Knotted cosmic strings in early universe

Yu Hamada (DESY)

arXiv: 2407.11731

based on collaboration. w/

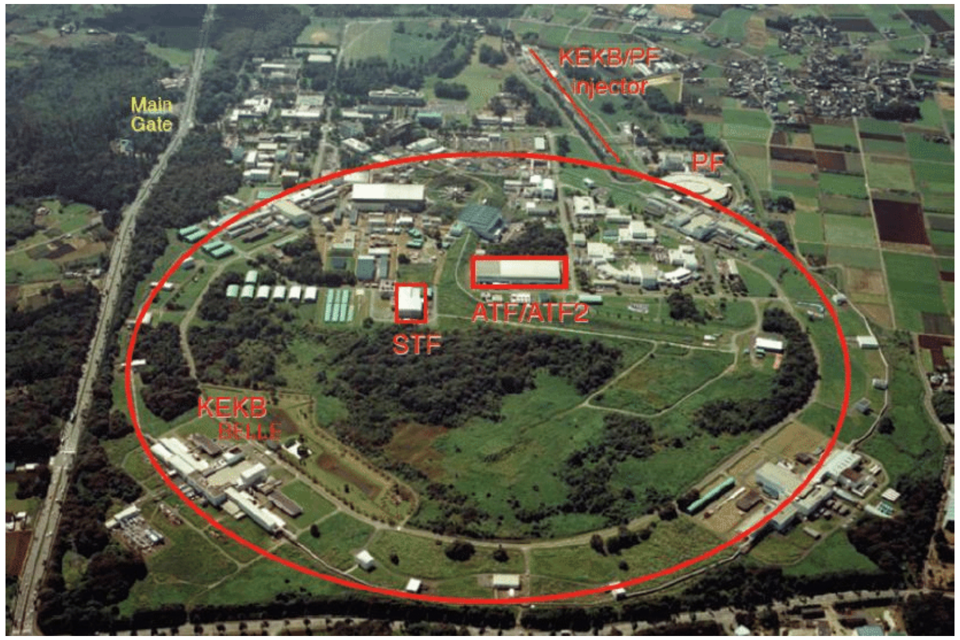
Minoru Eto (Yamagata U.) and Muneto Nitta (Keio U.)

Seminar@Kyoto U, 16th October 2024

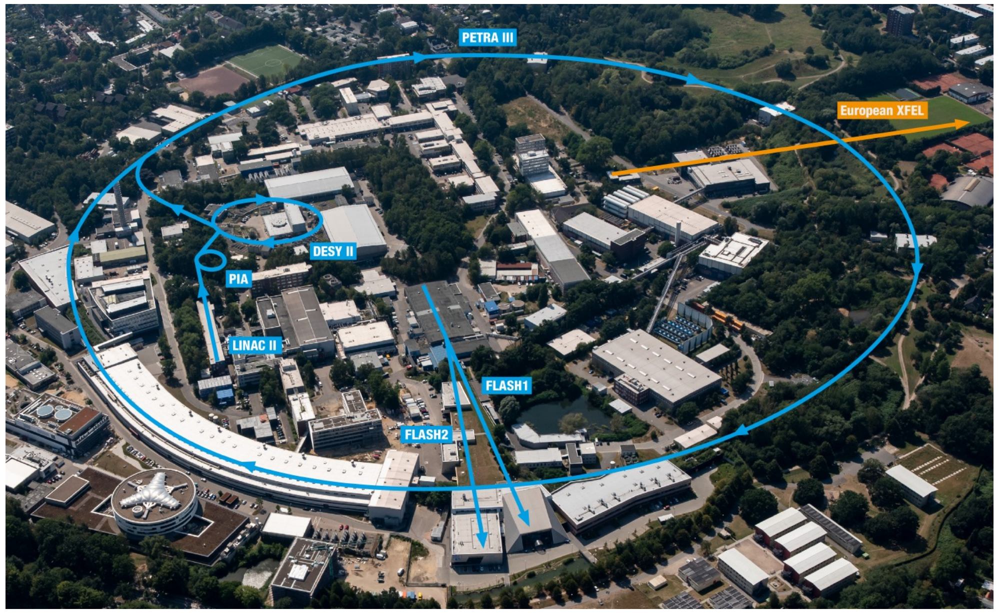
自己紹介

- 2016-2021 学生@京大物二素論
- 2021-2023 ポスドク@KEK (つくば)
- 2023- ポスドク@DESY (ハンブルク, ドイツ)

KEK

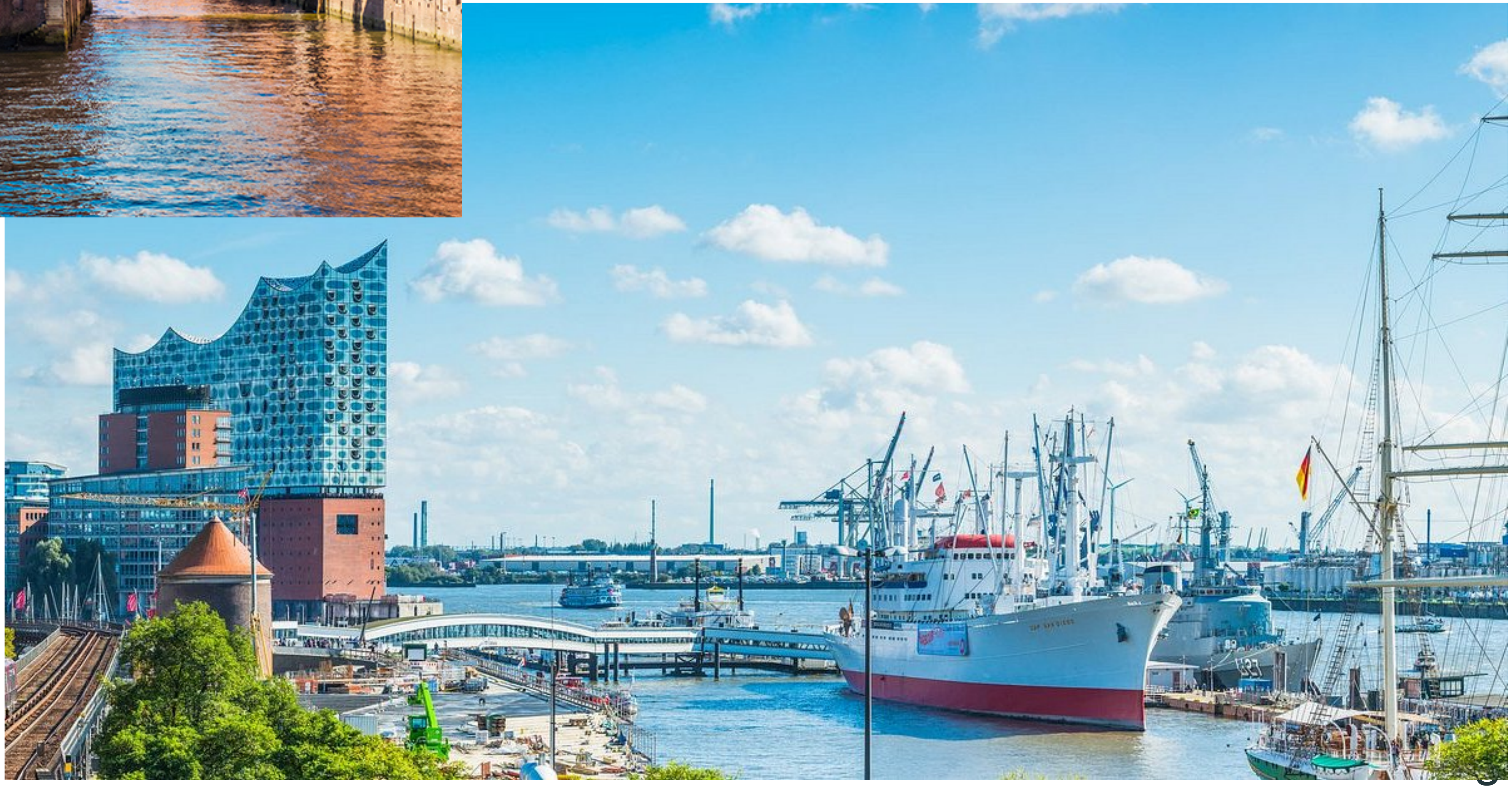


https://kegenpress.com/football-samurai-city-9/



DESY

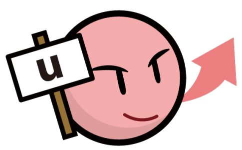
Hamburg



Introduction

Soliton

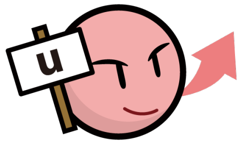
- **Particle** : fluctuation around vacuum



- **Soliton** : **classical and coherent excitation** (``lump'')

Soliton

- **Particle** : fluctuation around vacuum

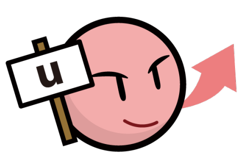


- **Soliton : classical and coherent excitation** (``lump'')

Tsunami

Soliton

- **Particle**：fluctuation around vacuum



- **Soliton**：**classical and coherent excitation** (``lump'')

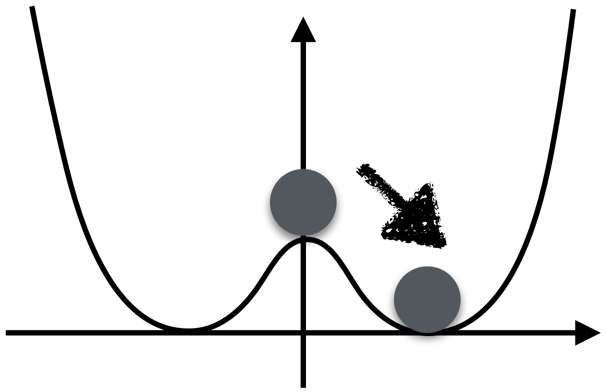
Tsunami

KdV soliton: solution of non-linear wave eq.

``Collision of KdV solitons'' (from YouTube)

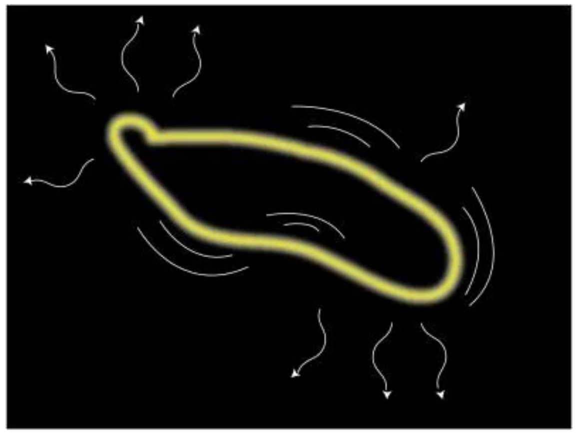
Soliton in QFT

One of the key phenomena in QFT is **spontaneous symmetry breaking (SSB).**



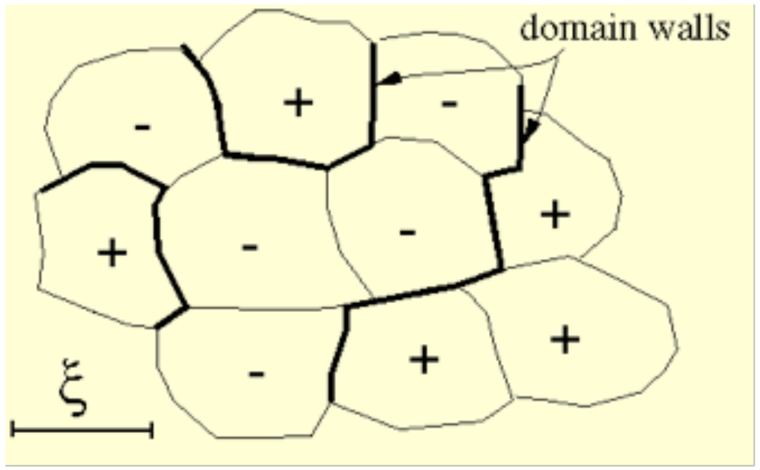
→ **SSB often leads to existence of solitons**

- magnetic monopole: $G \to U(1) \times G'$



(Image credit: Matt DePies/UW)

- Cosmic (vortex) string: $U(1) \to 1$
- Domain wall: $\mathbb{Z}_n \to 1$



Particularly, such solitons are called topological solitons

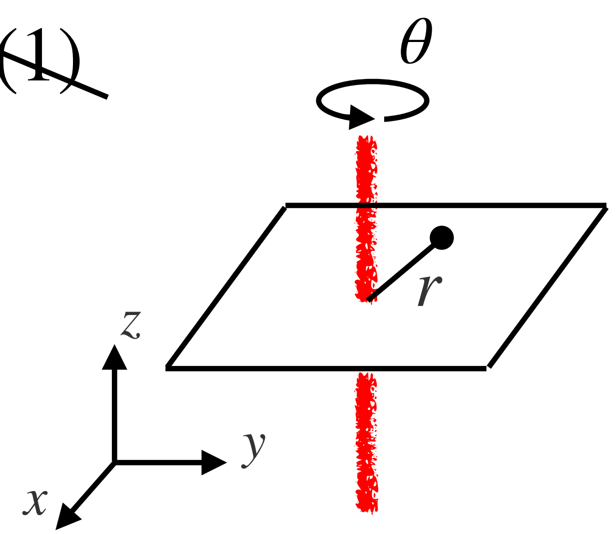
Example of cosmic string

[Abrikosov '58]

