SMEFTにおける弾性正値性制限 VS EXTREMAL RAYによる正値性制限

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Effective Field Theory (EFT) and Standard Model Effective Field Theory (SMEFT)

- EFT
  - heavy degrees of freedom decouple
  - for large-distance phenomena or small momentum scale
- EFT interaction terms:

$$\mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \mathcal{L}^{(8)} + \cdots$$

Wilson coefficients

$$\mathcal{L} = \sum_{i=1}^{n_d} \frac{c_i^5}{\Lambda}\mathcal{O}_i^{(5)} + \frac{c_i^6}{\Lambda^2}\mathcal{O}_i^{(6)} + \frac{c_i^7}{\Lambda^3}\mathcal{O}_i^{(7)} + \frac{c_i^8}{\Lambda^4}\mathcal{O}_i^{(8)} + \cdots$$

Effective Field Theory (EFT) and Standard Model Effective Field Theory (SMEFT)

SMEFT

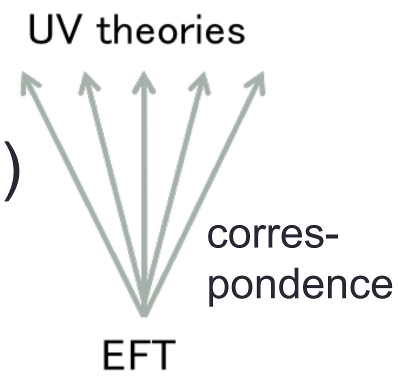
- Extending the SM by adding massive particles heavier than the energy scale of scatterings

- $SU_C(3) \times SU_L(2) \times U_Y(1)$ invariant higher-dimensional operator built by SM fields

- Model independent way to search for new physics

Positivity Bounds

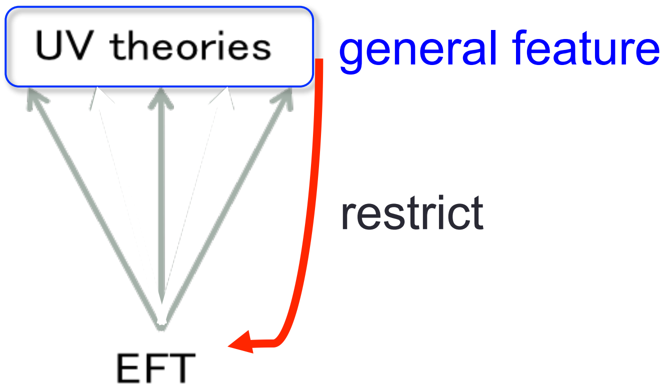
- EFT is for the energy scale of
  E << Λ (typical energy scale of the UV physics)
- Many UV models correspond with EFT



- From the general feature of UV theory,

  ⇒ Next next page

  can we bound on Wilson coefficients of EFT?



Positivity Bounds

- One of the way to do this is Positivity bounds

  A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, R.Rattazzi, JHEP **0610**, 014(2006)

- Positivity bounds: the signs of certain combinations of Wilson coefficients in EFT have to be positive, e.g. W^4 operators:

$$\frac{F_{T,0}}{\Lambda^4}\mathrm{Tr}[\hat{\mathrm{W}}_{\mu\nu}\hat{\mathrm{W}}^{\mu\nu}]\mathrm{Tr}[\hat{\mathrm{W}}_{\alpha\beta}\hat{\mathrm{W}}^{\alpha\beta}] \qquad \frac{F_{T,1}}{\Lambda^4}\mathrm{Tr}[\hat{\mathrm{W}}_{\alpha\nu}\hat{\mathrm{W}}^{\mu\beta}]\mathrm{Tr}[\hat{\mathrm{W}}_{\mu\beta}\hat{\mathrm{W}}^{\alpha\nu}]$$

$$\frac{F_{T,2}}{\Lambda^4}\mathrm{Tr}[\hat{\mathrm{W}}_{\alpha\mu}\hat{\mathrm{W}}^{\mu\beta}]\mathrm{Tr}[\hat{\mathrm{W}}_{\beta\nu}\hat{\mathrm{W}}^{\nu\alpha}] \qquad \frac{F_{T,10}}{\Lambda^4}\mathrm{Tr}[\hat{\mathrm{W}}_{\mu\nu}\tilde{\mathrm{W}}^{\mu\nu}]\mathrm{Tr}[\hat{\mathrm{W}}_{\alpha\beta}\tilde{\mathrm{W}}^{\alpha\beta}]$$

$$\hat{W}^{\mu\nu} \equiv ig\frac{\sigma^I}{2}W^{I,\mu\nu} \qquad \tilde{W}^{\mu\nu} \equiv ig\frac{\sigma^I}{2}\left(\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}W^{I,\rho\sigma}\right)$$

One of the positivity bounds:

$$2F_{T,0} + 2F_{T,1} + F_{T,2} \geq 0$$

Positivity Bounds

If we base on the local Quantum Field Theory(QFT) for the general feature of UV theory,

1. Special relativity
2. Conservation of probability
3. Causality

Positivity Bounds

If we base on the local Quantum Field Theory(QFT) for the general feature of UV theory,

1. Special relativity ⟶ Lorentz invariance
2. Conservation of probability ⟶ Unitarity
3. Causality - - -> Analyticity

Positivity Bounds

Forward limit positivity bounds are from:

1. Lorentz Invariance

2. Unitarity ⇨ Optical theorem:
   e.g., elastic case,

$$\mathrm{Im}\mathcal{M}(k_1, k_2 \to k_1, k_2) = \underline{\underline{s\sigma_{\mathrm{tot}}(k_1, k_2 \to \mathrm{anything})}}$$

Positive

3. Analyticity* ⇨ Froissart Bound:

$$|\mathcal{M}(s, \underline{\underline{\cos\theta = 1}})| < \mathrm{Const.}\ s(\ln s)^2$$

forward, elastic   Froissart, Martin 1960's

*Analyticity of the amplitude besides poles and branch cuts on real axis

Positivity Bounds

Even though the EFT as IR theory seems totally okay by seeing the Lagrangian and check it with

- ✓ Lorentz invariance
- ✓ Perturbative Unitarity
  (Unitary S-matrix at energies far lower than the cutoff)
  , etc.,

It can be excluded out when we suppose the UV completed theory!

