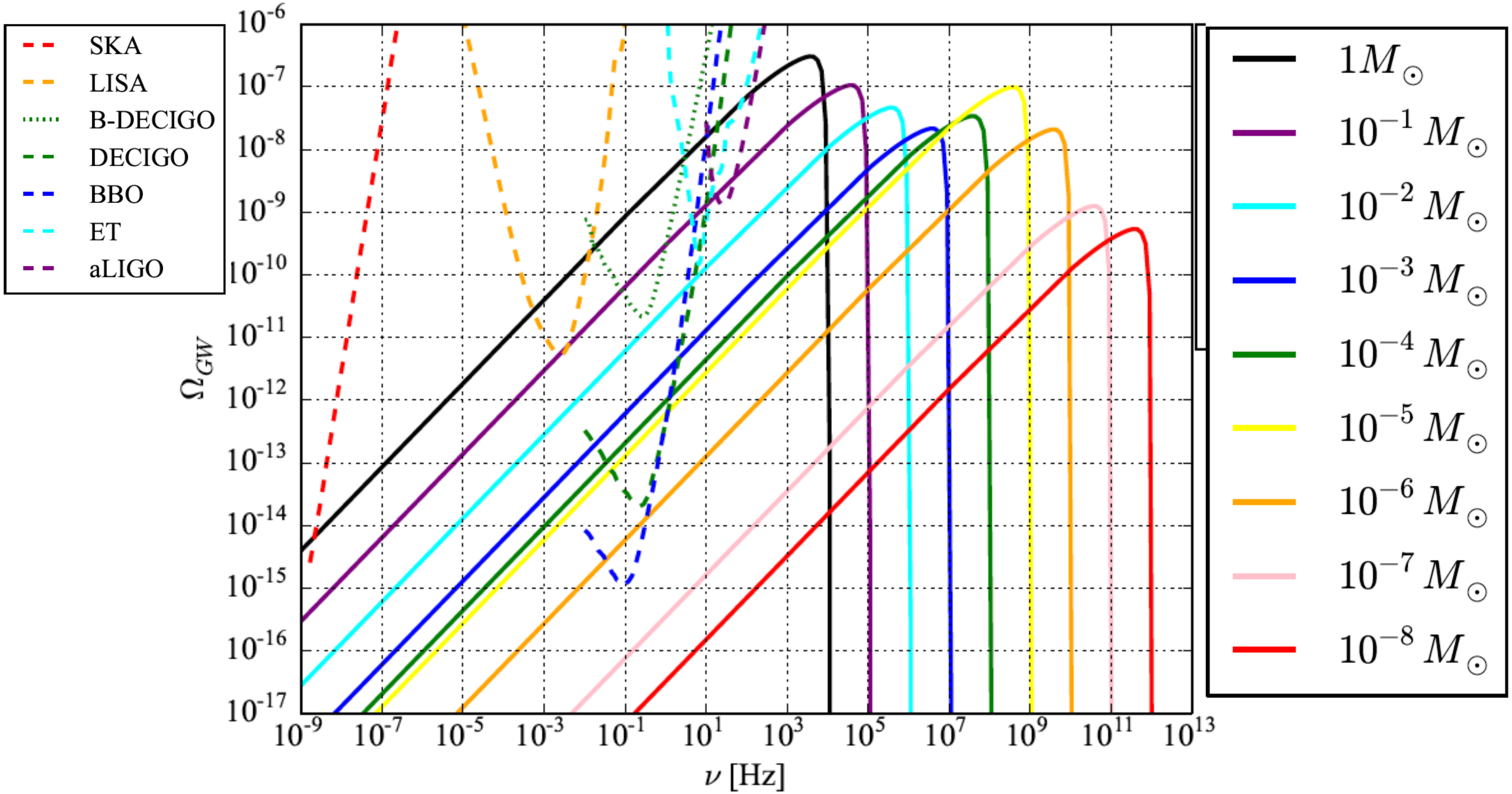


$1M_\odot = 2\times10^{33}$g

Merger signals from subsolar-mass binary PBHs

S. Wang, K. Kohri, and T. Terada, arXiv:1903.05924v2 [astro-ph.CO]



Memory Burden Effects in evaporating BHs

Gia Dvali, Lukas Eisemann, Marco Michel, Sebastian Zell, arXiv:2006.00011 [hep-th]

