Seminar @ Kyoto (素) N = 38?

23年度5月10日 15:30 〜 ?

Dressed State Formalism(s) in QFT Scattering

Hideo Furugori (古郡 秀雄)

PD @Kyoto University (素)

Introduction to Hideo Furugori

Name: Hideo Furugori

Excellent Hero Ancient County

秀 雄 古 郡

Home town : Tokyo (Monzennakacho)
門前仲町

Famous in Splash festival (水掛祭)

One of the 3-biggest festival in Tokyo

Fukagawa-meshi (深川飯)
is a speciality of our town

Figs from https://www.mitsui-mall.com/article/1826.html
https://www.afpbb.com/articles/-/2894743
https://f-kurashi.tokyo/fukagawameshi/

Introduction to Hideo Furugori

I got Ph.D. last March! @Nagoya Univ

s.v. Nojiri-san (Shin'ichi, NOT Mihoko-san)

My home lab： QG lab NOT E lab

- Classical/Quantum cosmology
- Black hole physics
- Quantum phenomena in curved spacetime
- GR/Modified Gravity

Quantum AND **G**ravity or

Quest for **G**ravity lab ?

Main interest： What is the Spacetime(Universe) ?

I want to investigate the quantum aspects of gravity

Especially, I desire a minimal but concrete progress

→ **Bottom up approach from Infrared properties**

Table of contents (for this talk)

Table of contents (for informal seminar?)

- **I: Conventional Scattering Theory in QM**
  - Quantum Mechanics Preliminaries
  - Free Dynamics and Scattering States
  - Møller operator and Scattering operators
- **II: QM Scattering with Long-Range Potential**
  - Asymptotic Dynamics and Wave Operator
  - Dollard Formalism
  - Our Formalism
- **III: Dressed States in QFT**
  - KF Dressed States
  - Our Dressed States

Table of contents (for this talk)

Conventional Quantum Scattering

Describe the transition: **in state** $|\Psi_\alpha^-\rangle \longrightarrow$ **out state** $|\Psi_\beta^+\rangle$

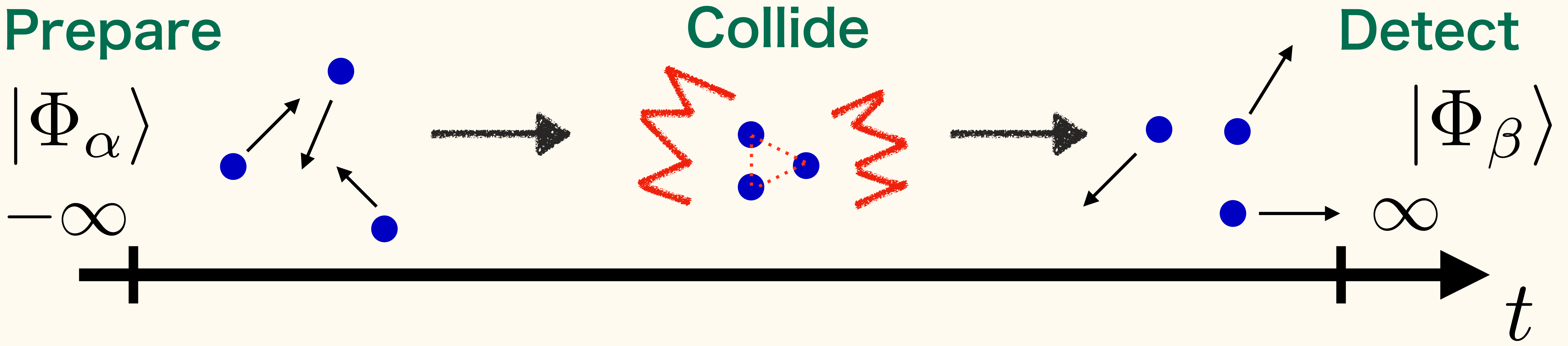
$\{\alpha\}, \{\beta\}$: sets of quantities characterizing the process

Assumption: Scat. occurs in a local region

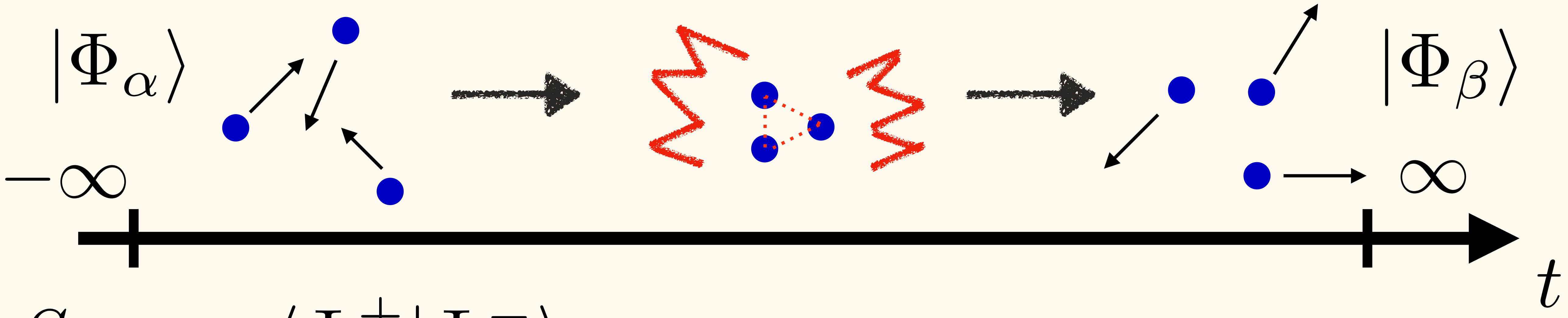
$\longrightarrow$ **in/out states asymptote to (free) particle states**

$$\lim_{t\to\pm\infty} \underset{\text{Interaction pic.}}{\underline{|\Psi_\gamma(t)\rangle_{\mathrm{I}}}} = \Omega_\pm \underset{\text{Heisenberg pic.}}{\underline{|\Psi_\gamma^\pm\rangle}} \simeq |\Phi_\gamma\rangle$$

Particle state



Scattering Problem with Particle Picture



$$\underline{S_{\beta\alpha} := \langle \Psi^+_\beta | \Psi^-_\alpha \rangle}$$ **S-matrix**

$$\simeq \langle \Phi_\beta | \underline{\mathcal{T} \exp \left[-i \int_{-\infty}^{\infty} d\tau V^{\mathrm{I}}(\tau) \right]} | \Phi_\alpha \rangle =: S^{\mathrm{D}}_{\beta\alpha}$$

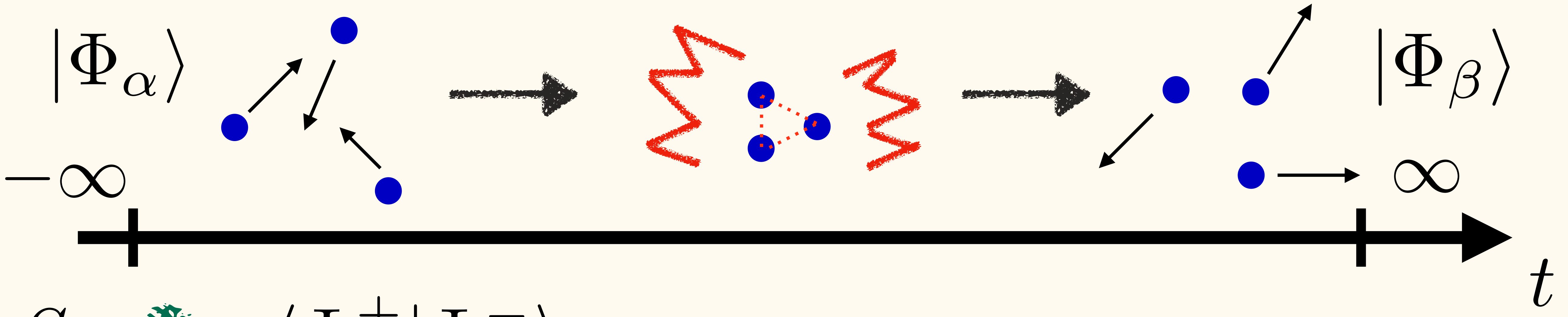
Dyson S-operator : $\mathcal{S}_{\mathrm{D}} := \Omega_+ \Omega_-^\dagger$

Dyson S-matrix $S^{\mathrm{D}}_{\beta\alpha} \longrightarrow \Gamma_{\beta\alpha}$ **Transition rate** (probability)

Theory : calculate S and predict Γ by considering interactions

Experiment : obtain Γ by analyzing data of scatterings

Scattering Problem with Particle Picture



$$S_{\beta\alpha} := \langle \Psi^{+}_{\beta} | \Psi^{-}_{\alpha} \rangle$$ S-matrix

$$\simeq \langle \Phi_\beta | \mathcal{T} \exp\left[-i \int_{-\infty}^{\infty} d\tau V^{\mathrm{I}}(\tau) \right] |\Phi_\alpha\rangle =: S^{\mathrm{D}}_{\beta\alpha}$$

IR divergence!