[Nielsen-Olesen '73]

- 3+1 D Abelian-Higgs model $\langle \phi \rangle = v \rightarrow \cancel{U(1)}$

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu \phi|^2 + m^2|\phi|^2 - \lambda|\phi|^4$$



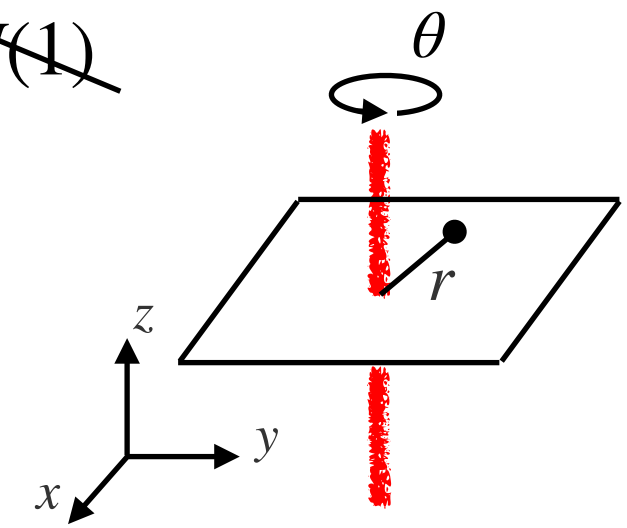
Example of cosmic string

[Abrikosov '58]

[Nielsen-Olesen '73]

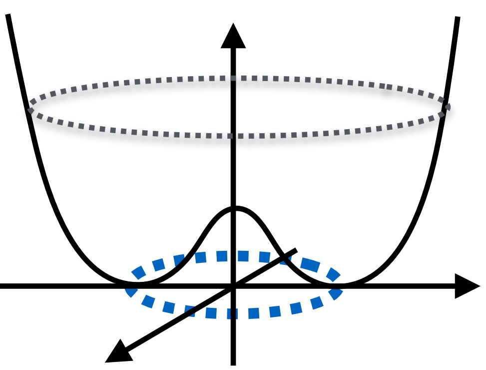
- 3+1 D Abelian-Higgs model

$$\langle \phi \rangle = v \rightarrow \cancel{U(1)}$$



$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 + m^2|\phi|^2 - \lambda|\phi|^4$$

- **z-independent field configuration:**



$$\phi(x) = v\, f(r)\, e^{i\theta} \qquad \vec{A}(x) = g^{-1}\, a(r)\, \vec{e}_\theta$$

ϕ's phase has winding # = 1

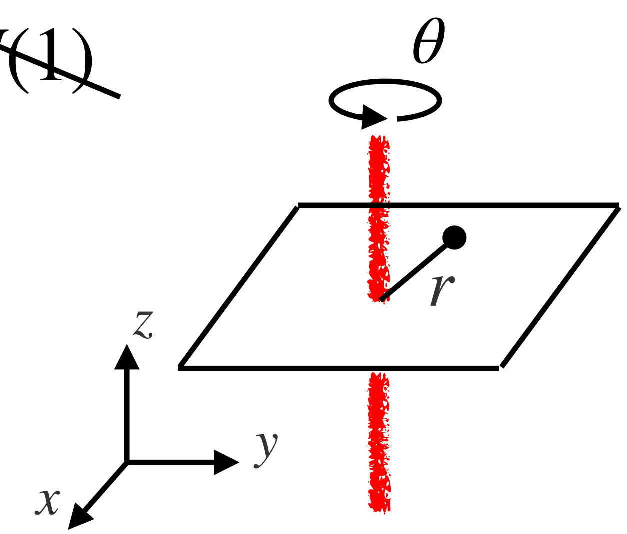
i.e., non-trivial map characterized by $\pi_1(S^1) = \mathbb{Z}$

Example of cosmic string

[Abrikosov '58]

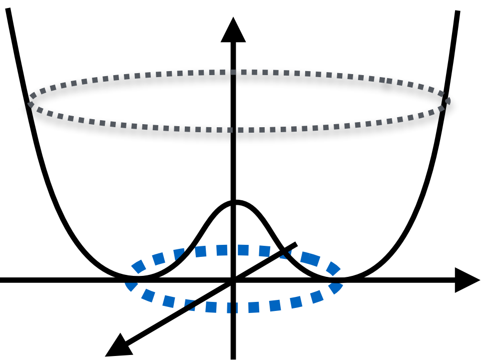
[Nielsen-Olesen '73]

- 3+1 D Abelian-Higgs model $\langle \phi \rangle = v \rightarrow \cancel{U(1)}$



$$\mathscr{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 + m^2 |\phi|^2 - \lambda |\phi|^4$$

- **z-independent field configuration:**



$$\phi(x) = v\, f(r)\, e^{i\theta} \qquad \vec{A}(x) = g^{-1}\, a(r)\, \vec{e_\theta}$$

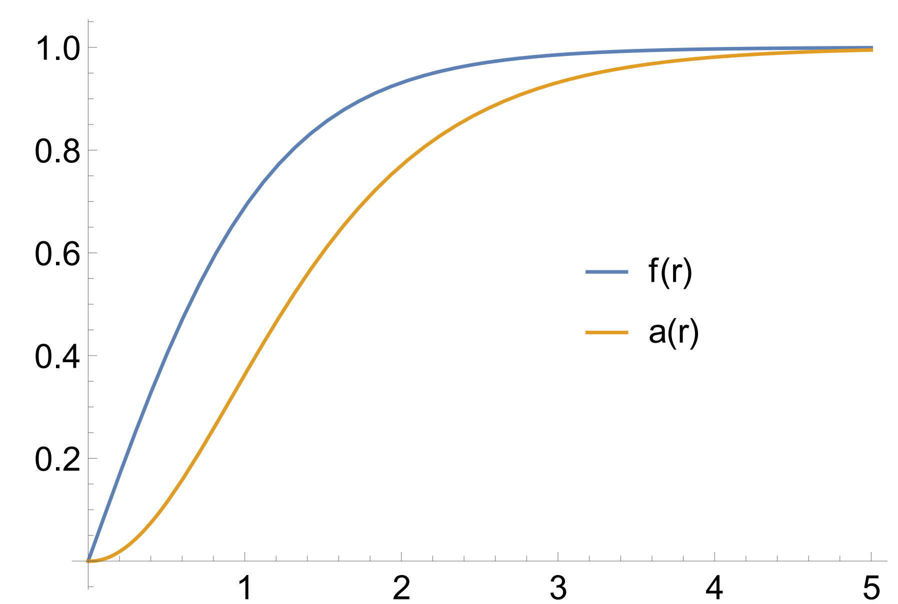
ϕ's phase has winding # = 1

i.e., non-trivial map characterized by $\pi_1(S^1) = \mathbb{Z}$

- **solving classical EOMs for $f(r)$ and $a(r)$:**

$$f'' + \frac{1}{r} f' - \frac{(1-a)^2}{r^2} f - \frac{1}{2} \frac{\partial V}{\partial f} = 0$$

$$a'' - \frac{1}{r} a' + 2(1-a) f^2 = 0$$

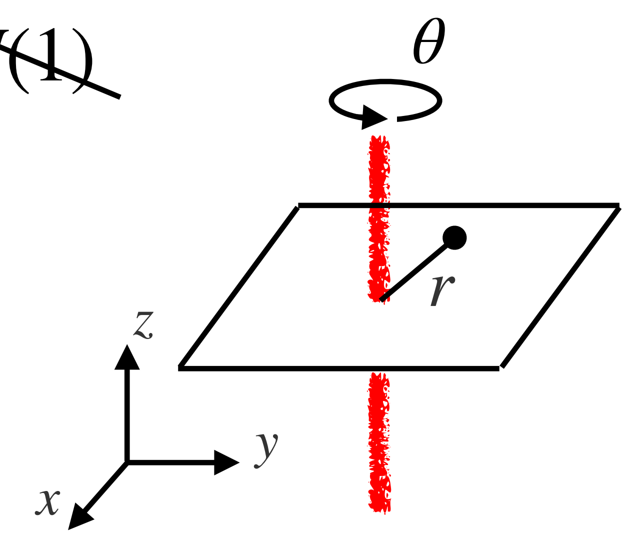


Example of cosmic string

[Abrikosov '58]

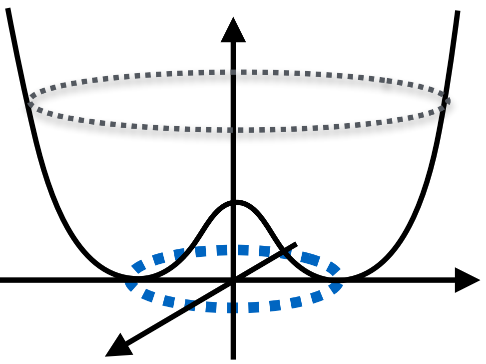
[Nielsen-Olesen '73]

- 3+1 D Abelian-Higgs model $\langle \phi \rangle = v \rightarrow \cancel{U(1)}$



$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu \phi|^2 + m^2|\phi|^2 - \lambda|\phi|^4$$

- **z-independent field configuration:**



$$\phi(x) = v\, f(r)\, e^{i\theta} \qquad \overrightarrow{A}(x) = g^{-1}\, a(r)\, \overrightarrow{e_\theta}$$

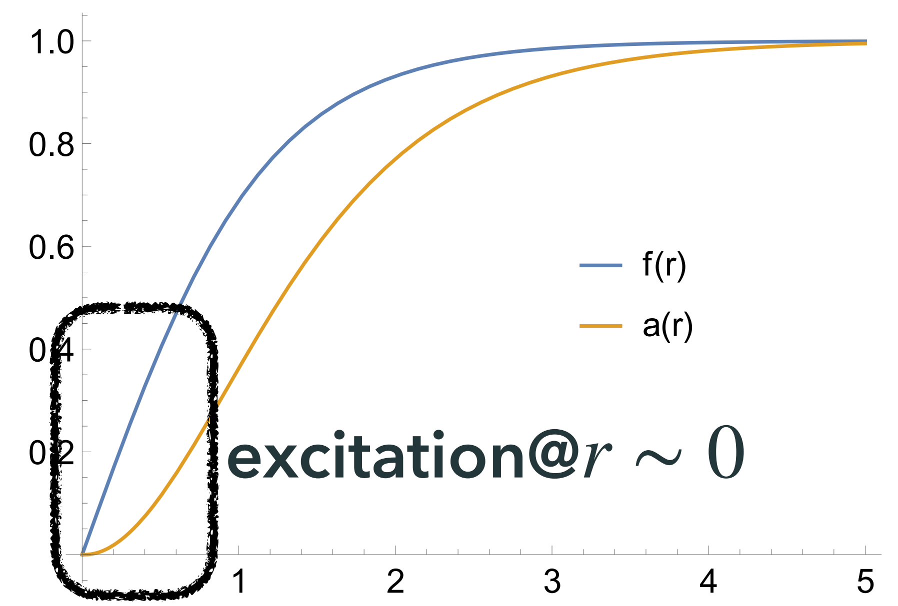
ϕ's phase has winding # = 1

i.e., non-trivial map characterized by $\pi_1(S^1) = \mathbb{Z}$

- **solving classical EOMs for $f(r)$ and $a(r)$:**

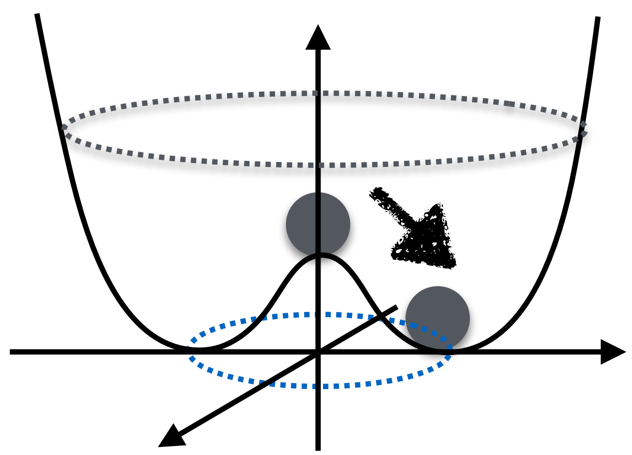
$$f'' + \frac{1}{r}f' - \frac{(1-a)^2}{r^2}f - \frac{1}{2}\frac{\partial V}{\partial f} = 0$$