Upper bounds on the fraction to CDM

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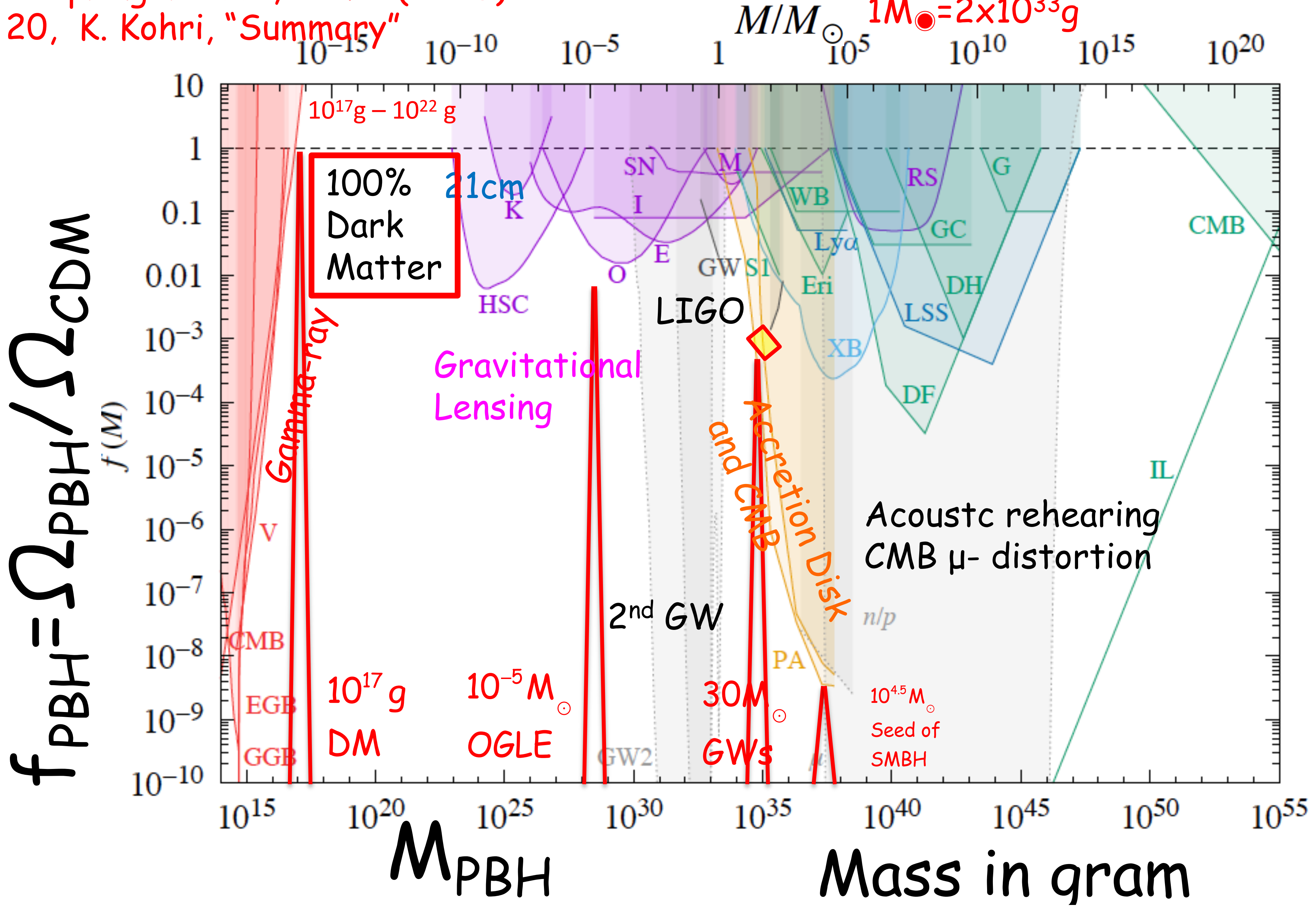


