一般化King関係式を用いた新しい力の探索

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Based on 1710.11443, 1911.05345, 2110.13544

Collaborators

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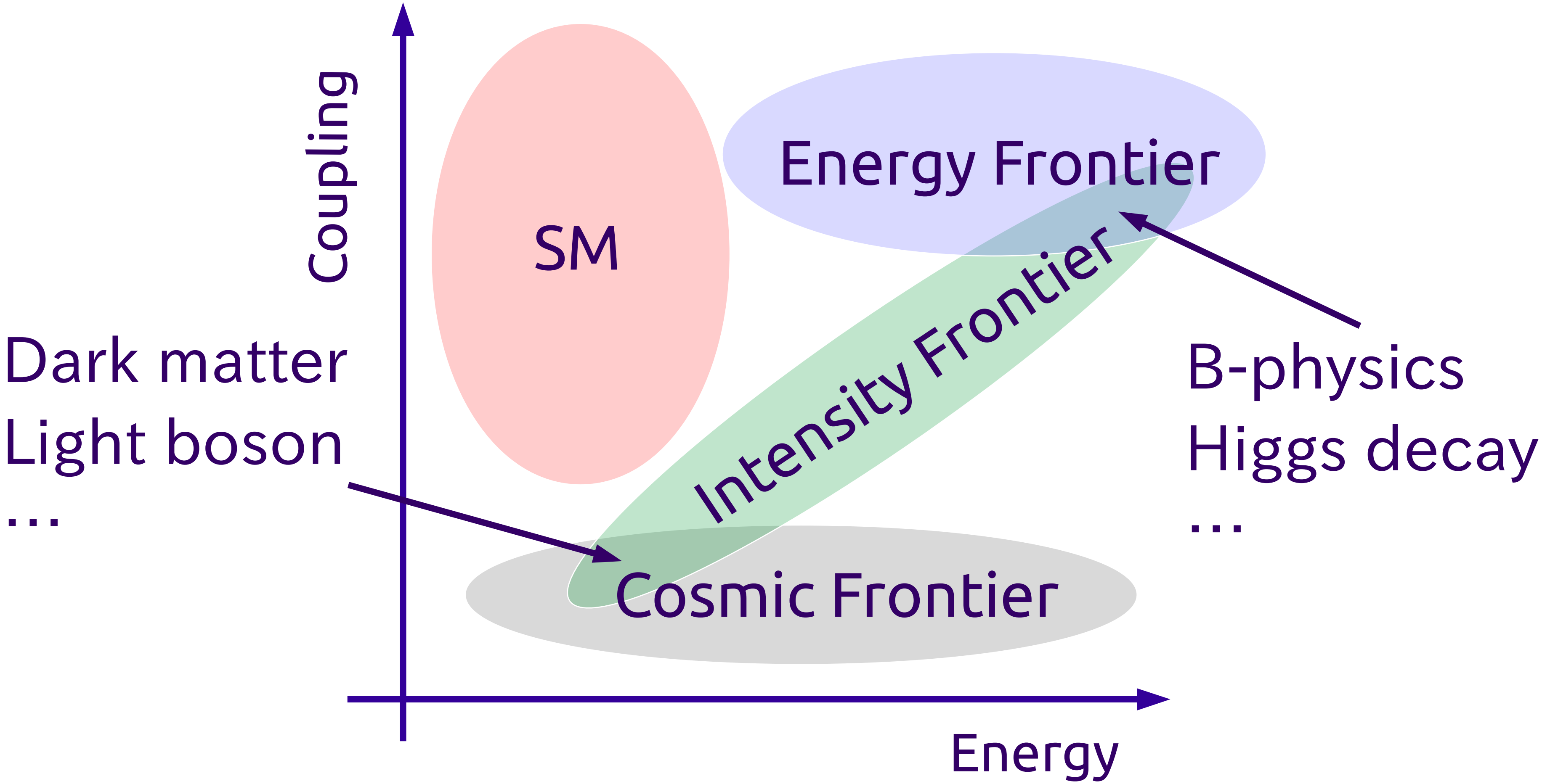
Map of phenomenology

- Interaction strength $\sim \frac{g}{M} : \frac{1}{1\ \mathrm{TeV}} = \frac{10^{-4}}{100\ \mathrm{MeV}}$

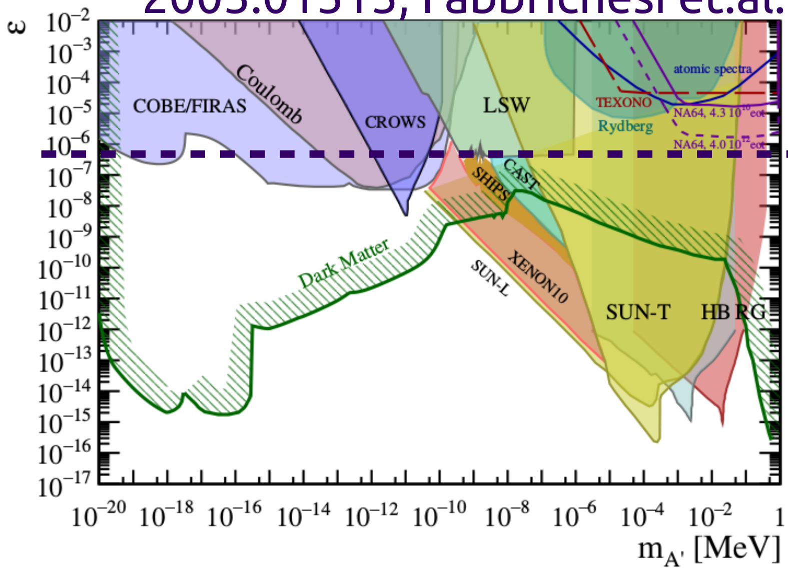


▶ Collider, Flavor, g-2, EDM, Dark matter, Inflation ...

