Wave Packets in AdS/CFT Correspondence

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based on the following papers:

2304.08478 [hep-th] ; JHEP11(2022)041, 2207.06455 [hep-th];

PTEP,2104.11743 [hep-th] ;PRD104 (2021) 8, 2005.05962 [hep-th];

JHEP02 (2018)019,1710.07298 [hep-th];JHEP02 (2020)021, 1907.05419 [hep-th]

Introduction

One way to study quantum gravity is AdS/CFT duality

Maldacena

Quantum gravity on AdS

= conformal field theory (CFT)

Highly non-trivial and important!

Of course, CFT is not quantum gravity in general.

Special class of CFT, called Holographic CFT, is dual to quantum gravity on AdS

Typical holographic CFT:
SU(N) gauge theory with conformal symmetry

$$\text{Newton constant } G_N \sim \frac{1}{N^2}$$

Large N is needed for the bulk spacetime picture

Important property:

(d+1)-dim gravity = d-dim CFT

Gravity theory on AdS_{d+1}
(bulk theory)

$\Updownarrow$

CFT_d on $\mathbf{R} \times S^{d-1}$
(boundary theory)

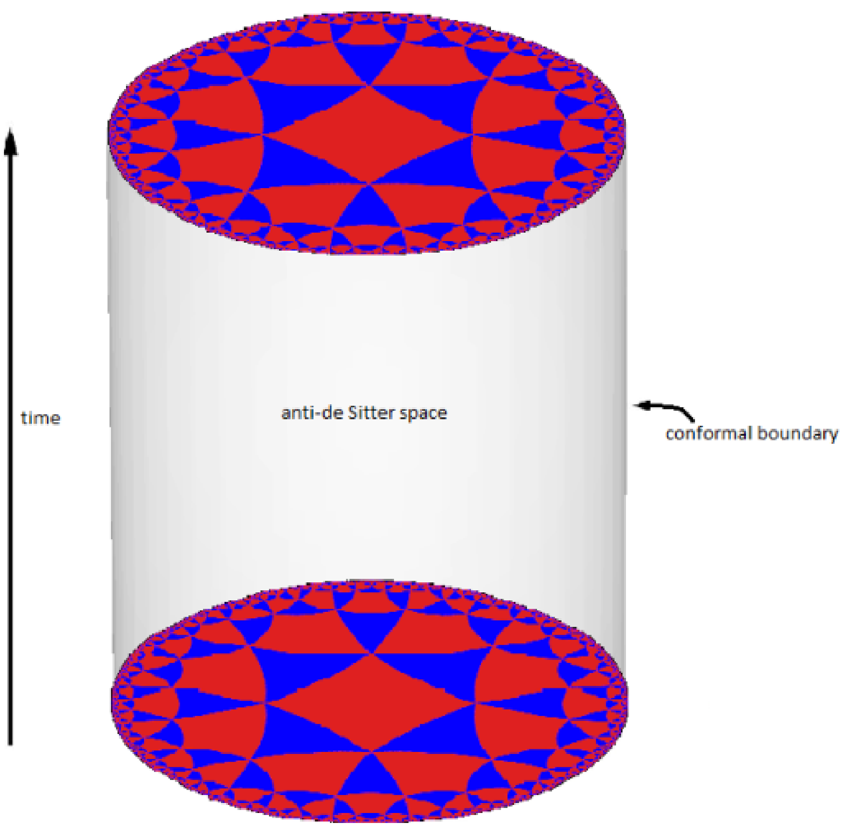


figure from Wikipedia

Duality = Equivalence (of QFT)

How to determine two QFT are equivalent or not?

Two ways:

1. All correlation functions are equivalent

2. Hilbert spaces and Hamiltonians are equivalent

1. All correlation functions are equivalent

=Partition function with sources are equivalent

(Euclidian) path-integral is convenient

2. Hilbert spaces and Hamiltonians are equivalent
=spectrums are equivalent

Operator formalism is convenient

Useful to study
time evolution and intuitive picture of states

Usual formulation of AdS/CFT

equivalences of partition function with source

$$Z_{bulk}(J) = Z_{CFT}(J)$$

J as boundary condition in AdS

⇕

J as source terms in CFT

This relation, called GKPW relation, is assumed

Another formulation of AdS/CFT

In operator formalism,

equivalence between
Hilbert spaces and Hamiltonians
of gravity on AdS and CFT

In this talk, we will study
AdS/CFT in operator formalism

This has not been studied so much,

and,

important to understand
(time-evolution of) states in AdS/CFT,

in particular, to understand
how bulk space-time emerges from CFT

We will focus on Large N limit, which is essential for AdS/CFT duality

First, we will determine

low energy spectrum of CFT_d

which is realized as

(generic) large N strong coupling gauge theory.

Then, we will explicitly show the duality, i.e.

$$\text{Low energy spectrum of large } N \ CFT_d$$

⇕ **equivalent!**

$$\text{Spectrum of free gravity on } AdS_{d+1}$$

- **Not assuming SUSY, string, D-brane**
- **Not assuming dual gravity, AdS space**
- **Not assuming AdS/CFT dictionary (GKPW)**

"a derivation of AdS/CFT"

The spectrum determine the theory itself.

⇩

From CFT, we can

- construct bulk local fields in AdS (bulk reconstruction)
- derive the GKPW relation

Including interactions in bulk theory:

Considering the classical limit
of generic large N gauge theory
with conformal symmetry.

we find that
it is Einstein gravity!

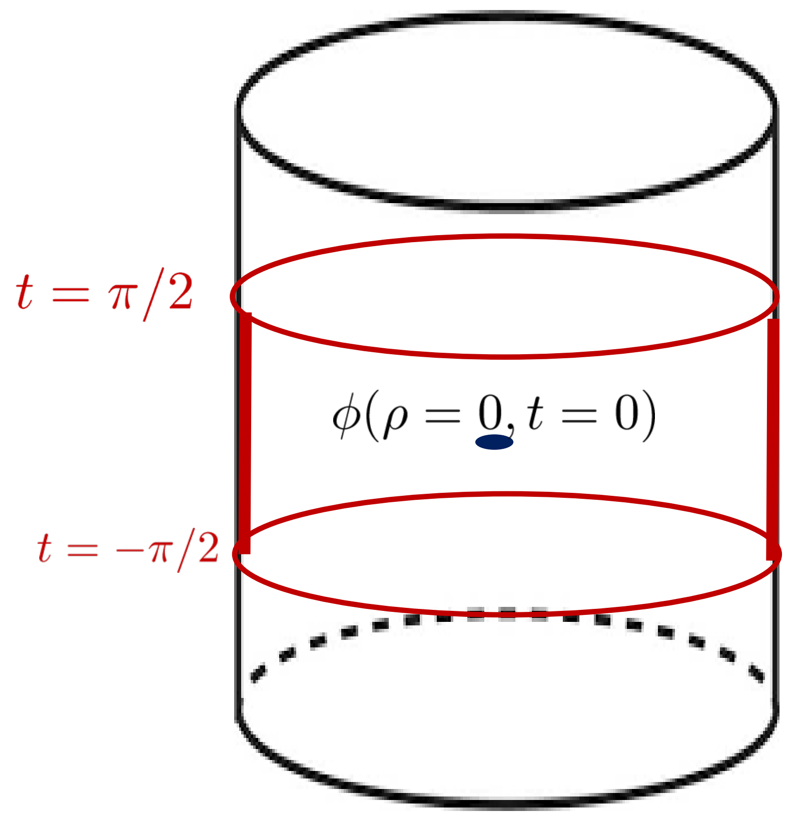
though I will NOT talk this topic

The explicit formula (HKLL recosntrcution) is known for a free theory on AdS,

(which is equivalent to strong coupling limit of large N CFT).

It is integration of CFT field over a region in the spacetime of CFT

This formula is not simple and rather counter intuitive with the causality.



In this talk, we will show that

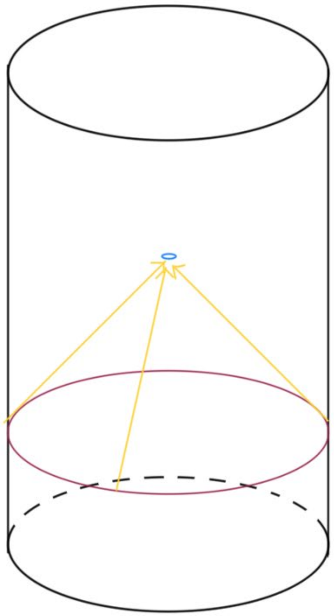
bulk reconstruction is rather simple

and has an intuitive picture

using alternative reconstruction formula

For example,

bulk local state at center is given by time evolution of uniformly distributed CFT primary state.



Only light-like trajectories appear!

Plan

1. **Introduction**
2. **Low energy spectrum of CFT**
3. **Simple picture of bulk reconstruction**
4. **Bulk wave packet**
5. **AdS-Rindler**
6. **Conclusion**

Low energy spectrum of CFT
and
bulk reconstruction in AdS/CFT

c.f. Balasubramanian-Kraus-Lawrence
Banks-Douglas-Horowitz-Martinec
Fitzpatrick-Kaplan

CFT_d on $\mathbf{R} \times S^{d-1}$

unit radious

Conformal mapping from the complex plane to the cylinder (for $d = 2$)

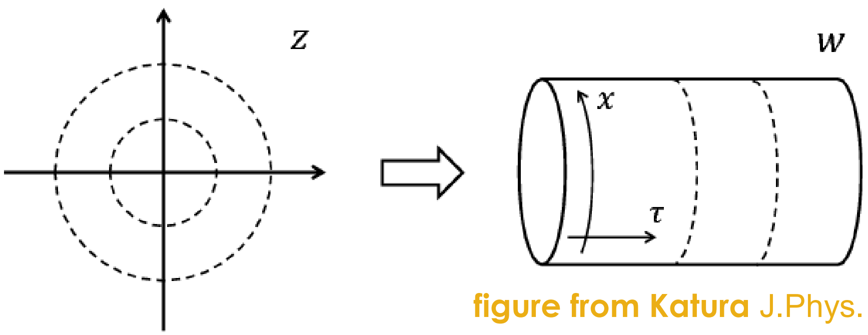


figure from Katura J.Phys. A45 (2012) 115003

CFT_d on $\mathbf{R}^d$ $\qquad$ CFT_d on $\mathbf{R} \times S^{d-1}$

$\hat{P}_\mu$ = translations $\longrightarrow$ $\hat{P}_\mu = ?$

$\hat{M}_{\mu\nu}$ = rotations $\longrightarrow$ $\hat{M}_{\mu\nu}$ = rotations in S^{d-1}

$\hat{D}$ = dilatation $\longrightarrow$ $\hat{D}$ = translation in $\mathbf{R} = \boxed{\hat{H}}$

$\hat{K}_\mu$ = special conformal $\longrightarrow$ $\hat{K}_\mu = ?$

CFT_d on $\mathbf{R} \times S^{d-1}$

unit radious

Conformal algebra

$$\left\{ \begin{aligned} &[\hat{D}, \hat{P}_\mu] = \hat{P}_\mu, \quad [\hat{D}, \hat{K}_\mu] = -\hat{K}_\mu, \\ &[\hat{K}_\mu, \hat{P}_\nu] = 2\delta_{\mu\nu}\hat{D} - 2\hat{M}_{\mu\nu} \\ &[\hat{D}, \hat{M}_{\mu\nu}] = 0, \ldots \end{aligned} \right.$$

Diagonalizing $\hat{H} = \hat{D}$ and "$\hat{M}_{\mu\nu}$",

$\hat{P}$, $\hat{K}$ are "creation" and "annihilation" operators

"Highest weight" represenation

Define primary state, $|\Delta\rangle$, s.t

$$\hat{K}_\mu|\Delta\rangle = 0, \quad \hat{D}|\Delta\rangle = \Delta|\Delta\rangle$$

Then, any state in CFT can be represented as

$$\hat{P}_{\mu_1}\hat{P}_{\mu_2}\cdots\hat{P}_{\mu_l}|\Delta\rangle$$

We can rewrite them as

$$\hat{P}_{\mu_1}\hat{P}_{\mu_2}\cdots\hat{P}_{\mu_l}|\Delta\rangle \quad \Longrightarrow \quad |nlm\rangle, \qquad n, l \in \mathbf{Z}_{\geq 0}$$

where l, m are labels for the angular mometa and

$$\hat{H}|nlm\rangle = (2n + l + \Delta)|nlm\rangle$$

We can rewrite them as

$$\hat{P}_{\mu_1}\hat{P}_{\mu_2}\cdots\hat{P}_{\mu_l}|\Delta\rangle \quad \Longrightarrow \quad |nlm\rangle, \qquad n,l \in \mathbf{Z}_{\geq 0}$$

where l, m are labels for the angular mometa and

$$\hat{H}|nlm\rangle = (2n + l + \Delta)|nlm\rangle$$

$$|nlm\rangle \equiv c_{nl}\, s^{\mu_1\mu_2\ldots\mu_l}_{(l,m)} P_{\mu_1}P_{\mu_2}\cdots P_{\mu_l}(P_\nu P^\nu)^n|\Delta\rangle$$

where
- c_{nl} is the normalization constant
- $s^{\mu_1\mu_2\ldots\mu_l}_{(l,m)}$ is a normalized rank l symmetric traceless constant tensor

⟹ Spectrum is determined by specifying primary states, $|\Delta\rangle$

State-operator correspondence:

Primary state $|\Delta\rangle$ ⟺ Primary field $\mathcal{O}_\Delta(x)$

$$|\Delta\rangle = \mathcal{O}_\Delta(x=0)|0\rangle$$

⟹ Spectrum is determined by specifying $\mathcal{O}_\Delta(x)$

Let us consider **large** N CFT_d

$$\hat{H}(\hat{P}_{\mu_1}\cdots\hat{P}_{\mu_l}|\Delta\rangle) = (\Delta + l)(\hat{P}_{\mu_1}\cdots\hat{P}_{\mu_l}|\Delta\rangle)$$

$\Delta \gg \mathcal{O}(N^0)$ for a generic state
because of the quatum corrections

But, symmetry currents are protected
from quantum corrections. ex. for $T_{\mu\nu}$, $\Delta = d$

⇓

Energy is expected to be $\mathcal{O}(N^0)$
only for symmetry currents, generically.

analogous to hydrodynamics

Thus, NOT so many low energy fields (=sparce spectrum)

Large N factorization 't Hooft

In the large N limit,

n-point func. is dominated by 2-point func.

i.e. $\langle \mathcal{O}_1 \mathcal{O}_2 \cdots \mathcal{O}_l \rangle = \sum \langle \mathcal{O}_1 \mathcal{O}_2 \rangle \langle \mathcal{O}_3 \mathcal{O}_4 \rangle \cdots \langle \mathcal{O}_{l-1} \mathcal{O}_l \rangle$

where $\langle \mathcal{O}_i \rangle = 0$

⇕ (Wick theorem)

(generalized) free theory

⇕

Fock space

Fock space

$$\mathcal{O}|0\rangle \Longrightarrow |nlm\rangle \text{ one particle state}$$

$$\mathcal{O}\mathcal{O}\cdots\mathcal{O}|0\rangle \Longrightarrow \text{multi particle states}$$

Fock space is generated by $\hat{a}^\dagger_{nlm}$

which satisfy $\begin{cases} [\hat{a}_{nlm}, \hat{a}^\dagger_{n'l'm'}] = \delta_{n,n'}\delta_{l,l'}\delta_{m,m'} \\ \hat{a}^\dagger_{nlm}|0\rangle = |nlm\rangle \end{cases}$

Energy is given by $[\hat{H}, \hat{a}^\dagger_{nlm}] = \Delta + 2n + l$

where $n = 0, 1, 2, \cdots$ and $l = 0, 1, 2, \cdots$

Large N CFT spectrum

We conclude that large N CFT spectrum is
Fock space generated by $\hat{a}^\dagger_{nlm}$

Energy is given by $[\hat{H}, \hat{a}^\dagger_{nlm}] = \Delta + 2n + l$
where $n = 0, 1, 2, \cdots$ and $l = 0, 1, 2, \cdots$

(This is valid for low energy spectrum)

Thus,
low energy limit of the large N CFT is
a free theory.