This "wrong" EFT leads to the superluminal fluctuations around non-trivial backgrounds, and making it impossible to define local and causal evolution, and implying an IR breakdown of the EFT.

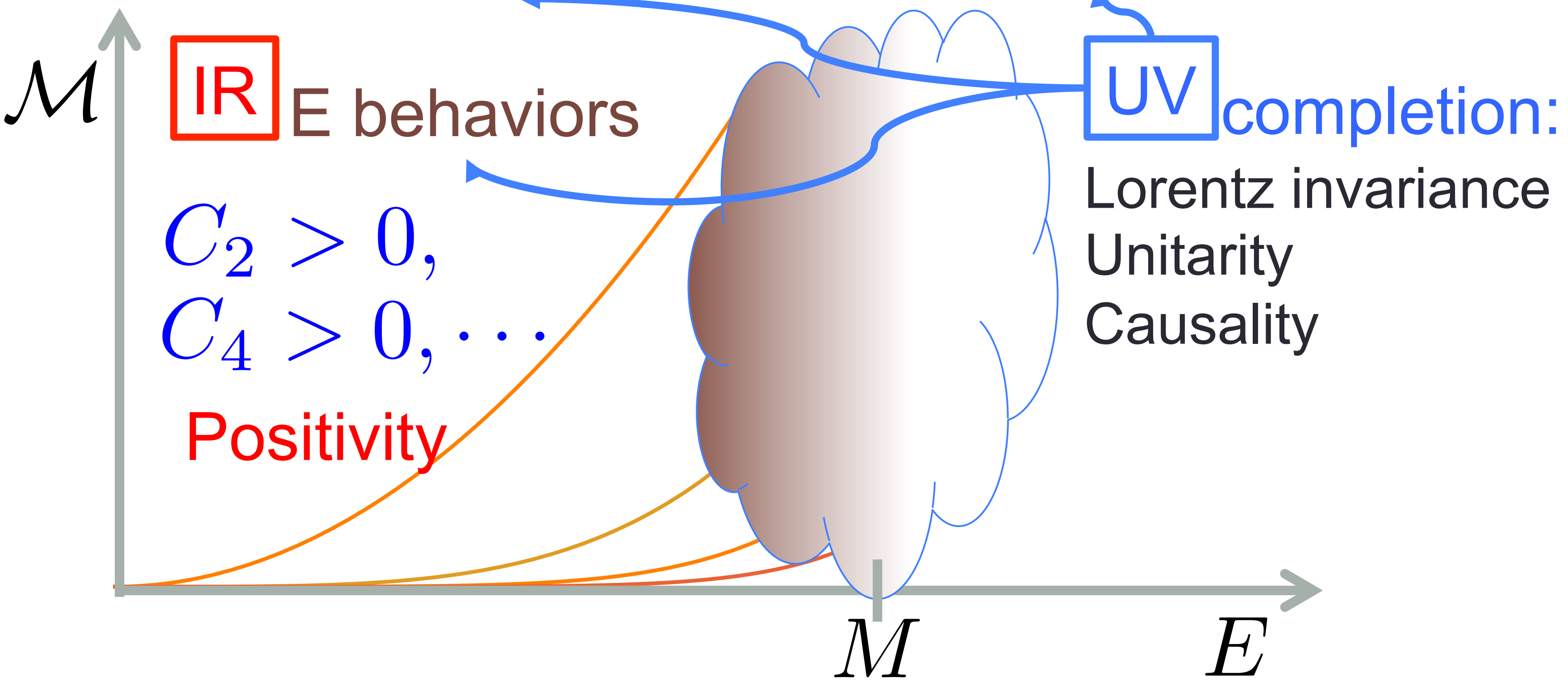
A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, R.Rattazzi, JHEP **0610**, 014(2006)

Positivity Bounds

*Ref: Slides by Francesco Riva https://indico.ph.tum.de/event/4408/contributions/3825/attachments/3292/3974/Berlin-2.pdf

- Effective Theory Forward Amplitude (IR):

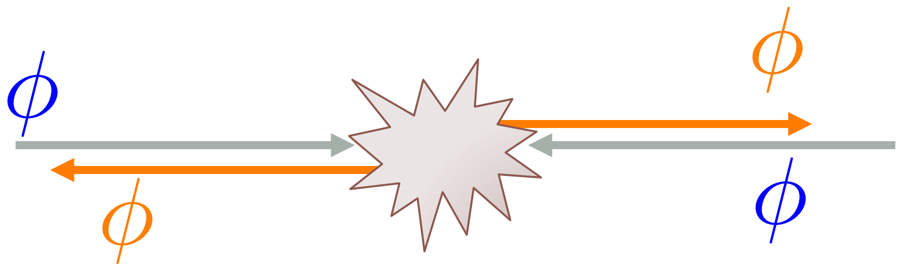
$$\mathcal{M} = C_0 + C_1 \frac{s}{M^2} + C_2 \frac{s^2}{M^4} + C_3 \frac{s^3}{M^6} + C_4 \frac{s^4}{M^8} + \cdots$$



Positivity Bounds

massless scalar 2-2 forward elastic scattering:

forward: t=0



I+II →I+II
elastic

Let us consider the amplitude (divided by s^{n+1}) of this:

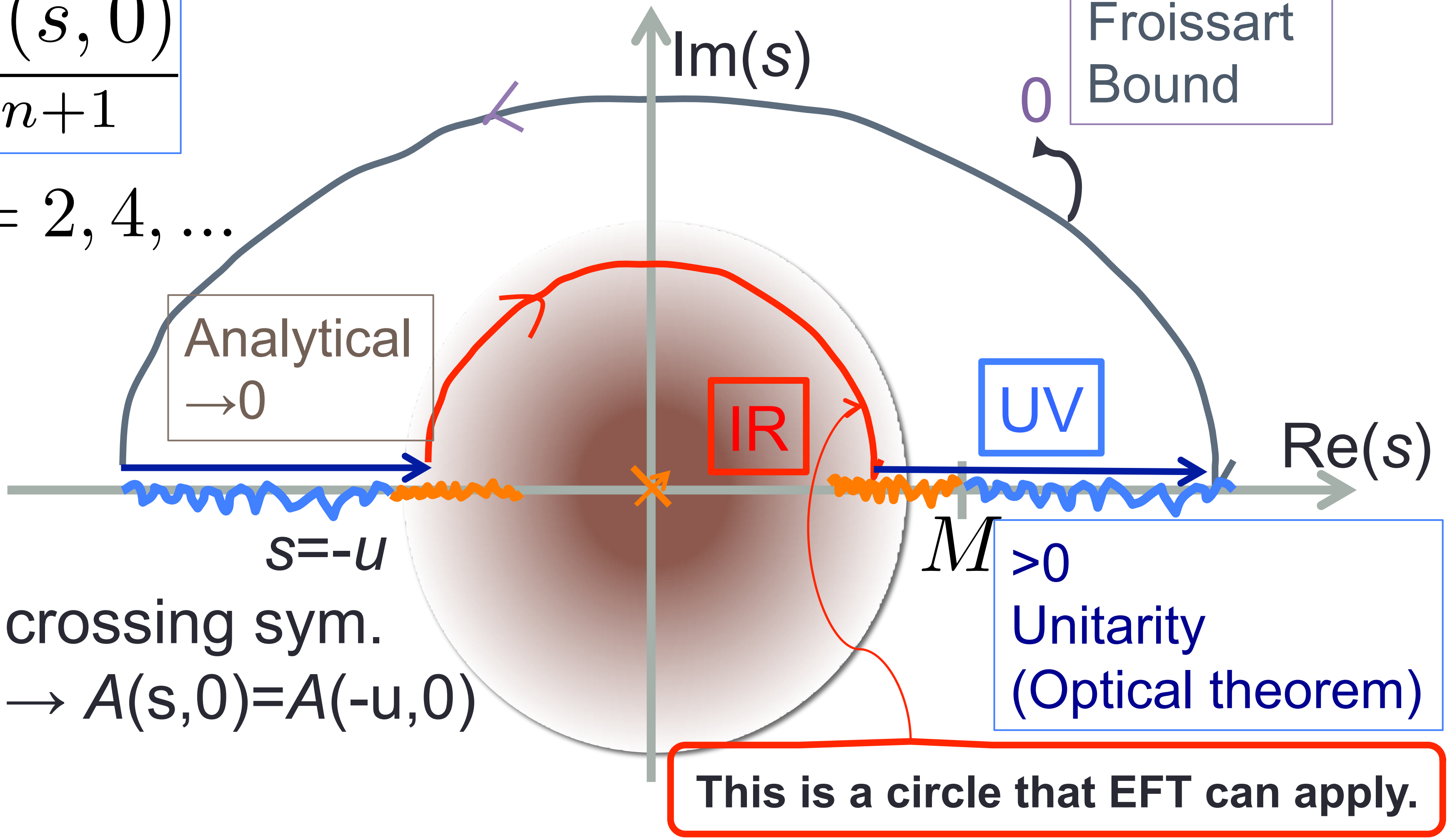
$$\frac{\mathcal{M}(s,0)}{s^{n+1}} \quad n = 2, 4, \ldots$$

Positivity Bounds

massless scalar 2-2 forward elastic scattering amplitude:

$$\frac{\mathcal{M}(s,0)}{s^{n+1}}$$

$$n = 2, 4, \ldots$$



Positivity Bounds

$$\frac{1}{\pi i}\int ds \frac{\mathcal{M}(s,0)}{s^{n+1}} = 0 = \frac{1}{\pi i}\int ds \frac{\mathcal{M}(s,0)}{s^{n+1}}$$

Analytical

=0

Froissart Bound

$$\frac{1}{\pi i}\int ds \frac{\mathcal{M}(s,0)}{s^{n+1}} = \frac{2}{\pi}\int_{s_{IR}}^{\infty} ds \frac{\mathrm{Im}\mathcal{M}(s)}{s^{n+1}} > 0$$

$\mathrm{Res}[\mathcal{M}(s,0)]_{s=0} = C_n$ $s_{IR} \ll M^2$

Unitarity
(Optical theorem)

Positivity

→ $C_n > 0 (n = 2, 4, \cdots)$ from UV general features

$$\mathcal{M} = C_0 + C_1 \frac{s}{M^2} + C_2 \frac{s^2}{M^4} + C_3 \frac{s^3}{M^6} + C_4 \frac{s^4}{M^8} + \cdots$$

