The holography of duality in $\mathcal{N} = 4$ SYM theory

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based on arXiv:2208.09396
in collaboration with
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An overview of this talk

$\mathcal{N}=4$ $SU(N)$ **SYM**

Type IIB on $AdS_5 \times S^5$

(gauge inv.) local operators

nonlocal operators
(e.g. Wilson & ’t Hooft loops)

(generating function of)
correlators

=

massless & massive
string excitation modes

branes
(e.g. F & D-strings)

boundary deformed
partition function via GKPW

$\mathcal{N}=4$ $su(N)$ **SYMs**
with
different 1-form symmetries
&
its associated line operator spectrum

Type IIB on $AdS_5 \times S^5$
with
all admissible boundary conditions
on (B_2, C_2)

(gauge inv.) local operators

nonlocal operators
(e.g. Wilson & ’t Hooft loops)

(generating function of)
correlators

=

massless & massive
string excitation modes

branes
(e.g. F & D-strings)

boundary deformed
partition function via GKPW

$\mathcal{N}=4$ $so(N/2)$ & $sp(N/2)$ **SYMs**
with
different 1-form (and 0-form) symmetries
&
line (and point) operator spectrum

Type IIB on $AdS_5 \times RP^5$
with discrete $(\theta_{NS}, \theta_{RR})$
all admissible boundary conditions
on (b, c) & (b', c')

(gauge inv.) local operators

nonlocal operators
(e.g. Wilson & 't Hooft loops)

(generating function of)
correlators

=

massless & massive
string excitation modes

branes
(e.g. F & D-strings
wrapped NS5 & D5-branes)

boundary deformed
partition function via GKPW

Motivation

- Recently, nonlocal operators and higher form symmetries (under which they are charged) gained renewed interests in QFT.

- A new type of symmetry structure: higher group/category with non/invertible symmetry generators

  Gaiotto-Kapustin-Seiberg-Willett, Cordova-Dumitrescu-Intriligator
  Kaidi-Ohmori-Zheng, Choi-Cordova-Hsin-Lam-Shao…

- New insights into the phase structure of QFT via mixed 't Hooft anomaly involving higher form symmetries.

  Gaiotto-Kapustin-Komargodski-Seiberg…

- They can be used to reveal the intricate structure of the duality web.

- Meanwhile, string theory naturally hosts the ingredients for these developments:

(non)local operators = branes

its associated higher form gauge fields = RR and NSNS forms

higher group or category = combining (bound states of) branes

Albertini-Del Zotto-Garcia Etxebarria-Heidenreich-Hosseini-Regalado, Bah-Bonetti-Minasian Apruzzi-van Beest-Gould-Schafer-Nameki…

- Via holography (AdS/CFT), we can thus expect to gain a more elementary and intuitive understanding of the recent developments in higher form symmetries. Aharony-Tachikawa, Bergman-Tachikawa-Zafrir

- It is hoped that this line of study provides new perspectives both on QFT and string theory.

$\mathcal{N}=4\ su(N)$ SYM theories

1. Gauging center symmetry

- The $SU(N)$ group has the Z_N center symmetry. By gauging a subgroup of Z_N, we can construct new theories:

$$\mathcal{N}=4\ \ SU(N)/Z_k\ \ \text{SYM theories} \qquad \text{with}\ \ N=kk'$$

Note: The $SU(N)/Z_N$ theory is the S-dual of the $SU(N)$ theory

- This is not the end of the story and the life is more intricate:

  The $SU(N)/Z_k$ theory are further classified into sub-theories distinguished by the line operator spectrum

Aharony-Seiberg-Tachikawa

2. Line operator spectrum

- The line operators are Wilson, 't Hooft, and dyonic lines (determined by the mutual locality condition $z_e z'_m - z'_e z_m = 0 \bmod N$)

$$L_{k,\ell} := \{(z_e, z_m) = \underbrace{\boxed{e(k,0)}}_{k \text{ Wilson lines}} + \underbrace{\boxed{m(\ell, k')}}_{k' \text{ 't Hooft lines } + \ \ell \text{ Witten effect}} \bmod N\}$$

- The $su(N)$ theories (center symmetry gauging + line operator spectrum)

$$\boxed{\mathcal{N} = 4 \ \ [SU(N)/Z_k]_\ell \ \text{ SYM theories}}$$

where $\ell = 0, 1, \cdots, k-1$ with $N = kk'$

3. 1-form symmetry

- The most basic $SU(N)$ theory has an (electric) Z_N 1-form symmetry.

= gauging magnetic Z_N symmetry

$$W(L) \xrightarrow{Z_N} \underbrace{e^{\frac{2\pi i}{N}}}_{Z_N \text{ charge}} W(L)$$

- The $SU(N)/Z_k$ theories has an (electric) $Z_{k'}$ 1-form symmetry:

= gauging magnetic $Z_{k'}$ symmetry

$$\underbrace{W(L)^k}_{Z_k \text{ invariant}} \xrightarrow{Z_{k'}} e^{\frac{2\pi i k}{N}} W(L)^k = \underbrace{e^{\frac{2\pi i}{k'}}}_{Z_{k'} \text{ charge}} W(L)^k$$

- In fact, the $[SU(N)/Z_k]_0$ theory has $Z^e_{k'} \times Z^m_k$ 1-form symmetry:

$$\underbrace{T(L')^{k'}}_{Z_{k'} \text{ invariant}} \xrightarrow{Z_k} e^{\frac{2\pi i k'}{N}} T(L')^{k'} = \underbrace{e^{\frac{2\pi i}{k}}}_{Z_k \text{ charge}} T(L')^{k'}$$

- The 1-form symmetry of the general $[SU(N)/Z_k]_\ell$ theory

$$\Big(\underbrace{Z_{k'}}_{\text{electric}} \times \underbrace{Z_{N/gcd(k',\ell)}}_{\text{dyonic}}\Big)/Z_{k'/gcd(k',\ell)} = \boxed{Z_{N/\mathrm{gcd}(k,k',\ell)} \times Z_{\mathrm{gcd}(k,k',\ell)}}$$

Gaiotto-Kapustin-Seiberg-Willett

$$\Big\{\underbrace{W(L)^k}_{Z_{k'}\text{ charged}},\ \underbrace{W(L)^\ell T(L)^{k'}}_{Z_{N/gcd(k',\ell)}\text{ charged}}\Big\} \text{ w/ } \underbrace{\left(W(L)^k\right)^\ell = \left(W(L)^\ell T(L)^{k'}\right)^k}_{Z_{k'/gcd(k',\ell)}\text{ charged}}$$

for the line operator spectrum

$$L_{k,\ell} := \{(z_e, z_m) = \underbrace{\boxed{e(k,0)}}_{k\text{ Wilson lines}} + \underbrace{\boxed{m(\ell, k')}}_{k'\text{ 't Hooft lines } + \ \ell\text{ Witten effect}} \mod N\}$$

Holographic description for $\mathcal{N} = 4$ $su(N)$ SYM theories

- The line operator spectrum of all $[SU(N)/Z_k]_\ell$ theories and its center symmetry are encoded in the 5d topological theory (at low energies)

Witten & Aharony-Witten

$$S_{CS} = \int_{AdS_5 \times S^5} B_2 \wedge dC_2 \wedge dC_4 = \frac{N}{2\pi} \int_{AdS_5} B_2 \wedge dC_2$$

