Relations among Topological Solitons

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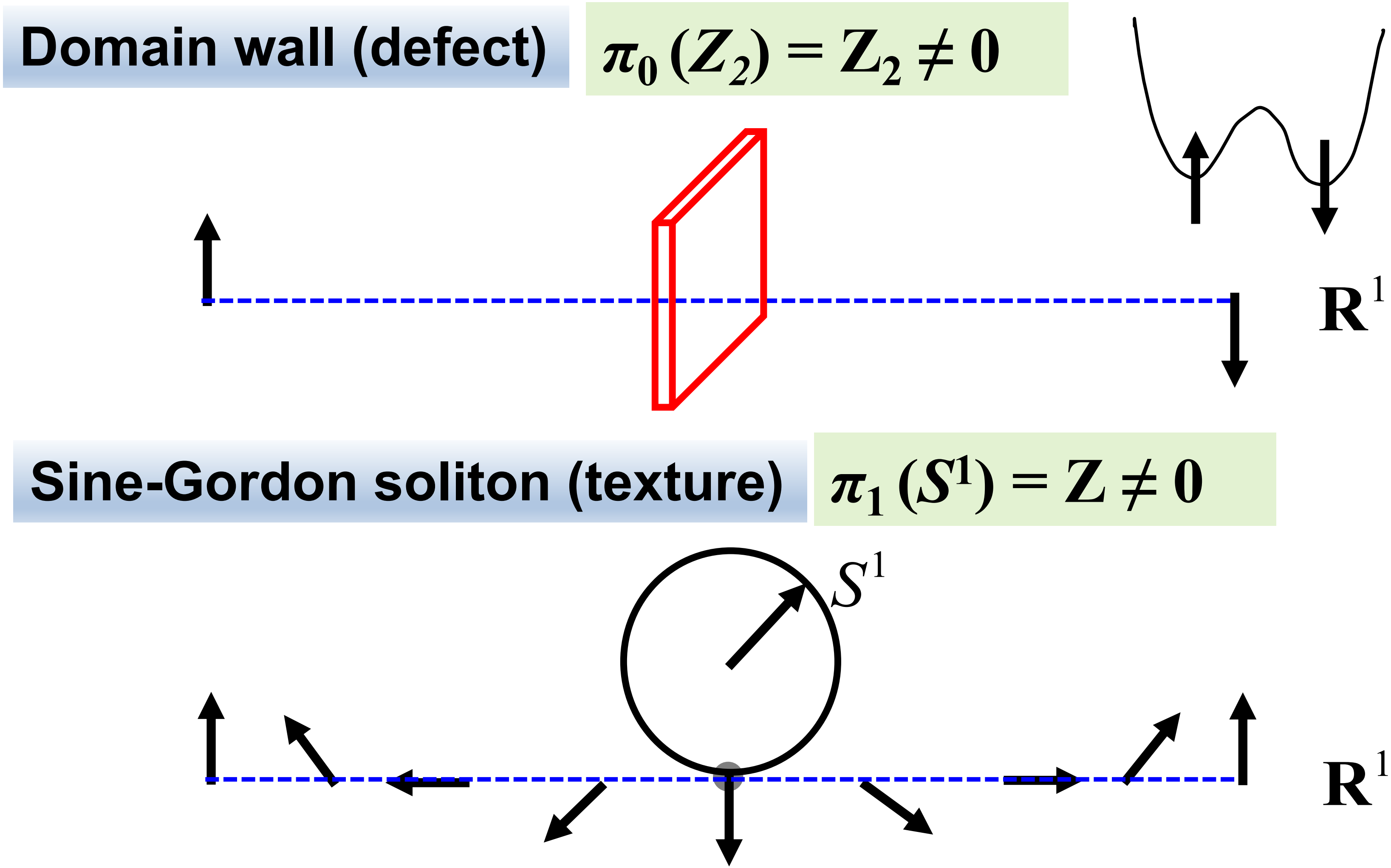
[e-Print: 2202.03929 [hep-th]]

Classification of topological solitons: 3 types

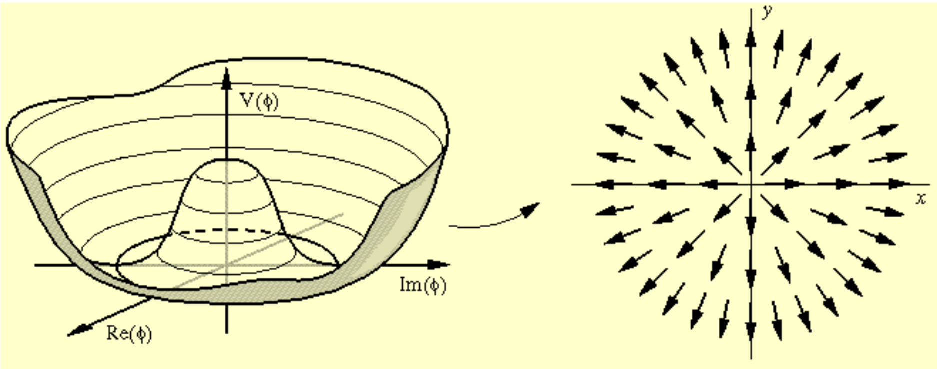
d	Defects		Textures		Gauge Structure	
1	Domain wall, Kink	π_0	Sine-Gordon soliton	π_1		
2	Vortex, Cosmic string	π_1	Lumps, Baby Skyrmion	π_2		
3	Monopole	π_2	Skyrmion, Hopfion	π_3		
4					YM instanton	π_3
	$\partial R^d \cong S^{d-1} \to G/H$		$R^d + \{\infty\} = S^d \to G/H$		$\partial R^d \cong S^{d-1} \to G$	
	$\pi_{d\text{-}1}(G/H) \neq 0$		$\pi_d(G/H) \neq 0$		$\pi_{d\text{-}1}(G) \neq 0$	

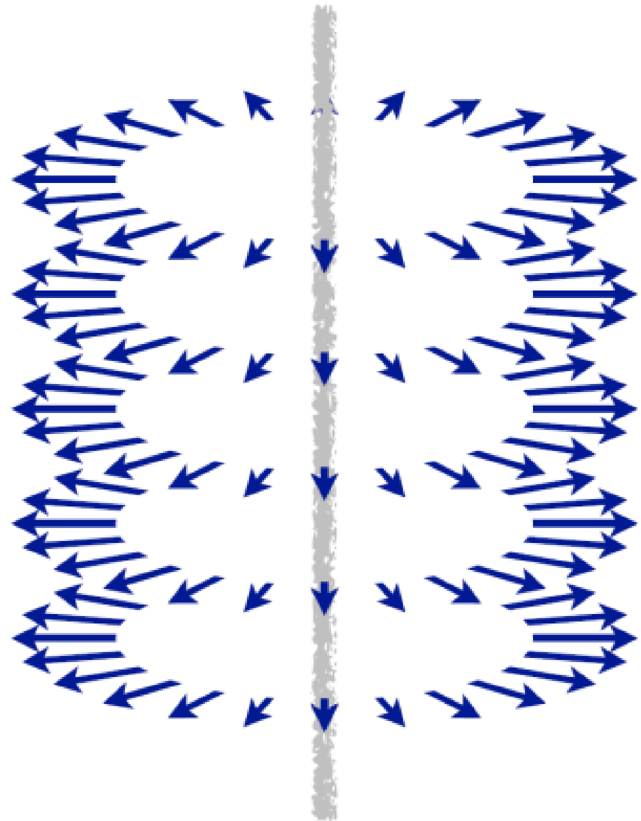
d: codimensions (in which solitons are particles, or on which solitons depend)


Domain wall (defect)
$\pi_0(Z_2) = Z_2 \neq 0$
R^1
Sine-Gordon soliton (texture)
$\pi_1(S^1) = Z \neq 0$
S^1
R^1

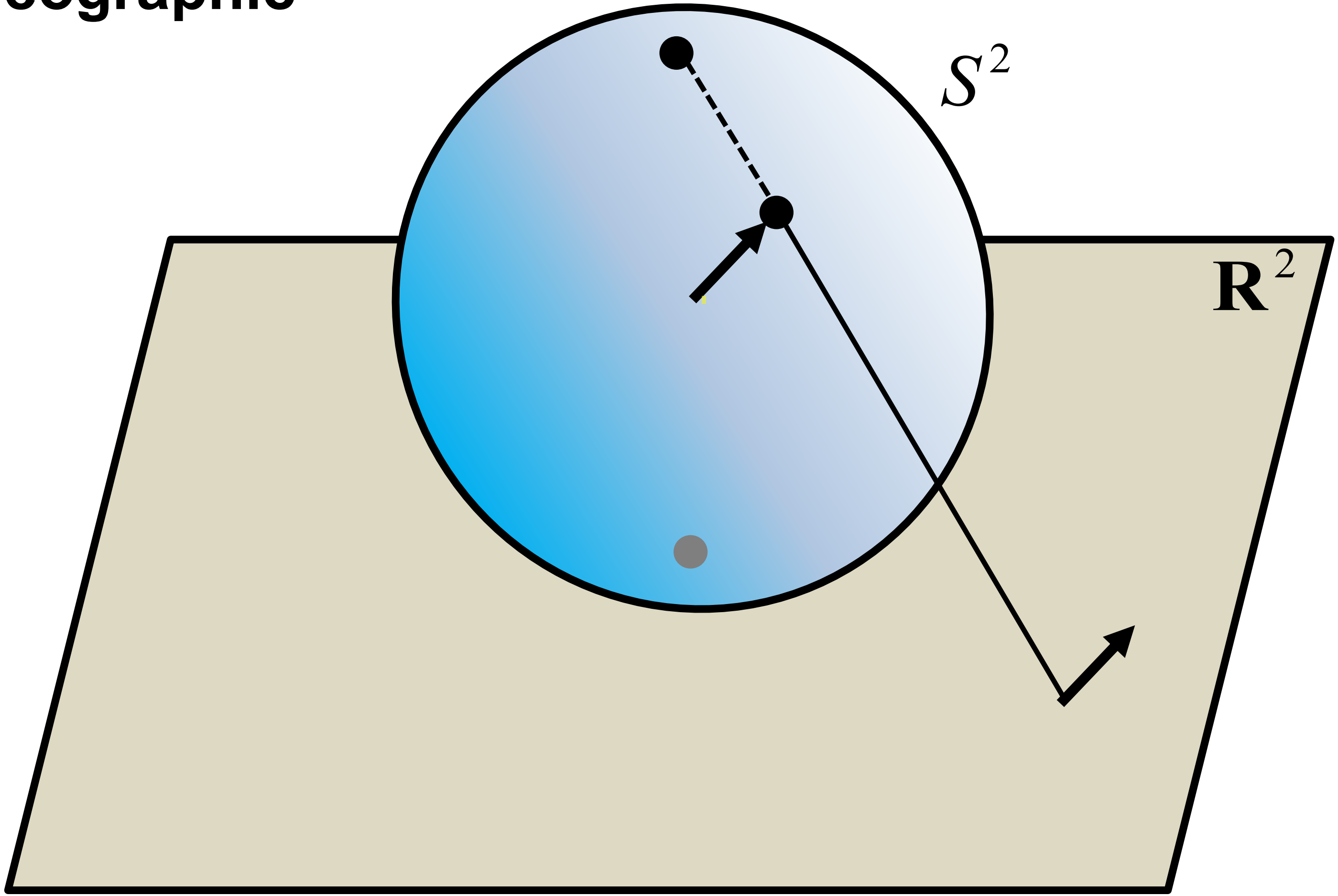
Vortex, cosmic string (defect)

$\pi_1(S^1) = Z \neq 0$



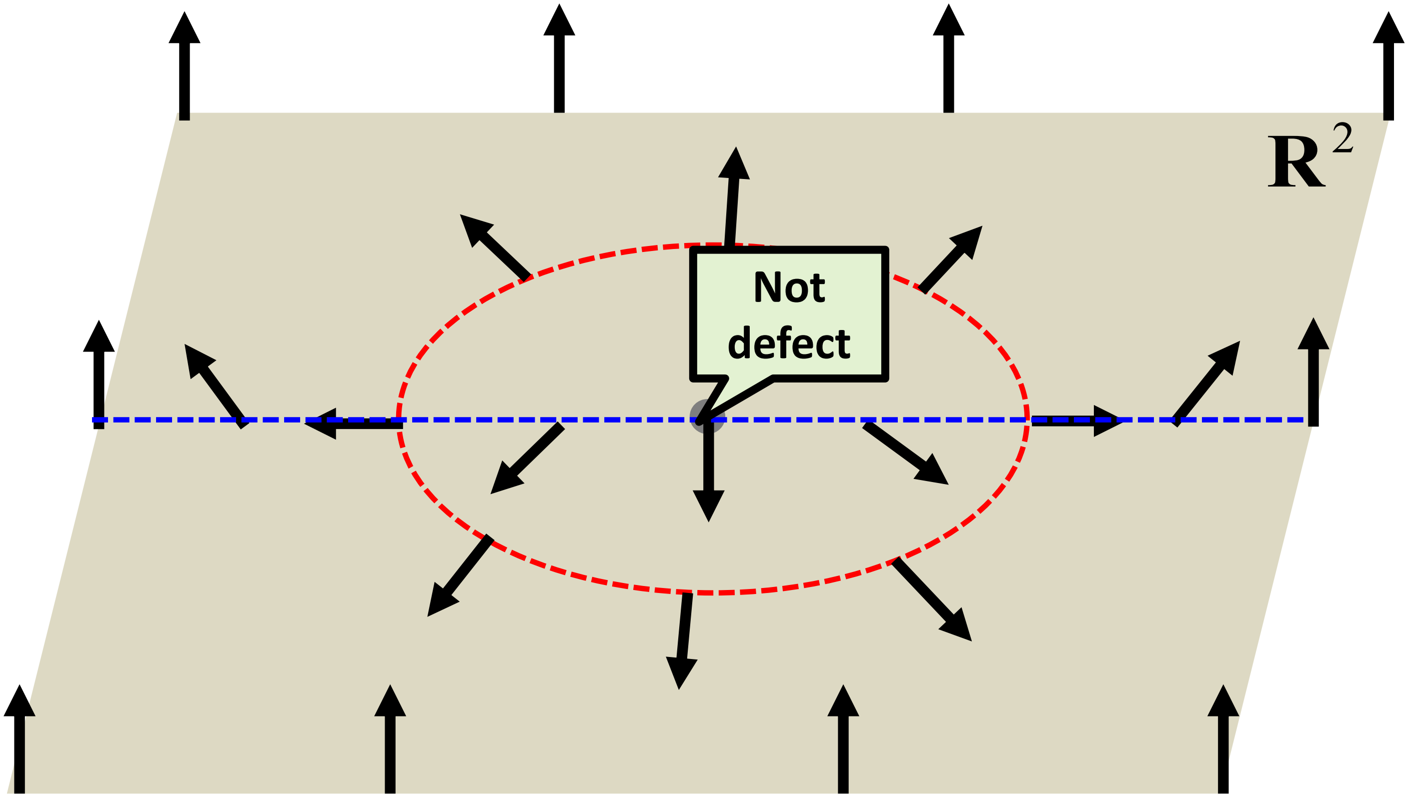


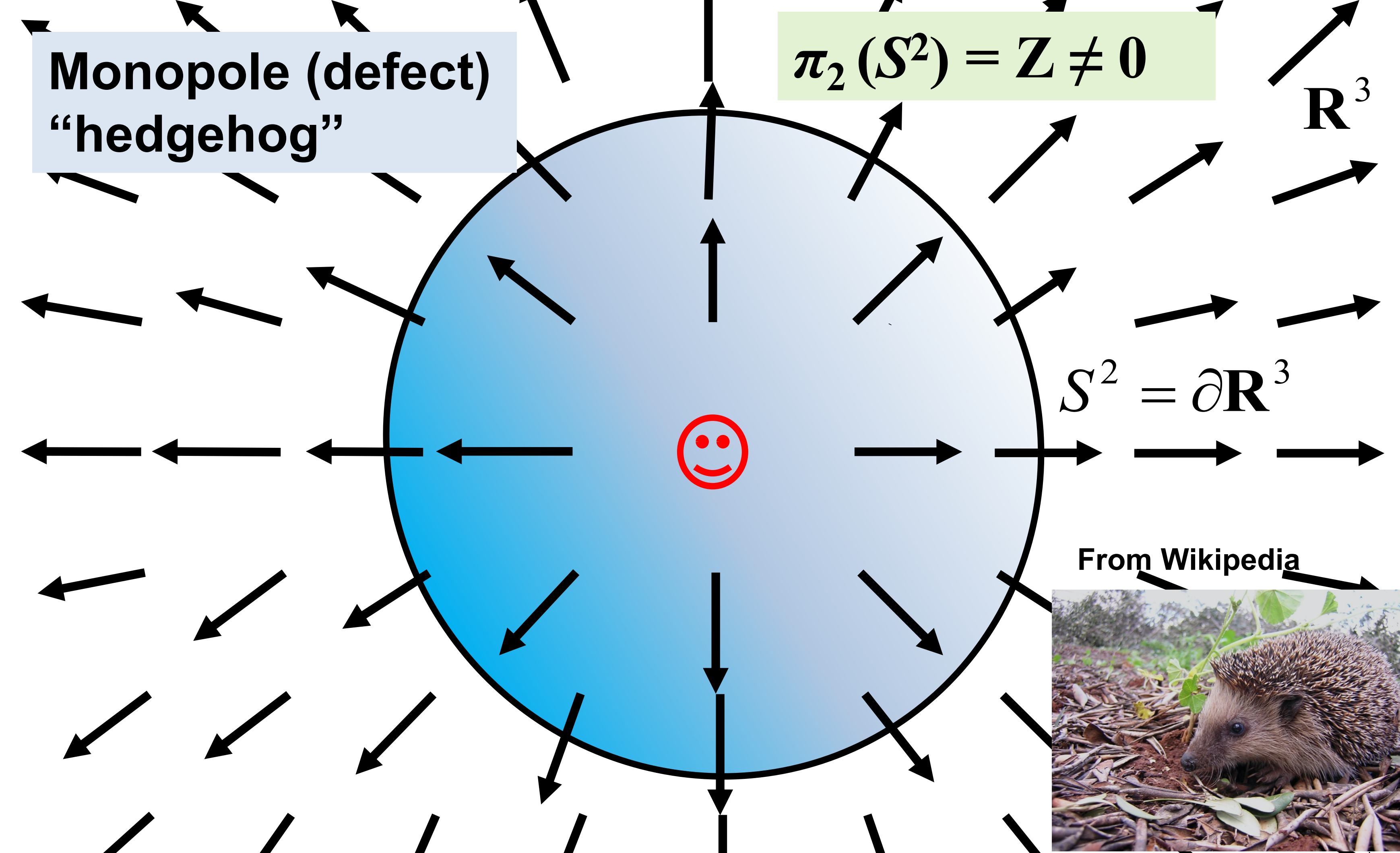
stereographic



Lump, baby Skyrmion (texture)

