Fluid model of black hole/string transition

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How to obtain analytic solution of black hole/string transition

Black hole/string transition near string scale (Hagedorn temp.)

String bound state (self-gravitating string) ⟺ (transition) ⟺ Black hole

Bound state of strings is described by Horowitz-Polchinski model

- How to solve EOM analytically?
- What is solution near black hole/string?

H-P model is solved only numerically even in linear gravity approx.

⇒ Some simplification is necessary

Strings in bound state can be approximated by perfect fluid

Black hole is fluid of strings

Black hole/string transition near string scale (Hagedorn temp.)

String bound state (self-gravitating string)	$\xleftrightarrow{\text{transition}}$	Black hole

Temperature of strings effectively exceeds Hagedorn temperature

$\Rightarrow$ Strings become fluid in the bound state

We derive geometry of string fluid

High temperature: bound state of strings

Low temperature: approximately black hole (horizonless)

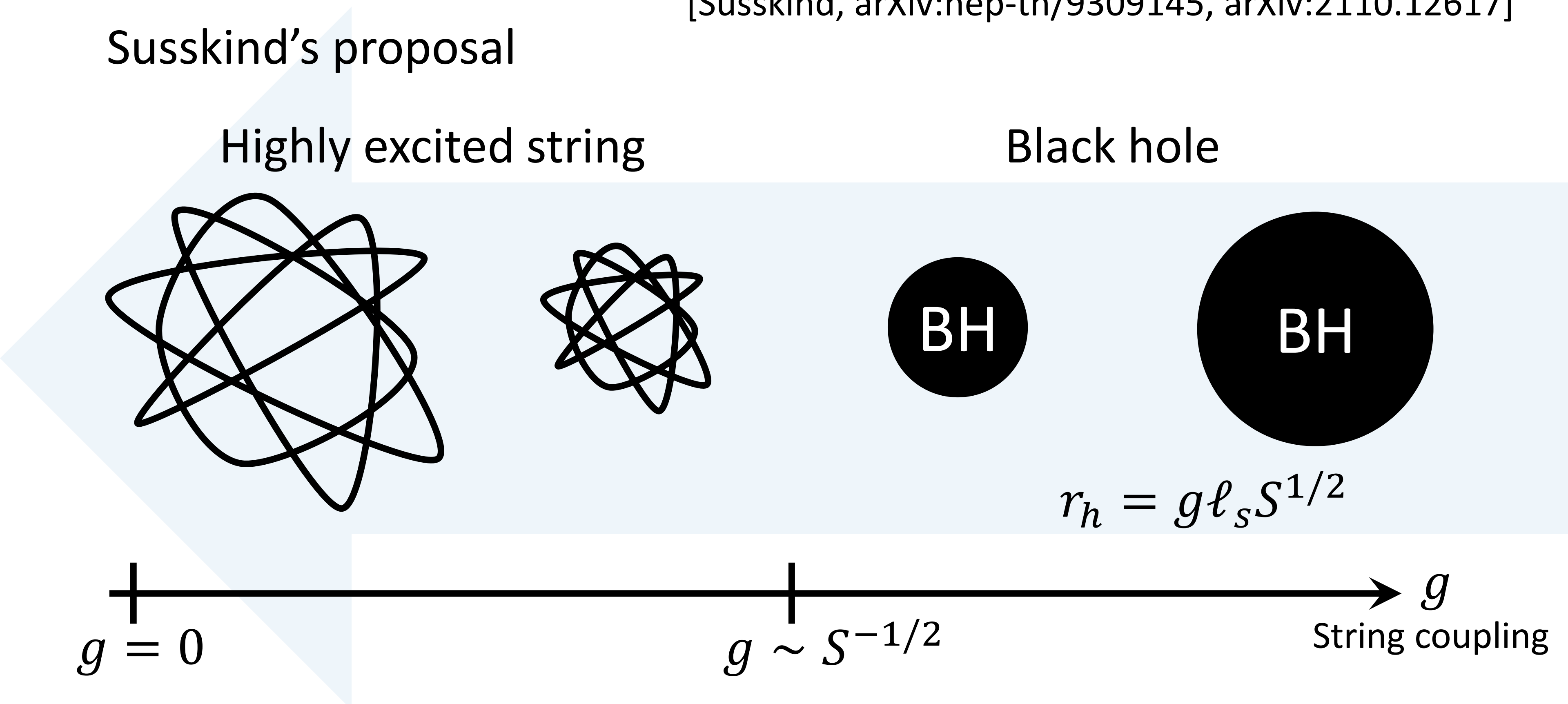
Plan of Talk

1. Black hole/string transition
    - String becomes black hole by self-gravitation
    - Horowitz-Polchinski model: Self-gravitating winding string
    - Temperature effectively exceeds Hagedorn temperature
2. Fluid model of self-gravitating strings
3. Geometry of winding string fluid
4. Black hole/string transition

String become black hole by self-gravitation

[Susskind, arXiv:hep-th/9309145, arXiv:2110.12617]

Susskind's proposal