Weakly interacting light boson

- Drastically change astrophysical observations.

2005.01515, Fabbrichesi et.al.

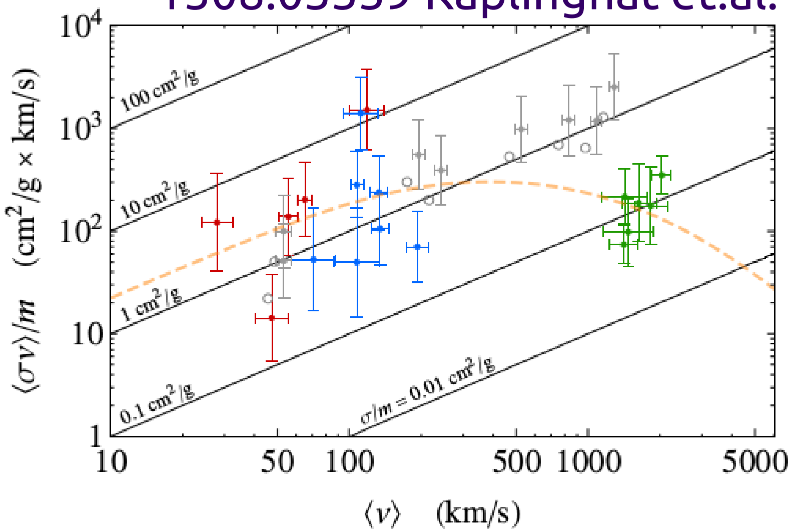


Terrestrial experiments

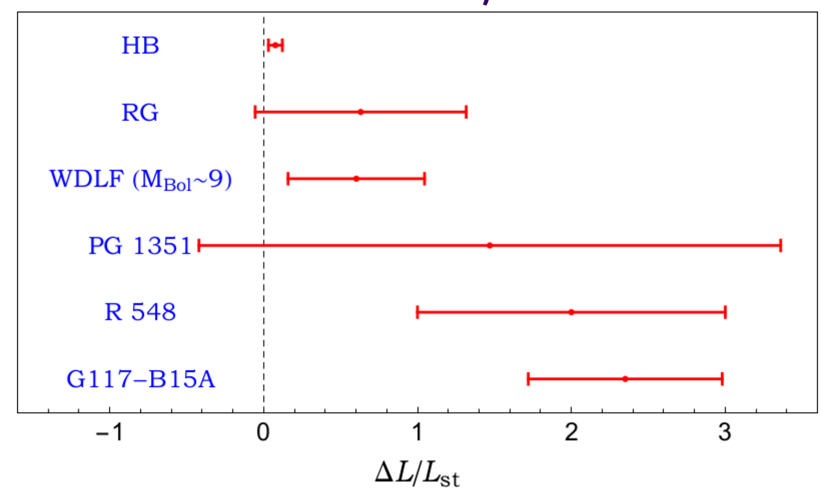
Astrophysical obs.

- Some implications in observations.

1508.03339 Kaplinghat et.al.

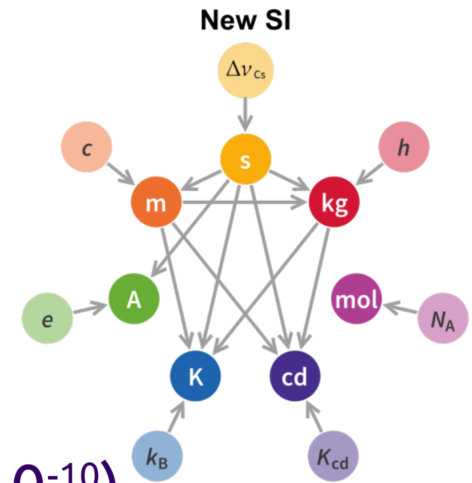


1512.08108, Giannotti et.al.



Atomic clock

- Extremely precise atomic spectroscopy.
  - The relative precision is $O(10^{-15})$.
    (Clock error ~ 410 sec./13By)
    - The relativistic effect of GPS is $O(10^{-10})$.
    - The error of the fine structure constant is $O(10^{-10})$.



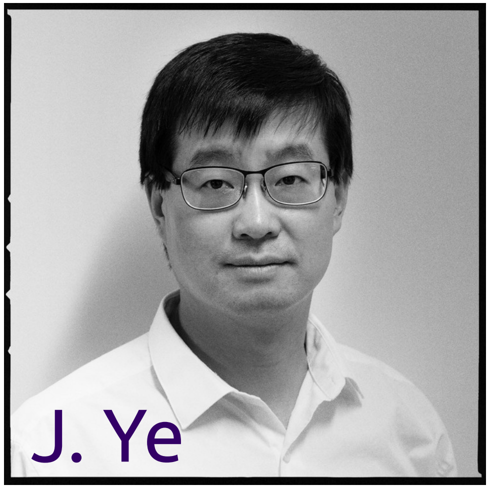
- At the frontier with Sr, Yb ⟶ $O(10^{-18})$ or better.

▼

Experimentally excellent but theoretically poor.

Breakthrough Prize 2022
Fundamental Physics

Invention and development
of the optical lattice clock



Isotope shift and King's relation

- Isotope dependence of spectrum.

H. Nagaoka

Inverse mass diff. — Nuclear size

$$\delta\nu_1 = \underbrace{K_1 \delta\mu}_{\text{Mass shift}} + \underbrace{F_1 \delta\langle r^2 \rangle}_{\text{Field shift}}$$

▶ (Electron wave fn) x (Isotope dependence)

- Isotope shift of Hg by Nagaoka in 1920s.
  (c.f. discovery of neutron by J. Chadwick in 1933)

▶ A linear relation among transitions.

