非超対称な弦理論と宇宙定数

Yuichi Koga

Osaka Metropolitan University

- Y.K., NPB 990 (2023) 116160
- H. Itoyama, Y.K. and Sota Nakajima, NPB 975(2022) 115667

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Introduction

■ Where is SUSY breaking scale?

➢ There is no evidence for SUSY in multi TeV scale according to the LHC.

➢ It is interesting to consider SUSY is already broken at very high energies.

■ We have more non-SUSY vacua than SUSY ones in 10D:

- Type IIA
- Type IIB
- Type I
- Heterotic $SO(32)$
- Heterotic $E_8 \times E_8$

- Type 0A
- Type 0B
- Heterotic $SO(32)$
- Heterotic $SO(16) \times E_8$
- Heterotic $SO(16) \times SO(16)$

- Heterotic $E_7^2 \times SU(2)^2$
- Heterotic $SO(24) \times SO(8)$
- …

- difficulty: very large cosmological constant (vacuum energy density)

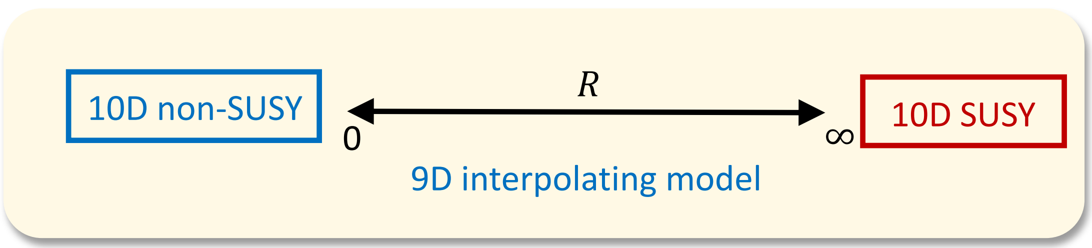
$$\Lambda^{(D)} \sim \mathcal{O}(M_s^D) \qquad M_s \text{ : string scale}$$

- We want to obtain small or vanishing cosmological const. without SUSY.

■ Our approach: "interpolating models"

- Low-dimensional models constructed by $\mathbb{Z}_2$ freely acting orbifolds
- The models are Non-SUSY where a radius R is finite.
- But SUSY can be restored in $R \to \infty$ (and/or 0) limits.

(Example)



Interpolate
between two 10D endpoints

■ In SUSY restored region ($R \approx \infty$),

$$\Lambda^{(9)} = \frac{\xi}{R^9}(n_F - n_B) + \mathcal{O}(e^{-R})$$

(ξ : positive constant; n_F, n_B : # (massless fermion,boson))

Itoyama, Taylor '86

➢ $n_F = n_B \Rightarrow$ exponentially suppressed cosmological constant

■The general heterotic models constructed by $\mathbb{Z}_2$-twists:

- d-dim. compactified with #($\mathbb{Z}_2$-twisted directions) arbitrary
- With a full set of moduli: $C_{ij} = G_{ij} + B_{ij}$ & A_i^I turned on

$(i = 10 - d, \dots, 9\,, I = 1, \dots, 16)$

↔ with all marginal deformations considered:

$$A_{Ii} \int d^2 z \partial X_L^I \bar{\partial} X_R^i + C_{ji} \int d^2 z \partial X_L^j \bar{\partial} X_R^i$$

➢Show various interpolation patterns in $d = 1, 2$

➢Find solutions of $n_F = n_B$ where cosmol. const. is exp. supp.

➢Analyze Wilson-line stability of the effective potential

Outline

1. Introduction
2. Non-SUSY heterotic strings with general Z_2 twists
3. Endpoint limits & interpolations
4. Cosmological constant
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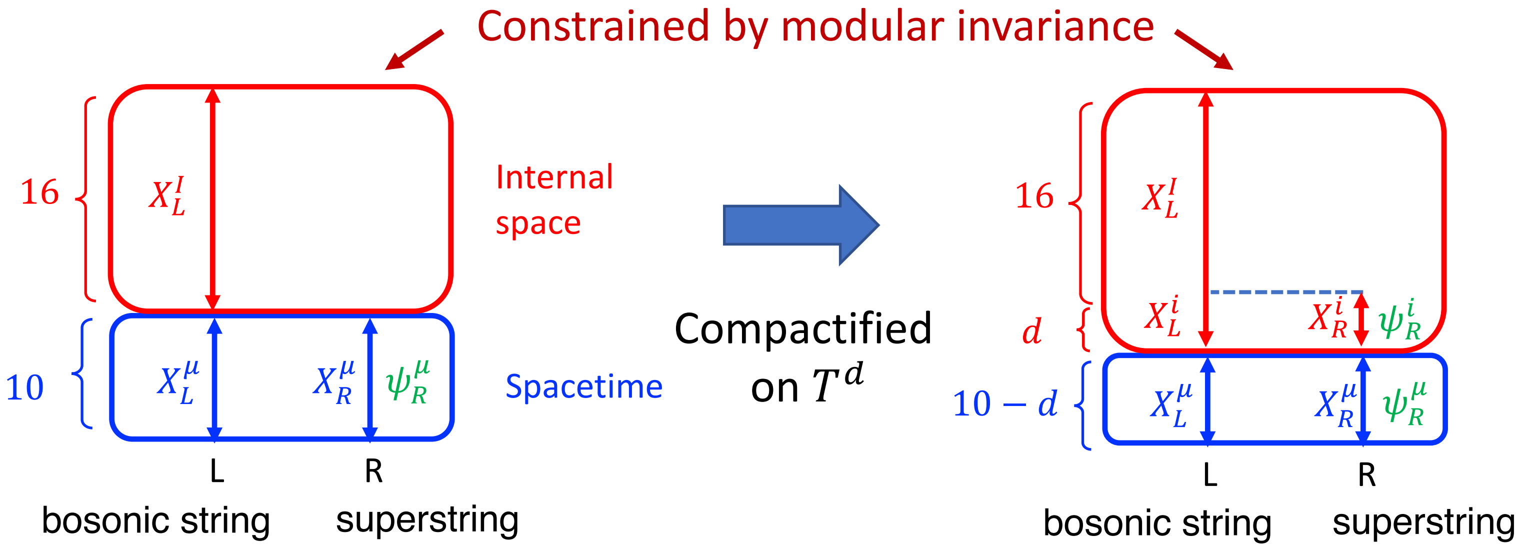
SUSY heterotic strings on T^d