If string coupling is so weak, gravity cannot trap string inside horizon

Similar phase structure can be found for temperature

Horowitz-Polchinski model:
Self-gravitating winding strings

[Horowitz-Polchinski, Phys.Rev.D 57 (1998)]

χ: winding string on Euclidean time circle

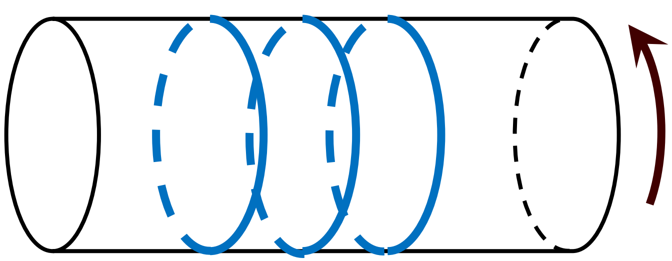
mass:

$$m^2 = \beta^2 - \beta_H^2 \quad \Rightarrow \quad m^2 = |g_{tt}|\beta^2 - \beta_H^2$$

in gravitational potential

Mass from tension

Tachyonic at ground state



Euclidean
time circle
Radius β

Wining strings wrapping on Euclidean time circle

Horowitz-Polchinski model: Self-gravitating winding strings

[Horowitz-Polchinski, Phys.Rev.D 57 (1998)]

χ: winding string on Euclidean time circle

mass:

$$m^2 = \beta^2 - \beta_H^2$$

Mass from tension

Tachyonic at ground state

in gravitational potential

$$\Rightarrow \quad m^2 = |g_{tt}|\beta^2 - \beta_H^2$$

Assumptions:

- High temperature: $\beta - \beta_H$ is small
- High temperature: φ is small, where $|g_{tt}| = e^{2\varphi}$

$\Rightarrow$ Spacetime is almost flat

dilaton is negligible

Horowitz-Polchinski model:
Self-gravitating winding strings

EOM for winding string χ and graviton φ $\quad (g_{tt} = -e^{2\varphi})$

$$0 = \nabla^2\chi - 2\varphi\beta^2\chi - (\beta^2 - \beta_H^2)\chi$$

$$0 = \nabla^2\varphi - \beta^2|\chi|^2$$

This is Schrödinger equation with self-interaction

$e^{2\varphi}\beta^2 \Rightarrow$ Potential $V(x)$ $\qquad$ $\beta_H^2 \Rightarrow$ Energy

Eigenvalue comes from normalization of χ

$$-\hat{\nabla}^2\hat{\chi}(\hat{x}) + \hat{V}(\hat{x})\hat{\chi}(\hat{x}) = \zeta\hat{\chi}(\hat{x})$$