1963, W.H.King

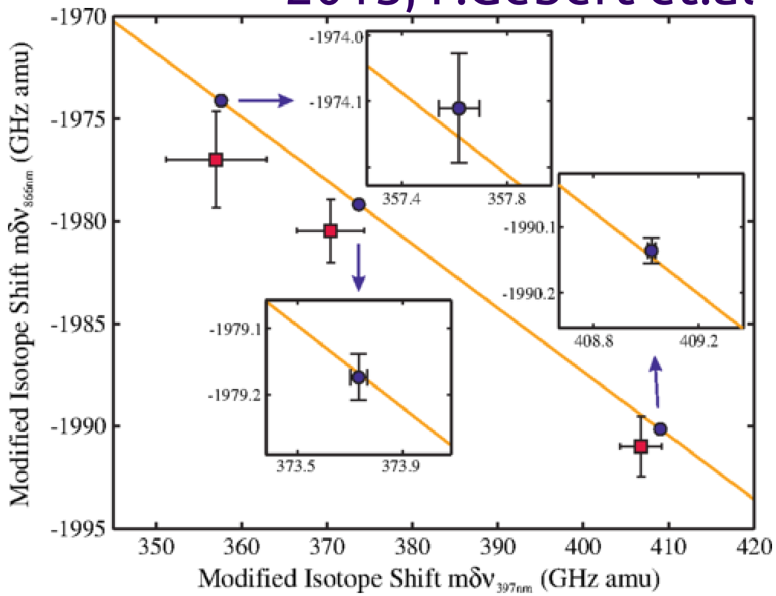
$$\frac{\delta\nu_2}{\delta\mu} = \boxed{\frac{F_2}{F_1}} \frac{\delta\nu_1}{\delta\mu} + \boxed{K_2 - \frac{F_2}{F_1} K_1}$$

Independent of isotopes

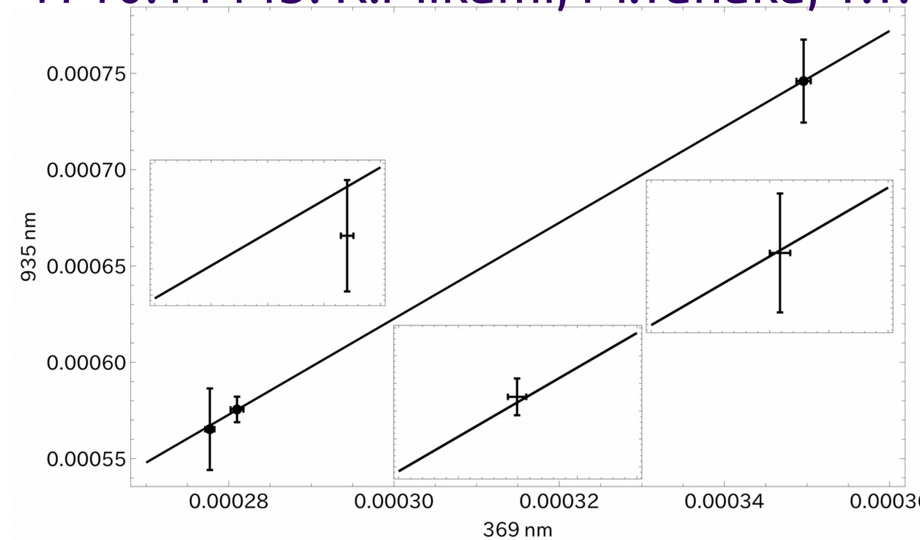
▶ Uncertainties to calculate spectra are suppressed.

Violation of the linear relation

2015, F.Gebert et.al



1710.11443: K.Mikami, M.Tanaka, Y.Y.



- Other higher order contributions.

1987, Blundell et. al.

$$\delta\nu_1 = K_1\delta\mu + F_1\delta\langle r^2\rangle + \underline{X_1\delta\eta}$$

- The linear relation is violated.

Additional fit parameter

$$\frac{\delta\nu_2}{\delta\mu} = \frac{F_2}{F_1}\frac{\delta\nu_1}{\delta\mu} + K_2 - \frac{F_2}{F_1}K_1 + \underline{\left(X_2 - \frac{F_2}{F_1}X_1\right)\frac{\delta\eta}{\delta\mu}}$$

- Which kinds of non-linearity may appear?

Violation of King's relation

- Higgs boson — 1601.05087: C.Delaunay et.al.
  - Heavy boson contribution is canceled.

- Weakly interacting light boson — 1704.05068: J.C.Berengut et.al.

$$\delta H = \delta K + \delta V + \alpha_{\mathrm{ne}} \delta A \frac{e^{-mr}}{r}$$

- Second order perturbation — 1709.00600: V.V. Flambaum et.al.

$$\delta \nu_1 = G_1 \delta \mu + F_1 \delta \langle r^2 \rangle + H_1 [\delta \langle r^2 \rangle^2] + \cdots$$

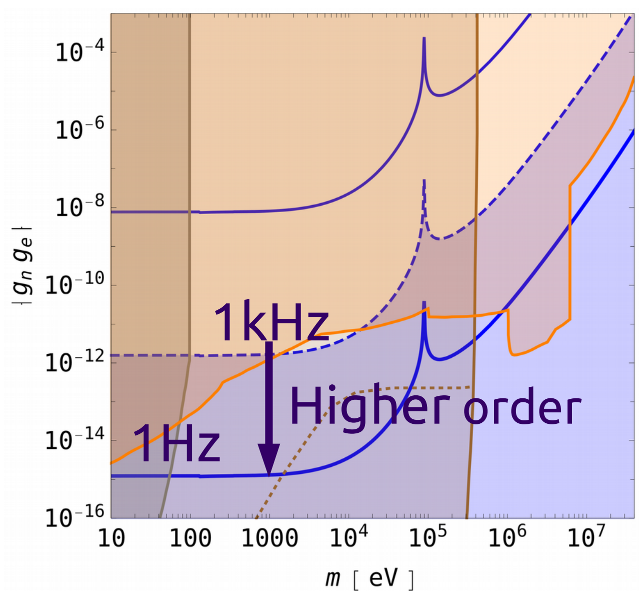
- Higher order moment

$$\delta \nu_1 = G_1 \delta \mu + F_1 \delta \langle r^2 \rangle + \tilde{F}_1 \delta \langle r^4 \rangle + \cdots$$

1710.11443: K.Mikami, M.Tanaka, Y.Y.