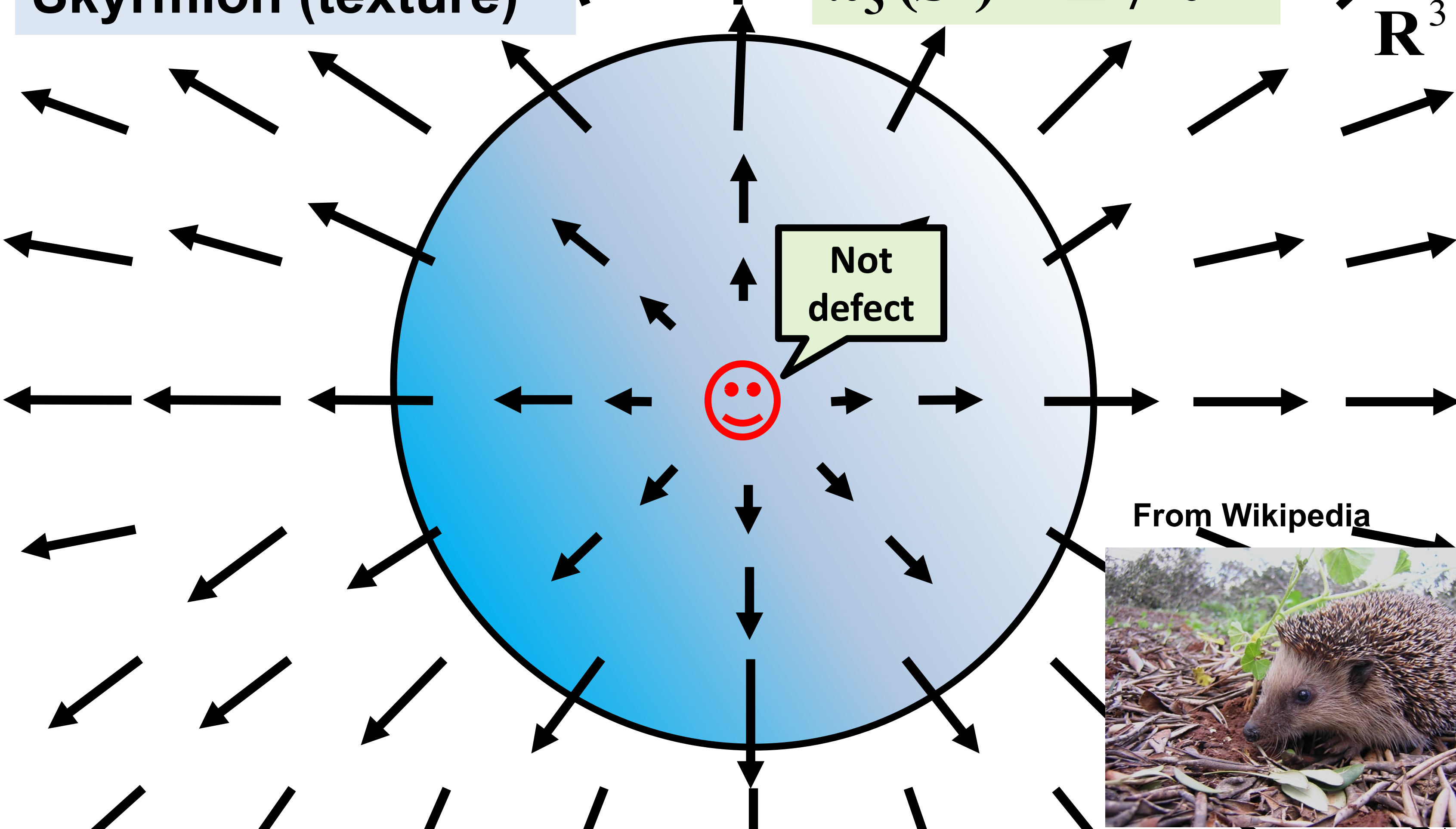
$\pi_2(S^2) = \mathbb{Z} \neq 0$




Monopole (defect)
“hedgehog”
$\pi_2(S^2) = Z \neq 0$
$\mathbf{R}^3$
$S^2 = \partial \mathbf{R}^3$
From Wikipedia

Skyrmion (texture)

$\pi_3(S^3) = \mathbb{Z} \neq 0$



From Wikipedia

Topological solitons appear in various fields, playing significant roles.

(1) **Quantum field theory & string theory**:

Non-perturbative effects, resurgence, supersymmetry, higher-form symmetry, ….

(2) **Particle physics**: BSM, GUTs, axion, …

(3) **Cosmology**: cosmic string, axion, dark matter …

(4) **Nuclear physics**: nuclear matter, quark matter

(5) **Astrophysics**: neutron stars

(6) **Condensed matter and others**

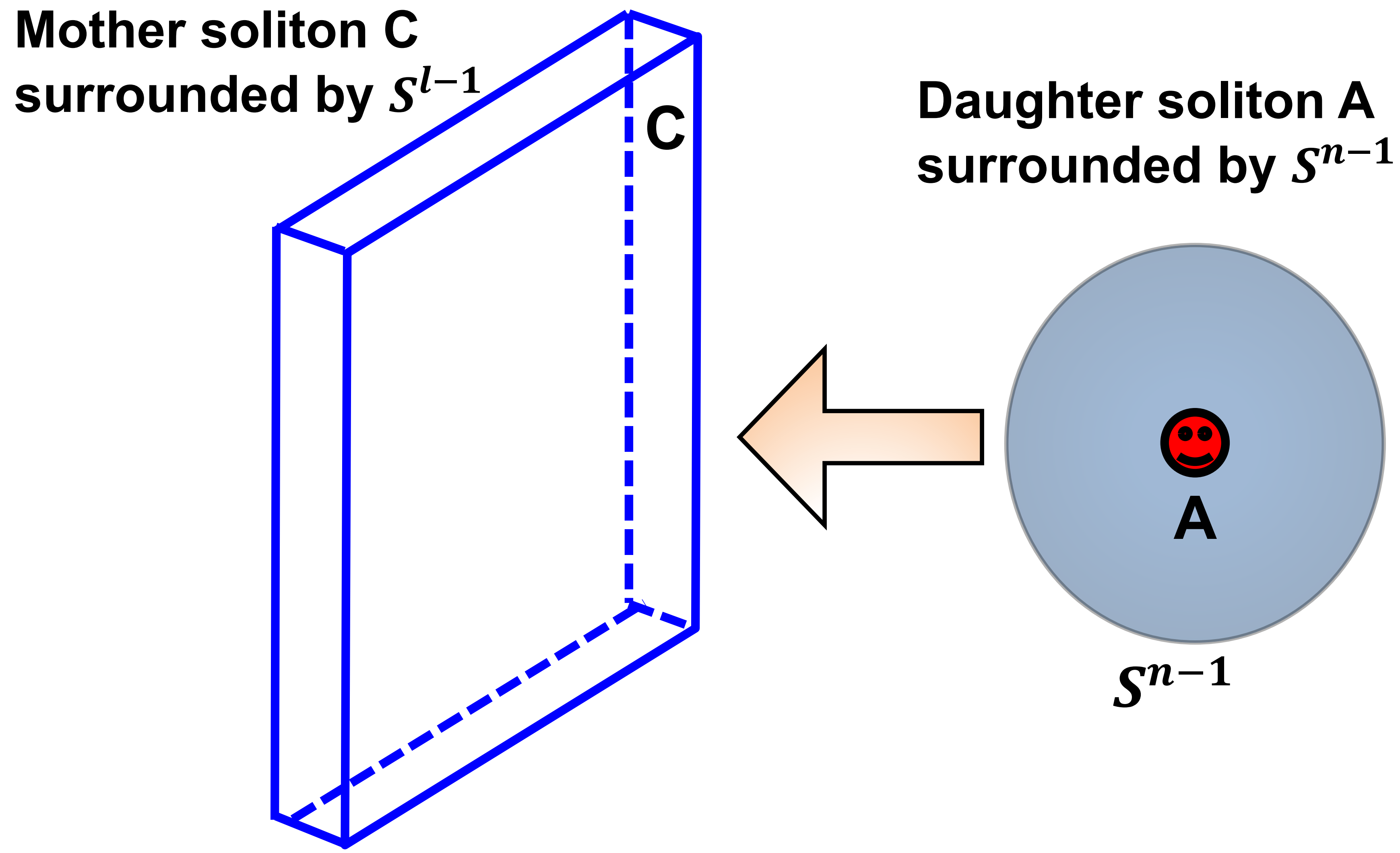
Superconductors, Superfluids, Magnets, Crystals, Nematic liquids, Cold atomic gases, Statistical phys, Optics, Soft matter, Biophysics

(7) **Nonlinear science, Biology, Medicine**

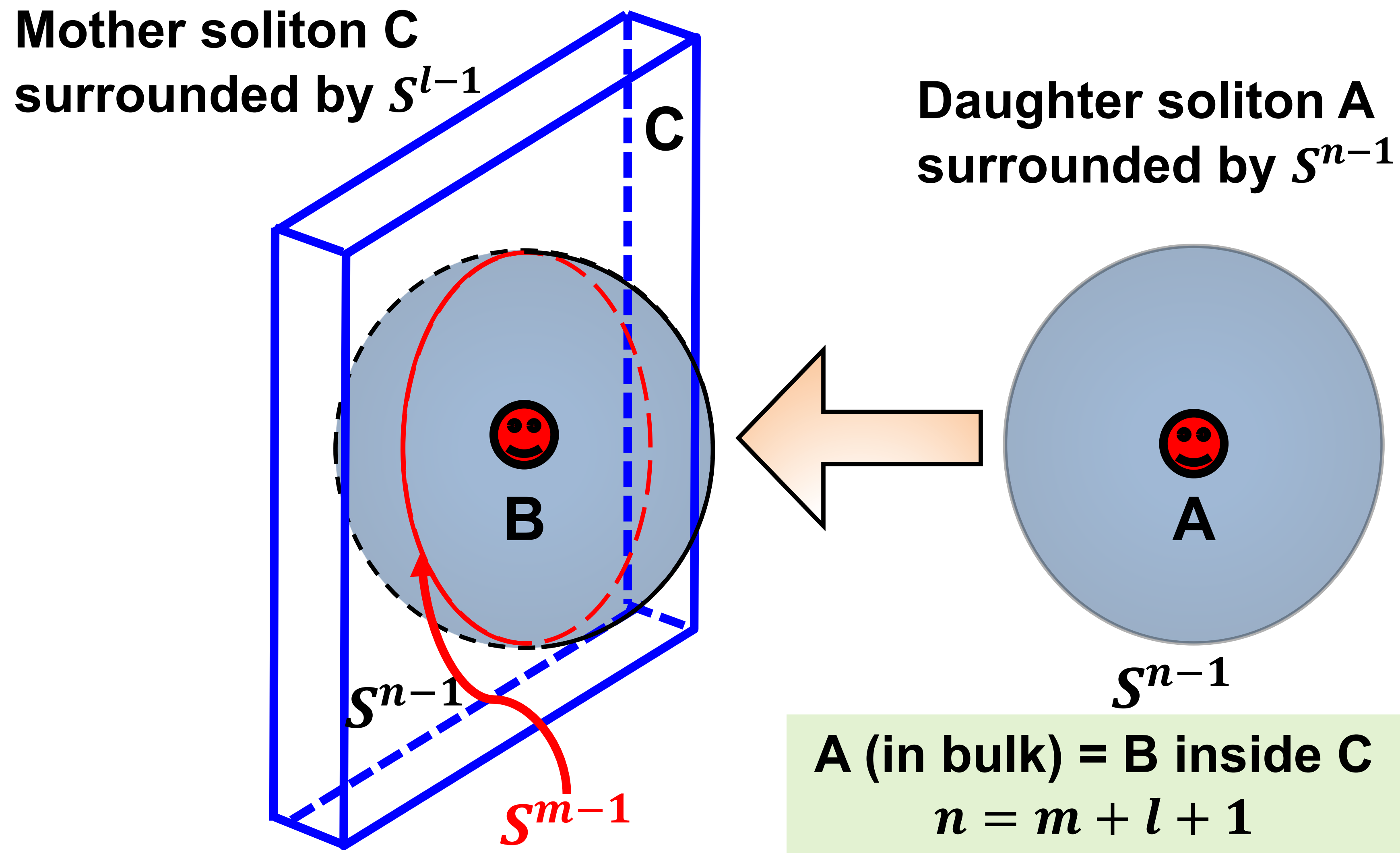
Usually, these solitons were studied separately.

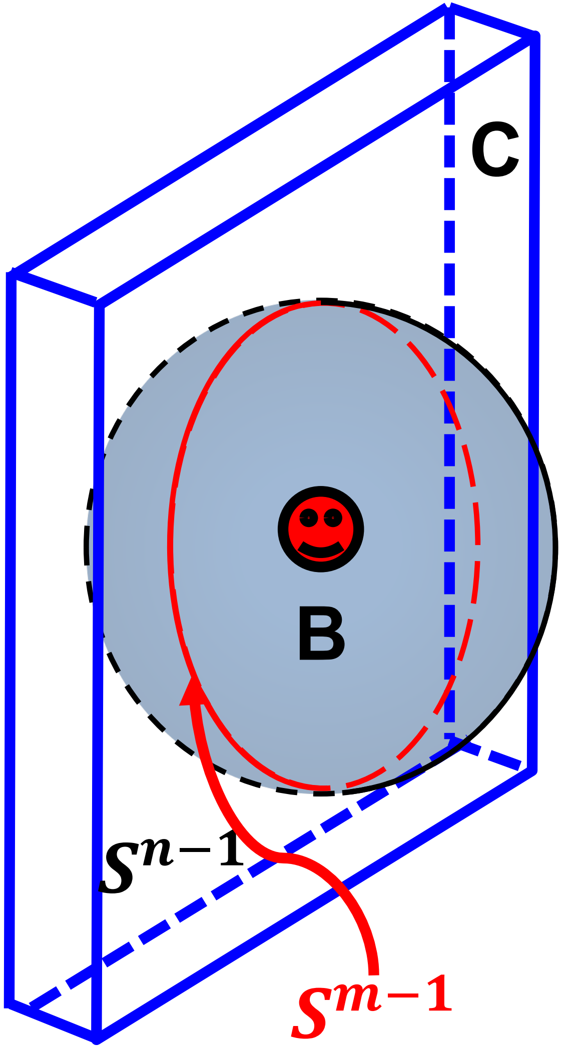
Are there any relations among them?

YES !!



Mother soliton C
surrounded by S^{l-1}
C
Daughter soliton A
surrounded by S^{n-1}
A
S^{n-1}


Mother soliton C
surrounded by S^{l-1}
C
Daughter soliton A
surrounded by S^{n-1}
B
A
S^{n-1}
S^{n-1}
S^{m-1}
A (in bulk) = B inside C
$n = m + l + 1$



1. The mother C usually has moduli *M* (collective coordinates).
2. Low energy effective theory (LEET) of C is a nonlinear σ model with a target space *M*.
3. The daughter B can be realized as a soliton in the LEET.
4. Matching of the topological charges of B in the bulk and LEET.