$$\hat{x} = \frac{x}{\ell\sqrt{\zeta}} \qquad \hat{\chi} = \ell^2\zeta\chi$$

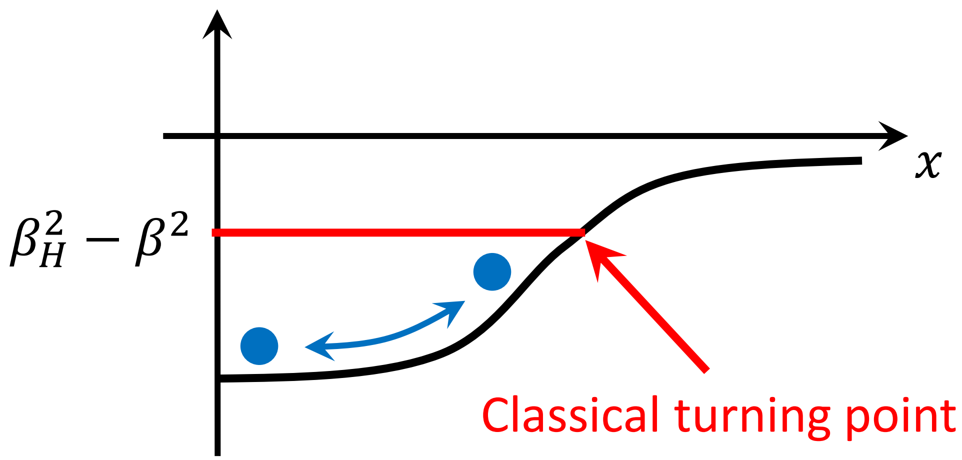
Typical scale

$$\ell = (\beta - \beta_H)^{-1/2}$$

Temperature of winding strings effectively exceeds Hagedorn temperature

Classical turning point is at $|g_{tt}|\beta^2 - \beta_H^2 = 0$

$V_{\text{eff}} = (|g_{tt}| - 1)\beta^2$



Outside turning point:

$$T_p = |g_{tt}|^{-1/2} T < T_H$$

Inside turning point:

$$T_p = |g_{tt}|^{-1/2} T > T_H$$

Winding string is located where local temp. exceeds Hagedorn temp.

⇨ Winding condensate beyond Hagedorn temperature

Strings will be fluid in the bound state by self-gravitation

Plan of Talk

1. Black hole/string transition

2. Fluid model of self-gravitating strings

   - Winding strings can be approximated by perfect fluid
   - Thermodynamic relation of winding string fluid

3. Geometry of winding string fluid

4. Black hole/string transition

Winding strings can be approximated by perfect fluid

Bound state of strings is ground state of Horowitz-Polchinski model

⇨ Naively, only mass of winding strings is important

$$m^2 = |g_{tt}|\beta^2 - \beta_H^2$$

Mass terms in energy-momentum tensor (ignoring kinetic terms)

$$T^t{}_t = -\underbrace{(3|g_{tt}|\beta^2 - \beta_H^2)|\chi|^2}_{=\rho} \qquad T^i{}_i = \underbrace{(\beta_H^2 - |g_{tt}|\beta^2)|\chi|^2}_{=P}$$