1911.05345: M.Tanaka, Y.Y.

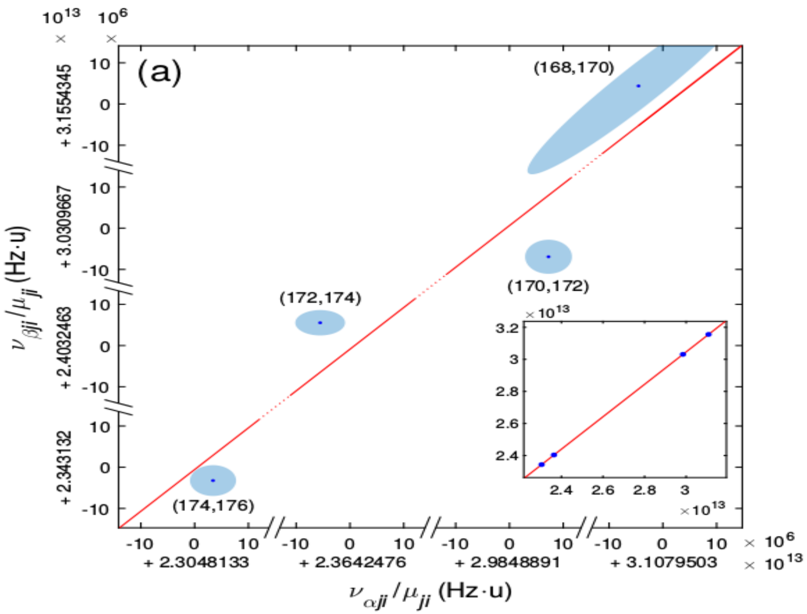
Yb+

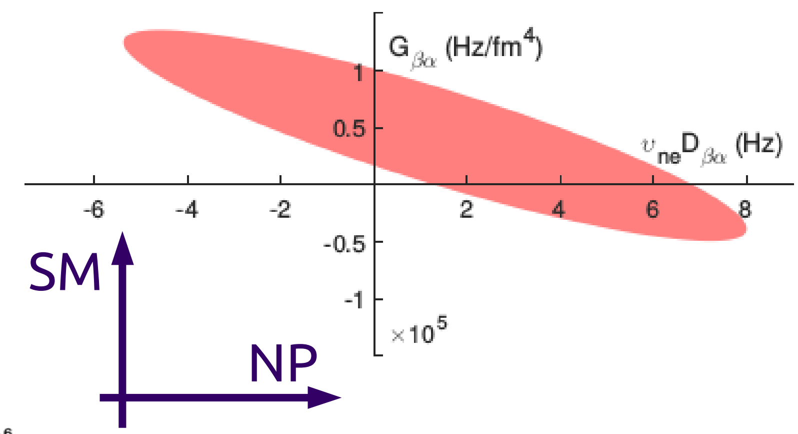


Beyond King’s relation

- Non-linearity is observed around 300 Hz of precision.

2004.11383: I.Counts, et. al.





  - What is the origin of this non-linearity?

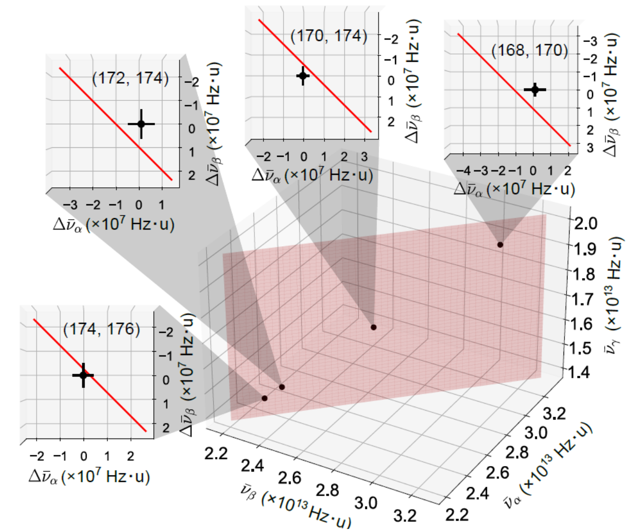
- King’s relation is generalized. 1710.11443: K.Mikami, M.Tanaka, Y.Y.

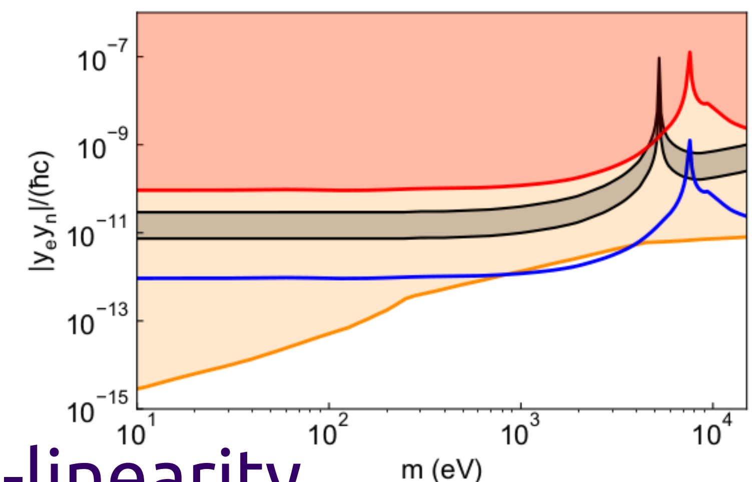
$$\frac{\delta\nu_3}{\delta\mu} = c_1\frac{\delta\nu_1}{\delta\mu} + c_2\frac{\delta\nu_2}{\delta\mu} + c_0 + c_X\frac{\delta\eta}{\delta\mu}$$

  - An additional isotope dependence is canceled.

