# Axion Fragmentation

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N. Fonseca, E. Morgante, RS, G. Servant, 1911.08472, JHEP 04 (2020) 010
N. Fonseca, E. Morgante, RS, G. Servant, 1911.08473, JHEP 05 (2020) 080
E. Morgante, W. Ratzinger, RS, B.A. Stefanek, 2109.13823, JHEP 12 (2021) 037

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# Axion (-like) particle

## Axion field : $\phi$

- Shift symmetry (NG boson) + Chern-Simons coupling

$$\phi \;\to\; \phi + \delta\phi \qquad\qquad \frac{1}{f}\phi G_{\mu\nu}\widetilde{G^{\mu\nu}}$$

$$\Downarrow$$

- Shift symmetry breaking by strong dynamics

$$V(\phi) = \Lambda_b^4 \cos\frac{\phi}{f}$$

- Theoretical motivation, interesting phenomenology, …
  - Strong CP problem, QCD axion
  - Naturalness of electroweak scale, Relaxion
  - Axion monodromy
  - Axion inflation
  - …

# Axion (-like) particle & cosmology

Dynamics of axion field is interesting

- Axion dark mattter
- Relaxion : dynamical expanation of electroweak scale
- …

Solving EOM $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$ with some initial condition

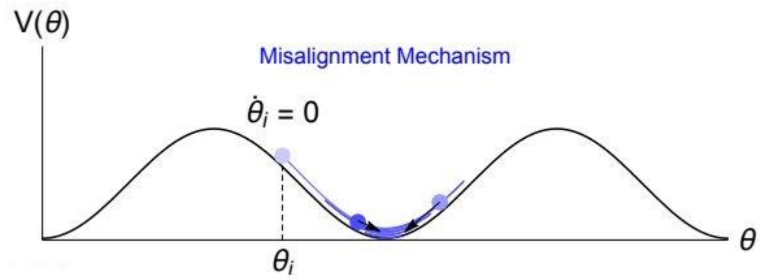
# ex1) Axion (-like) particle DM scenario

- **Misalignment mechanism**

[Preskill, Wise, Wilczek (1983)]
[Abbott, Sikivie (1983)]
[Dine, Fischler (1983)]

Initial condition $$\phi = \phi_0 \neq 0, \quad \dot{\phi} = 0$$

EOM $$\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda_b^4(T)}{f}\sin\frac{\phi}{f} = 0$$



[taken from Co, Hall, Harigaya (2019)]

The axion starts to oscillate when $3H(T) \sim m(T)$

$$\rho_{DM} \sim m_a \times \left(\frac{a(T_{osc})}{a_0}\right)^3 \times \frac{\Lambda_b(T_{osc})^4\theta_i^2}{m_a(T_{osc})}$$

w/ $m_a(T_{osc}) \sim 3H(T_{osc})$

mass | Dilution factor | Number density at $T = T_{osc}$

# ex1) Axion (-like) particle DM scenario

- Misalignment mechanism

[Preskill, Wise, Wilczek (1983)]
[Abbott, Sikivie (1983)]
[Dine, Fischler (1983)]

Initial condition $\phi = \phi_0 \neq 0$, $\dot{\phi} = 0$

What happens if $\dot{\phi} > \Lambda_b^2$?

EOM

The axion starts to o

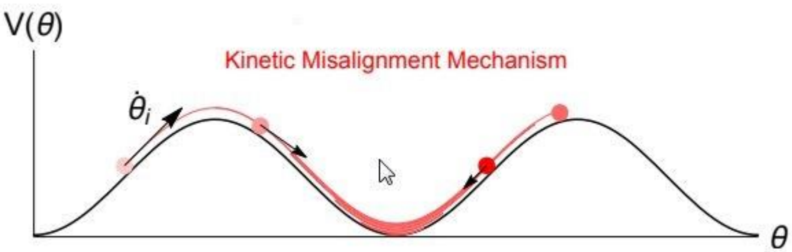
$m_a(T_{osc}) \sim 3H(T_{osc})$

# ex1) Axion (-like) particle DM scenario

- **Kinetic Misalignment mechanism** [Co, Hall, Harigaya (2019)] [Chang, Cui (2019)]

Initial condition $\dot{\phi} > \Lambda_b^2$

EOM $$\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda_b^4(T)}{f}\sin\frac{\phi}{f} = 0$$




[taken from Co, Hall, Harigaya (2019)]

The axion starts to oscillate when $\dot{\phi}^2(T) \sim \Lambda_b^4(T)$

$$\Rightarrow \quad \rho_{DM} \sim m_a \times \left(\frac{a(T_{osc})}{a_0}\right)^3 \times \frac{\Lambda_b(T_{osc})^4\theta_i^2}{m_a(T_{osc})} \qquad \text{w/} \quad \dot{\phi}^2(T_{osc}) \sim \Lambda_b^4(T_{osc})$$

mass — Dilution factor — Number density at $T = T_{osc}$

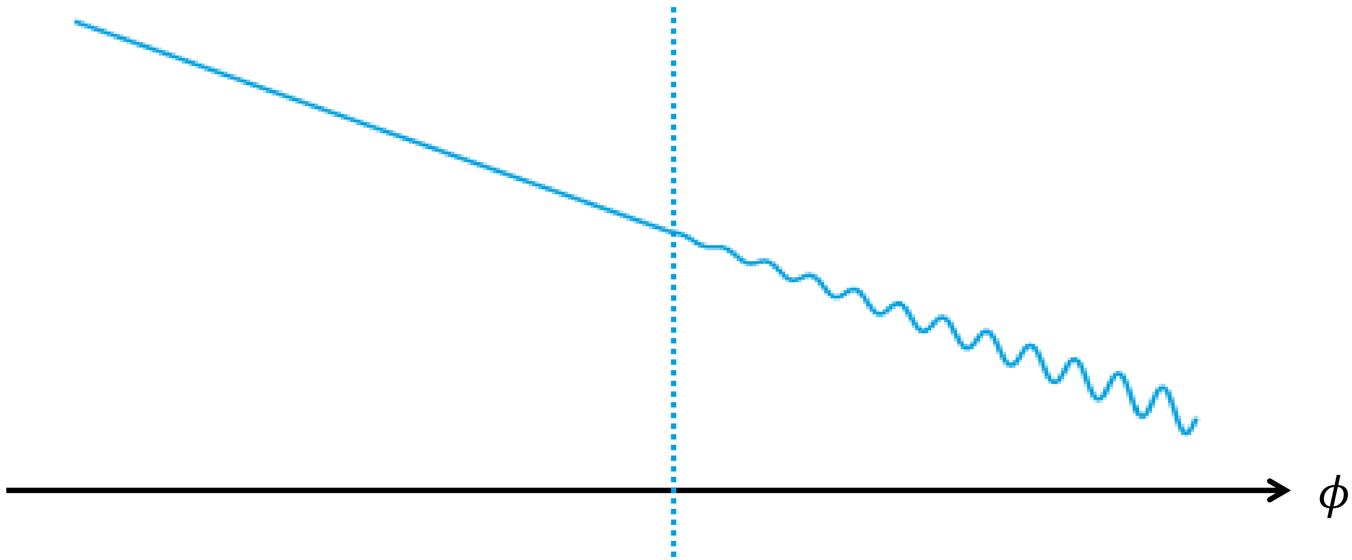
Delay of onset of oscillation → larger $\rho_{DM}$

# ex2) Relaxion scenario

[Graham, Kaplan, Rajendran (2015)]

$$V = -(\Lambda^2 - g'\Lambda\phi)H^2 + \lambda H^4 \quad + \quad g\Lambda^3\phi \quad + \quad \Lambda_b^4(H)\cos\frac{\phi}{f}$$

Higgs potential　　slope　　Wiggles




$$\Lambda^2 - g'\Lambda\phi < 0 \qquad \Lambda^2 - g'\Lambda\phi > 0$$

$$\Lambda_b(H) = 0 \qquad \Lambda_b(H) > 0$$

$$H = 0 \qquad |H| > 0$$

# Axion fluctuation?

## What people usually do

Solving EOM for spatially homogeneous field : $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

## However...

Even we start from (almost) homogeneous field configuration,
fluctuations can grow later.

# Velocity as U(1) charge

Velocity $\dot{\phi}$ is U(1) charge : $\rho_{\rm shift} = f \frac{\partial L}{\partial_0 \phi} = f \dot{\phi}$

$\phi \to \phi + f\,\delta$

Shift transf.

Explicit breaking of U(1) : $V(\phi) = \Lambda_b^4 \cos\frac{\phi}{f} + \cdots$

⇨ U(1) charge will be lose = energy dissipation

**Axion fragmentation** [Fonseca, Morgante, RS, Servant (2019)]

1. Introduction
2. **Axion Fragmentation**
3. Axion Fragmentation on the lattice

# Axion fragmentation

Let us investigate the simplest case.

