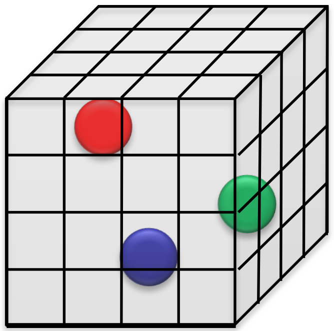
Quantum sampling for the Euclidean path integral of lattice gauge theory

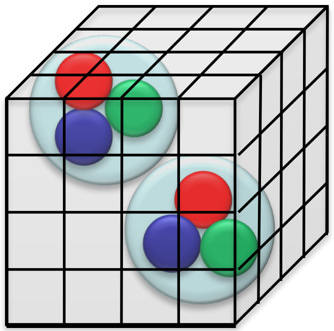
Arata Yamamoto (University of Tokyo)

arXiv:2201.12556 [quant-ph]

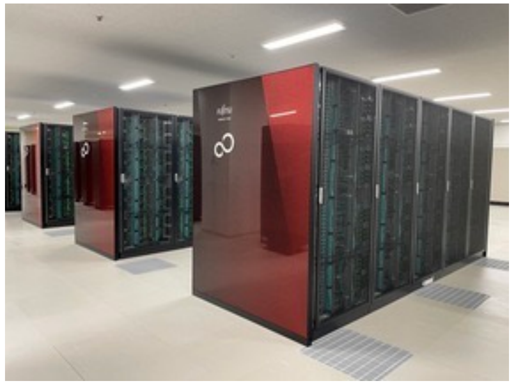
Self-introduction

my research subject: lattice gauge theory





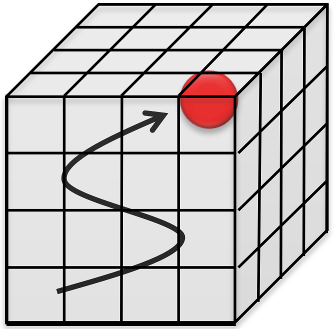
particle & nuclear physics

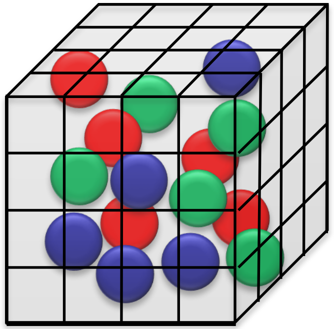


supercomputer

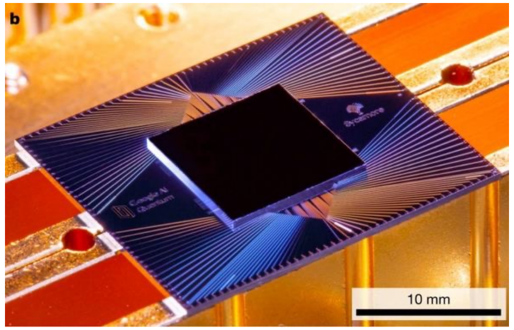
Self-introduction

my research subject: lattice gauge theory





open problems in lattice gauge theory



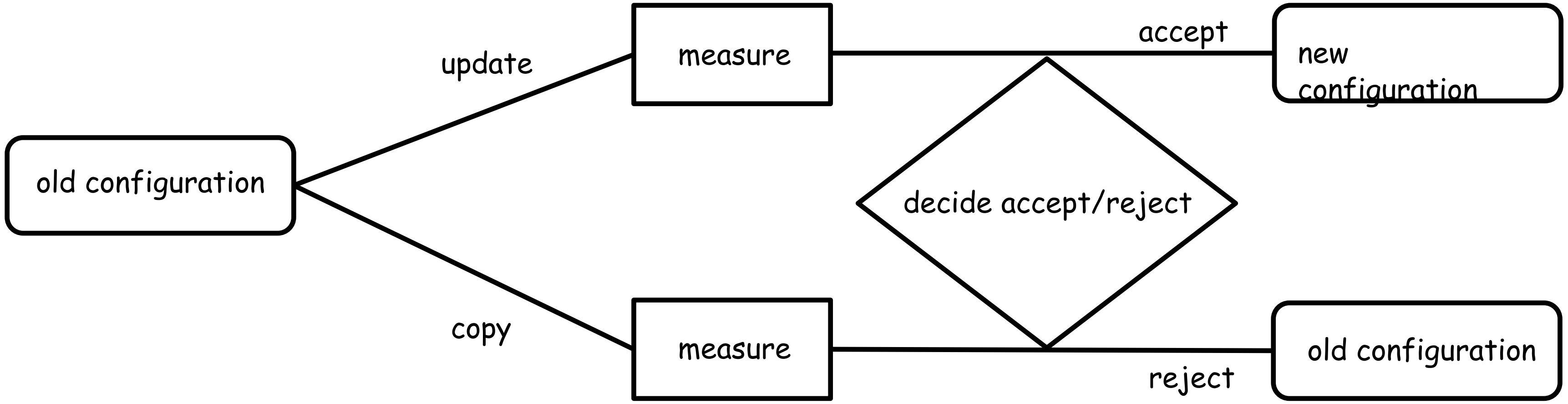
quantum computer

What's a quantum computer?

classical computer	quantum computer
c-bit $= 0$ or 1	q-bit $= a\|0\rangle + b\|1\rangle$
operation $\sim \{+, -, \times, \cdots\}$	operation $\sim \begin{pmatrix} U_1 & U_2 \\ U_3 & U_4 \end{pmatrix}$
	superposition of 2^N states!!