- Hybrid("heterotic") theory including closed strings:
  - Left: bosonic string (26D)
  - Right: superstring (10D)
- Compactified on T^d (with maximal SUSY)



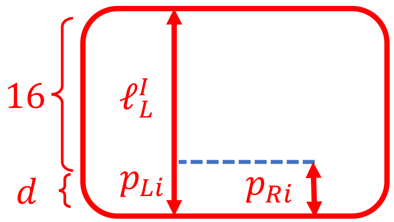
- $X_{L,R}^{\mu}, X_{L,R}^{I,i}$ / $\psi_R^{\mu,i}$: bosonic / fermionic coordinates
  $(\mu = 0, \ldots 9-d\ ,\ i = 10-d, \ldots, 9\ , I = 1, \ldots, 16)$

Narain lattice

- Modular inv. ⇒ internal momenta $P = Z\mathcal{E} \in \Gamma^{16+d,d}$
  - Narain lattice: even self-dual lattice w/ Lorentz. sign. $(16+d,d)$
  - P is labeled by an integer vector $Z = (q, \underline{m}, \underline{n}) \in \boldsymbol{Z}^{16} \times \boldsymbol{Z}^{d} \times \boldsymbol{Z}^{d}$
    winding numbers (m), KK momenta (n)
  - Turn on full moduli: $d(d+16) = d^2 + 16d \quad \Rightarrow (G_{ij} + B_{ij})$ & A_i^I
  - Consider a rectangular d-torus: $G_{ij} = R_i^2 \delta_{ij}$

➢ $P = (\ell_L, p_L; p_R)$ [Narain, Sarmadi, Witten '86]

$$
\begin{cases}
\ell_L^I = \pi^I - m^i A_i^I, \\
p_{Li} = \dfrac{1}{\sqrt{2}R_i}\left(\pi \cdot A_i + n_i + m^j \left(G_{ij} + B_{ij} - \dfrac{1}{2} A_i \cdot A_j\right)\right) \\
p_{Ri} = \dfrac{1}{\sqrt{2}R_i}\left(\pi \cdot A_i + n_i - m^j \left(G_{ij} - B_{ij} + \dfrac{1}{2} A_i \cdot A_j\right)\right)
\end{cases}
$$



$\pi^I \equiv q^I \alpha_{16} \in \underline{\Gamma^{16}} \Leftarrow Spin(32)/\boldsymbol{Z}_2$ or $E_8 \times E_8$ lattice

Construction of non-SUSY heterotic strings

[Ginsparg, Vafa '86]

- Z_2 freely acting orbifold (stringy Scherk-Schwarz comp.)

  - Project out SUSY hetero on T^d by $\frac{1+(-)^F\alpha}{2}$ (+ twisted sec. added)

    Z_2 generator : $(-)^F\alpha$ $\begin{cases} F\text{: spacetime fermion \#} \\ \alpha\text{: shift of order 2 such as } \alpha\,|P\rangle = e^{2\pi i P\cdot\delta}\,|P\rangle \end{cases}$

  - δ is called a shift vector : $2\delta \in \Gamma^{16+d,d} \Rightarrow 2\delta = \hat{Z}\mathcal{E}$

[Itoyama, Koga, Nakajima '21]

    - labeled by a vector $\hat{Z} = (\hat{q}^I, \hat{m}^i, \hat{n}_i)$ whose components are 0 or 1

      → Non-SUSY strings depend on $\hat{Z}$

    - Splitting the Narain lattice $\Gamma^{16+d,d}$ into $\Gamma_+^{16+d,d}$ and $\Gamma_-^{16+d,d}$:

$$\Gamma_+^{16+d,d} = \{P \in \Gamma^{16+d,d} \,|\, \delta\cdot P \in \mathbb{Z}\}$$
$$\Gamma_-^{16+d,d} = \{P \in \Gamma^{16+d,d} \,|\, \delta\cdot P \in \mathbb{Z}+1/2\}$$

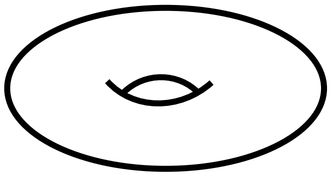
$$\Rightarrow \alpha\,|P\rangle = \begin{cases} +\,|P\rangle & \text{for} \quad P \in \Gamma_+^{16+d,d} \\ -\,|P\rangle & \text{for} \quad P \in \Gamma_-^{16+d,d} \end{cases}$$

Bosons/Fermions live in $\Gamma_+^{16+d,d}$ / $\Gamma_-^{16+d,d}$ respectively → SUSY breaking