EM tensor of perfect fluid

$$T^t{}_t = -\rho \qquad T^i{}_i = P$$

ρ: energy density $\quad P$: pressure

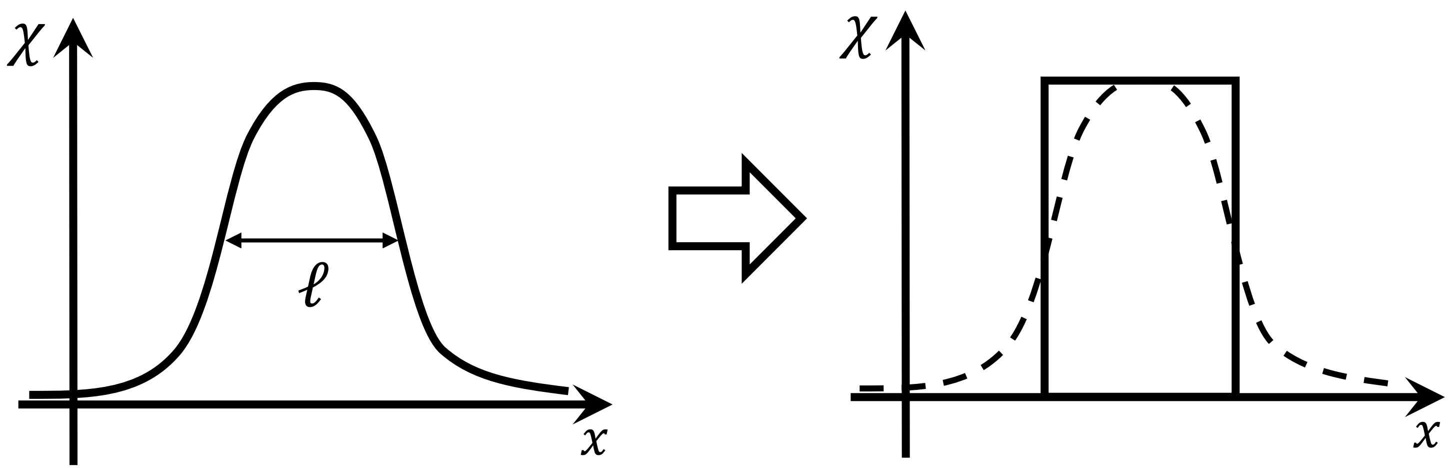
Fluid is in the resion $P > 0$

⇨ Local temp. exceeds Hagedorn temp.

Winding strings can be approximated by perfect fluid

What is this approximation?

We ignored kinetic term $\quad m^2|\chi|^2 \gg |\partial\chi|^2 \sim 0$



EOM: $\quad 0 = \nabla^2\chi - m^2\chi$ $\qquad$ Near Hagedorn temp. $m^2 \sim \ell^{-2} \sim \epsilon$

For $r \ll \ell$

$$\nabla^2\chi = \partial_r^2\chi - \frac{2}{r}\partial_r\chi \sim -\frac{2}{r}\partial_r\chi \sim \epsilon \quad \Rightarrow \quad |\partial\chi|^2 \sim \epsilon^2 \ll m^2|\chi|^2$$

Thermodynamic relation of winding string fluid

Entropy from Free energy

$$\int s = \left(\beta\partial_\beta - 1\right)I$$

Entropy density

action

$$s = e^{3\varphi}\beta^3|\chi|^2$$

Entropy satisfies local thermodynamic relation

$$s = e^{\varphi}\beta(\rho + P)$$

$$\rho = (3|g_{tt}|\beta^2 - \beta_H^2)|\chi|^2$$

$$P = (\beta_H^2 - |g_{tt}|\beta^2)|\chi|^2$$

ADM mass is different from energy density

$$M = \int(\rho + 3P)$$

$$M = -\partial_\beta I$$

We can also calculate energy density and pressure from entropy and ADM mass

Plan of Talk

1. Black hole/string transition

2. Fluid model of self-gravitating strings

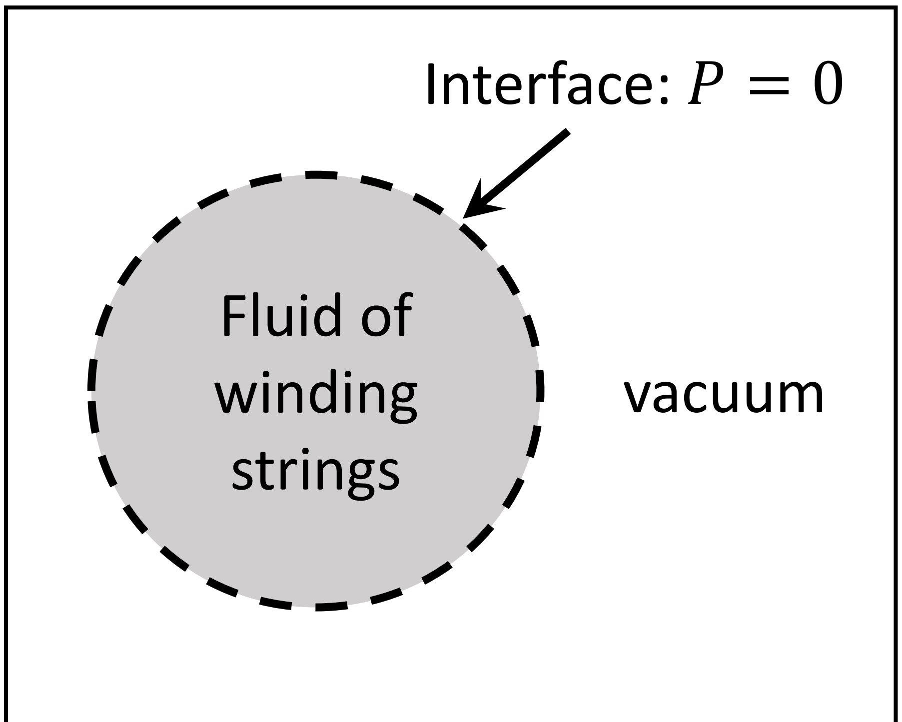
3. Geometry of winding string fluid
   - Geometry is junction of interior and exterior geometries
   - Interior is solution for winding string fluid
   - Exterior is Schwarzschild spacetime
   - Quantization condition