What's a quantum computer?

ex. classical Markov-chain Monte Carlo



What's a quantum computer?

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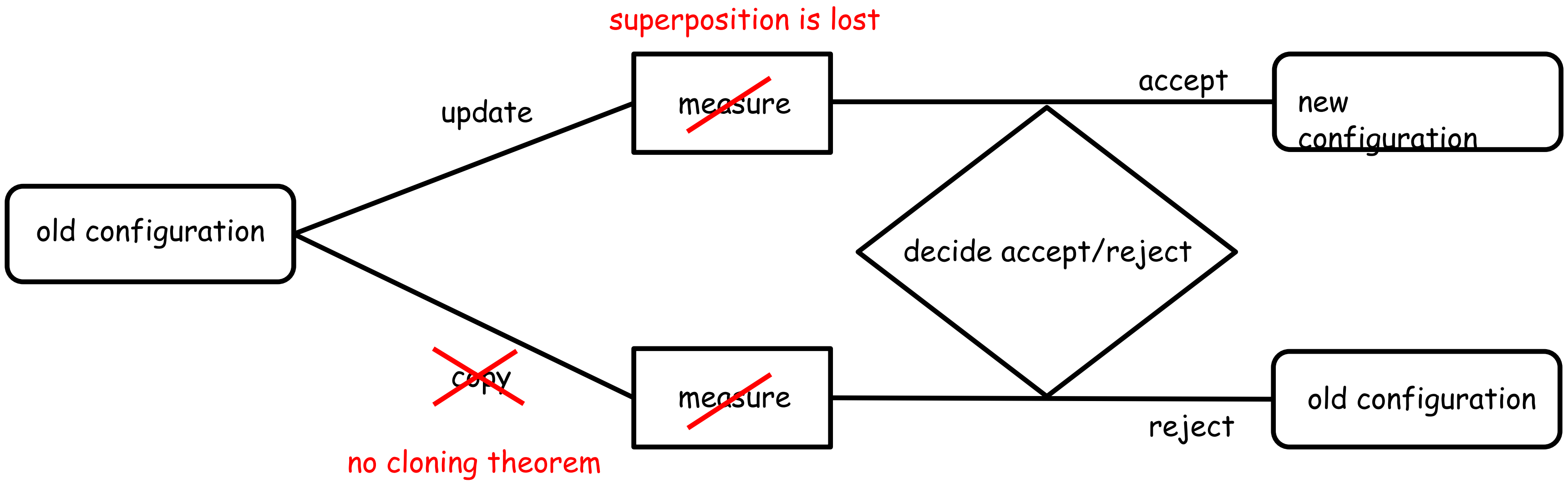
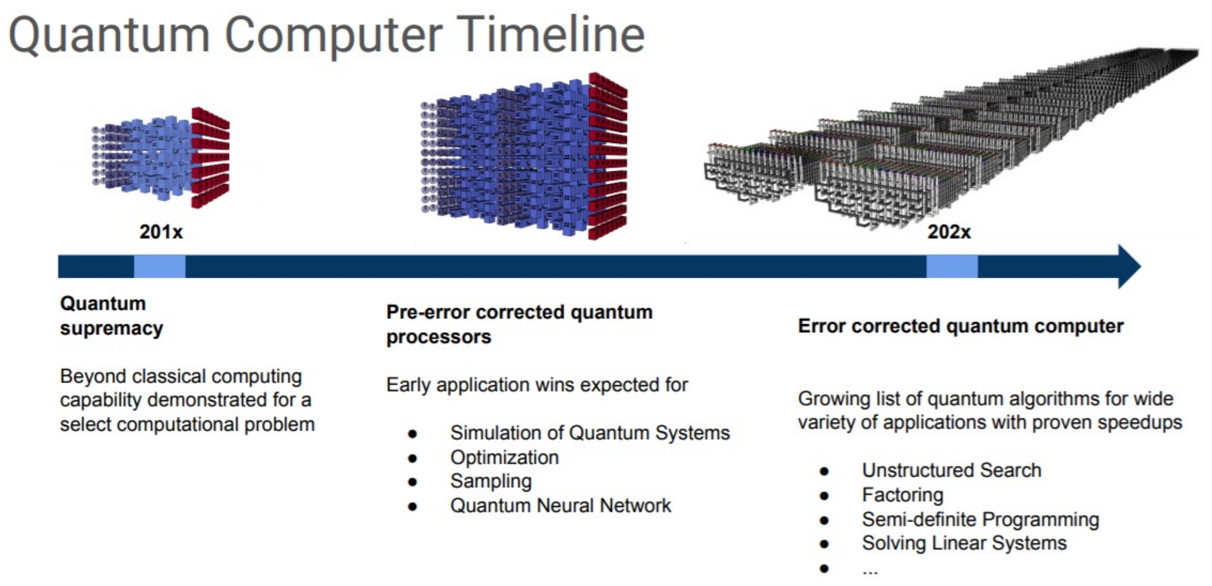