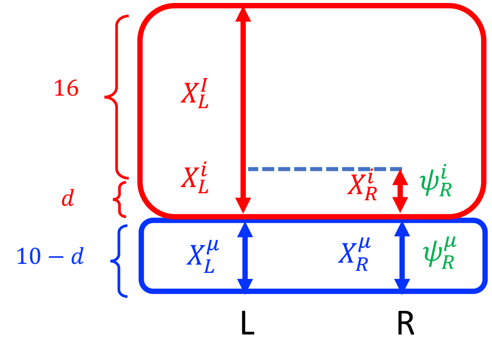
1-loop partition function

- Heterotic strings on T^d (with maximal SUSY)

$$Z^{T^d} = \underbrace{Z_B^{(8-d)}}_{X_L^\mu, X_R^\mu} \underbrace{(\bar{V}_8 - \bar{S}_8)}_{\psi_R^\mu, \psi_R^i} \underbrace{Z_{\Gamma^{16+d,d}}}_{X_L^I, X_L^i, X_R^i}$$



$\tau = \tau_1 + i\tau_2$



orbifolding by $(-)^F\alpha$

- Non-SUSY Heterotic strings

$$Z^{\cancel{SUSY}}_{(\hat{Z})} = Z_B^{(8-d)} \left\{ \underbrace{\bar{V}_8 Z_{\Gamma_+^{16+d,d}}}_{\text{vector}} - \underbrace{\bar{S}_8 Z_{\Gamma_-^{16+d,d}}}_{\text{spinor}} + \underbrace{\bar{O}_8 Z_{\Gamma_\pm^{16+d,d}+\delta}}_{\text{scalar}} - \underbrace{\bar{C}_8 Z_{\Gamma_\mp^{16+d,d}+\delta}}_{\text{co-spinor}} \right\}$$

$$Z_B^{(8-d)} = \tau_2^{-\frac{8-d}{2}} (\eta\bar{\eta})^{-(8-d)}$$

$$Z_{\Gamma^{16+d,d}} = \eta^{-(16+d)} \bar{\eta}^{-d} \sum_{p \in \Gamma^{16+d,d}} q^{\frac{1}{2}p_L^2} \bar{q}^{\frac{1}{2}p_R^2} \quad , q = e^{2\pi i\tau}$$

$$\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(z,\tau) = \sum_{n=-\infty}^{\infty} \exp\left(\pi i(n+\alpha)^2\tau + 2\pi i(n+\alpha)(z+\beta)\right)$$

- $SO(2n)$ characters :

$$\begin{pmatrix} O_{2n} \\ V_{2n} \end{pmatrix} = \frac{1}{2\eta^n}\left(\vartheta\begin{bmatrix}0\\0\end{bmatrix}^n \pm \vartheta\begin{bmatrix}0\\1/2\end{bmatrix}^n \right)$$

$$\begin{pmatrix} S_{2n} \\ C_{2n} \end{pmatrix} = \frac{1}{2\eta^n}\left(\vartheta\begin{bmatrix}1/2\\0\end{bmatrix}^n \pm \vartheta\begin{bmatrix}1/2\\1/2\end{bmatrix}^n \right)$$

Simplest case : $d = 0$

■10D non-SUSY heterotic models

- shift vector $\delta = \frac{\hat{\pi}}{2}$ $(\hat{\pi} \in \Gamma^{16})$, Γ^{16} : 16D Narain lattice

- $\Gamma^{16} = E_8 \times E_8$ root lattice

$\delta \in \frac{1}{2}\Gamma^{16}$	$(\mathbf{1}, (\mathbf{0})^7; (\mathbf{0})^8)$	$\left(\left(\frac{1}{2}\right)^2, (\mathbf{0})^6; \left(\frac{1}{2}\right)^2, (\mathbf{0})^6\right)$	$(\mathbf{1}, (\mathbf{0})^7; \mathbf{1}, (\mathbf{0})^7)$
Gauge sym.	$SO(16)\times E_8$	$(SU(2)\times E_7)^2$	$SO(16)\times SO(16)$

tachyon-free

[Dixon, Harvey '86]

- $\Gamma^{16} = Spin(32)/\boldsymbol{Z}_2$ root lattice

$\delta \in \frac{1}{2}\Gamma^{16}$	$(\mathbf{1}, (\mathbf{0})^{15})$	$\left(\left(\frac{1}{2}\right)^4, (\mathbf{0})^{12}\right)$	$\left(\left(\frac{1}{4}\right)^{16}\right)$	$\left(\left(\frac{1}{2}\right)^8, (\mathbf{0})^8\right)$
Gauge sym.	$SO(32)$	$SO(24)\times SO(8)$	$SU(16)\times U(1)$	$SO(16)\times SO(16)$

※ Modular invariance $\Longrightarrow$ δ^2 : integer

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Endpoint limits & interpolations

- Consider $d = 1{,}2$ cases (9,8D models) with $A = B = 0$

➢ 1-loop partition function:

$$Z_{(\hat{Z})}^{\not{SUSY}} = Z_B^{(8-d)} \left\{ \bar{V}_8 \underline{Z_{\Gamma_+^{16+d,d}}} - \bar{S}_8 \underline{Z_{\Gamma_-^{16+d,d}}} + \bar{O}_8 \underline{Z_{\Gamma_\pm^{16+d,d}+\delta}} - \bar{C}_8 \underline{Z_{\Gamma_\mp^{16+d,d}+\delta}} \right\}$$

➢ The behavior of $Z_{\Gamma_\pm^{16+d,d}(+\delta)}$ in the limits of $R_i \to 0, \infty \ \ (i = 1{,}2)$