2110.13544: K.Ono, Y.Y., et.al.

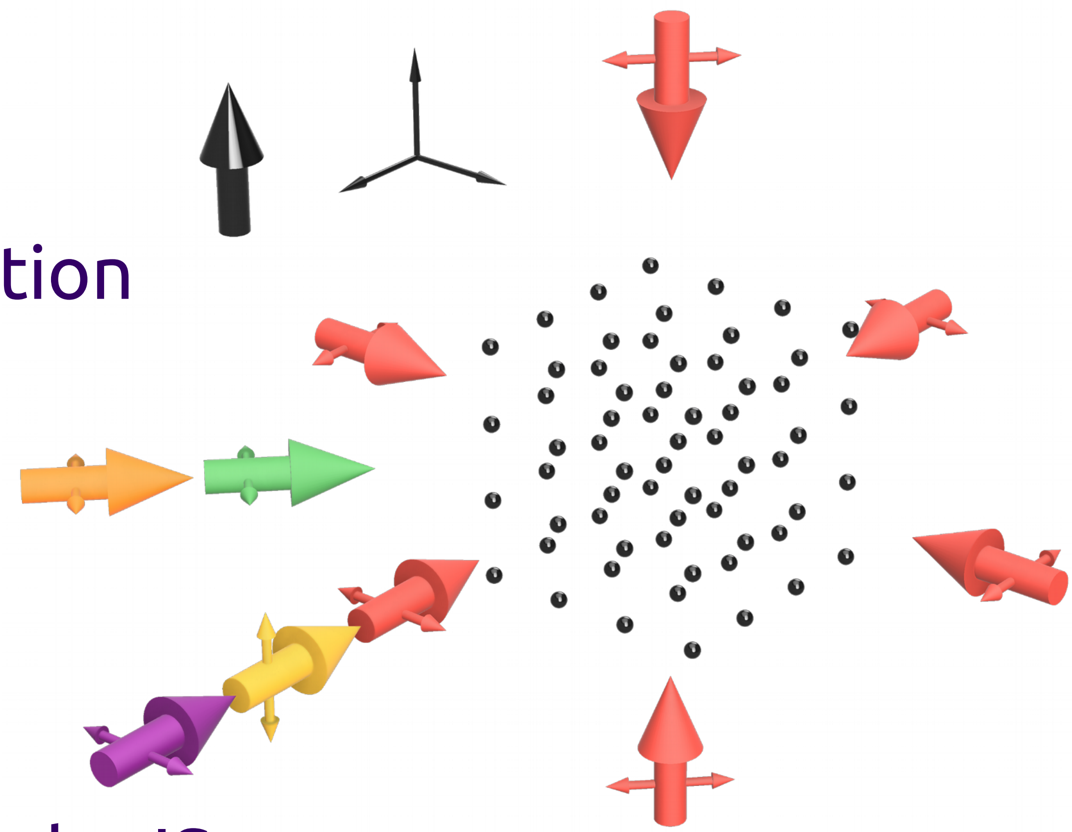
- $^1S_0$-$^3P_0$ transition of neutral Yb with precision of a few Hz.
- Additional 2D King plot studies with Yb^+ transitions.
  - Implication of two higher order isotope shifts.
- First time to study 3D King plot
  - Further insight to origin of non-linearity
  - New bound and future prospect to new physics





Contents

- Introduction
- Some formulas
  - Field and particle shift
  - Generalized King's relation
- 2D King's plot
  - Review of MIT results
  - Combined analyses
- 3D King's plot
  - Implication to higher order IS
  - Upper bounds
- Conclusion



Experimental set up drawn by K. Ono

Field shift

- Seltzer moment expansion:

1969, E. C. Seltzer

$$\int d\vec{r}\,\left(|\psi_j(\vec{r})|^2 - |\psi_i(\vec{r})|^2\right)\delta V(\vec{r})$$

$$-Z\alpha\int d\vec{r'}\,\frac{\delta\rho(\vec{r'})}{|\vec{r}-\vec{r'}|}$$

$$\propto \int_0^\infty dr' \int_0^{r'} dr\, r^2 \sum_k \xi_k r^k \left(r' - \frac{r'^2}{r}\right)\delta\rho(r')$$

$$\delta\langle r^k\rangle = \int d\vec{r}\, r^k \delta\rho(r)$$

$$Z\alpha\sum_k \frac{\xi_k}{(k+3)(k+2)}\delta\langle r^{k+2}\rangle = F\delta\langle r^2\rangle + \tilde{F}\delta\langle r^4\rangle + \cdots$$

- Second order perturbation:

$$\sum_m a_m \langle n|\delta V|m\rangle\langle m|\delta V|n\rangle \propto [\delta\langle r^2\rangle^2]$$