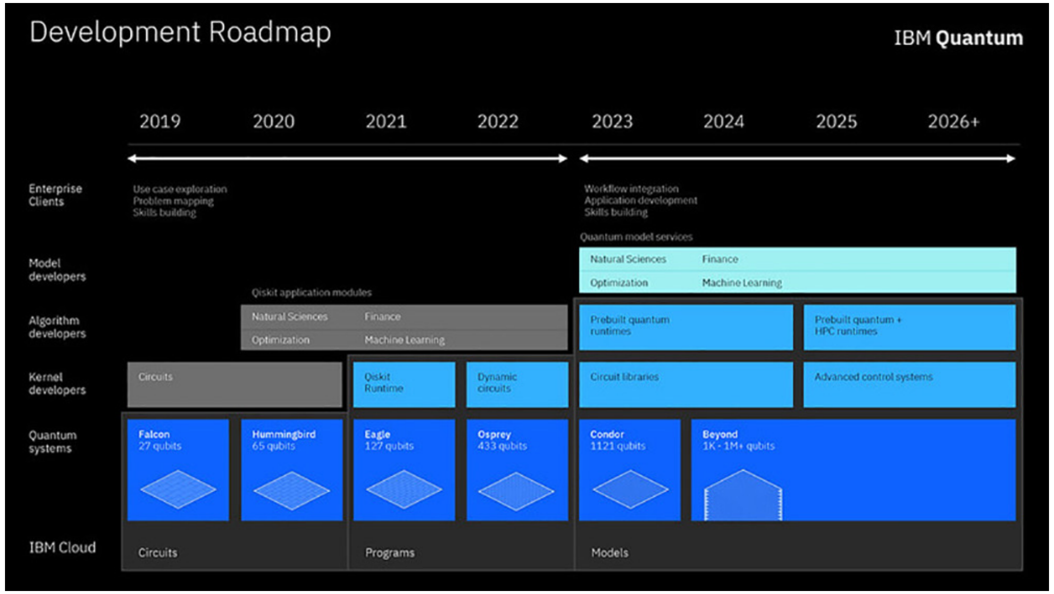
classical algorithm $\neq$ quantum algorithm

What's a quantum computer?

noisy intermediate-scale quantum (NISQ)

$\underbrace{\text{noisy}}_{\text{device error}}$ $\underbrace{\text{intermediate-scale}}_{\text{limited resource}}$





Introduction

Let’s make algorithms for lattice gauge theory!!

unfortunately, no physics discussion...

Introduction

Hamiltonian formalism

$$E = \langle U|H|U\rangle$$

Hamiltonian operator

quantum state

Lagrangian formalism

$$\mathcal{Z} = \int dU\, e^{-S}$$

c-number

classical action

Introduction

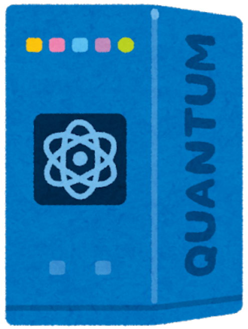
contents of this presentation:

1. Hamiltonian formalism (prelude)

2. Lagrangian formalism

1. Hamiltonian formalism (prelude)

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle \quad \Rightarrow$$



real-time simulation

quantum computer

1. Hamiltonian formalism (prelude)

Z_2 lattice gauge theory

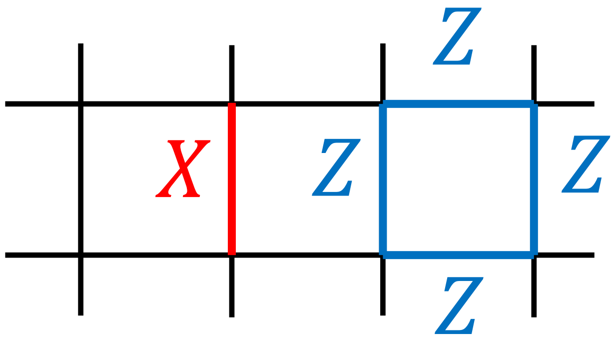
classical $\quad +1 \text{ or } -1$

quantum $\quad a|+1\rangle + b|-1\rangle \;\leftrightarrow\; 1 \text{ qubit}$

1. Hamiltonian formalism (prelude)

2-dim. Hamiltonian

$$H = -\sum_{\mathrm{link}} X - \sum_{\mathrm{plaq}} ZZZZ$$



1. Hamiltonian formalism (prelude)

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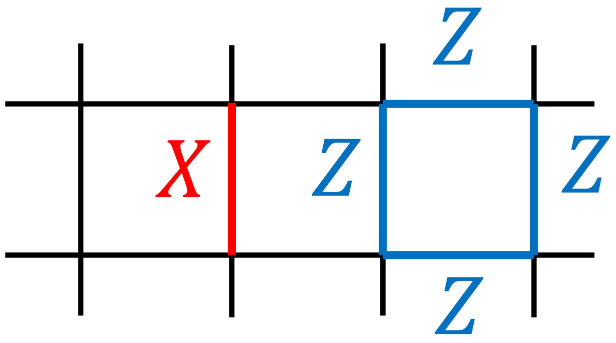
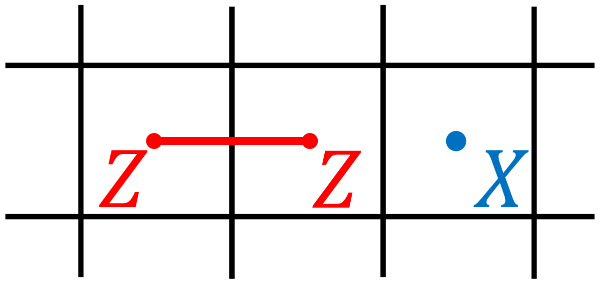