What is this free theory?

Thus,
low energy limit of the large N CFT is
a free theory.

What is this free theory?

Answer: free theory on AdS space

Free scalar field in AdS_{d+1}

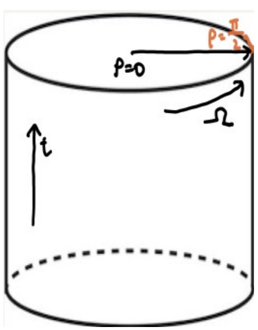
c.f. Breitenlohner-Freedman

The metric of global AdS_{d+1} ($l_{AdS} = 1$) is

$$ds^2_{AdS} = -(1+r^2)dt^2 + \frac{1}{1+r^2}dr^2 + r^2 d\Omega^2_{d-1}$$

where $0 \leq r < \infty$, $-\infty < t < \infty$ and

$d\Omega^2_{d-1}$ is the metric for round unit sphere S^{d-1}



$$= \frac{1}{\cos^2(\rho)}\left(-dt^2 + d\rho^2 + \sin^2(\rho)d\Omega^2_{d-1}\right)$$

where $r = \tan\rho$, $0 \leq \rho < \pi/2$

Boundary of AdS_{d+1} is located at $\rho = \pi/2$

Free scalar field in AdS_{d+1}

The action is

$$S_{scalar} = \int d^{d+1}x\sqrt{-\det(g)}\left(\frac{1}{2}g^{MN}\nabla_M\phi\nabla_N\phi + \frac{m^2}{2}\phi^2\right)$$

The e.o.m. is

$$0 = -g^{MN}\nabla_M\nabla_N\phi + m^2\phi^2.$$

We expand ϕ with spherical harmonics $Y_{lm}(\Omega)$,

$$\phi(t,\rho,\Omega) = \sum_{n,l,m}\left(a^\dagger_{nlm}e^{i\omega_{nl}t} + a_{nlm}e^{-i\omega_{nl}t}\right)\psi_{nlm}(\rho)Y_{lm}(\Omega)$$

Then, normalized solution for the e.o.m. is given as

$$\psi_{nlm}(\rho) = \frac{1}{N_{nl}}\sin^l(\rho)\cos^\Delta(\rho)\,{}_2F_1\left(-n, \Delta+l+n, l+\frac{d}{2}, \sin^2(\rho)\right)$$

$$\omega_{nl} = \Delta + 2n + l$$

Gauss's hyper geometric function

where $\Delta = d/2 \pm \sqrt{m^2 + d^2/4}$

Free scalar field in AdS_{d+1}

In summary, quantized field is

$$\hat{\phi}(t,\rho,\Omega) = \sum_{n,l,m} \left(\hat{a}^\dagger_{nlm} e^{i\omega_{nl}t} + \hat{a}_{nlm} e^{-i\omega_{nl}t} \right) \psi_{nlm}(\rho) Y_{lm}(\Omega)$$

$$\begin{cases} \omega_{nl} = \Delta + 2n + l \\ \text{where } \Delta = d/2 \pm \sqrt{m^2 + d^2/4} \end{cases}$$

The commutation relation and Hamiltonian are

$$[\hat{a}_{nlm}, \hat{a}^\dagger_{n'l'm'}] = \delta_{n,n'}\delta_{l,l'}\delta_{m,m'} \qquad [\hat{H}, \hat{a}_{nlm}] = -\omega_{nl}$$

Same spectrum as the CFT!

Therefore,

(low energy theory of)
large N CFT is equivalent to
a free theory on AdS

with the natural assumptions.

- **Not assuming SUSY, string, D-brane**
- **Not assuming dual gravity, AdS space**
- **Not assuming AdS/CFT dictionary (GKPW)**

The spectrum determine the theory itself.

⇩

From CFT, we can

- construct bulk local fields in AdS
- derive the GKPW relation and BDHM

For energy momentum tensor, instead of scalar, we can also show that

low energy theory of large N CFT is equivalent to (free) graviton on AdS, under the natural assumptions.

c.f. Ishibashi-Wald

Including interactions in bulk theory:

Considering the classical limit
of generic large N gauge theory
with conformal symmetry.

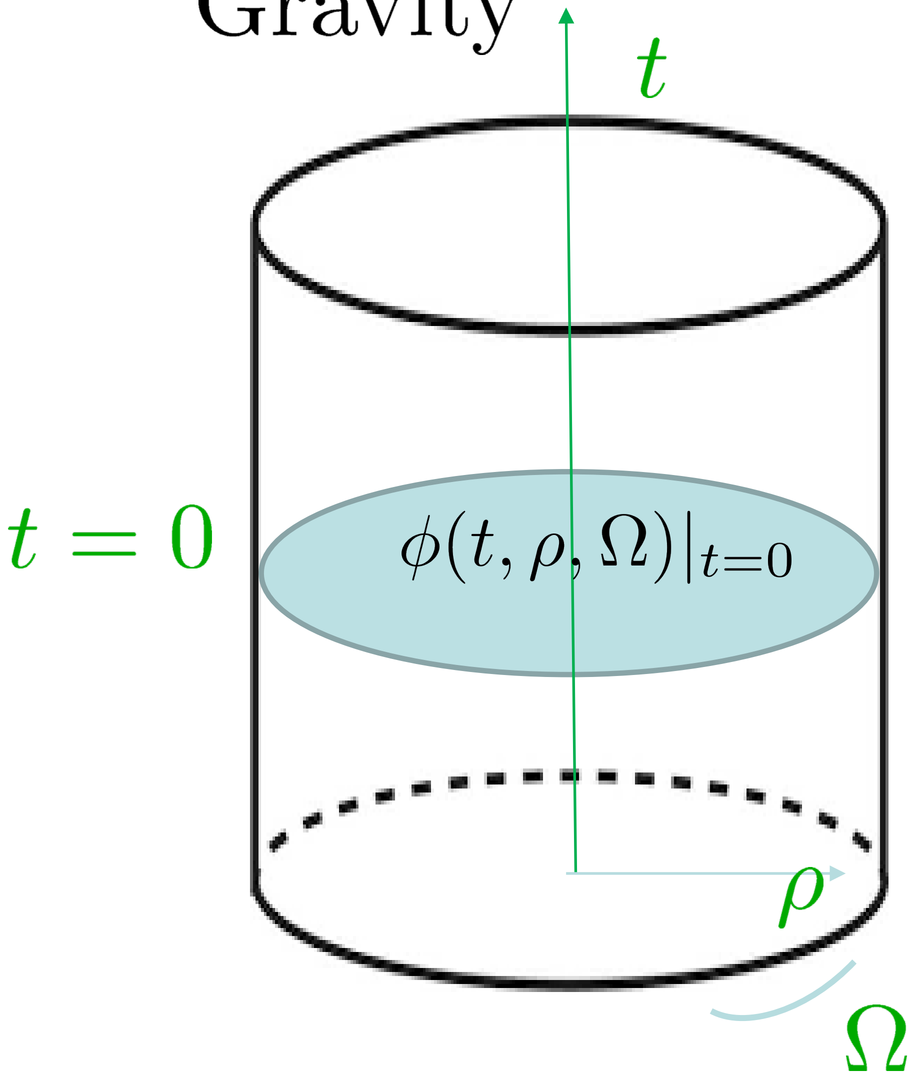
we find that
it is Einstein gravity!

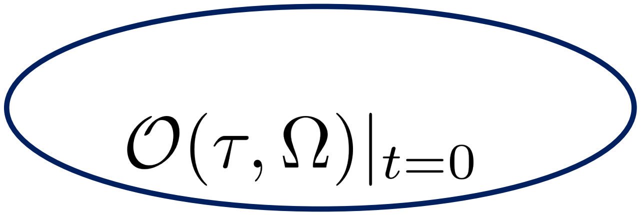
“a derivation of AdS/CFT”

I will skip this topic

Gravity

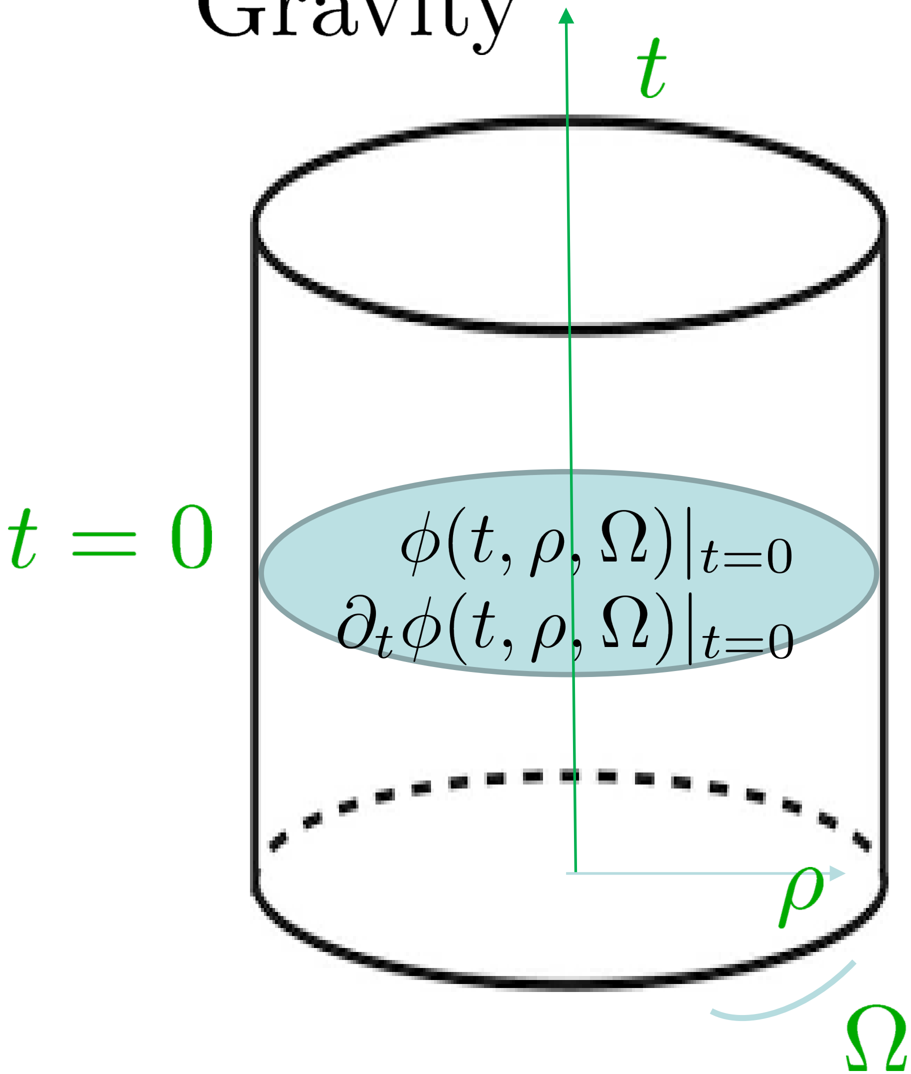
CFT





Shortage of d.o.f?

Gravity



CFT

N^2 independent fields!

$$\mathcal{O}(\tau, \Omega)|_{t=0}$$

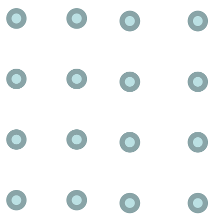
$$\partial_t \mathcal{O}(t, \Omega)|_{t=0}$$
$$(\partial_t)^2 \mathcal{O}(t, \Omega)|_{t=0}$$
$$(\partial_t)^3 \mathcal{O}(t, \Omega)|_{t=0}$$

⇓

$$e^{\bar{t}\partial_t} \mathcal{O}(t, \Omega)|_{t=0} = \mathcal{O}(t, \Omega)|_{t=\bar{t}}$$

Similarity between Lattice field theory and AdS/CFT

Lattice field theory	AdS/CFT
QFT	CFT → Gravity
4-dim	d-dim → (d+1)-dim
Low energy	Low energy

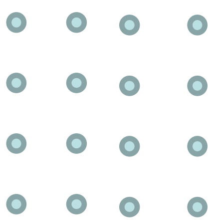


0-dim theory

Large number of sites (d.o.f)	Large N
Symmetry: dicretized Poincare	Symmetry: Conformal
nearest neighbour interaction (locality)	large N factorization

Similarity between Lattice field theory and AdS/CFT

Lattice field theory



QFT
4-dim
Low energy

0-dim theory

Large number of sites (d.o.f)

Symmetry: dicretized Poincare

nearest neighbour interaction
(locality)

AdS/CFT

CFT
d-dim

Gravity
(d+1)-dim

Low energy

Large N

Symmetry: Conformal

large N factorization

Qunatum gravity with UV cut-off.
String theory as asymptotic expansion.