A (in bulk) = B inside C

$$n = m + l + 1$$

An example of the mother soliton: *non-Abelian* vortex

$U(N)$ Non-Abelian Higgs model

Hanany-Tong('03), Auzzi et.al('03)

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{2g^2} F_{\mu\nu}F^{\mu\nu} - D_\mu H D^\mu H^\dagger - \frac{g^2}{4}\left(c\mathbf{1}_N - HH^\dagger\right)^2 \right]$$

H: $N\times N$ complex scalars $F_{\mu\nu}$: $U(N)$ gauge field

$U(N)_C$ color $\times SU(N)_F$ flavor symmetry

$$H \to g_{\mathrm{C}}(x) H g_{\mathrm{F}}, \qquad F_{\mu\nu} \to g_{\mathrm{C}}(x) F_{\mu\nu} g_{\mathrm{C}}(x)^{-1}$$

$$g_{\mathrm{C}}(x) \in SU(N)_{\mathrm{C}}, \qquad g_{\mathrm{F}} \in SU(N)_{\mathrm{F}}$$

Vacuum $\langle H \rangle = \sqrt{c}\,\mathbf{1}_N$ color-flavor locking (CFL) vac

SSB $U(N)_{\mathrm{C}} \times SU(N)_{\mathrm{F}} \to SU(N)_{\mathrm{C+F}}$ CFL symmetry

$$SU(N)_{\mathrm{C+F}}: \left(1, g, g^\dagger\right) \in U(1)_{\mathrm{D}} \times SU(N)_{\mathrm{C}} \times SU(N)_{\mathrm{F}}$$

Non-Abelian vortex

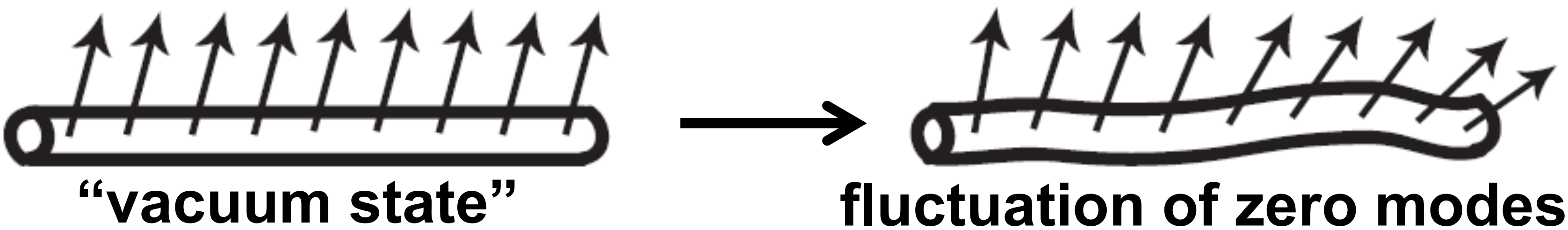
$$H = \begin{pmatrix} f(r)e^{i\theta} & \\ & g(r)\mathbf{1}_{N-1} \end{pmatrix} \qquad F_{12} = \begin{pmatrix} * & \\ & \mathbf{0}_{N-1} \end{pmatrix}$$

$$SU(N)_{\mathrm{C+F}} \to SU(N-1)\times U(1)$$ **SSB around the vortex**

Nambu-Goldstone modes around the vortex = moduli

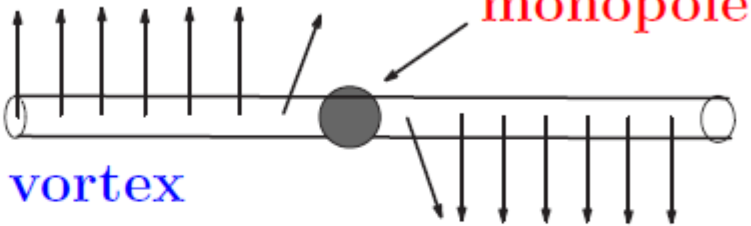
$$\mathbb{C}P^{N-1} \cong \frac{SU(N)_{\mathrm{C+F}}}{SU(N-1)\times U(1)}$$

The LEET is the $\mathbb{C}P^{N-1}$ model

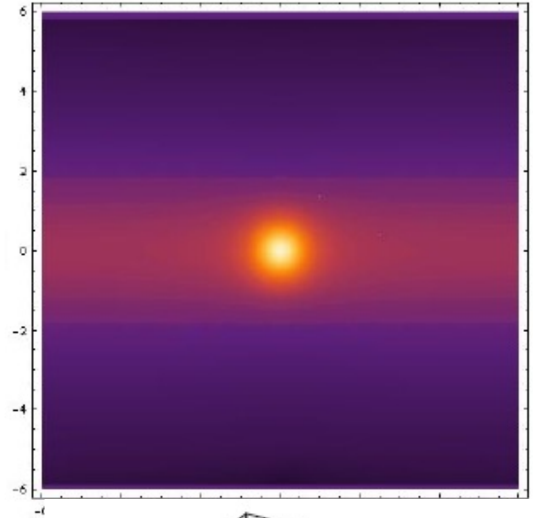


Confined monopole $= \mathbb{C}P^{N-1}$ kink

"Vortex-monopole"



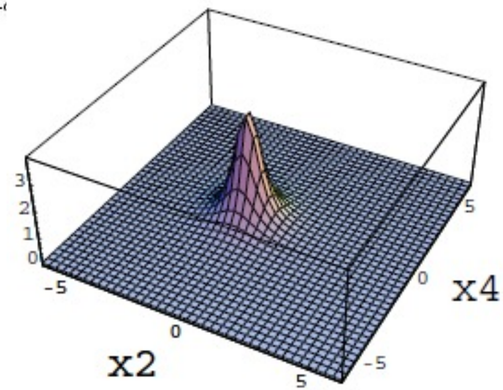
Tong('03), Hanany-Tong, Shifman-Yung ('04),
Non-Abelian monopole: Vinci-MN ('10)



Instanton trapped inside a vortex

$= \mathbb{C}P^{N-1}$ lump **"Vortex-instanton"**

Hanany-Tong,
Eto-Isozumi-MN-Ohashi-Sakai('04)



We deal with instanton particle in Euclidean $\mathbb{R}^4$ in $d = 4 + 1$.

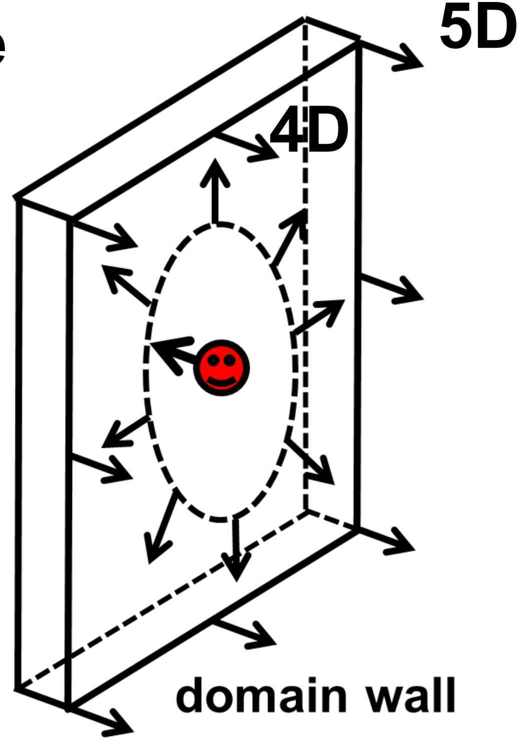
$\Longrightarrow$ Relation between non-perturbative effects in 4D & 2D

Yang-Mills instanton trapped inside a *non-Abelian* domain wall = **Skyrmion**

LEET of the wall = Skyrme model

"Domain wall-instanton"
(We called it "Domain wall-Skyrmion" before)

$\Longrightarrow$ **Physical proof of Atiyah-Manton ansatz (Skyrmion from instanton holonomy)**



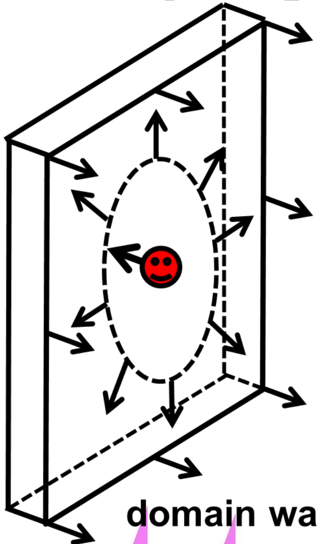
cf. holographic QCD

We deal with instanton particle in Euclidean $\mathbb{R}^4$ in $d = 4 + 1$.

Eto, Ohashi, MN, Tong, ***Phys.Rev.Lett.*** **95 (2005) 252003 [hep-th/0508130 [hep-th]]**

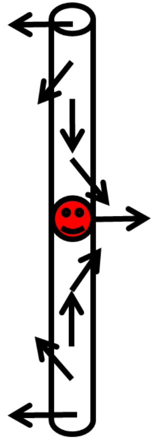
Incarnation of Skyrmions Gudnason & MN ('14- '16)

(1) $m^2(n_1^2+n_2^2+n_3^2)$



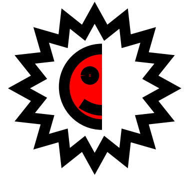
Lump in DW, "Domain wall Skyrmion"

(2) $m^2(n_1^2+n_2^2)$

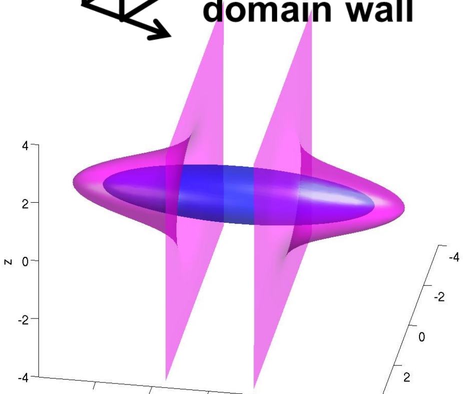


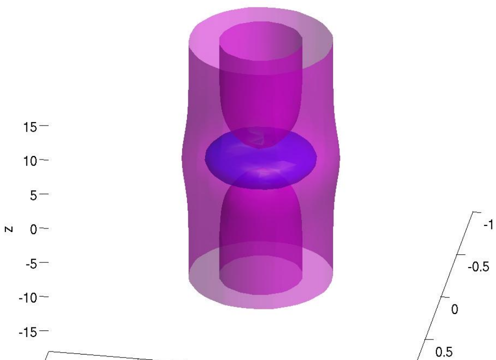
SG soliton in vortex, "Vortex Skyrmion"

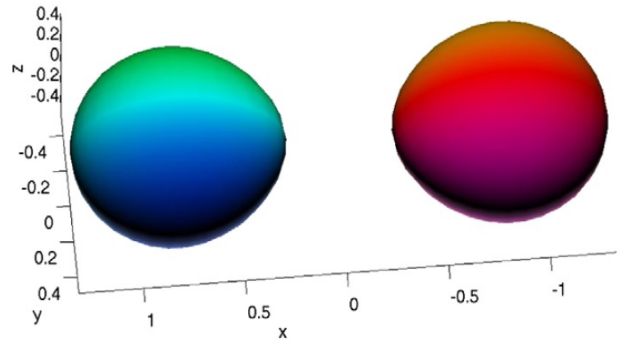
(3) $m^2 n_1^2$



Ising spin in monopole, "Monopole Skyrmion"







Plan of My Talk

§ 1 Introduction

§ 2 Sketch of the results

§ 3 Proof

§ 4 Summary

Plan of My Talk

§ 1 Introduction

§ 2 Sketch of the results

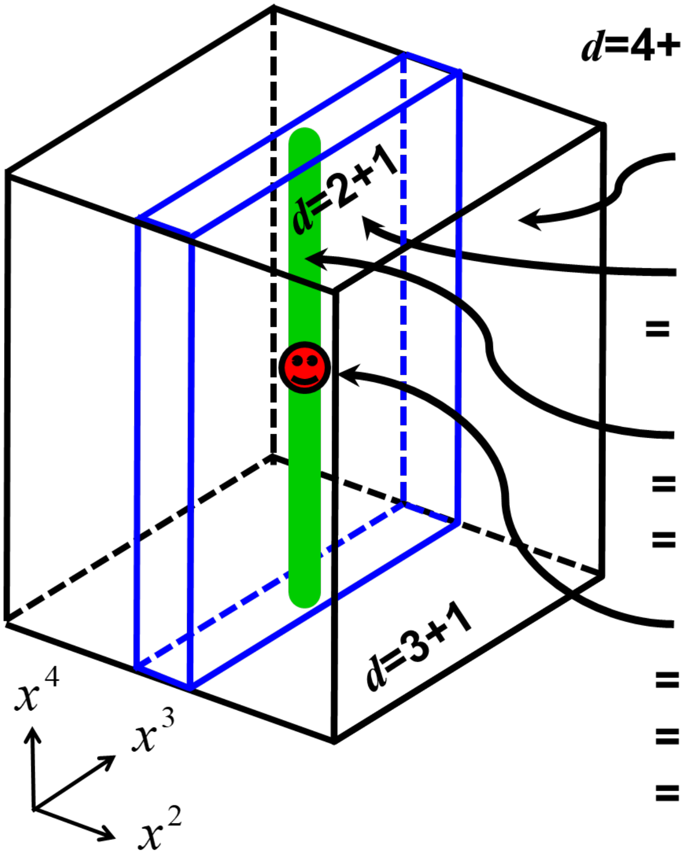
§ 3 Proof

§ 4 Summary

Instanton-monopole-vortex-domain wall in 5D

(1) reducing to all the known composite solitons,
(2) giving relations among (almost) all topological solitons