$\updownarrow$ Wigner duality

$$H = -\sum_{\text{link}} ZZ - \sum_{\text{site}} X$$



1. Hamiltonian formalism (prelude)

2-dim. Hamiltonian

$$H = -\sum_{\text{link}} X - \sum_{\text{plaq}} \boxed{ZZZZ} \qquad N \sim 2L_x L_y$$

4-qubit operation

$\updownarrow$ Wigner duality

$$H = -\sum_{\text{link}} \boxed{ZZ} - \sum_{\text{site}} X \qquad N \sim L_x L_y$$

2-qubit operation

1. Hamiltonian formalism (prelude)

real-time simulation

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

$$\sim \left(\prod e^{-iZZ\delta t}\prod e^{-iX\delta t}\right)^{l} \ |\Psi(0)\rangle$$

$$= (\text{gates})^{l} \ |\Psi(0)\rangle$$

1. Hamiltonian formalism (prelude)

real-time simulation

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

$$\sim \left(\prod e^{-iZZ\delta t}\prod e^{-iX\delta t}\right)^{l} \; |\Psi(0)\rangle$$

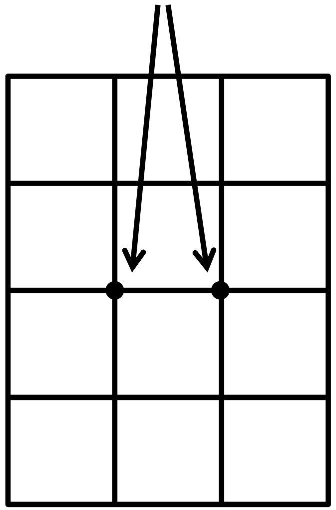
$$= (\mathrm{gates})^{l} \; |\Psi(0)\rangle$$

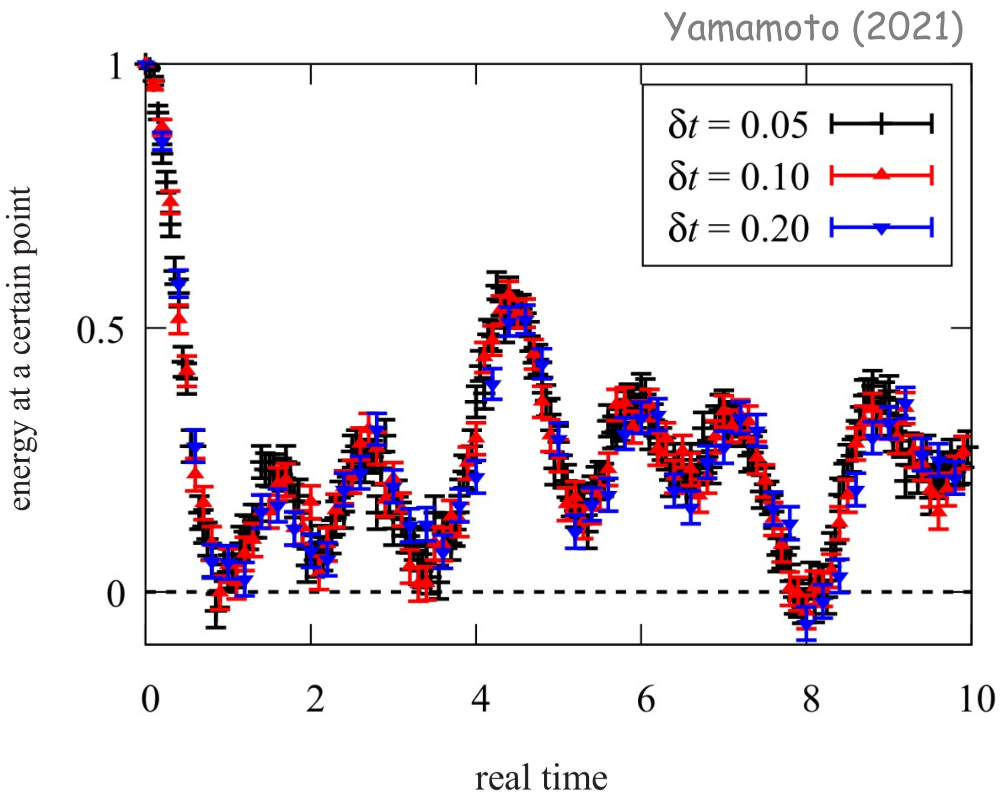
classical computational cost　　　quantum computational cost

$$O(2^N) \quad \gg \quad O(N^p)$$

1. Hamiltonian formalism (prelude)

two static charges (two Wilson lines)