- (Global) 1-form symmetries (on the boundary)

$$B_2 \to B_2 + d_z \lambda_1^{NS} \;, \qquad C_2 \to C_2 + d_z \lambda_1^{RR} \quad \textbf{with} \quad d_x \lambda_1 = 0 \quad \& \quad N\lambda_1 = 0$$

- By canonical quantization, (B_2, C_2) are a canonical conjugate pair like (x, p)

$$[b, c] = \frac{2\pi i}{N} \qquad \textbf{with} \qquad b = \int_S B_2 \;, \quad c = \int_{S'} C_2 \qquad (S \cdot S' \neq 0)$$

1. Boundary conditions

S-dual $SU(N)/Z_N$ theory $\quad (\underbrace{c=0}_{\text{Dirichlet}} \, , \, \underbrace{b \text{ free}}_{\text{Neumann}})$

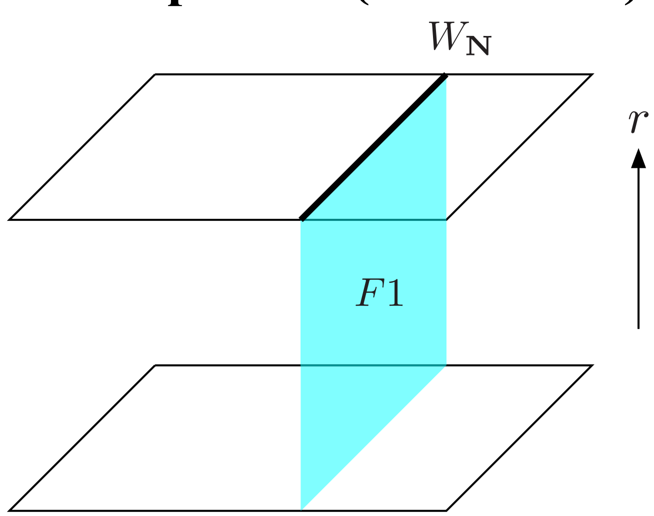
- The simplest case dual to the $SU(N)$ theory

Witten, Aharony-Tachikawa, Bergman-SH

$(\underbrace{b=0}_{\text{Dirichlet}} \, , \, \underbrace{c \text{ free}}_{\text{Neumann}})$ analogous to fixed x and fluctuating p

$Nc = 0$ equivalent to Neumann

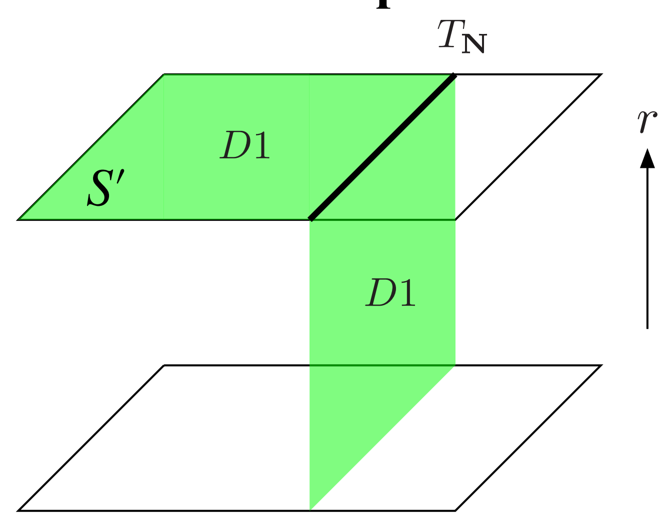
line operator (Wilson line)



a

$b = 0$

surface operator



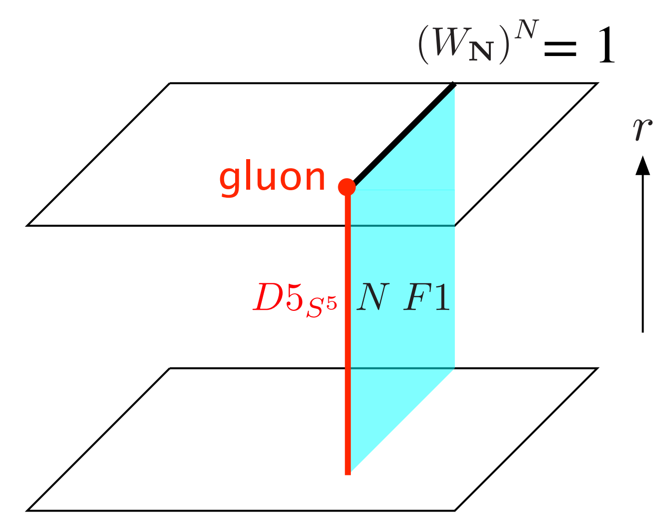
c

$nc \neq 0 \; (n = 0,1,\cdots,N-1)$

$Nc = 0$

$D1$ surface $= AdS_3 \times S^1$ $D3$ with F

Gukov-Witten, Drukker-Gomis-Matsuura



b

$Nb = 0$

C_6 (Dirichlet), B_6 (Neumann)

- The all admissible boundary conditions dual to the $[SU(N)/Z_k]_\ell$ theory

mutual locality = mutually commuting pair

$$z_e b + z_m c = 0 \,, \quad z'_e b + z'_m c = 0 \qquad \textbf{with} \qquad \underbrace{z_e z'_m - z'_e z_m = 0 \bmod N}_{\text{two BCs commute mod } N}$$

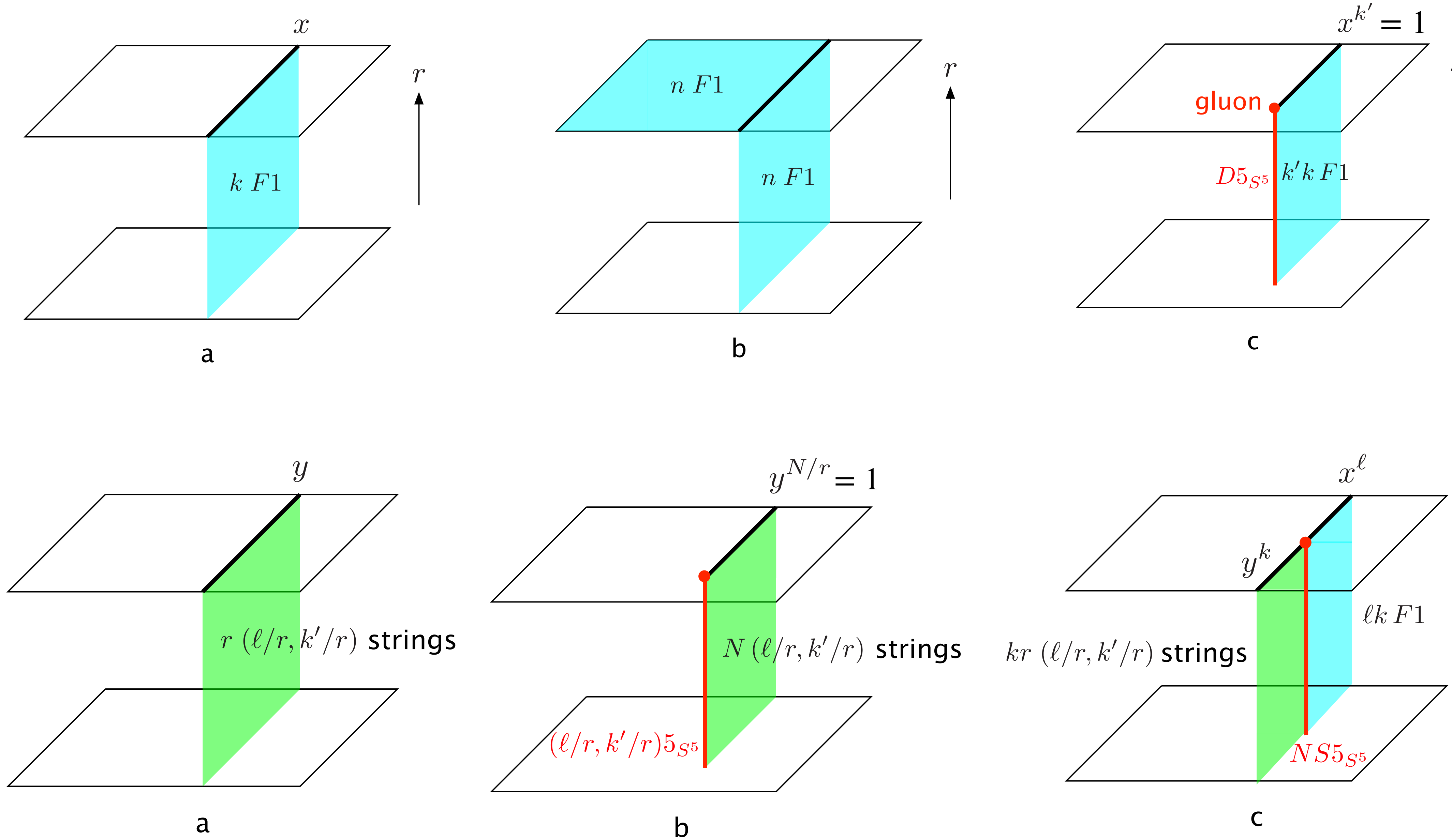
equivalent to Dirac quantisation condition