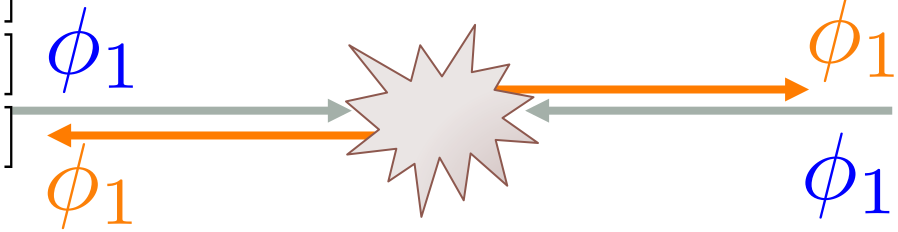
e.g., From Dim.8 operators

Positivity Bounds: Example of Positivity for Dim.8

- EFT Lagrangian →Amplitudes
- $\mathrm{SU}(2)_L$ Higgs doublet $\Phi : \Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi_3 + i\phi_4\end{pmatrix}$

$$O_{S,0} = [(D_\mu\Phi)^\dagger D_\nu\Phi][(D^\mu\Phi)^\dagger D^\nu\Phi]$$
$$O_{S,1} = [(D_\mu\Phi)^\dagger D^\mu\Phi][(D_\nu\Phi)^\dagger D^\nu\Phi]$$
$$O_{S,2} = [(D_\mu\Phi)^\dagger D_\nu\Phi][(D^\nu\Phi)^\dagger D^\mu\Phi]$$

forward: t=0



I+II →I+II, elastic

$$\mathcal{L}^{(8)} = \frac{c_{S,0}}{\Lambda^4}\mathcal{O}_{S,0} + \frac{c_{S,1}}{\Lambda^4}\mathcal{O}_{S,1} + \frac{c_{S,2}}{\Lambda^4}\mathcal{O}_{S,2}$$

$\phi_1\phi_1 \to \phi_1\phi_1$ scattering

$$\mathcal{M} = (c_{S,0} + c_{S,1} + c_{S,2})s^2/\Lambda^4$$

Positivity Bounds: Example of Positivity for Dim.8

- EFT Lagrangian →Amplitudes
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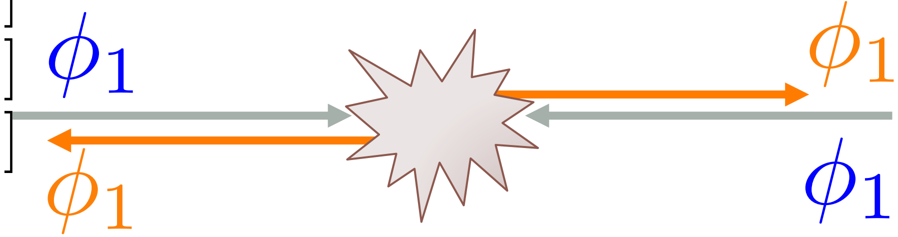


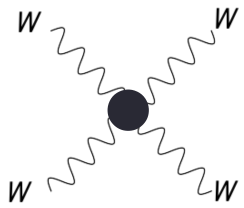
$$\mathcal{L}^{(8)} = \frac{c_{S,0}}{\Lambda^4}\mathcal{O}_{S,0} + \frac{c_{S,1}}{\Lambda^4}\mathcal{O}_{S,1} + \frac{c_{S,2}}{\Lambda^4}\mathcal{O}_{S,2}$$

$\phi_1\phi_1 \to \phi_1\phi_1$ scattering

$$\mathcal{M} = \underbrace{(c_{S,0} + c_{S,1} + c_{S,2})}_{>0} s^2/\Lambda^4$$

Positivity: the signs of certain linear combinations of effective operators have to be positive

Positivity Bounds

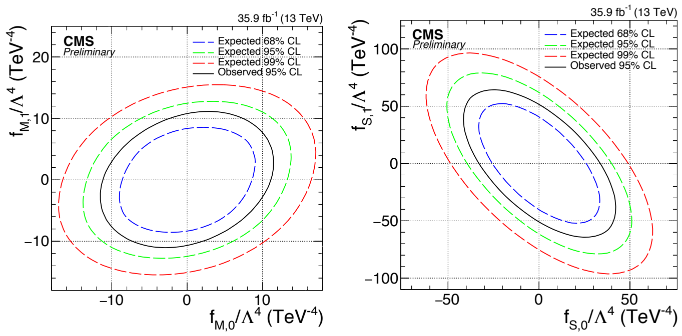


[FFFF]=Dim 8

- Positivity bounds can apply for dim-8 operators in tree-level SMEFT← Froissart Bound (⇦Analyticity)

- Dim-8 operators are more suppressed by Λ than lower dimensional ones, however, for dim-8 aQGC operators, LHC experimentalists have been and currently working on constraining them

CMS_PAS_SMP_18_001



- In the future, more dim-8 effects may become accessible

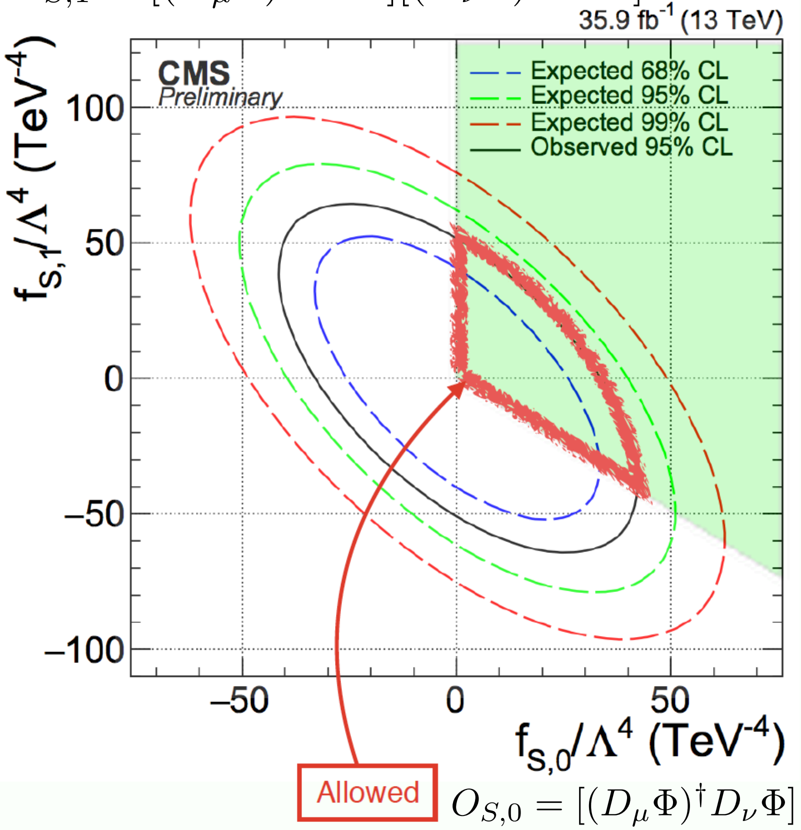
(e.g. new observable proposed for DY process [Alioli, Boughezal, Mereghetti, Petriello, 2003.11615])