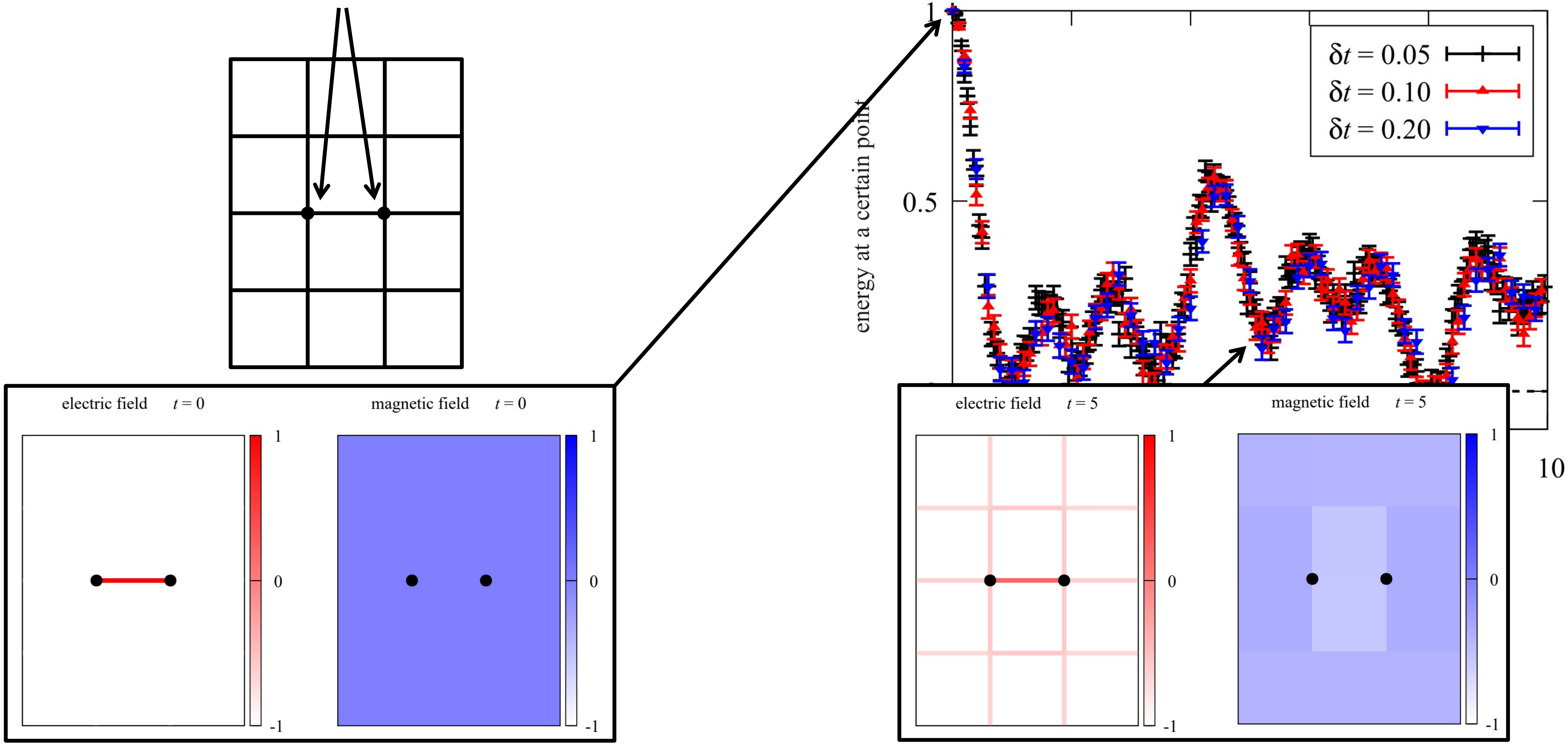




1. Hamiltonian formalism (prelude)

two static charges (two Wilson lines)

Yamamoto (2021)