4. Black hole/string transition

Geometry is given by junction of interior and exterior geometries

We solve Einstein equation (for $D = 4$)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$



Interior solution (winding strings)

$$T^t{}_t = -(3|g_{tt}|\beta^2 - \beta_H^2)|\chi|^2$$

$$T^i{}_i = (\beta_H^2 - |g_{tt}|\beta^2)|\chi|^2$$

Exterior solution (Schwarzschild)

$$T_{\mu\nu} = 0$$

Fluid solution of Einstein equation

Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

EM tensor of perfect fluid

$$T^t{}_t = -\rho \qquad\qquad T^i{}_i = P$$

Einstein equation gives solution of metric

$\Rightarrow$ EM tensor should be given by other conditions

- Conservation law of EM tensor
- Equation of state of fluid (relation between ρ and P)

Interior solution of winding string fluid

Conservation law ⇨ $|\chi|^2 \equiv D_0 = \text{const.}$

metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)h(r)} + r^2 d\Omega^2$$

Solution

$$f(r) = \frac{\beta_h^2}{\beta^2}\left\{\frac{1}{h_0} + 1 - \frac{\sqrt{r_m^2 - r^2}}{r}\left[\sin^{-1}\left(\frac{r}{r_m}\right)\right]\right\}$$

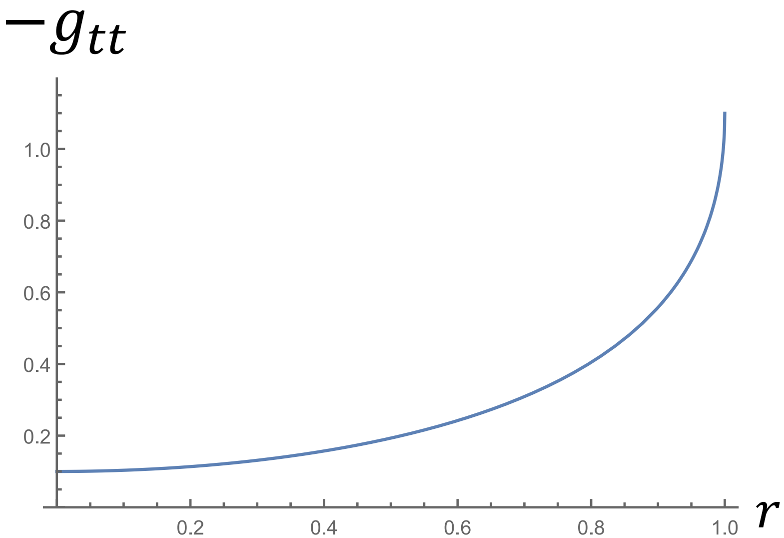
$$h(r) = \frac{\beta^2}{\beta_H^2} h_0 \left(1 - \frac{r^2}{r_m^2}\right)$$