- $H = 0$ (no cosmic expansion)
- $V(\phi) = \Lambda_b^4 \cos(\phi/f)$

We have only three parameters :

| | |
|---|---|
| $\dot{\phi}_0$ | : initial velocity |
| $f$ | : decay constant |
| $\Lambda_b^4$ | : height of barrier |

EOM of axion :

$$\frac{d^2\phi}{dt^2} - \nabla^2\phi - \frac{\Lambda_b^4}{f}\sin\frac{\phi}{f} = 0$$

# EOM of axion

We decompose $\phi(\vec{x},t) = \bar{\phi}(t) + \left[\int \frac{d^3k}{(2\pi)^3}\delta\phi_k(t)e^{ikx} + h.c.\right]$

$$\frac{d^2\phi}{dt^2} - \nabla^2\phi - \frac{\Lambda_b^4}{f}\sin\frac{\phi}{f} = \quad 0$$

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At the leading order of $\delta\phi_k$,

$$\frac{d^2\bar{\phi}}{dt^2} - \frac{\Lambda_b^4}{f}\sin\frac{\bar{\phi}}{f} = \underbrace{\frac{1}{2}\frac{\Lambda_b^4}{f^3}\sin\frac{\bar{\phi}}{f}\int\frac{d^3x}{V_{vol}}\langle\delta\phi(x)\rangle^2}_{\text{Back reaction}}$$

$$\frac{d^2\delta\phi}{dt^2} - \nabla^2\delta\phi - \frac{\Lambda_b^4}{f^2}\cos\frac{\bar{\phi}}{f}\delta\phi = 0$$

# EOM of axion

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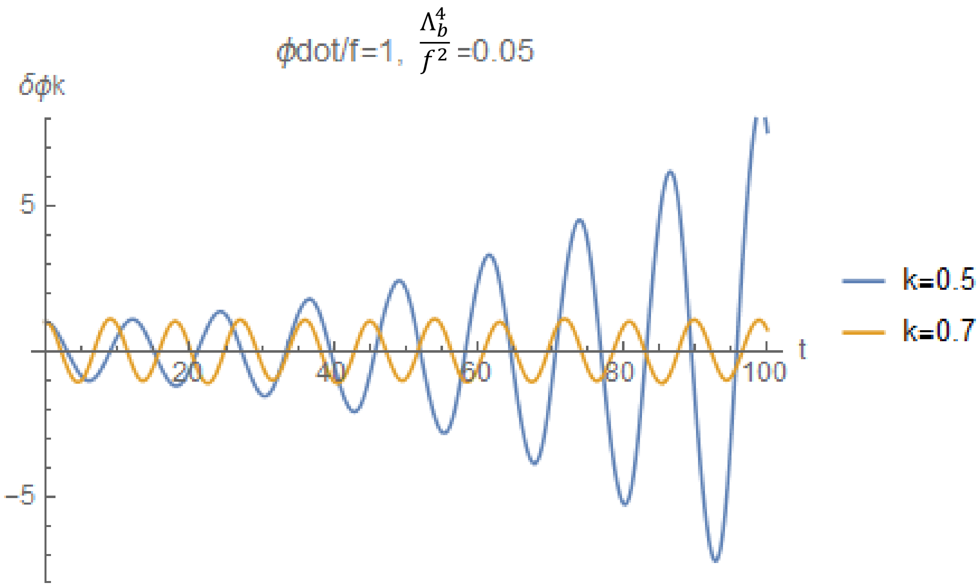
$$\frac{d^2\delta\phi_k}{dt^2} + \left(k^2 - \frac{\Lambda_b^4}{f^2}\cos\frac{\dot{\bar{\phi}}t}{f}\right)\delta\phi_k = 0$$

Mathieu equation

# EOM of axion



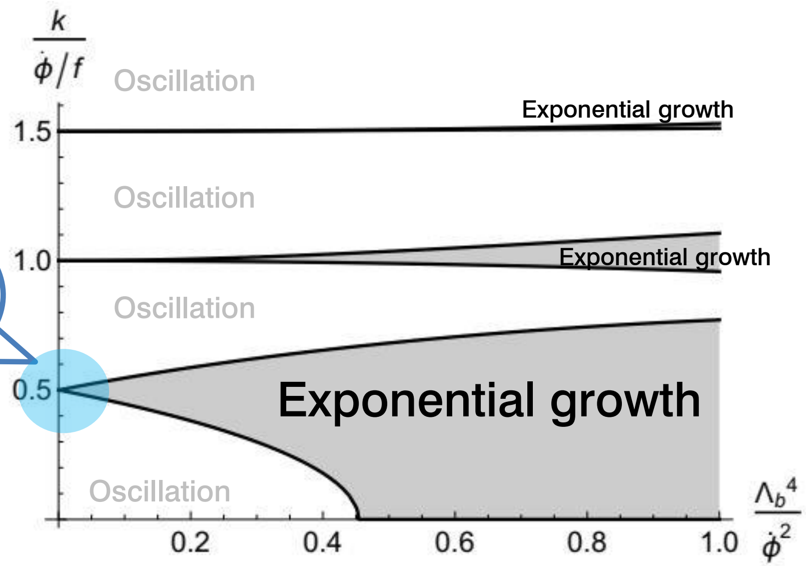

There exist resonant solutions for this.
It's like a swing!

$$\frac{d^2\delta\phi_k}{dt^2}+\left(k^2-\frac{\Lambda_b^4}{f^2}\cos\frac{\dot{\phi}t}{f}\right)\delta\phi_k \quad = \quad 0$$

Mathieu equation

# EOM of axion

$$\left|k - \frac{\dot{\phi}}{2f}\right| < \frac{\Lambda_b^4}{2f\dot{\phi}}$$




Large velocity ← → Small velocity

There exist resonant solutions for this.
It's like a swing!

$$\frac{d^2\delta\phi_k}{dt^2} + \left(k^2 - \frac{\Lambda_b^4}{f^2}\cos\frac{\bar{\dot{\phi}}t}{f}\right)\delta\phi_k = 0$$

Mathieu equation

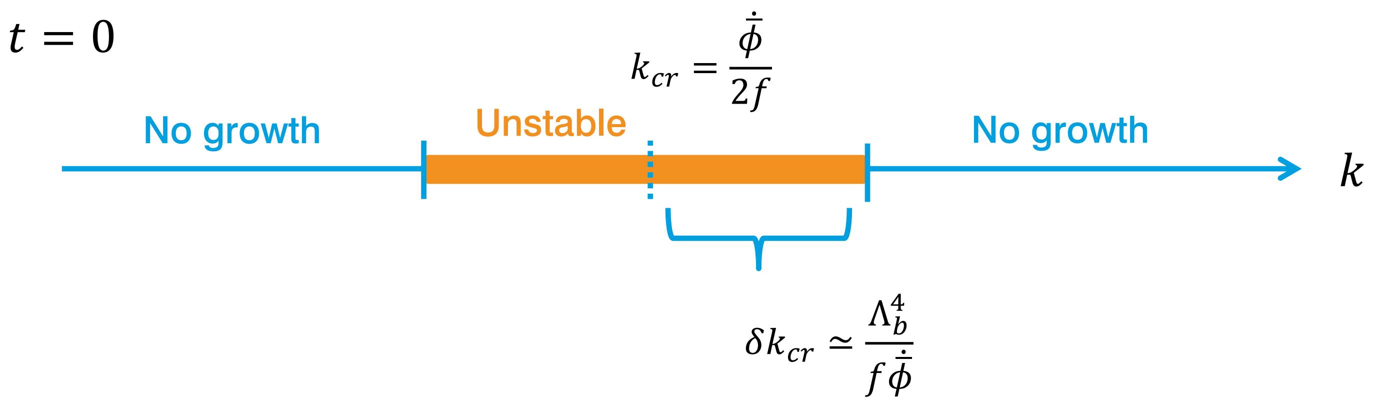
Growth of fluctuation

⇩

Back reaction to zeromode

# Naïve estimation on back reaction

As long as $\dot{\phi}$ is constant, $\delta\phi_k \sim \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$ for $\left|k - \frac{\dot{\phi}}{2f}\right| < \frac{\Lambda_b^4}{2f\dot{\phi}}$




By using dimensional analysis

$$\rho_{fluc}(t) \sim k_{cr}^3 \delta k_{cr} \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$$

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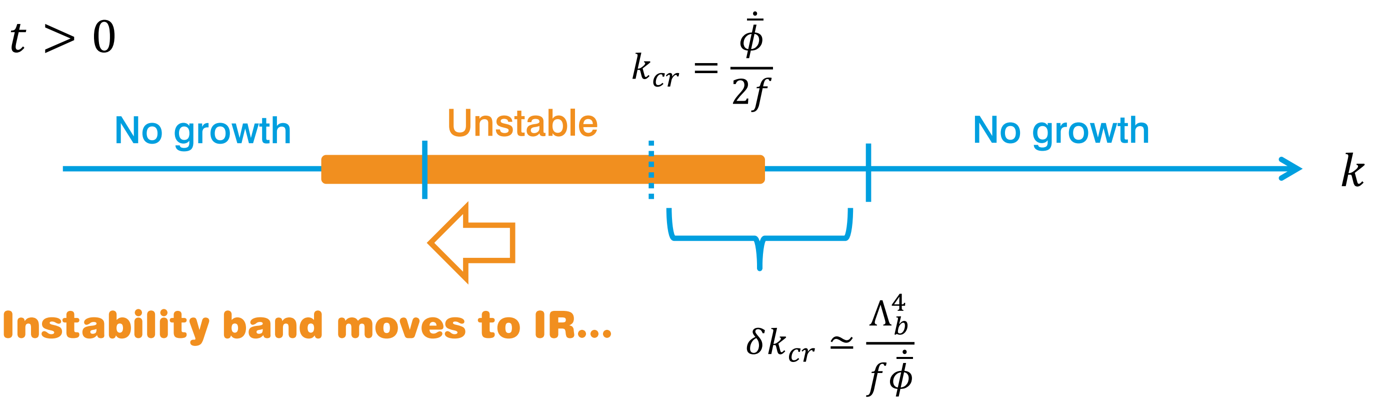