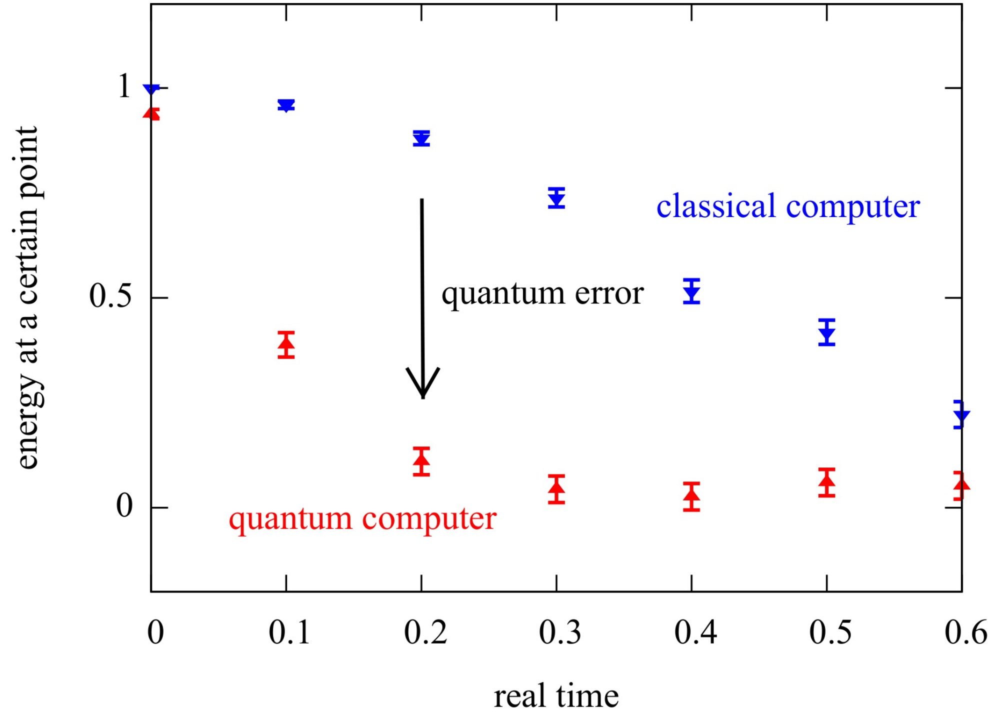


1. Hamiltonian formalism (prelude)

noiseless "simulater" is used

(classical computer to emulate quantum computer)

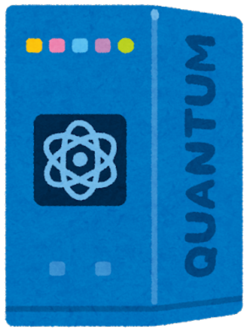
Yamamoto (2021)



2. Lagrangian formalism

$$\mathcal{Z} = \int DU\, e^{-S}$$

how to encode?



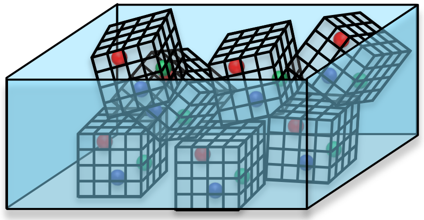
Euclidean path integral

quantum computer

2. Lagrangian formalism

classical sampling (Monte Carlo)

$$\mathcal{Z} = \int DU\, e^{-S} \sim$$



with weight e^{-S}

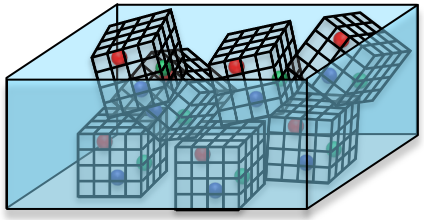
random number → gauge configuration 1 → ensemble average

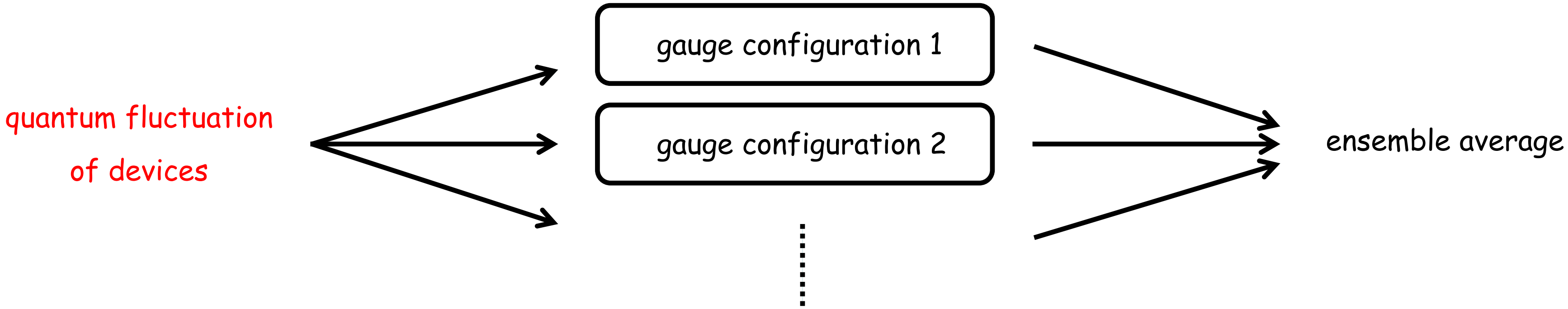
random number → gauge configuration 2 → ensemble average

⋮

2. Lagrangian formalism

quantum sampling

$$\mathcal{Z} = \int DU\, e^{-S} \sim$$  with weight e^{-S}



2. Lagrangian formalism

quantum sampling algorithm Wild *et al.* (2021)

matrix representation of classical action

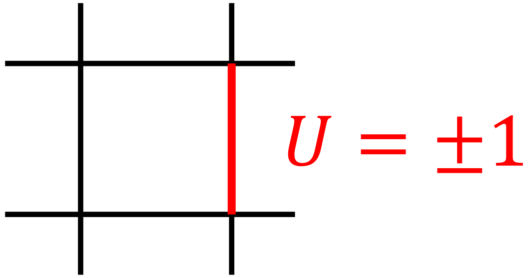
$$H = N\left(I - e^{-\mathcal{S}/2} M e^{\mathcal{S}/2}\right)$$