Dyson S-matrix $S^{\mathrm{D}}_{\beta\alpha} \to \Gamma_{\beta\alpha}$ Transition rate (probability)

Theory : calculate S and predict Γ by considering interactions

Experiment : obtain Γ by analyzing data of scatterings

IR Divergence Problem in QED

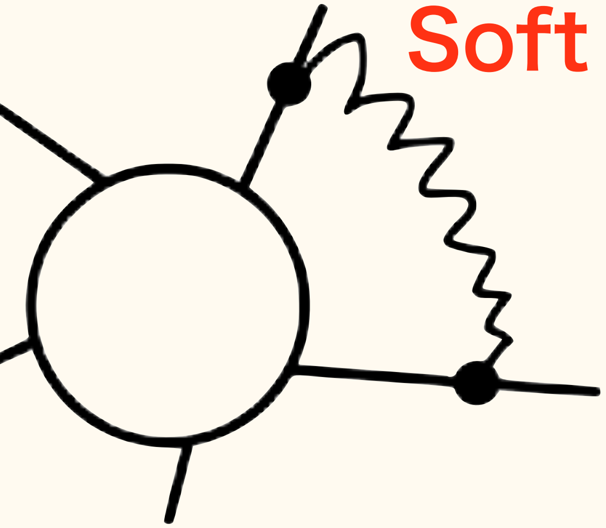
QED： Photons mediate EM forces bet. charged particles

IR div. **Low energy photon corrections make S diverge**

Soft photon $a_\mu(k,h)$ w/ $k^0 = |\vec{k}| \equiv \omega \in [\underline{\lambda}, \underline{\Lambda_s}]$, h : helicity

IR cutoff "softness" parameter

$$S^{\mathrm{D}}_{\beta\alpha} = \underline{(\lambda/\Lambda)^{A_{\beta\alpha}/2} \mathrm{e}^{-i\theta^{\lambda}_{\beta\alpha}(\Lambda)}}\, \underline{S^{\mathrm{hard}}_{\beta\alpha}(\Lambda)}$$ **S w/o soft photons**



Soft correction $C^{\lambda}_{\beta\alpha}(\Lambda)$, $A > 0$

Phase: $\theta^{\lambda}_{\beta\alpha}(\Lambda) \xrightarrow{\lambda\to 0} \infty$ **Limit does not exist**

→ **S-matrix cannot be defined**

By ignoring the phase, we can derive predictions

Rate: $\Gamma^{\lambda}_{\beta\alpha} \propto \left|C^{\lambda}_{\beta\alpha}(\Lambda)\right|^2 \xrightarrow{\lambda\to 0} 0$ **Nothing occurs in QED !**

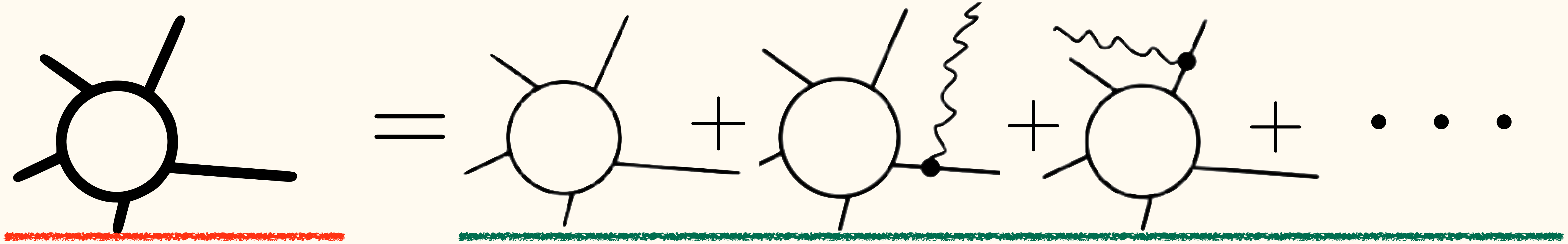
Conventional prescription for IR div.

We have limitations of detector's resolution

→ We cannot distinguish a process w/ or w/o soft photons

Inclusive prescription :

out states contain any soft emission processes



Physical transition amplitude $\alpha \to \beta$

Sum of $n \geq 0$ -soft photons emitted amplitudes

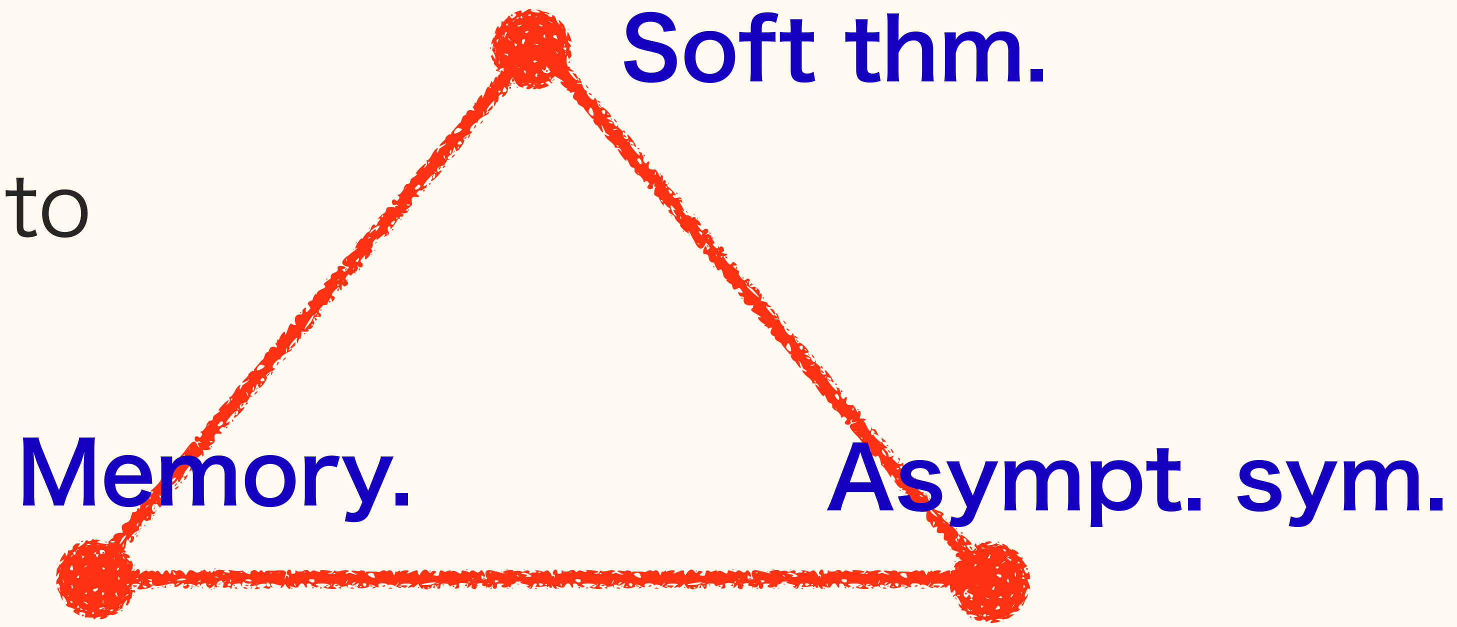
Soft theorem : $M_{\alpha\to\beta+a_{\mathrm{soft}}} = M_{\alpha\to\beta} \times \underline{F_{\beta\alpha}} + \mathcal{O}(\omega^0)$

soft factor

→ $\Gamma^{\mathrm{obs}}_{\beta\alpha}(\Lambda_{\mathrm{D}})$ w/ Λ_{D} : detection limit **finite rate !**

Introduction to IR triangle physics

IR triangle relation :



Soft theorem is equivalent to

- **Memory effect**
- **Asymptotic symmetry**

Strominger ’14

Triangles have been discovered/supported in various fields

Gauge, Gravity, SUSY, String, ・・・

Leading, Sub-leading, ・・・

Higher dimensions, Asympt.-flat/dS spacetimes, ・・・

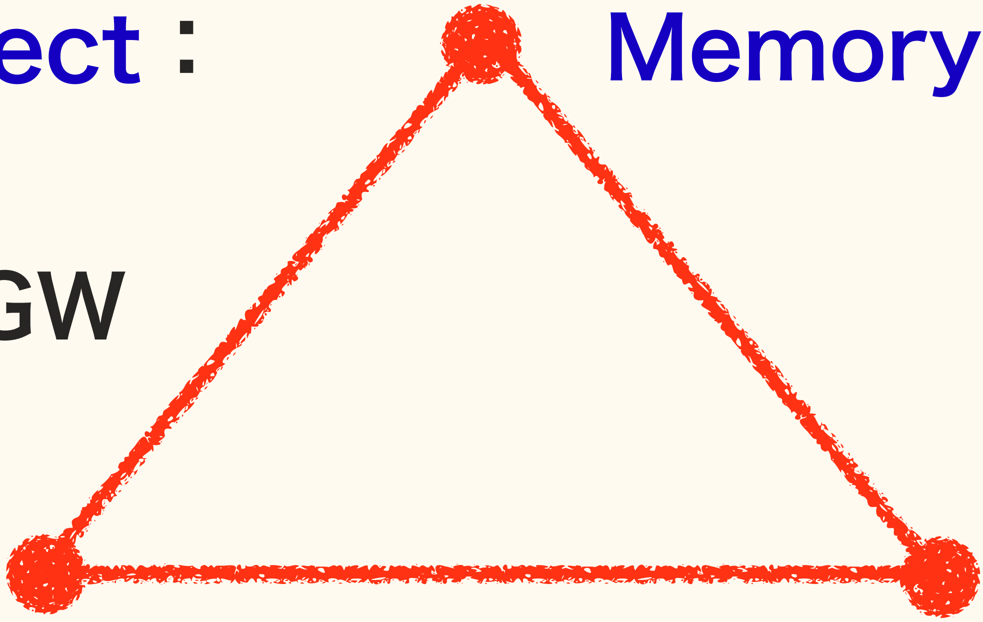
Let’s see the case of 4dim. asympt.- flat Einstein gravity