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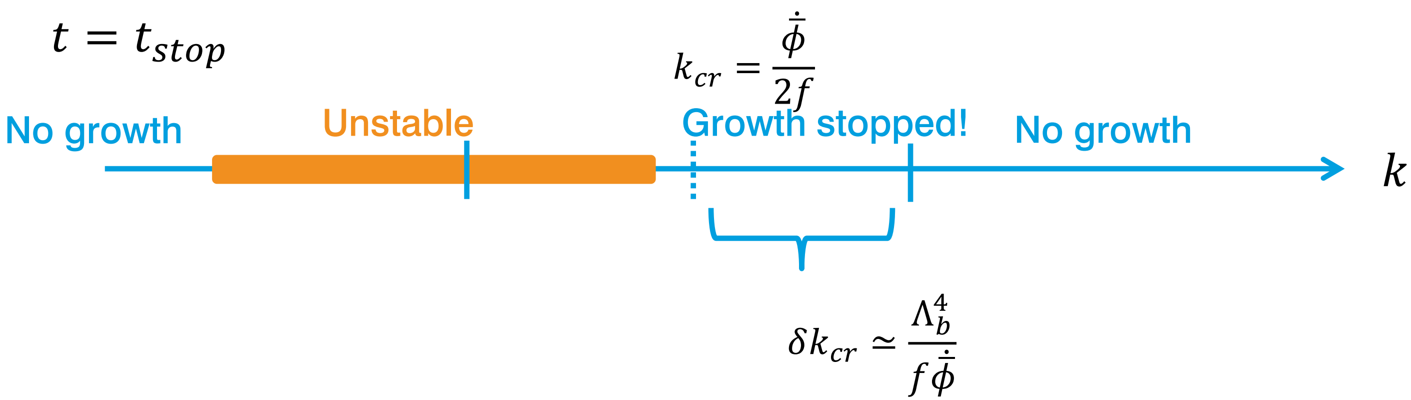


By using dimensional analysis

$$\rho_{fluc}(t) \sim k_{cr}^3 \delta k_{cr} \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$$

of mode with k=kcr

The growth stops when

$$\rho_{fluc}(t_{stop}) \sim \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\left(\dot{\phi} - 2f\delta k_{cr}\right)^2$$

# Naïve estimation on back reaction

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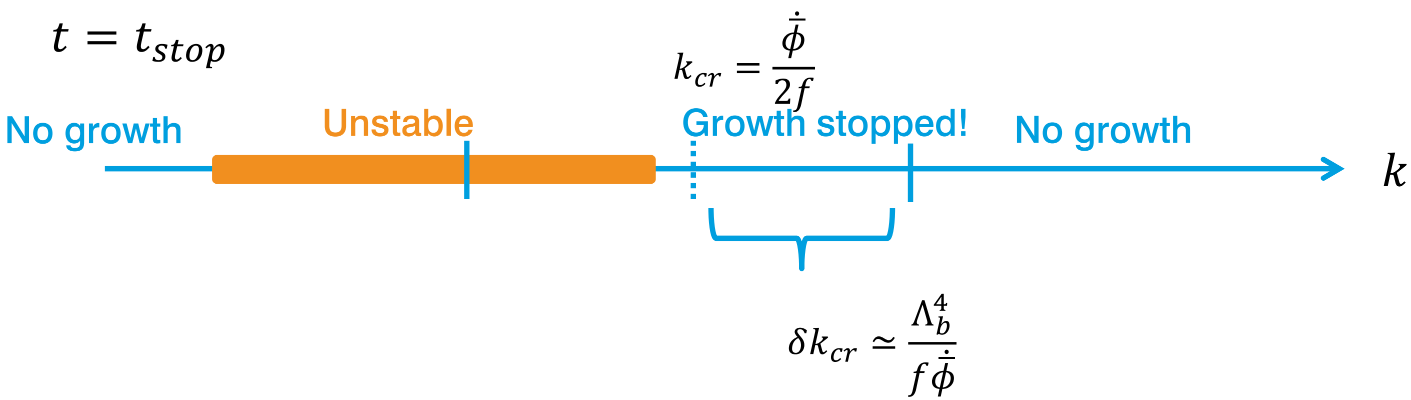


By using dimensional analysis

$$\rho_{fluc}(t) \sim k_{cr}^3 \delta k_{cr} \exp\left(\frac{\Lambda_b^4 t}{f\bar{\dot{\phi}}}\right)$$

of mode with k=kcr

The growth stops when

$$\rho_{fluc}(t_{stop}) \sim \bar{\dot{\phi}}^2 \times \frac{\delta k_{cr}}{k_{cr}}$$

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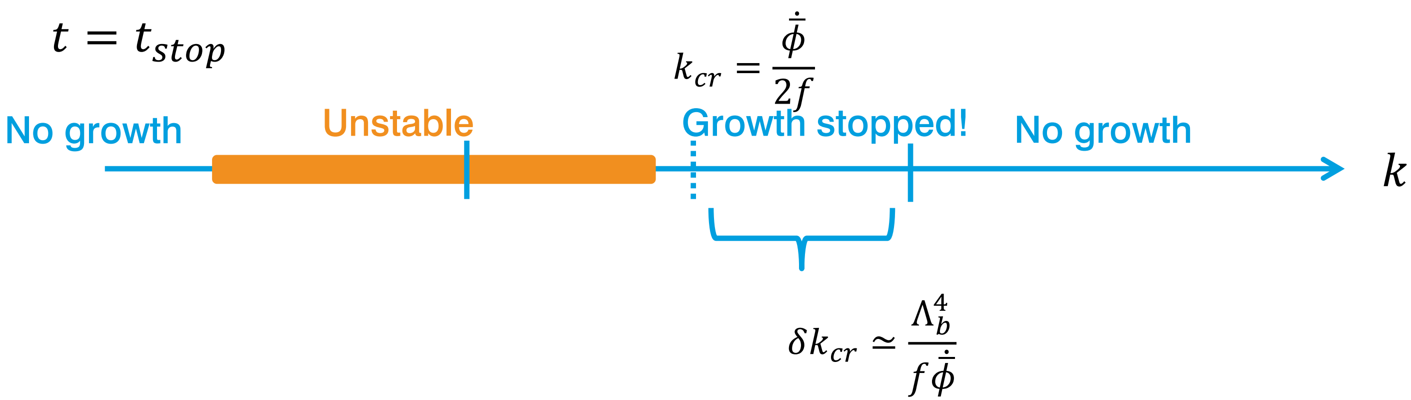


By using dimensional analysis

$$\rho_{fluc}(t) \sim k_{cr}^3 \delta k_{cr} \exp\left(\frac{\Lambda_b^4 t}{f\dot{\bar{\phi}}}\right)$$

The growth stops when (of mode with k=kcr)

$$\rho_{fluc}(t_{stop}) \sim \dot{\bar{\phi}}^2 \times \frac{\delta k_{cr}}{k_{cr}}$$

⇨ $$t_{stop} \sim \frac{f\dot{\bar{\phi}}}{\Lambda_b^4} \log \frac{f^4}{\dot{\bar{\phi}}^2}$$

# Naïve estimation on back reaction

$$\rho_{fluc}(t_{stop}) \sim \bar{\dot{\phi}}^2 \times \frac{\delta k_{cr}}{k_{cr}}, \qquad t_{stop} \sim \frac{f\bar{\dot{\phi}}}{\Lambda_b^4} \log \frac{f^4}{\bar{\dot{\phi}}^2}$$

$$\Rightarrow \quad \frac{d}{dt}\dot{\phi}^2 \sim -\frac{\rho_{fluc}(t_{stop})}{t_{stop}}$$

# Naïve estimation on back reaction

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$$\Rightarrow \quad \frac{d}{dt}\dot{\phi}^2 \sim -\frac{\Lambda_b^8}{f\dot{\phi}} \left(\log \frac{f^4}{\dot{\phi}^2}\right)^{-1}$$

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$$\Rightarrow \quad \frac{d}{dt}\dot{\phi} \sim -\frac{\Lambda_b^8}{f\dot{\phi}^2}\left(\log\frac{f^4}{\dot{\phi}^2}\right)^{-1}$$