Particle shift

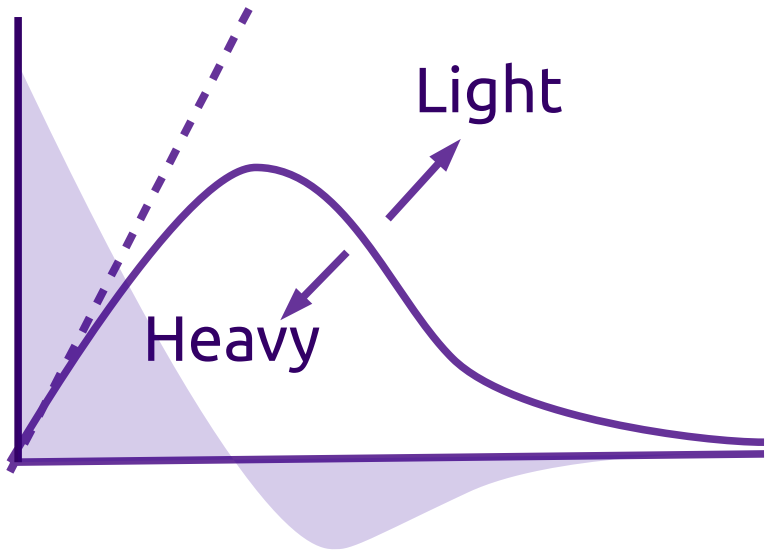
- Leading new physics effect

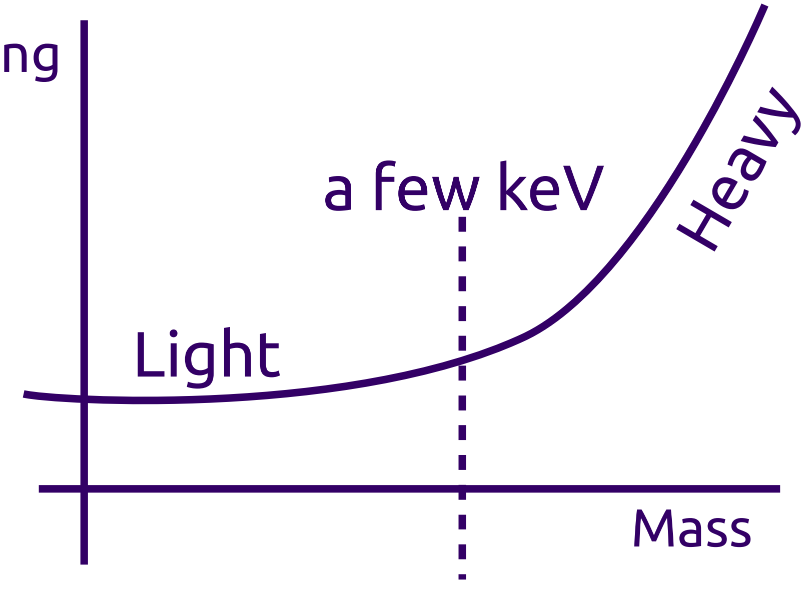
$$\int d\vec{r}\,\left(|\psi_j(\vec{r})|^2 - |\psi_i(\vec{r})|^2\right)\alpha_{\rm ne}\delta A\frac{e^{-mr}}{r}$$

Sensitive to only e-n coupling

Heavy → $\alpha_{\rm ne}\delta A\sum_k \frac{k!}{m^{k+2}}\xi_k$

Light → Binding energy





Cancellation of PS

- Heavy mediator expansion is similar to FS.

$$\alpha_{\rm ne}\delta A \sum_k \frac{k!}{m^{k+2}}\xi_k = a_0 F/m^2 + a_2 \tilde{F}/m^4 + \cdots$$

Same   Same

$$Z\alpha \sum_k \frac{\xi_k}{(k+3)(k+2)}\delta\langle r^{k+2}\rangle = F\delta\langle r^2\rangle + \tilde{F}\delta\langle r^4\rangle + \cdots$$

- $\delta\nu = K\delta\mu + F\left(\delta\langle r^2\rangle + a_0/m^2\right) + \tilde{F}\left(\delta\langle r^4\rangle + a_2/m^4\right) + \cdots$

Keep the linearity   Non linearity

- Sensitivity to new heavy boson is $1/m^4$ or worse.
  - The target is not Higgs but a light particle.

Generalized King's relation

- Considering only the Seltzer moment,

$$\delta\nu_i = G_i \delta\mu + \sum F_i^{(k)} \delta\langle r^{2+k}\rangle + X_i$$ (FS2, PS, …)

$$\delta\vec{\nu} - \vec{X} = \begin{pmatrix} \vdots & \vdots & \vdots & \\ G_i & F_i^{(0)} & F_i^{(2)} & \cdots \\ \vdots & \vdots & \vdots & \end{pmatrix} \begin{pmatrix} \delta\mu \\ \delta\langle r^2\rangle \\ \delta\langle r^4\rangle \\ \vdots \end{pmatrix}$$

(Electronic factors : T) x (Isotope dependence)

- King's linearity is

$$\sum (T^{-1})_{1k} \left(\delta\vec{\nu} - \vec{X}\right)_k = \delta\mu$$

Precision of the linearity

- Other linearities, e.g., PS induces linearity of δA
- Irreducible formulas to determine the nuclear shape.

Transitions

- Yb^+ ([Xe] $4f^{14}$ $6s^1$) 2004.11383: I.Counts, et. al.

  α: $^2S_{1/2} - {}^2D_{5/2}$ β: $^2S_{1/2} - {}^2D_{3/2}$

  - Error ~ 300 Hz
  - Split by the relativistic effect.

- Yb ([Xe] $4f^{14}$ $6s^2$) 2110.13544: K.Ono, Y.Y., et.al.

  γ: $^1S_0 - {}^3P_0$

  - Error ~ 3 Hz

    (c.f. $^1S_0 - {}^1D_2$ of Yb with 300 Hz by another group.)