Bulk reconstruction formula

Bulk local operator:

$$\phi(t=0,\rho,\Omega) = \sum_{n,l,m} \psi_{nl}(\rho) Y_{lm}(\Omega) \hat{a}^{\dagger}_{nlm} + h.c.$$

CFT primary operator:

$$\mathcal{O}_{\Delta}(\Omega,t) = \sum_{n,l,m} \psi^{CFT}_{nl} Y_{lm}(\Omega) e^{i\omega_{nl}t} \hat{a}^{\dagger}_{nlm} + h.c.$$

where $\psi^{CFT}_{nl} \equiv \sqrt{\frac{2}{\pi}\frac{\Gamma(d/2)}{\Gamma(\Delta)\Gamma(\Delta+1-d/2)}}\sqrt{\frac{\Gamma(n+\Delta+1-d/2)\Gamma(n+l+\Delta)}{\Gamma(n+1)\Gamma(n+l+d/2)}}$

Bulk recpnstruction for bulk local operator at center:

$$\phi(t=0,\rho=0)|0\rangle = \int d\Omega \, (F_{\Delta}(\partial_t)\, \mathcal{O}_{\Delta}(\Omega,t))\,|_{t=-\frac{\pi}{2}}|0\rangle$$

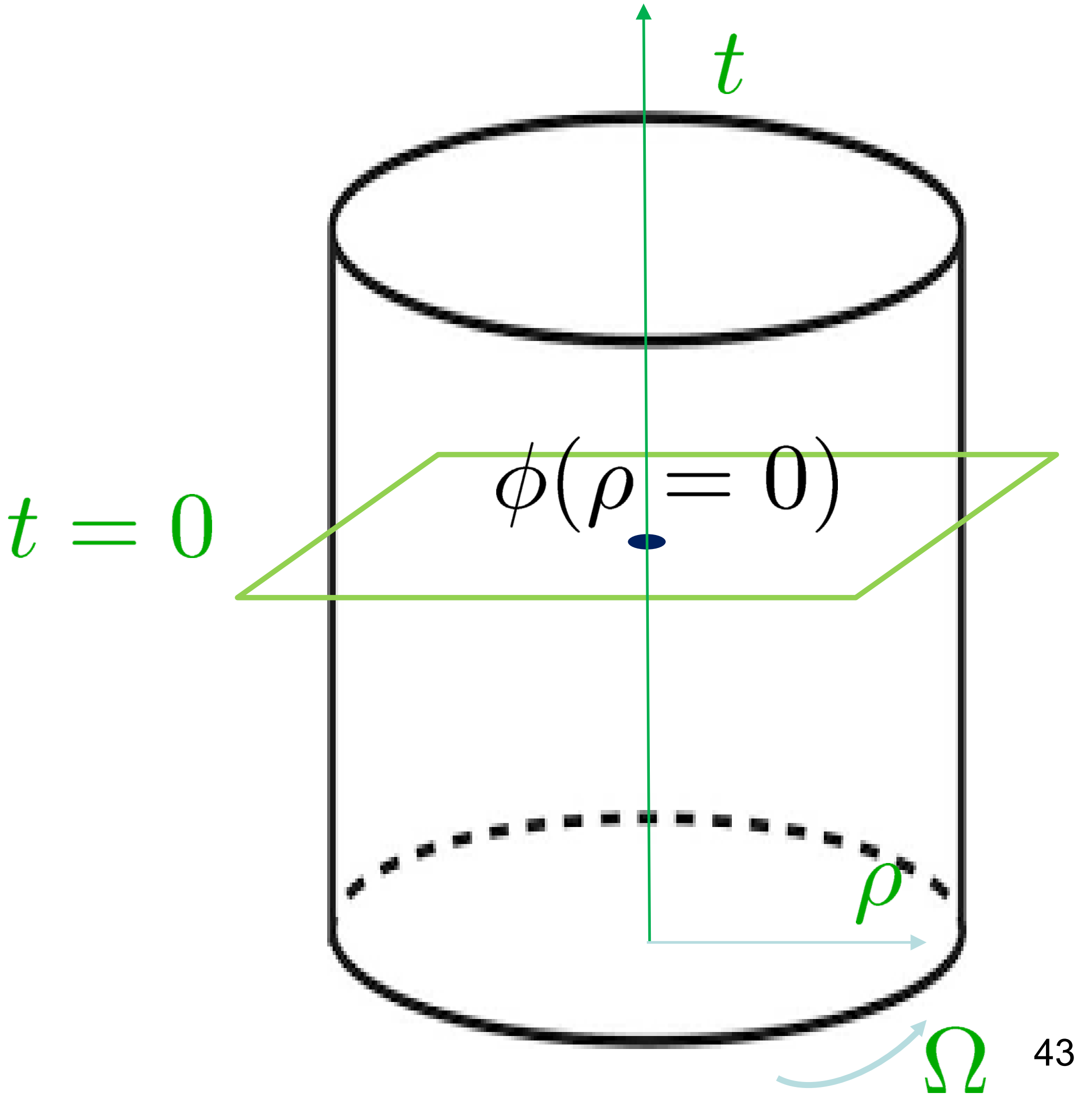
where $F_{\Delta}(x) = \frac{\Gamma((-ix-\Delta+d)/2)}{\Gamma((-ix+\Delta-d+2)/2)}$.

This should be equivalent to HKLL

For $\Delta = d-1$, $\phi(t=0,\rho=0)|0\rangle = \int d\Omega \, (\, \mathcal{O}_{\Delta}(\Omega,t))\,|_{t=-\frac{\pi}{2}}|0\rangle$

Simple picture of bulk reconstruction

Bulk local operator at the center

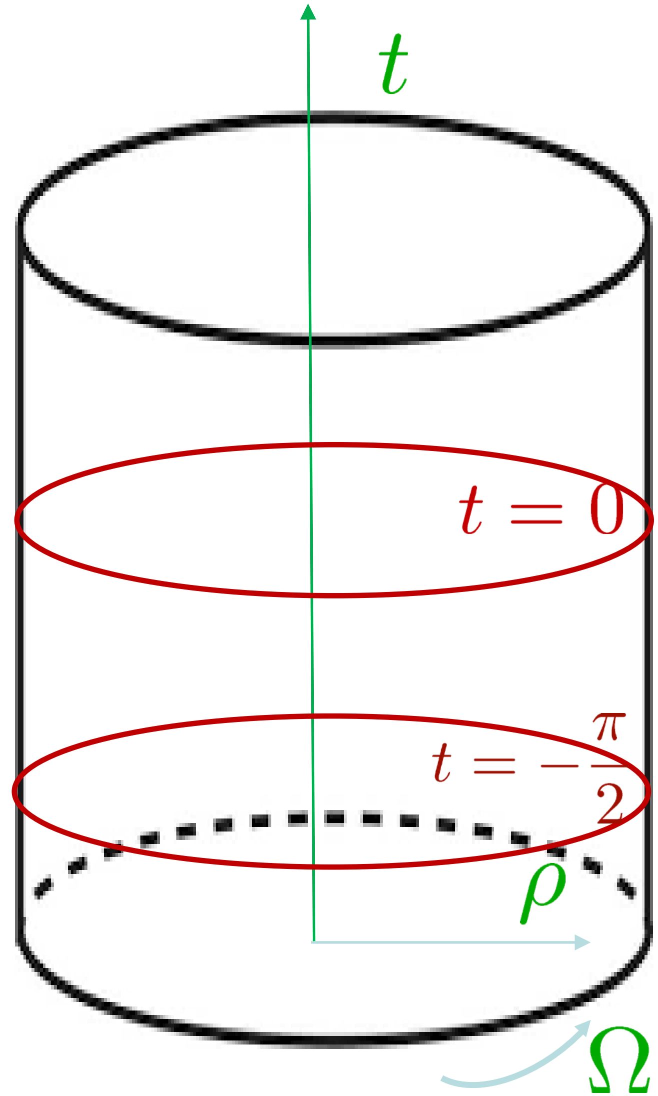


CFT state created at $t = -\pi/2$

$$\int d\Omega \, e^{i\frac{\pi}{2}H} \mathcal{O}_\Delta(\Omega)|0\rangle$$

time evolved

$$\int_{S^{d-1}} d\Omega \, \mathcal{O}_\Delta(\Omega)|0\rangle$$

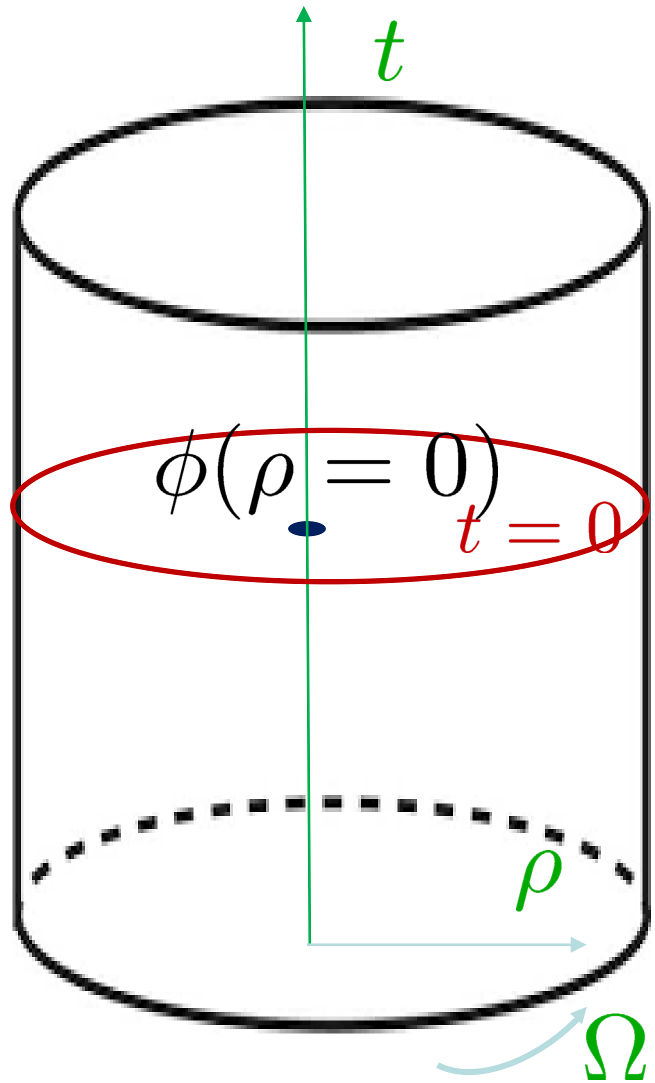


CFT state created at $t=-\pi/2$ is same as bulk local state at the center !

$$\phi(\rho=0)|0\rangle =$$

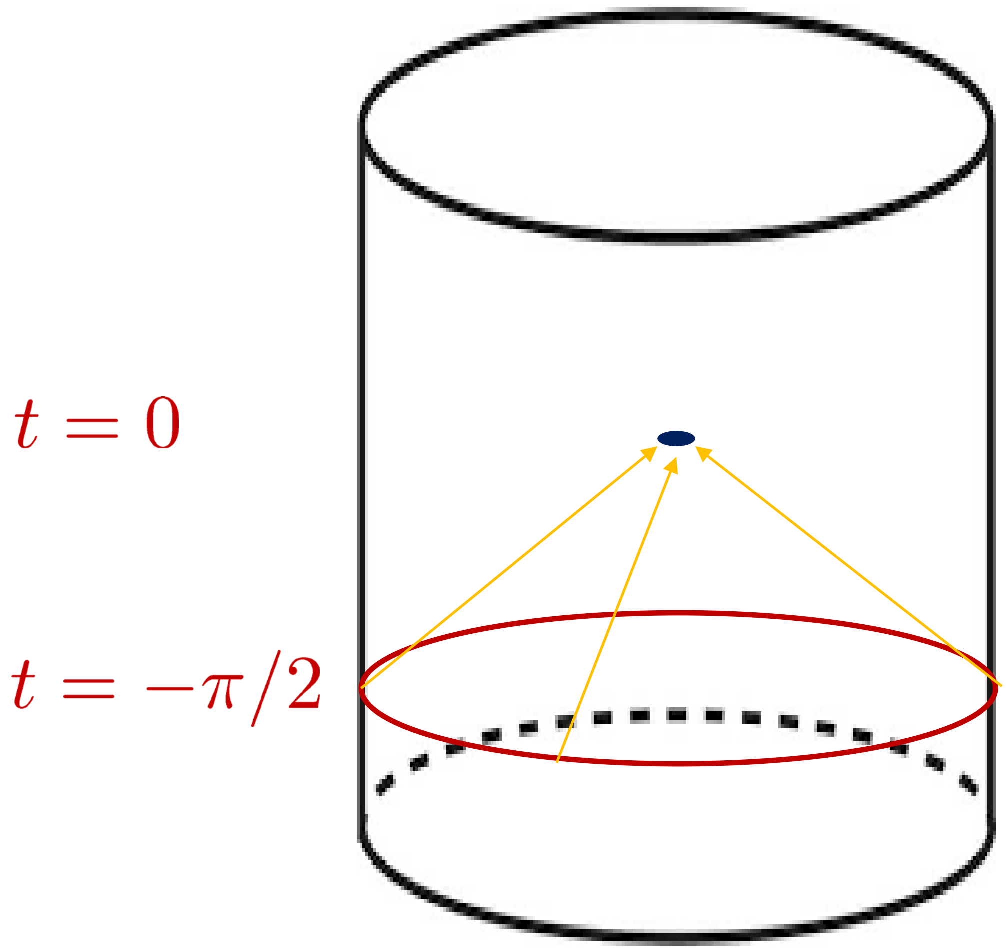
$$\int d\Omega\, e^{i\frac{\pi}{2}H}\mathcal{O}_\Delta(\Omega)|0\rangle$$

(for $\Delta = d-1$. For other values, derivative corrections are needed)

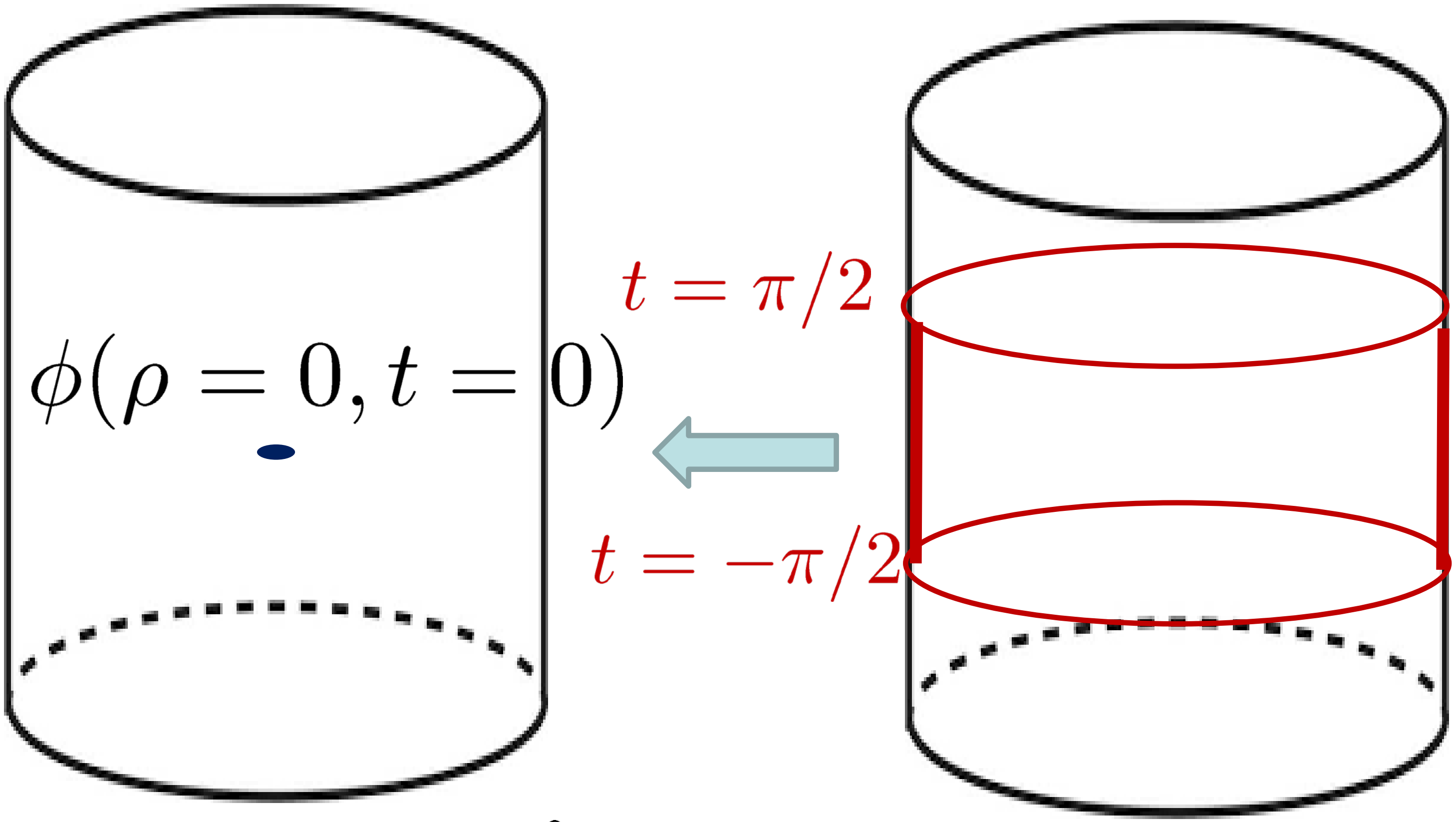


Simple picture:

Light rays from S^{d-1} go to the center



We will see this from HKLL reconstruction



$$\phi(\rho = 0, t = 0) = \int_{-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}} dt' d\Omega' K(\Omega', t') \mathcal{O}(\Omega', t')$$

$$K(\Omega, t) \sim \frac{1}{(\cos t)^{d-\Delta}}$$

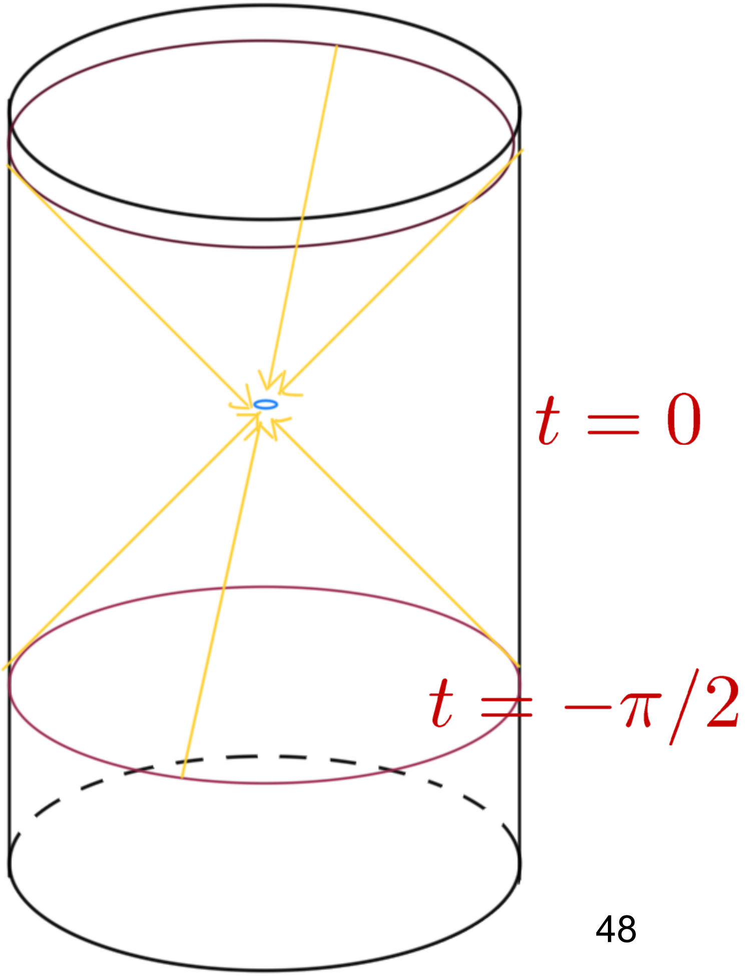
$$K(\Omega, t) \sim \frac{1}{(\cos t)^{d-\Delta}} \text{ is divergent at } t = \pm\pi/2$$

$$\phi(\rho = 0, t = 0)$$

$$= \int dt' d\Omega' K(\Omega', t') \mathcal{O}(\Omega', t')$$

$$\sim \int d\Omega' \mathcal{O}(\Omega', t = -\frac{\pi}{2})$$

$$+ \int d\Omega' \mathcal{O}(\Omega', t = \frac{\pi}{2})$$

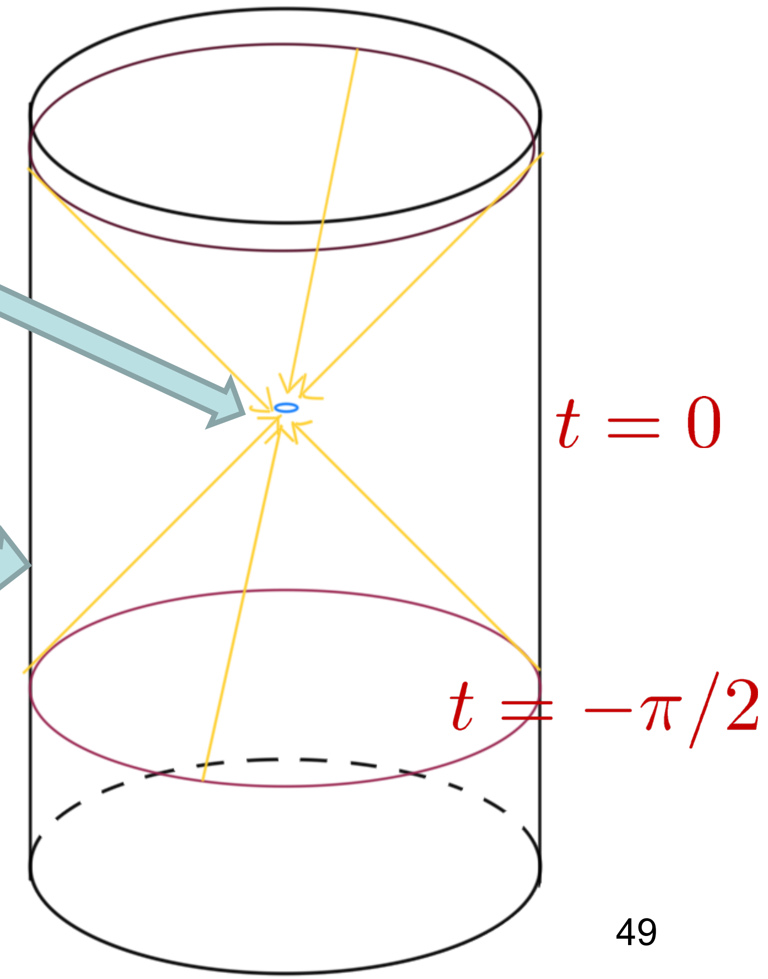


In fact, this localization is required by causality!

These two should be indpendent

$\phi(\rho = 0, t = 0)$

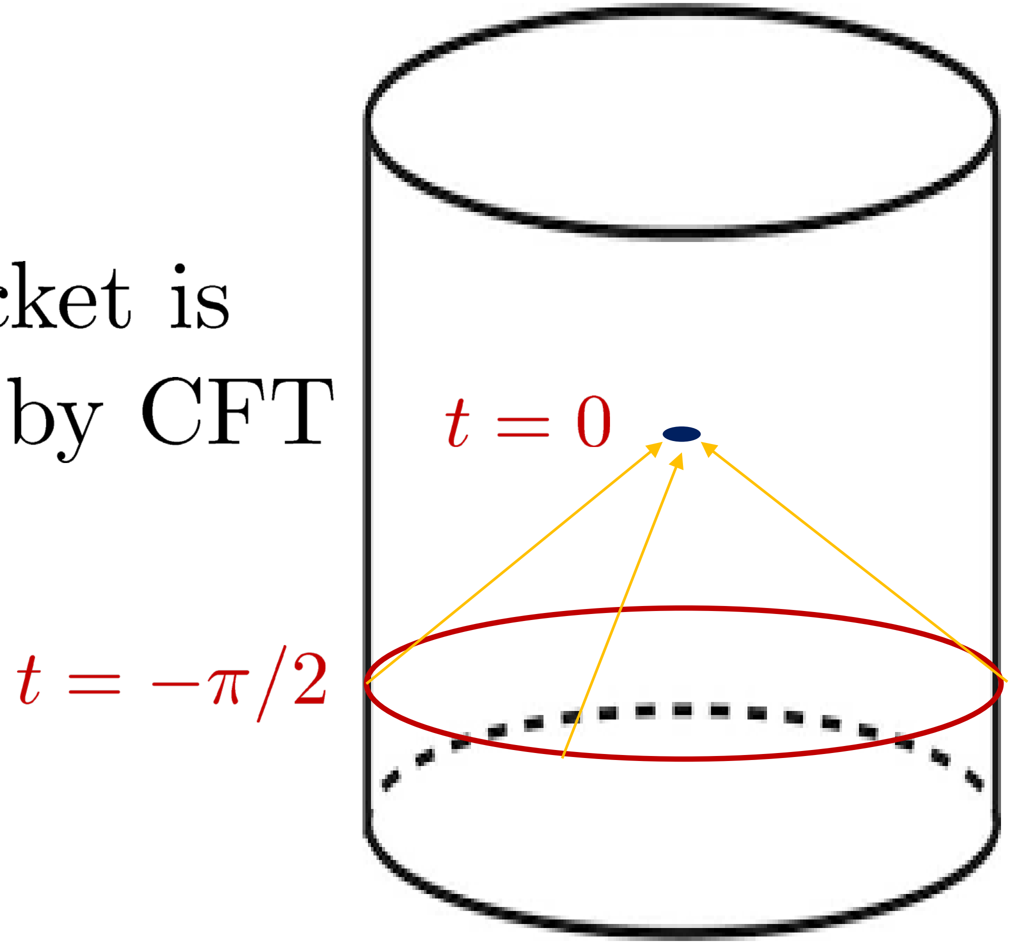
$\mathcal{O}(\Omega', t')$where $|t'| < \pi/2$



Bulk wave packet

Bulk local state is a sum of the light rays, which are wave packets moving to different directions.

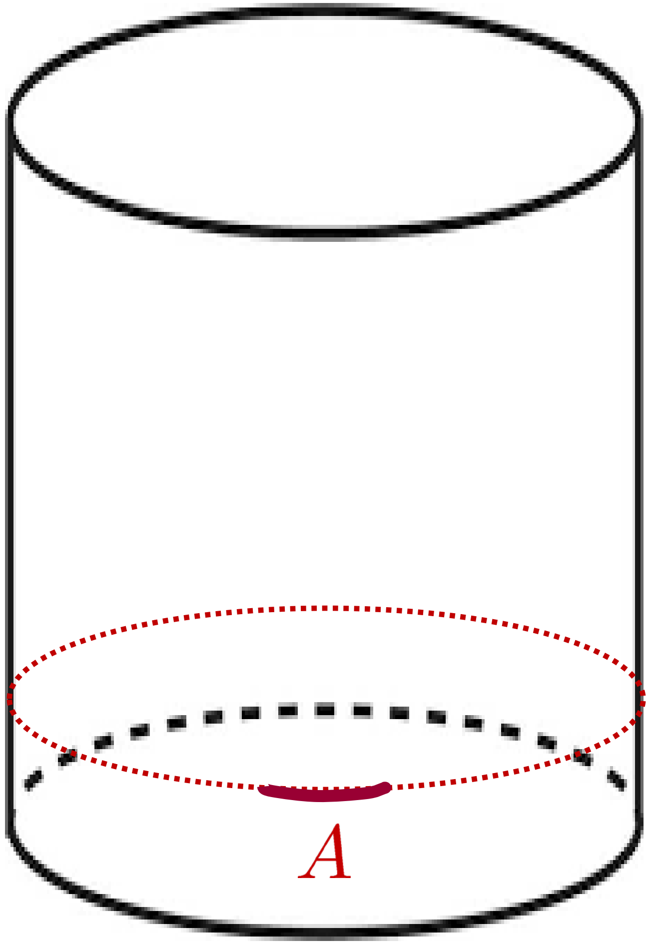
Each wave packet is reconstructed by CFT