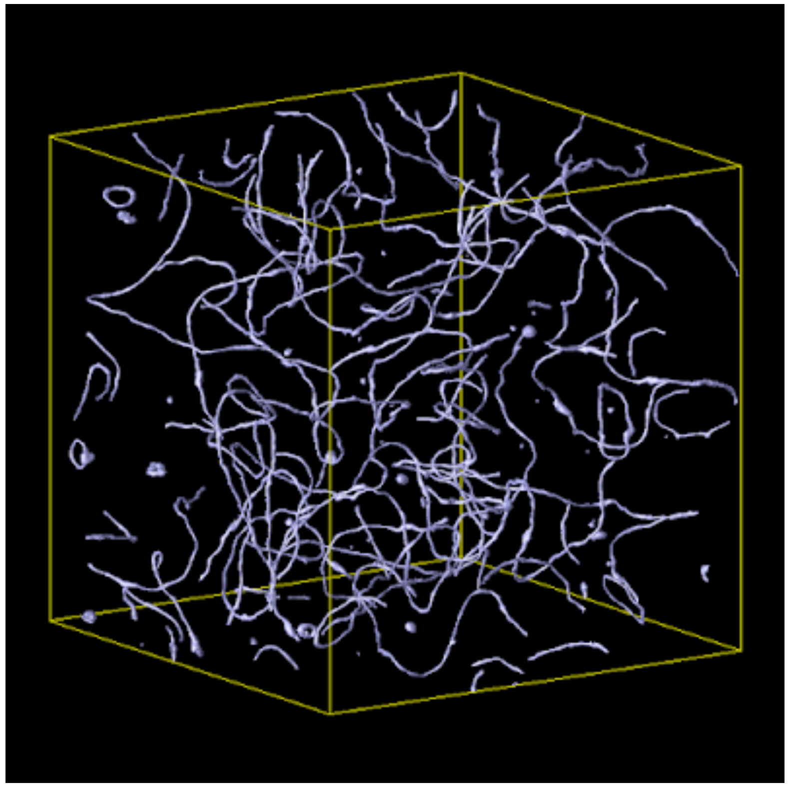
$$a'' - \frac{1}{r}a' + 2(1-a)f^2 = 0$$



Cosmic string network in universe

- SSB in early universe → network of cosmic string in universe



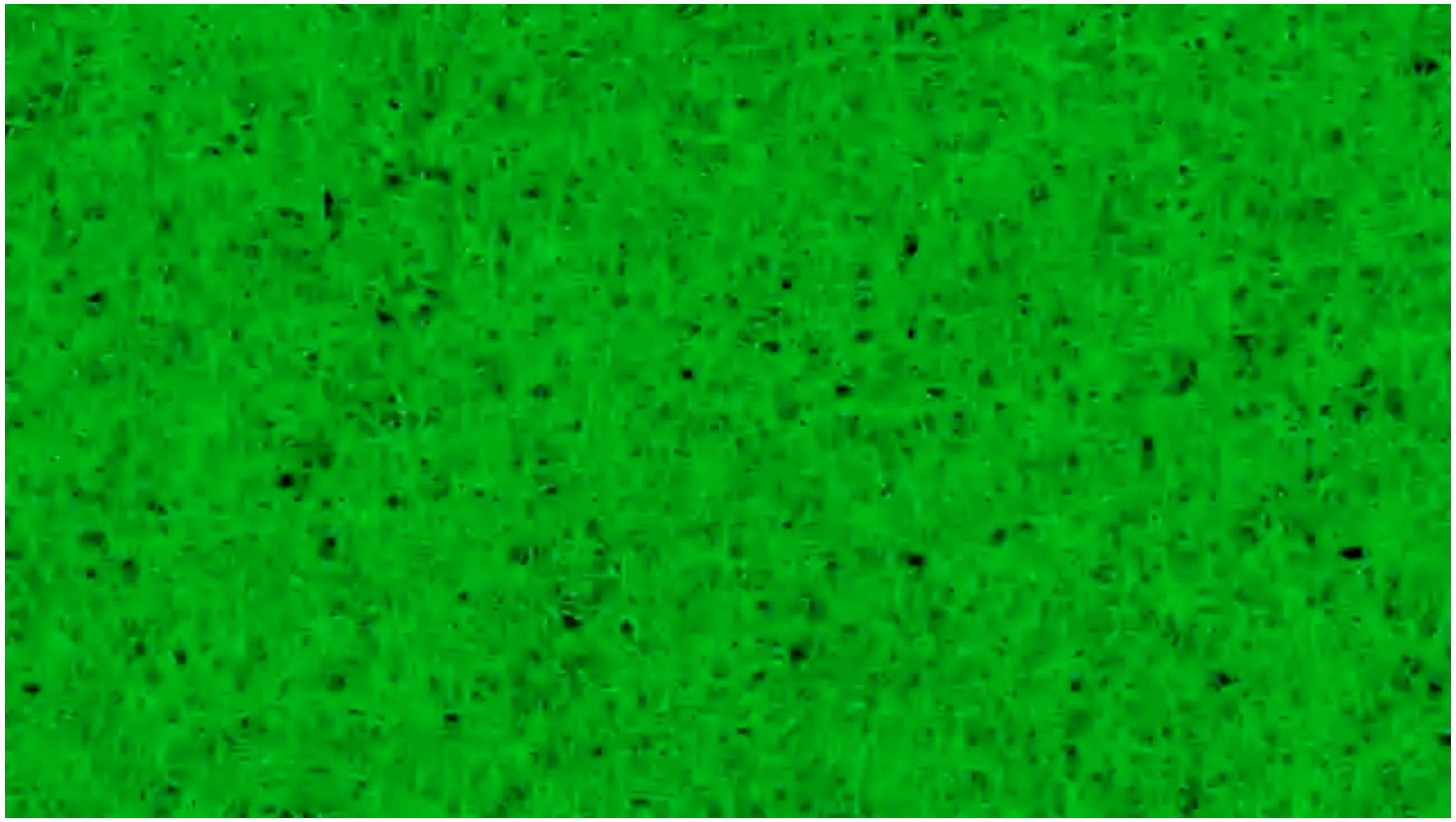


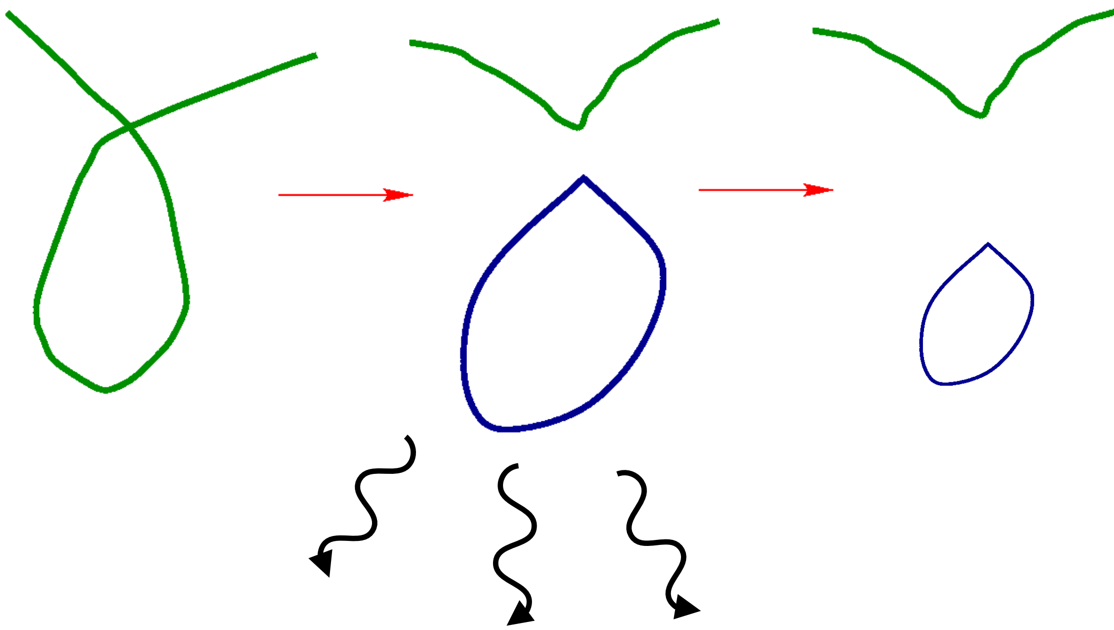
from slide by Takashi Hiramatsu

- might be detected by CMB observation, gravitational wave
- **strong evidence of new physics**, but not discovered so far
- was popular in '90s, and **is getting popular again!**

GW & string network

- The network continuously produces small loops of strings, which decay by **radiating gravitational wave** or particles

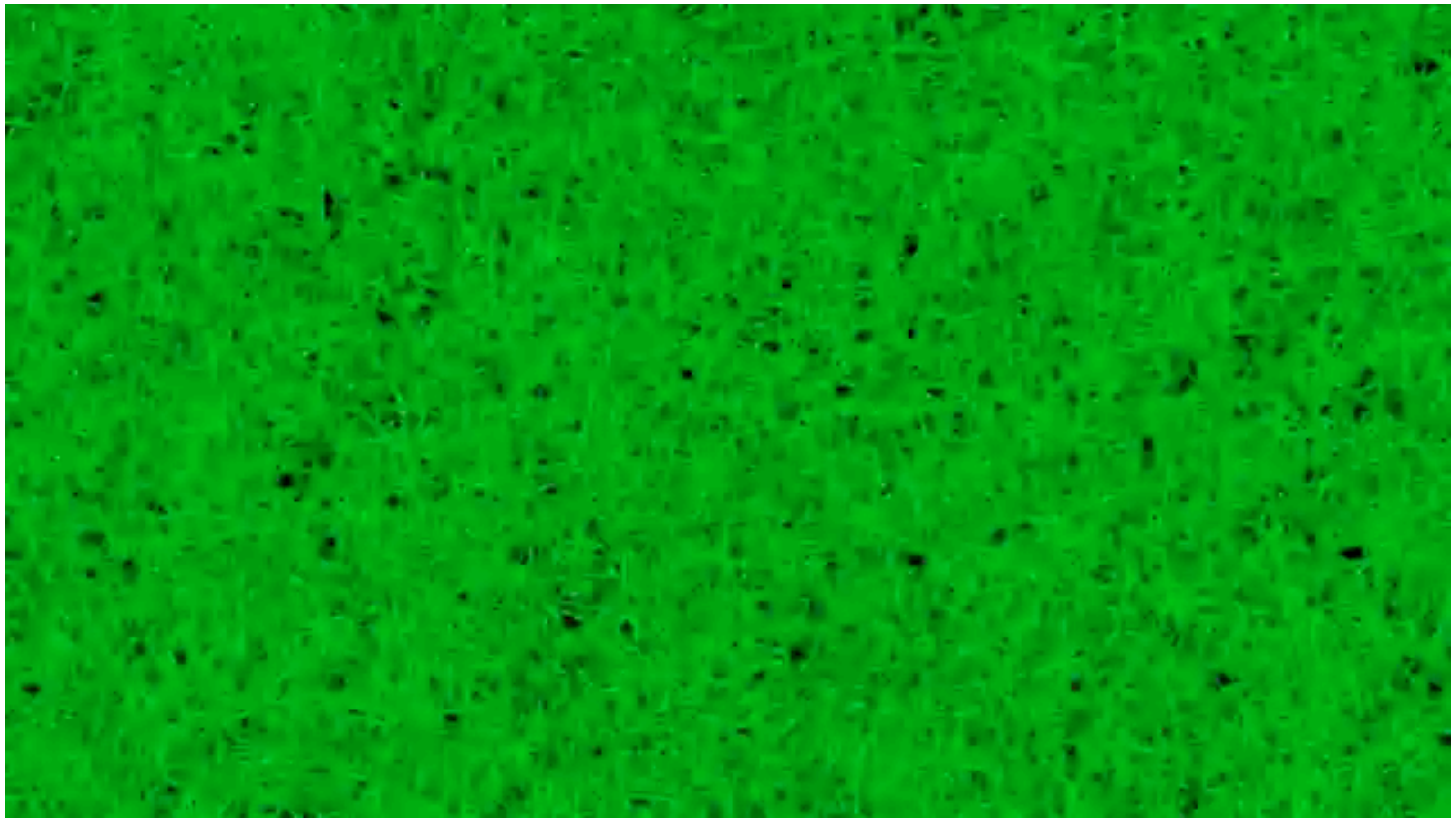


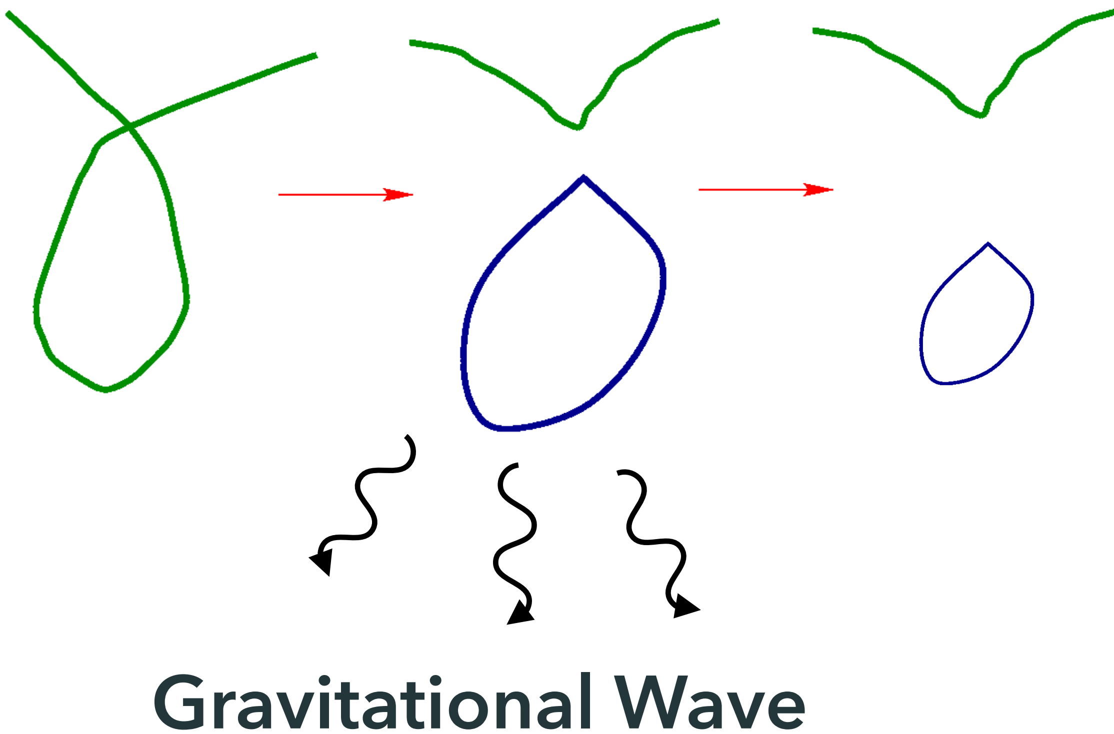


Gravitational Wave

GW & string network

- The network continuously produces small loops of strings, which decay by **radiating gravitational wave** or particles