By an $SL(2,Z)$ rotation, the B.C.s can be brought into a canonical form

$$\begin{pmatrix} z_e & z_m \\ z'_e & z'_m \end{pmatrix} \longrightarrow \begin{pmatrix} k & 0 \\ \ell & k' \end{pmatrix} \Longrightarrow \boxed{\underbrace{kb = 0}_{k \text{ F1}} \,, \quad \underbrace{k'c + \ell b = 0}_{(\ell,k') \text{ string}}}$$

where $\ell = 0,1,\cdots,k-1$

“Dirichlet” = line operator **“Neumann” = surface operator**



$$x \leftrightarrow kb = 0 \qquad r = gcd(k', \ell)$$

$$y \leftrightarrow k'c + \ell b = 0 \qquad n \neq mk\ , N$$

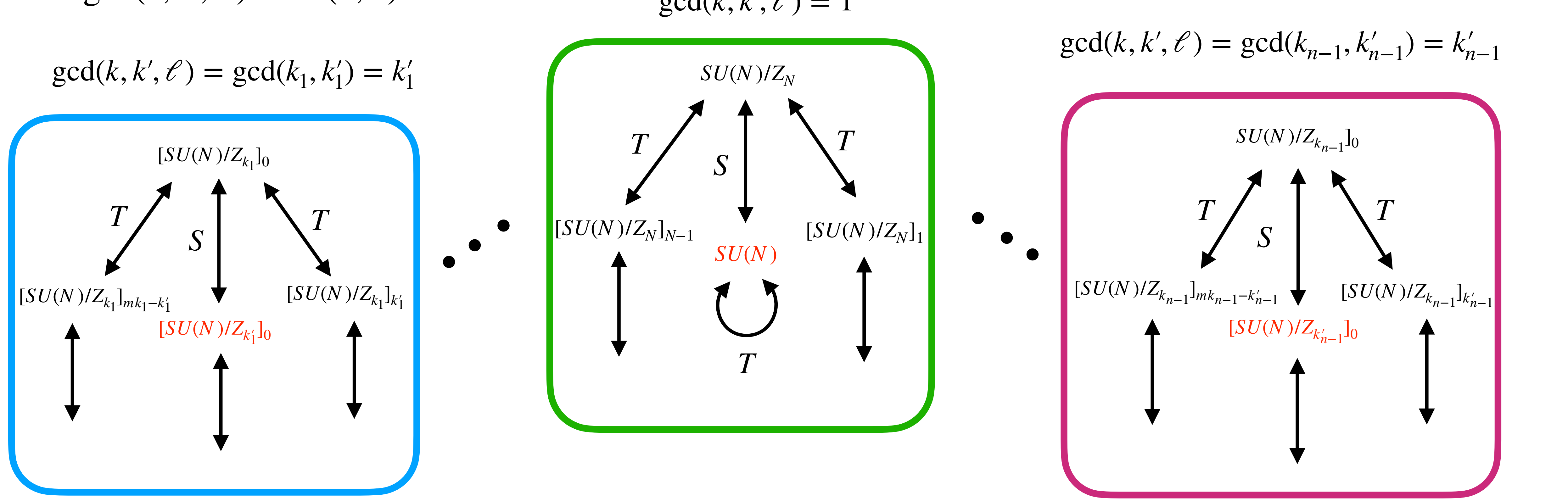
2. $SL(2,Z)$ duality orbits

Bergman-SH

The $SL(2,Z)$ rotates (b, c), and so the duality web of the $su(N)$ theories can be understood by analysing how diff. B.C.s are related by $SL(2,Z)$

$$\boxed{\#\text{ of } SL(2,Z) \text{ orbits} \;=\; \#\text{ of } k \text{ with } k = \gcd(k, N/k) \;=\; \#\text{ of square divisors of } N}$$

Note: $\gcd(k, k', \ell)$ **is** $SL(2,Z)$ **invariant**



$SL(2,Z)$ duality web of $\mathcal{N} = 4$ $su(N)$ SYM theories Aharony-Seiberg-Tachikawa

3. Mixed 't Hooft anomaly from type IIB SUGRA

- There are two types of mixed anomalies in the $su(N)$ (S)YM theories:

  (1) The 1-form symmetries $Z_{N/\mathrm{gcd}(k,k',\ell)} \times Z_{\mathrm{gcd}(k,k',\ell)}$ are anomalous if we try to gauge both 1-form symmetries (or equivalently, in the presence of the background 2-form gauge fields).
  Gaiotto-Kapustin-Seiberg-Willett, Hsin-Lam

  (2) The electric $Z_{k'}$ subgroup of the 1-form symmetries combined with the shift $\theta \to \theta + 2\pi k$ is anomalous if we try to gauge the $Z_{k'}$ 1-form symmetry.
  Gaiotto-Kapustin-Kormagodski-Seiberg

- Both are accounted for, holographically, by the the same type IIB CS action

$$\boxed{S_{CS} = \underbrace{\int_{AdS_5 \times S^5} B_2 \wedge dC_2 \wedge F_5}_{\text{former}} = \int_{AdS_5 \times S^5} B_2 \wedge \tilde{F}_3 \wedge F_5 + \underbrace{\int_{AdS_5 \times S^5} C_0 B_2 \wedge dB_2 \wedge F_5}_{\text{latter}}}$$

(continuous version, to be precise)