IR Triangle - Memory Effect

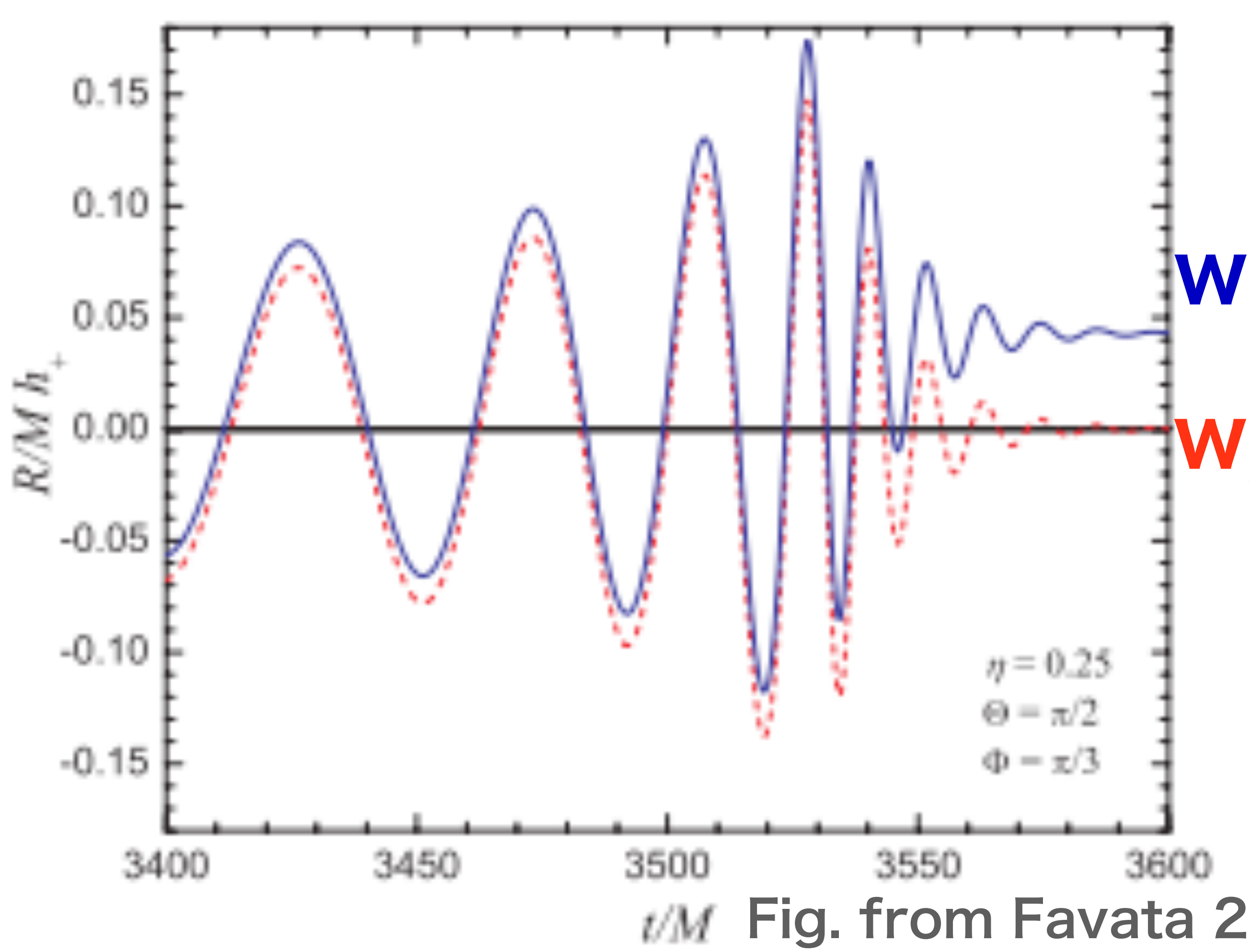
Gravitational memory effect :

Permanent displacement remains after passage of GW

Zel'dovich-Polnarev '74



Strain



w/ memory

w/o memory

Fig. from Favata 2010

GW detection

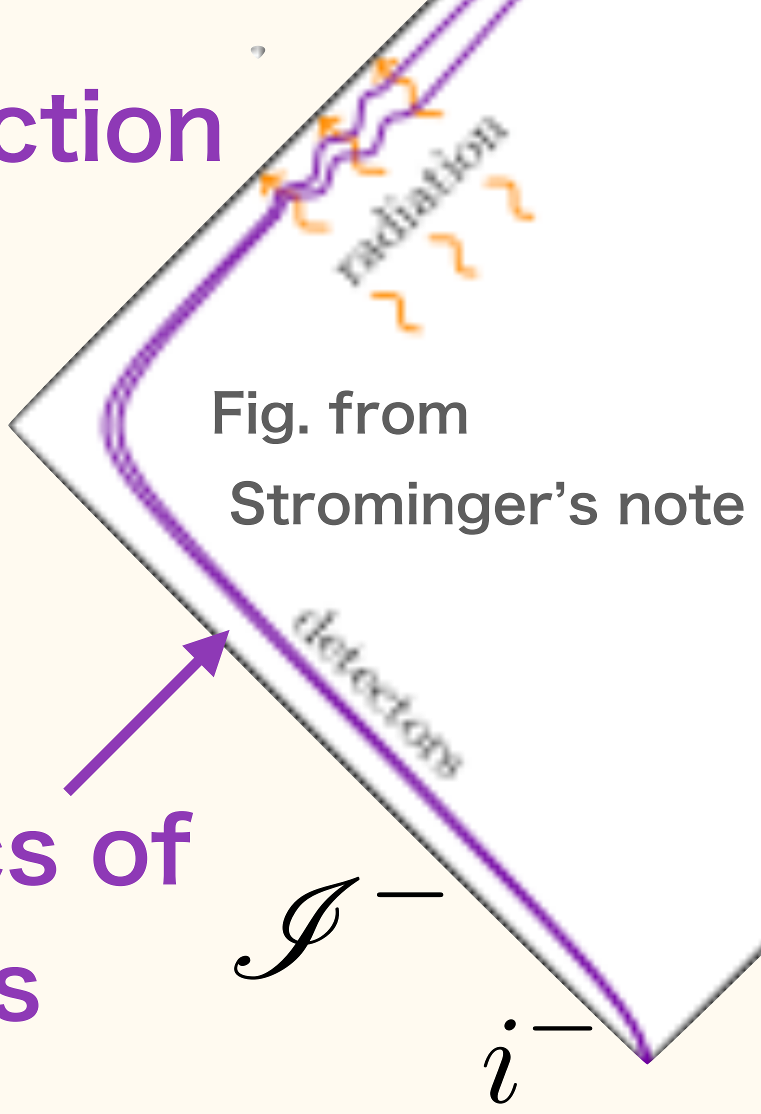


Fig. from Strominger's note

geodesics of detectors

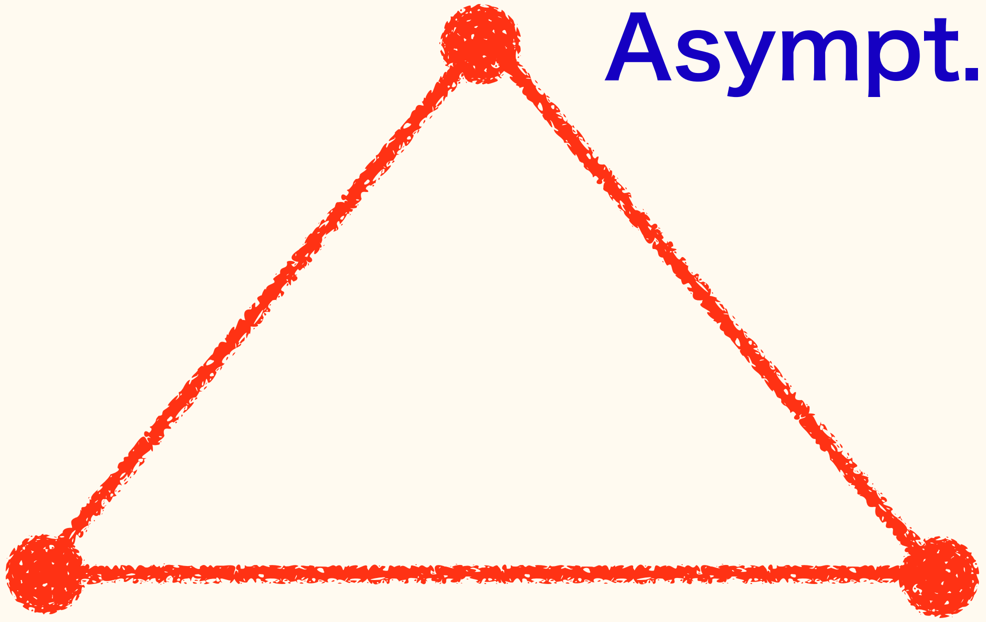
IR Triangle - Asymptotic symmetry

BMS asympt. symmetry :

Transformation preserving

- Gauge conditions
- Fall-off conditions

Bondi '60
Bondi-Burg-Metzner '62 , Sachs '62



retarded Bondi coordinates $x^\mu = (u, r, x^A)$

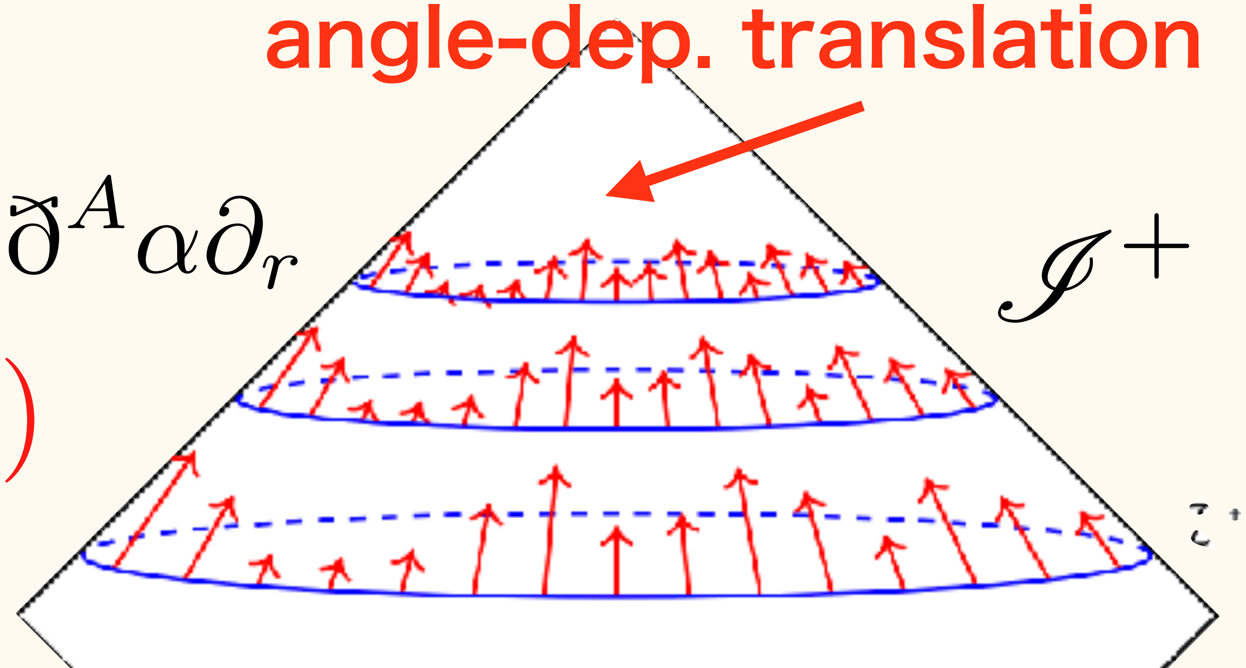
u : out-going null , r: radius , $x^A \in \mathbb{S}^2$ $\eth$: cov. deriv. on $\mathbb{S}^2$

$\mathscr{I}^+ : r \to \infty$ w/ fixed u

$$\underline{\boldsymbol{\xi}_\alpha} = \alpha\partial_u - r^{-1}\eth^A\alpha\partial_A + \frac{1}{2}\eth_A\eth^A\alpha\partial_r$$

BMS generator w/ $\alpha = \alpha(x^A)$

Infinite d.o.f. on infinity $\mathscr{I}^+$



IR Triangle - Connections to Soft Thm.