GW & string network

- current GW spectrum:

$$\boxed{\frac{\rho_{\mathrm{GW},0}(f)}{\rho_{\mathrm{tot},0}} \sim (G\mu)^2 \int_{t_i}^{t_0} dt \left(\frac{a(t)}{a(t_0)}\right)^4 \Delta(t, f_{\mathrm{emit}})}$$

$$f = \frac{a(t)}{a(t_0)} f_{\mathrm{emit}} \qquad ds^2 = -dt^2 + a(t)^2 \, dr_3^2$$

GW spectrum function

$$G\mu \simeq v_{\mathrm{st.}}^2 / M_{\mathrm{pl.}}^2$$

depends on cosmology

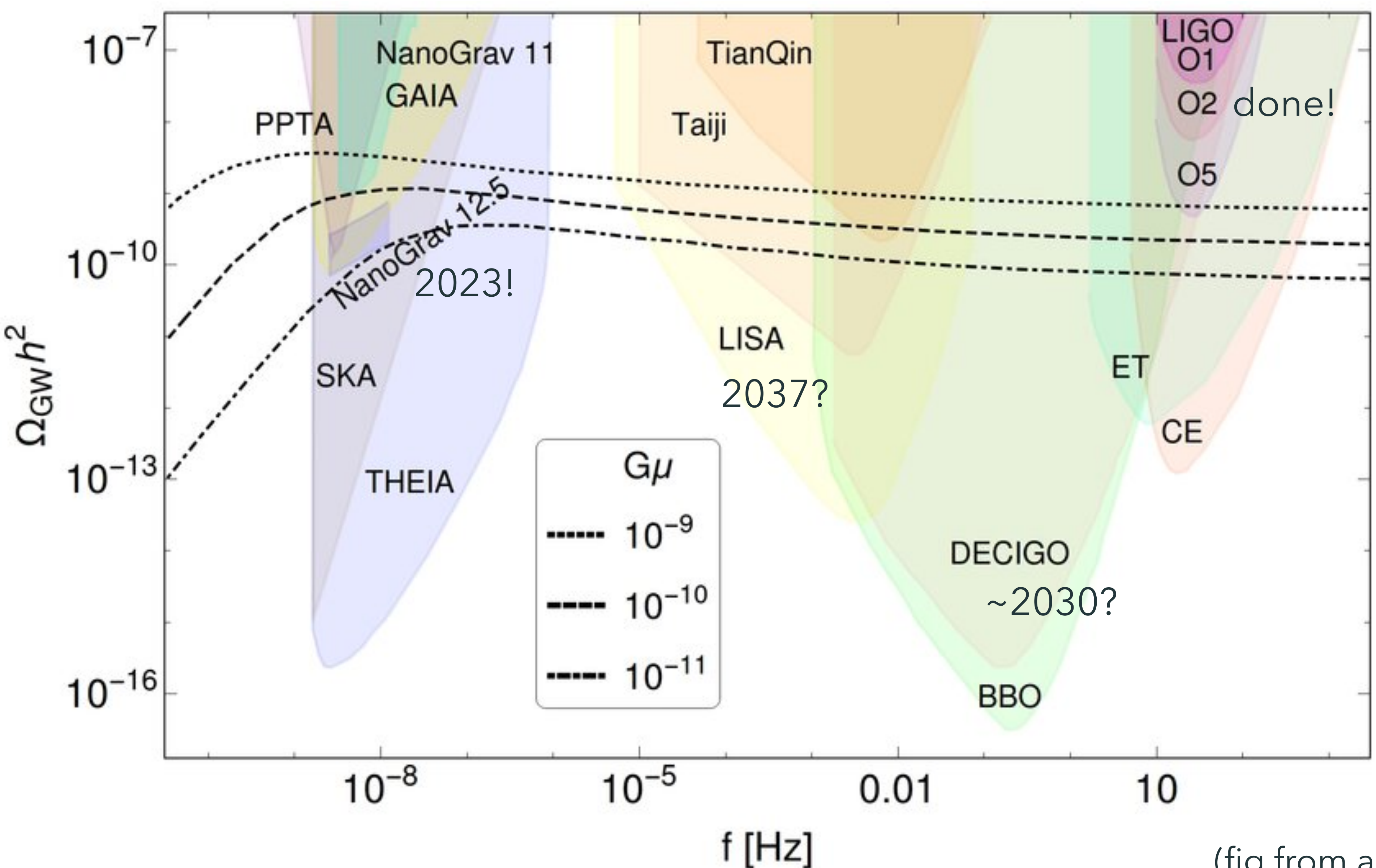
- scale factor $a(t)$: $\dot{a}(t)/a(t) \simeq \sqrt{\rho_{\mathrm{tot}}(t)}/M_{\mathrm{pl}}$

→ GW from cosmic string "knows" what happened in past universe

→ **if detected, new probe of cosmological history**

Future prospect of GW

$G\mu \simeq v_{\rm st.}^2/M_{\rm pl.}^2$



(fig from arXiv: 2212.05291)

age of GW & cosmic string!?

我々のやったこと:

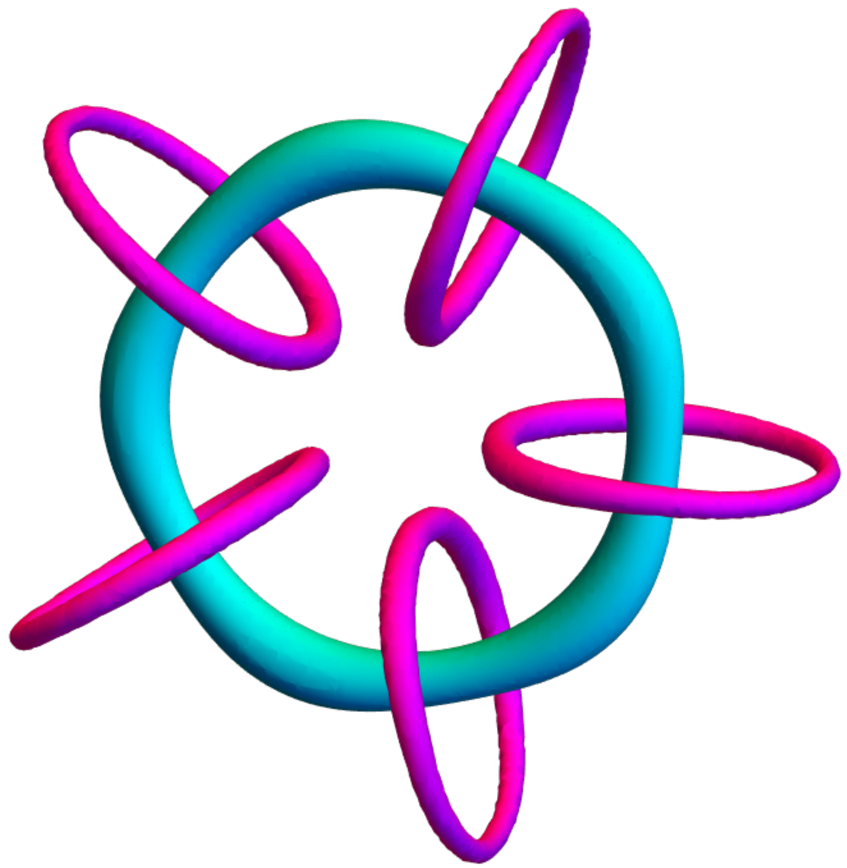
2種類のcosmic stringを使って新しいソリトンを作りました

→初期宇宙に存在したかもしれない！

Our result in a nutshell

Knot soliton:

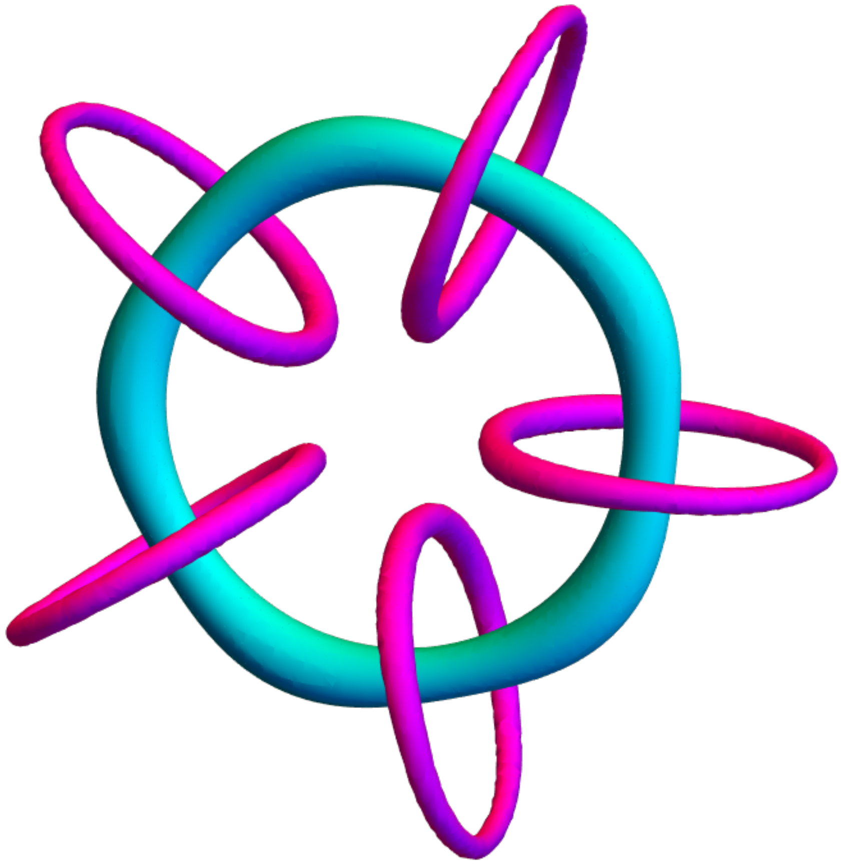
NEW stable object made of two kinds of cosmic strings!



Our result in a nutshell

Knot soliton:

NEW stable object made of two kinds of cosmic strings!



remained abundant in early universe

→ **can be probed by gravitational wave!**

Plan of talk

- Introduction
- Knot soliton
- Application to cosmology
- Summary

Knot soliton

Local vs Global strings

- SSB of **gauged** $U(1)$ sym → **local** string

  → magnetic flux in string $\int d^2x\, B = 2\pi/g$

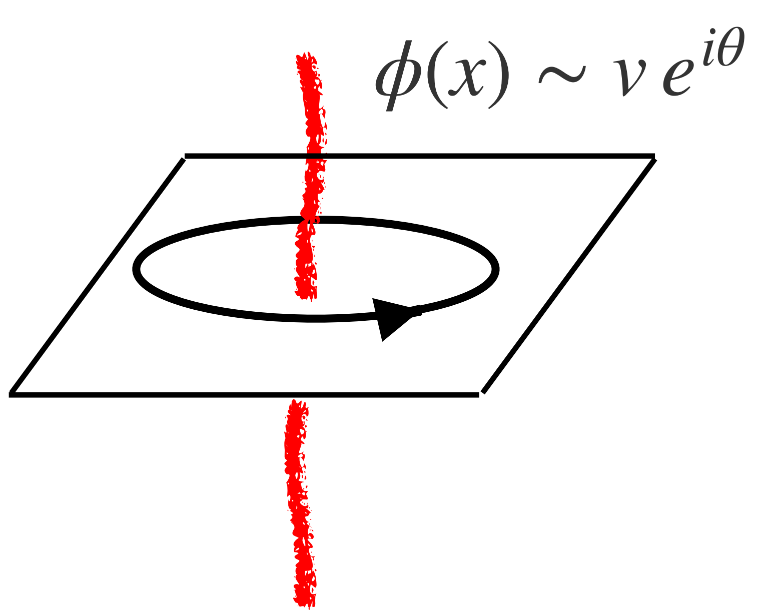
  (eg. magnetic flux in supercond.)