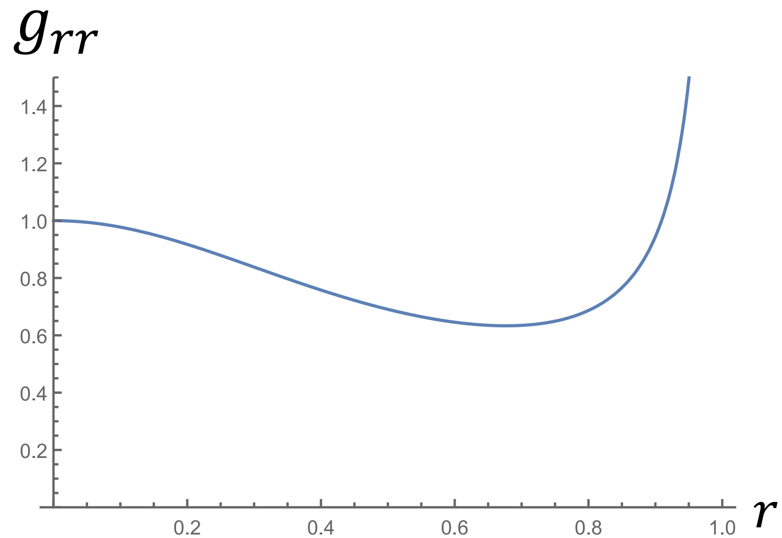
h_0: integration constant

$$r_m^2 = \frac{h_0}{\beta_H^2 D_0}$$

This solution was first studied in [Wyman, Phys.Rev.75 (1949)]

Interior solution of winding string fluid





Exterior solution is Schwarzschild spacetime

Schwarzschild metric

$$ds^2 = -\left(1 - \frac{r_h}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{r_h}{r}\right)} + r^2 d\Omega^2$$

1st junction cond.: induced metric on interface is identical

2nd junction cond.: extrinsic curvature is the same on both side

Position of interface is determined by junction conditions

$$r = r_0 \equiv \frac{\beta^2 r_h}{\beta^2 - \beta_H^2} \quad \Rightarrow$$ approaches r_h in low temp.

Constants h_0 and D_0 are given as functions of β.

Quantization condition

Solution of fluid model has two parameters: β and r_h.

Horowitz-Polchinski model has one parameter: β.

⇨ Solution satisfies quantization condition

We need to impose quantization condition by hand

$$\oint p_r dr = \pi \qquad (n = 1 \text{ for ground state})$$

⇨ Quantization cond. gives relation between β and r_h

Plan of Talk

1. Black hole/string transition
2. Fluid model of self-gravitating strings
3. Geometry of winding string fluid
4. Black hole/string transition
   - High temperature: bound state of strings, agrees with HP
   - Low temperature: approximately black hole but no horizon
   - Horizonless geometry is realized by negative energy of true vacuum

High temperature limit: bound state of strings
Results consistent with Horowitz-Polchinski model

Near Hagedorn temperature: small $(\beta - \beta_H)$ expansion

$$f(r) \simeq 1 \qquad \Rightarrow \qquad \text{Spacetime is almost flat}$$

$$h(r) \simeq 1 \qquad \qquad \text{Gravitation is weak}$$

Higher order corrections: $f(r) \simeq 1 - \frac{3}{2}(\beta^2 - \beta_H^2) + \frac{1}{2}(\beta^2 - \beta_H^2)^3 \frac{r^2}{r_h^2} + \cdots$

Quantization condition gives $r_h \sim \sqrt{\beta^2 - \beta_h^2}$

Size of bound state:

$$r_0 = \frac{\beta^2 r_h}{\beta^2 - \beta_H^2} \quad \Rightarrow \quad r_0 = (\beta^2 - \beta_H^2)^{-1/2}$$

ADM mass $M \sim \frac{1}{G_N}\sqrt{\beta^2 - \beta_H^2}$ Entropy $S \sim \frac{\beta_H}{G_N}\sqrt{\beta^2 - \beta_H^2}$

Low temperature limit: approximately black hole but geometry has no horizon

Low temperature: large β expansion $(\beta \gg \beta_H)$

Surface of star at $r = r_0$ (where $P \sim \beta_H^2 - f(r)\beta^2 = 0$)

$\beta^2 f(r) = \beta_H^2$ ⇨ $f(r) \ll 1$ Almost frozen in star

$r_0 = r_h\left(1 + \frac{\beta_H^2}{\beta^2}\right)$ ⇨ Surface is slightly outside the horizon