Positivity Bounds

Positivity bounds are important as they offer complementary bounds to the experiments Q. Bi, C. Zhang, S.-Y. Zhou JHEP 1906 (2019) 137

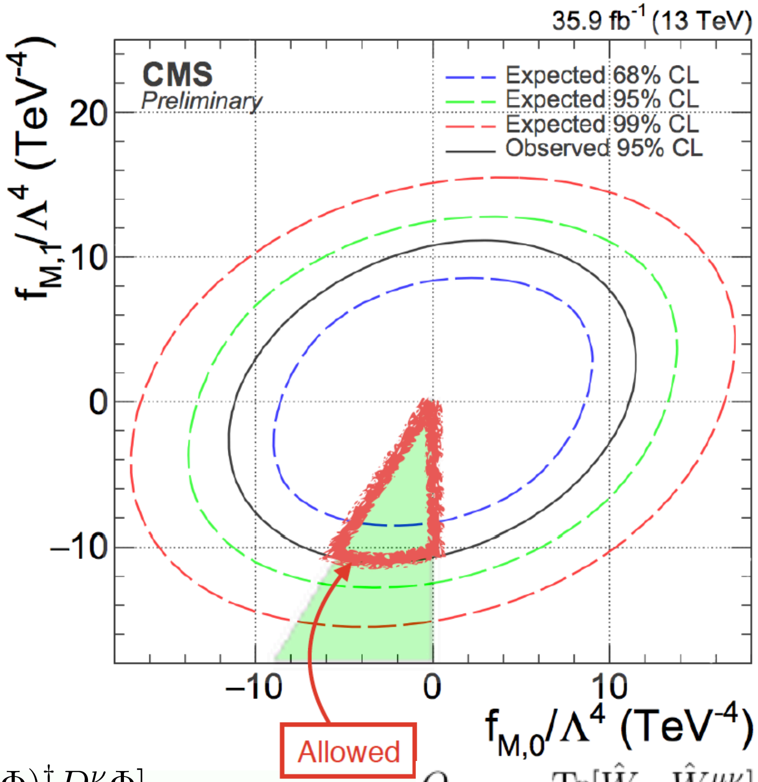
E.g. WZjj (CMS-PAS-SMP-18-001)

$O_{S,1} = [(D_\mu\Phi)^\dagger D^\mu\Phi][(D_\nu\Phi)^\dagger D^\nu\Phi]$



$O_{S,0} = [(D_\mu\Phi)^\dagger D_\nu\Phi][(D^\mu\Phi)^\dagger D^\nu\Phi]$

$O_{M,1} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}][(D_\beta\Phi)^\dagger D^\mu\Phi]$



$O_{M,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}][(D_\beta\Phi)^\dagger D^\beta\Phi]$

Positivity restricts the directions in which SM deviation is possible

Positivity Bounds

- Positivity bounds can be obtained traditionally by considering ①elastic forward scattering by using EFT operators
- Recently new approach for positivity bounds is established: ②extremal positivity bounds
  C. Zhang, S.-Y. Zhou, arXiv: 2005.03047 [hep-ph]
- Comparing ② with ① should be useful
- To see this, we use F^4 operators (next page) as a concrete example
- Deriving bounds on F^4 operators itself is also useful as offering complementary bounds to the experiments

Positivity Bounds

- F^4 operators (this work)

- WWWW

$$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}] \qquad O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$

$$O_{T,2} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}] \qquad O_{T,10} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$$

- WWBB

$$O_{T,5} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \qquad O_{T,6} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\hat{B}_{\mu\beta}\hat{B}^{\alpha\nu}$$

$$O_{T,7} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha} \qquad O_{T,11} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\hat{B}_{\alpha\beta}\tilde{B}^{\alpha\beta}$$

- BBBB

$$O_{T,8} = \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \qquad O_{T,9} = \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta}\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha}$$

$$\hat{W}^{\mu\nu} \equiv ig\frac{\sigma^I}{2}W^{I,\mu\nu}, \quad \hat{B}^{\mu\nu} \equiv ig'\frac{1}{2}B^{\mu\nu}$$

$$\tilde{W}_{\mu\nu} \equiv ig\frac{\sigma^I}{2}\left(\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}W^{I,\rho\sigma}\right), \tilde{B}_{\mu\nu} \equiv ig'\frac{1}{2}\left(\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}B^{\rho\sigma}\right)$$

Dispersion Relation (for Positivity Bounds)

Forward scattering amp, at low energy (EFT)

$$M^2 = m_i^2 + m_j^2 + m_k^2 + m_l^2$$

$$M_{ijkl} = \frac{1}{2}\frac{d^2}{ds^2}M_{ij\to kl}\left(s = \frac{1}{2}M^2, t = 0\right) + c.c.$$

$$= \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} \frac{ds M_{ij\to X} M^*_{kl\to X}}{2\pi(s - \frac{1}{2}M^2)}$$