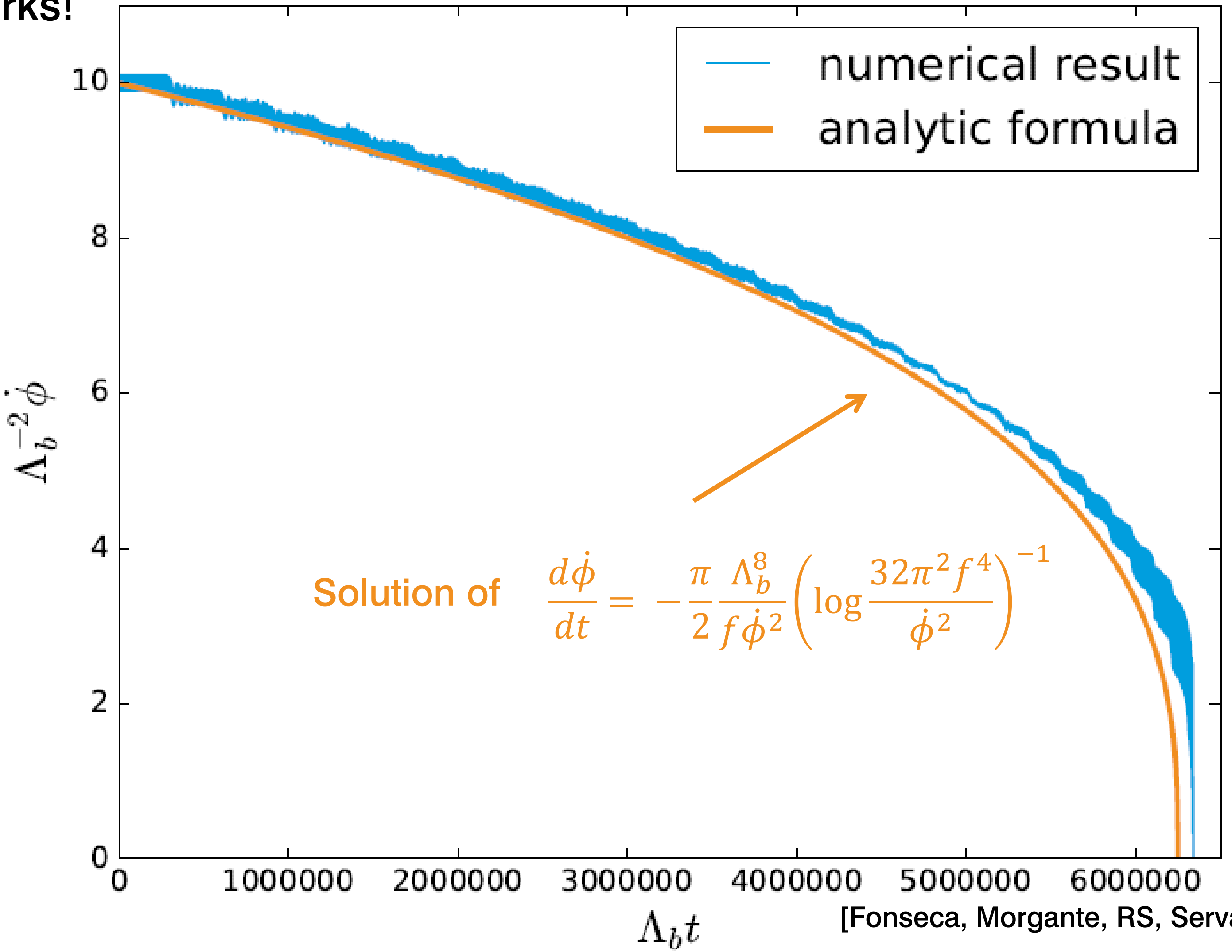
c.f.) WKB approx. with $\dot{\phi} \gg \Lambda_b^2$ gives

$$\frac{d\dot{\phi}}{dt} = -\frac{\pi}{2}\frac{\Lambda_b^8}{f\dot{\phi}^2}\left(\log\frac{32\pi^2 f^4}{\dot{\phi}^2}\right)^{-1}$$

(see 1911.08472 for details)

# Numerical Example

It works!




$$\frac{f}{\Lambda_b} = 1000$$

[Fonseca, Morgante, RS, Servant (2019)]

# Field excursion

- Field excursion $\Delta\phi_{frag}$
- time scale of relaxation $\Delta t_{frag}$

are given as a function of $\Lambda_b, f,\ \dot{\phi}_0$

$$\frac{d\dot{\phi}}{dt} \sim -\frac{\Lambda_b^8}{f\dot{\phi}^2}\left(\log\frac{f^4}{\dot{\phi}^2}\right)^{-1}$$

⇨

$$\Delta t_{frag} \sim f\frac{\dot{\phi}_0^3}{\Lambda_b^8}\log\frac{f^4}{\dot{\phi}_0^2}$$

$$\Delta\phi_{frag} \sim \dot{\phi}_0\Delta t_{frag} \sim f\frac{\dot{\phi}_0^4}{\Lambda_b^8}\log\frac{f^4}{\dot{\phi}_0^2}$$

Number of wiggles : $n \sim \frac{\Delta\phi_{frag}}{f} \sim \frac{\dot{\phi}_0^4}{\Lambda_b^8}\log\frac{f^4}{\dot{\phi}_0^2}$

# Non-zero slope & Hubble expansion

What happens for non-zero $\mu^3$ & non-zero $H$?

- Fragmentation $\ddot{\phi}_{frag} = -\frac{\pi \Lambda_b^8}{2f\dot{\phi}^2}\left(\log \frac{32\pi^2 f^4}{\dot{\phi}^2}\right)^{-1}$
- Acceleration by slope $\mu^3$
- Hubble expansion $3H\dot{\phi}$

Fragmentation works if

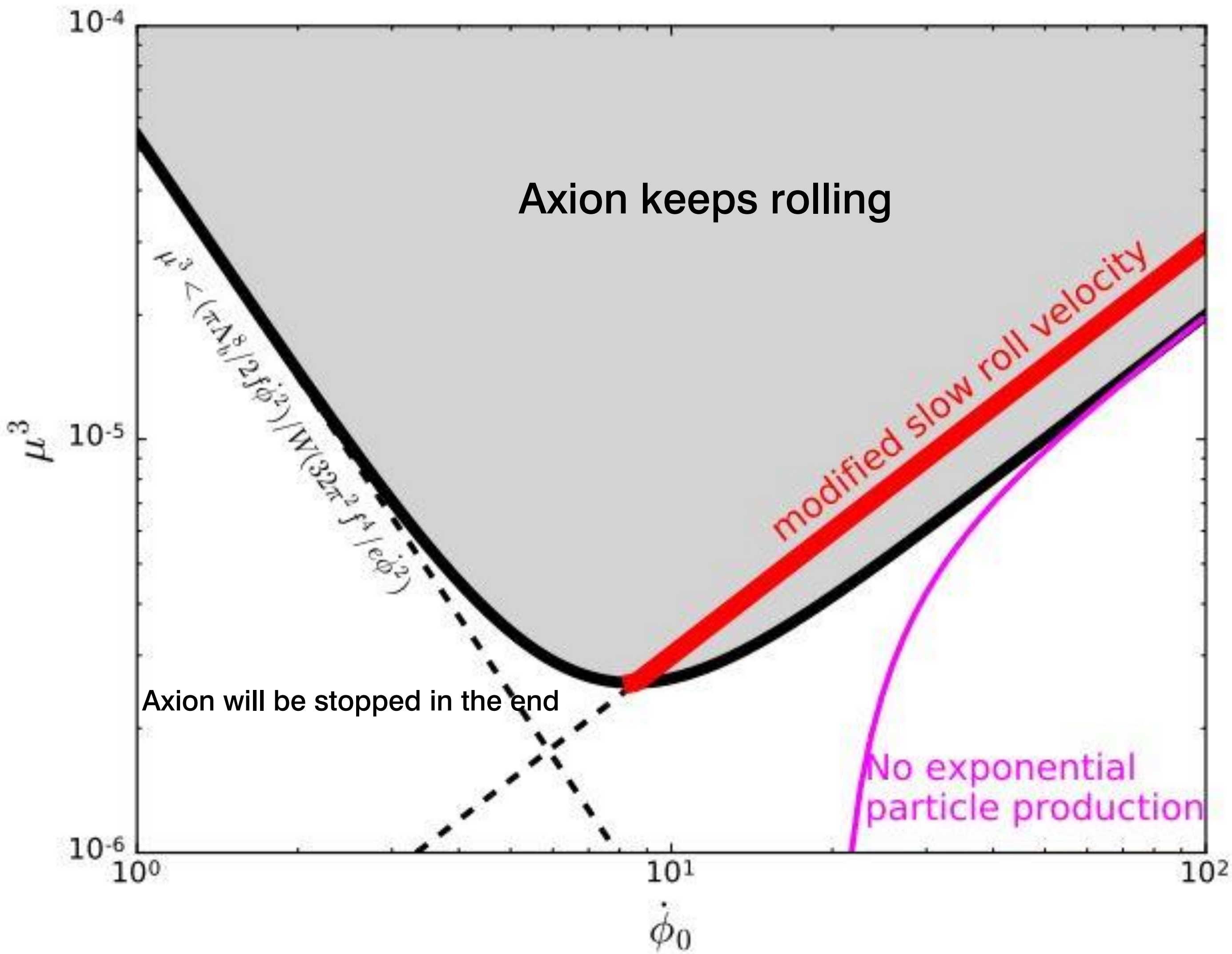
- During inflation ($3H\dot{\phi} \sim \mu^3$)

  $3H\dot{\phi} < \sim |\ddot{\phi}_{frag}|$ If not, axion keeps rolling with slow-roll velocity

- Not during inflation ($3H\dot{\phi} \ll \mu^3$)