IR triangle： **Asympt. sym. dictates IR div.** He et.al. '14

$$\underline{J^\mu_\alpha = \alpha J^\mu_{\mathrm{mat}} + F^{\mu\nu}\partial_\nu\alpha}\ , \quad \lim_{r\to\infty}\alpha(x^i) = \alpha(x^A) \neq Const.$$

Noether current for large gauge sym.

"Large charge"

hard/soft part

$$Q^\pm_\alpha = \int_{\underline{\Sigma^\pm_{\mathrm{h}}}\cup\underline{\Sigma^\pm_{\mathrm{s}}}} d\Sigma_\mu J^\mu_\alpha \quad =: \underline{Q^\pm_{\mathrm{h}}[\alpha]} + \underline{Q^\pm_{\mathrm{s}}[\alpha]}$$

See also 平井・杉下 '18

Assuming large charge conservation in quantum theory

$$0 = \langle \mathrm{out}|[Q_\alpha, \mathcal{S}]|\mathrm{in}\rangle \Rightarrow \Big(\Delta Q_{\mathrm{h}}[\alpha] + \Delta Q_{\mathrm{s}}[\alpha]\Big) S_{i\to f} = 0$$

$\Delta Q_\alpha \neq 0 \Rightarrow S_{i\to f} = 0$ **IR div. protects to asympt. sym.**

$$S_{i\to f} \neq 0 \Rightarrow \Delta Q_{\mathrm{s}}[\alpha] S_{i\to f} = -\Delta Q_{\mathrm{h}}[\alpha] S_{i\to f} \sim S_{i\to f+\underline{\gamma_{\mathrm{s}}(\alpha)}}$$

Soft theorem implicates memory effect **Memory**

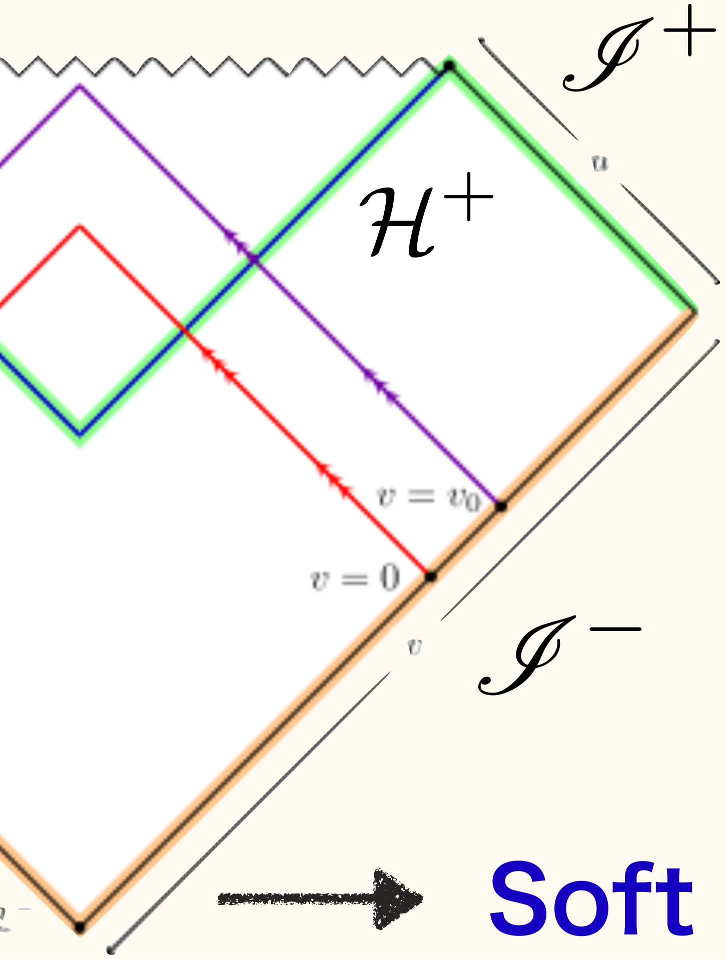
IR Triangle and BH Information Paradox

Suppose IR triangle in quantum gravity exist

→ **Asympt. sym. needs soft gravitons w/ memories**

“Soft hair implants” on BH Hawking, Perry, Strominger ’16

See also “soft hair as a soft wig” ’17



Fall shockwave w/ (ℓ, m) **mode into BH**

They claimed that information

can be stored on the horizon and infinity

via large charge conservation

$$Q^{\mathscr{I}^-}_{\ell m} = Q^{\mathscr{I}^+}_{\ell m} + \underline{Q^{\mathcal{H}^+}_{\ell m}}$$

Soft hair on BH

→ **Soft particles rescue the unitarity problem?**

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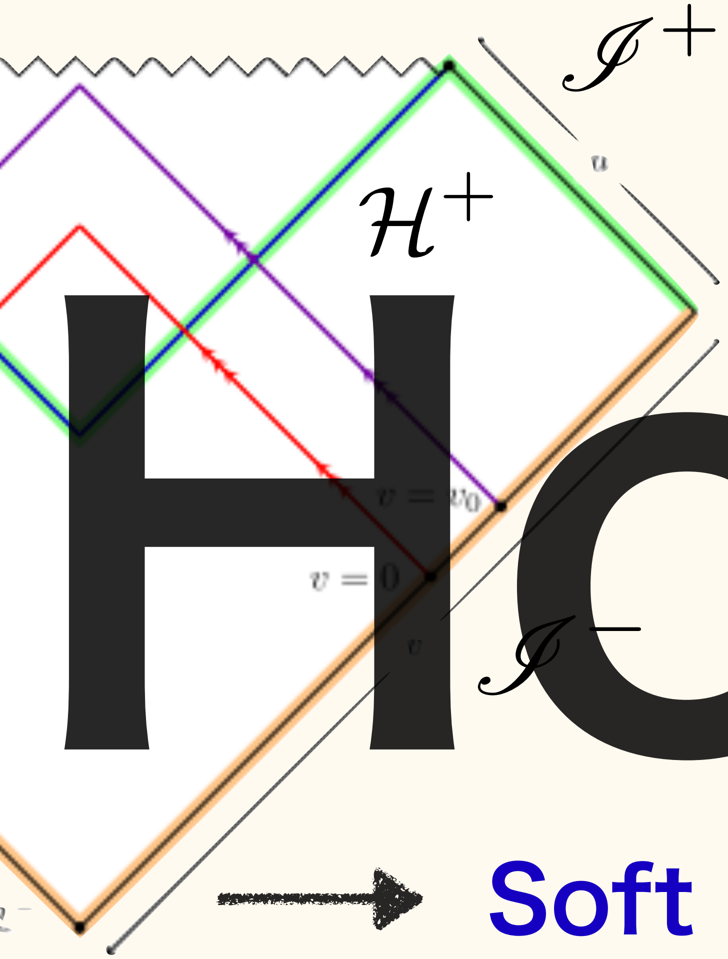

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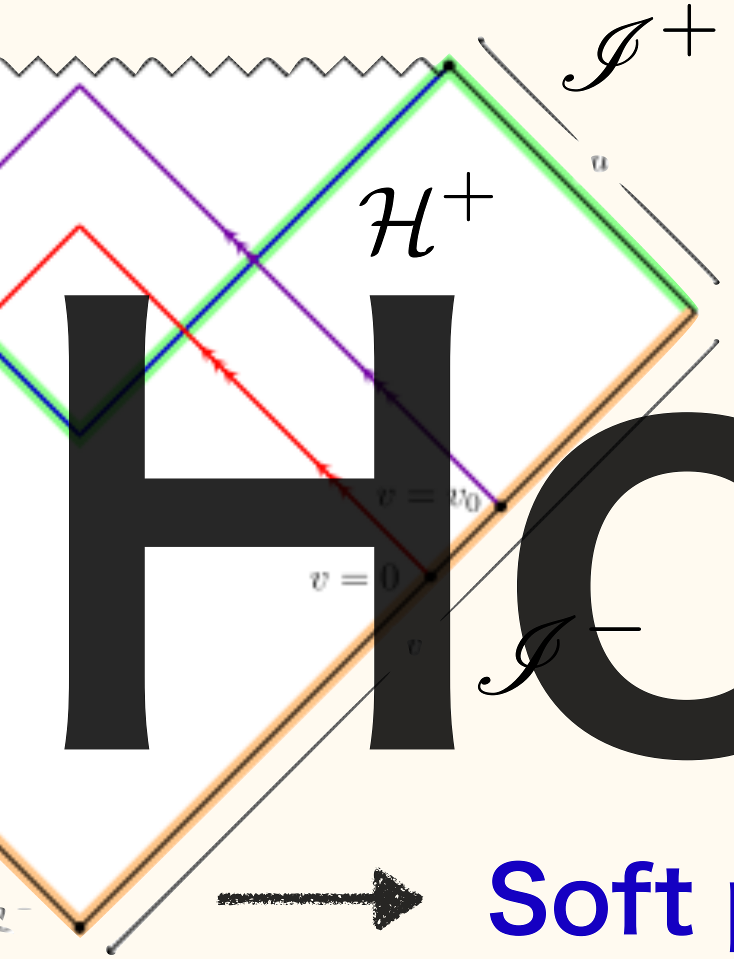

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<u>Soft thm. cannot save the IR div. in S-matrix</u>

S-matrix is important !

The unitarity of time evolution is
a fundamental principle of quantum theory

S describing the transition process must be unitary

$$\int d\beta\,\overline{S_{\beta\alpha}}\,S_{\beta\gamma} = \delta(\alpha-\gamma) \quad \text{or} \quad \mathcal{S}^{\dagger}\mathcal{S} = I = \mathcal{S}\mathcal{S}^{\dagger}$$

→ Gives us info. for theories to be discovered

e.g.

Restrictions to symmetry — Coleman-Mandula ’67

Restrictions to interaction — Adams et.al. ’06

S-matrix is a clue in exploring the BSM and/or QG !

We need well-defined unitary S-matrix in QFT

Or, we have “info. paradox” even in the flat spacetime

Table of contents (for this talk)

Conceptual Idea of Dressing

Dyson S-matrix is not defined in QFT due to IR div.