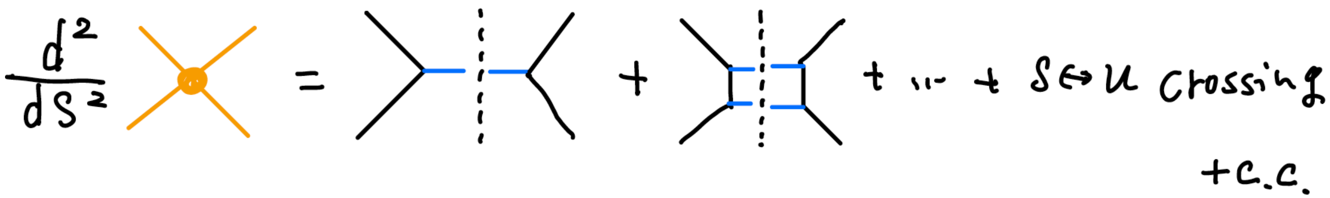
Amplitude of SM →X

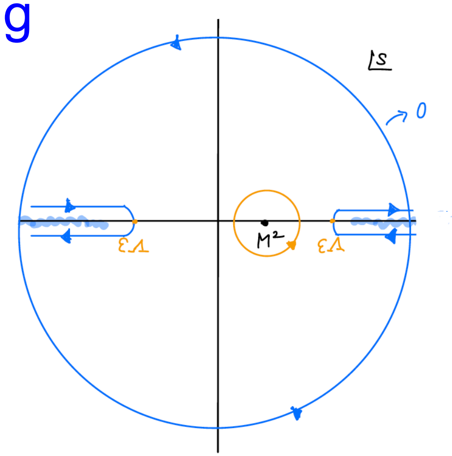
$\epsilon \leq 1$

$$+(j \leftrightarrow l) + c.c$$

Σ_X: BSM states, X summation& LIPS integration

s⇆u crossing





Dispersion Relation

- Useful to rewrite Dispersion Relation for Positivity Bounds

(Amp by Dim.8)
∝ $(F/\Lambda^4)\, s^2$

$$M_{ijkl} = \frac{F_\alpha M_\alpha^{ijkl}}{\Lambda^4}$$

$$M^{ijkl} = \int_{(\epsilon\Lambda)^2}^{\infty} \sum_X{}' \sum_{K=R,I} \frac{d\mu\, m_{K\,X}^{\;ij} m_{K\,X}^{\;kl}}{\pi\mu^3} + (j \leftrightarrow l)$$

where $M(ij \to X) \equiv m_{R_X}^{ij} + i m_{I_X}^{ij}$

- When i=k, j=l, RHS complete squares >=0

$M^{ijij} \geq 0$ because $m_{K\,X}^{\;ij} m_{K\,X}^{\;ij\,*} \geq 0$

- More generally,
  Elastic Forward Scattering between Superposed States :

$M(ab \to ab)$ with $|a\rangle = u^i\,|i\rangle\,, \quad |b\rangle = v^i\,|i\rangle$

‖

$$u^i v^j u^{*k} v^{*l} M^{ijkl} = \int_{(\epsilon\Lambda)^2}^{\infty} \sum_X{}' \sum_{K=R,I} \frac{d\mu}{\pi\mu^3} \left[|u \cdot m_{K_X} \cdot v|^2 + |u \cdot m_{K_X} \cdot v^*|^2 \right] \geq 0$$

(generalized) Elastic Positivity Bounds

Elastic Positivity Bounds on F^4 operators (Result)

- All possible superposed states for the elastic forward scattering are considered: $W_x^1, W_y^1, W_x^2, W_y^2, W_x^3, W_y^3, B_x, B_y$
- Linear Positivity Bounds

bounds	channel $(\|1\rangle + \|2\rangle \to \|1\rangle + \|2\rangle)$
$F_{T,2} \geq 0,$	$\|1\rangle = \|W_x^1\rangle\,,\ \|2\rangle = \|W_y^2\rangle$
$4F_{T,1} + F_{T,2} \geq 0,$	$\|1\rangle = \|W_x^1\rangle\,,\ \|2\rangle = \|W_x^2\rangle$
$F_{T,2} + 8F_{T,10} \geq 0,$	$\|1\rangle = \|W_x^1\rangle + \|W_y^2\rangle\,,\ \|2\rangle = \|W_y^1\rangle - \|W_x^2\rangle$
$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0,$	$\|1\rangle = \|W_x^1\rangle + \|W_y^2\rangle\,,\ \|2\rangle = \|W_x^1\rangle + \|W_y^2\rangle$
$2F_{T,8} + F_{T,9} \geq 0,$	$\|1\rangle = \|B_x\rangle\,,\ \|2\rangle = \|B_x\rangle$
$F_{T,9} \geq 0,$	$\|1\rangle = \|B_x\rangle\,,\ \|2\rangle = \|B_y\rangle$
$4F_{T,6} + F_{T,7} \geq 0,$	$\|1\rangle = \|B_x\rangle\,,\ \|2\rangle = \|W_x^1\rangle$
$F_{T,7} \geq 0,$	$\|1\rangle = \|B_x\rangle\,,\ \|2\rangle = \|W_y^1\rangle$

other superposed states don't give new bounds

- Some Quadratic and Cubic Bounds
- Parameter space is constrained to ~0.7% of total

Extremal Positivity Bounds on F^4 operators

C. Zhang, S-Y. Zhou, arXiv: 2005.03047 [hep-ph]

- Dispersion Relation

Dynamics

Symmetry

$$M^{ijkl} = \int_{(\epsilon\Lambda)^2}^{\infty} \mathrm{d}\mu \sum_{X \text{ in } \mathbf{r}}{}' \frac{|\langle X|\mathcal{M}|\mathbf{r}\rangle|^2}{\pi\mu^3} P_{\mathbf{r}}^{i(j|k|l)}$$