  ϕ's phase is not physical (gauge redundant)

- SSB of **global** $U(1)$ sym → **global** string

  → w/o magnetic flux

  ϕ's phase is physical NG boson



The model

Lagrangian:

$$\boxed{\begin{aligned}\mathcal{L} &= |D_\mu \phi_1|^2 + |\partial_\mu \phi_2|^2 - \frac{1}{4}F_{\mu\nu}^2 - V(\phi_1, \phi_2) \\ V(\phi_1, \phi_2) &= \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4\end{aligned}}$$

- Symmetries:

$$U(1)_{gauge} : \phi_1 \to e^{i\theta_1}\phi_1 \qquad U(1)_{global} : \phi_2 \to e^{i\theta_2}\phi_2$$

$$D_\mu \phi_1 \equiv (\partial_\mu - igA_\mu)\phi_1$$

- Both symmetries are broken at the vacuum:

$$\langle \phi_1 \rangle = v_1, \ \langle \phi_2 \rangle = v_2$$

→ **co-existence of local string (ϕ_1, A_μ) & global string (ϕ_2)**

Chern-Simons coupling

Lagrangian:

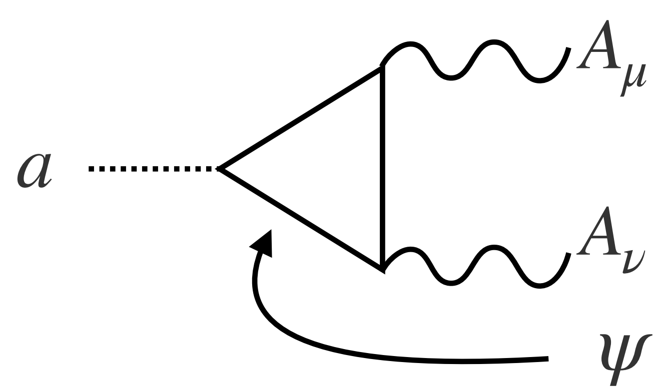
Chern-Simons coupling

$$\mathcal{L} = |D_\mu \phi_1|^2 + |\partial_\mu \phi_2|^2 - \frac{1}{4}F_{\mu\nu}^2 - V(\phi_1, \phi_2) \boxed{+\frac{c}{16\pi^2} a F_{\mu\nu}\tilde{F}^{\mu\nu}}$$

$$V(\phi_1, \phi_2) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

$$a \equiv -i \arg(\phi_2) \qquad D_\mu \phi_1 = (\partial_\mu - igA_\mu)\phi_1$$

- At the broken phase, CS coupling is induced by triangle anomaly.



The coefficient c depends on matter sector,
but we take it as free parameter in this talk.

Linking configuration

- Rewriting CS coupling: $\frac{c}{16\pi^2}\, aF_{\mu\nu}\tilde{F}^{\mu\nu} \longrightarrow -\frac{c}{16\pi^2}\left(\partial_i a\right) A_0 B^i$

$$B_i \equiv \epsilon_{ijk}\partial^j A^k$$

Linking configuration

- Rewriting CS coupling: $\dfrac{c}{16\pi^2}\, aF_{\mu\nu}\tilde{F}^{\mu\nu} \longrightarrow -\dfrac{c}{16\pi^2}\left(\partial_i a\right) A_0 B^i$

$$B_i \equiv \epsilon_{ijk}\partial^j A^k$$

→ Gauss law:

$$\frac{\delta \mathscr{L}}{\delta A_0} = \partial_i E_i - g^2 J^0 + \frac{g^2 c}{16\pi^2}\vec{\nabla} a \cdot \vec{B} = 0$$

Linking configuration

- Rewriting CS coupling: $\dfrac{c}{16\pi^2} aF_{\mu\nu}\tilde{F}^{\mu\nu} \longrightarrow -\dfrac{c}{16\pi^2} \left(\partial_i a\right) A_0 B^i$

$B_i \equiv \epsilon_{ijk}\partial^j A^k$

→ Gauss law:

Noether charge

$$\frac{\delta \mathscr{L}}{\delta A_0} = \partial_i E_i - g^2 J^0 + \frac{g^2 c}{16\pi^2} \overrightarrow{\nabla} a \cdot \overrightarrow{B} = 0$$

"electric field"

Linking configuration

- Rewriting CS coupling: $\frac{c}{16\pi^2} aF_{\mu\nu}\tilde{F}^{\mu\nu} \longrightarrow -\frac{c}{16\pi^2}\left(\partial_i a\right) A_0 B^i$

$B_i \equiv \epsilon_{ijk}\partial^j A^k$

→ Gauss law:

Noether charge

$$\frac{\delta \mathscr{L}}{\delta A_0} = \partial_i E_i - g^2 J^0 + \underline{\frac{g^2 c}{16\pi^2}\vec{\nabla} a \cdot \vec{B}} = 0$$

$\vec{\nabla} a \cdot \vec{B}$ **sources "electric field"**

"electric field"

Linking configuration

- Rewriting CS coupling: $\frac{c}{16\pi^2} a F_{\mu\nu}\tilde{F}^{\mu\nu} \longrightarrow -\frac{c}{16\pi^2}\left(\partial_i a\right) A_0 B^i$

$B_i \equiv \epsilon_{ijk}\partial^j A^k$

→ Gauss law:

Noether charge

$$\frac{\delta \mathscr{L}}{\delta A_0} = \partial_i E_i - g^2 J^0 + \underline{\frac{g^2 c}{16\pi^2}\vec{\nabla} a \cdot \vec{B}} = 0$$

$\vec{\nabla} a \cdot \vec{B}$ **sources "electric field"**

"electric field"

- $\int d^3x\, \vec{\nabla} a \cdot \vec{B}$ corresponds to linking number of the strings

Linking configuration

- Rewriting CS coupling: $\dfrac{c}{16\pi^2} aF_{\mu\nu}\tilde{F}^{\mu\nu} \longrightarrow -\dfrac{c}{16\pi^2}\left(\partial_i a\right) A_0 B^i$

$B_i \equiv \epsilon_{ijk}\partial^j A^k$

→ Gauss law:

Noether charge

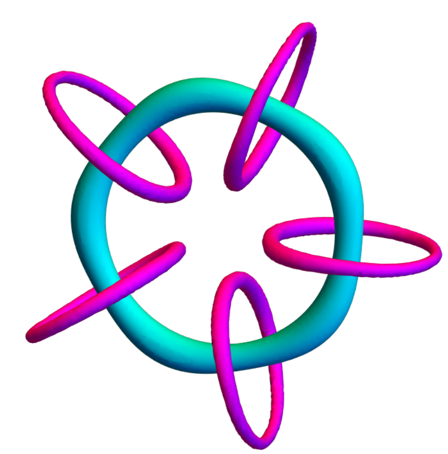
$$\frac{\delta \mathscr{L}}{\delta A_0} = \partial_i E_i - g^2 J^0 + \underline{\frac{g^2 c}{16\pi^2}\vec{\nabla} a \cdot \vec{B}} = 0$$

"electric field"

$\vec{\nabla} a \cdot \vec{B}$ **sources "electric field"**

- $\int d^3x\, \vec{\nabla} a \cdot \vec{B}$ corresponds to linking number of the strings

→ The linking loops gets "electric charge", stabilizing this object!

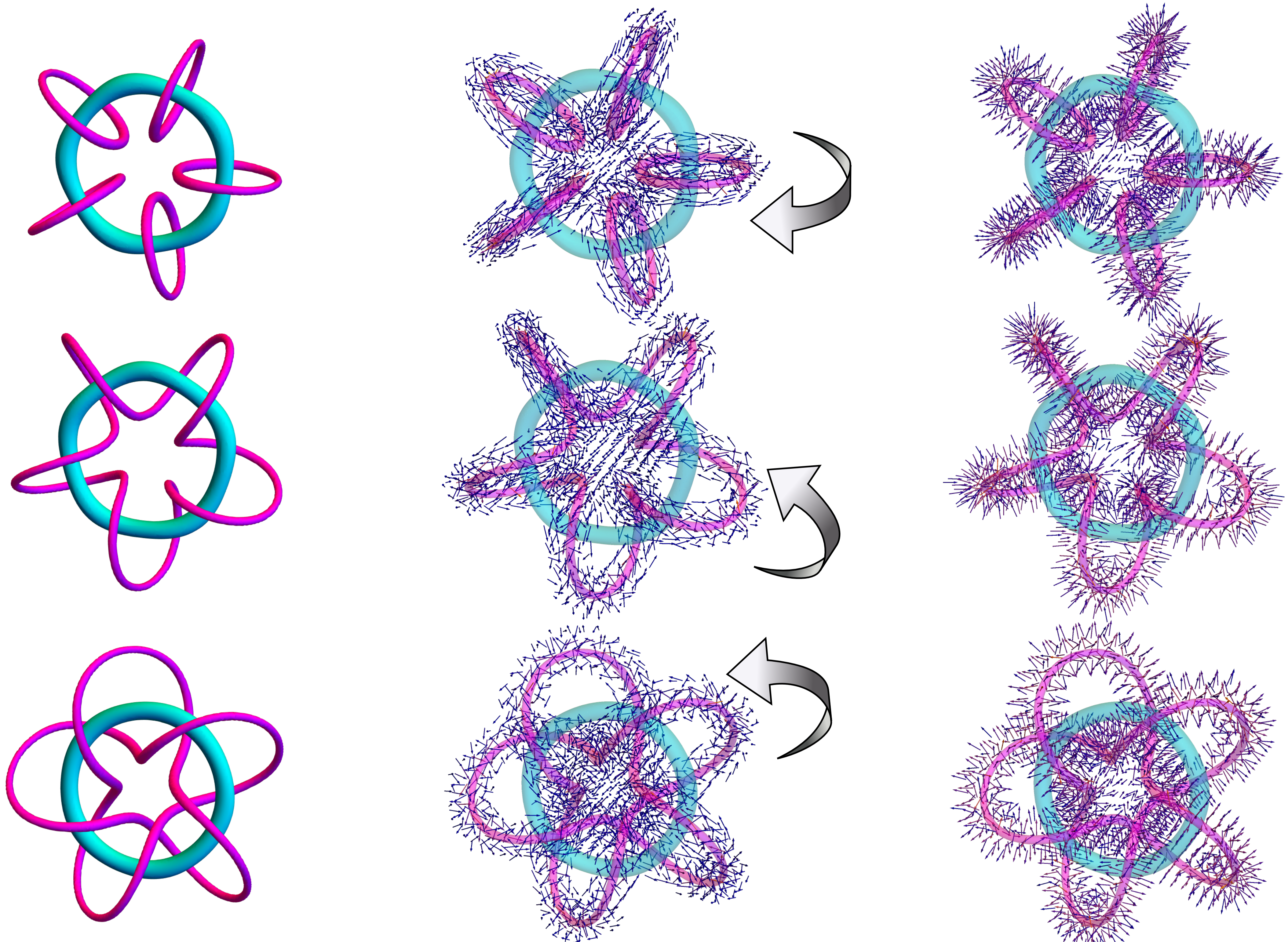


Other solutions w/ linking # 5

string core

$\vec{B}$

$\vec{E} = \vec{\nabla} A_0$

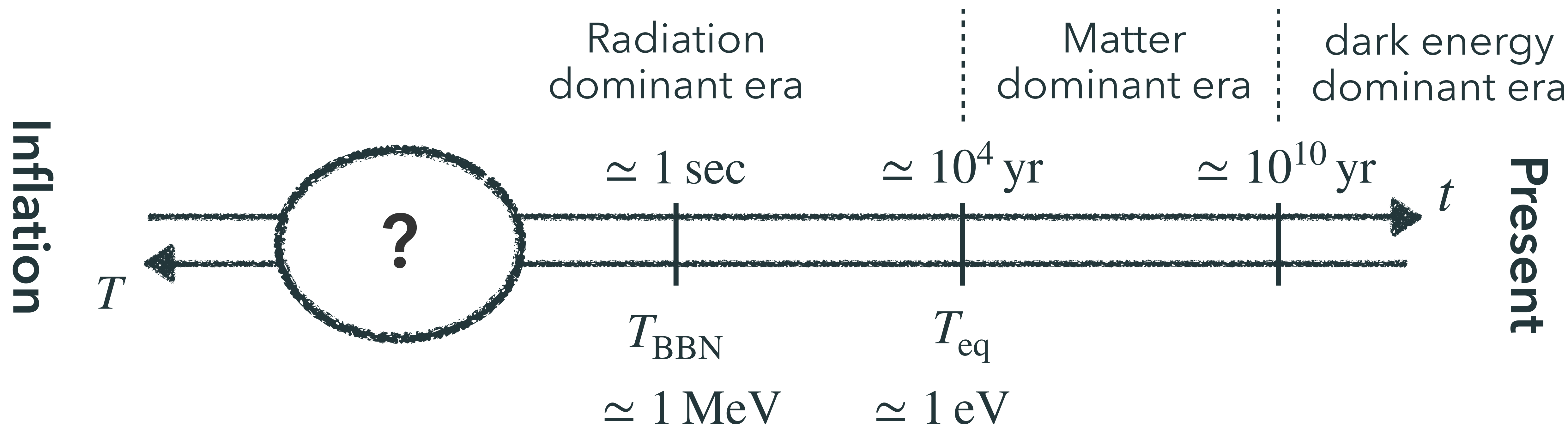


Plan of talk

- Introduction
- Knot soliton
- Application to cosmology
- Summary

Application to cosmology

Standard cosmology



- The universe starts from hot big-bang (end of inflation)
- We know what happened after Big-Bang Nucleosynthesis, but do not know before that.
- Our scenario:  = **knot dominant era!**