d=4+1 bulk

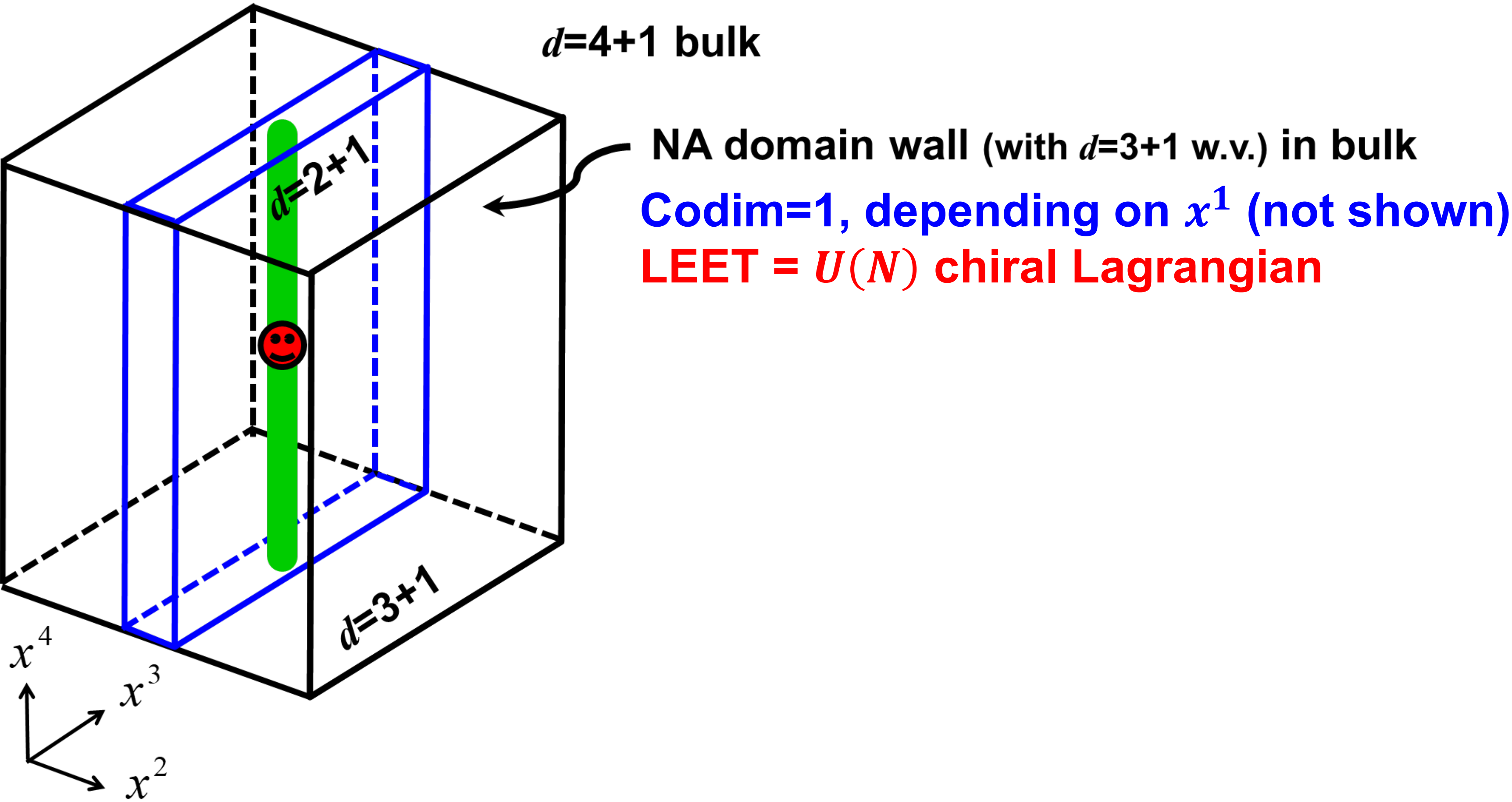


NA domain wall (with d=3+1 w.v.) **in bulk**

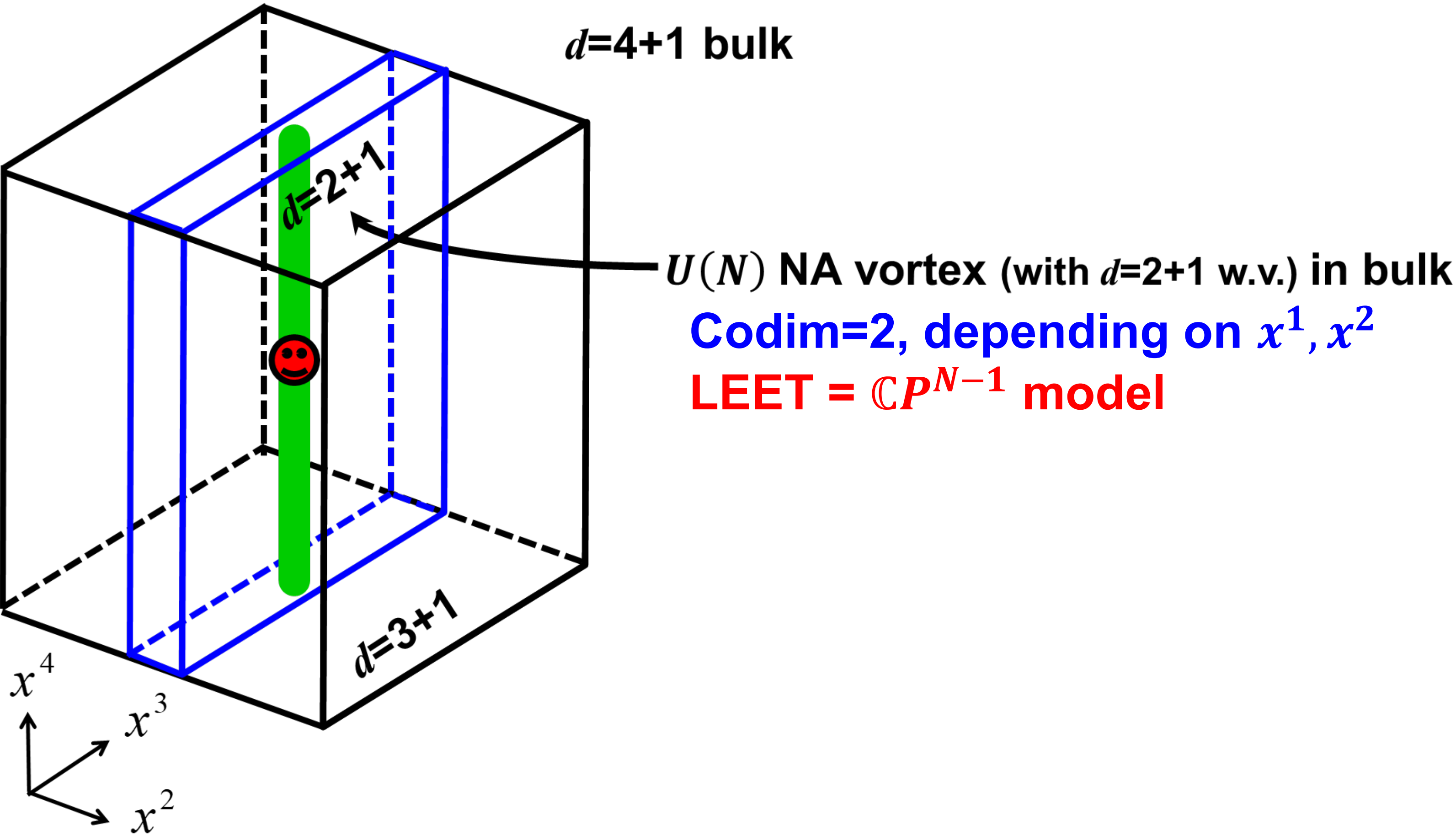
$U(N)$ NA vortex (with d=2+1 w.v.) **in bulk**
= $U(N)$ NA SG soliton in d=3+1 wall w.v.

$SU(N)/U(1)^{N-1}$ monopole (with d=1+1 w.v.)
= $U(1)^{N-1}$global vortex in d=3+1 wall w.v.
= $\mathbb{C}P^{N-1}$ kink in d=2+1 vortex w.v.

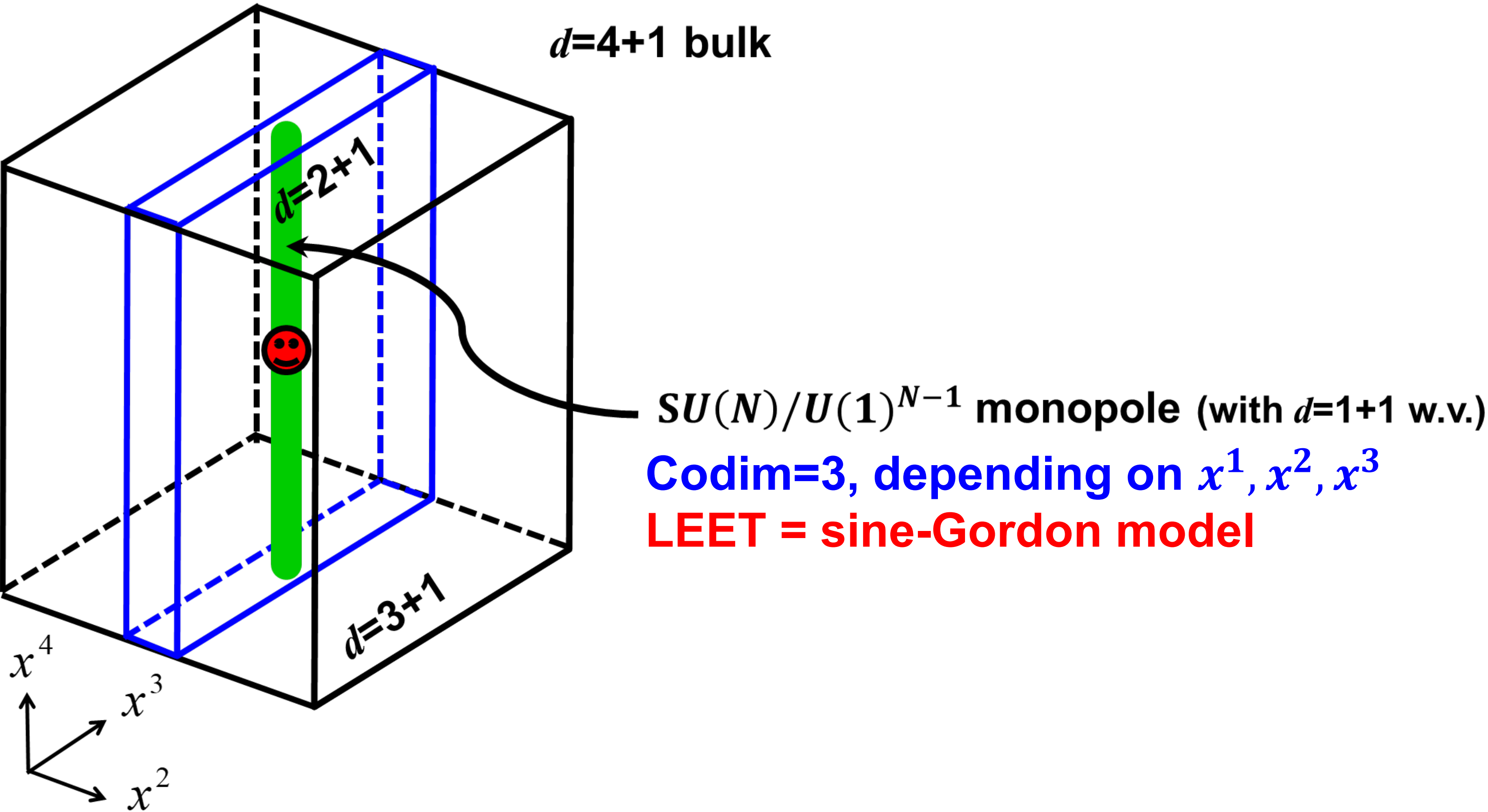
$SU(N)$ YM instanton in bulk
= $SU(N)$ Skyrmion in d=3+1 wall w.v.
= $\mathbb{C}P^{N-1}$ lump in d=2+1 vortex w.v.
= $U(1)^{N-1}$ SG soliton in d=1+1 monopole w.v.



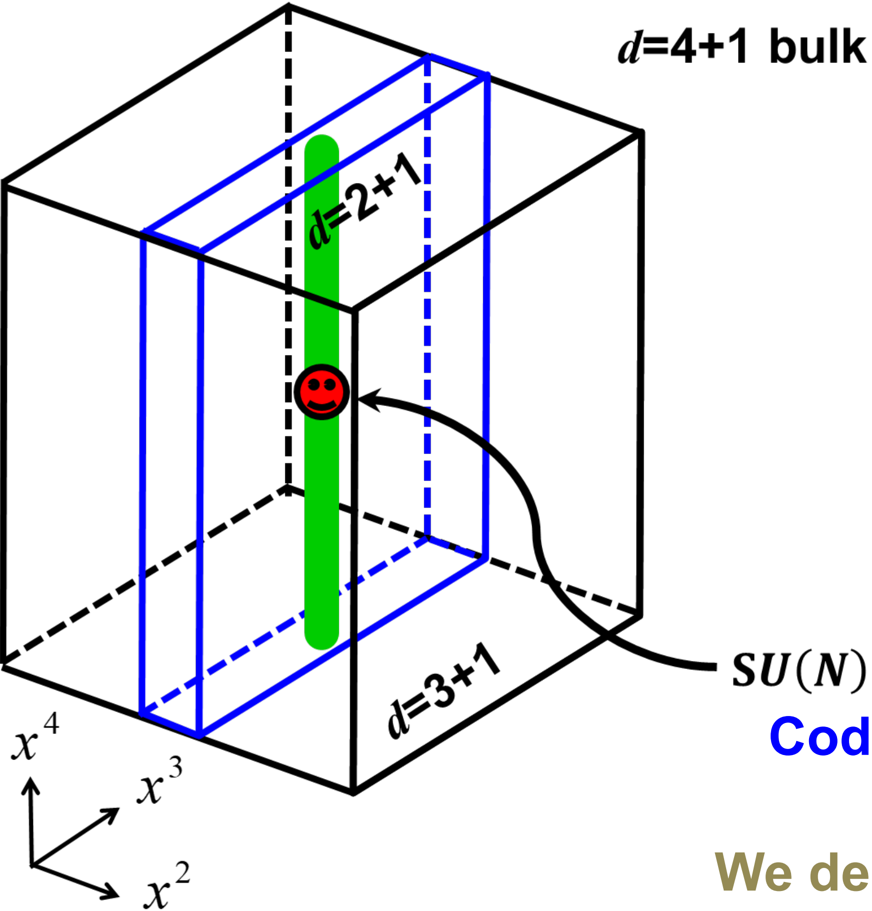
d=4+1 bulk
NA domain wall (with d=3+1 w.v.) in bulk
Codim=1, depending on x^1 (not shown)
LEET = $U(N)$ chiral Lagrangian
d=2+1
d=3+1
x^4
x^3
x^2



d=4+1 bulk
d=2+1
$U(N)$ NA vortex (with d=2+1 w.v.) in bulk
Codim=2, depending on x^1, x^2
LEET = $\mathbb{C}P^{N-1}$ model
d=3+1
x^4
x^3
x^2



d=4+1 bulk
d=2+1
d=3+1
$SU(N)/U(1)^{N-1}$ monopole (with d=1+1 w.v.)
Codim=3, depending on x^1, x^2, x^3
LEET = sine-Gordon model
x^4
x^3
x^2

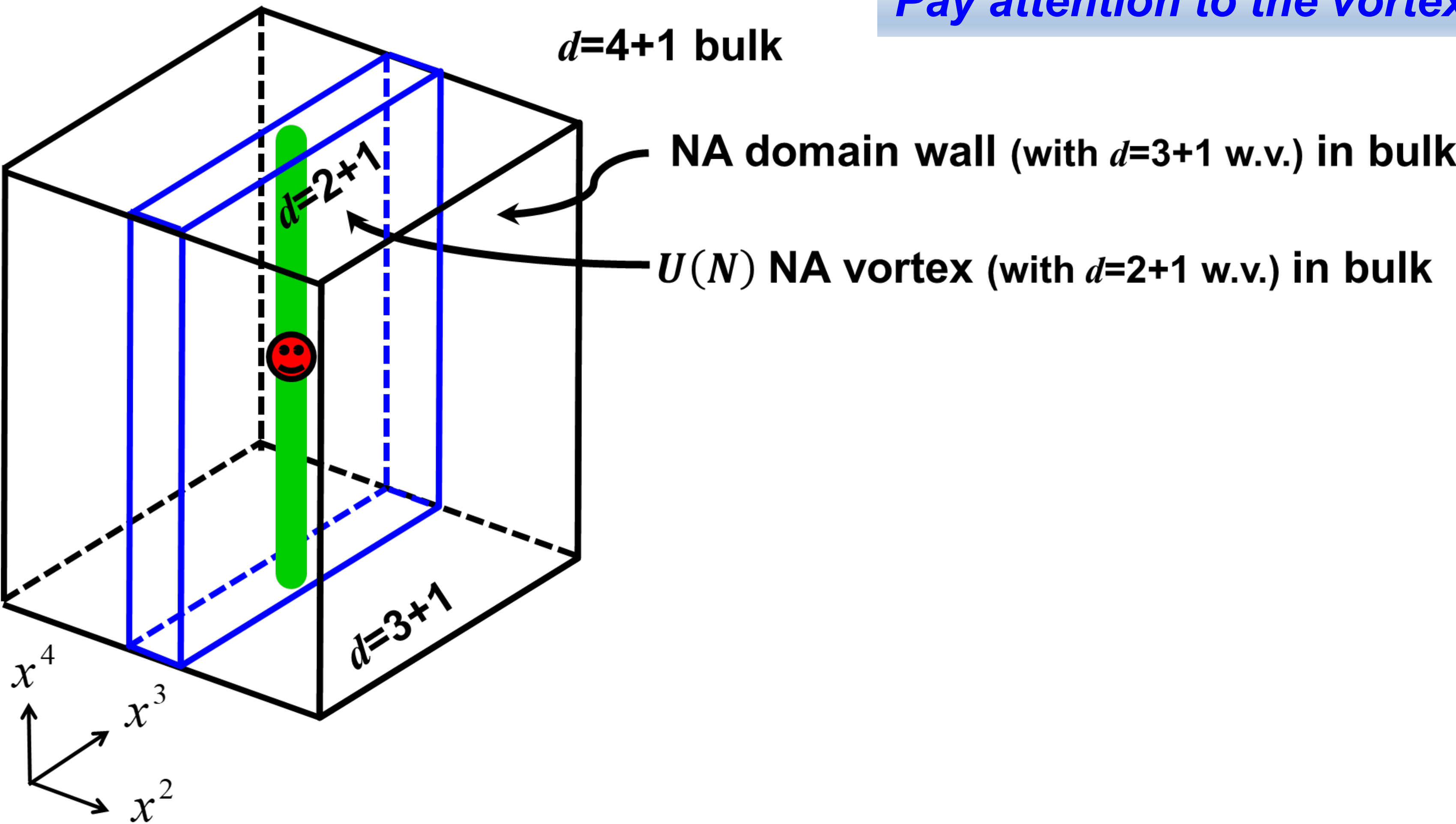


$SU(N)$ **YM instanton in bulk**

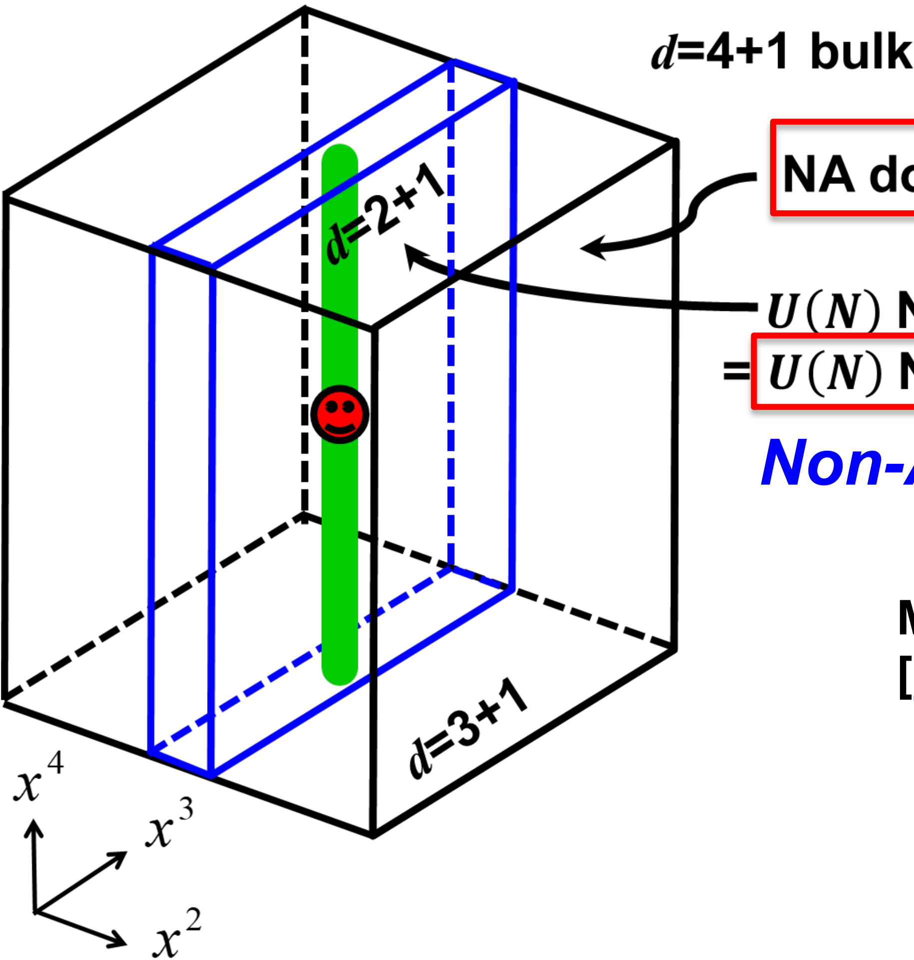
Codim=4, depending on x^1, x^2, x^3, x^4

We deal with instanton particle in Euclidean $\mathbb{R}^4$ in $d = 4 + 1$.