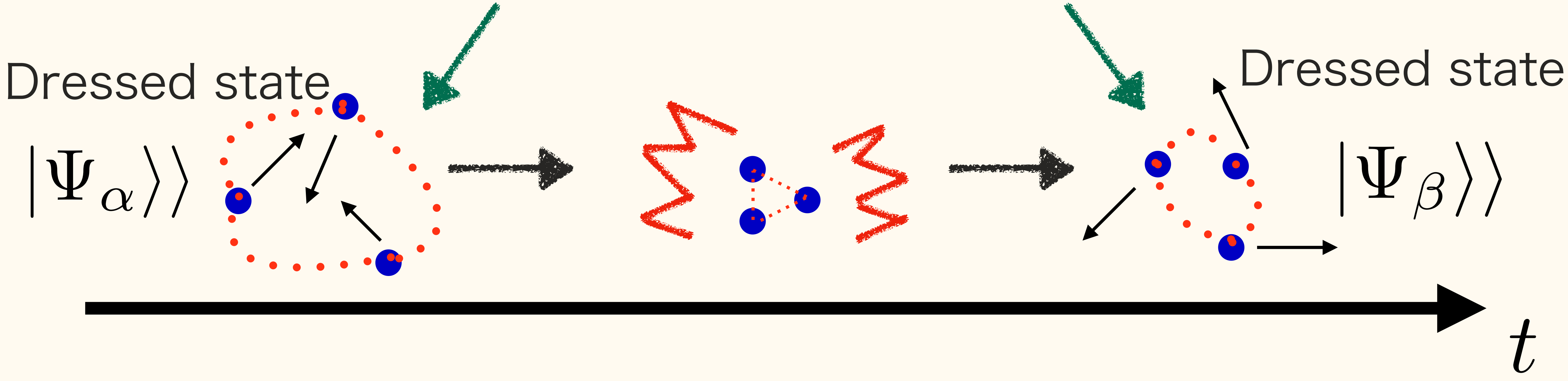
Dressed state formalism(DF) is a good candidate

DF : IR div. is breaking down of the particle picture

→ incorporates interaction to the asymptotic states

Asympt. interaction V_{as}

Charged particles are dressed by soft photons



Chung Dress -Coherent States

Chung looked for a state s.t. $\left|\langle\langle\Psi_\beta^{\rm Ch}|\mathcal{S}_D|\Psi_\alpha^{\rm Ch}\rangle\rangle\right| < \infty$

Chung dress $|\Psi_\alpha^{\rm Ch}\rangle\rangle = \underline{D_{f_\alpha^{\rm Ch}}}\,|\Phi_\alpha\rangle$ **Chung '65**

Counter part of displacement op. on phase space in QM.

Disp.op. w/ para. $\alpha \in \mathbb{C}$ $D_\alpha = \exp\left[\alpha a^\dagger - \alpha^* a\right]$

Coherent state $|\alpha\rangle = D_\alpha\,|0\rangle$, $\underline{a\,|\alpha\rangle = \alpha\,|\alpha\rangle}$ **Klauder '60**

Eigenstate for annihilation op.

Averaged number $\langle n\rangle = \langle\alpha|a^\dagger a|\alpha\rangle = |\alpha|^2$ Polarization vector

$$f_\alpha^{\rm Ch} = \sum_{h=\pm}\int d^3k\,\underline{f_\alpha^{\rm Ch}(\vec{k},h)} \equiv \sum_{n\in\alpha}\frac{e_n}{(2\pi)^{3/2}\sqrt{2\omega}}\,\frac{p_n\cdot\underline{\bar{\epsilon}(\vec{k},h)}}{p_n\cdot k}$$

$\longrightarrow$ $\langle n\rangle = \infty$ **Infinite # of photon** $|\Psi_\alpha^{\rm Ch}\rangle\rangle \notin \underline{\mathcal{H}_{\rm F}}$

Fock space：space for particle states

Chung’s Limitations and Kibble Dress

- 😄 $|\Psi_\alpha^{\mathrm{Ch}}\rangle\rangle = D_{f_\alpha^{\mathrm{Ch}}}|\Phi_\alpha\rangle \Rightarrow \left|\langle\langle\Psi_\beta^{\mathrm{Ch}}|\mathcal{S}_D|\Psi_\alpha^{\mathrm{Ch}}\rangle\rangle\right|$ **IR finite**
- 😩 **No guiding rule for determining asymptotic states**
- 😩 **No ability to deal with phase divergence**

Kibble formalism **Kibble ’68**

By analyzing the structure of N-pt Green function G_N

Redefine both

asymp. states $|\Phi_\alpha\rangle \rightarrow |\Psi_\alpha^{\mathrm{Ki}}\rangle\rangle = D_{f_\alpha^{\mathrm{Ki}}}|\Phi_\alpha\rangle$

S-operator $\mathcal{S}_{\mathrm{D}} \rightarrow \mathcal{S}_{\mathrm{Ki}}$

$\Rightarrow \langle\langle\Psi_\beta^{\mathrm{Ki}}|\mathcal{S}_{\mathrm{Ki}}|\Psi_\alpha^{\mathrm{Ki}}\rangle\rangle$ **IR finite** 😆

We need Kibble reduction formula

- 😅 **Impractical due to complexity of** G_N
- 😩 **We can not describe** $\alpha \rightarrow \alpha$ **process**

Coulomb Scat in QM : Dollard Formalism

Consider Coulomb scattering process $\vec{p} \to \vec{p'}$

Long-range interaction $V(\vec{x}) = C|\vec{x}|^{-1}$

$S_{p'p} \overset{!}{=} \langle \vec{p}' | \Omega_+ \Omega_-^\dagger | \vec{p} \rangle$ **has divergent phase !**

$$\lim_{t \to \pm\infty} \Omega^\dagger(t) \equiv \underline{\Omega_\pm^\dagger} \,, \quad \Omega^\dagger(t) = \mathrm{e}^{iHt} \mathrm{e}^{-iH_0 t} \qquad \Omega_+ |\Psi_\beta^+\rangle \underline{\simeq} |\Phi_\beta\rangle$$

Møller op connects full theory and free theory

$\Omega_\pm H = H_0 \Omega_\pm$: Intertwiner

In a long-range interaction, this limit is not exist !

Dollard formalism Dollard '64

Relace $H_0 \to H_{\mathrm{as}}(t) = H_0 + \underline{V_{\mathrm{as}}(t)}$ in $\Omega^\dagger(t)$ to define the limit

Asympt. interaction

→ **Dollard op.** $\Omega_{\mathrm{D}}^\dagger(t) = \mathrm{e}^{iHt} \mathrm{e}^{-iH_{\mathrm{as}}(t)t} \xrightarrow{t \to \pm\infty} \Omega_\pm^\dagger$

Møller op can be defined and S-matrix exist !

Kulish-Faddeev(KF) Formalism

Kibble

Analyze N-pt Green function → states / S-operator → S-matrix

$\sim$ full dynamics of charged particles : tough

😆 **Trans amplitudes are finite including the phase**

😩 We can not describe $\alpha \longrightarrow \alpha$ process

Dollard

Derive proper $V_{\rm as}$ to define Dollard op

Analyze asympt. dynamics → S-operator → S-matrix

Kulish-Faddeev

- Incorporates $V_{\rm as}$ into S-op as Dollard
- Choose coherent states for asympt. states as Chung
- Define S-op as a map $\mathcal{H}_{\rm F} \rightarrow \mathcal{H}_{\rm F}$

Guiding rule for asympt. states? Enables to describe $\alpha \rightarrow \alpha$?

KF -Asymptotic Interaction

Extract surviving terms from $V^{\mathrm{I}}(t)$ in $|t| \to \infty$

$$V^{\mathrm{I}}(t) \ni \int d^3k d^3p \hat{\mathcal{A}}(\vec{k}, \vec{p}) \mathrm{e}^{i\mathcal{E}(\vec{k},\vec{p})t}$$

Assuming that terms with $\mathcal{E}(\vec{k}, \vec{p}) = 0$ can survive

$$\xrightarrow{\mathrm{e}^{i\mathcal{E}(\vec{k},\vec{p})t} \sim \delta(\omega)} \quad V^{\mathrm{I}}_{\mathrm{as}}(t) = \int d^3x \underline{a^{\mathrm{IR}}_{\mu}(x)} j^{\mu}_{\mathrm{cl}}(x)$$

Photon with $\omega \sim 0$

$$\underline{j^{\mu}_{\mathrm{cl}}(x) = \int d^3p e \hat{\rho}(\vec{p}) v^{\mu} \delta^3(\vec{x} - \vec{v}t)}$$

$\hat{\rho}$: charge density

$v^{\mu} = p^{\mu}/E_p$

Current as particle flows

$$\mathcal{S}_{\mathrm{as}}(t) = Z^{\dagger}(t) \mathcal{S}_{\mathrm{D}} Z(-t) \,, \; Z(t) = \mathcal{T} \exp\left[-i \int_{\underline{\partial_{\mathrm{KF}}}}^{t} d\tau \, V^{I}_{\mathrm{as}}(\tau) \right]$$

B.C. ：any quanta can not depend on lower limit of integral

KF -Asymptotic States

$$Z(t) = \mathcal{T}\exp\left[-i\int_{\partial_{\mathrm{KF}}}^{t} d\tau\, V_{\mathrm{as}}^{I}(\tau)\right] = \mathrm{e}^{i\hat{\theta}(t)} D_{\hat{f}(t)}$$

KF Dressed state $|\Psi_{\alpha}^{\mathrm{KF}}\rangle\rangle = D^{\dagger}_{f_{\alpha}^{\mathrm{KF}}}|\Phi_{\alpha}\rangle$ with function f_{α}^{KF}

$$\longrightarrow \quad S^{\mathrm{as}}_{\beta\alpha}(t) = \langle\Phi_{\beta}|\underline{D_{f_{\beta}^{\mathrm{KF}}} Z^{\dagger}(t)\mathcal{S}_{\mathrm{D}} Z(-t) D^{\dagger}_{f_{\alpha}^{\mathrm{KF}}}}|\Phi_{\alpha}\rangle$$

$\mathcal{S}^{\mathrm{as}}_{\mathrm{KF}}(t)$

Impose $\mathcal{S}^{\mathrm{as}}_{\mathrm{KF}}(t)$, $\mathcal{S}_{\mathrm{D}}$ as operators on Fock space

$\longrightarrow$ **Convergence condition** $\displaystyle\int d^3k\left|f_{\alpha}(t) - f_{\alpha}^{\mathrm{KF}}\right|^2 < \infty$