  $\mu^3 < \sim |\ddot{\phi}_{frag}|$ If not, axion is just accelerated by slope

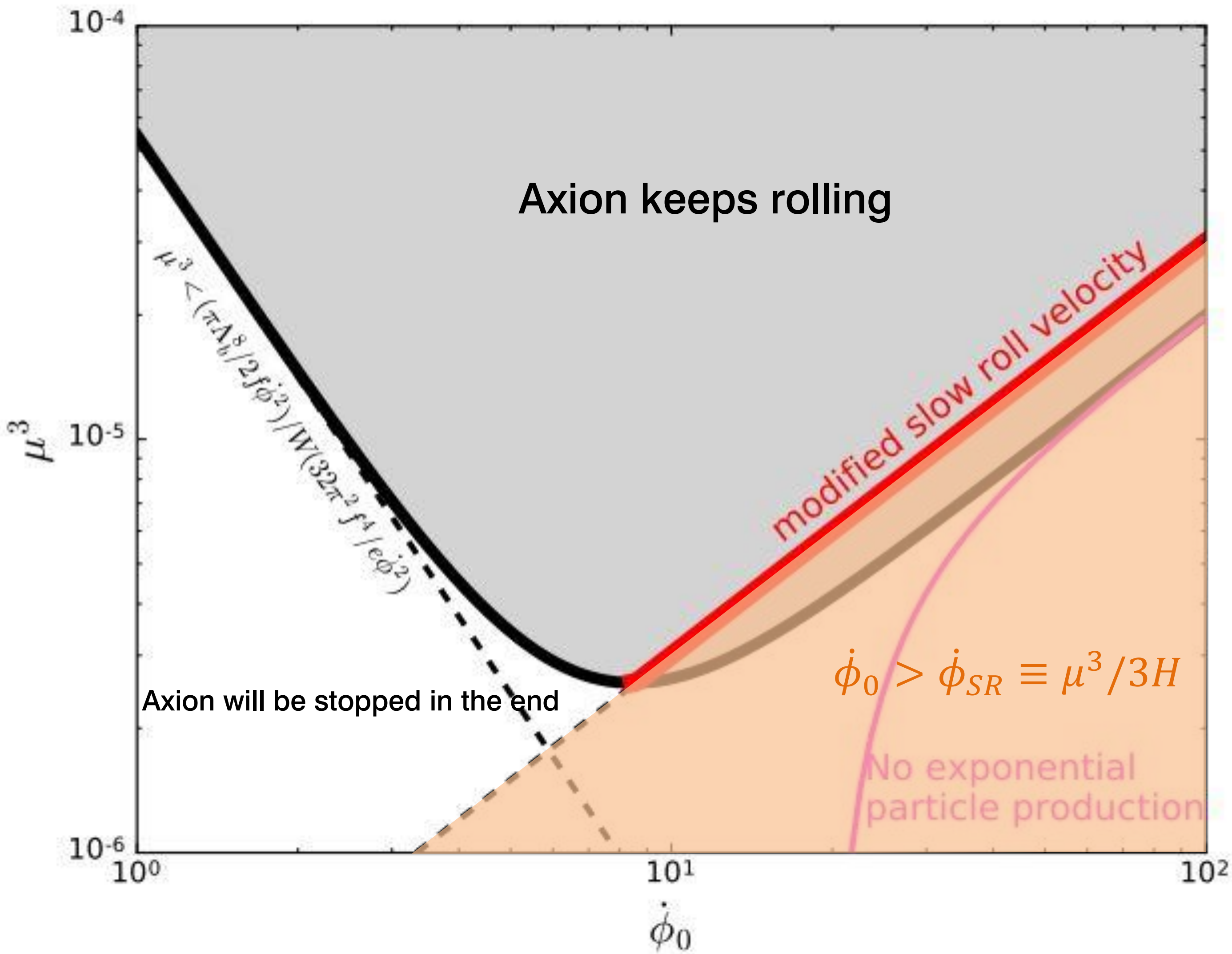
# Fragmentation




$f = 10^3\Lambda_b$,
$H = 10^{-7}\Lambda_b$

[Fonseca, Morgante, RS, Servant (2019)]

# Fragmentation

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# Non-linear analysis

## In perturbative analysis OK?

Inittial kinetic energy : $\dot{\phi}_0^2/2$

Typical wavenumber : $\dot{\phi}_0/f$

Energy conservation : $(\delta\phi)^2 \times \left(\dot{\phi}_0/f\right)^2 \sim \dot{\phi}_0{}^2$

⇨ Typical field variation : $\delta\phi \sim f$

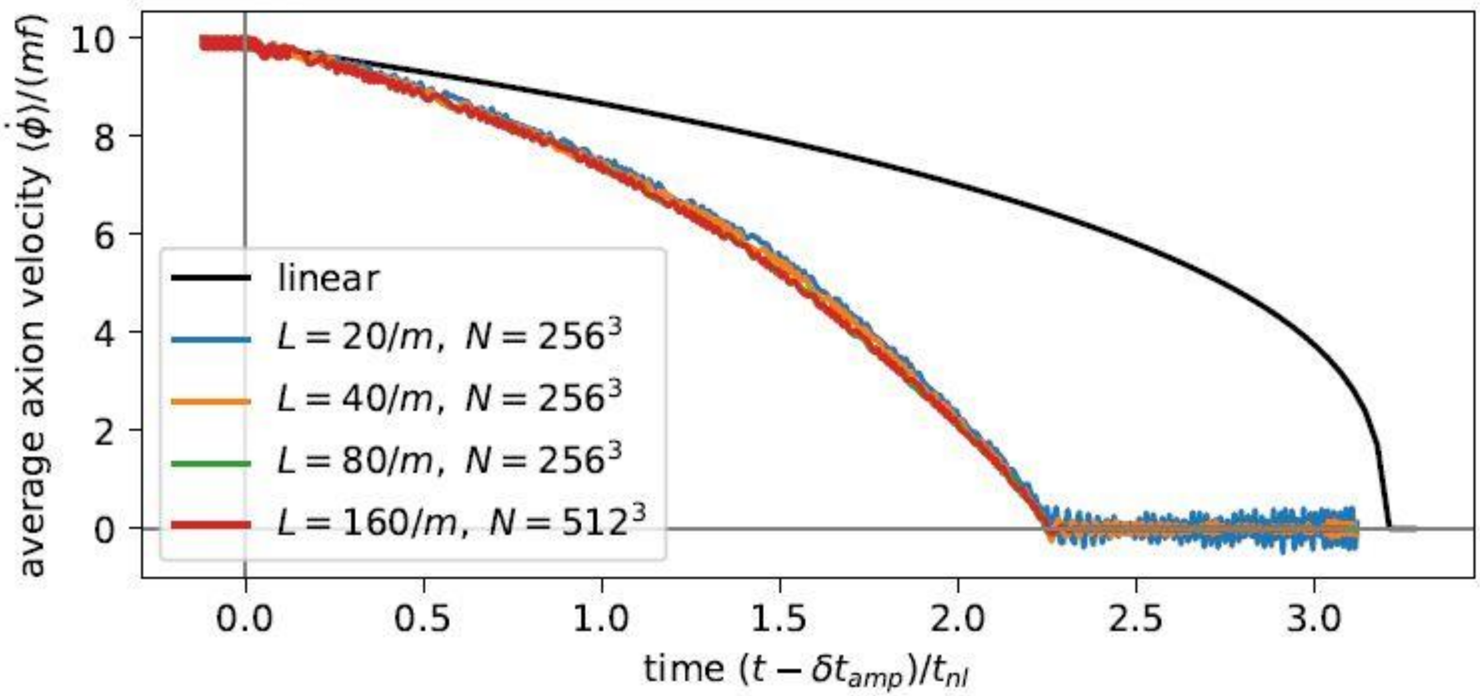
## Classical lattice simulation

$$\ddot{\phi} = \nabla^2\phi + \frac{\Lambda_b^4}{f}\sin\frac{\phi}{f}$$

⇨

$$\begin{aligned}\frac{d^2\phi_{i,j,k}}{dt^2} =& \frac{1}{a^2}\left(\phi_{i+1,j,k} - 2\phi_{i,j,k} + \phi_{i-1,j,k}\right) \\ &+ \frac{1}{a^2}\left(\phi_{i,j+1,k} - 2\phi_{i,j,k} + \phi_{i,j-1,k}\right) \\ &+ \frac{1}{a^2}\left(\phi_{i,j,k+1} - 2\phi_{i,j,k} + \phi_{i,j,k-1}\right) \\ &+ \frac{\Lambda_b^4}{f}\sin\frac{\phi_{i,j,k}}{f}.\end{aligned}$$

# Velocity of zeromode




$\left(t_{nl}=\frac{f\dot{\phi}_0^3}{\Lambda_b^8}\right)$

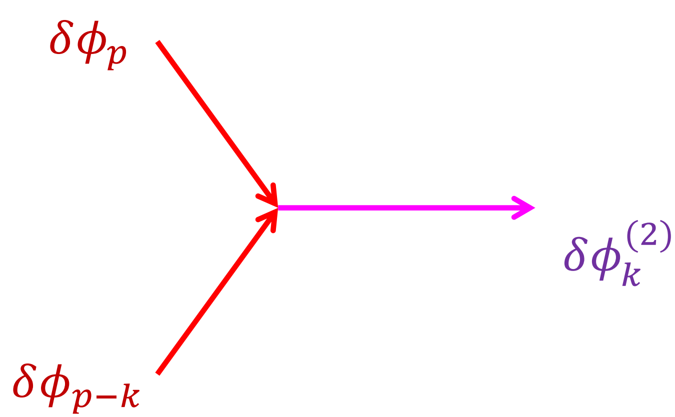
- Confirmed energy dissipation in non-linear calculation.
- Dissipation effect is stronger than linear analysis.