(1) The mixed $Z_{\mathrm{gcd}(k,k',\ell)}$ anomaly between 1-form symmetries $Z_{N/\mathrm{gcd}(k,k',\ell)} \times Z_{\mathrm{gcd}(k,k',\ell)}$

$$S_{CS} = \frac{N}{2\pi}\int_{AdS_5} B_2' \wedge dC_2' \ + \ ''\text{local counterterms}'' \quad \textbf{w/} \quad \int B_2' \in \frac{\mathbb{Z}}{N}\ , \int dC_2' \in \frac{\mathbb{Z}}{\mathrm{gcd}(k,k',\ell)}$$

where (B_2', C_2') is an $SL(2,Z)$ rotation of (B_2, C_2) and diagonal w.r.t. the 1-form symmetries

(2) The mixed anomaly between 1-form symmetries $\theta \to \theta + 2\pi k$ and the electric $Z_{k'}$ 1-form symmetry

$$S_{CS} = -\frac{N}{4\pi}\int_{AdS_5} dC_0 \wedge B_2 \wedge B_2 \quad \textbf{w/} \quad C_0 = \theta/2\pi \ \& \ k'B_2 = \mathsf{B}_{\mathrm{FT}} \in H^2(X, Z_{k'})$$

To be more precise, the $\mathcal{O}(N^2)$ term is missing and it agrees only on spin manifolds X

4. Axionic Janus as interfaces between different θ angles

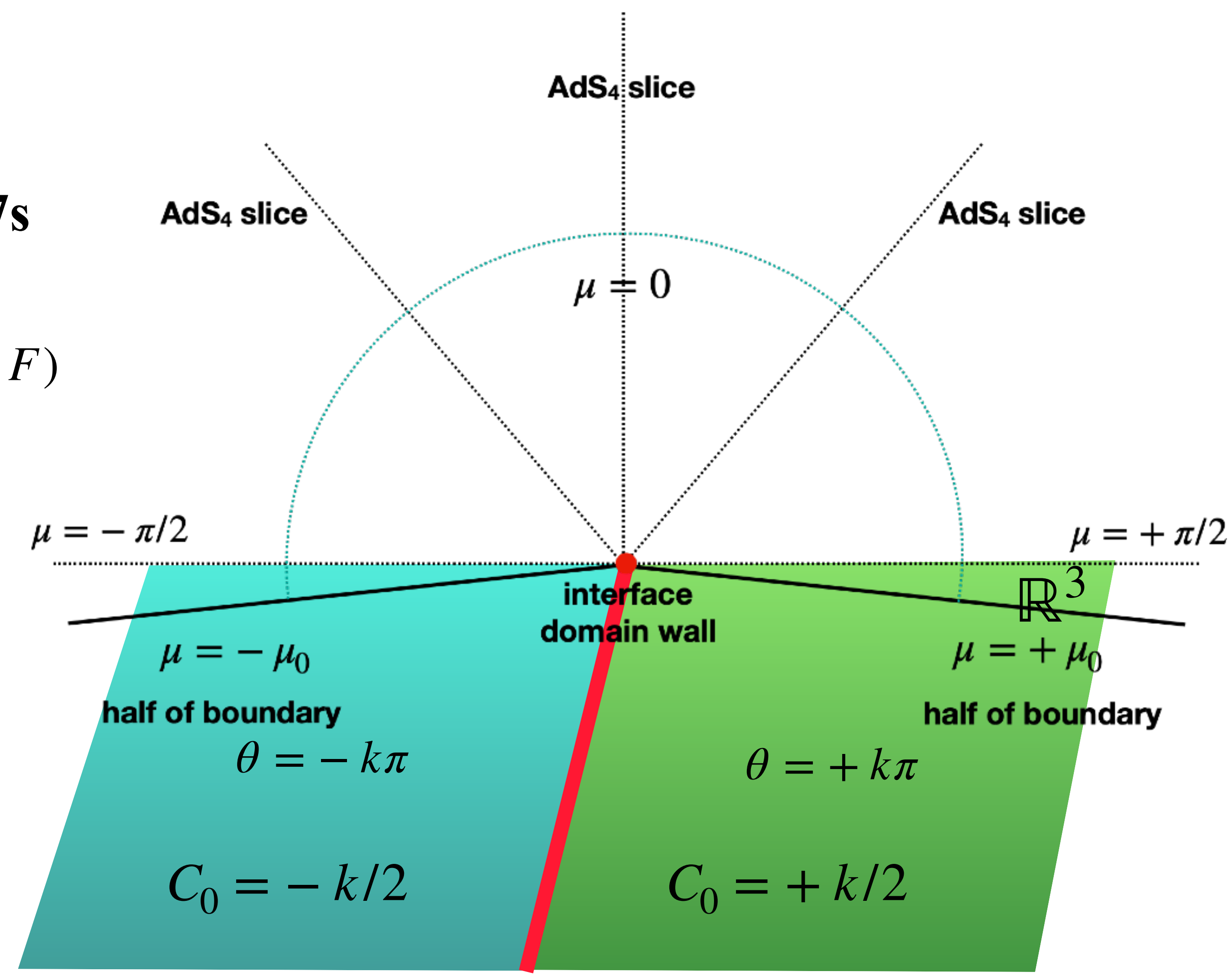
D'Hoker-Estes-Gutperle   Bak-Gutperle-SH

- For pure $SU(N)$ YM, the mixed 't Hooft anomaly implies SSB CP at $\theta = \pi$ (out of options, nontrivial gapless theory, gapped TFT, or SSB) and the existence of domain walls. Meanwhile, $\mathcal{N} = 4$ $SU(N)$ SYM is conformal and a nontrivial gapless theory w/o SSB. No dynamical DW, but interfaces separating $\theta = \pm \pi$ exist.

The interface theory = $U(k)_{-N}$ CS theory on D7s

$$S_{D7} = \int_{R^{1,2}\times S^5} C_4 \wedge \mathrm{Tr}_{U(k)}(F \wedge F) = -N \int_{R^{1,2}} \mathrm{Tr}_{U(k)}(A \wedge F)$$

Janus = deformed AdS_5
(in AdS_4 slices) with two faces



k D7 branes wrapping $S^5 = (2+1)$d wall in $\mathbb{R}^{3,1}$

Holographic description for $\mathcal{N}=4$ $so(N/2)$ SYM theories

- The basic differences from $su(N)$:

(1) N D3s $\longrightarrow$ even N D3s + $O3^-$ for $so(2n)$ w/ $(\theta_{RR},\theta_{NS})=(0{,}0)$

+ $\widetilde{O3}^-$ for $so(2n+1)$ w/ $(\theta_{RR},\theta_{NS})=(1/2{,}0)$ $\quad S$

+ $O3^+$ for $sp(n)$ w/ $(\theta_{RR},\theta_{NS})=(0{,}1/2)$ $\quad T$

+ $\widetilde{O3}^+$ for $\widetilde{sp}(n)$ w/ $(\theta_{RR},\theta_{NS})=(1/2{,}1/2)$

where (B_2, C_2) is projected out under orientifold $\Omega(-1)^{F_L}$ except for Z_2 holonomies $\theta_{RR}=\int C_2$ and $\theta_{NS}=\int B_2$ that leave the wavefunction $e^{2\pi i\theta}$ invariant under $\Omega(-1)^{F_L}$

(2) The internal space: $S^5 \longrightarrow RP^5 = S^5/Z_2$ (and $\int_{S^5} F_5 = N \longrightarrow \int_{RP^5} F_5 = N/2$)