Taking limit $t \to \infty$, $\hat{f}(t) \to 0$, $S^{\mathrm{as}}_{\beta\alpha}(t) \to S^{\mathrm{KF}}_{\beta\alpha}$

$$S^{\mathrm{KF}}_{\beta\alpha} = \underline{\langle\Phi_{\beta}|D_{f_{\beta}^{\mathrm{KF}}}}\,\underline{\mathrm{e}^{-i\theta_{\beta}(\infty)}\mathcal{S}_{\mathrm{D}}\mathrm{e}^{i\theta_{\alpha}(-\infty)}}\,\underline{D^{\dagger}_{f_{\alpha}^{\mathrm{KF}}}|\Phi_{\alpha}\rangle}$$

Dollard-like phase contribution Chung-like dressed state

Additional Condition for Asympt. States

KF analysis is in apparent covariant gauge

→ **We should deal with unphysical states**

$\tilde{h}=s$: scalar mode , $\tilde{h}=l$: longitudinal mode

Free Gupta-Bleuler(GB) condition $k^{\mu}a_{\mu}(\vec{k},\tilde{h})\,|\Psi\rangle=0$

$$f_{\alpha}^{\mathrm{KF}}=\sum_{\tilde{h}=\pm,s,l}\int d^3k f_{\alpha}^{\mathrm{KF}}(\vec{k},\tilde{h}) \quad , f_{\alpha}^{\mathrm{KF}}(\vec{k},\tilde{h})=f_{\alpha}^{\mathrm{KF}^{\mu}}(\vec{k})\epsilon_{\mu}^{*}(\vec{k},\tilde{h})$$

$$f_{\alpha}^{\mathrm{KF}^{\mu}}(\vec{k})=\sum_{n\in\alpha}\frac{\phi_n(\vec{k},\vec{p}_n)e_n}{(2\pi)^{3/2}\sqrt{2\omega}}\left(\frac{p_n^{\mu}}{p_n\cdot k}+\bar{k}^{\mu}\right)$$

$\phi_n(\vec{k},\vec{p}_n)$: arbitrary

if Conv cond satisfy

$\bar{k}^{\mu}$: $\bar{k}\cdot k=-1$

FK claimed that $|\Psi_{\alpha}^{\mathrm{KF}}\rangle\rangle=D^{\dagger}_{f_{\alpha}^{\mathrm{KF}}}|\Phi_{\alpha}\rangle$ is equiv to Chung dress

IR finite

However, their discussion has a flaw

Gauge Invariance, Convergence Cond.

In modern view, we should consider **BRS invariance**

Sym between fields and ghosts

Conventional assumption: **asympt. states are free states**

→ **BRS condition is equivalent to free GB condition**

Is it true with long-range interaction? 平井・杉下 ’19

No : $\lim_{t\to\infty}\left[k^{\mu}a_{\mu}(\vec{k},\tilde{h}) + \underline{\hat{\varrho}(t,\vec{k})}\right]|\Psi_{\alpha}^{\rm as}\rangle\rangle = 0$

Correction term with charge density

→ **unphysical modes should dress asympt. states**

Convergence condition $\int d^3k\left|f_{\alpha}(t) - f_{\alpha}^{\rm KF}\right|^2 < \infty$

- **No restriction for IR :ambiguity remains in predictions**
- **Seems it depends on t : Is it OK to take limit naively ?**

Other Dressed Staes Formalisms

Most DF mainly based on KF as is

→ **These DF can not resolve issues in DF of KF**

☑平井・杉下 ’21： **BRS & large charge**

→ **Correct DF of KF in Gauge sym & IR finiteness**

- Functional dof remains in observables
- Nature of large charges remain unclear

$H_0 \to H_{\mathrm{as}} \;/\; H \to H_{\mathrm{as}}$

☑Hannesdottir-Schwartz ’20：**Divide to hard/soft regions**

→ **DF with predictability based on Dollard formalism**

- Intricate calculation due to numerous Feynman diagrams
- No implications to IR triangle relations

Table of contents (for this talk)

散乱問題の定式化のアイディア

関数自由度が予言能力を損ねる

時空と相互作用のみで時間発展の議論は閉じないか？

赤外三角関係を満足には反映しない

大電荷(漸近対称性)の意味を明らかにする定式化はあるか？

メモリー効果の検証可能な定量的予言は得られるか？

→ 時空に超曲面を導入して漸近相互作用を議論する

明白な共変ゲージの理論： 平井・杉下のドレス状態

→ 非物理的自由度は何らかの役割を持つ $\lim_{t\to\infty}\left[k^{\mu}a_{\mu}(\vec{k},\tilde{h})+\hat{\varrho}(t,\vec{k})\right]|\Psi_{\alpha}^{\rm as}\rangle\rangle=0$

ゴーストを初めから導入しない理論ではどうなる？

→ Coulombゲージの正準形式でも議論してみる

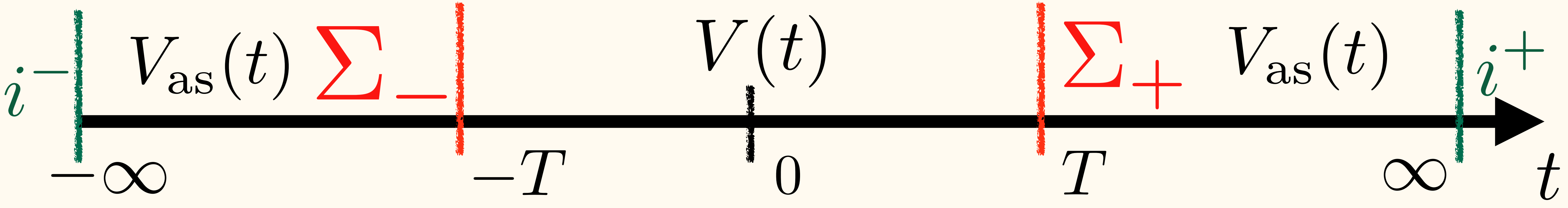
散乱問題の設定

検出機の静止系を用意し十分大きな時間スケール T を導入

$1/T \equiv \lambda_T \ll \mathcal{E}_{\mathrm{sys}}$ 系の適当なエネルギースケール e.g. 質量

↔ **時刻 $t = \pm T$ の時間一定超曲面 $\Sigma_\pm$ を定める**

遷移：入射状態 $|\Psi^-_\alpha\rangle$ **on** $\Sigma_- \longrightarrow$ **放射状態** $|\Psi^+_\beta\rangle$ **on** Σ_+



H **の理論と** H_{as} **の理論を繋ぐ** Cf. Møller op $\Omega_\pm H = H_0 \Omega_\pm$

$i^\pm$ の仮想的粒子状態から V_{as} **を通して漸近状態を構成**

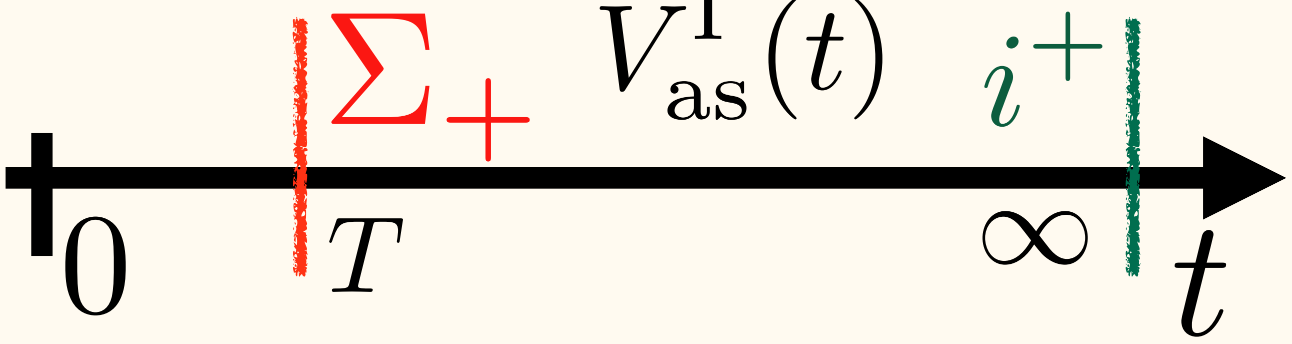
$\lim_{|t|\to\infty} V^{\mathrm{I}}(t) = 0$ は一般に成り立つ

漸近相互作用の定義

漸近的ダイナミクス：λ_T の摂動論で扱う

→ $\lambda_T/\mathcal{E}_{\rm sys}$ **の次数を定める**

ここではLOまで取り入れる。



$$V^{\rm I}(t) \ni \int d^3k d^3p \hat{\mathcal{A}}(\vec{k},\vec{p}) \mathrm{e}^{i\mathcal{E}(\vec{k},\vec{p})t}$$

$V^{\rm I}_{\rm as}(t)$: $\mathcal{E}(\vec{k},\vec{p}) \le \lambda_T$ となる部分のうち $\lambda_T/\mathcal{E}_{\rm sys}$ の主要項

→ $V^{\rm I}_{\rm as}(t) \underline{\overset{\rm L}{=}} V^{\rm I}(t)\ ,\ |t| \ge T$