Positive

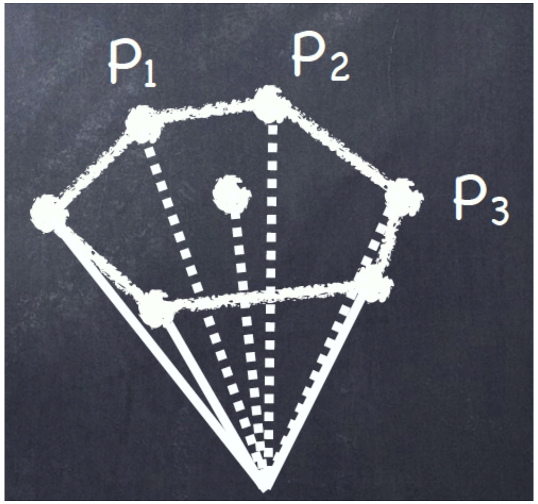
i(j|k|l): *j*, *l* indices are symmetrized

$$P_{\mathbf{r}}^{ijkl} \equiv \sum_\alpha C_{i,j}^{\mathbf{r},\alpha} (C_{k,l}^{\mathbf{r},\alpha})^*$$

- **r** runs all irreps of the SO(2) rotation around the forward scattering axis and SM gauge symmetries

- M must stay inside the cone

A convex cone *C*:
if ***x***, ***y*** ∈ *C* then α***x***+β***y*** ∈ *C*
for α,β>=0



Edges of this cone are extremal rays

Extremal Positivity Bounds on F^4 operators

C. Zhang, S-Y. Zhou, arXiv: 2005.03047 [hep-ph]

- Dispersion Relation

Dynamics Symmetry

$$M^{ijkl} = \int_{(\epsilon\Lambda)^2}^{\infty} d\mu \sum_{X \text{ in } \mathbf{r}}{}' \frac{|\langle X|\mathcal{M}|\mathbf{r}\rangle|^2}{\pi\mu^3} P_{\mathbf{r}}^{i(j|k|l)}$$

Positive

i(j|k|l): *j*, *l* indices are symmetrized

$$\mathcal{C} \equiv \text{cone}\left(\left\{P_{\mathbf{r}}^{i(j|k|l)}\right\}\right)$$

- Dynamics by the SMEFT determine:

$M^{ijkl} = \sum_\alpha C_\alpha M_\alpha^{ijkl} \longrightarrow$ *S* of $\{M^{ijkl}\}$

- The positivity bounds are obtained by

$$\mathcal{C}_S = \mathcal{C} \cap \mathcal{S}$$

Extremal Positivity Bounds on F^4 operators

- WWWW case
  Find projection operators:

1. $\mathbf{3}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{3}\oplus\mathbf{5}$ in $\mathrm{SU}(2)_L$ (N=3)
2. $\mathbf{2}\otimes\mathbf{2}=\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{2}$ in SO(2) rotation around the forward direction (N=2)

$$P^{(1)}_{\alpha\beta\gamma\sigma}=\frac{1}{N}\delta_{\alpha\beta}\delta_{\gamma\sigma},\qquad P^{(2)}_{\alpha\beta\gamma\sigma}=\frac{1}{2}\left(\delta_{\alpha\gamma}\delta_{\beta\sigma}-\delta_{\alpha\sigma}\delta_{\beta\gamma}\right),$$

$$P^{(3)}_{\alpha\beta\gamma\sigma}=\frac{1}{2}\left(\delta_{\alpha\gamma}\delta_{\beta\sigma}+\delta_{\alpha\sigma}\delta_{\beta\gamma}\right)-\frac{1}{N}\delta_{\alpha\beta}\delta_{\gamma\sigma},$$

Combining both 1) and 2) projectors with
β, σ indices symmetrized→ 9 potential ERs
→ 8 ERs for *C* ER: cannot be a sum of two other elements in the cone

Calculate the EFT amplitudes→*S* and find the intersections:

- WWWW

$$\mathcal{C}_S=\mathcal{C}\cap\mathcal{S}$$

$$O_{T,0}=\mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}]\qquad O_{T,1}=\mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$

$$O_{T,2}=\mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}]\qquad O_{T,10}=\mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$$

Extremal Positivity Bounds on F^4 operators (Result)

- WWWW case

- WWWW

$$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}] \quad O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$
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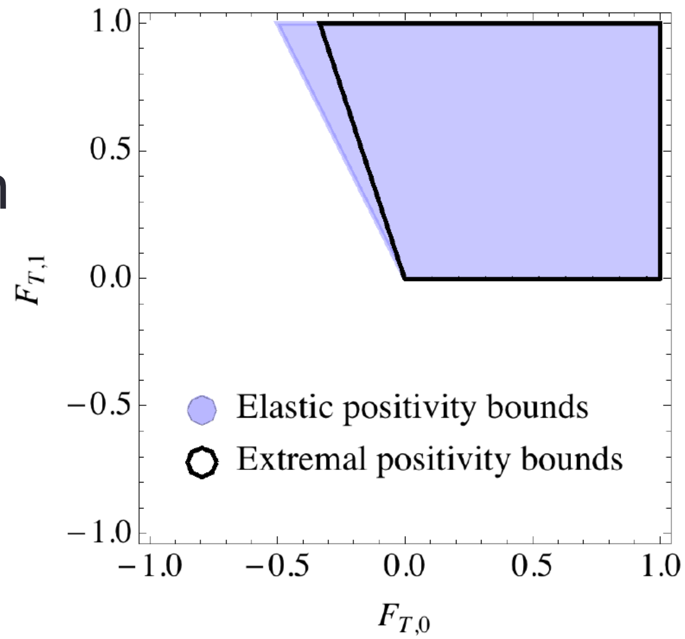
$$F_{T,2} \geq 0,$$
$$4F_{T,1} + F_{T,2} \geq 0,$$
$$F_{T,2} + 8F_{T,10} \geq 0,$$
$$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0,$$
$$12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0,$$
$$4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 0.$$

Better than Elastic Positivity Approach



Extremal Positivity Bounds on F^4 operators (Result)