$G^{(1)}$ **= 1-form symmetries**

- The groups for $so(N/2)$ and $sp(N/2)$

$so(4k+2)$

theory	(z_e, z_m)	$G^{(1)}$
$Spin(4k+2)$	$(S, I)^n$	$\mathbb{Z}_4$
$SO(4k+2)_0$	(V^n, V^m)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
$SO(4k+2)_1$	$(S, V)^n$	$\mathbb{Z}_4$
$(Spin(4k+2)/\mathbb{Z}_4)_\ell$	$(S^\ell, S)^n$	$\mathbb{Z}_4$

$\ell = 0,1,2,3$

$so(2n+1)$ & $sp(n)$

theory	(z_e, z_m)	$G^{(1)}$
$Spin(2n+1)$	$(S, I)^n$	$\mathbb{Z}_2$
$SO(2n+1)_0$	$(I, V)^n$	$\mathbb{Z}_2$
$SO(2n+1)_1$	$(S, V)^n$	$\mathbb{Z}_2$
$Sp(n)$	$(V, I)^n$	$\mathbb{Z}_2$
$(Sp(n)/\mathbb{Z}_2)_0$	$(I, S)^n$	$\mathbb{Z}_2$
$(Sp(n)/\mathbb{Z}_2)_1$	$(V, S)^n$	$\mathbb{Z}_2$

$so(4k)$

theory	$(z_{e_S}, z_{e_C}, z_{m_S}, z_{m_C})$	$G^{(1)}$
$Spin(8k+4j)$	(S^n, C^m, I, I)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
$SO(8k+4j)_{\ell_V}$	$(S^{n+\ell_V m}, C^n, S^m, C^m)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
$Ss(8k+4j)_{\ell_S}$	$(S^n, C^{\ell_S m}, S^{(j+1)m}, C^{jm})$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
$Sc(8k+4j)_{\ell_C}$	$(S^{\ell_C m}, C^n, S^{jm}, C^{(j+1)m})$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
$(SO(8k+4j)/\mathbb{Z}_2)_{\substack{\ell_{SS}\ell_{SC}\\ \ell_{CS}\ell_{CC}}}$	$(S^{\ell_{SS}n+\ell_{CS}m}, C^{\ell_{SC}n+\ell_{CC}m}, S^n, C^m)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$

$\ell = 0,1$ & $j = 0,1$

1. Line operator spectrum & strings

$so(4k)$ theories

$G^{(1)}$ = 1-form symmetries

theory	$(z_{e_S}, z_{e_C}, z_{m_S}, z_{m_C})$	$G^{(1)}$
$Spin(8k+4j)$	(S^n, C^m, I, I)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
$SO(8k+4j)_{\ell_V}$	$(S^{n+\ell_V m}, C^n, S^m, C^m)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
$Ss(8k+4j)_{\ell_S}$	$(S^n, C^{\ell_S m}, S^{(j+1)m}, C^{jm})$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
$Sc(8k+4j)_{\ell_C}$	$(S^{\ell_C m}, C^n, S^{jm}, C^{(j+1)m})$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
$(SO(8k+4j)/\mathbb{Z}_2)_{\substack{\ell_{SS}\ell_{SC}\\ \ell_{CS}\ell_{CC}}}$	$(S^{\ell_{SS}n+\ell_{CS}m}, C^{\ell_{SC}n+\ell_{CC}m}, S^n, C^m)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$

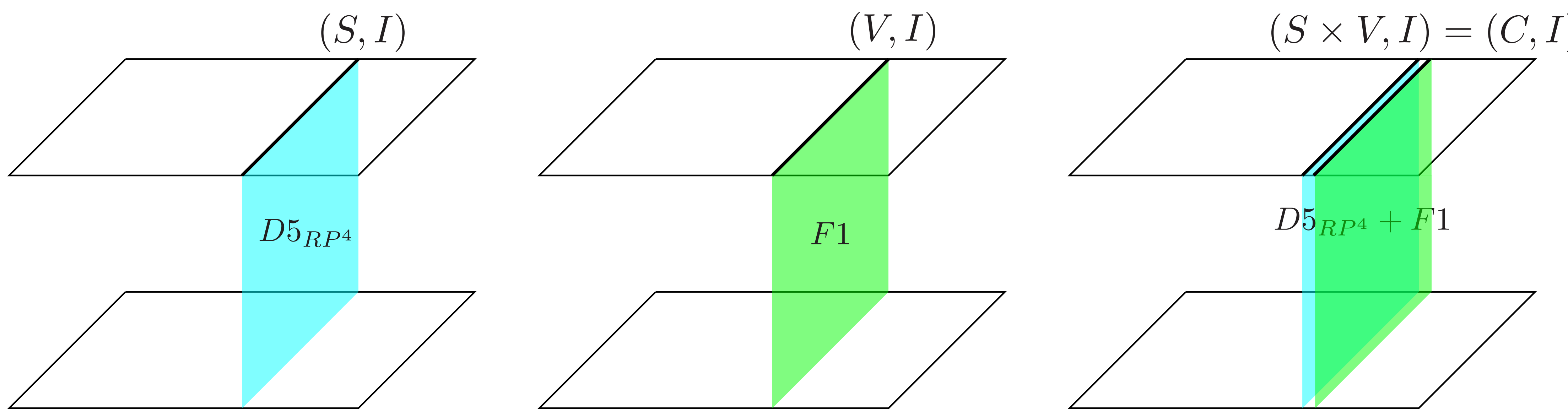
(1) S $\times$ C =V vector (Wilson, 't Hooft) lines = (F1, D1)

(2) S-spinor (Wilson, 't Hooft) lines = (D5 on RP^4, NS5 on RP^4)

(3) S $\times$ V = C-spinor (W, T) lines = (D5 on RP^4 + F1, NS5 on RP^4 + D1)

$\boxed{\text{3-5 F-string ground states = electric fermions}}$

all Z_2 charged: 2$\times$(branes) = nothing



(1) S $\times$ C =V vector (Wilson, ’t Hooft) lines = (F1, D1)

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(3) S $\times$ V = C-spinor (W, T) lines = (D5 on RP^4 + F1, NS5 on RP^4 + D1)

$\boxed{\text{3-5 F-string ground states = fermions}}$

all Z_2 charged: 2×(branes) = nothing

2. Boundary conditions

Bergman-SH

- The line operator spectrum of all these theories and its center symmetry are encoded in the 5d topological theory (at low energies)

$$S_{CS}[B_2, C_2, \tilde{B}_2, \tilde{C}_2] = \int_{AdS_5} \left(\underbrace{\frac{n}{2\pi} \overset{b}{B_2} \wedge d\overset{c}{C_2}}_{\text{CS}} + \underbrace{\frac{1}{\pi} B_2 \wedge d\overset{\tilde{b}}{\tilde{B}_2}}_{B_2 \text{ kinetic}} + \underbrace{\frac{1}{\pi} C_2 \wedge d\overset{\tilde{c}}{\tilde{C}_2}}_{C_2 \text{ kinetic}} \right)$$

where $(\tilde{B}_2, \tilde{C}_2) = \left(\int_{\mathbb{R}P^4} B_6, \int_{\mathbb{R}P^4} C_6 \right)$ and $(dB_6, dC_6) = ({}^*dB_2, {}^*dC_2)$