matrix representation of Markov chain

ground state $\longleftrightarrow$ classical ensemble

2. Lagrangian formalism

Z_2 lattice gauge theory

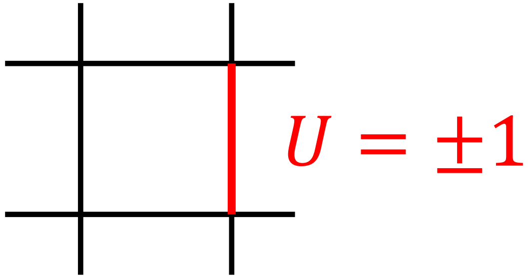


$$S = -\beta \sum_{\mathrm{plaq}} UUUU$$

$$\{U_1, U_2, \cdots, U_N\}$$

2. Lagrangian formalism

Z_2 lattice gauge theory



$$S = -\beta \sum_{\mathrm{plaq}} UUUU \xrightarrow[\text{matrix representation}]{} \mathcal{S} = -\beta \sum_{\mathrm{plaq}} ZZZZ$$

$$\{U_1, U_2, \cdots, U_N\} \qquad |U_1\rangle |U_2\rangle \cdots |U_N\rangle$$

2. Lagrangian formalism

$$H = \sum_n \frac{1}{2}\left(I - \tanh(\beta C_n)\, Z_n - \frac{1}{\cosh(\beta \underset{\uparrow}{C_n})} X_n \right)$$

"staple" operator

2. Lagrangian formalism

$$H = \sum_n \frac{1}{2}\left(I - \tanh(\beta C_n)\, Z_n - \frac{1}{\underbrace{\cosh(\beta C_n)}_{\text{"staple" operator}}} X_n\right)$$

$$|\Psi\rangle = \frac{1}{\sqrt{\mathcal{Z}}} \sum e^{-\mathcal{S}/2} |U_1\rangle |U_2\rangle \cdots |U_N\rangle \xrightarrow[\text{measurement}]{} \{U_1, U_2, \cdots, U_N\}$$

gauge configuration !!

2. Lagrangian formalism

quantum adiabatic algorithm Farhi *et al.* (2000)

ground state of **full Hamiltonian**

ground state of **solvable Hamiltonian**

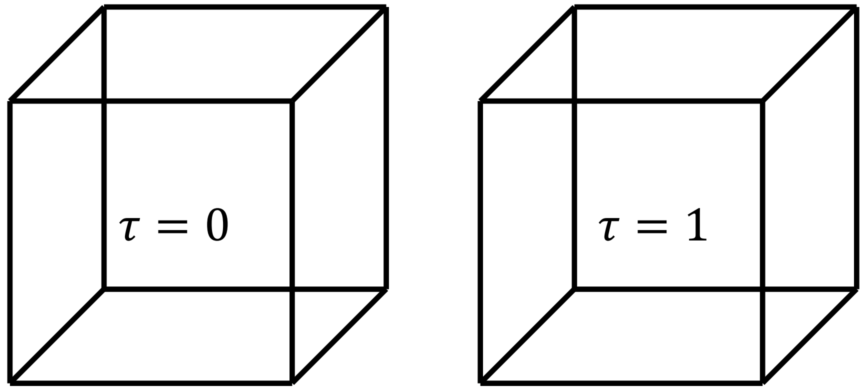
$$|\Psi\rangle = \prod_{l=1,\cdots,L} U(l)\,|\Psi_0\rangle$$