Integration over small region A, instead of whole space

$$e^{i\frac{\pi}{2}H}\int_A d\Omega\, \mathcal{O}_\Delta(\Omega)|0\rangle$$

This state is like wave packet moving in radial direction

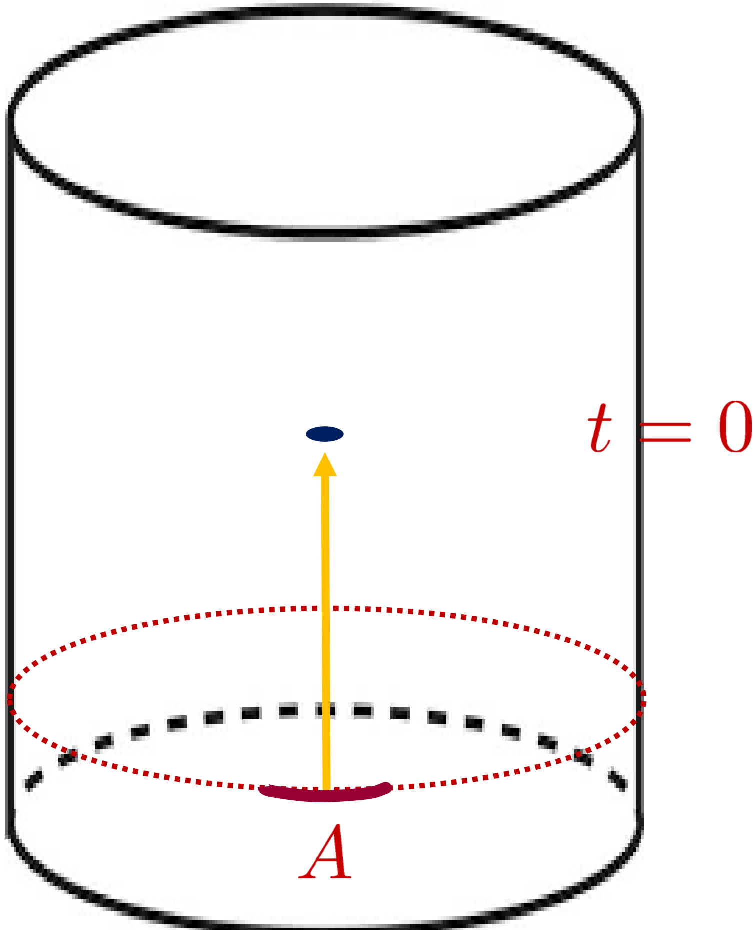


A bulk local state,
but moving in radial direction only

$$e^{i\frac{\pi}{2}H}\int_A d\Omega\,\mathcal{O}_\Delta(\Omega)|0\rangle$$

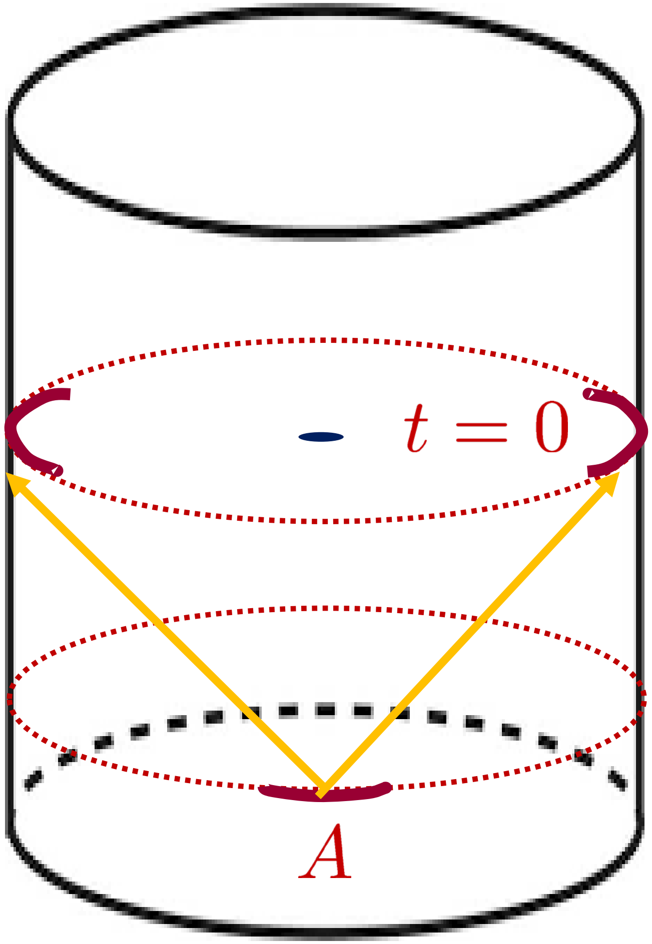
This is like wave packet moving in radial direction.

Linear combinations of them gives (usual) bulk local state

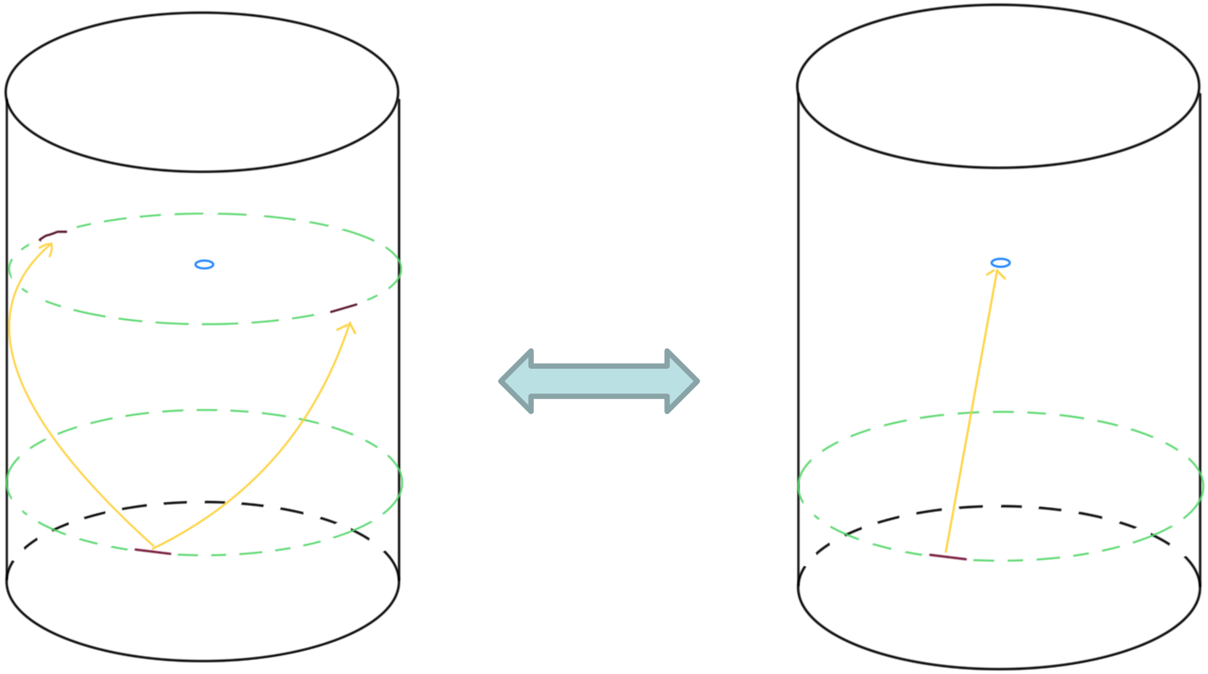


In CFT picture at t=0

the bulk local state
moving in radial direction
is
entangled state in CFT

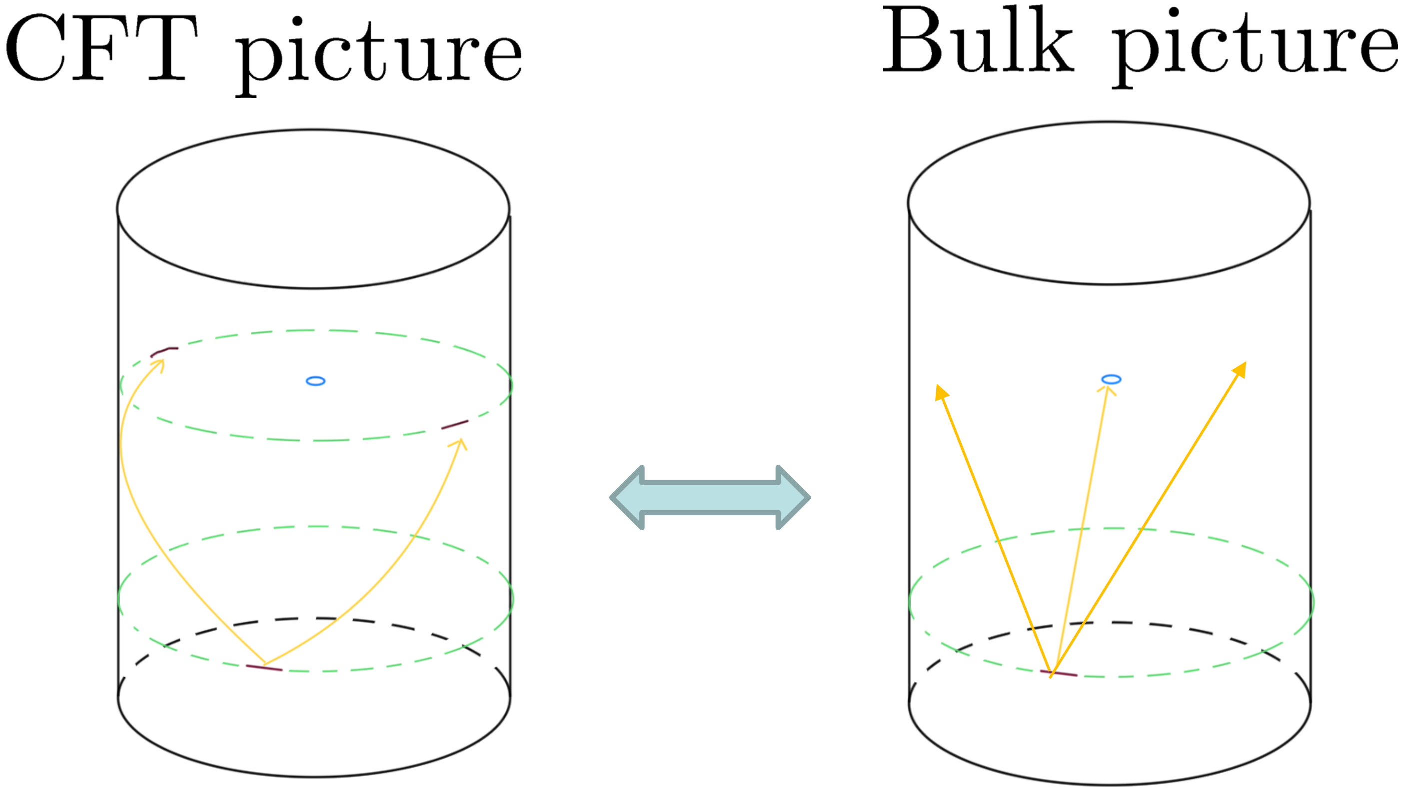


CFT picture | Bulk picture



This intuitive picture is obtained just by BDHM and bulk free theory time-evolution.

Bulk wavepacket moving in any direction

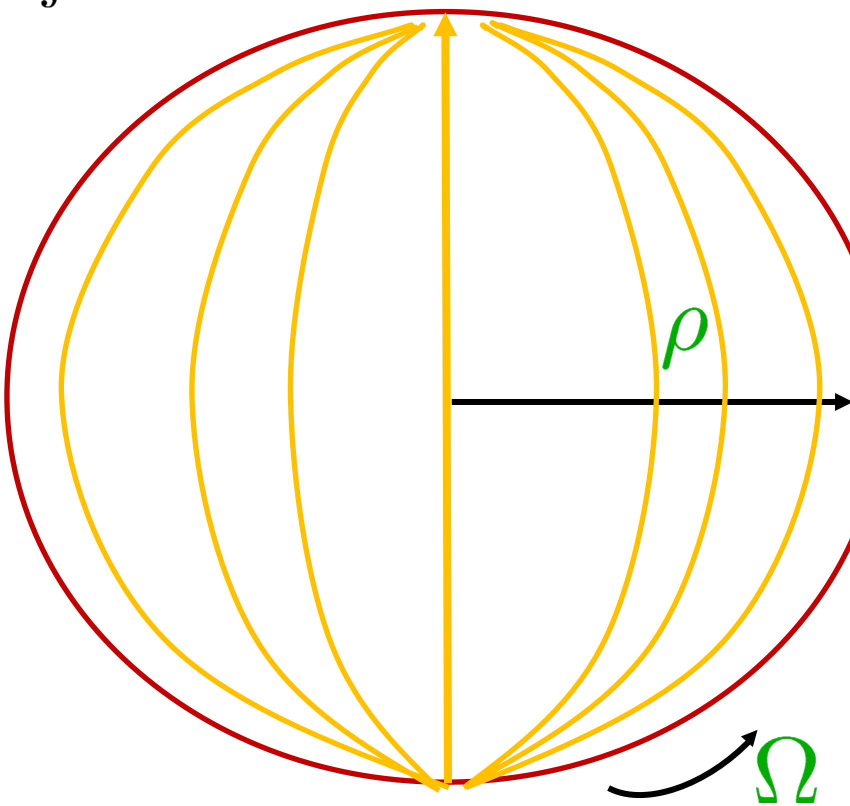


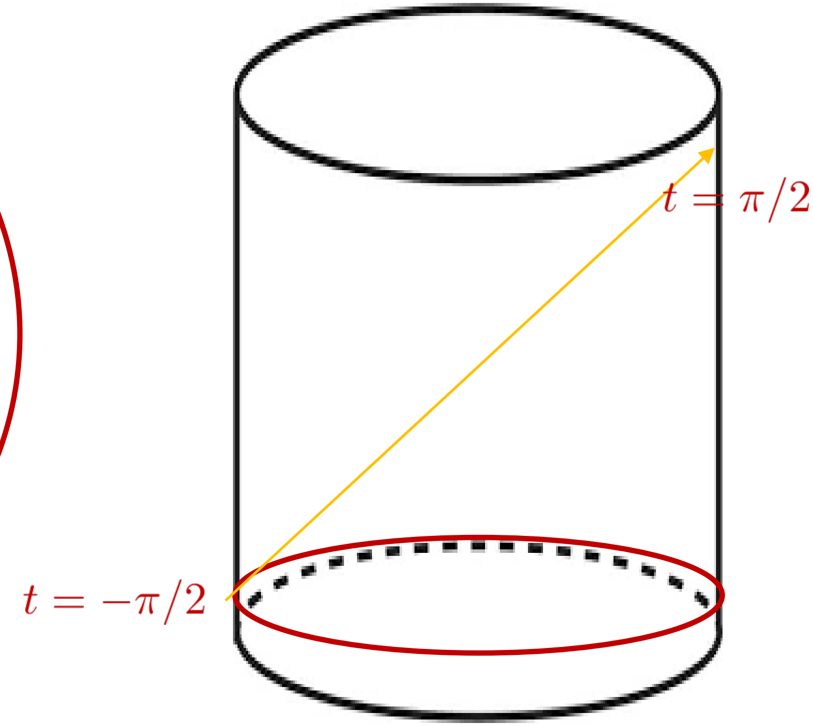
In CFT picture, all are in same trajectries
(in this example, 3 wave packets in bulk picture)

Causality and duality

Consider light rays from a same spacetime point.

Projected to a time slice of AdS space:





Any light ray reaches same spacetime point,
i.e. $\Delta t = \pi$ and at antipodal point. **Gao-Wald**

Wave packets in Minkowski space

Wave packet of a free scalar field $\phi(t,\vec{x})$ in $d+1$ dimension at $t=\vec{x}=0$ with momentum $\vec{p}$:

$$\int d\vec{x}\, e^{-\frac{\vec{x}^2}{2a^2}+i\vec{p}\cdot\vec{x}}\phi(t,\vec{x})|_{t=0}|0\rangle \propto \int d\vec{k}\, e^{-\frac{a^2(\vec{k}-\vec{p})^2}{2}} a^\dagger_{\vec{k}}|0\rangle$$

It is required that $a^2(\vec{p})^2 \gg 1$

Instead of this, we can use

$$\int dt \prod_{i=2,\cdots,d} dx^i\, e^{-\frac{x^i x_i+t^2}{2a^2}+ip_i x^i+i\omega t}\phi(t,\vec{x})|_{x_1=0}|0\rangle$$

$$\propto \int d\vec{k}\, e^{-\frac{a^2}{2}\left((k^i-p^i)(k_i-p_i)+(\sqrt{(k_1)^2+k^i k_i}-\omega)^2\right)} a^\dagger_{\vec{k}}|0\rangle$$

where i runs only for $2,\cdots,d$.