- The canonical quantization yields the commutation relations

$$[c, \tilde{c}] = [b, \tilde{b}] = \pi i \mod 2\pi i \,, \qquad [\tilde{b}, \tilde{c}] = \frac{n\pi i}{2} \mod 2\pi i$$

- The all admissible boundary conditions dual to these theories

mutual locality = mutually commuting pair

$$n_b b + n_c c + n_{\tilde{b}} \tilde{b} + n_{\tilde{c}} \tilde{c} = 0$$
$$n'_b b + n'_c c + n'_{\tilde{b}} \tilde{b} + n'_{\tilde{c}} \tilde{c} = 0$$

with $n_b n'_{\tilde{b}} + n_{\tilde{b}} n'_b + n_c n'_{\tilde{c}} + n_{\tilde{c}} n'_c + j(n_{\tilde{b}} n'_{\tilde{c}} + n_{\tilde{c}} n'_{\tilde{b}}) = 0 \text{ mod } 2$

$Z_2 \times Z_2$

theory	boundary conditions
$Spin(8k+4j)$	$\tilde{c}=0$ $b=0$
$SO(8k+4j)_{\ell_V}$	$b=0$ $c+\ell_V\tilde{c}=0$
$Ss(8k+4j)_{\ell_S}$	$\tilde{c}=0$ $\tilde{b}+jc+\ell_S b=0$
$Sc(8k+4j)_{\ell_C}$	$\tilde{c}+b=0$ $c+\tilde{b}+jc+\ell_C\tilde{c}=0$
$(SO(8k+4j)/\mathbb{Z}_2)_{\ell_{CS}\ell_{CC}}^{\ell_{SS}\ell_{SC}}$	$\tilde{b}+(\ell_{SS}+\ell_{SC})\tilde{c}+\ell_{SC}b=0$ $c+\tilde{b}+(\ell_{CS}+\ell_{CC})\tilde{c}+\ell_{CC}b=0$

Z_4 or $Z_2 \times Z_2$

theory	BC's
$Spin(4k+2)$	$\tilde{c}=0$
$SO(4k+2)_0$	$2\tilde{b}=0,\ 2\tilde{c}=0$
$SO(4k+2)_1$	$\tilde{c}+2\tilde{b}=0$
$(Spin(4k+2)/\mathbb{Z}_4)_\ell$	$\tilde{b}+\ell\tilde{c}=0$

$b = 2\tilde{c}$, $c = 2\tilde{b}$

Z_2

theory	boundary conditions
$Spin(2n+1)$	$\tilde{c}=0$
$SO(2n+1)_0$	$c=0$
$SO(2n+1)_1$	$c+\tilde{c}=0$
$Sp(n)$	$b=0$
$(Sp(n)/\mathbb{Z}_2)_0$	$\tilde{b}=0$
$(Sp(n)/\mathbb{Z}_2)_1$	$b+\tilde{b}=0$

$b, \tilde{b}$ absent

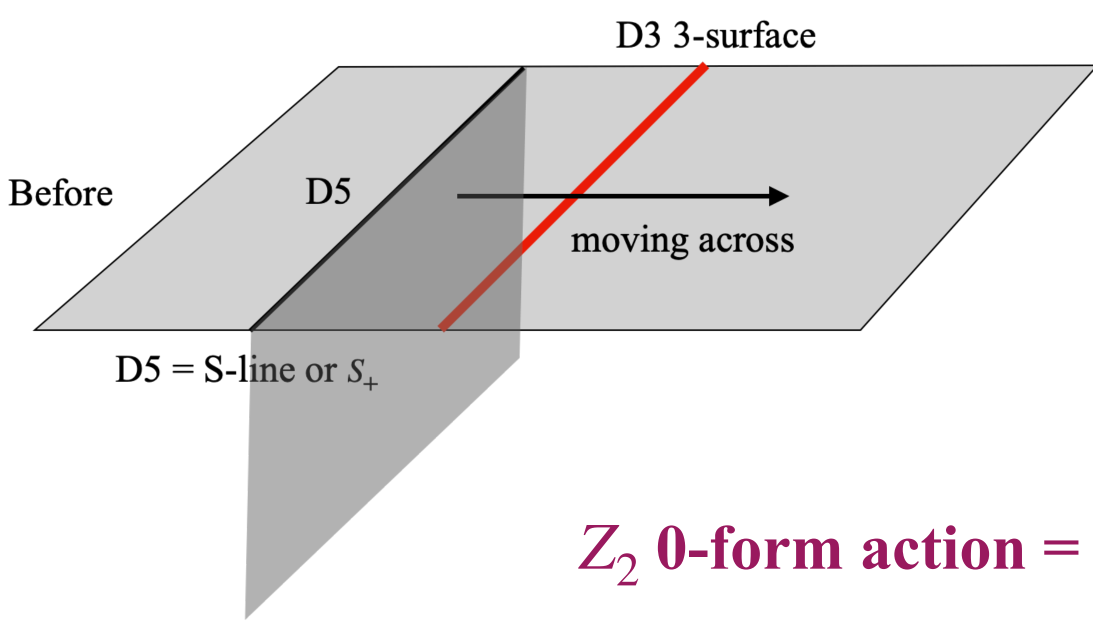
Holographic description of higher group/category

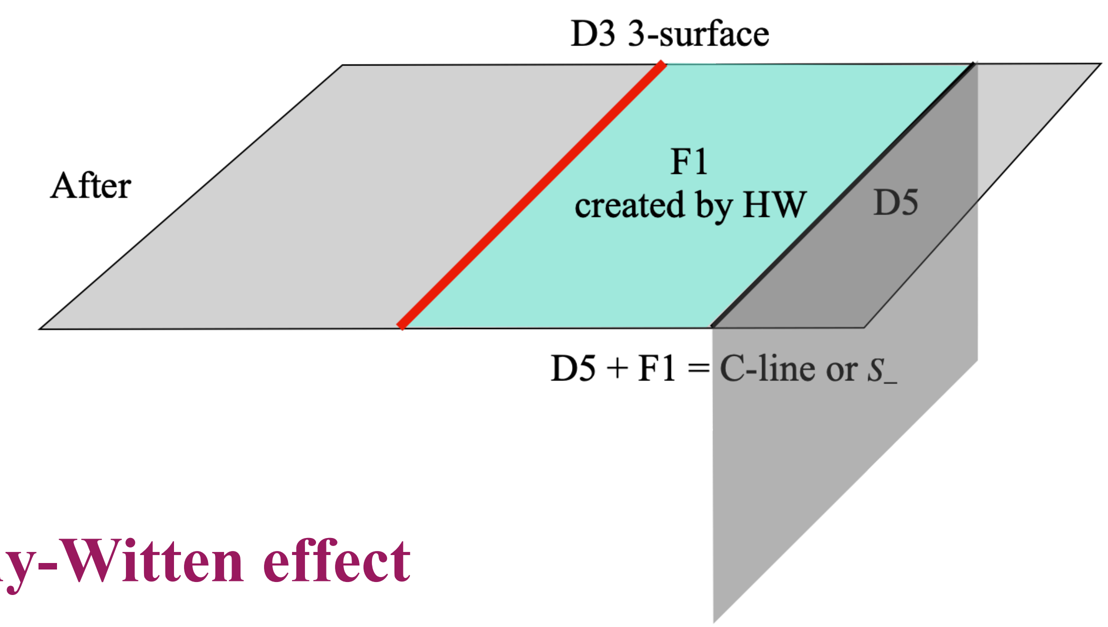
Garcia Etxebarria, Apruzzi-Bah-Bonetti-Schafer-Nameki