# 2 to 1 process

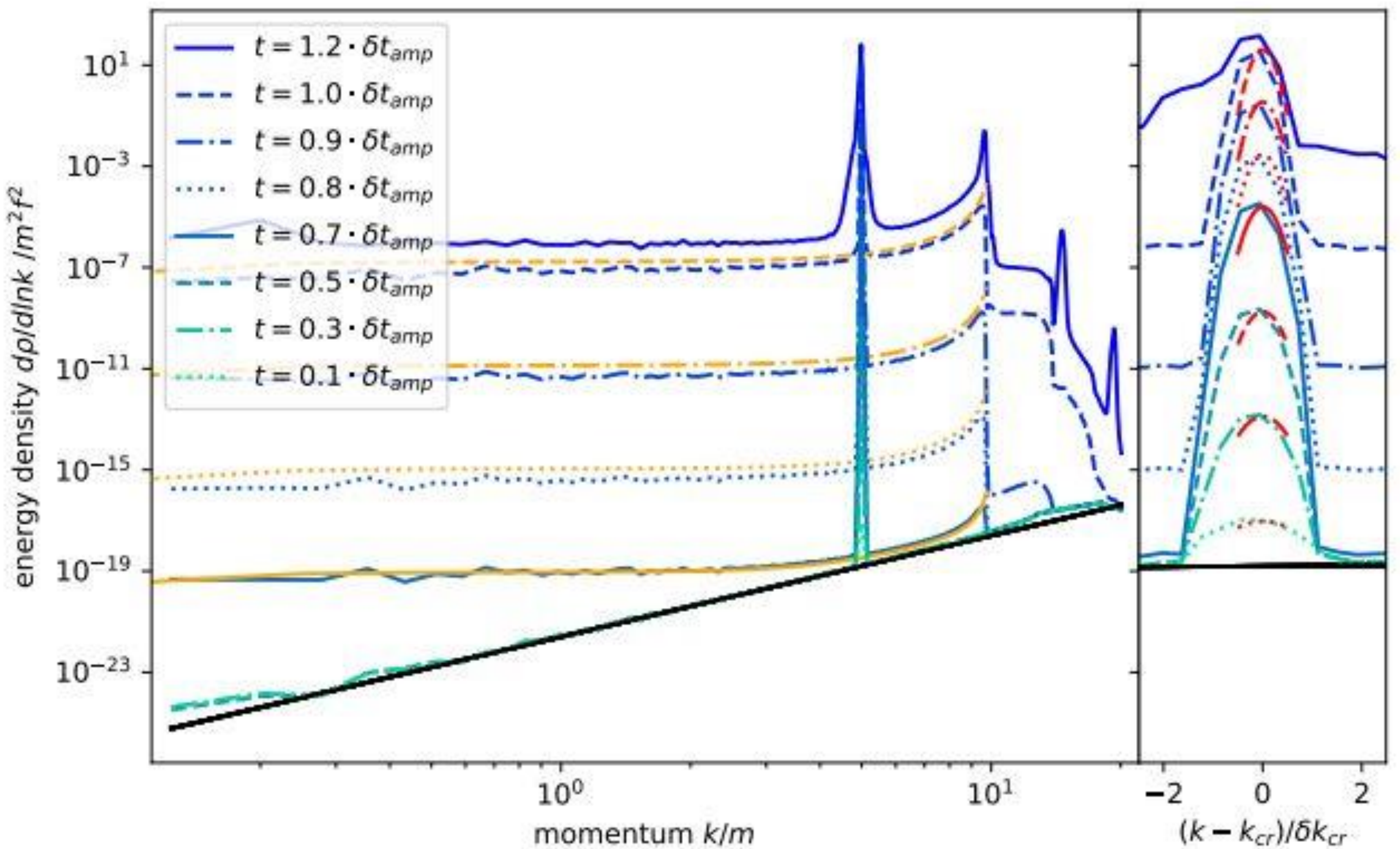
$$\phi(x,t) = \phi(t) + \delta\phi(x,t) + \delta\phi^{(2)}(x,t) + \ \ldots$$

$$\ddot{\phi} - \nabla^2\phi = V'(\phi) \quad \Rightarrow \quad \delta\ddot{\phi}^{(2)} + (k^2 + V'')\delta\phi^{(2)} = -\frac{1}{2}V'''\int d^3p\ \delta\phi_p\delta\phi_{k-p}$$

- $\delta\phi_p$ with $|p| = \dot{\phi}/2f$ is amplified by resonance
- $\delta\phi$ becomes source term for $\delta\phi^{(2)}$



# Growth of spectrum (early stage)




$$\delta t_{amp} \equiv \frac{f\dot{\phi}}{\Lambda_b^4}\log\frac{16f^4}{\dot{\phi}^2}$$

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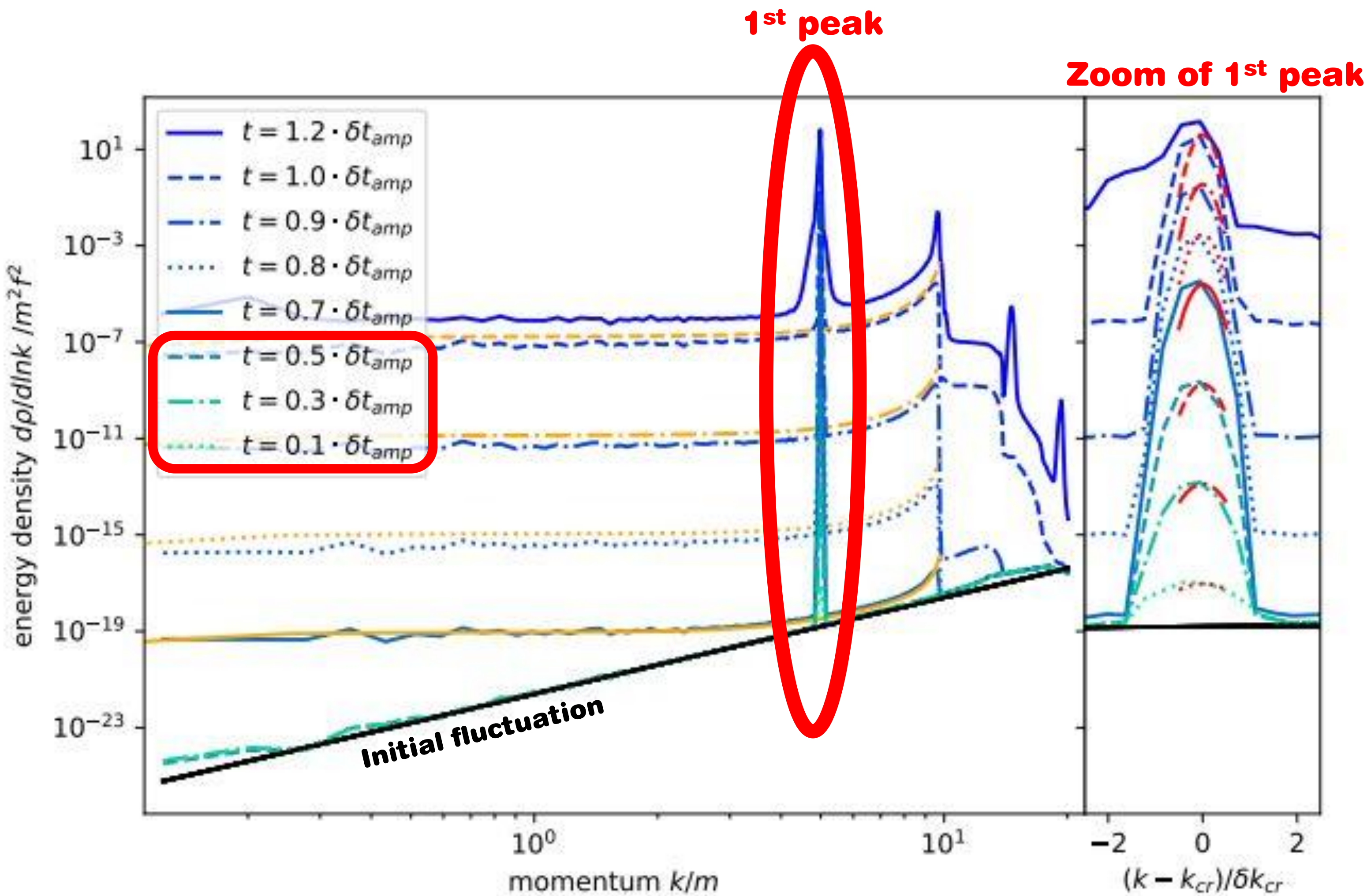