主要項まで等しいという記号

$\lambda_T/\mathcal{E}_{\rm sys}$ の準主要項以降は無視する

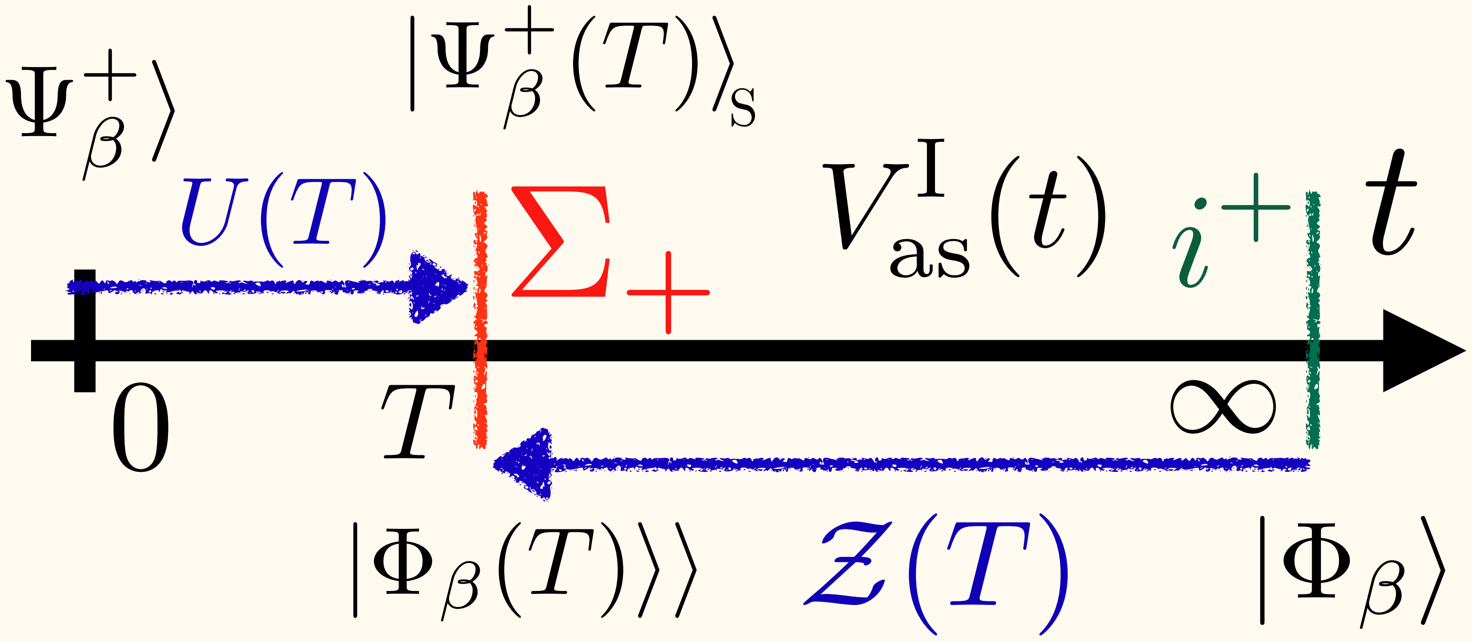
時間スケールの摂動の次数を合わせる

漸近状態と漸近S行列

$$|\Psi_\beta^+\rangle \equiv \Omega^\dagger(T)\Omega(T)\Omega_+^\dagger\Omega_+\,|\Psi_\beta^+\rangle$$

$$\simeq \Omega^\dagger(T)\mathcal{Z}(T)\,|\Phi_\beta\rangle$$

$$\mathcal{Z}(T) = \mathcal{T}\exp\left[i\int_T^\infty d\tau V_{\rm as}^{\rm I}(\tau)\right]$$



$$|\Psi_\beta^+\rangle \simeq \Omega^\dagger(T)\underline{|\Phi_\beta(T)\rangle\rangle}$$ **放射漸近状態**

漸近S行列 $\underline{S_{\beta\alpha}^{\rm as}(T)} = \langle\langle\Phi_\beta(T)|\underline{\mathcal{S}_{\rm as}(T)}|\Phi_\alpha(-T)\rangle\rangle$

$-T \to T$ の遷移を記述　$\Omega(T)\Omega^\dagger(-T) = \mathcal{T}\exp\left[-i\int_{-T}^{T} d\tau V^{\rm I}(\tau)\right]$

$$S_{\beta\alpha}^{\rm as}(T) \to S_{\beta\alpha} = \langle\Psi_\beta^+|\Psi_\alpha^-\rangle,\ \lambda_T/\mathcal{E}_{\rm sys} \to 0$$

Table of contents (for this talk)

QED -漸近相互作用

Coulombゲージの正準形式

ゴーストなどの非物理的自由度なく構築可能

$$\epsilon^0(k,h)=0$$

$$\vec{k}\cdot\vec{\epsilon}(k,h)=0$$

しわ寄せ $\left[a_\mu(\vec{k},h),a_\nu(\vec{k}',h')\right]=\left[\underline{\eta_{\mu\nu}}-\underline{Q_{\mu\nu}(\vec{k})}\right]\delta_{hh'}\delta^3(\vec{k}-\vec{k}')$

共変　非共変　$Q_{\mu\nu}k^\nu=k_\mu$

QED相互作用 $V^I(t)=ie\int d^3x a_\mu(x)\bar{\psi}(x)\gamma^\mu\psi(x)\underline{+V^{\mathrm{I}}_{\mathrm{C}}(t)}$

$V^{\mathrm{I}}_{\mathrm{C}}(t)$ **非共変項を打ち消すCoulomb項**

→ **伝搬関数は共変と考えてよい**

Cf. Heisenberg描像で
共変なラグランジアンから
このような正準形式に移れる
See Weinberg text

漸近相互作用 $V^{\mathrm{I}}_{\mathrm{as}}(t)=\int d^3x a^{\mathrm{soft}}_\mu(x)\underline{j^\mu_{\mathrm{cl}}(x)}$

Current as
a particle flow

$a^{\mathrm{soft}}_\mu(x)$: $\underline{\omega\leq 1/T}$ **の光子** 軟光子

Cf. KF : $\omega\sim 0$

QED-漸近S行列

$$S^{\rm as}_{\beta\alpha}(T) = \underline{\langle f_\beta(T)|f_\alpha(-T)\rangle}\langle\Phi_\beta|\mathcal{S}(T,-T)|\Phi_\alpha\rangle$$

軟光子のコヒーレント状態の内積

$$f_\alpha(\vec{k},h;t) := \sum_{n\in\alpha} \frac{e_n}{(2\pi)^{3/2}\sqrt{2\omega}} \frac{p_n\cdot\epsilon^*(k,h)}{k\cdot p_n} \mathrm{e}^{-ik\cdot v_n t}$$

$$\overset{\rm L}{=} \underline{S^{\rm D}_{\beta\alpha}(\lambda_T)}$$

赤外切断 λ_T のDyson S行列

発散なし

Fock空間 $\mathcal{H}_{\rm F}$ ：軟光子は含まれないものとして再定義

Frame毎に粒子概念を再定義

$$\longrightarrow \int d\beta\, S^{\rm as^*}_{\beta\gamma}(T) S^{\rm as}_{\beta\alpha}(T) \overset{\rm L}{=} \int d\beta\, S^{\rm D^*}_{\beta\gamma}(\lambda_T) S^{\rm D}_{\beta\alpha}(\lambda_T)$$

$$= \int_{\rm hard} d\beta \,\langle\Phi^{\rm hard}_\gamma|\mathcal{S}_D^\dagger|\Phi^{\rm hard}_\beta\rangle\,\langle\Phi^{\rm hard}_\beta|\mathcal{S}_D|\Phi^{\rm hard}_\alpha\rangle = \delta(\alpha-\gamma)$$

Unitarity

QED-遷移確率

検出器の設定 を与える

$\omega < \underline{\Lambda_D}$ のエネルギーを持つ光子は検出できない

1光子検出限界

$\sum_n \omega_n < \underline{\Lambda_E}$ のエネルギーを持つ光子群の放出は気付けない

エネルギー分解能

状況1: 軟光子が検出できない場合 $\Lambda_D \geq 1/T$

→ $\alpha \to \beta$ と**区別できない過程を全て考慮**する

T **非依存**

$$\Gamma^{\rm as}_{\beta\alpha}(\Lambda_D, \Lambda_E) = \mathcal{F}(\Lambda_D/\Lambda_E, A_{\beta\alpha})(\Lambda_D T)^{A_{\beta\alpha}}\Gamma^D_{\beta\alpha}(1/T) = \Gamma^{\rm obs}_{\beta\alpha}\underline{(\Lambda_{\rm D})}$$

従来の結果(inclusive)を再現

状況2 : 軟光子が検出できる場合 $\Lambda_D < 1/T$

→ $\alpha \to \beta$ と確定

$$\Gamma^{\rm as}_{\beta\alpha}\underline{(T)} = (1/\Lambda_D T)^{A_{\beta\alpha}}\,\Gamma^D_{\beta\alpha}(\Lambda_D)$$

Λ_D **非依存**

QED-ゲージ対称性と漸近対称性

漸近S行列 $S^{\mathrm{as}}_{\beta\alpha}(T) = \langle f_\beta(T)|f_\alpha(-T)\rangle\langle\Phi_\beta|\mathcal{S}(T,-T)|\Phi_\alpha\rangle$

理論のゲージ対称性 ：硬い部分は通常のWT identity

軟部分 $\underline{\epsilon_\mu(k,h) \to \epsilon_\mu(k,h) + \varepsilon^a(k,h)k^\mu}$ $\quad a = +/-$ 放射/入射

漸近状態のゲージ変換　　ゲージ不変条件

$$S^{\mathrm{as}}_{\beta\alpha}(T) \to \Delta(\varepsilon^+,\varepsilon^-)S^{\mathrm{as}}_{\beta\alpha}(T) \overset{\mathrm{L}}{=} \begin{cases} \underline{S^{\mathrm{as}}_{\beta\alpha}(T), \quad \varepsilon^+(k,h) \overset{\mathrm{L}}{=} \varepsilon^-(k,h)} \\ 0, \quad \text{Otherwise} \end{cases}$$

漸近ゲージ変換 $\epsilon_\mu(k,h) \to \epsilon_\mu(k,h) + \omega^{-1}\xi(\hat{\vec{k}},h)k_\mu$

$$|\Psi_\alpha(\pm T)\rangle\rangle \to |\Psi^\xi_\alpha(\pm T)\rangle\rangle \equiv |\Phi_\alpha\rangle \otimes |f^\xi_\alpha(\pm T)\rangle \in \mathcal{H}^\pm_{\mathrm{as},\xi}$$

漸近対称性 $\underline{\mathcal{H}^-_{\mathrm{as},\xi} \to \mathcal{H}^+_{\mathrm{as},\eta}, \xi \neq \eta}$ **の遷移を禁止**

超選択則 $\mathcal{H}^\pm = \oplus_\xi \mathcal{H}^\pm_{\mathrm{as},\xi}$

厳密には直交性の計算から
$\alpha \to \alpha$ **も議論可能**

QED-漸近対称性とLorentz不変性/共変性

時間の箱の導入は一見Lorentz対称性を損ねそう

3軸へLorentz boost $x^\mu \to \Lambda^\mu{}_\nu\, x^\nu$ (回転対称性は明らか)

$$a^{\rm s}_\mu(x) \to \Lambda_\mu{}^\nu a^{{\rm s}_\Lambda}_\nu(\Lambda x) + \partial_\mu \lambda^{\xi^\Lambda}(x) \qquad \xi^\Lambda(\vec{k}_\Lambda, h) = \Lambda_\mu{}^0 \epsilon^\mu(\vec{k}_\Lambda, h)$$

$$\lambda^{\xi^\Lambda}(x) := \int_{\rm S} \frac{d^3k}{(2\pi)^{3/2}\sqrt{2\omega}} \sum_h \left[\underline{i\frac{\xi^\Lambda(\vec{k}_\Lambda, h)}{\omega}} {\rm e}^{ik\cdot x} a(\vec{k}_\Lambda, h) + ({\rm H.c.}) \right]$$