- Take $R_i \to \infty \Rightarrow$ only $m^i = 0$ contributes
- Take $R_i \to 0 \Rightarrow$ only $n_i = 0$ contributes

$$p_{L/Ri} = \frac{1}{\sqrt{2}} \left(\frac{n_i}{R_i} + / - m_i R_i \right)$$

$$Z_{\Gamma_\pm^{16+d,d}(+\delta)} = \eta^{-(16+d)} \bar{\eta}^{-d} \sum_{p \in \Gamma_\pm^{16+d,d}(+\delta)} \underline{q^{\frac{1}{2}\left(\ell_L^2 + p_L^2\right)} \bar{q}^{\frac{1}{2} p_R^2}}$$

$$\to e^{-\pi\tau_2(\ell_L^2 + p_L^2 + p_R^2)} e^{i\pi\tau_1(\ell_L^2 + p_L^2 - p_R^2)}$$

- Recall: $\Gamma_+^{16+d,d} = \{ P \in \Gamma^{16+d,d} \,|\, \underline{\delta \cdot P} \in \mathbb{Z} \}$

$$\Gamma_-^{16+d,d} = \{ P \in \Gamma^{16+d,d} \,|\, \underline{\delta \cdot P} \in \mathbb{Z} + 1/2 \}$$

$d = 1$ case

■ Example with $(\widehat{m}^1, \hat{n}_1) = (1,0)$

$(\hat{Z} = (\hat{q}, \widehat{m}, \hat{n}) \in \boldsymbol{Z}^{16} \times \boldsymbol{Z} \times \boldsymbol{Z})$

$\hat{\pi} = \hat{q}\alpha_{16} \in \Gamma^{16}$

- Inner product: $\delta \cdot p = \frac{1}{2}\left(\hat{\pi} \cdot \pi + n_1\right)$

$$\Gamma^{17,1}_{\pm} = \left\{ p = Z\tilde{\mathcal{E}} \,\middle|\, (\pi, m, n) \in (\Gamma^{16}_{\pm}, \mathbb{Z}, 2\mathbb{Z}) \right\} \oplus \left\{ p = Z\tilde{\mathcal{E}} \,\middle|\, (\pi, m, n) \in (\Gamma^{16}_{\mp}, \mathbb{Z}, 2\mathbb{Z}+1) \right\},$$

$$\Gamma^{17,1}_{\pm} + \delta = \left\{ p = Z\tilde{\mathcal{E}} \,\middle|\, (\pi, m, n) \in \left(\Gamma^{16}_{\pm} + \frac{\hat{\pi}}{2}, \mathbb{Z} + \frac{1}{2}, 2\mathbb{Z} \right) \right\}$$

$$\oplus \left\{ p = Z\tilde{\mathcal{E}} \,\middle|\, (\pi, m, n) \in \left(\Gamma^{16}_{\mp} + \frac{\hat{\pi}}{2}, \mathbb{Z} + \frac{1}{2}, 2\mathbb{Z}+1 \right) \right\}.$$

where $\Gamma^{16}_{+}(\hat{\pi}) = \left\{ \pi \in \Gamma^{16} \,\middle|\, \hat{\pi} \cdot \pi \in 2\mathbb{Z} \right\}, \quad \Gamma^{16}_{-}(\hat{\pi}) = \left\{ \pi \in \Gamma^{16} \,\middle|\, \hat{\pi} \cdot \pi \in 2\mathbb{Z}+1 \right\}.$

$$Z_{\Gamma^{17,1}_{\pm}} \xrightarrow[m_1 = 0]{R_1 \to \infty} \frac{R_1}{\sqrt{\tau_2}} \left(\eta\bar{\eta}\right)^{-1} Z_{\Gamma^{16}}, \quad Z_{\Gamma^{17,1}_{\pm}+\delta} \xrightarrow[m_1 = 0]{R_1 \to \infty} 0, \quad \text{SUSY}$$

$$Z_{\Gamma^{17,1}_{\pm}} \xrightarrow[n_1 = 0]{R_1 \to 0} \frac{1}{R_1\sqrt{\tau_2}} \left(\eta\bar{\eta}\right)^{-1} Z_{\Gamma^{16}_{\pm}}, \quad Z_{\Gamma^{17,1}_{\pm}+\delta} \xrightarrow[n_1 = 0]{R_1 \to 0} \frac{1}{R_1\sqrt{\tau_2}} \left(\eta\bar{\eta}\right)^{-1} Z_{\Gamma^{16}_{\pm}+\frac{\hat{\pi}}{2}}. \quad \text{Non-SUSY}$$

$$Z^{\cancel{SUSY}}_{(\hat{Z})} = Z_B^{(8-d)} \left\{ \bar{V}_8 Z_{\Gamma^{16+d,d}_{+}} - \bar{S}_8 Z_{\Gamma^{16+d,d}_{-}} + \bar{O}_8 Z_{\Gamma^{16+d,d}_{\pm}+\delta} - \bar{C}_8 Z_{\Gamma^{16+d,d}_{\mp}+\delta} \right\} \qquad Z^{T^d} = Z_B^{(8-d)} \left(\bar{V}_8 - \bar{S}_8 \right) Z_{\Gamma^{16+d,d}}$$

■ 9D Non-SUSY heterotic models ($d = 1$)

[Itoyama, Koga, Nakajima ’21]

- $2^2 = 4$ classes exist

$$|\hat{\pi}|^2 + 2\hat{m}\hat{n}^t = 0 \pmod 4$$

※Modular inv. $\Rightarrow \delta^2 \in \mathbb{Z}$

- class (1): $|\hat{\pi}|^2 = 0 \pmod 4$, $(\hat{m}; \hat{n}) = (0; 0)$

10D non-SUSY ← R: 0 → ∞ → 10D non-SUSY

non-SUSY heterotic strings on a circle

- class (2): $|\hat{\pi}|^2 = 0 \pmod 4$, $(\hat{m}, \hat{n}) = (1; 0)$

10D non-SUSY ← R: 0 → ∞ → 10D SUSY

- class (3): $|\hat{\pi}|^2 = 0 \pmod 4$, $(\hat{m}, \hat{n}) = (0; 1)$

10D SUSY ← R: 0 → ∞ → 10D non-SUSY

interpolation between SUSY and non-SUSY vacua

- class (4): $|\hat{\pi}|^2 = 2 \pmod 4$, $(\hat{m}, \hat{n}) = (1; 1)$

10D SUSY ← R: 0 → ∞ → 10D SUSY

SUSY restored at both of the endpoints

$d = 2$ case

■Example with $(\widehat{m}^1, \widehat{m}^2; \hat{n}_1, \hat{n}_2) = (1, 0; 0, 0)$