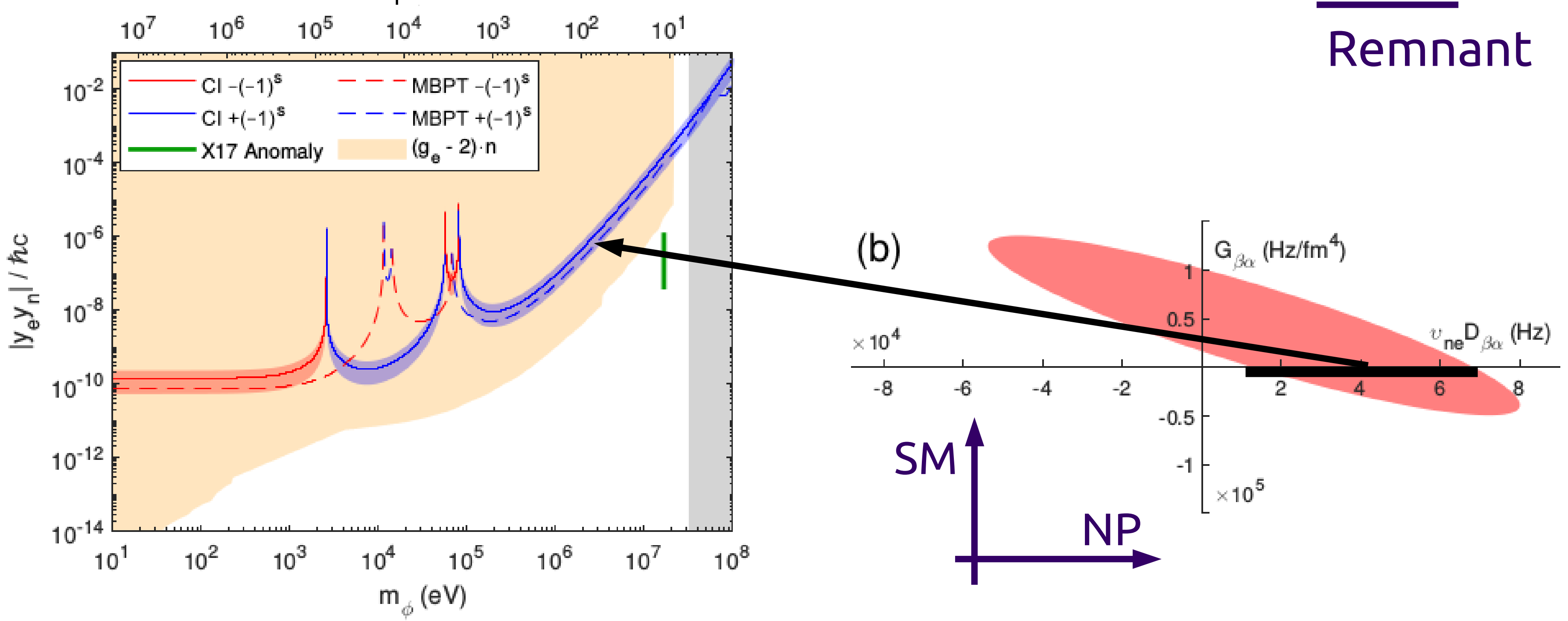
- 4 isotope pairs (168, 170, 172, 174, 176).
- 3 King's plots + 3D King's plot.

2D King relation by Yb^+

- α: $^2S_{1/2} - {}^2D_{5/2}$ & β: $^2S_{1/2} - {}^2D_{3/2}$
- $\delta\langle r^4\rangle$ ➡ $\delta\langle r^2\rangle$ & $[\delta\langle r^2\rangle^2]$: SM effects is given by FS2

χ^2 (p-value)	King	FS_2	PS	FS_2 + PS
(α, β)	15(0.004)	4.2(0.23)	5.4(0.15)	4.037

Remnant



- NP is excluded by other exps.

2D King's relation with Yb

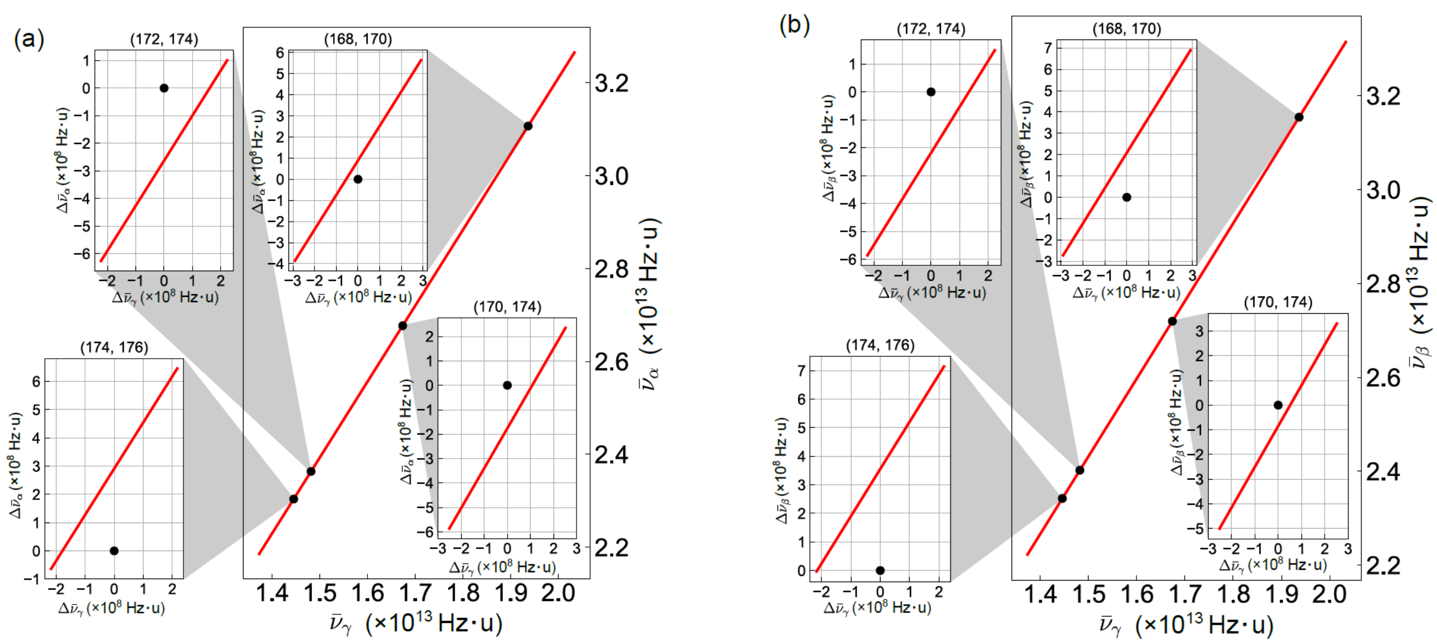
- γ: $^1S_0$-$^3P_0$ & α / β

χ^2	King	FS_2	PS	FS_2 + PS
(α, γ)	$\sim 10^4$	$\sim 10^4$	$\sim 10^4$	0.126
(β, γ)	$\sim 10^4$	$\sim 10^4$	$\sim 10^4$	3.911

} 4.037 (0.126 + 3.911, the FS_2 + PS column)

Cannot fit the data (King, FS_2 and PS columns)

▶ Need two or more higher order IS to fit the results.