Pay attention to the vortex



Pay attention to the vortex



LEET = $U(N)$ Chiral Lagrangian

NA domain wall (with d=3+1 w.v.) in bulk

$U(N)$ NA vortex (with d=2+1 w.v.) in bulk

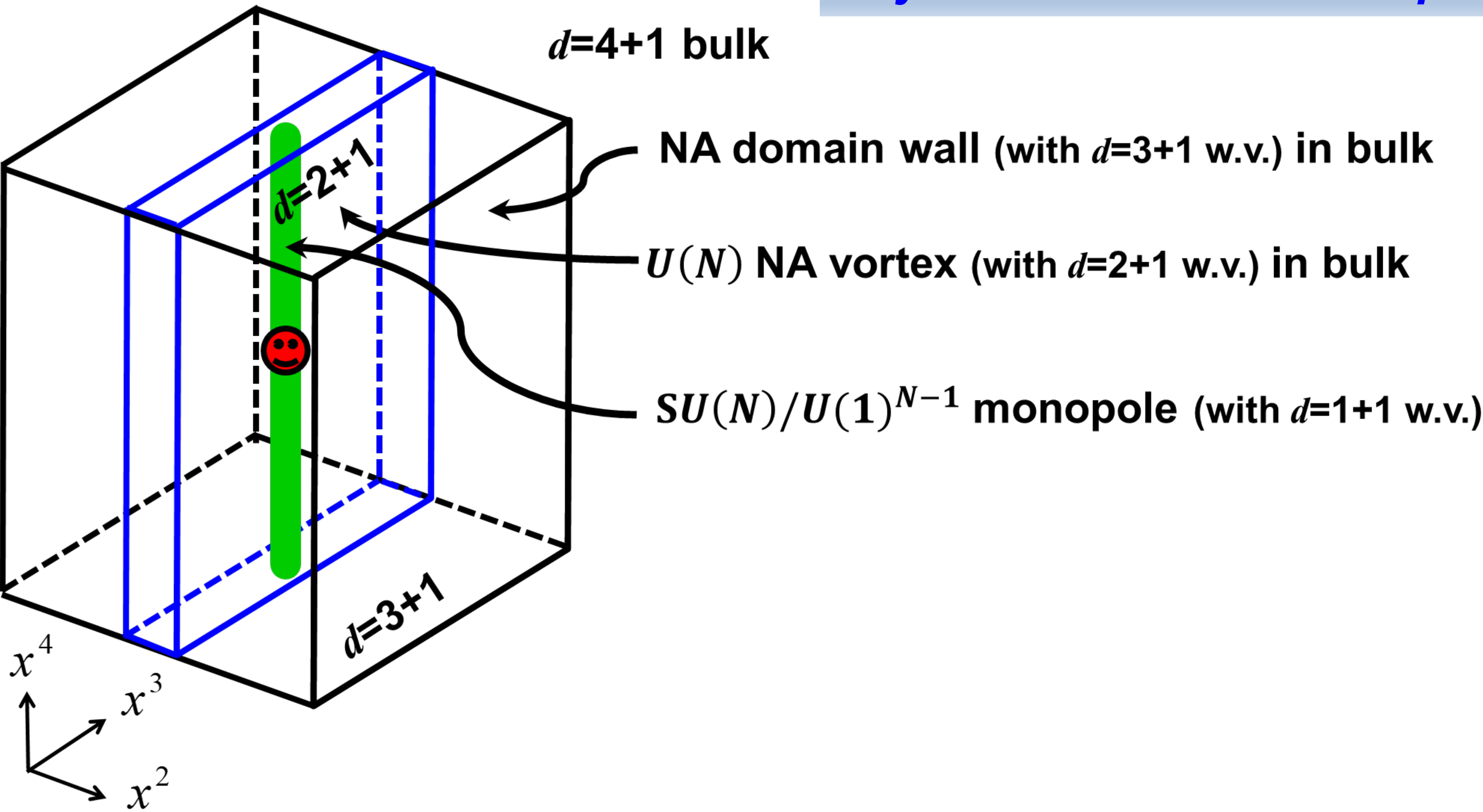
= $U(N)$ NA SG soliton in d=3+1 wall w.v.

Non-Abelian Josephson vortex

(Domain wall-vortex)

MN, *Nucl.Phys.B* 899 (2015) 78-90 [1502.02525 [hep-th]]

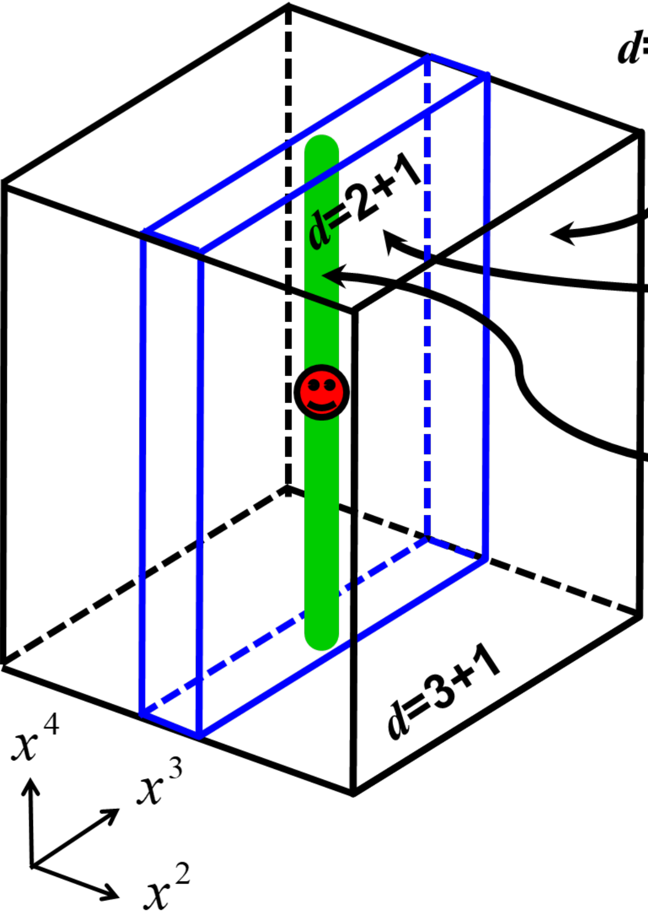
Pay attention to the monopole



Pay attention to the monopole

d=4+1 bulk

LEET = $U(N)$ Chiral Lagrangian



NA domain wall (with *d*=3+1 w.v.) in bulk

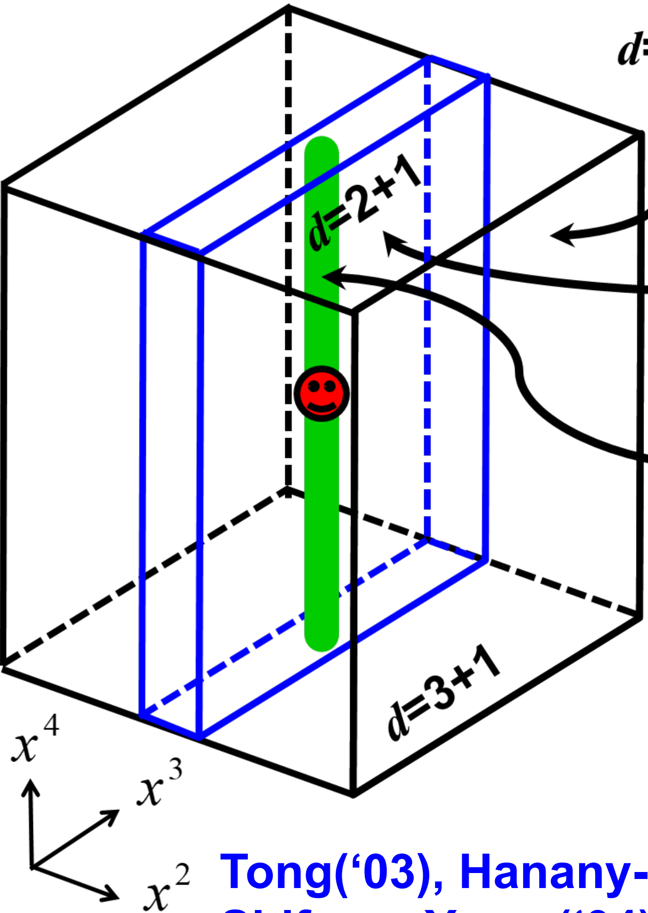
$U(N)$ NA vortex (with *d*=2+1 w.v.) in bulk

$SU(N)/U(1)^{N-1}$ monopole (with *d*=1+1 w.v.)
= $U(1)^{N-1}$ global vortex in *d*=3+1 wall w.v.

Josephson monopole
(domain wall monopole)

MN, Phys.Rev.D 92 (2015) 4, 045010
[1503.02060 [hep-th]]

Pay attention to the monopole



***d*=4+1 bulk**

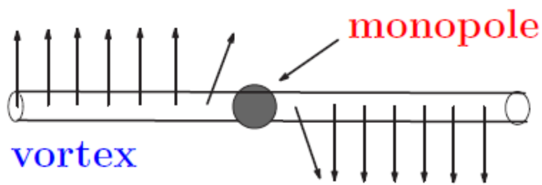
NA domain wall (with *d*=3+1 w.v.) in bulk

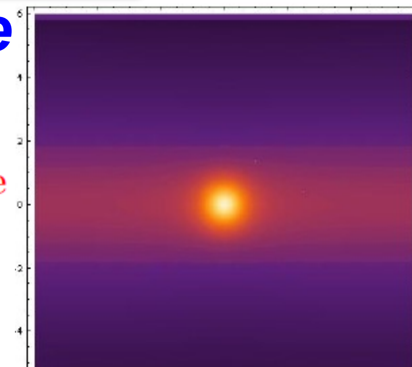
$U(N)$ NA vortex (with *d*=2+1 w.v.) in bulk

LEET = $\mathbb{C}P^{N-1}$ model

$SU(N)/U(1)^{N-1}$ monopole (with *d*=1+1 w.v.)
= $U(1)^{N-1}$ global vortex in *d*=3+1 wall w.v.

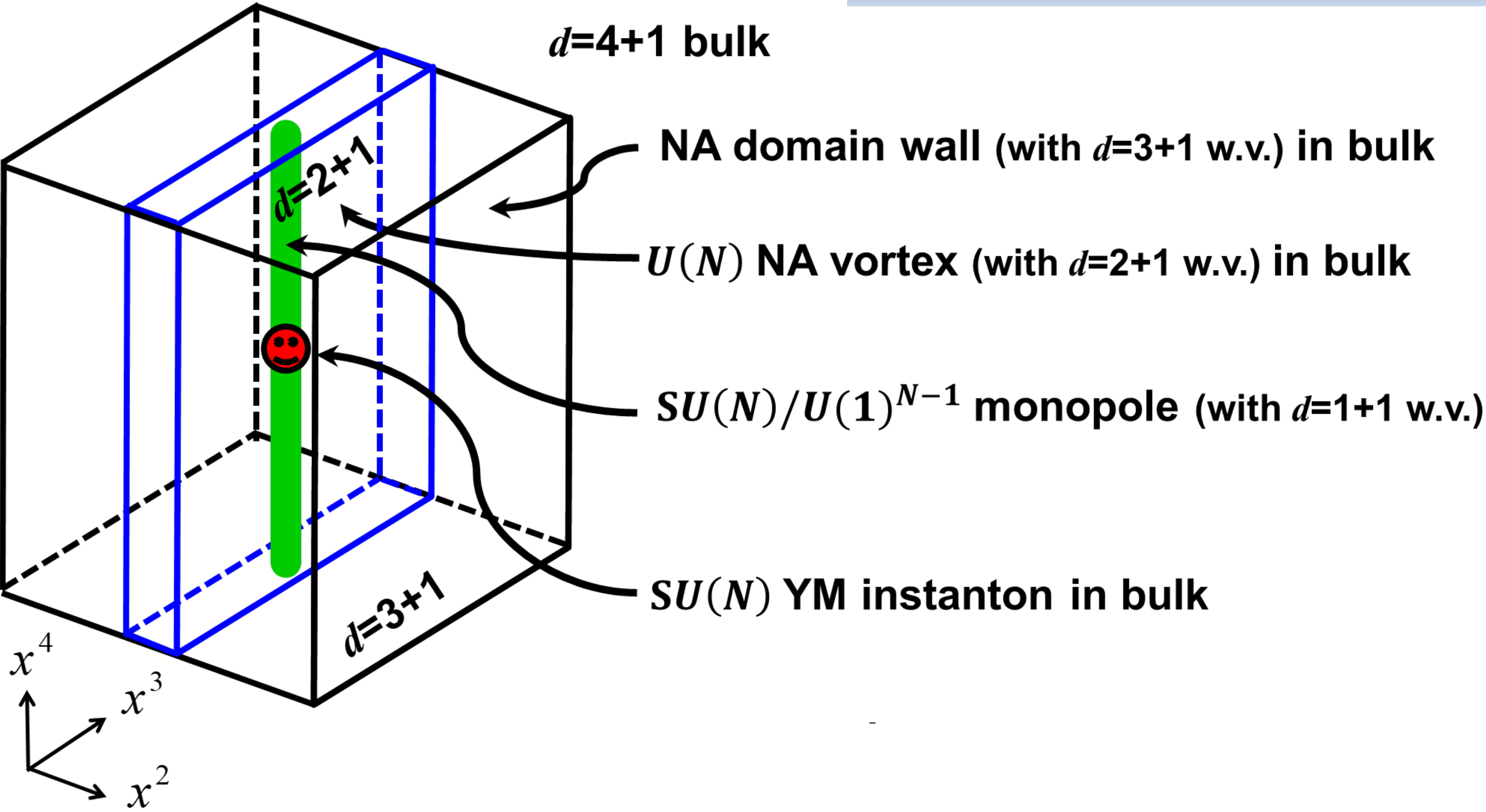
Confined monopole
(vortex monopole)



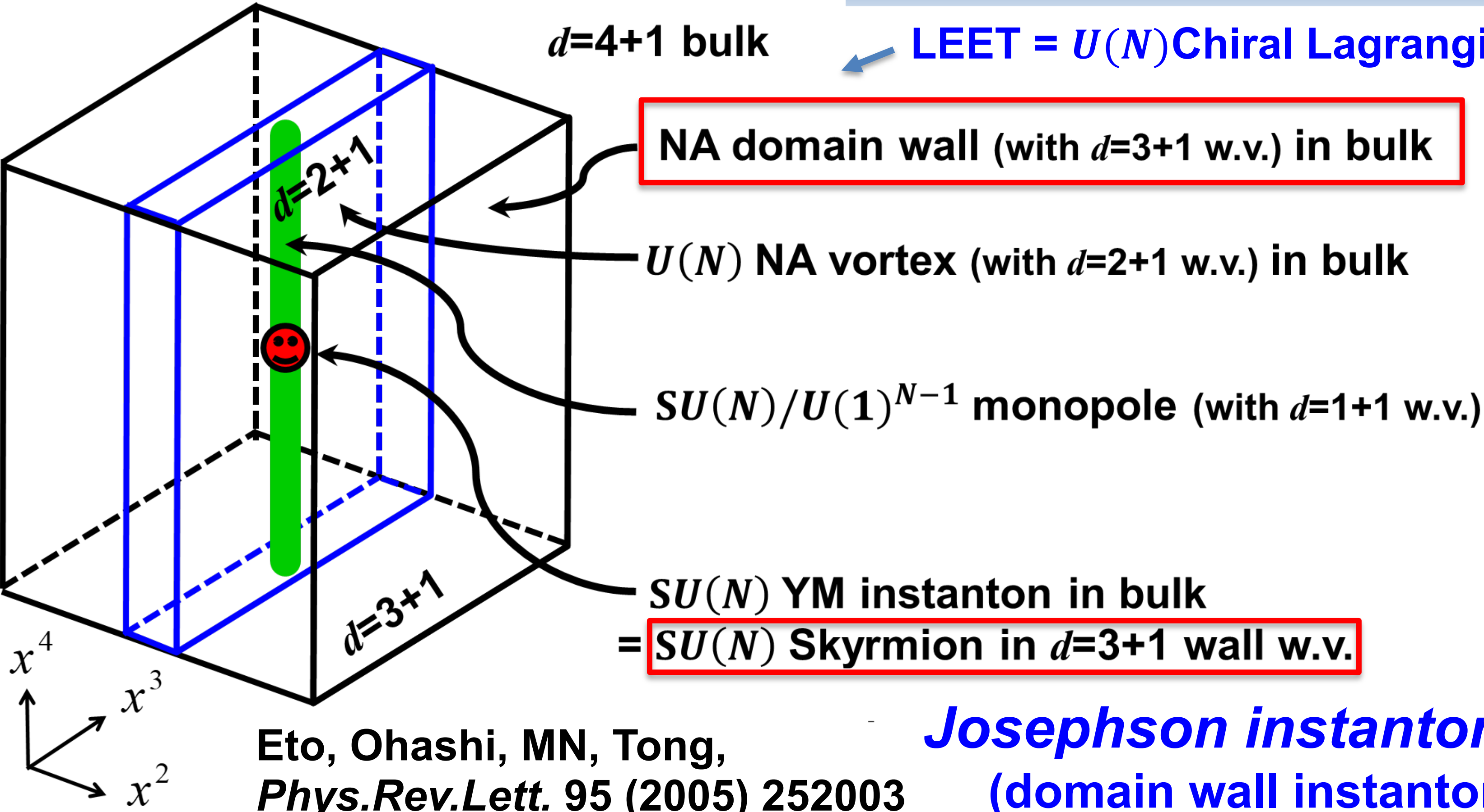


Tong('03), Hanany-Tong, Shifman-Yung ('04)