Class(1) in 9D

Class(2) in 9D

- Inner product: $\delta \cdot p = \frac{1}{2}(\hat{\pi} \cdot \pi + n_1)$

$$\Gamma^{18,2}_{\pm} = \left\{ p = Z\tilde{\mathcal{E}} \,\middle|\, (\pi, m, n) \in \left(\Gamma^{16}_{\pm}, \mathbb{Z}^2, 2\mathbb{Z} \times \mathbb{Z}\right) \right\}$$
$$\oplus \left\{ p = Z\tilde{\mathcal{E}} \,\middle|\, (\pi, m, n) \in \left(\Gamma^{16}_{\mp}, \mathbb{Z}^2, (2\mathbb{Z}+1) \times \mathbb{Z}\right) \right\}$$

$$Z_{\Gamma^{18,2}_{\pm}} \xrightarrow[m_1 = 0]{R_1 \to \infty} \frac{R_1}{\sqrt{\tau_2}} (\eta\bar{\eta})^{-1} Z_{\Gamma^{17,1}} \xrightarrow{R_2 \to \infty} \frac{R_1 R_2}{\tau_2} (\eta\bar{\eta})^{-2} Z_{\Gamma^{16}},$$

$$Z_{\Gamma^{18,2}_{\pm}} \xrightarrow[m_2 = 0]{R_2 \to \infty} \frac{R_2}{\sqrt{\tau_2}} (\eta\bar{\eta})^{-1} Z_{\Gamma^{17,1}_{\pm}}|_{(2)} \xrightarrow{R_1 \to \infty} \frac{R_1 R_2}{\tau_2} (\eta\bar{\eta})^{-2} Z_{\Gamma^{16}},$$

SUSY

$$Z_{\Gamma^{18,2}_{\pm}} \xrightarrow[n_1 = 0]{R_1 \to 0} \frac{1}{R_1\sqrt{\tau_2}} (\eta\bar{\eta})^{-1} Z_{\Gamma^{17,1}_{\pm}}|_{(1)} \xrightarrow{R_2 \to 0} \frac{1}{R_1 R_2 \tau_2} (\eta\bar{\eta})^{-2} Z_{\Gamma^{16}_{\pm}},$$

$$Z_{\Gamma^{18,2}_{\pm}} \xrightarrow[n_2 = 0]{R_2 \to 0} \frac{1}{R_2\sqrt{\tau_2}} (\eta\bar{\eta})^{-1} Z_{\Gamma^{17,1}_{\pm}}|_{(2)} \xrightarrow{R_1 \to 0} \frac{1}{R_1 R_2 \tau_2} (\eta\bar{\eta})^{-2} Z_{\Gamma^{16}_{\pm}},$$

Non-SUSY

$$Z_{\Gamma^{18,2}_{\pm}} \xrightarrow{R_1 \to \infty, R_2 \to 0} \frac{R_1}{R_2 \tau_2} (\eta\bar{\eta})^{-2} Z_{\Gamma^{16}},$$ SUSY

Class (#) in 9D

$$Z_{\Gamma^{18,2}_{\pm}} \xrightarrow{R_1 \to 0, R_2 \to \infty} \frac{R_2}{R_1 \tau_2} (\eta\bar{\eta})^{-2} Z_{\Gamma^{16}_{\pm}},$$ Non-SUSY

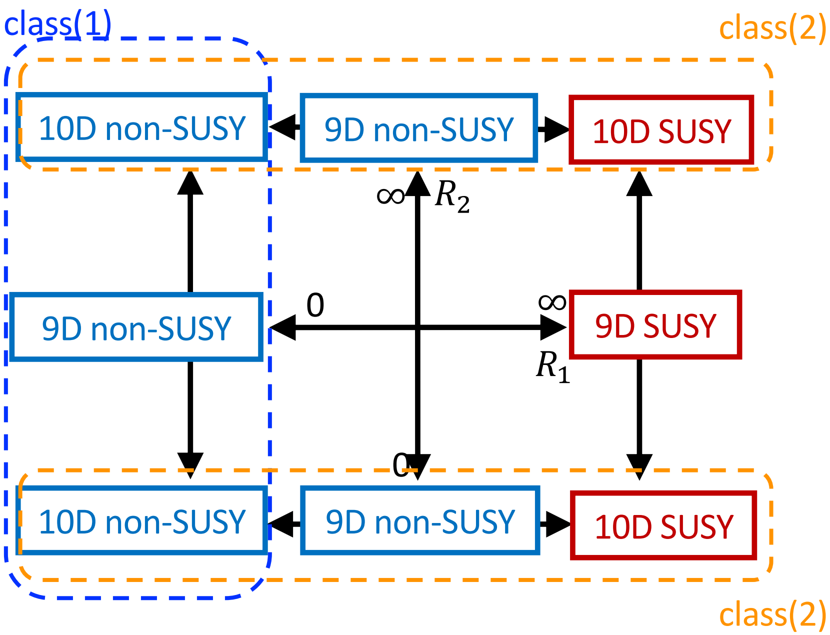
■ 8D Non-SUSY heterotic models ($d = 2$)