Memory Burden in evaporating BHs

Gia Dvali, Lukas Eisemann, Marco Michel, Sebastian Zell, arXiv:2006.00011 [hep-th]

Valentin Thoss, Andreas Burkert, Kazunori Kohri, arXiv:2402.17823 [astro-ph.CO]

$$\frac{\mathrm{d}^2 N_{i,\mathrm{MB}}}{\mathrm{d}E\mathrm{d}t}(E, M, s_i) = \frac{1}{S(M)^k} \frac{\mathrm{d}^2 N_{i,\mathrm{SC}}}{\mathrm{d}E\mathrm{d}t}(E, M, s_i)$$

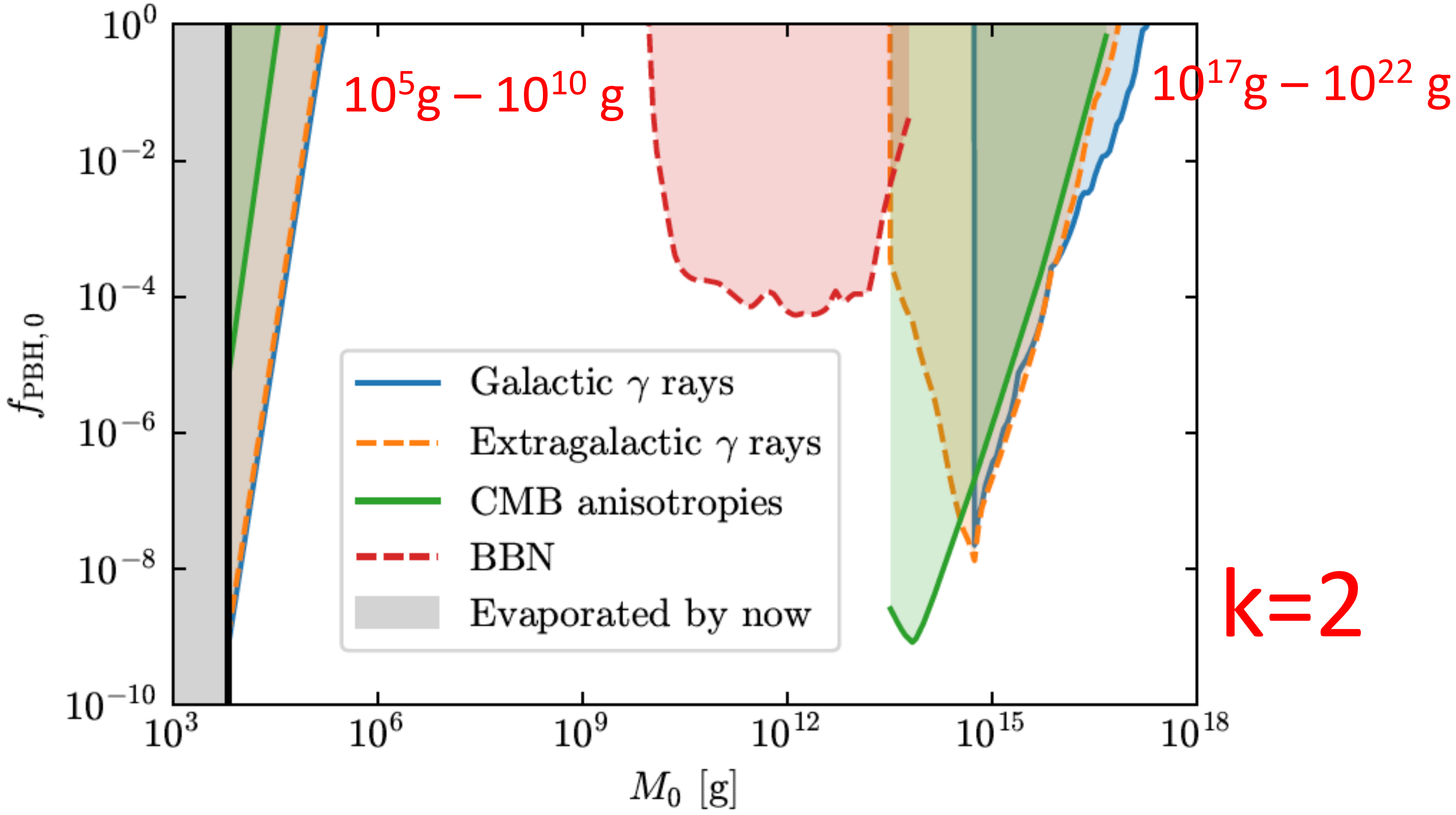
k=2

$$S = \frac{4\pi M^2 G}{\hbar c} \approx 2.6 \times 10^{10} \left(\frac{M}{1\,\mathrm{g}}\right)^2$$

$$\dot{M}_{\mathrm{PBH}} \sim \begin{cases} -\dfrac{M_{\mathrm{pl}}^4}{M_{\mathrm{PBH}}^2} & \left(M_{\mathrm{PBH}} \geq \frac{1}{2} M_{\mathrm{PBH,ini}}\right) \\ -\dfrac{1}{S^k} \dfrac{M_{\mathrm{pl}}^4}{M_{\mathrm{PBH}}^2} & \left(M_{\mathrm{PBH}} < \frac{1}{2} M_{\mathrm{PBH,ini}}\right) \end{cases}$$

Breakdown of Hawking Evaporation opens new Mass Window PBHs as DM

Valentin Thoss, Andreas Burkert, Kazunori Kohri, arXiv:2402.17823 [astro-ph.CO]



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Induced Gravitational Wave probing Primordial Black Hole Dark Matter with Memory Burden

K. Kohri. T. Terada. T. Yanagida. arXiv:2409.06365

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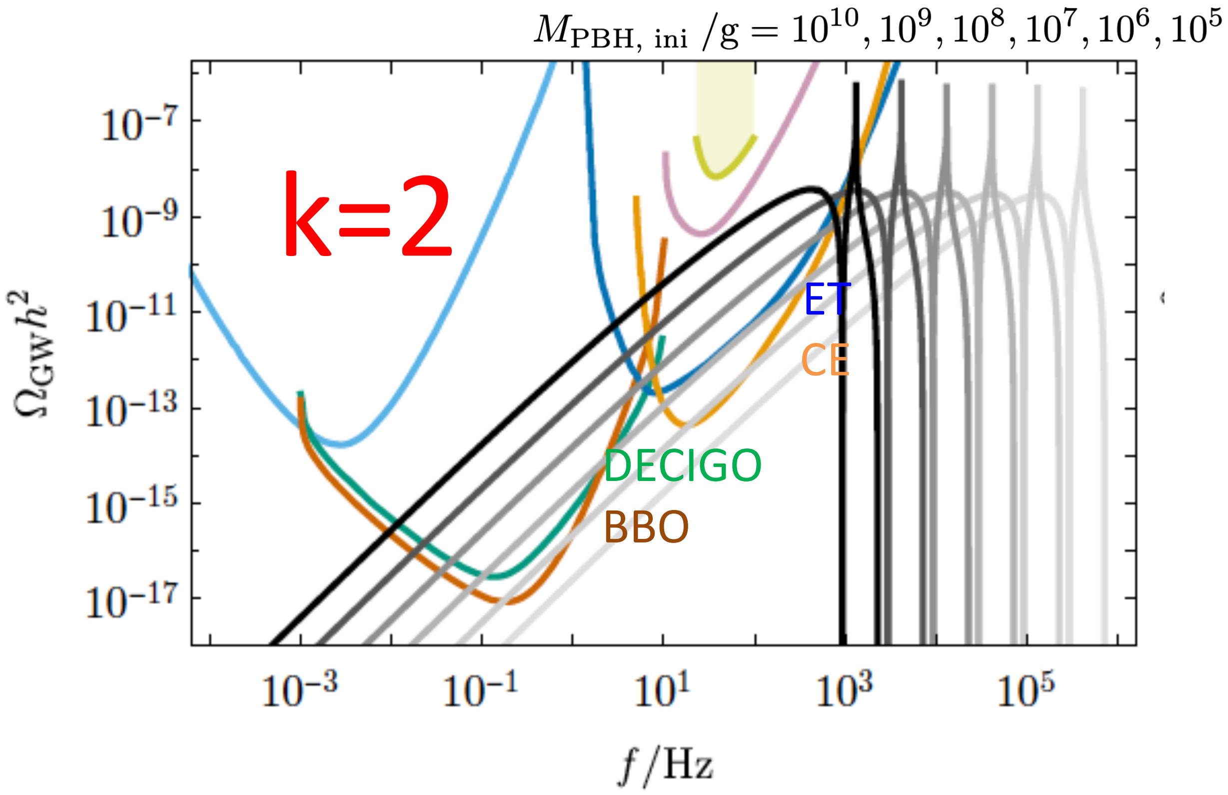
K.Kohri and T.Terada, arXiv:1804.08577