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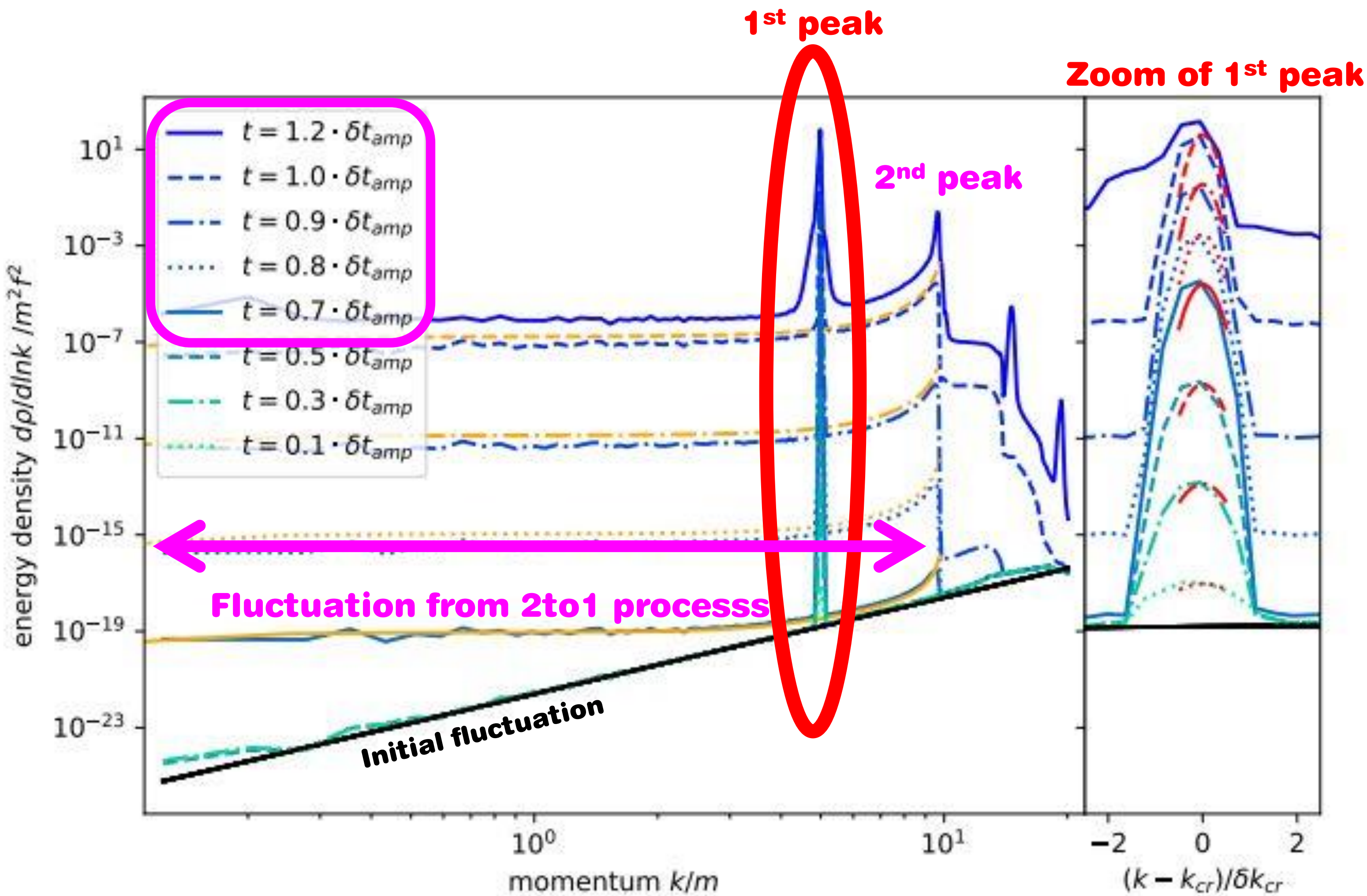


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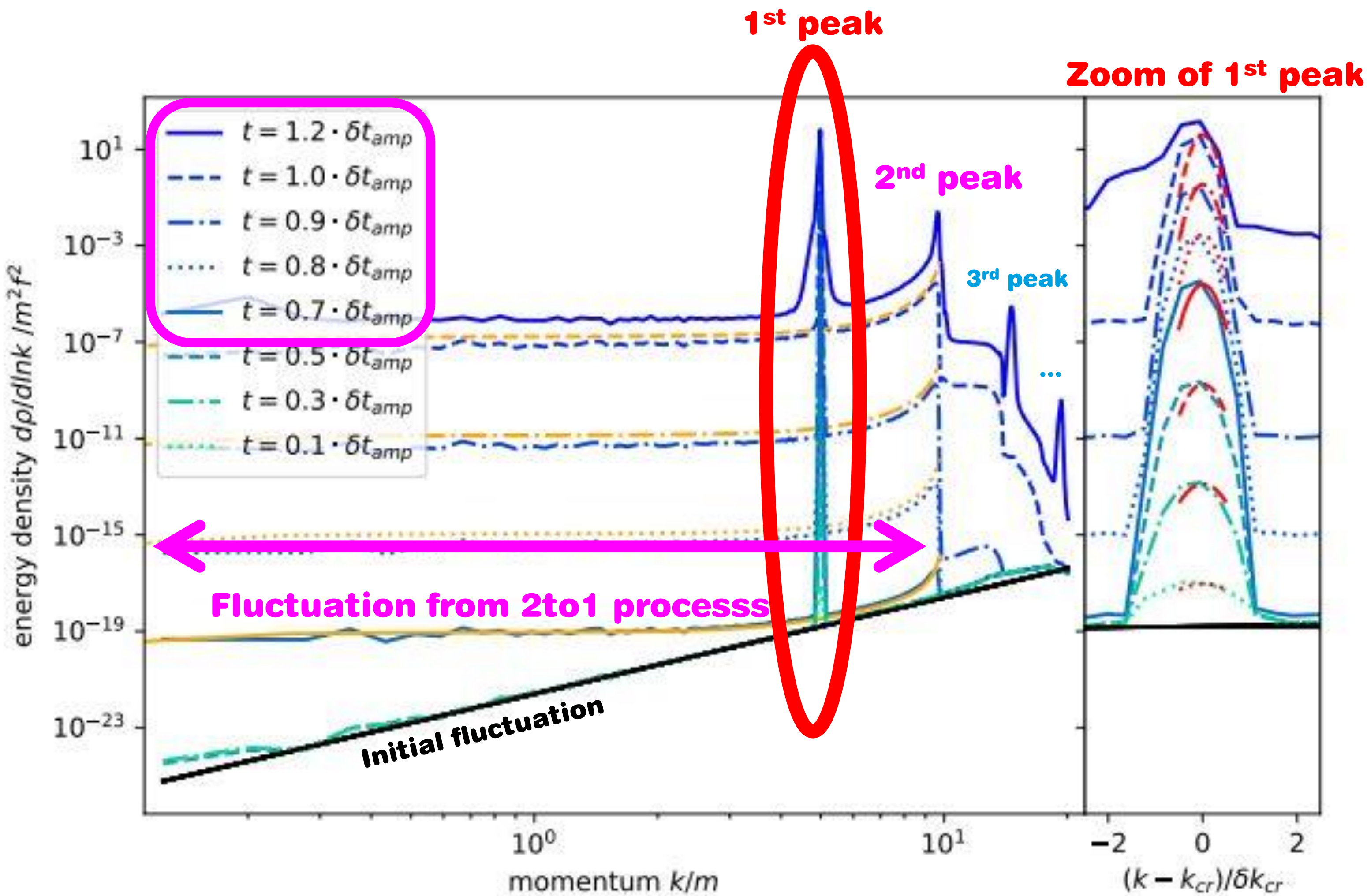
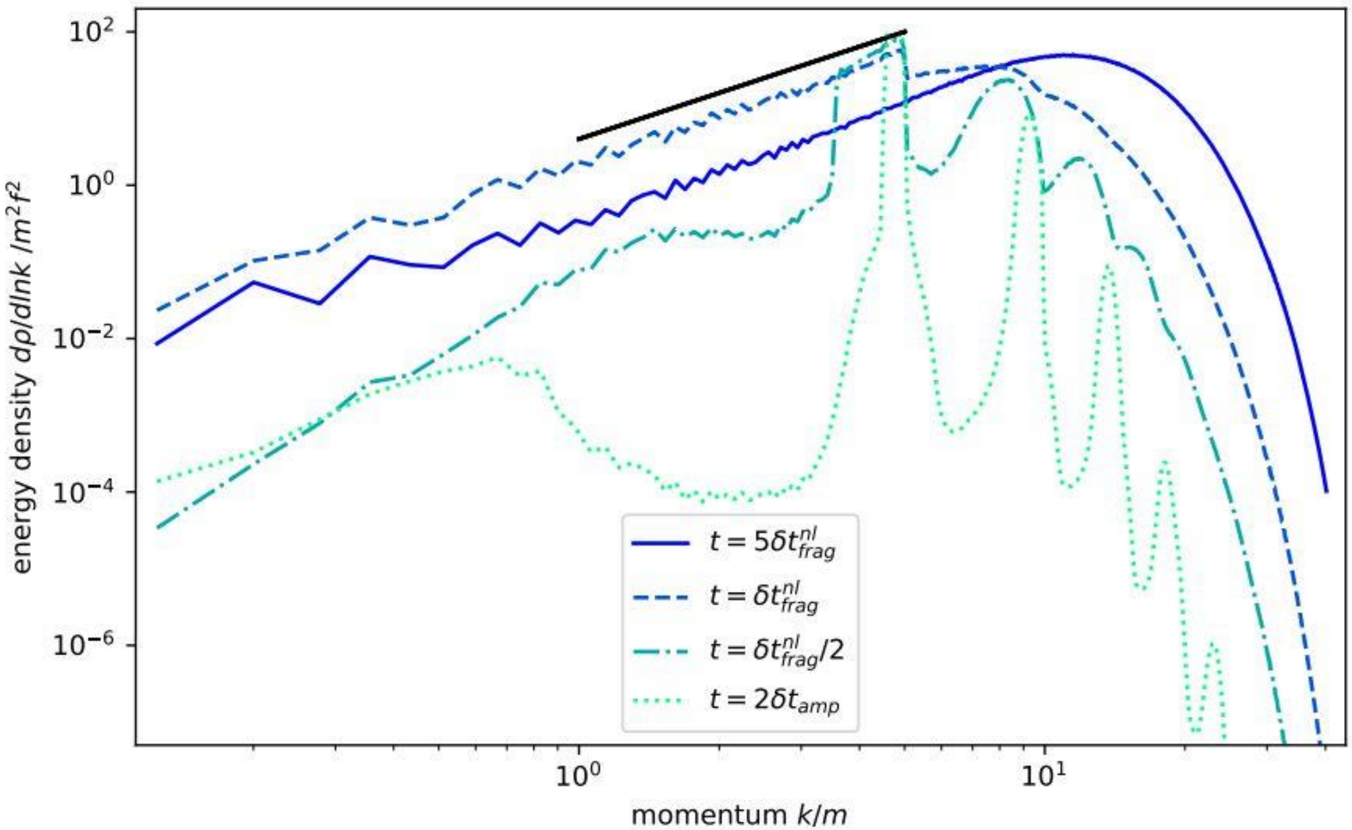



$$\delta t_{amp} \equiv \frac{f\dot{\phi}}{\Lambda_b^4} \log \frac{16 f^4}{\dot{\phi}^2}$$