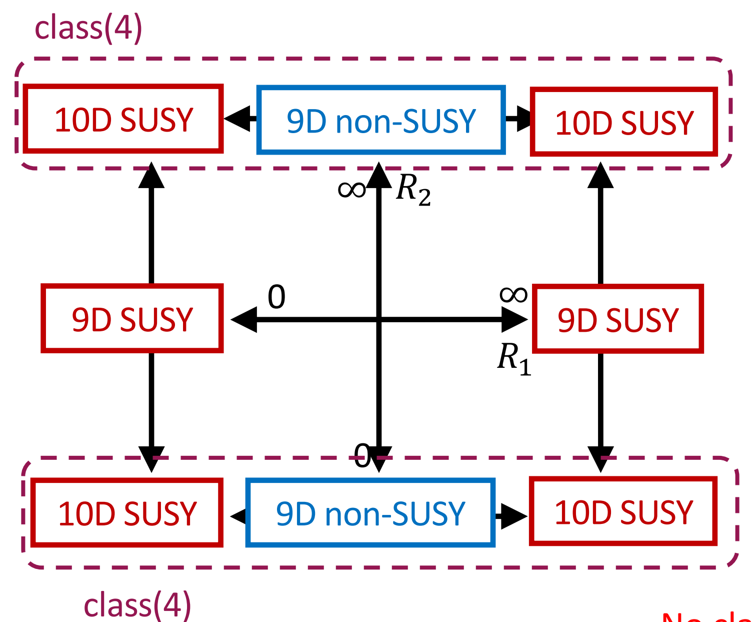
[Koga '22]

➢ $2^4 = 16$ classes exist

- class (2) & (1) :
  $|\hat{\pi}|^2 = 0$ (mod 4), $(\hat{m}; \hat{n}) = (1,0; 0,0)$

- class (4) & (1) :
  $|\hat{\pi}|^2 = 2$ (mod 4), $(\hat{m}; \hat{n}) = (1,0; 1,0)$





No class(1)

10D (Non-)SUSY condition:

Limits of R_1, R_2	10D SUSY model	10D Non-SUSY model
$(R_1, R_2) \to (\infty, \infty)$	$\hat{m}^1 + \hat{m}^2 > 0$	$\hat{m}^1 + \hat{m}^2 = 0$
$(R_1, R_2) \to (\infty, 0)$	$\hat{m}^1 + \hat{n}_2 > 0$	$\hat{m}^1 + \hat{n}_2 = 0$
$(R_1, R_2) \to (0, \infty)$	$\hat{n}_1 + \hat{m}^2 > 0$	$\hat{n}_1 + \hat{m}^2 = 0$
$(R_1, R_2) \to (0, 0)$	$\hat{n}_1 + \hat{n}_2 > 0$	$\hat{n}_1 + \hat{n}_2 = 0$

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Cosmological Constant

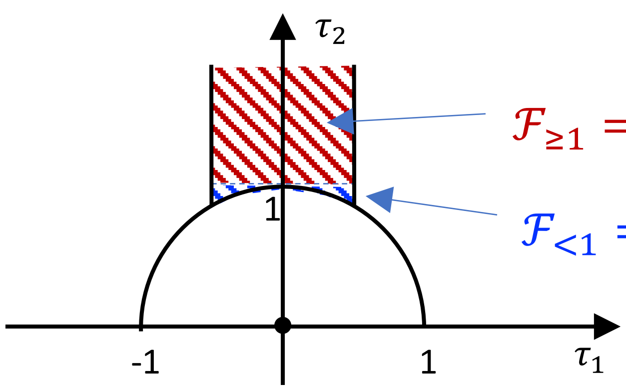
- 1-loop cosmological constant (effective potential) :

$$\Lambda^{(10-d)} = -\frac{1}{2}(2\pi\sqrt{\alpha'})^{-(10-d)} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z^{\cancel{SUSY}}_{(\hat{Z})}$$

<u>Fundamental Region</u> :

$$\mathcal{F} = \left\{ \tau = \tau_1 + i\tau_2 \in \mathbb{C} \,\middle|\, -\frac{1}{2} \leq \tau_1 \leq \frac{1}{2},\ |\tau| \geq 1 \right\}$$

- decompose $\mathcal{F}$ into $\mathcal{F}_{\geq 1} = \{\tau \in \mathcal{F} | \tau_2 \geq 1\}$ and $\mathcal{F}_{<1} = \{\tau \in \mathcal{F} | \tau_2 < 1\}$



$\mathcal{F}_{\geq 1} = \{\tau \in \mathcal{F} | \tau_2 \geq 1\}$ The integral can be evaluated

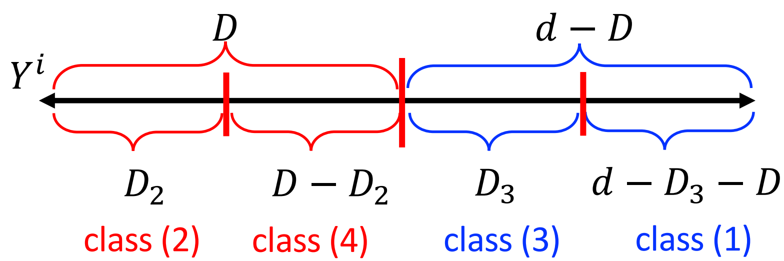
$\mathcal{F}_{<1} = \{\tau \in \mathcal{F} | \tau_2 < 1\}$ exp. suppressed