Pay attention to the instanton



Pay attention to the instanton

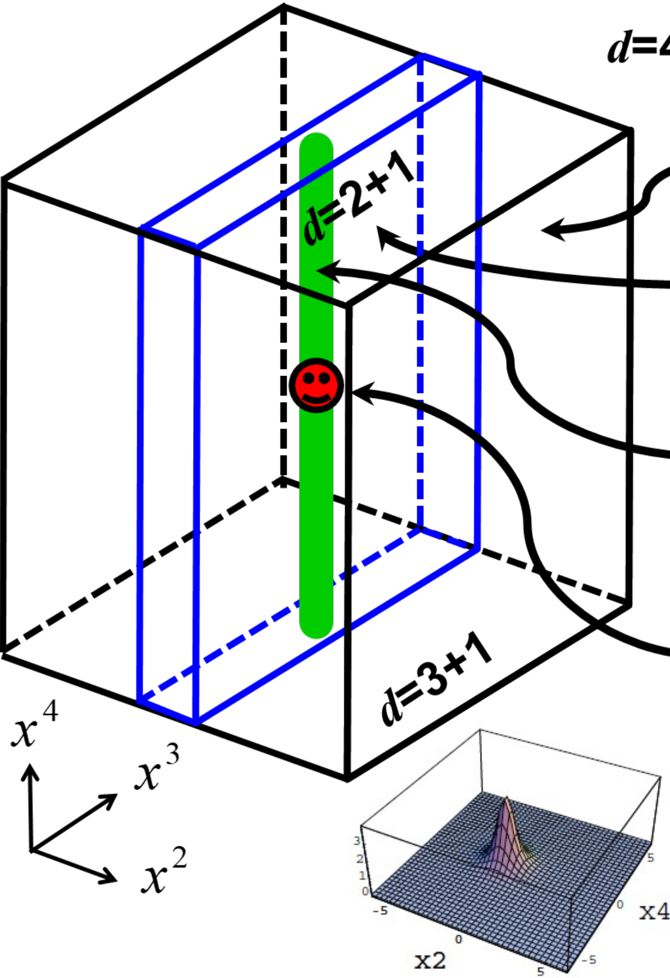


Eto, Ohashi, MN, Tong, *Phys.Rev.Lett.* 95 (2005) 252003 [hep-th/0508130 [hep-th]]

Josephson instanton

(domain wall instanton)

Pay attention to the instanton



d=4+1 bulk

NA domain wall (with d=3+1 w.v.) in bulk

$U(N)$ NA vortex (with d=2+1 w.v.) in bulk

LEET = $\mathbb{C}P^{N-1}$ model

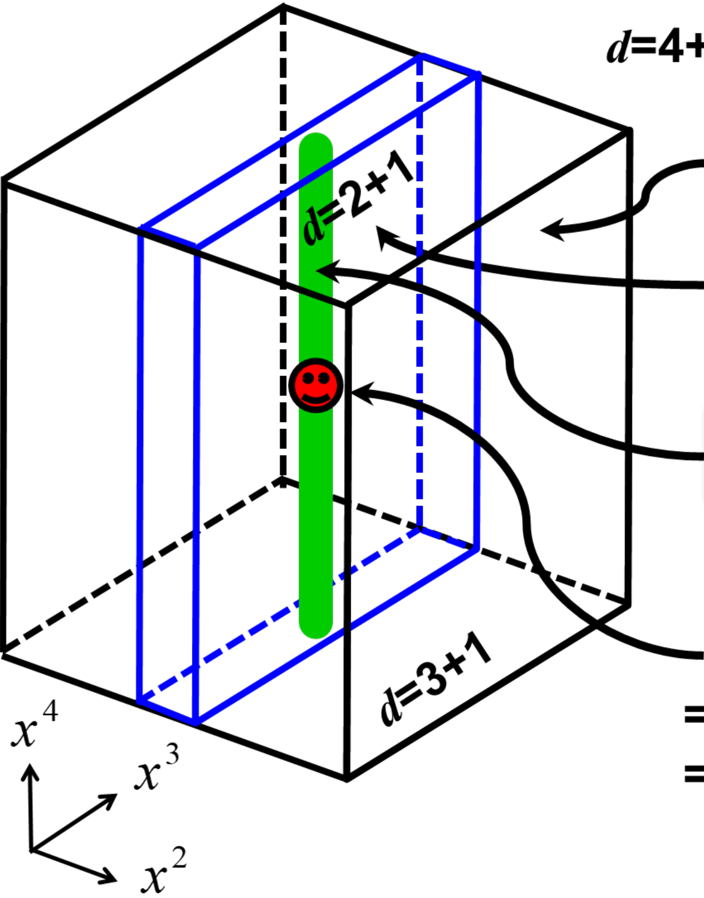
$SU(N)/U(1)^{N-1}$ monopole (with d=1+1 w.v.)

$SU(N)$ YM instanton in bulk
= $SU(N)$ Skyrmion in d=3+1 wall w.v.

Trapped instanton (vortex instanton)

Hanany-Tong,
Eto-Isozumi-MN-Ohashi-Sakai('04)

Pay attention to the instanton



NA domain wall (with d=3+1 w.v.) **in bulk**

$U(N)$ **NA vortex** (with d=2+1 w.v.) **in bulk**

$SU(N)/U(1)^{N-1}$ **monopole** (with d=1+1 w.v.)

LEET = sine-Gordon model

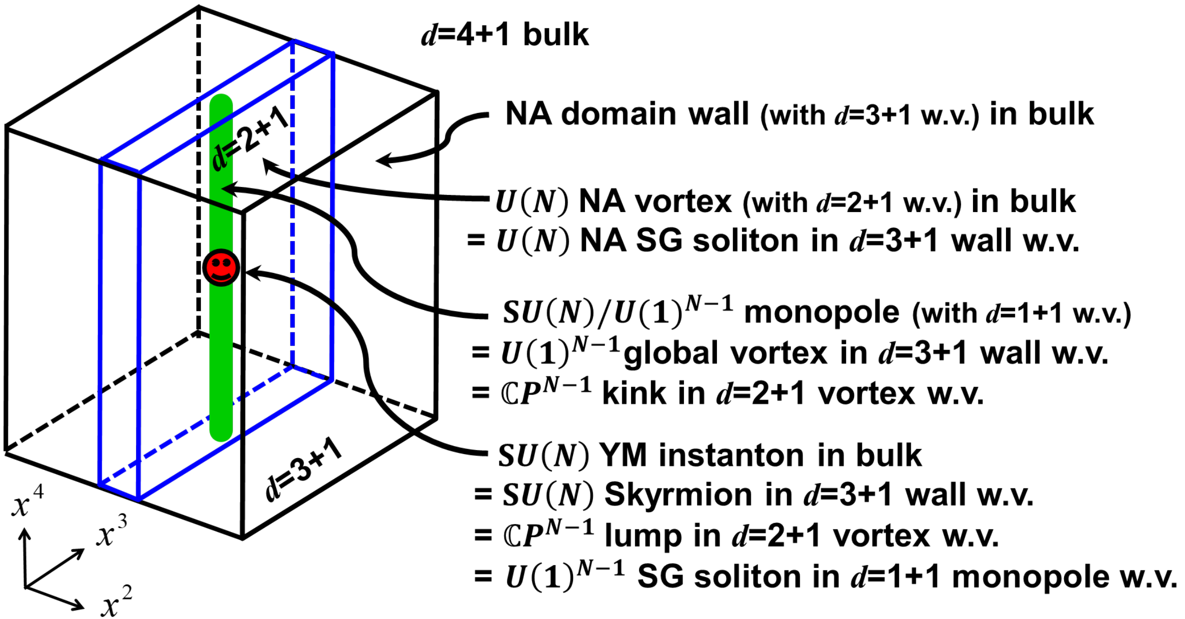
$SU(N)$ **YM instanton in bulk**
= $SU(N)$ **Skyrmion in** d**=3+1 wall w.v.**
= $\mathbb{C}P^{N-1}$ **lump in** d**=2+1 vortex w.v.**

Monopole instanton

MN, *Phys.Rev.D* 87 (2013) 6, 066008 [1301.3268 [hep-th]]

Summary

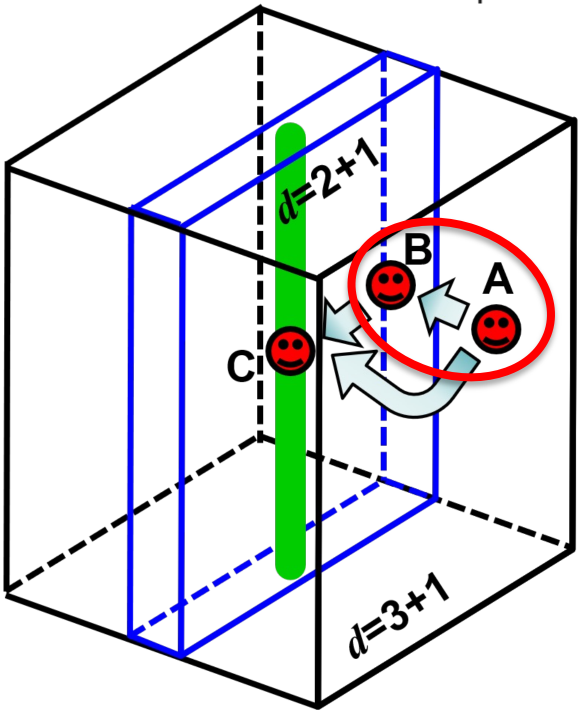
		Daughters		
Mothers	Moduli	$U(N)$ NA vortex	$\frac{SU(N)}{U(1)^{N-1}}$ monopole	$SU(N)$ YM instanton
$U(N)$ NA wall	$U(N)$	$U(N)$ NA SG soliton	$U(1)^{N-1}$ global vortex	$SU(N)$ Skyrmion
$U(N)$ NA vortex	$\mathbb{C}P^{N-1}$	–	$\mathbb{C}P^{N-1}$ wall	$\mathbb{C}P^{N-1}$ lump
$\frac{SU(N)}{U(1)^{N-1}}$ monopole	$U(1)$	–	–	SG soliton



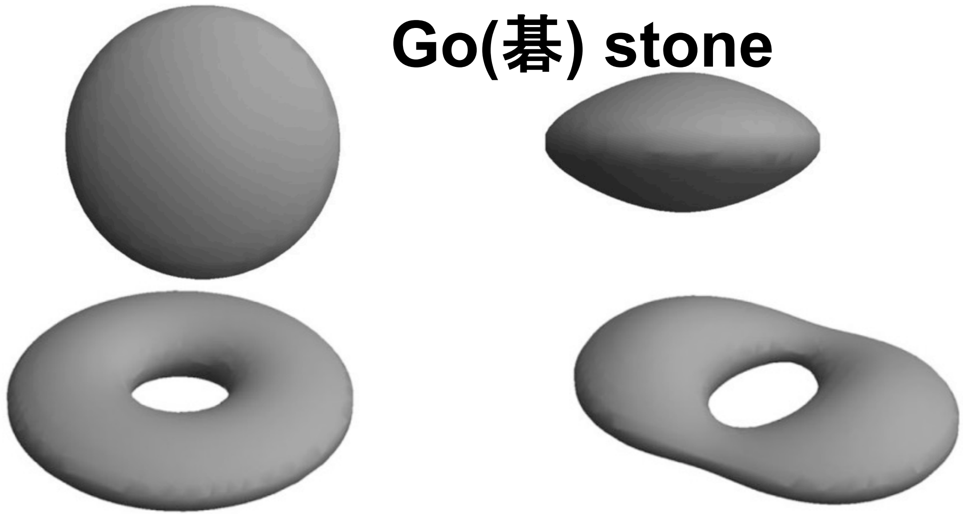
Further relations

Daughters **In the $U(N)$ Chiral Lagrangian**

Mothers	Moduli	$U(1)^{N-1}$ global vortex	$SU(N)$ Skyrmion
$U(N)$ NA SG soliton	$\mathbb{C}P^{N-1}$	$\mathbb{C}P^{N-1}$ wall	$\mathbb{C}P^{N-1}$ lump
$U(1)^{N-1}$ global vortex	$U(1)$	–	SG soliton



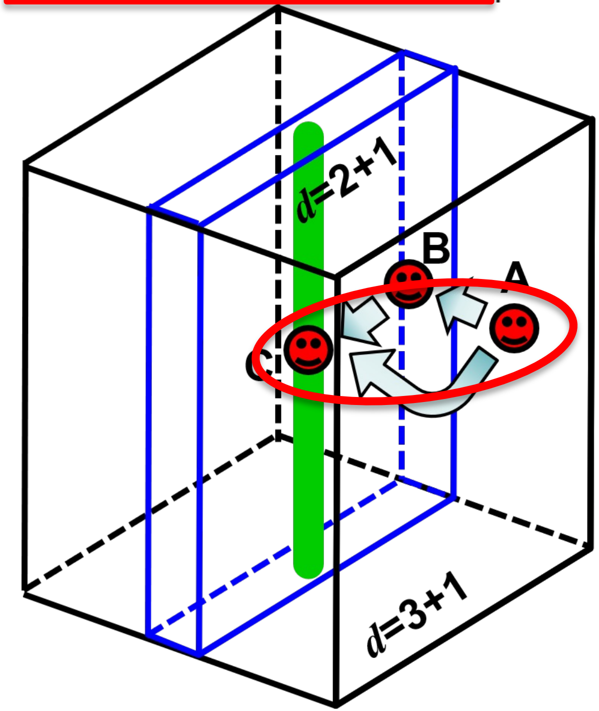
Eto & MN, *Phys.Rev.D*91 (2015) 085044 [1501.07038 [hep-th]]



Further relations

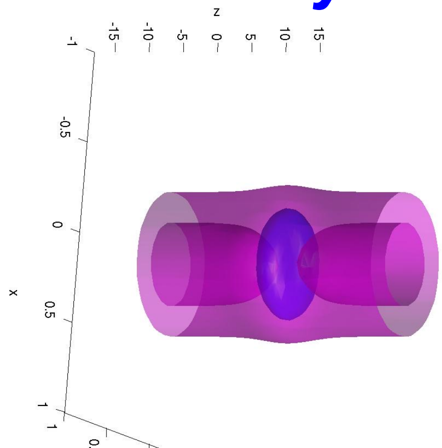
In the $U(N)$ Chiral Lagrangian

Mothers	Moduli	*Daughters*	
		$U(1)^{N-1}$ global vortex	$SU(N)$ Skyrmion
$U(N)$ NA SG soliton	$\mathbb{C}P^{N-1}$	$\mathbb{C}P^{N-1}$ wall	$\mathbb{C}P^{N-1}$ lump
$U(1)^{N-1}$ global vortex	$U(1)$	–	SG soliton



Gudnason & MN, *Phys.Rev.D*94 (2016) 025008 [1606.00336 [hep-th]] (for *N*=2)

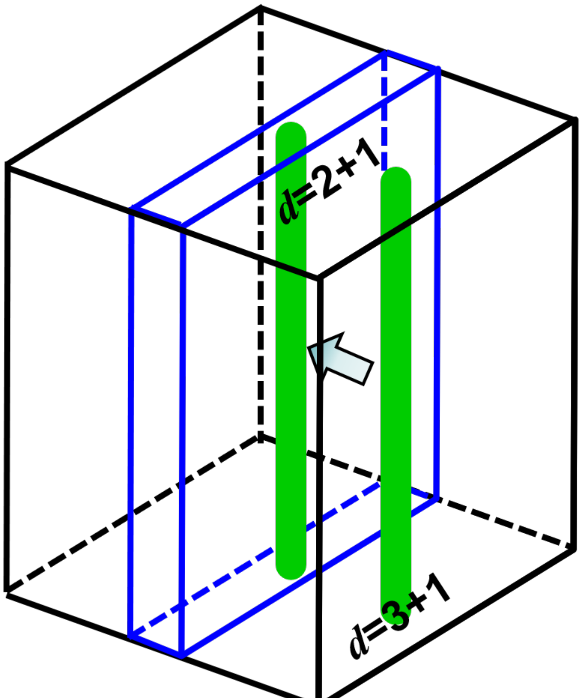
Confined Skyrmion



Further relations

In the $U(N)$ Chiral Lagrangian

Mothers	Moduli	*Daughters*	
		$U(1)^{N-1}$ global vortex	$SU(N)$ Skyrmion
$U(N)$ NA SG soliton	$\mathbb{C}P^{N-1}$	$\mathbb{C}P^{N-1}$ wall	$\mathbb{C}P^{N-1}$ lump
$U(1)^{N-1}$ global vortex	$U(1)$	–	SG soliton