- In addition to the $Z_2 \times Z_2$ 1-form symmetry, the $so(4k)$ theories have a Z_2 0-form symmetry that exchanges S and C-spinors. These two together form a 2-category symmetry, in which the 0-form generator $G^{(0)}$ is noninvertible, with the following fusions:

$$G^{(0)}(M_3) \times G^{(0)}(M_3) = 1 + G^{(1)}(M_2)$$
$$G^{(0)}(M_3) \times G^{(1)}(M_2) = G^{(0)}(M_3)$$

where $G^{(0)}(M_3) = Z_{D3\,DW\,\mathrm{on}\,RP^1}(F=0) + Z_{D3\,DW\,\mathrm{on}\,RP^1}(F=1)$ and $G^{(1)}(M_2) = Z_{D1\,\mathrm{surface}}$





Z_2 0-form action = Hanany-Witten effect

Discussions

- An ensemble of $\mathcal{N}=4$ SYM theories? Could there be the factorization problem in AdS_{d+1}/CFT_d with $d \geq 3$?

  As an example, consider $\mathrm{CFT}_L[SU(N)/Z_N] \times \mathrm{CFT}_R[SU(N)/Z_N]$ and identify two magnetic flux vectors $m_L = m_R = m \in H^2(X, Z_2)$. The resulting theory is the self S-dual $(SU(N) \times SU(N))/Z_N$ theory with the S-duality invariant partition function

  $$Z_{SU(N)\times SU(N)/Z_N} = \sum_{m\in H^2(X,Z_2)} Z_m(e^{2\pi i\tau_1})Z_m(e^{2\pi i\tau_2}) \qquad \text{Vafa-Witten}$$

  which is not factorized.

  This is different from the TFD state, so it is not quite the two-sided AdS Schwarzschild. Is there a dual geometry and is it a spacetime wormhole?

Thank you!

Back-up: diagonal line operators

- In the diagonal basis, the 1-form symmetry of the $[SU(N)/Z_k]_\ell$ theory

$$\boxed{Z_{N/\mathrm{gcd}(k,k',\ell)} \times Z_{\mathrm{gcd}(k,k',\ell)}}$$

under which the following line operators are charged:

$$W(L)^{pk+\ell}T(L)^{k'} \xrightarrow{Z_{N/\mathrm{gcd}(k,k',\ell)}} e^{\frac{2\pi i}{N/\mathrm{gcd}(k,k',\ell)}} \left(W(L)^{pk+\ell}T(L)^{k'}\right)$$

Bergman-SH

$$W(L)^{\frac{\delta N}{\mathrm{gcd}(k,k',\ell)}}T(L)^{\frac{\gamma N}{\mathrm{gcd}(k,k',\ell)}} \xrightarrow{Z_{\mathrm{gcd}(k,k',\ell)}} e^{\frac{2\pi i}{\mathrm{gcd}(k,k',\ell)}} \left(W(L)^{\frac{\delta N}{\mathrm{gcd}(k,k',\ell)}}T(L)^{\frac{\gamma N}{\mathrm{gcd}(k,k',\ell)}}\right)$$

where there always exists $^\exists p \in Z$ such that $\mathrm{gcd}(pk+\ell, k') = \mathrm{gcd}(k,k',\ell)$

$$\delta \frac{k'}{\mathrm{gcd}(k,k',\ell)} - \gamma \frac{pk+\ell}{\mathrm{gcd}(k,k',\ell)} = 1$$

Back-up: $SL(2,Z)$ duality

- The different $[SU(N)/Z_k]_\ell$ theories are connected by $SL(2,Z)$.

- However, not all the $[SU(N)/Z_k]_\ell$ theories belong to a single $SL(2,Z)$ orbit. There are "islands" of $SL(2,Z)$ orbits distinguished by the 1-form symmetry:

$$Z_{N/\mathrm{gcd}(k,k',\ell)} \times Z_{\mathrm{gcd}(k,k',\ell)}$$

- This can be most manifestly understood from the $SL(2,Z)$ transformations of the boundary conditions (in the diagonal basis of the 1-form symmetry):

$$\boxed{\begin{array}{r} Z_{N/gcd(k,k',\ell)} : \\ Z_{gcd(k,k',\ell)} : \end{array} \quad \begin{pmatrix} pk+\ell & k' \\ \frac{\delta N}{\mathrm{gcd}(k,k',\ell)} & \frac{\gamma N}{\mathrm{gcd}(k,k',\ell)} \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} \equiv M_D \begin{pmatrix} b \\ c \end{pmatrix} = 0}$$

$$\begin{array}{lll} T: & B \to B\,, & C \to C+B \\ S: & B \to -C\,, & C \to B \end{array}$$

- The duality orbit can be understood from the following relation:

$$\underbrace{M_D \begin{pmatrix} \frac{pk+\ell}{gcd(k,k',\ell)} & \frac{k'}{gcd(k,k',\ell)} \\ \delta & \gamma \end{pmatrix}}_{SL(2,Z)\text{ duality rotation}} = \underbrace{\begin{pmatrix} gcd(k,k',\ell) & 0 \\ 0 & N/gcd(k,k',\ell) \end{pmatrix}}_{[SU(N)/Z_{gcd(k,k',\ell)}]_0\text{ theory}}$$

Back-up: Mixed anomaly

- There is a mixed 't Hooft anomaly by gauging the (electric) $Z_{k'}$ subgroup of the 1-form symmetry in the presence of θ, which breaks CP at $\theta = \pi$.
  For the $SU(N)$ theory ($k' = N$)

Gaiotto-Kapustin-Seiberg-Willett

$$Z[\theta + 2\pi] = Z[\theta]\exp\left[2\pi i \frac{N-1}{N}\int_X \frac{\mathcal{P}(\mathsf{B})}{2}\right]$$

where $\mathsf{B} \in H^2(X, Z_N)$ the background 2-form gauge field, $\mathcal{P}(\,\cdot\,)$ the Pontryagin square operation; $\mathcal{P}(\mathsf{B})/N \simeq$ fractional instanton number by Z_N gauging.