(10–d)D non-SUSY heterotic models

- Compact coordinates $Y^i \ (i = 1, \ldots, d)$:

$$i = a_{(2)} + a_{(4)} + b_{(3)} + b_{(1)}$$

class (#) in 9D



- Assignment of $(\hat{m}, \hat{n})$:

$$\begin{aligned}
(\hat{m}^{a_{(2)}}, \hat{n}_{a_{(2)}}) &= (1, 0) \quad \text{for} \quad a_{(2)} = 1, \ldots, D_2 \\
(\hat{m}^{a_{(4)}}, \hat{n}_{a_{(4)}}) &= (1, 1) \quad \text{for} \quad a_{(4)} = D_2 + 1, \ldots, D
\end{aligned} \Bigg\} \ \text{SUSY at } R_a \to \infty$$

$$\begin{aligned}
(\hat{m}^{b_{(3)}}, \hat{n}_{b_{(3)}}) &= (0, 1) \quad \text{for} \quad b_{(3)} = D + 1, \ldots, D + D_3 \\
(\hat{m}^{b_{(1)}}, \hat{n}_{b_{(1)}}) &= (0, 0) \quad \text{for} \quad b_{(1)} = D + D_3 + 1, \ldots, d
\end{aligned} \Bigg\} \ \text{Non-SUSY at } R_b \to \infty$$

Formula for cosmological constant

■ Consider $D \geq 1$, all $R_i \approx \infty$ ⇒SUSY is asymptotically restored

$$Z_{(\hat{Z})}^{SUSY} = Z_B^{(8-d)} \left\{ \bar{V}_8 Z_{\Gamma_+^{16+d,d}} - \bar{S}_8 Z_{\Gamma_-^{16+d,d}} + \bar{O}_8 Z_{\Gamma_\pm^{16+d,d}+\delta} - \bar{C}_8 Z_{\Gamma_\mp^{16+d,d}+\delta} \right\}$$

Exponentially suppressed

- Up to exponentially suppressed terms,

$$\Lambda^{(10-d)} \sim -\frac{4! \cdot 2^{d-1}}{\pi^{15-d}(\sqrt{\alpha'})^{10-d}} \left(\prod_{i=1}^{d} R_i \right) \sum_{\boldsymbol{n}} \left\{ \sum_{a=1}^{D} (2n_a - 1)^2 R_a^2 + \sum_{b=D+1}^{d} (2n_b)^2 R_b^2 \right\}^{-5}$$
$$\times 8 \left(24 + \sum_{\pi \in \Delta_g} \exp \left[2\pi i \left\{ \sum_{a=1}^{D} (2n_a - 1)(\pi \cdot A_a) + \sum_{b=D+1}^{d} n_b (\pi \cdot A_b) \right\} \right] \right)$$

Δ_g: nonzero roots of $SO(32)$ or $E_8 \times E_8$, not $\Delta_\pm$

⇒ CC does not depend on all the other endpoint models

massless condition

$$2\pi \cdot A_a \in \mathbb{Z}$$
$$\pi \cdot A_b \in \mathbb{Z}$$

$$\Lambda^{(10-d)} \sim \frac{4! \cdot 2^{d-1}}{\pi^{15-d}} \left(\prod_{i=1}^{d} R_i \right) \sum_{\vec{n}} \left\{ \sum_{a} (2n_a - 1)^2 R_a^2 + \sum_{b} (2n_b)^2 R_b^2 \right\}^{-5} (n_F - n_B)$$

Solutions of $n_F = n_B$ (1)

■SUSY $SO(32)$ endpoint model:

$$\Delta_{SO(32)} = \left\{ \left(\underline{\pm, \pm, 0^{14}} \right) \right\}$$

■Simplest configurations:

- A_a^I $(a = 1, \dots, D)$ are the same configuration
- $A_b^I (b = D+1, \dots, d)$ are taken to be 0

$$A_a = \left(0^p, \left(\frac{1}{2} \right)^q \right) \quad (p + q = 16), \quad A_b = \left(0^{16} \right)$$

- $D \in 2\boldsymbol{Z}$: $\quad n_F - n_B = -504 \neq 0$
- $D \in 2\boldsymbol{Z} + 1$: $\quad n_F - n_B = 4pq - \{2p(p-1) + 2q(q-1)\} - 24$

$$n_F = n_B \Rightarrow (p, q) = (9,7), (7,9)$$

Cosmological constant is exponentially suppressed when the gauge group is $SO(18) \times SO(14)$

Wilson-line Moduli Stability(1)

■SUSY $SO(32)$ endpoint models:

$$\Lambda^{(10-d)} \sim -\sum_{\boldsymbol{n}} C_{\boldsymbol{n}} \left(24 + 4 \sum_{1\leq I<J\leq 16} \cos\left[2\pi\theta^I\right] \cos\left[2\pi\theta^J\right] \right)$$

$$\begin{cases} \theta^I = \sum_{a=1}^{D} (2n_a - 1) \underline{A_a^I} + \sum_{b=D+1}^{d} n_b \underline{A_b^I} \\ C_{\boldsymbol{n}} = \frac{4! \cdot 2^{d+2}}{\pi^{15-d} (\sqrt{\alpha'})^{10-d}} \left(\prod_{i=1}^{d} R_i \right) \left\{ \sum_{a=1}^{D} (2n_a - 1)^2 R_a^2 + \sum_{b=D+1}^{d} (2n_b)^2 R_b^2 \right\}^{-5} \end{cases}$$

sum of WLs

■Simplest configurations are critical points:

$$A_a = \left(0^p, \left(\frac{1}{2}\right)^q \right) \quad (p+q=16), \quad A_b = (0^{16})$$

$$\Rightarrow \quad \frac{\partial \Lambda^{(10-d)}}{\partial A_i^I} \sim 0 \qquad (I = 1, \ldots, 16, \; i = 1, \ldots, d)$$