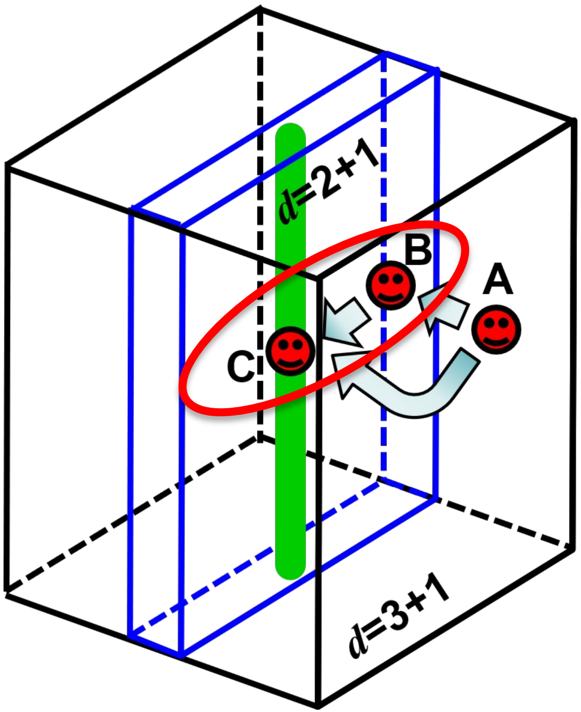


New relation not studied so far

Further relations

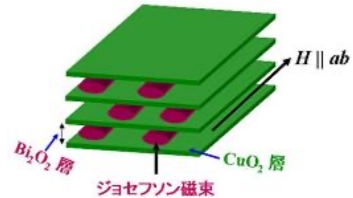
In the $\mathbb{C}P^{N-1}$ model

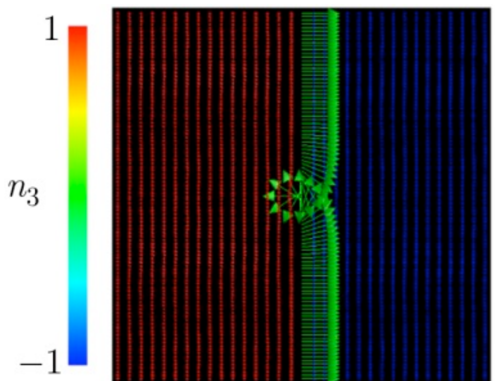
Mother	Moduli	Daughter
		$\mathbb{C}P^{N-1}$ lump (baby skyrmion)
$\mathbb{C}P^{N-1}$ wall	$U(1)$	SG soliton

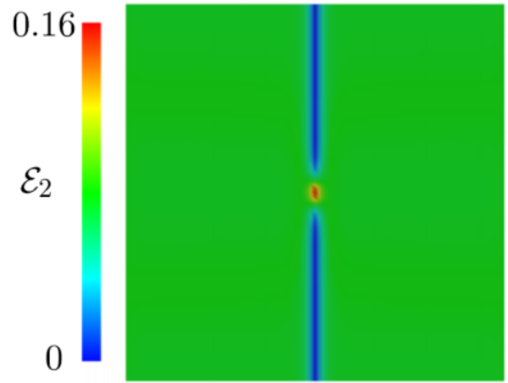


MN, *Phys.Rev.D* 86 (2012) 125004 [1207.6958 [hep-th]]
Kobayashi & MN, *Phys.Rev.D* 87 (2013) 085003 [1302.0989 [hep-th]]

Josephson vortex or Domain wall Skyrmion

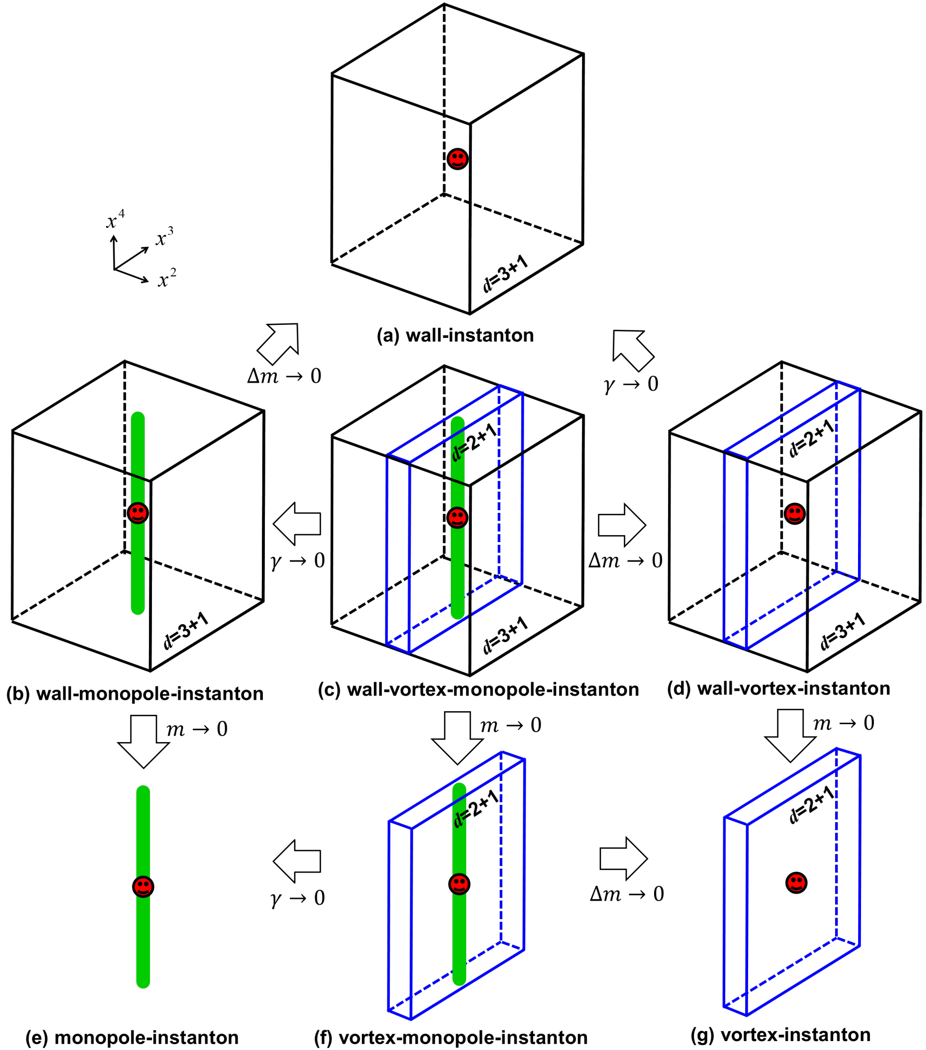






Various limits

→ All possible composite solitons

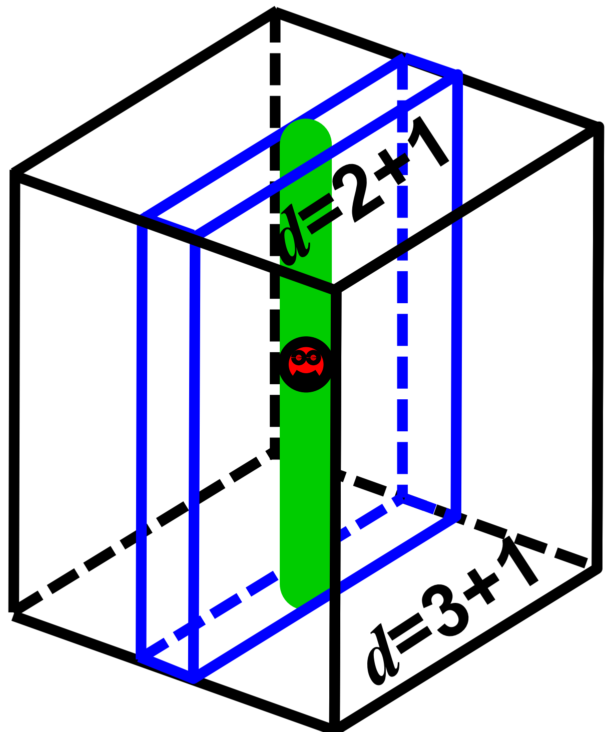


Hierarchy

m

wall

***Sketch of a proof*: analytically proved using the effective theory technique**



Plan of My Talk

§ 1 Introduction

§ 2 Sketch of the results

§ 3 Proof

§ 4 Summary

Matter contents

$H(x) = (H_1, H_2)$: $N \times 2N$ complex scalars

$\Sigma(x)$: $U(N)$ adjoint real scalar fields

$A_\mu(x)$: $U(N)$ gauge fields

$U(N)$ gauge transformation

$$A_\mu \to g A_\mu g^{-1} + i g \partial_\mu g^{-1}, \quad H \to gH, \quad \Sigma \to g \Sigma g^{-1}, \quad g \in U(N)_{\mathrm{C}}.$$

$SU(N)_L \times SU(N)_R$ flavor symmetry

$$H(x) = (H_1, H_2) \to \left(H_1 U_L^\dagger U, H_2 U_R^\dagger\right)$$

Lagrangian

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} + \frac{1}{g^2}\mathrm{tr}\,(D_\mu\Sigma)^2 + \mathrm{tr}\,|D_\mu H|^2 + \mathcal{L}_J - V$$

$$V = \frac{g^2}{4}\mathrm{tr}\,(HH^\dagger - v^2\mathbf{1}_N)^2 + \mathrm{tr}\,|\Sigma H - HM|^2$$

$\mathcal{N} = 2$ SUSY except for $\mathcal{L}_J$

Mass matrix ($N\times 2N$) $M = \mathrm{diag.}(m\mathbf{1}_N + \Delta M, -m\mathbf{1}_N - \Delta M)$

$$\Delta M = \mathrm{diag.}(m_1, m_2, \cdots, m_N)$$

Josephson terms (breaking SUSY)

$$\mathcal{L}_J = \mathcal{L}_{J,1} + \mathcal{L}_{J,2} \qquad \mathcal{L}_{J,1} = -\gamma\mathrm{tr}\,(H_1^\dagger H_2 + H_2^\dagger H_1)$$

$$\mathcal{L}_{J,2} = -\sum_{r=1}^{N-1}\frac{\beta_r^2}{v^2}[\mathrm{tr}\,(H_1 X_r H_2^\dagger) + \mathrm{tr}\,(H_2 X_r H_1^\dagger)]$$

$$X_r = X_r^\dagger \ (r = 1, \cdots, N-1)$$

Vacua

(1) $m = 0,\ \Delta M = 0,\ \gamma = 0,\ \beta_r = 0,$ **Continuous vac: Grassmannian**

$$H = (v\mathbf{1}_N, \mathbf{0}_N)\,, \qquad \Sigma = \mathbf{0}_N \qquad Gr_{2N,N} \simeq \frac{SU(2N)}{SU(N) \times SU(N) \times U(1)}$$

(2) $m \neq 0 \quad \Delta M = 0$ **2 disjoint vacua: Color-flavor locked(CFL) vac**

$$H = (v\mathbf{1}_N, \mathbf{0}_N)\,, \quad \Sigma = +m\mathbf{1}_N : \quad SU(N)_{\mathrm{C+L}},$$

$$H = (\mathbf{0}_N, v\mathbf{1}_N)\,, \quad \Sigma = -m\mathbf{1}_N : \quad SU(N)_{\mathrm{C+R}}$$

(3) $m \neq 0 \quad \Delta M \neq 0$

$$H = (v'\mathbf{1}_N, \mathbf{0}_N)\,, \quad \Sigma = +m\mathbf{1}_N + \Delta M : \quad U(1)^{N-1}_{\mathrm{C+L}},$$

$$H = (\mathbf{0}_N, v'\mathbf{1}_N)\,, \quad \Sigma = -m\mathbf{1}_N - \Delta M : \quad U(1)^{N-1}_{\mathrm{C+R}},$$

Strong coupling limit $g \to \infty$: Grassmannian σ model

$$HH^\dagger = v^2 \mathbf{1}_N, \quad \Sigma = v^{-2} HMH^\dagger, \quad A_\mu = \frac{i}{2} v^{-2} [H\partial_\mu H^\dagger - (\partial_\mu H) H^\dagger],$$

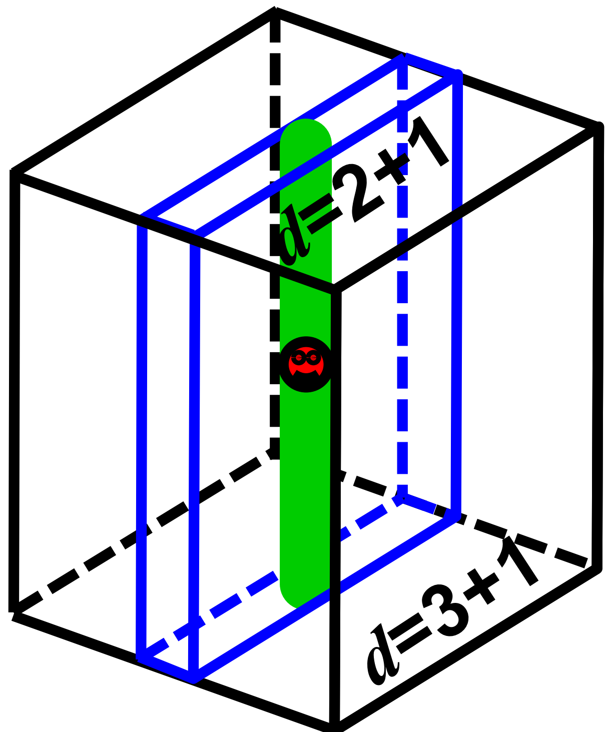
$$Gr_{2N,N} \simeq \frac{SU(2N)}{SU(N) \times SU(N) \times U(1)}$$

Just for technical reason
(One can calculate everything analytically.)

Hierarchy

m

wall



(1) Non-Abelian domain wall

$$H = H_{\rm wall}(x^1) = \frac{v}{\sqrt{1+e^{\mp 2m(x^1-X^1)}}}\left(\mathbf{1}_N, e^{\mp m(x^1-X^1)}U\right)$$

Moduli: $(X^1, U) \in \mathcal{M}_{\rm wall} \simeq \mathbb{R}\times U(N)$

Effective field theory(EFT) = $U(N)$ chiral Lagrangian

$$\mathcal{L}_{\rm wall} = \int dx^1 \mathcal{L}(H = H_{\rm wall}(x^1; X^1(x^i), U(x^i)))$$

$$= \frac{v^2}{2m}\partial_i X^1 \partial^i X^1 - f_\pi^2 {\rm tr}\left(U^\dagger \partial_i U U^\dagger \partial^i U\right), \quad f_\pi^2 \equiv \frac{v^2}{4m}$$

Turning on γ ⇒ **Pion mass term on wall EFT**

$$\Delta\mathcal{L}_{{\rm wall},J} = \int dx^1 \mathcal{L}_{J,1}(H = H_{\rm wall}(x^1; X^1(x^i), U(x^i)))$$

$$= -m'^2({\rm tr}\, U + {\rm tr}\, U^\dagger), \quad m'^2 \equiv \frac{\pi\gamma}{2m}.$$

(2) Non-Abelian vortex

MN, Nucl.Phys.B 895 (2015) 288-302, [e-Print: 1412.8276 [hep-th]]

Non-Abelian sine-Gordon soliton in d.w. EFT

$$U = U_{\rm vortex}(x^2) = V {\rm diag}\,(u(x^2), 1, \cdots, 1) V^\dagger = \mathbf{1}_N + (u-1)\phi\phi^\dagger$$

$$u(x^2) = \exp i\theta_{\rm SG}(x^2 - X^2) = \exp\left(4i \arctan \exp[m''(x^2 - X^2)]\right)$$

= non-Abelian vortex in the bulk $\quad m''^2 = \frac{m'^2}{f_\pi^2} = \frac{2\pi\gamma}{v^2}$

Moduli: $\mathcal{M}_{\rm vortex} \simeq \mathbb{R} \times \mathbb{C}P^{N-1}$

Vortex EFT = $\mathbb{C}P^{N-1}$ model

$$\mathcal{L}_{\rm vortex} = C_X \partial_\alpha X^2 \partial^\alpha X^2 + C_\phi \left[\partial_\alpha \phi^\dagger \partial^\alpha \phi + (\phi^\dagger \partial_\alpha \phi)(\phi^\dagger \partial^\alpha \phi)\right]$$

Turning on Δm_r ⇨ Masses on wall.EFT & vortex EFT

$$V_{\rm wall} = \frac{v^2}{4m} {\rm tr}\left([\Delta M, U]^\dagger [\Delta M, U]\right) \qquad V_{\rm vortex} = C_\phi \left[(\phi^\dagger \Delta M \phi)^2 - \phi^\dagger (\Delta M)^2 \phi\right]$$

(3) Monopole $\mathbb{C}P^{N-1}$ kink in massive $\mathbb{C}P^{N-1}$ model

$\mathbb{C}P^1$ submanifold, massive $\mathbb{C}P^1$ model

$$\mathcal{L}_{\text{vortex},\mathbb{C}\text{P}^1} = C_X \partial_\alpha X^2 \partial^\alpha X^2 + C_\phi \left[\frac{\partial_\alpha u^* \partial^\alpha u - \delta m_r^2 |u|^2}{(1+|u|^2)^2} \right]$$

$\mathbb{C}P^{N-1}$ kink: $u = u_{\text{monopole}}(x^3) = e^{\mp \delta m_r (x^3 - X^3) + i\varphi}$ **= monopole in the bulk**

Moduli: $\mathcal{M}_{\text{monopole}} \simeq \mathbb{R} \times U(1)$

Monopole EFT = $\mathbb{R}\times U(1)$ scalar field theory

$$\mathcal{L}_{r-\text{th monopole}} = \frac{C_\phi}{2\delta m_r} [(\partial_m X^3)^2 + (\partial_m \varphi)^2]$$