The model

- Natural setup: $\boxed{U(1)_{gauge} = U(1)_{B-L} \text{ \& } U(1)_{global} = U(1)_{PQ}}$

- requires right-handed neutrino coupled w/ ϕ_1: $y_R \phi_1^* \bar{\nu}_R \nu_R^c$

  → $\langle \phi_1 \rangle$ gives Majorana mass → type-I seesaw

[Minkowski '77] [Yanagida '79] [Gell-Mann+ '79] [Mohapatra-Senjanovic+ '80]

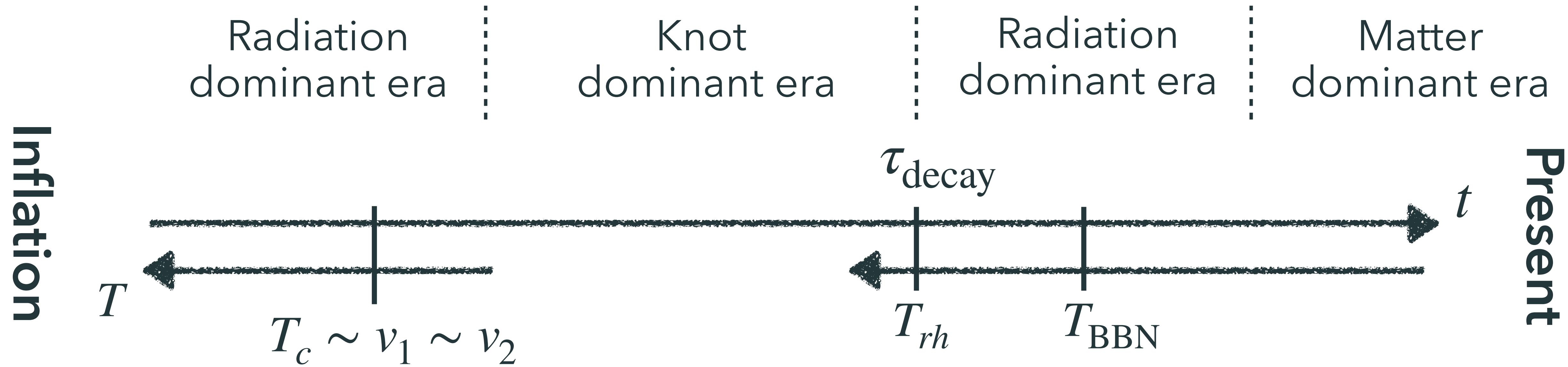
- phase of ϕ_2 (a) is identified as QCD axion

[Peccei-Quinn '77] [Weinberg '78] [Wilczek '78]

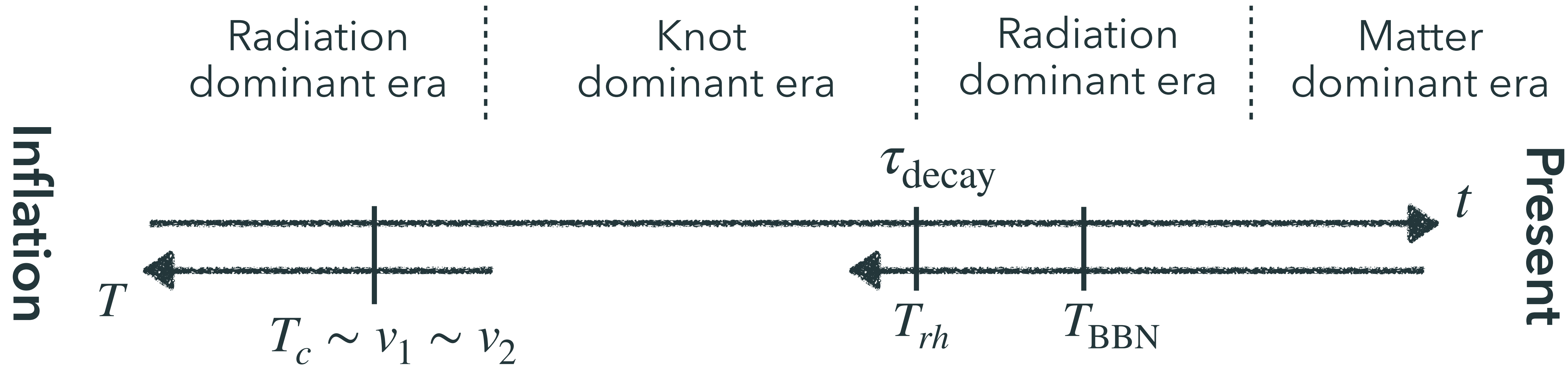
  → solution of strong CP problem & Dark matter

$$\Rightarrow v_1 \sim v_2 \sim 10^{9-12}\,\mathrm{GeV}$$

Fate of knot soliton

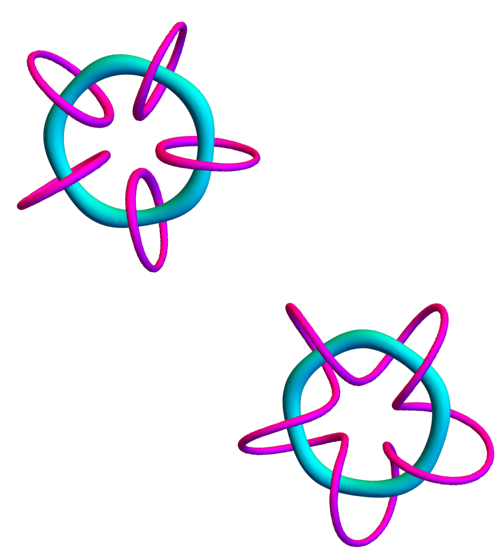


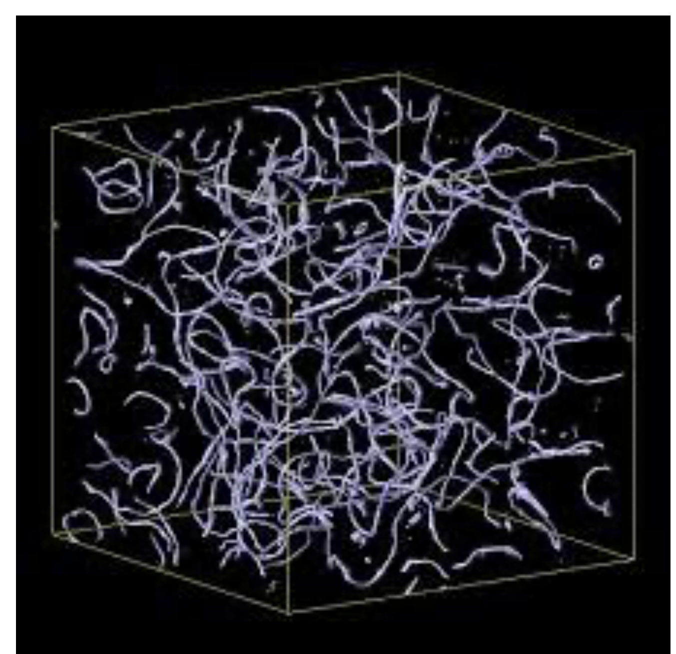
Fate of knot soliton



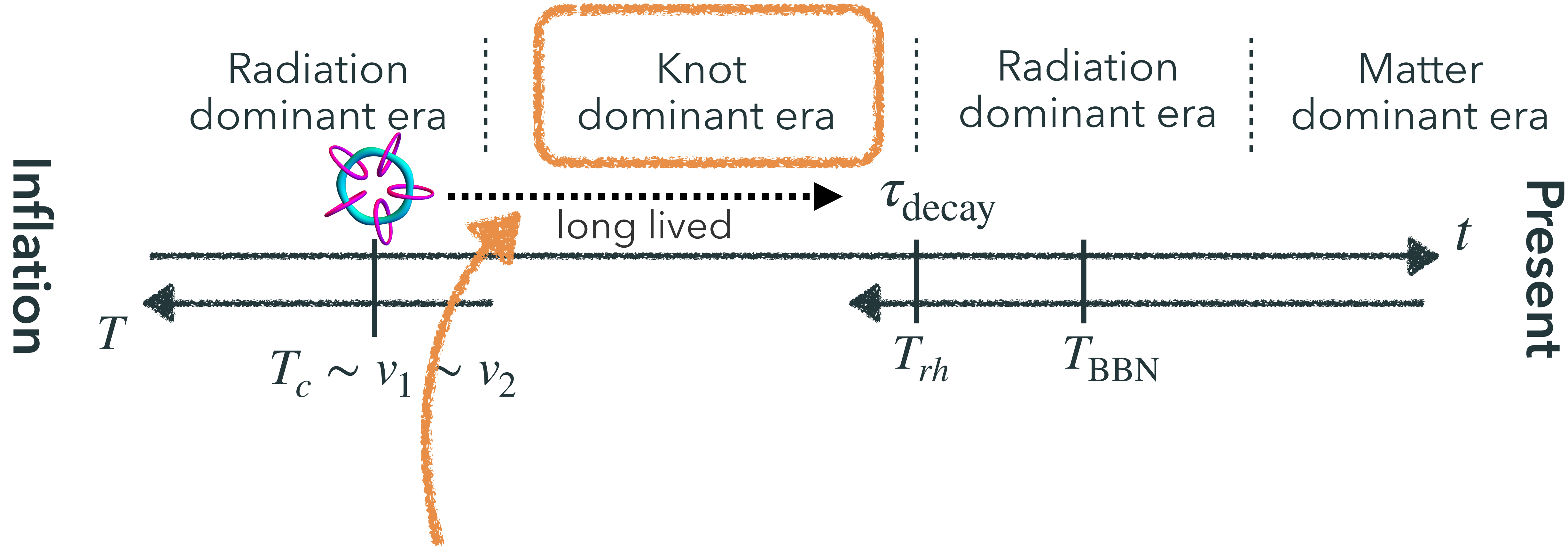
SSB: $\cancel{U(1)_{gauge} \times U(1)_{global}}$

→produce **knots &** ϕ_1, ϕ_2 **cosmic strings**



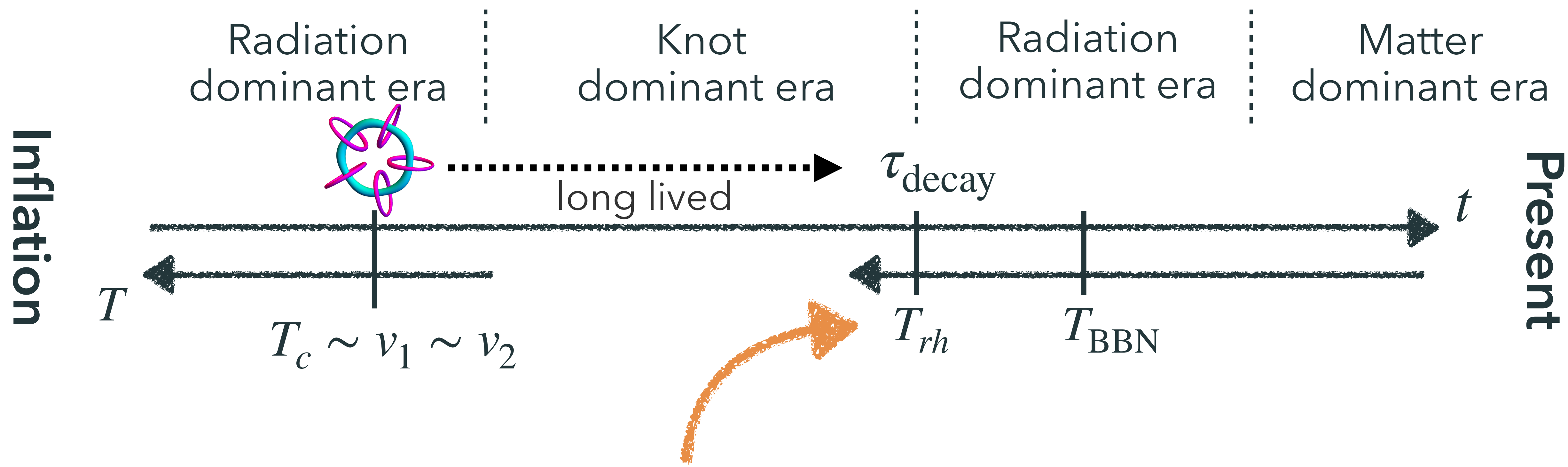


Fate of knot soliton



Knots behave as long-lived heavy matter (such as GUT monopole), and eventually dominate the energy density of universe