Induced Gravitational Waves probing Primordial Black Hole Dark Matter with Memory Burden

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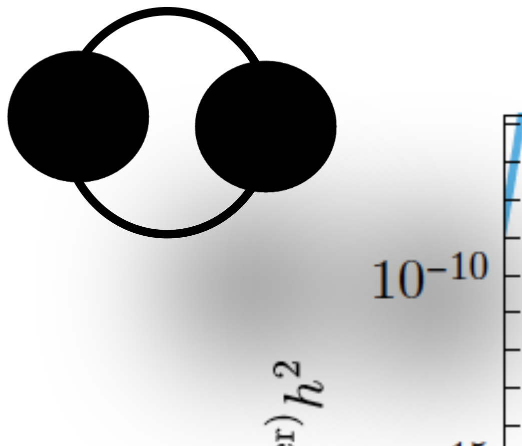
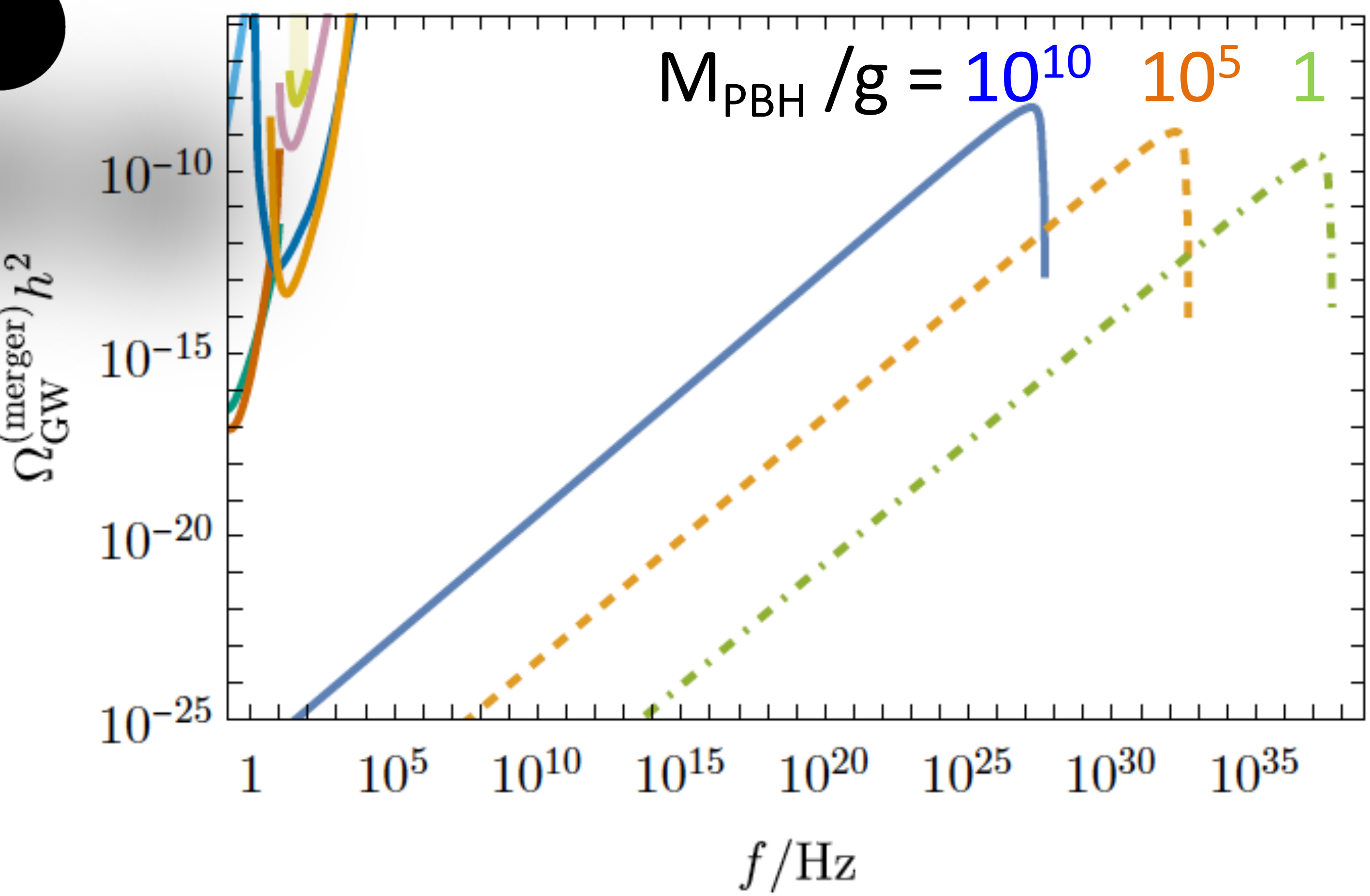