Turning on β_r ⇨ **Masses on wall,vortex&monopole EFTs**

$$\Delta \mathcal{L}_{\text{wall},\beta} = \sum_r \frac{\pi \beta_r^2}{2m''} \text{tr}\left[X_r (U + U^\dagger)\right] \quad \Delta \mathcal{L}_{\text{vortex},\beta} = -\sum_r \frac{4\pi \beta_r^2}{m''^2} (\phi^\dagger X_r \phi)$$

$$\Delta \mathcal{L}_{r-\text{th monopole},\beta} = -C_r \cos\varphi$$ **Sine-Gordon** $$C_r \equiv \frac{\pi^2 \beta_r^2}{2\delta m_r m''^2} = \frac{\pi \beta_r^2 v^2}{4\delta m_r \gamma}$$

(4) Instanton

Monopole EFT = sine-Gordon model

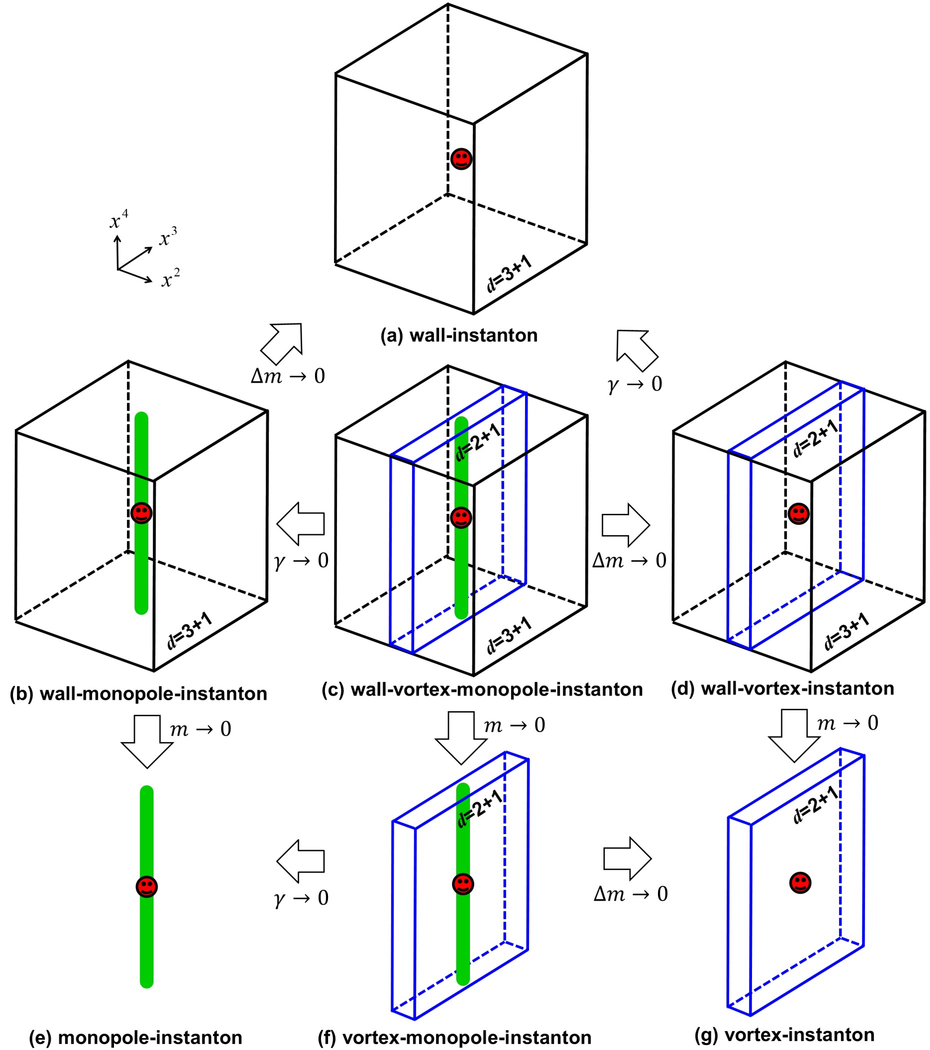
Sine-Gordon soliton

$$\varphi = \varphi_{\mathrm{instanton}}(x^4) = 4 \arctan \exp \sqrt{\frac{D_r}{2}}(x^4 - X^4) + \pi$$

= instanton in the bulk

Various limits

➡ **All possible composite solitons**



Plan of My Talk

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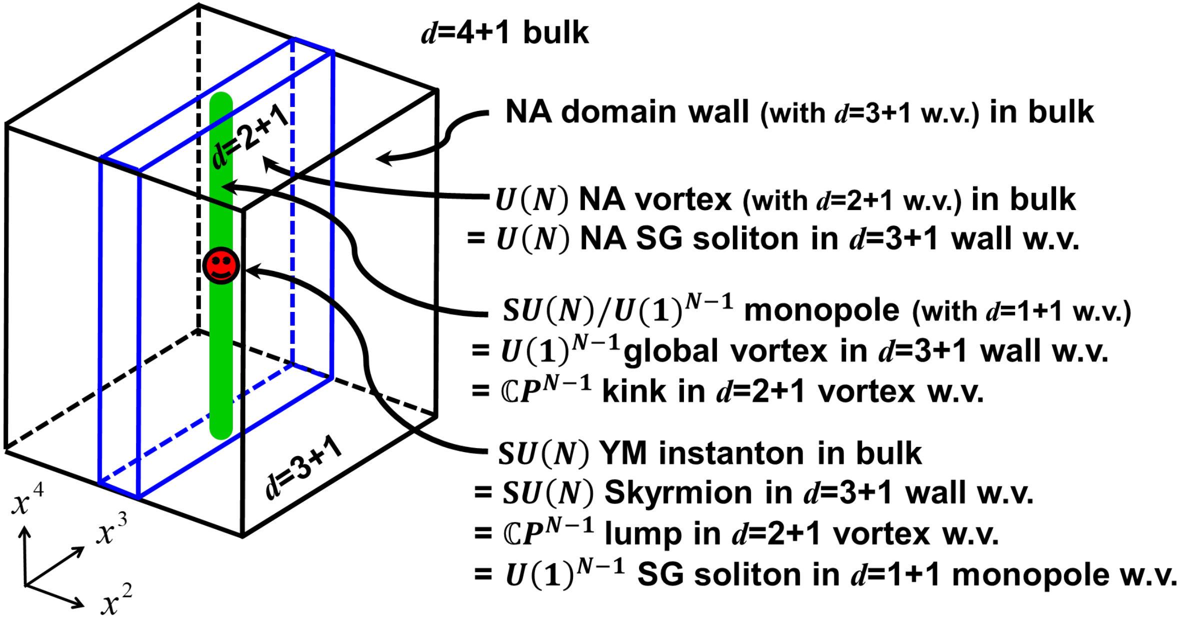
Summary

Mothers	Moduli	*Daughters*		
		$U(N)$ NA vortex	$\frac{SU(N)}{U(1)^{N-1}}$ monopole	$SU(N)$ YM instanton
$U(N)$ NA wall	$U(N)$	$U(N)$ NA SG soliton	$U(1)^{N-1}$ global vortex	$SU(N)$ Skyrmion
$U(N)$ NA vortex	$\mathbb{C}P^{N-1}$	–	$\mathbb{C}P^{N-1}$ wall	$\mathbb{C}P^{N-1}$ lump
$\frac{SU(N)}{U(1)^{N-1}}$ monopole	$U(1)$	–	–	SG soliton



Discussion

1. Non-Abelian monopoles
2. Hopfions
3. Non-perturbative effects in different dimensions
4. Dyonic extension
5. Higher-form symmetries, higher groups
6. Fermions: Fermion zero modes, Non-Abelian anyons, Anomaly inflow, Anomaly matching

Symmetry breaking: $G \to H$
Either gauge or global symmetries

↓

Nambu-Goldstone modes
Vacuum manifold or Order parameter space(OPS): G/H

↓

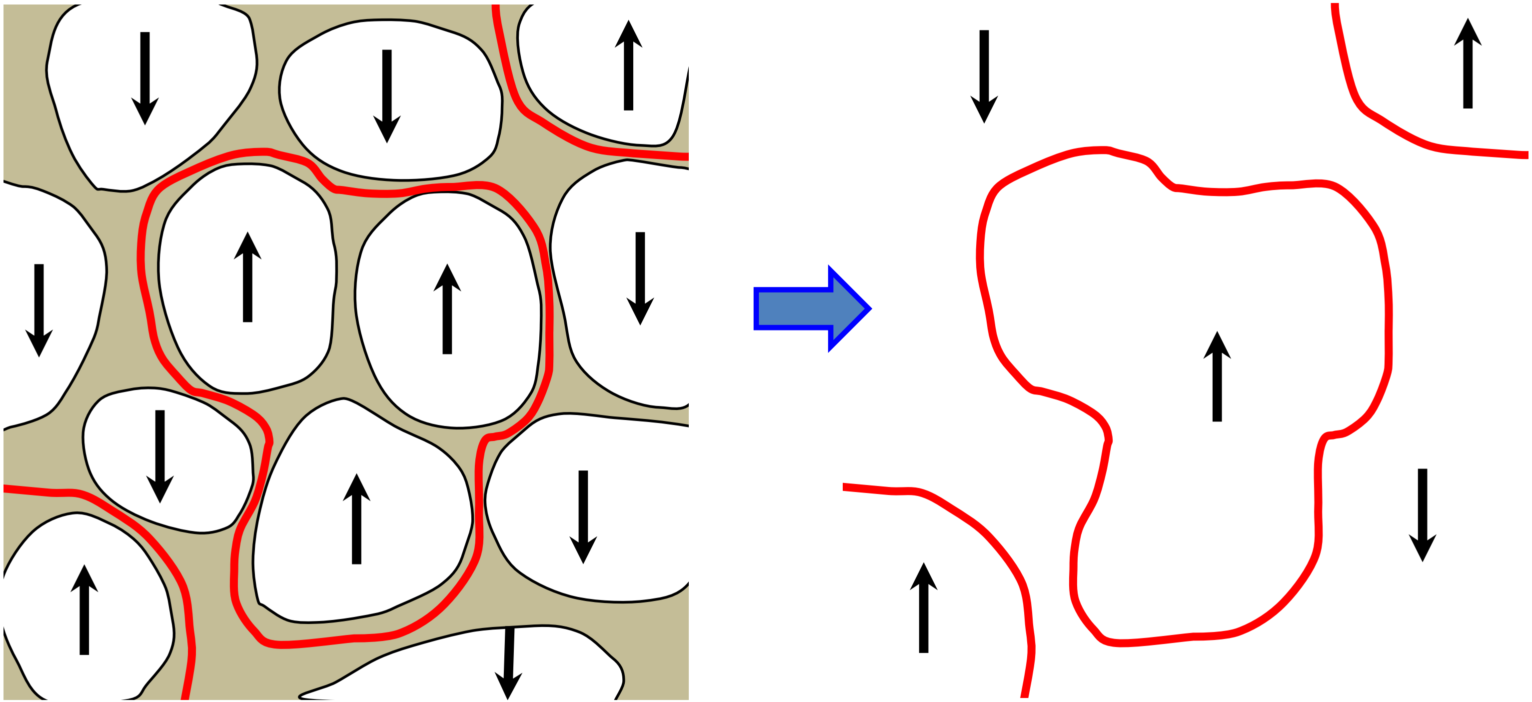
Topology of OPS: $\pi_n (G/H)$

↓

Topological solitons, defects/textures

T.Kibble,
N.D.Mermin
Rev.Mod.Phys.('79),
G.E.Volovik
Universe in a helium droplet

How are they created?
e.g. Kibble-Zurek mechanism @ phase transition

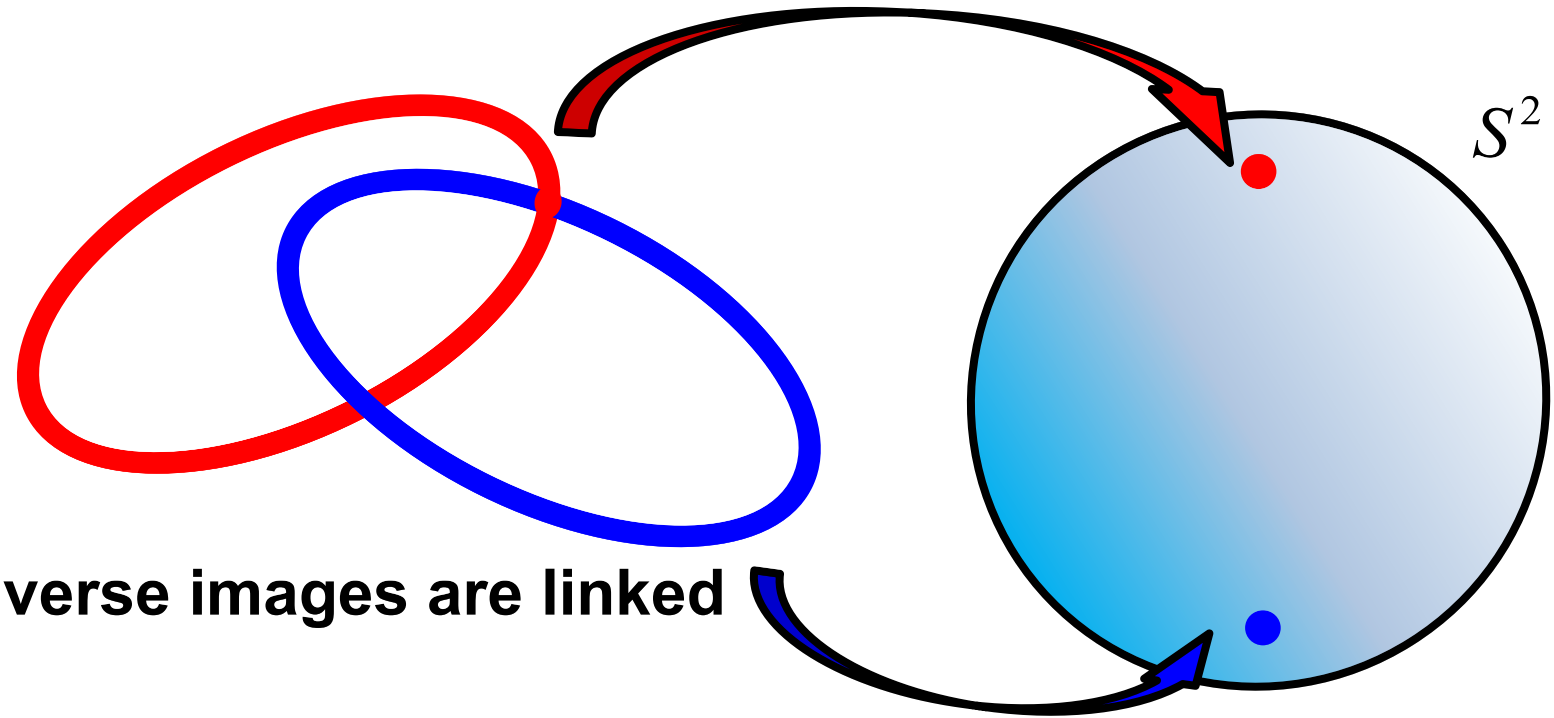


Domain walls

Hopfion (texture)

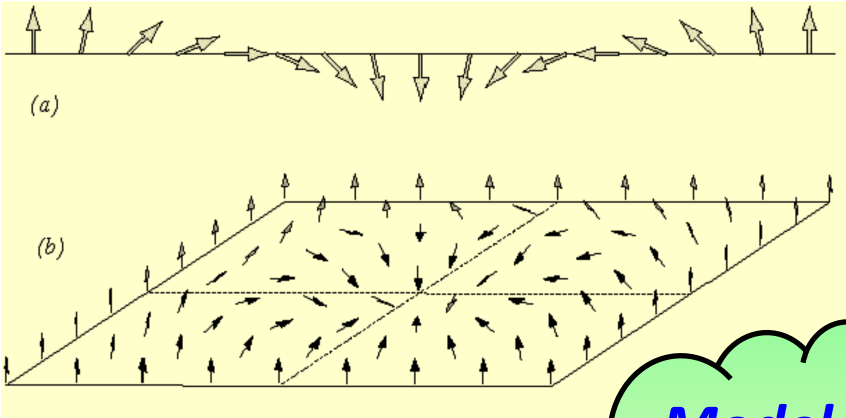
$$\pi_3(S^2) \cong \mathbf{Z}$$

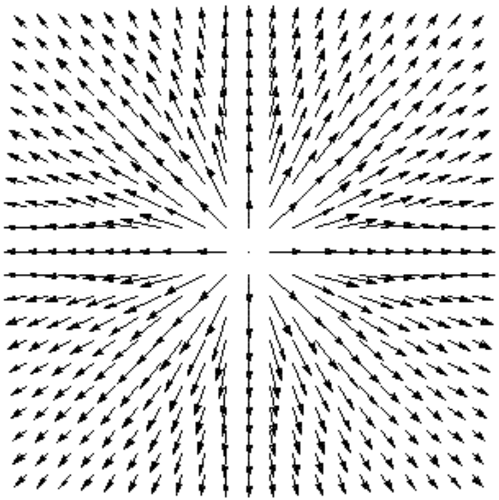
Hopf map



2 inverse images are linked

What is a Skyrmion?





3dim
hedgehog

Model of nucleon in HEP

1D Skyrmion

=Sine-Gordon kink

$$\pi_1(S^1) = \mathbf{Z}$$

2D Skyrmion

$$\pi_2(S^2) = \mathbf{Z}$$

3D Skyrmion

$$\pi_3(S^3) = \mathbf{Z}$$

T.H.R. Skyrme

A Nonlinear theory of strong interactions
Proc.Roy.Soc.Lond. A247 (1958) 260-278
A Unified Field Theory of Mesons and Baryons
Nucl.Phys. 31 (1962) 556-569