■Hessian matrix:

- Simplest configurations

$$A_a = \left(0^p, \left(\frac{1}{2}\right)^q\right) \quad (p+q=16)\,, \quad A_b = \left(0^{16}\right)$$

- $D \in 2\boldsymbol{Z}$:

$$\frac{\partial^2 \Lambda^{(10-d)}}{\partial A_i^I \partial A_j^J} \sim \xi \delta_{IJ} \delta_{ij} \qquad (I, J = 1, \ldots, 16,\ i, j = 1, \ldots, d) \qquad \xi > 0$$

⇒ Hessian is positive definite

➢ A global minimum when the gauge group is $SO(32)$
and no massless fermions exist ($\Lambda < 0$)

■Hessian matrix:

- Simplest configurations

$$A_a = \left(0^p, \left(\frac{1}{2}\right)^q\right) \quad (p+q=16)\,, \quad A_b = (0^{16})$$

- $D \in 2\boldsymbol{Z}+1$:

$$\frac{\partial^2 \Lambda^{(10-d)}}{\partial A_i^I \partial A_j^J} \sim \begin{cases} (2p-17)\,\xi' \delta_{IJ}\delta_{ij} & (I=1,\ldots,p), \\ (-2p+15)\,\xi' \delta_{IJ}\delta_{ij} & (I=p+1,\ldots,16) \end{cases} \qquad \xi' > 0$$

⇒ Hessian is positive/negative definite for $p = 0{,}16 / \; p = 8$

➢ A global minimum when the gauge group is $SO(32)$
while a local maximum when the gauge group is $SO(16)\times SO(16)$

➢ $p = 7{,}9 \; (n_F = n_B) \Rightarrow$ saddle points

Solutions of $n_F = n_B$(2)

■SUSY $E_8 \times E_8$ endpoint model:

$$\Delta_{E_8\times E_8} = \left\{ \left(\underline{\pm,\pm,0^6};0^8\right), \frac{1}{2}\left(\underline{\pm,\pm,\pm,\pm,\pm,\pm,\pm,\pm}_+;0^8\right) \right\}$$
$$+ \left\{ \left(0^8;\underline{\pm,\pm,0^6}\right), \frac{1}{2}\left(0^8;\underline{\pm,\pm,\pm,\pm,\pm,\pm,\pm,\pm}_+\right) \right\}$$

■Simplest configurations: $A_a = (A_1; A_2)\,,\ A_b = (A_1'; A_2')$

$$A_k = \left(0^{p_k}, \left(\frac{1}{2}\right)^{q_k}\right) \ (p_k + q_k = 8)\,, \quad A_k' = \left(0^8\right)\,, \quad \text{for } k = 1, 2$$ (p_k: even)

- $D \in 2\boldsymbol{Z}$: $n_{Fk} - n_{Bk} = -252 \neq 0$
- $D \in 2\boldsymbol{Z} + 1$:

  $n_{Fk} - n_{Bk} = -252$ for $p_k = 0, 8$

  $n_{Fk} - n_{Bk} = -28$ for $p_k = 2, 6$

  $n_{Fk} - n_{Bk} = +4$ for $p_k = 4$

$$n_{F(B)} = \sum_{k=1,2} n_{F(B)k}$$

No solutions for $n_F = n_B$

Wilson-line Moduli Stability(2)

■ SUSY $E_8 \times E_8$ endpoint models:

$$\Lambda^{(10-d)} \sim -\sum_{\boldsymbol{n}} C_{\boldsymbol{n}} \left(24 + \sum_{k=1,2} \left\{ 4 \sum_{I_k > J_k} \cos\left[2\pi\theta^{I_k}\right] \cos\left[2\pi\theta^{J_k}\right] + 128 \left(\prod_{I_k} \cos\left[\pi\theta^{I_k}\right] + \prod_{I_k} \sin\left[\pi\theta^{I_k}\right] \right) \right\} \right)$$

■ Simplest configurations are critical points.

$$A_k = \left(0^{p_k}, \left(\frac{1}{2}\right)^{q_k} \right) \quad (p_k + q_k = 8)\,, \quad A'_k = \left(0^8\right)\,, \quad \text{for } k = 1, 2 \quad (\,p_k\text{: even})$$

■ $p_1, p_2 = 0, 8 \Rightarrow$ global minima when the gauge group is $E_8 \times E_8$

$p_1, p_2 = 4 \Rightarrow$ a local maximum when the gauge group is $SO(16) \times SO(16)$

■ $p_1, p_2 = 2, 6 \Rightarrow$ saddle points of $\Lambda^{(10-d)}$

Outline

Summary

- $(10-d)$D Non-SUSY models are constructed by orbifolding by $(-)^F\alpha$

(α : shift of order 2 in Narain lattice)

- Various interpolations are shown in $d=2$ case
- Cosmological constant of $(10-d)$D Non-SUSY models in $R_i \approx \infty$ is

$$\Lambda^{(10-d)} \sim \frac{4!\cdot 2^{d-1}}{\pi^{15-d}}\left(\prod_{i=1}^{d} R_i\right)\sum_{\vec{n}}\left\{\sum_a (2n_a-1)^2 R_a^2 + \sum_b (2n_b)^2 R_b^2\right\}^{-5}(n_F-n_B) + \mathcal{O}(e^{-R/\sqrt{\alpha'}})$$

- Find the configurations of WLs which give exp. supp. cosmol. const.
- Analyze WL-moduli stability: $n_F = n_B \leftrightarrow$ saddle points

<u>Out look</u>

Higher-loop/sub-leading corrections, (meta)stable vacua, cosmology, …