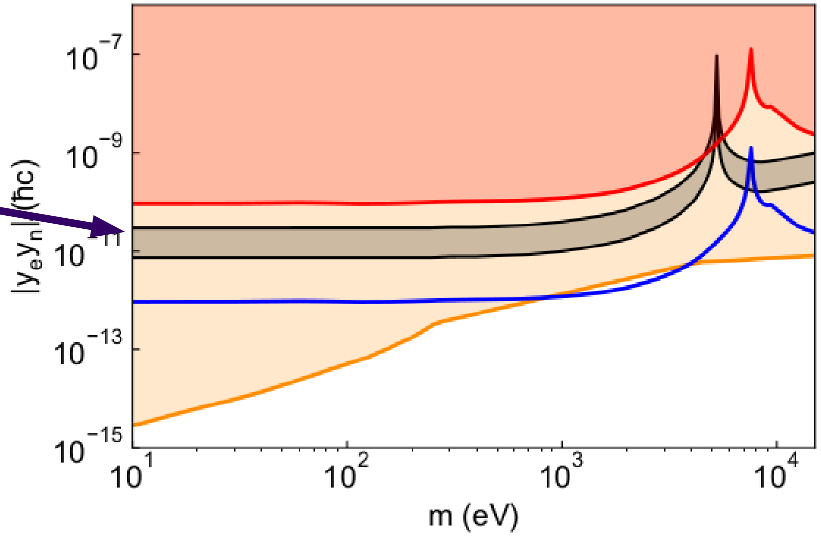
3D King's relation

- The linear relation is recovered by an additional source.

χ^2 (p-value)	King	FS_2	PS
(α, β, γ)	$15(2.3\times10^{-3})$	4.037	4.037

  - Need two or more higher order isotope shifts.

- PS fit : excluded by other experiments.
- FS2 fit : inconsistent with FS2 + PS hypothesis



	f_α	f_γ	k_μ	h
Fit	1.018(13)	-0.010(21)	124.1(7.7)	70(21)
Calc.	0.975	0.063	217.7	-81

  - Another higher order IS is observed.

Upper bound

- Cancel unknown IS with 3D King relation in FS2 fit.

$$\delta\nu_\beta = f_\alpha \delta\nu_\alpha + f_\gamma \delta\nu_\gamma + k\delta\mu + h[\delta\langle r^2\rangle^2]$$

▶ Assuming that it gives us the correct constraint to h.

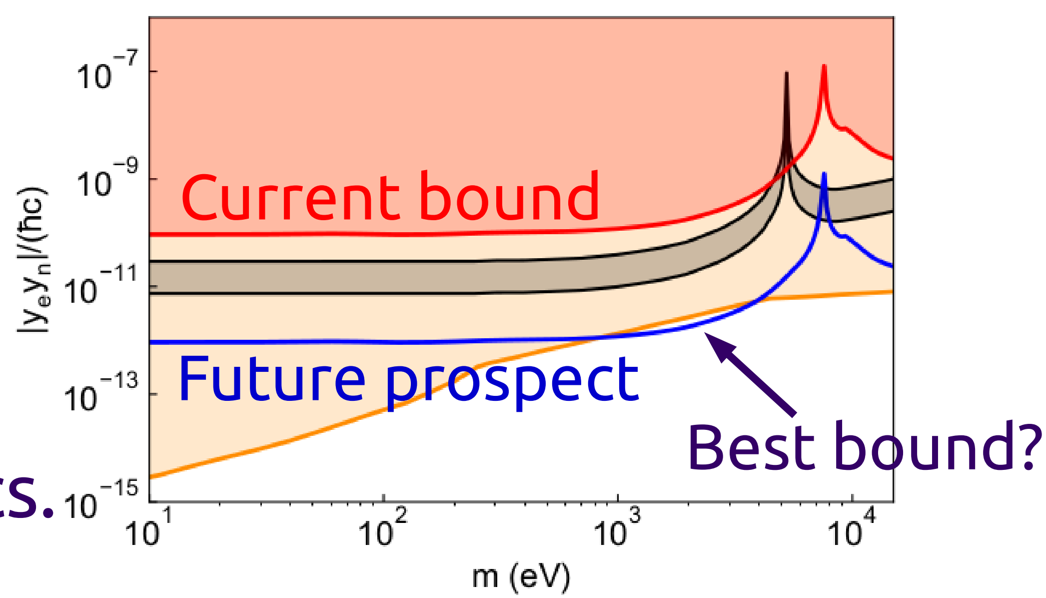
▼

Upper bound to PS within the given h.

$$\delta\nu_\beta = f_\alpha \delta\nu_\alpha + f_\gamma \delta\nu_\gamma + k\delta\mu + h[\delta\langle r^2\rangle^2] + \underline{X\delta A}$$

- Future prospect
  - α, β : 1/100 errors (a few Hz level)
  - $\delta\langle r^2\rangle$: 1/10 errors

▶ Need accurate numerics.



Conclusion

- Search for light new boson with isotope shifts.
  - How can we understand new data with higher order IS?
- First attempt of the generalized King relation to data.
  - New measurement of a few Hz level IS with Yb.
  - Extract information of higher order IS from data.
  - New bound and prospect excluding possible BG.