$$U(1) = \exp[-i\delta t H_0]\,,\cdots,U(L) = \exp[-i\delta t H]$$

2. Lagrangian formalism

benchmark test

- ✓ noiseless simulator
- ✓ Z_2 pure gauge theory
- ✓ 2^4-site lattice

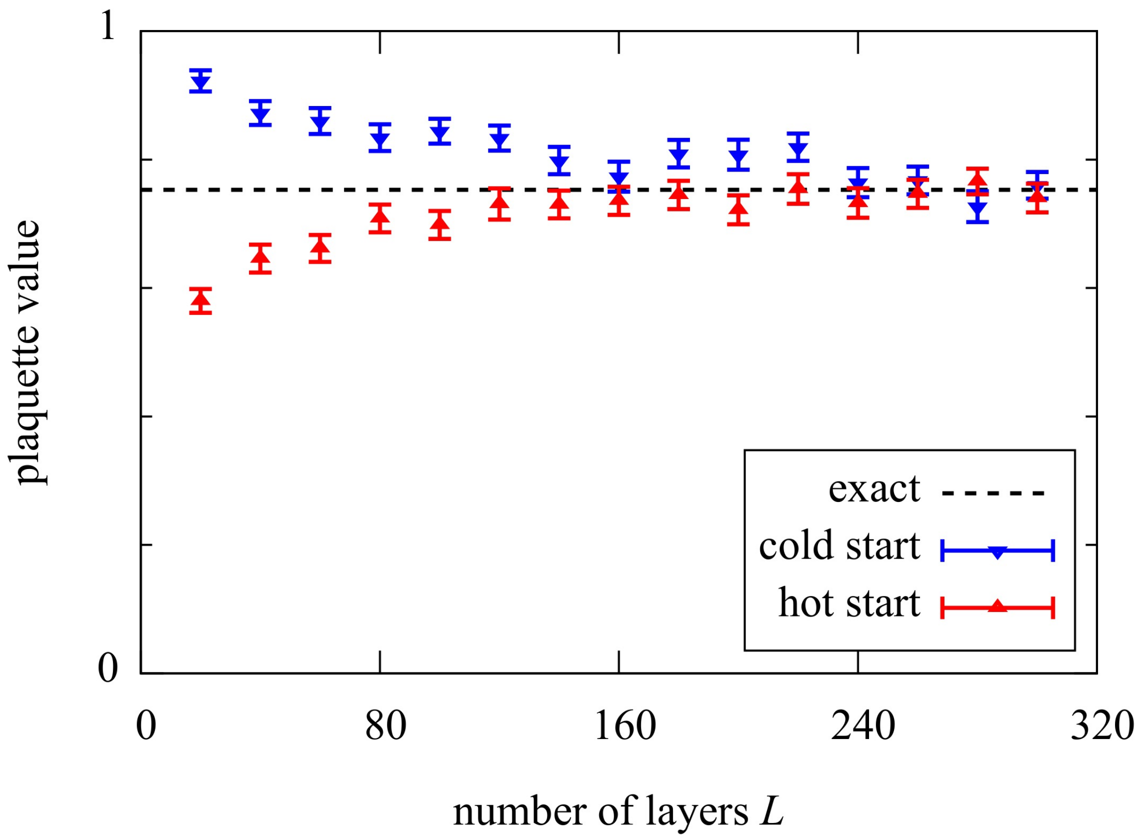


2. Lagrangian formalism

$$|\Psi(\beta)\rangle = \prod_{l=1,\cdots,L} U(l)\,|\Psi_0\rangle$$

cold start $\quad |\Psi_0\rangle = |\Psi(\beta = \infty)\rangle$

hot start $\quad |\Psi_0\rangle = |\Psi(\beta = 0)\rangle$



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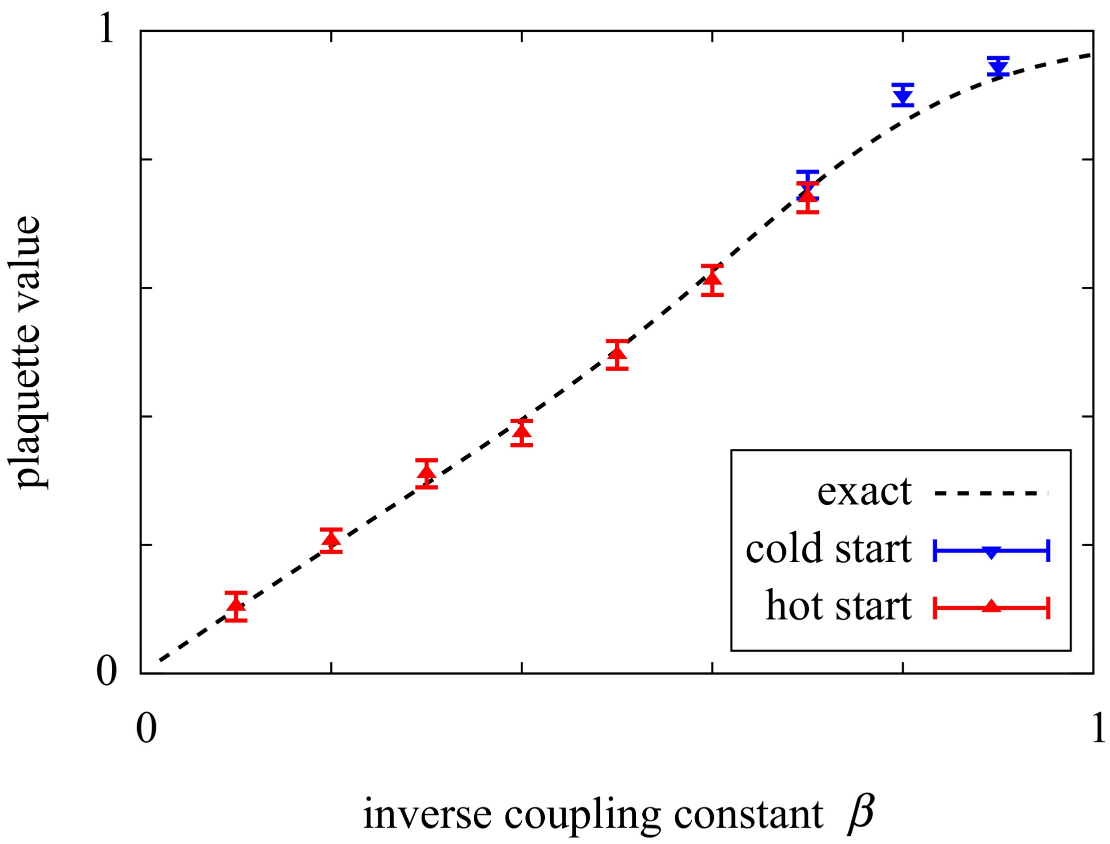

2. Lagrangian formalism

Compared with **Hamiltonian formalism**...

past experience, Lorentz invariance, classical storage

Compared with **classical simulation**...

quadratic speedup Wild *et al.* (2021)

fermion is more important

Summary

- ✓ quantum sampling for Euclidean path integral
- ✓ benchmark of Z_2 pure gauge theory
- ✓ useful someday in the future...