k=2

Gravitational wave search through electromagnetic telescopes

M.E.Gertsenshtein, JETP15 (1962) 84.
A. Ito, K. Kohri, K. Nakayama, arXiv:2309.14765 [gr-qc]
See also, M. E. Gertsenshtein, Sov. Phys. JETP 14 (1962) 84.
V. Domcke, C. Garcia-Cely, arXiv:2006.01161 [astro-ph.CO]
T. Fujita, K. Kamada, Y. Nakai, arXiv:2002.07548 [astro-ph.CO]

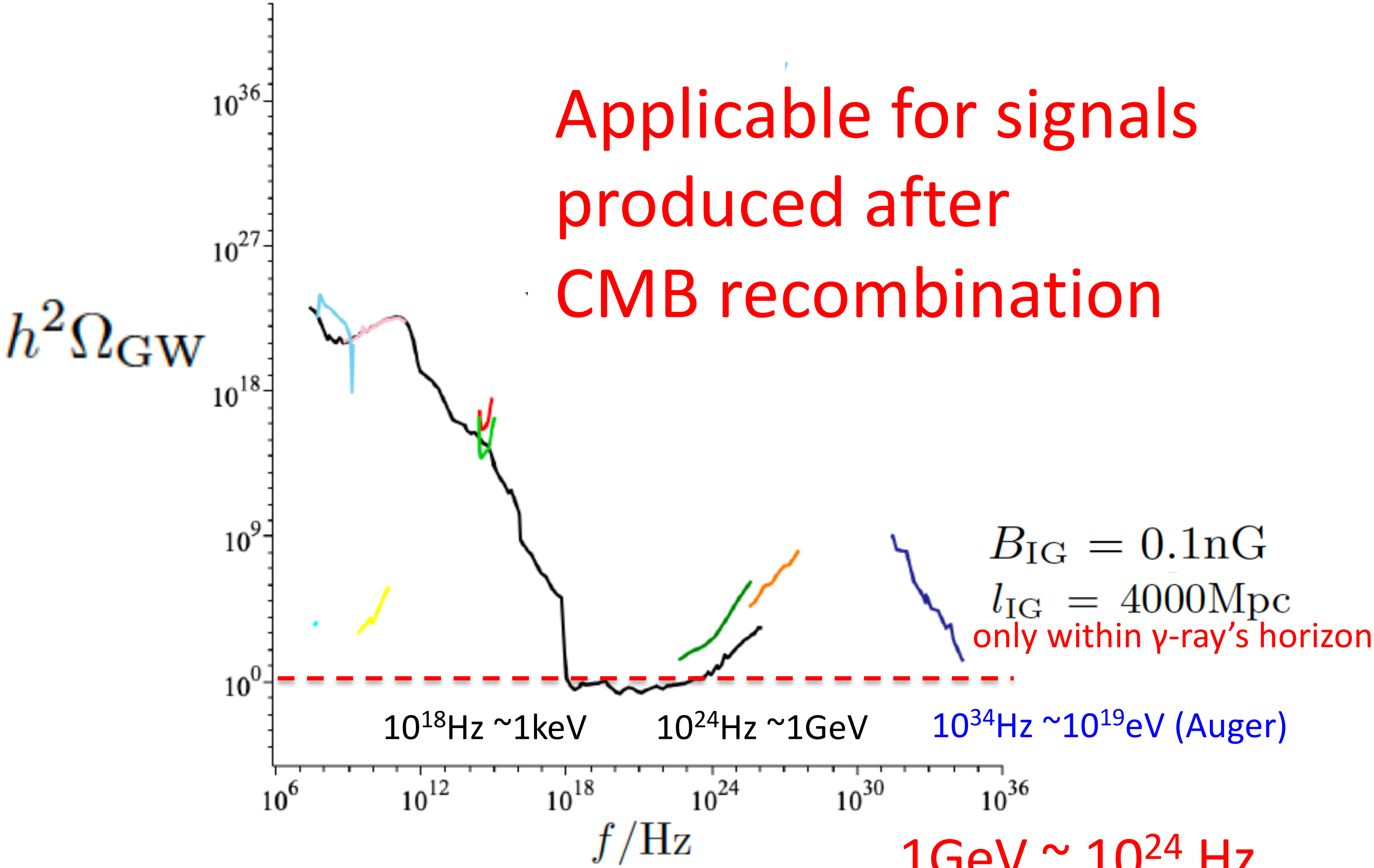
- Action of EM + gravity

$$S = \int d^4x \sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

$$\begin{aligned}\delta S^{(2)} &= \int d^4x \Bigg[-\frac{1}{2}\left(\partial_\mu h_{ij}\right)^2 - \frac{1}{2}\left(\partial_\mu A_i\right)^2 + \frac{2}{M_{\rm pl}}\epsilon_{ijk}\bar{B}^k h^{jl}\partial_i A^l \\ &+ \frac{\alpha^2}{90 m_e^4}\Big(16\bar{B}^i\bar{B}^j\Big(\delta_{ij}(\partial_k A_l)^2 - (\partial_k A_i)(\partial_k A_j) - (\partial_i A_k)(\partial_j A_k)\Big) + 28\Big((\partial_0 A_i)\bar{B}_i\Big)^2\Big)\Bigg].\end{aligned}$$

Gravitational wave search through electromagnetic telescopes

Asuka Ito, Kazunori Kohri, Kazunori Nakayama, arXiv:2309.14765 [gr-qc]



Applicable for signals produced after CMB recombination

1GeV ~ 10^{24} Hz

Conclusion

- The Memory Burden (MB) effect completely change the mass ranges for PBHs to be dark matter (10^5g-10^{10}g)

- The search for high-frequency GWs is a new direction for investigating phenomena in the early Universe.
- The targets are so many:

  1. Induced GW to produce dark matter PBHs with MB
  2. GWs from merging binary PBHs with subsolar mass
  3. Thermal/nonthermal graviton just after inflation,
  4. 1^{st}-order phase transition at E >> weak scale
  5. …

- We can test high-frequency GWs by observing the electromagnetic wave converted from the GWs