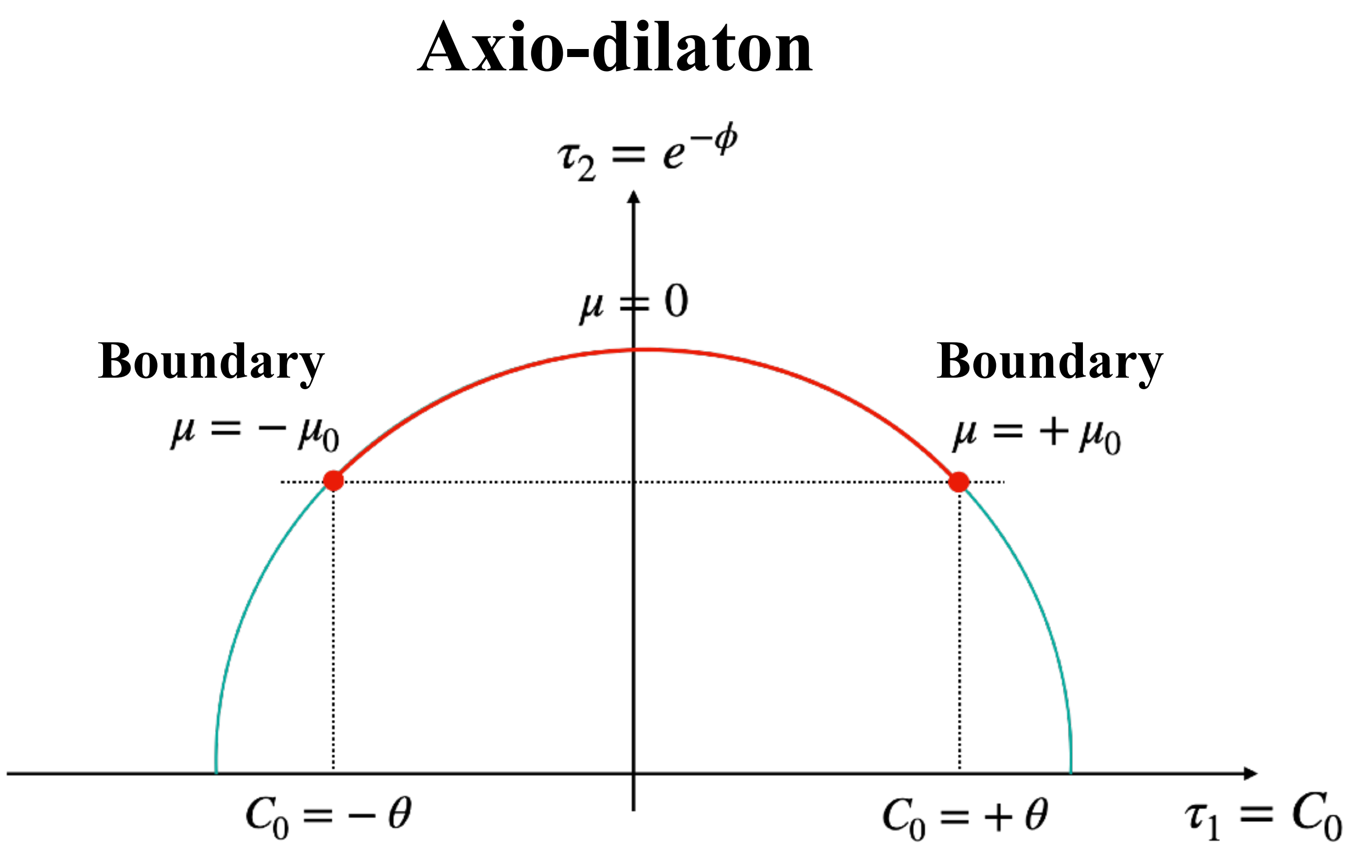
- The anomaly action (to be reproduced by gravity dual)

$$S_{5d} = 2\pi i \frac{N-1}{N}\int \frac{d\theta}{2\pi}\frac{\mathcal{P}(\mathsf{B})}{2} \xrightarrow{SU(N)/Z_k} 2\pi i \frac{N(N-1)}{k'^2}\int \frac{d\theta}{2\pi}\frac{\mathcal{P}(\mathsf{B})}{2}$$

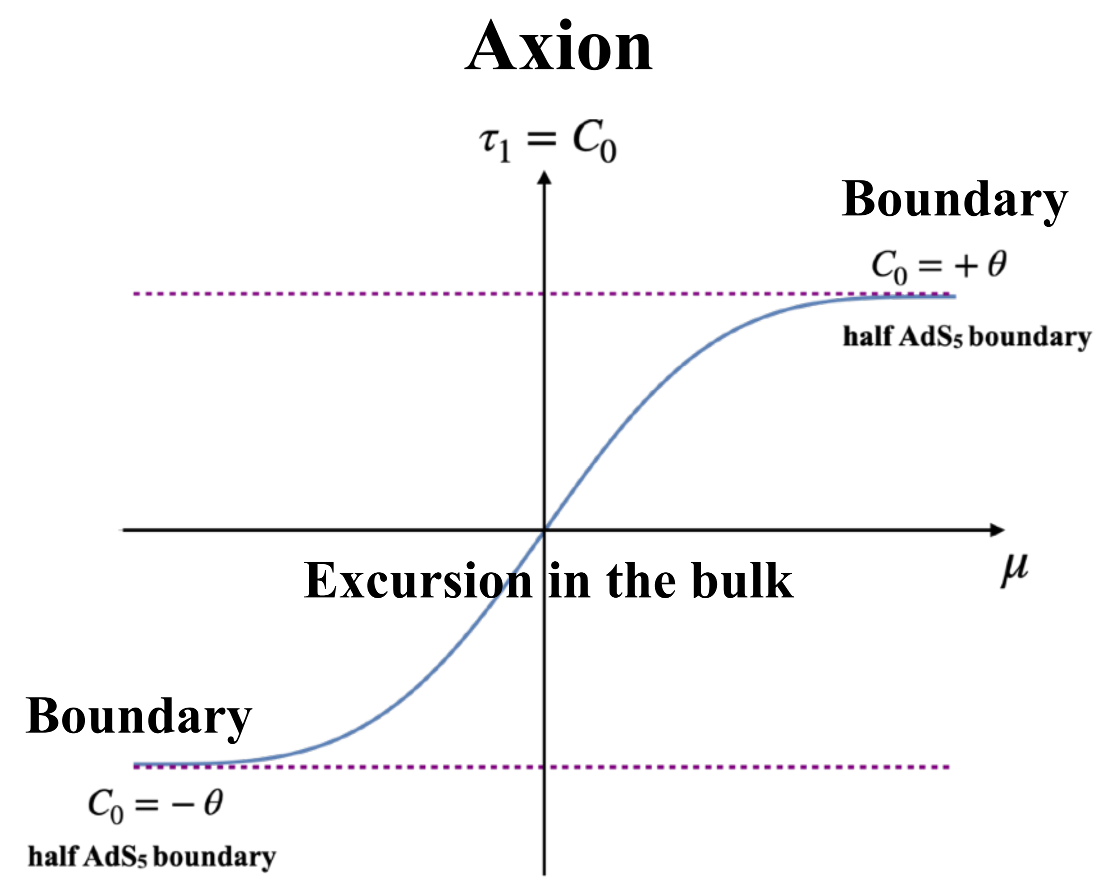
where $\mathsf{B} \in H^2(X, Z_{k'})$ for the latter

Back-up: Axionic Janus

- The relevant part of type IIB SUGRA is that of gravity $g_{\mu\nu}$ and axio-dilaton $\tau = C_0 + ie^{-\phi}$



The dilaton = YM coupling does not vary in the boundary, whereas the axion = θ angle jumps across the interface



Another illustration of a sadden jump of the axion across the interface in the boundary

Ansatz

$$ds^2 = h(\mu)\left(d\mu^2 + ds^2_{AdS_4}\right) + d\Omega_5^2$$

$$F_5 = 2h(\mu)^{5/2} d\mu \wedge \omega_{AdS_4} + 2\omega_{S^5}$$

$$\frac{\tau''}{\tau'} + \frac{3h'}{2h} + i\frac{\tau'}{\mathrm{Im}(\tau)} = 0$$

Solution

$$h'^2 - 4h^3 + 4h^2 = \frac{c_0^2}{6h}$$

$$|\tau'|^2/(\mathrm{Im}(\tau))^2 = c_0^2/h^3$$

$$|\tau(\mu)|^2 = r^2 \qquad (r \in \mathbb{R})$$