General wave packets in AdS/CFT

For the Poincare patch of AdS_{d+1}, metric is

$$ds^2 = \frac{1}{z^2}\left(-dt^2 + dz^2 + \delta_{ij}dx^i dx^j\right),$$

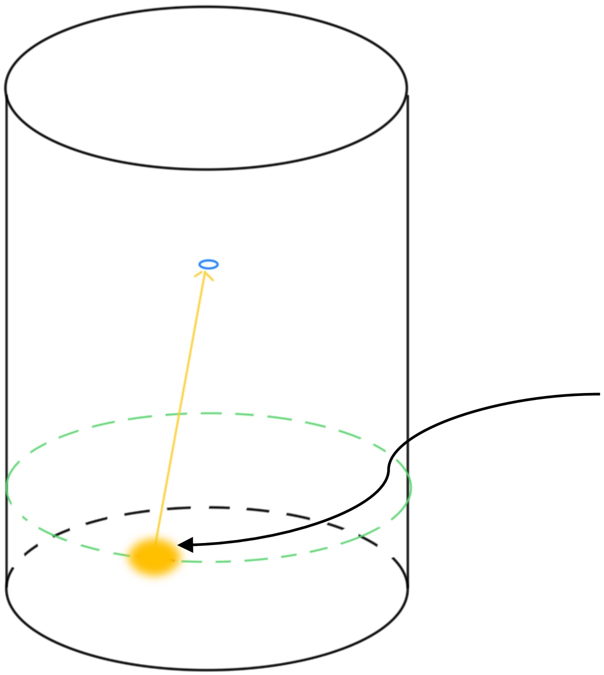
where $z > 0$ and $i, j = 1, 2, \cdots, d-1$.

General wave packet in AdS/CFT is given by

$$|p, \bar{\omega}\rangle = \lim_{z\to 0} \frac{1}{z^\Delta} \int dt\, dx^i\, e^{-\frac{x^i x_i + t^2}{2a^2} + ip_i x^i - i\bar{\omega}t} \phi(t, z, x^i)|0\rangle$$

General wave packet in AdS/CFT is given by

$$|p,\bar{\omega}\rangle = \lim_{z\to 0}\frac{1}{z^{\Delta}}\int dt\,dx^i\, e^{-\frac{x^i x_i + t^2}{2a^2} + ip_i x^i - i\bar{\omega}t}\phi(t,z,x^i)|0\rangle$$



$t = 0, x^i = 0$, size a

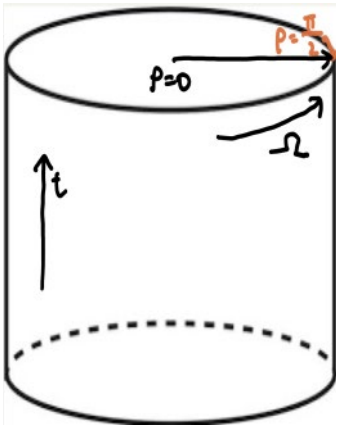
Bulk local field near boundary
(BDHM extrapolation formula):

Bulk operator at boundary is CFT primary field:

$$\lim_{z\to 0} \frac{\phi(t,z,\Omega)}{z^{\Delta}} \sim \mathcal{O}_{\Delta}(t,\Omega), \quad \text{where } z = \pi/2 - \rho$$

BDHM

$\mathcal{O}_{\Delta}(t,\Omega)$ is CFT primary field



General wave packets in AdS/CFT

General wave packet in AdS/CFT is given by

$$|p,\bar{\omega}\rangle = \lim_{z\to 0}\frac{1}{z^{\Delta}}\int dt\,dx^i\,e^{-\frac{x^i x_i+t^2}{2a^2}+ip_i x^i-i\bar{\omega}t}\phi(t,z,x^i)|0\rangle$$
$$= \int dt\,dx^i\,e^{-\frac{x^i x_i+t^2}{2a^2}+ip_i x^i-i\bar{\omega}t}\mathcal{O}(t,x)|0\rangle$$

in CFT picture

where we have used BDHM relation

Energy density of the wave packet

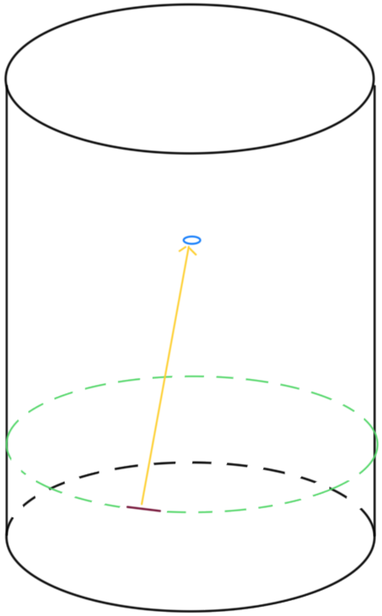
$$\mathcal{E}(t,x) \sim \langle p,\bar{\omega}|T_{00}(t=\bar{t},x^i=\bar{x}^i)|p,\bar{\omega}\rangle$$
$$= \int dt_1\, dx_1^i\, e^{-\frac{(x_1^i)^2+t_1^2}{2a^2}-ip_ix_1^i+i\bar{\omega}t_1} \int dt_2\, dx_2^i\, e^{-\frac{(x_2^i)^2+t_2^2}{2a^2}+ip_ix_2^i-i\bar{\omega}t_2}$$
$$\times \langle 0|\mathcal{O}(t_1,x_1)T_{00}(t=\bar{t},x^i=\bar{x}^i)\mathcal{O}(t_2,x_2)|0\rangle$$

For $d=2$,

$$\mathcal{E}(t,x) \simeq \frac{1}{2\sqrt{2\pi}a}\left(e^{-\frac{(x+t)^2}{2a^2}}(\bar{\omega}-p)+e^{-\frac{(x-t)^2}{2a^2}}(\bar{\omega}+p)\right).$$

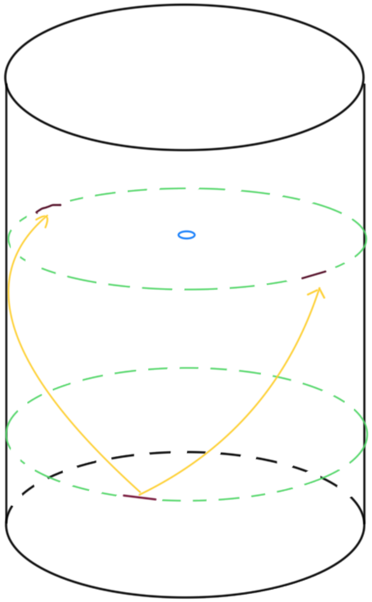
Bulk picture

An example of the bulk wave packet
(moving toward the center).



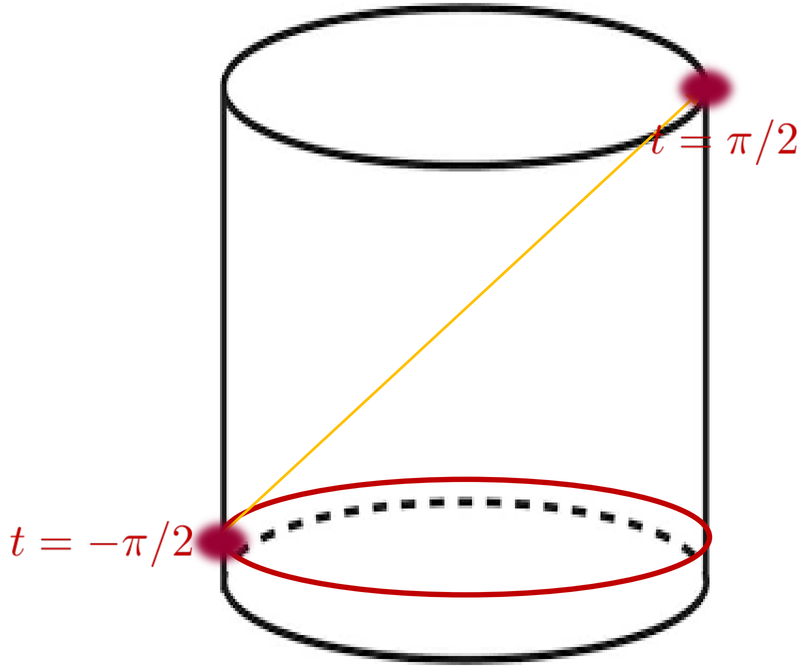
CFT picture

The corresponding
two "particles" in the CFT picture



Overlap between the wave packet state and CFT local state

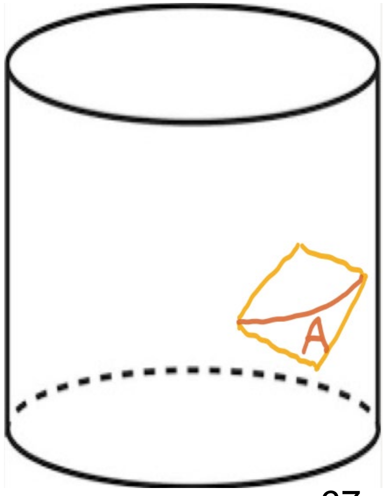
$$\langle 0|\mathcal{O}(\tau,\theta)|p,\bar{\omega}\rangle \simeq a^4 e^{-i\bar{\omega}\tau+ip\theta}\delta(\tau+\pi\mathbf{Z})\delta(\theta-\tau+2\pi\mathbf{Z})$$



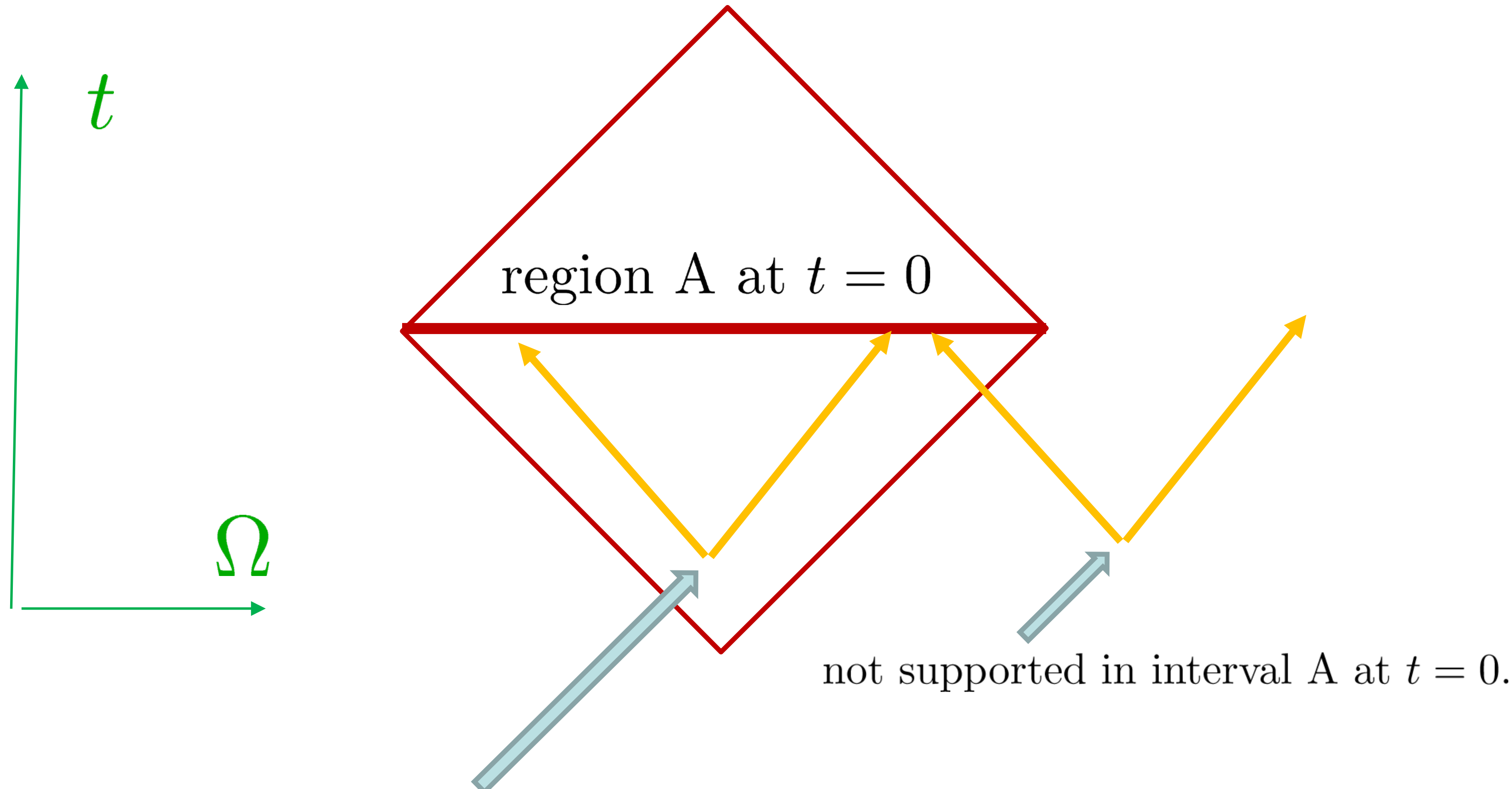
CFT states supported in a region

CFT states supported in a region

Let us consider bulk states correspond to CFT states supported in interval A at $t = 0$.



Causal diamond in CFT

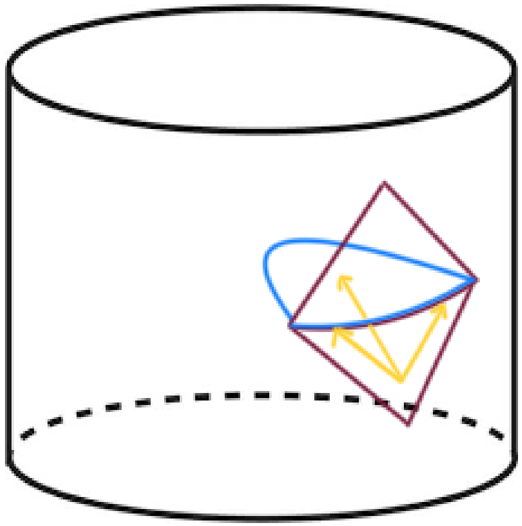


CFT state supported in interval A at $t = 0$.