Quantization condition gives $r_h \sim \beta$

ADM mass $M \sim \frac{r_h}{G_N}$ Entropy $S \sim \frac{r_h^2}{G_N}$

Solution has no horizon but size, temperature, ADM mass and entropy approximately agree with Schwarzschild BH

Horizonless geometry is realized by negative energy in true vacuum

Buchdahl theorem: No static star has size of $r_0 \leq \frac{9}{8} r_h$

Assumptions:
- Static Spherically symmetric
- Pressure is isotropic (perfect fluid)
- Energy density is non-increasing outwards

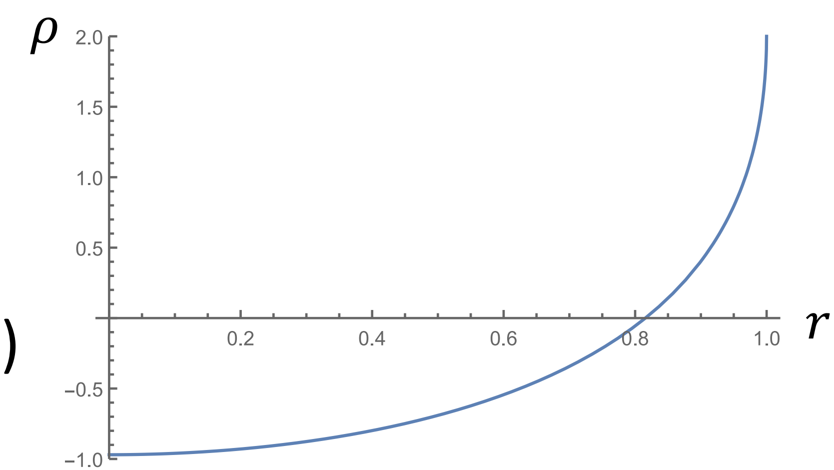
$\Rightarrow$ No negative energy is assumed

Energy density of winding string fluid

$$\rho = (3\beta^2 f(r) - \beta_H^2)|\chi|^2$$

$$\Rightarrow \begin{cases} \rho = 2\beta_H^2|\chi|^2 & (r = r_0) \\ \rho \simeq -\beta^2|\chi|^2 & (r = 0, \beta \gg \beta_H) \end{cases}$$

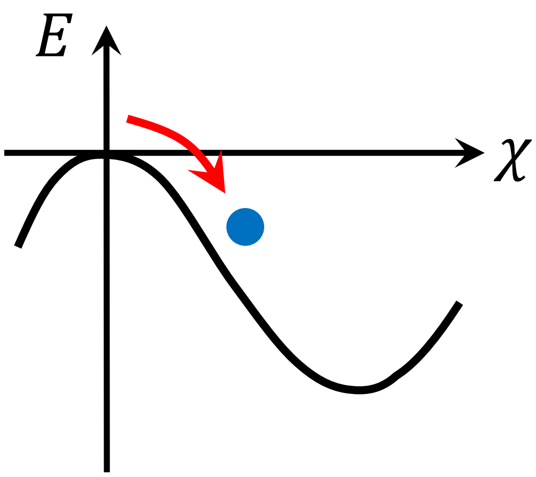
Negative energy



Horizonless geometry is realized by negative energy in true vacuum

Energy density of winding string fluid

$$\rho = (3\beta^2 f(r) - \beta_H^2)|\chi|^2 = \underbrace{2\beta^2|\chi|^2}_{\text{Thermal energy}} + \underbrace{m^2|\chi|^2}_{\text{tachyonic mass}}$$



Energy is renormalized as

$$E = 0 \text{ for } \chi = 0 \text{ (in } r \to \infty)$$

Temp. exceeds Hagedorn temp. by blue shift

Negative energy for true vacuum
at Hagedorn temp.

Horizonless geometry is realized by stringy effect

Summary

Black hole is fluid of strings

Black hole/string transition near Planck scale

String bound state $\xLeftrightarrow{\text{transition}}$ Black hole

Temperature of strings effectively exceeds Hagedorn temperature

$\Rightarrow$ Strings become fluid in the bound state

We derive geometry of string fluid

High temperature: bound state of strings

Low temperature: approximately black hole (horizonless)

Thank you