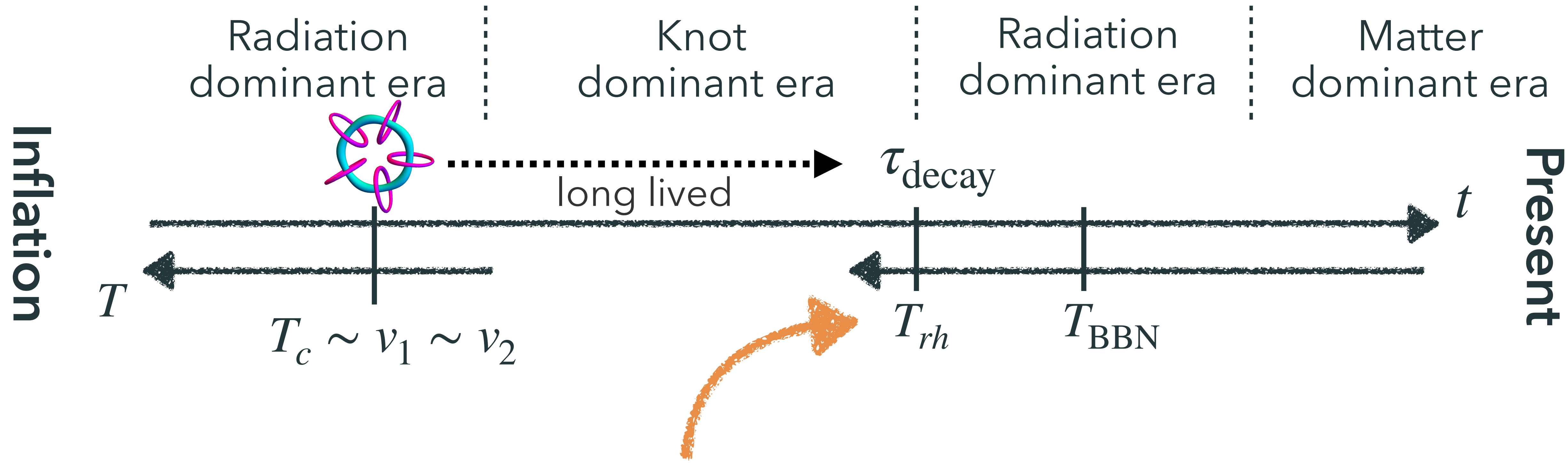
Fate of knot soliton



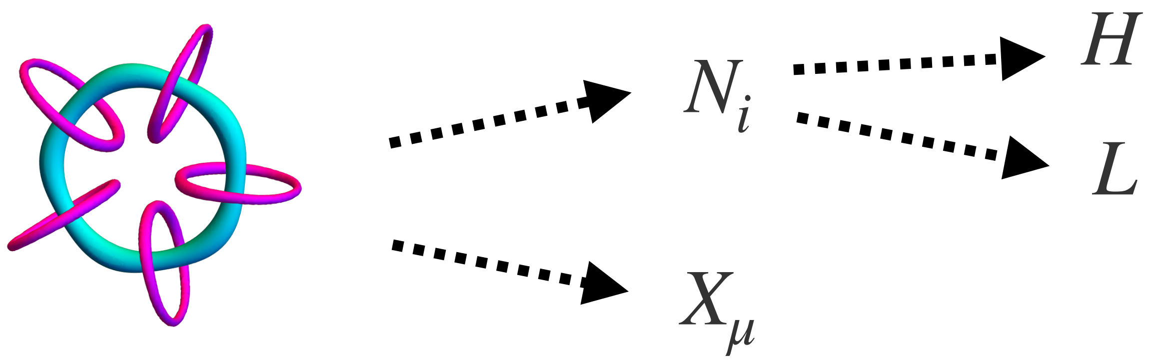
Knots decay into light particles via quantum tunneling
→ **reheat the thermal bath**
(secondary reheating)

$$T_{rh} \sim \sqrt{\frac{M_{\text{pl}}}{\tau_{\text{decay}}}}$$

Non-thermal Leptogenesis via knot



- Decay of knot solitons produce RH neutrinos

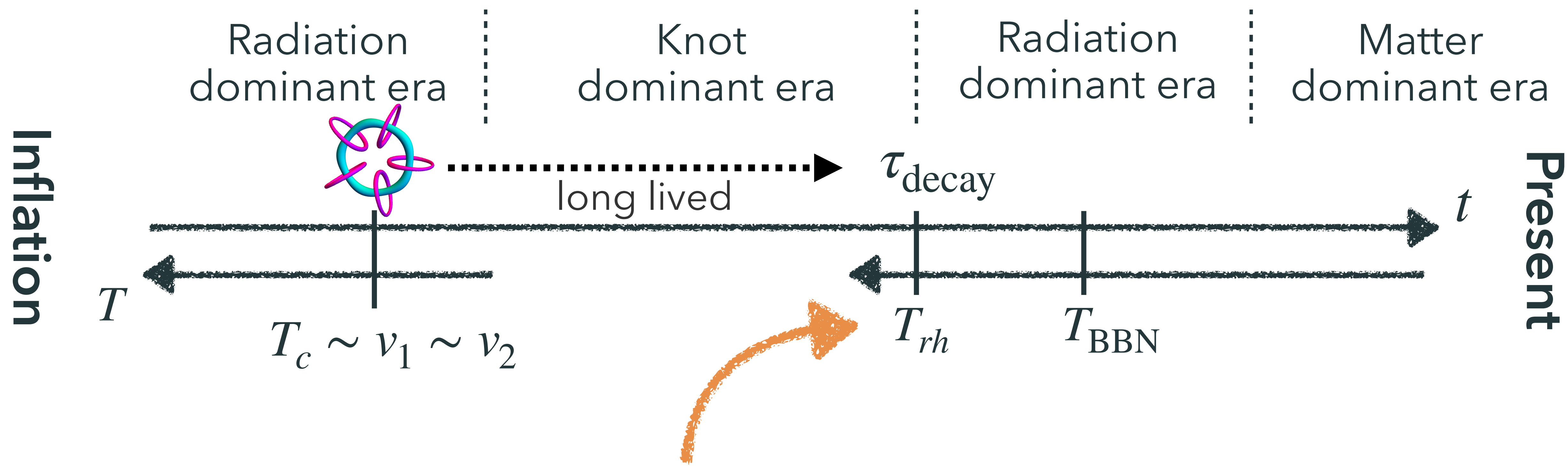


$M_{N_1} \gg T_{rh}$

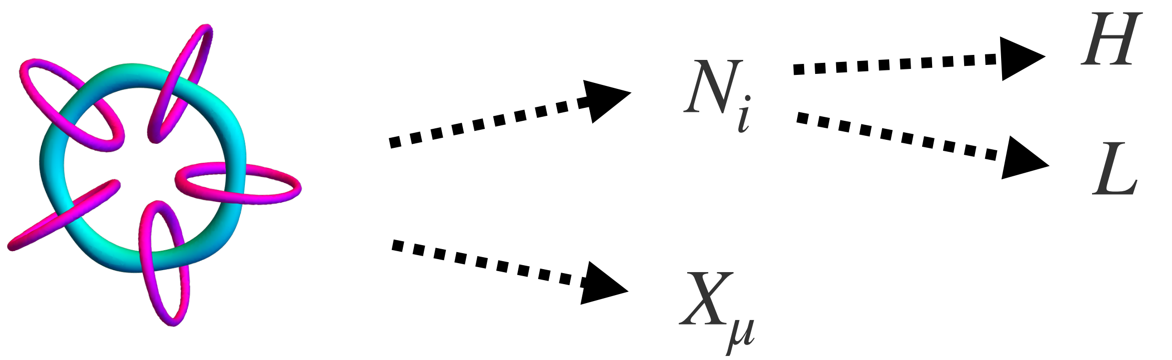
$f_{N_1} \in [0,1]$:
energy fraction of N_1

$$Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} \simeq 8.2 \times 10^{-11} f_{N_1} \left(\frac{T_{rh}}{10^6\,\text{GeV}} \right) \left(\frac{m_3}{0.05\,\text{eV}} \right) \delta_{\text{eff}}$$

Non-thermal Leptogenesis via knot



- Decay of knot solitons produce RH neutrinos

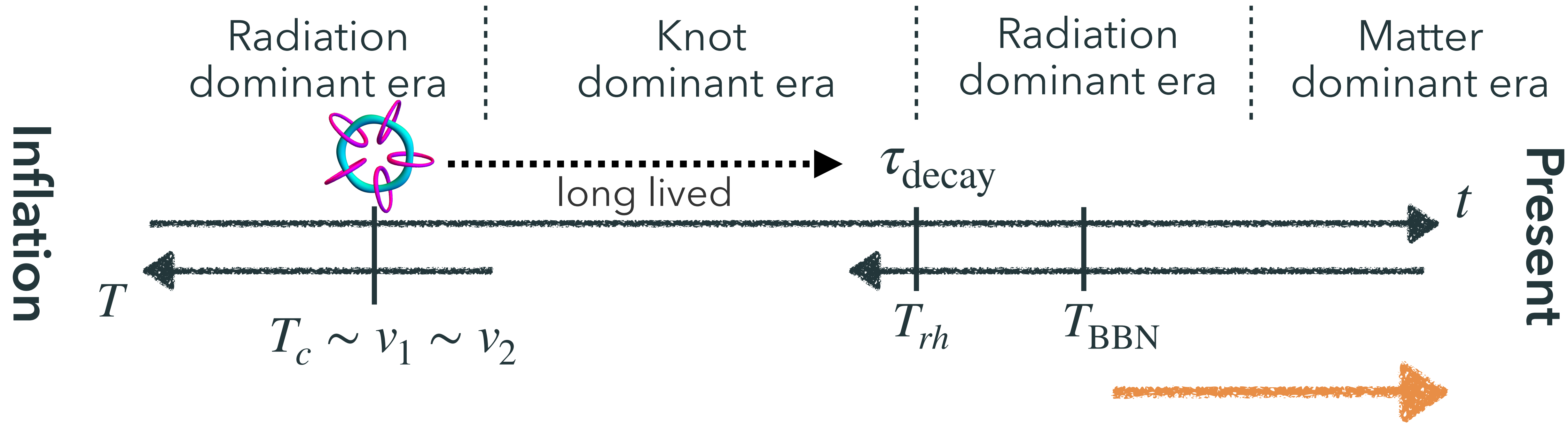


$M_{N_1} \gg T_{rh}$

$f_{N_1} \in [0,1]$: energy fraction of N_1

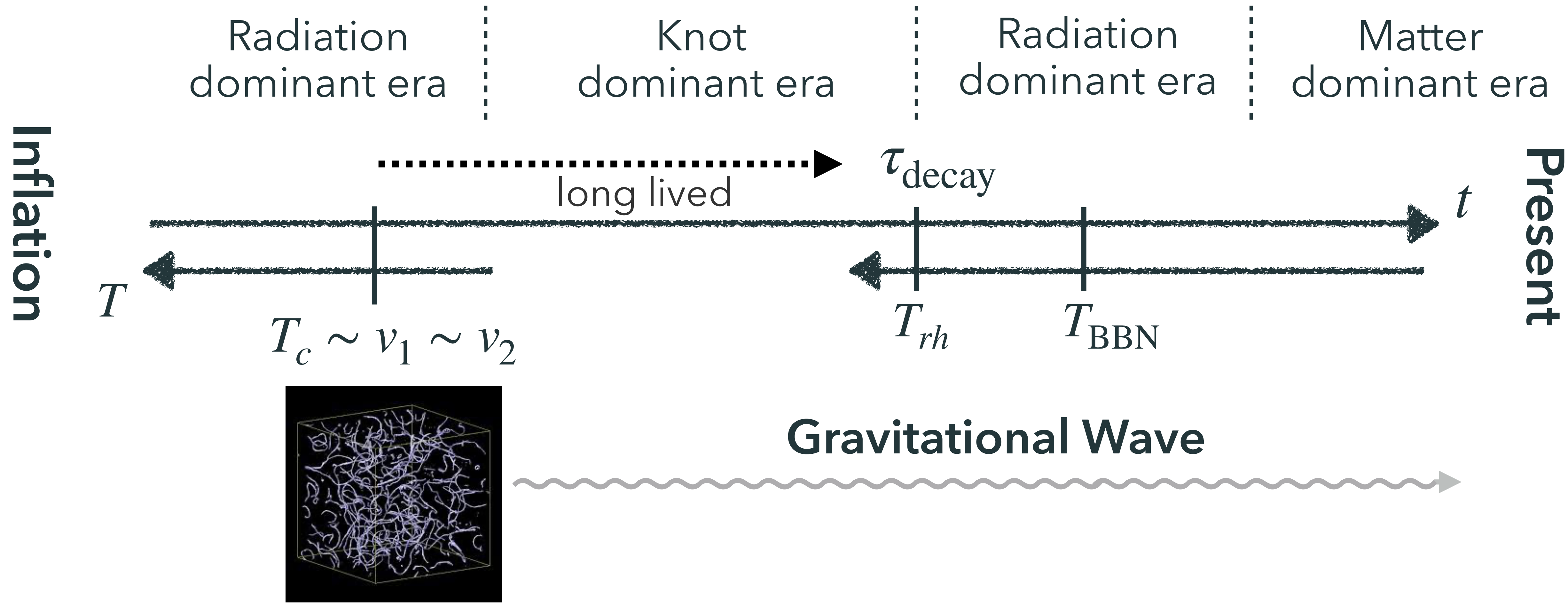
resonant case: $Y_B \lesssim 8.0 \times 10^{-11} f_{N_1} \left(\dfrac{T_{rh}}{10^2\,\text{GeV}} \right) \left(\dfrac{10^{12}\,\text{GeV}}{M_{R1}} \right)$

Fate of knot soliton



Afterwards Big-Bang Nucleosynthesis occurs and the later history is same as standard cosmology.