CFT states supported in a region

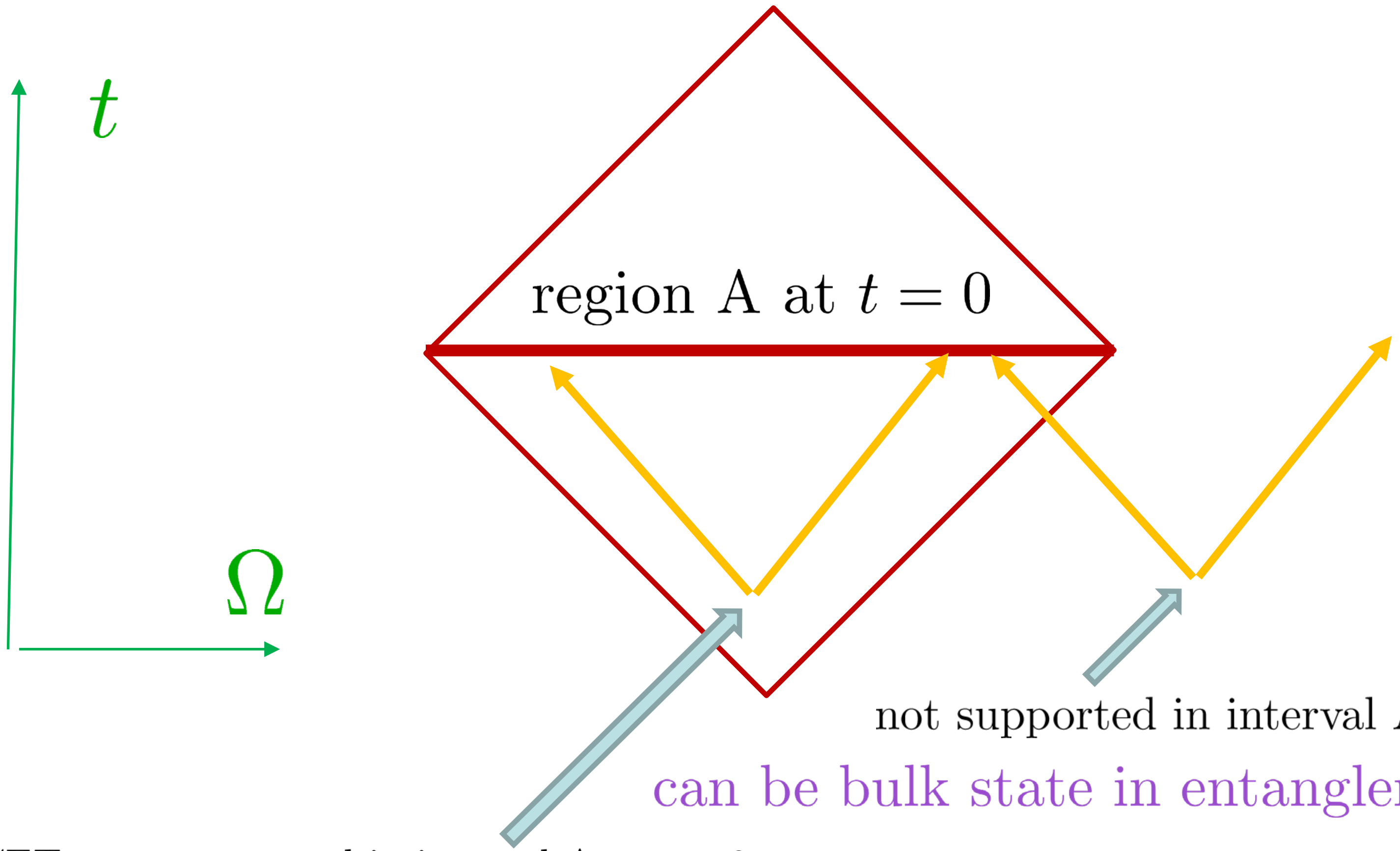
CFT states supported in region A are given by bulk states supported in the causal wedge of A.

The causal wedge of A on $t = 0$ is bulk region inside of blue curve.



Ryu-Takayanagi surface appears!

Causal diamond in CFT



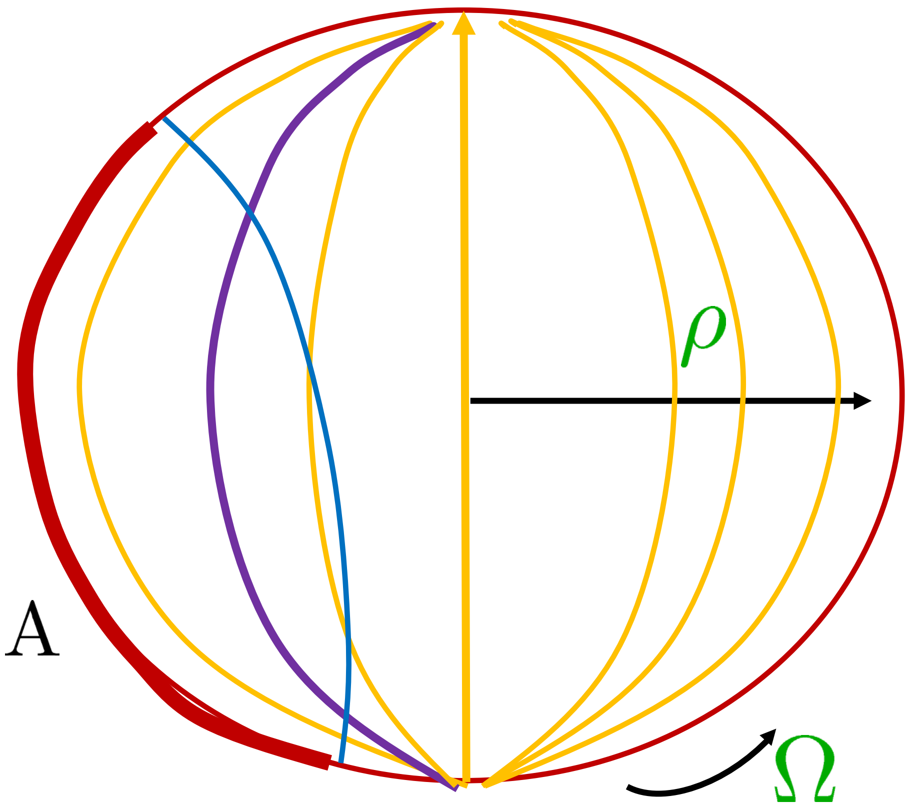
not supported in interval A at $t = 0$.

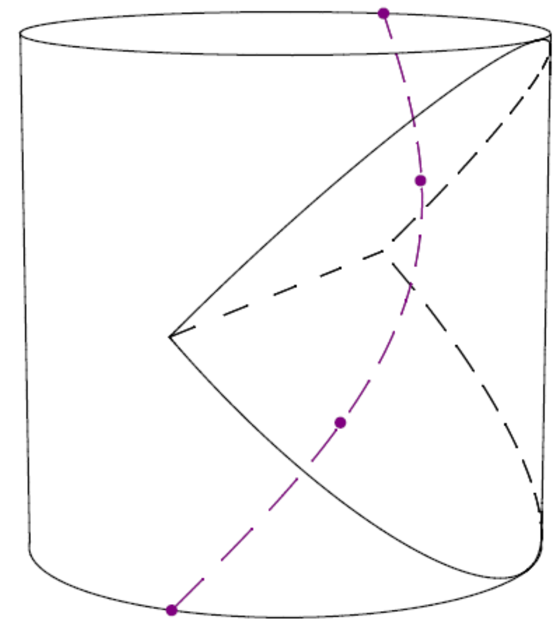
can be bulk state in entanglement wedge

CFT state supported in interval A at $t = 0$.

Null-geodesics connecting horizons

However, some bulk state supported in causal wedge of A can not be CFT state supported in region A !



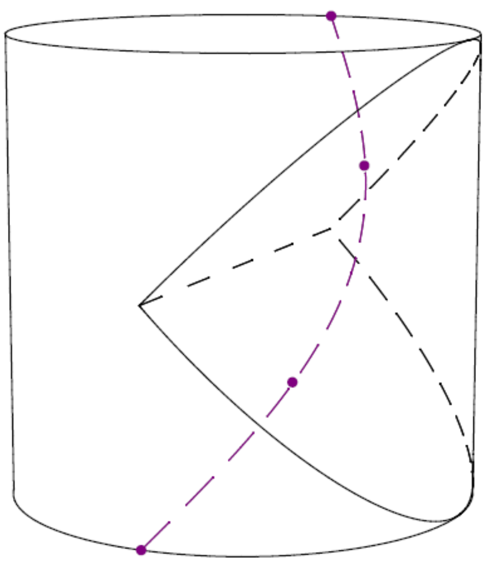


(strong) subregion duality is NOT valid.

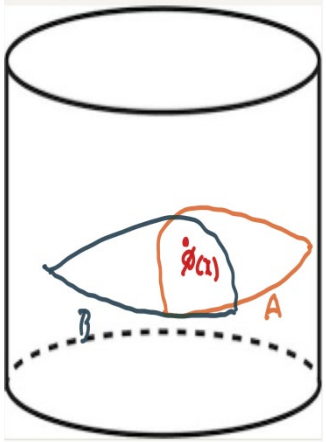
(strong) subregion duality is NOT valid

This problem associated with the null-geodesics was already raised by Bousso-Freivogel-Leichenauer-Rosenhaus-Zukowski in arXiv:1209.4641

Note that
entanglement wedge reconstruction is based on this subregion duality.



Bulk local states at a same bulk point constructed from CFT states supported in different regions are different even in the low energy (gravity) theory

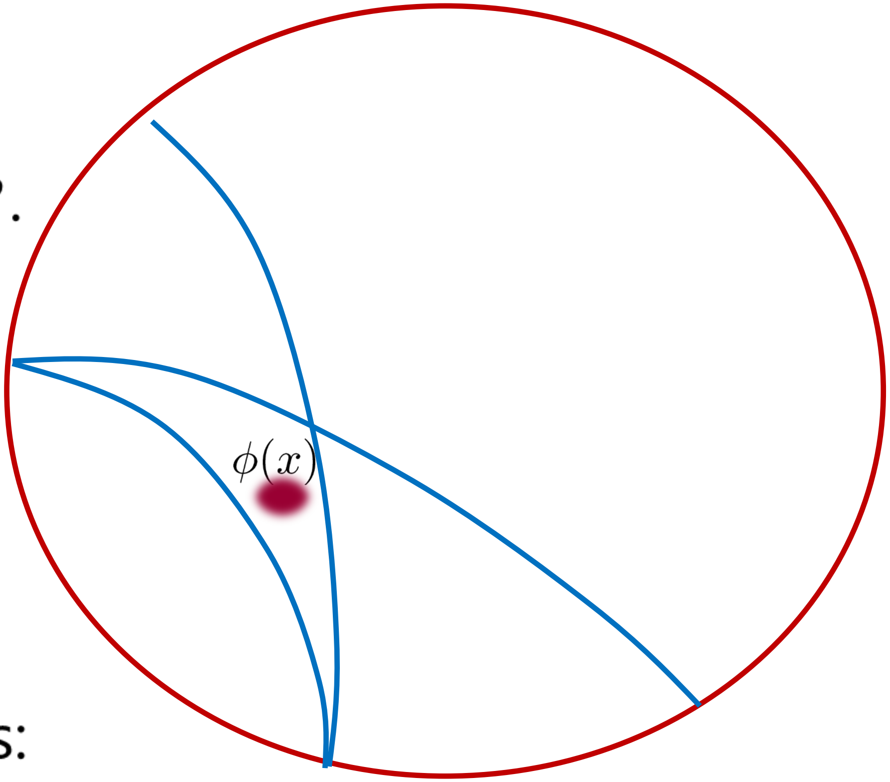


Quantum error correction (QEC) code

If bulk local operators $\phi(x)$ constructed from CFT operators supported in regions A and B are same, It should be constructed from the ones supported in regions $A \cap B$.

However, $\phi(x)$ is outside causal wedge of $A \cap B$.

$\phi(x)$ are same only in low energy theory (called code subspace) in QEC proposal.



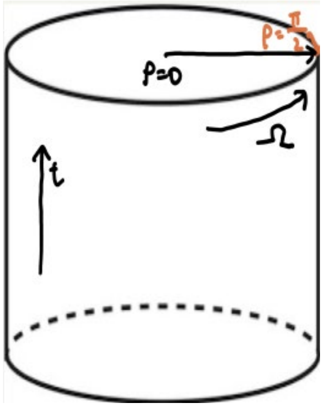
Our picture is opposite to this:
$\phi(x)$ are different even in low energy theory

Ad*S*-Rindler

(Global) AdS_{d+1}

The metric of global AdS_{d+1} ($l_{AdS} = 1$) is

$$ds^2_{AdS} = \frac{1}{\cos^2(\rho)} \left(-dt^2 + d\rho^2 + \sin^2(\rho) d\Omega^2_{d-1}\right)$$

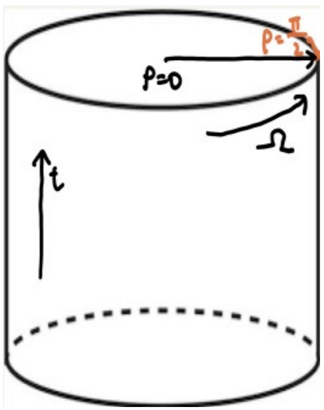


where $0 \le \rho < \pi/2$

Boundary of AdS_{d+1} is located at $\rho = \pi/2$

Boundary of (Global) AdS_3

The boundary of AdS_3 is the cylinder



$$ds^2_{cylinder} = -dt^2 + d\theta^2$$

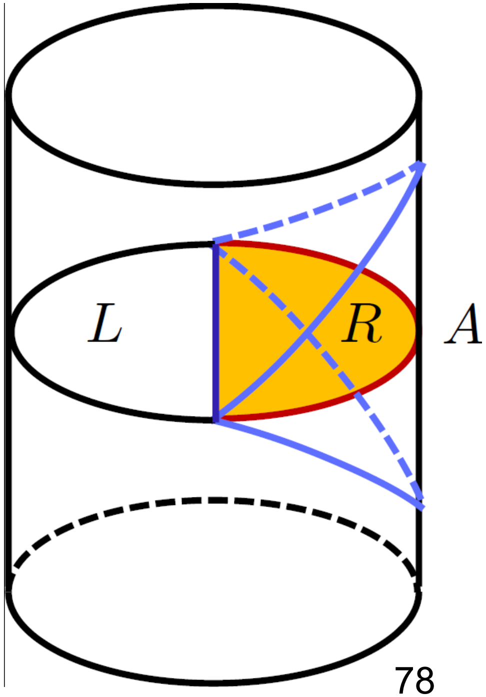
Rindler AdS_3

The metric of Rindler patch of AdS_3 ($l_{AdS} = 1$) is

$$ds^2 = -\xi^2 dt_R^2 + \frac{d\xi^2}{1+\xi^2} + (1+\xi^2)d\chi^2$$

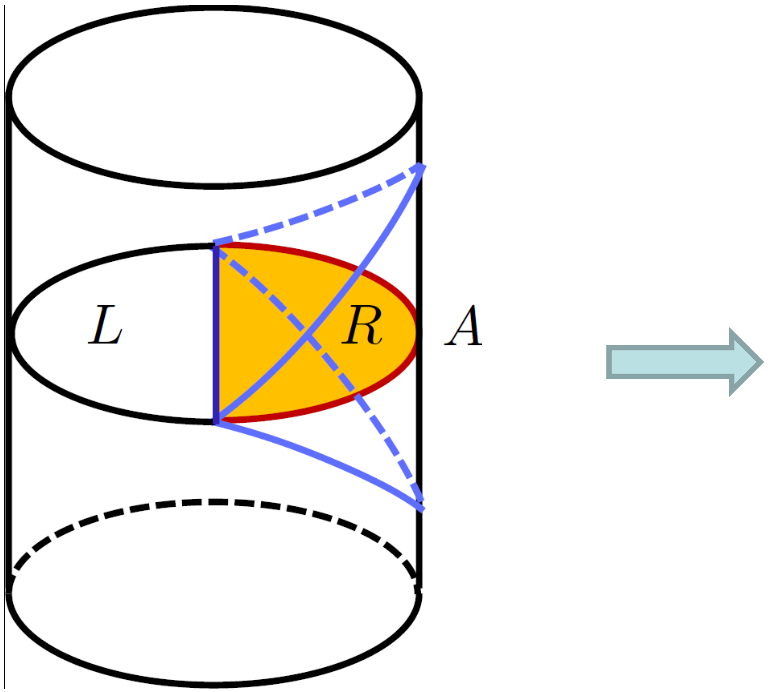
where $-\infty < t_R < \infty$, $-\infty \leq \chi < \infty$