ブーストに付随する漸近対称性変換

- **Lorentz対称性の破れは漸近対称性によって状態では回復する**
- **漸近対称性はframeの情報を少なくとも持つ**

Cf. $\mathcal{H}^\pm = \oplus_\xi \mathcal{H}^\pm_{{\rm as},\xi}$ 各frameのHilbert空間を束ねている

QED-メモリー効果

メモリー効果 ：運動の痕跡が境界のゲージ場の変化として残る

$$\langle \delta \vec{a}(\vec{x}) \rangle := \langle f_\beta(T) | \vec{a}(\vec{x}, T) | f_\beta(T) \rangle - \langle f_\alpha(-T) | \vec{a}(\vec{x}, -T) | f_\alpha(-T) \rangle$$

$$\langle \delta \vec{a}(\vec{0}) \rangle = -\sum_n \frac{e_n \eta_n G(v_n)}{8\pi^2 T} \hat{\vec{v}}_n \qquad \eta_n = \begin{cases} -1 & \text{for } n \in \alpha \\ 1 & \text{for } n \in \beta \end{cases}$$

これは主要項の寄与　T有限なら無視できない

$\vec{x} = \pm \vec{x}_0$ での位相差のメモリー？

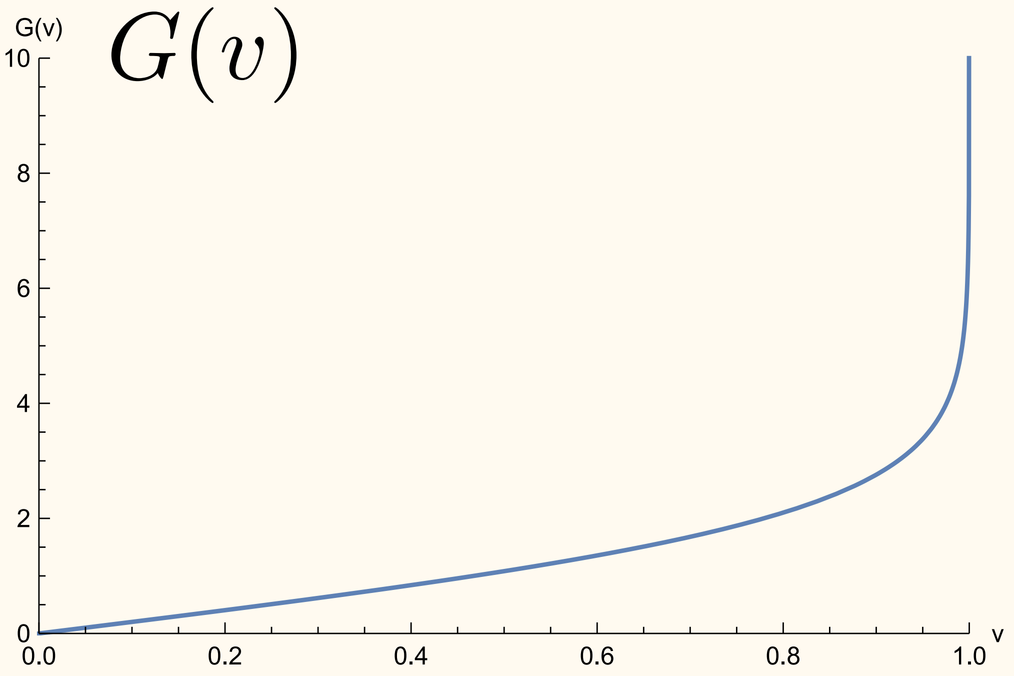
→ 散乱を見ずに散乱の情報が得られる？

電場・磁場 $\langle \delta \vec{E}(\vec{x}) \rangle$, $\langle \delta \vec{B}(\vec{x}) \rangle = 0$

Cf.

古典的Kickモデルとは設定・想定が異なる

e.g. Hamada, Sugishita ’18



$G(v)$

$G(v) \to -\log(1-v), \; v \to 1$

Table of contents (for this talk)

Summary-1

散乱行列は
現象論
形式論
の両側面に重要な役割を担う

しかし慣習的なDyson S行列は赤外発散により存在しない！

→ ドレス状態形式が必要である

漸近状態は自由粒子状態ではなくドレス状態

赤外発散は
漸近対称性
メモリー効果
と赤外三角関係で関連する

軟粒子定理はドレス状態の1次近似といえる

赤外三角関係は情報損失問題に迫る糸口としても注目される

ドレス状態形式なしでは平坦時空でもユニタリ性は非自明

→ ドレス状態形式が必要である！

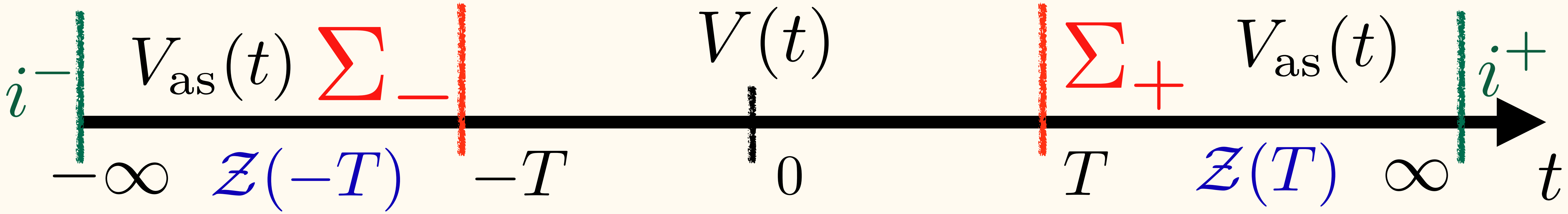
Summary-2

漸近相互作用の存在を考慮した散乱理論を提案した

時間パラメータ T を導入

→ 空間的超曲面 $\Sigma_\pm$ によって散乱問題を規定

理論の相互作用を T を用いて摂動的に解析し $V_{\rm as}$ を定義



H の理論と $H_{\rm as}$ の理論を繋ぐ演算子の構成を目指す

$|\Psi_\alpha^\pm\rangle \simeq |\Phi_\alpha(\pm T)\rangle\rangle = \mathscr{Z}(\pm T)\,|\Phi_\alpha\rangle$ **漸近放射/入射状態**

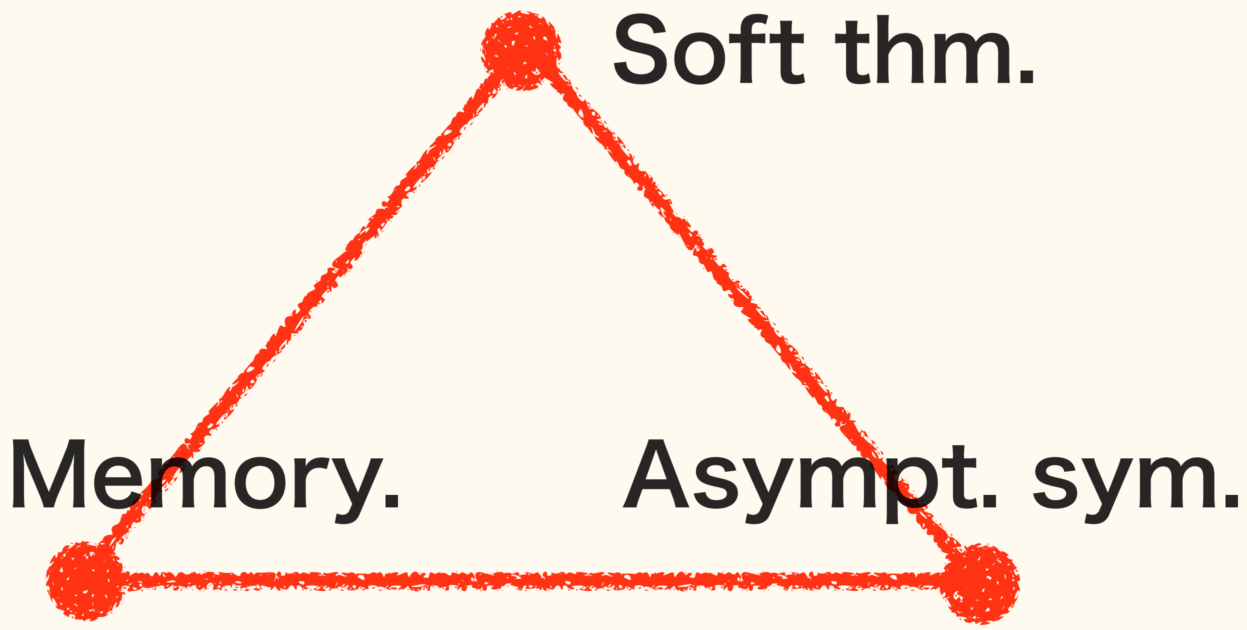
漸近S行列 $S^{\rm as}_{\beta\alpha}(T) = \langle\langle\Phi_\beta(T)|\mathcal{S}_{\rm as}(T)|\Phi_\alpha(-T)\rangle\rangle \overset{\rm L}{=} S_{\beta\alpha}$

Summary-3

散乱理論をCoulombゲージのQEDに応用した

$$V^{\mathrm{I}}_{\mathrm{as}}(t) = \int d^3x a^{\mathrm{soft}}_\mu(x) j^\mu_{\mathrm{cl}}(x)$$

$a^{\mathrm{soft}}_\mu(x)$: $\omega \leq 1/T$ の光子



- 非物理的自由度は現れない
- 散乱行列は発散を持たない
- 従来の結果を含む遷移確率の予言が可能
- 漸近対称性によってLorentz対称性は回復
- ベクトルポテンシャルの期待値変化が計算可能

赤外三角関係を反映したドレス状態形式である！

Prospects

QED

メモリー効果の定量的予言

Soft dofからの情報の読み出し
実験・観測への窓口

ドレスの重ね合わせ状態

量子効果を探る Cf. dress code ’22

NLOの議論

NLOドレスの存在とその帰結

散乱振幅の解析

DispersionなどへのIRの寄与

重力

Massless スピン2

軟重力子の果たす役割
非線形レベルではどうか

異なる背景時空

赤外三角は普遍的
-dS, 高次元など

時空と量子情報

ドレス状態は普遍的？

その他

Soft Collinear Effective Theory　共線発散のドレス？

Scalar QED　非線形重力のヒント？

などなど・・・