Testability by gravitational wave

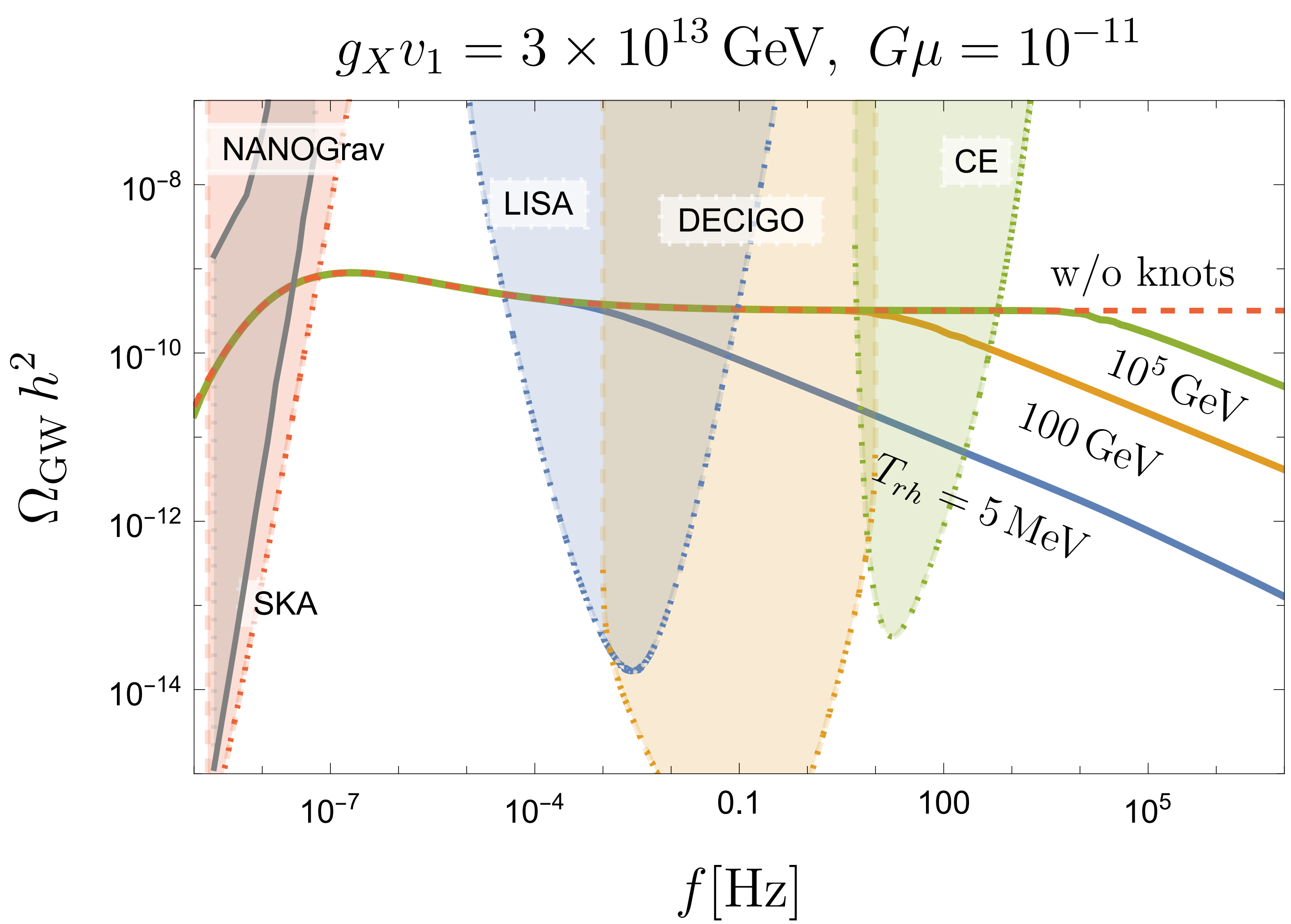


- Cosmic strings of ϕ_1, ϕ_2 emit stochastic GW background
- The existence of knot dominant era affects the GW spectrum

[Cui+, 1711.03104]

→ **We can test this scenario in terms of GW observation.**

Testability by gravitational wave



- GW spectrum w/o knot solitons is flat at high-frequency region.
- Knot dominant era makes the spectrum fall with $f^{-1/3}$

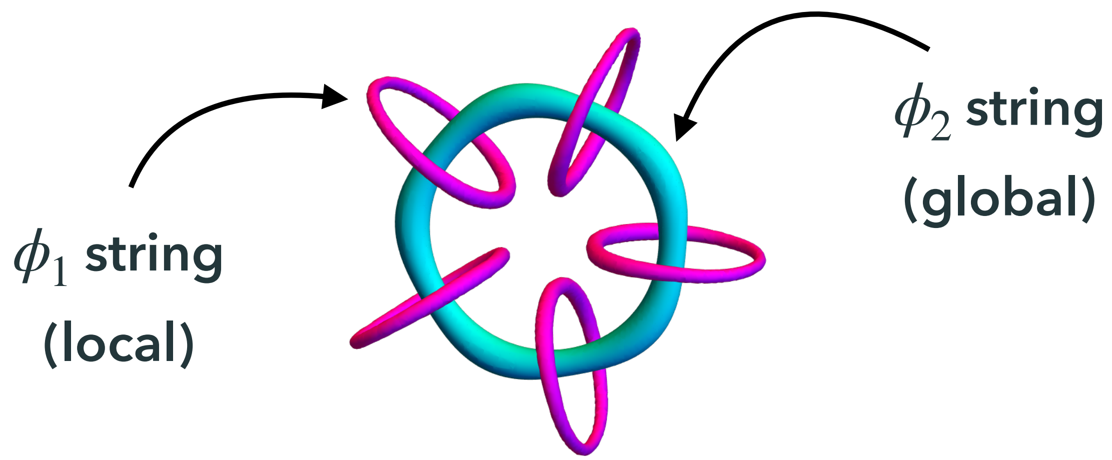
→ GW observation can distinguish these from case w/o knots

Summary

- Massage of this talk:

Knot soliton is a new stable object made of two cosmic strings!

- Key: Chern-Simons coupling $\frac{c}{16\pi^2}\int d^4x\, aF\tilde{F}$
- Linking strings obtain **electric charges**
- stable and remain in early universe



- can implement this setup in motivative models → dark matter, ν-mass
- **can be probed by gravitational wave**

Backup

Linking number

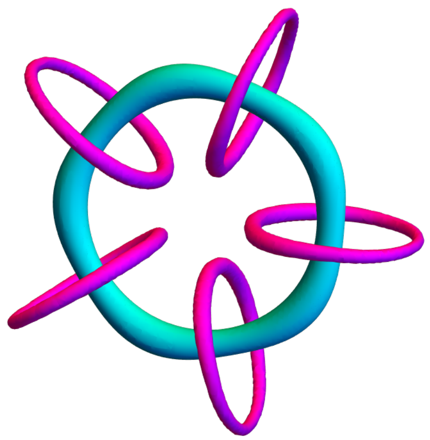
$$\epsilon^{ijk}\partial_i\partial_j a(x) = 2\pi N_a \oint_{C_a} d\vec{r}_a \cdot \overrightarrow{e}^{(k)}\,\delta^{(3)}(\overrightarrow{x} - \vec{r}_a)$$

$$\int d^3x\,\overrightarrow{\nabla} a \cdot \overrightarrow{B} = \int d^3x\,\epsilon^{ijk}\partial_i\partial_j a\,A_k$$

$$\phi_1 \propto e^{i\chi} \qquad \overrightarrow{A} \propto \overrightarrow{\partial}\chi$$

$$= 2\pi N_a \int d^3x \oint_{C_a} d\overrightarrow{r_a} \cdot \overrightarrow{A}(x)\,\delta^{(3)}(\overrightarrow{x} - \vec{r}_a)$$

$$= 2\pi N_a \oint_{C_a} d\overrightarrow{r_a} \cdot \overrightarrow{\partial}\chi(\overrightarrow{r_a})$$



- This counts how many times the phase of ϕ_1 winds along the ϕ_1 string. → definition of linking number

The model

	$U(1)_{B-L}$	$U(1)_{\rm PQ}$
ϕ_1	2	0
ϕ_2	0	1
KSVZ-like Q	Q^f_{B-L}	$Q^f_{\rm PQ}$
ν_R	-1	0
SM	q: $1/3$ l: -1	0

$v_1 \sim v_2 \sim 10^{9-12}\,\mathrm{GeV}$

$\leftarrow$not specified

$$\Rightarrow c = \sum_f Q^f_{global}(Q^f_{gauge})^2$$

- $\mathscr{L} \supset y_R\,\phi_1^*\,\bar{\nu}_R\nu_R^c$ → $\langle\phi_1\rangle$ gives Majorana mass → type-I seesaw

  [Minkowski '77] [Yanagida '79] [Gell-Mann+ '79] [Mohapatra-Senjanovic+ '80]

- phase of ϕ_2 (a) is identified as QCD axion

  [Peccei-Quinn '77] [Weinberg '78] [Wilczek '78]

  → solution of strong CP problem & Dark matter

Numerical calculation

Static energy in Coulomb gauge:

$$\boxed{\begin{aligned}\mathcal{E} = |D_i\phi_1|^2 + |\partial_i\phi_2|^2 + V(\phi_1, \phi_2) + \frac{1}{2g^2}(\partial_i A_j)^2 \\ -g^2|\phi_1|^2 A_0^2 - \frac{1}{2g^2}(\partial_i A_0)^2 - \frac{g^2 c}{16\pi^2}\, aF_{\mu\nu}\tilde{F}^{\mu\nu}\end{aligned}}$$

- Not positive definite → remove A_0 by solving Gauss law:

$$\frac{\delta\mathcal{L}}{\delta A_0} = \partial_i^2 A_0 - 2g^2|\phi_1|^2 A_0 + \frac{g^2 c}{16\pi^2}(\vec{\nabla} a)\cdot\vec{B} = 0$$

Substitute $A_0 = \dfrac{g^2 c}{16\pi^2}\dfrac{(\vec{\nabla} a)\cdot\vec{B}}{-\partial_i^2 + 2g^2|\phi_1|^2}$ into energy functional.

Numerical calculation

Energy in Coulomb gauge:

$$\mathcal{E} = |D_i\phi_1|^2 + |\partial_i\phi_2|^2 + V(\phi_1, \phi_2) + \frac{1}{2g^2}(\partial_i A_j)^2 + \frac{g^2 c}{32\pi^2}(\vec{\nabla} a \cdot \vec{B})A_0$$

$$\text{w/ } A_0 = \frac{g^2 c}{16\pi^2}\frac{(\vec{\nabla} a)\cdot \vec{B}}{-\partial_i^2 + 2g^2|\phi_1|^2}$$

- positive definite -> no obstacle
- Minimizing energy via gradient-flow method
- CPU 3584-cores parallelizing on YITP computer cluster
- lattice spacing $=0.8/gv_1$, $N = 320^3$, converged w/ O(1) days

Relation to Skyrmion

For $\lambda \gg g^2, \kappa, \chi$,

$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

$$\to \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2$$

→ non-linear sigma model w/ $O(4)$ symmetry,

which breaks into $O(3)$

There exists Skyrmion defined by winding number:

$$N_{sk} = \int d^3x \, \epsilon^{ijk} \, \mathrm{Tr} \left[U^\dagger \partial_i U U^\dagger \partial_j U U^\dagger \partial_k U \right] \qquad U \equiv \begin{pmatrix} \mathrm{Re}\, \phi_1 & \mathrm{Im}\, \phi_2 \\ -\mathrm{Im}\, \phi_1 & \mathrm{Re}\, \phi_2 \end{pmatrix}$$

The link is nothing but the Skyrmion!

[Gudnason-Nitta '20]