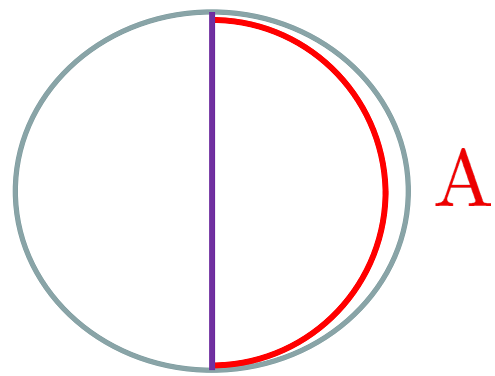
$$0 \leq \xi < \infty$$



Boundary of AdS_3 is located at $\xi = \infty$

Rindler horizon is at $\xi = 0$





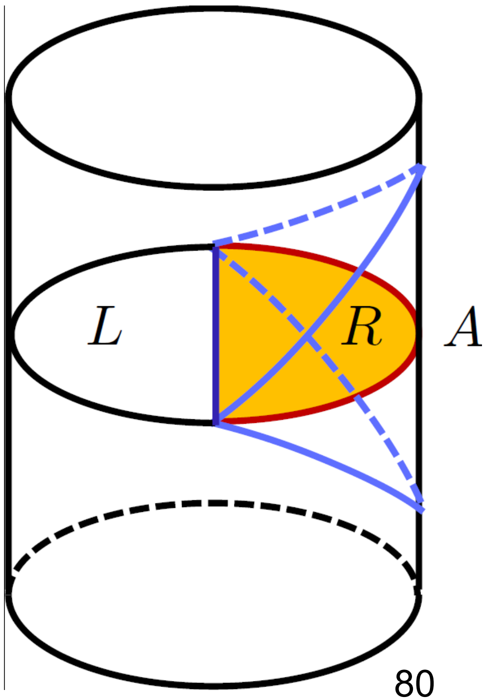
$t = 0$ slice of AdS_3

Boundary of Rindler AdS_3

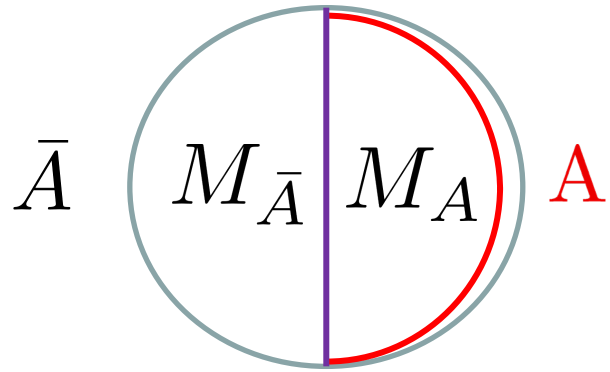
The boundary of Rindler patch of AdS_3
is (conformally) Minkowski space

$$ds^2 = e^{2\Phi}(-dt_R^2 + d\chi^2)$$

where $-\infty < t_R < \infty,\ -\infty \le \chi < \infty$



Decompositions for Bulk and CFT



$$\text{Bulk space} = M_A + M_{\bar{A}}$$

$$\text{CFT space } (= S^1) = A + \bar{A}$$

CFT operator in Rindler patch

By the conformal transformation,

CFT primary operator in Rindler patch is same as

CFT primary operator in Minkowski space

Free scalar in Bulk Rindler patch

We expand ϕ by the modes $v_{\omega,\lambda,\mu}(t_R,\xi,\chi)$,

$$\phi(t_R,\xi,\chi)=\int_0^\infty d\omega\int_{-\infty}^{\infty}d\lambda\frac{1}{\sqrt{2\pi}}\tilde{\psi}_{\omega,\lambda}(\xi)\left[a_{\omega,\lambda}e^{-i\omega t_R+i\lambda\chi}+a^\dagger_{\omega,\lambda}e^{i\omega t_R-i\lambda\chi}\right].$$

Modes are given as

$$\tilde{\psi}_{\omega,\lambda}(\xi)=\frac{N_{\omega,\lambda}}{\Gamma(\nu+1)}\xi^{i\omega}(1+\xi^2)^{-\frac{i\omega}{2}-\frac{\Delta}{2}}\ {}_2F_1\left(\frac{i\omega-i\lambda+\nu+1}{2},\frac{i\omega+i\lambda+\nu+1}{2};\nu+1;\frac{1}{1+\xi^2}\right)$$

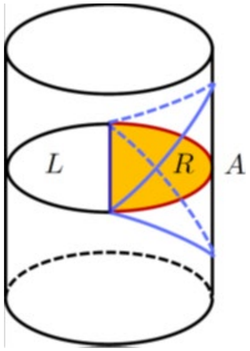
$$N_{\omega,\lambda}=\frac{\left|\Gamma\left(\frac{i\omega-i\lambda+\nu+1}{2}\right)\right|\left|\Gamma\left(\frac{i\omega+i\lambda+\nu+1}{2}\right)\right|}{\sqrt{4\pi\omega}|\Gamma(i\omega)|}$$

Here, energy ω is any real number

BDHM relates Bulk and CFT pictures

In global and Rindler, fields are identical.

$$\lim_{\xi \to \infty} \xi^{\Delta} \phi(t_R, \xi, \chi) = O_{\Delta}(t_R, \chi).$$



$$O_{\Delta}(t_R, \chi) = \int_0^{\infty} d\omega \int_{-\infty}^{\infty} d\lambda \frac{N_{\omega,\lambda}}{\sqrt{2\pi}\Gamma(\nu+1)} \left[a_{\omega,\lambda} e^{-i\omega t_R + i\lambda\chi} + a^{\dagger}_{\omega,\lambda} e^{i\omega t_R - i\lambda\chi} \right]$$

Modes with $\omega < |\lambda|$ are tachyonic!

But, this is not consistent!

What is wrong?

Bulk free theory is only the low energy and large N limit of the (finite N) CFT.

Free theory on the bulk Rindler patch M_A is incorrect as an approximation of the CFT, i.e. the quantum gravity, even though the free theory on the bulk Rindler patch is consistent if it is the UV complete theory.

Failure of low-energy effective theory(=bulk gravity)!
Asymptotic $1/N$ expansion vs Unitarity

Bulk gravity theory can be invalid if we consider a subregion of spacetime, which implies that there are "horizons".This is because of the UV cut-off, typically the Planck mass, of this effective theory.

We stress that this can be seen by considering finite N because 1/N expansion (i.e. semi-classical expansion) is based on the leading order spectrum. In this sense, this is the non-perturbative quantum gravity effect.

This is very surprising because the semi-classical expansion of the bulk gravity theory is expected to be valid even for a subregion of spacetime and have been used in many works of literature, including entanglement wedge construction, sub-region duality, JLMS, Haking radiation,,,,

Thus, these works are based on the wrong assumption.

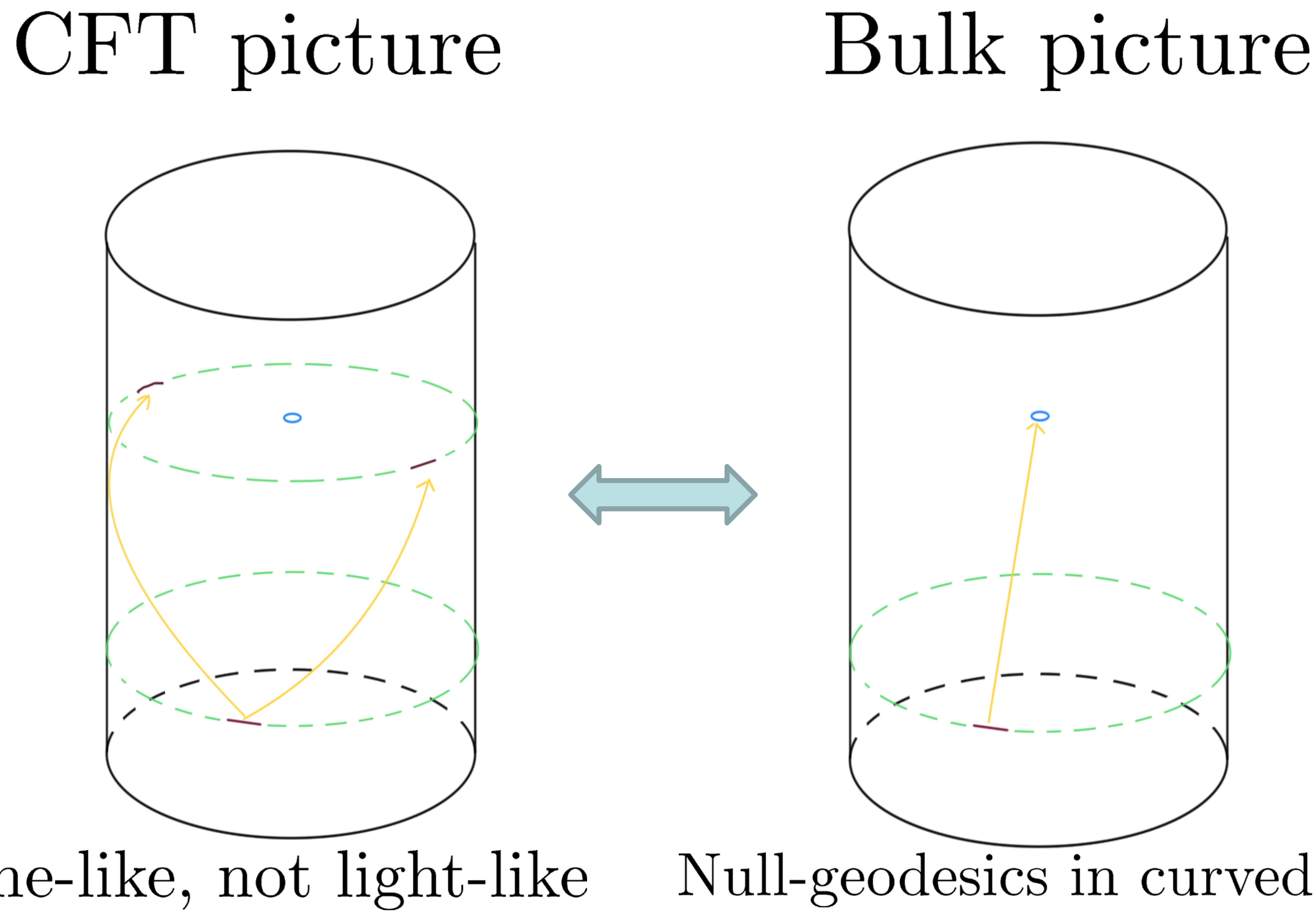
(Holographic error correction code is like “aether”)

Such a violation is an essential property of (black hole) horizon, which is universal to general black hole horizons.

(related to “Brick wall”)

Generalization to asymptotic AdS

For asymptotic AdS, assuming BDHM, we have same picture:



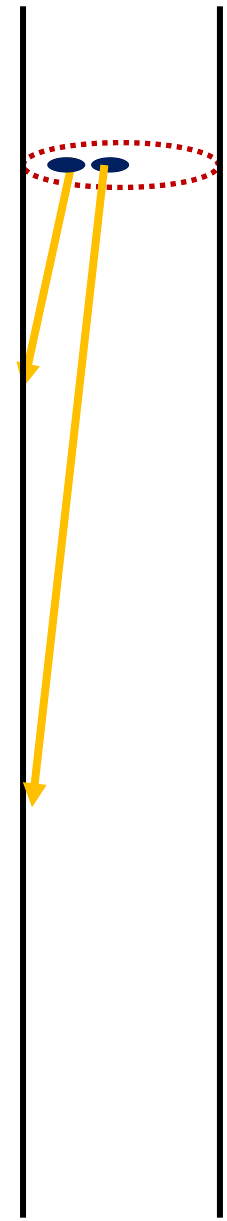
Always time-delay by Gao-Wald theorem

For very heavy star, we need huge number of independent light fields because of the time delay.

For black hole,
we need infinitely many fields.

However,
CFT has N^2 degrees of freedom.
Thus, No inside of Black hole.
(finite N effects are important.)

This is a realization of brick wall argument without boundary condition by hand.
(Dual of black hole is deconfinement phase (QGP) which has N^2 light fields.)



't Hooft, Iizuka-ST

Conclusion

- **Spectrum of large N CFT is identical to spectrum of free gravitational theory in AdS, i.e. "derivation" of AdS/CFT**

- **Bulk reconstruction in AdS/CFT is rather simple and has an intuitive picture.**

- **We also reconstruct the wave packets in bulk theory from CFT primary operators.**

- **Our picture of the bulk reconstruction can be applied to asymptotic AdS spacetime**

Future directions

- **Generalizations in many directions**
- **Non-CFT case**
- **Evaluating (entanglement) entropy**
- **Information loss paradox**
- **Principle of quantum gravity?**

Fin.

Null geodesics in the AdS-Rindler patch

We will regard

tachyonic modes as the bulk local field

Null geodesics in AdS-Rindler patch

There are two types:

(1) horizon ($\xi = 0$) to boundary ($\xi = \infty$), $|b| < 1$

$$\xi(t_R) = \frac{1}{\sqrt{1-b^2}\,|\sinh(t_R - t_0)|}, \quad \chi(t_R) = \chi_0 + \frac{1}{2}\log\frac{1 + b\tanh(t_R - t_0)}{1 - b\tanh(t_R - t_0)}$$

(2) horizon to horizon, $|b| > 1$

$$\xi(t_R) = \frac{1}{\sqrt{b^2-1}\,\cosh(t_R - t_0)}, \quad \chi(t_R) = \chi_0 + \frac{1}{2}\log\frac{b + \tanh(t_R - t_0)}{b - \tanh(t_R - t_0)}$$

Null geodesics in AdS-Rindler patch

For well-localized wave packet

along null-geodesics with b,

modes $a_{\omega,\lambda}$ with $\lambda/\omega = b$ are dominantly contribute

$\Downarrow$

(1) horizon ($\xi = 0$) to boundary ($\xi = \infty$), $|b| < 1$

non-tachyonic modes $\omega^2 > \lambda^2$

(2) horizon to horizon, $|b| > 1$

tachyonic modes $\omega^2 < \lambda^2$

Thus, from the CFT on Rindler patch,

wave packet along horizon to horizon

null-geodesics can not reconstructed