- Full F^4 operators

Linear

$$F_{T,2} \geq 0$$
$$4F_{T,1} + F_{T,2} \geq 0$$
$$F_{T,2} + 8F_{T,10} \geq 0$$
$$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0$$
$$12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0$$
$$4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 0$$

WWWW

$$4F_{T,6} + F_{T,7} \geq 0$$
$$F_{T,7} \geq 0$$

WWBB

$$2F_{T,8} + F_{T,9} \geq 0$$
$$F_{T,9} \geq 0$$

BBBB

Extremal Positivity Bounds on F^4 operators (Result)

- Full F^4 operators

  Quadratic

$$F_{T,9}\left(F_{T,2}+4F_{T,10}\right)\geq F_{T,11}^{2}$$

$$16\left(2\left(F_{T,0}+F_{T,1}\right)+F_{T,2}\right)\left(2F_{T,8}+F_{T,9}\right)\geq\left(4F_{T,5}+F_{T,7}\right)^{2}$$

$$32\left(2F_{T,8}+F_{T,9}\right)\left(3F_{T,0}+F_{T,1}+2F_{T,2}+4F_{T,10}\right)\geq 3\left(4F_{T,5}+F_{T,7}\right)^{2}$$

$$2\sqrt{2}\sqrt{F_{T,9}\left(F_{T,2}+8F_{T,10}\right)}\geq\max\left(-F_{T,7}-4F_{T,11},-4F_{T,6}-F_{T,7}+4F_{T,11}\right)$$

$$4\sqrt{\left(8F_{T,0}+4F_{T,1}+3F_{T,2}\right)\left(2F_{T,8}+F_{T,9}\right)}$$
$$\geq\max\left(-8F_{T,5}-4F_{T,6}-3F_{T,7},8F_{T,5}+F_{T,7}\right)$$

$$4\sqrt{F_{T,9}\left(12F_{T,0}+4F_{T,1}+5F_{T,2}+4F_{T,10}\right)}$$
$$\geq\max\left(-F_{T,7}-4F_{T,11},-4F_{T,6}-F_{T,7}+4F_{T,11}\right)$$

$$4\sqrt{6}\sqrt{\left(2F_{T,8}+F_{T,9}\right)\left(12F_{T,0}+4F_{T,1}+5F_{T,2}+4F_{T,10}\right)}$$
$$\geq\max\left[3\left(8F_{T,5}+F_{T,7}\right),-3\left(8F_{T,5}+4F_{T,6}+3F_{T,7}\right)\right]$$

$$\sqrt{6}\sqrt{\left(4F_{T,8}+3F_{T,9}\right)\left(6F_{T,0}+2F_{T,1}+3F_{T,2}+6F_{T,10}\right)}$$
$$\geq\max\left(3\left(2F_{T,5}+F_{T,11}\right),-3\left(2F_{T,5}+F_{T,7}+F_{T,11}\right)\right)$$

$$2\sqrt{\left(12F_{T,8}+7F_{T,9}\right)\left(12F_{T,0}+4F_{T,1}+5F_{T,2}+4F_{T,10}\right)}$$
$$\geq\max\left(-12F_{T,5}-4F_{T,6}-5F_{T,7}-2F_{T,11},12F_{T,5}+F_{T,7}-2F_{T,11},\right.$$
$$\left.12F_{T,5}+F_{T,7}+2F_{T,11},12F_{T,5}-4F_{T,6}+F_{T,7}+2F_{T,11}\right)$$

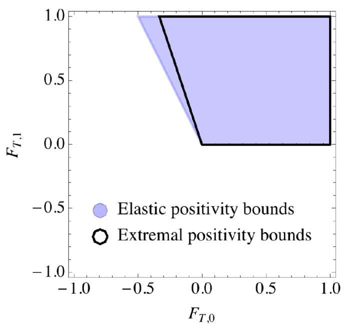
Comparison

- Elastic Positivity Approach
- Extremal Positivity Approach

Allowed Parameter Space

$$\Omega(C_S^{el}) = 0.693\%$$

✅ $\Omega(\mathcal{C}_S) = 0.681\%$



Differences mainly arise in W sector

Potential Ability to Apply for More Complicated Case

Not yet algorithmized
It should become
more complicated

✅ algorithmized
we directly wrote down using group theory as a guideline to know the edge of the cone

Numerical Calculation Speed

Single-core time [second]	Elastic $\mathcal{C}_N^{el}$	Extremal $\mathcal{C}_N$		
		$N = 50$	$N = 100$	$N = 200$
No filter	0.1	0.002	0.004	0.007
With filter	0.001	0.00019	0.00021	0.00023
Error on $\Omega(\mathcal{C}_N)$	N/A	3%	0.8%	0.2%

✅ faster

← the error is negligible

Summary

- If the UV theory possesses Unitarity and Analyticity of the S-matrix
  → the signs of certain linear combinations
  of Wilson coefficients in EFT have to be positive

- This positivity can be applied for many kind of EFTs

- Here we applied the positivity to SMEFT, especially, F^4 operators by using elastic & new extremal positivity bounds
  → bounds are very strong (99.3% excluded) and give a useful guide to experimental searches

- Extremal positivity approach has a potential ability to apply for more complicated case

Other progresses

(Almost) after our work, there are progresses on improving positivity bounds, e.g.,

- N. Arkani-Hamed, T. C. Huang, Y.T. Huang
  The EFT-Hedron
  arXiv: 2012.15849 [hep-ph]
- B. Bellazzini, J. Elias Miro, R. Rattazzi, M. Riembau, F. Riva
  Positive Moments for Scattering Amplitudes
  arXiv: 2011.00037 [hep-ph]
- X. Li, C. Yang, H. Xu, C. Zhang, S. Y. Zhou
  Positivity in Multi-Field EFTs
  arXiv: 2101.01191 [hep-ph]

Positivity is one of the active areas!