# Growth of spectrum (late stage)




$$\delta t_{amp} \equiv \frac{f\dot{\phi}}{\Lambda_b^4}\log\frac{16f^4}{\dot{\phi}^2}$$

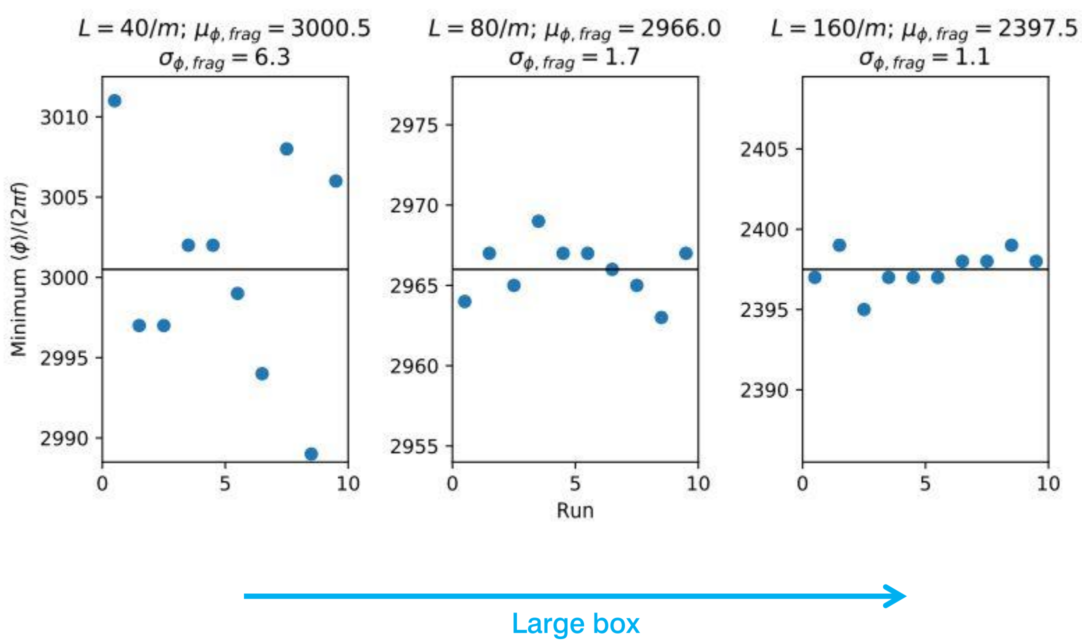
- We can see peak-like structure in the early stage
- The spectrum becomes broad
- Cascading towards UV (early stage of thermalization)

# Domain wall?

Field variance after fragmentation is not so small : $\delta\phi \sim f$

Multiple run with finite size box

- $\delta\phi$ in multiple run = $\delta\phi$ of causally disconnected area
- Extrapolation to $V^{1/3} \approx \delta t_{\rm frag}$



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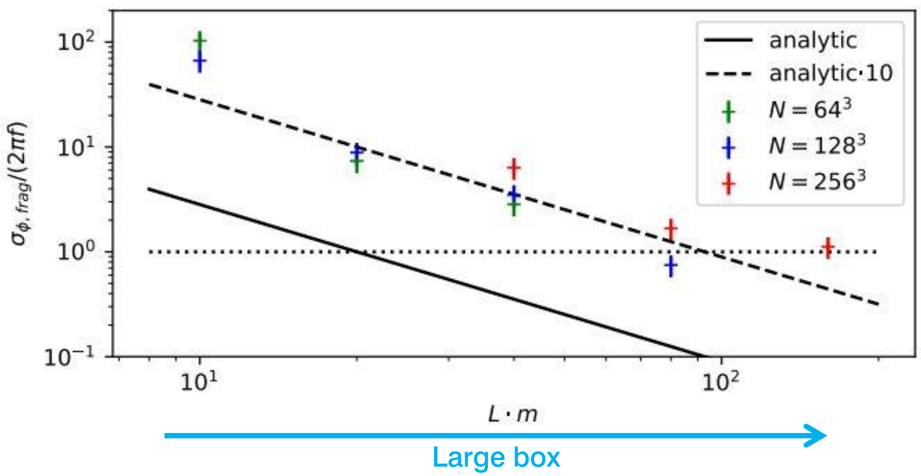



Empirical formula of variance:

$$\frac{\delta\phi}{2\pi f} \sim O(10) \times V^{-1/2} \times \left(\frac{f\dot{\phi}_0}{\Lambda_b^2}\right)^{3/2}$$

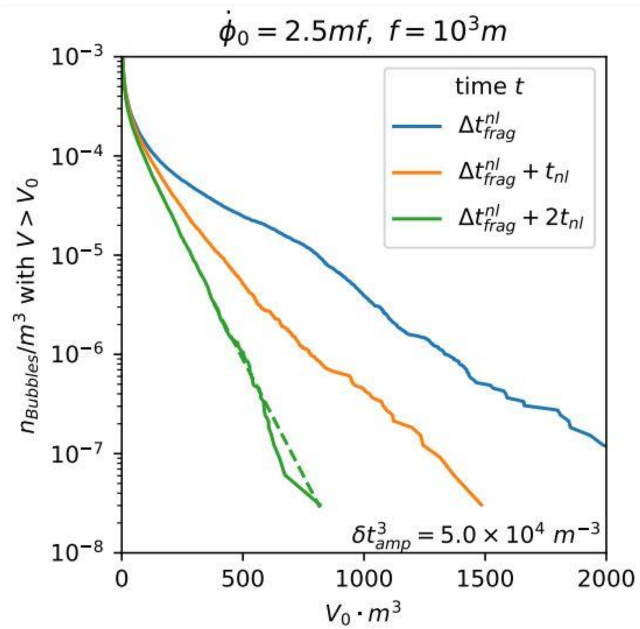
Naïve extrapolation to $V^{1/3} \sim t_{amp}$ :

$$\frac{\sigma}{2\pi f} \sim O(10) \times \left(\log\frac{8\pi f^2}{\dot{\phi}_0}\right)^{-\frac{3}{2}} \sim 0.01 - 0.1$$

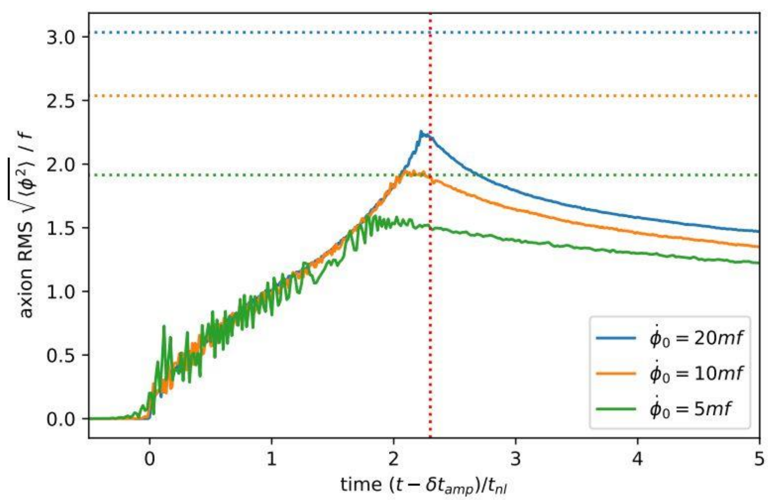
Domain wall formation probability is $\sim e^{-100} - e^{-10}$

# Energy cascade into UV

Number counting of "bubble"




Time evolution of variance $\langle \delta\phi^2 \rangle$




- Fluctuation with long wave-length is exponentially suppressed.
- The size of variance decreases in time.

# Summary

- Large axion velocity → growth of fluctuation
- Zeromode kinetic energy dissipates into fluctuations
- Domain wall formation is unlikely

- Generic phenomena w/ periodic potential and large velocity.
- Applications

  - Relaxion
  - Axion dark matter scenario
  - …

# Backup

# References

**Green, Kofman, Starobinsky, hep-ph/9808477**

Parametric resonance from large amplitude

**Flauger, McAllister, Pajer, Westphal, Xu, 0907.2916**

Cosine + linear term, monodromy infl.

**Jaeckel, Mehta, Witkowski, 1605.01367**

Cosine + quadratic term, linear

**Berges, Chatrchyan, Jaeckel, 1903.03116**

Cosine + quadratic term, non-perturbative

**Arvanitaki, Dimopoulos, Galanis, Lehner, Thompson, Van Tilburg, 1909.11665**

Parametric resonance from large amplitude

# Details on back reaction (1).

$$\phi(x,t) = \bar{\phi}(t) + \delta\phi(x,t) \qquad \delta\phi(x,t) = \int \frac{d^3k}{(2\pi)^3} a_k u_k(t) e^{ikx} + h.c.$$

Creation annihilation op : $\left[a_k, a^*_{k'}\right] = (2\pi)^3 \delta^{(3)}(k-k')$

Boundary condition for Wave function : $t \to -\infty \qquad u_k \to \frac{e^{-ikt}}{\sqrt{2k}}$

Bunch-Davies vacuum: $a|0\rangle = 0$

$$\Rightarrow \int \frac{dx^3}{V_{vol}} \langle \delta\phi(x)^2 \rangle = \int \frac{d^3k}{(2\pi)^3} |u_k|^2$$

# Details on back reaction (2).

Asymptotic behavior of wave function $u_k$ :

$$t \to -\infty \qquad u_k \to \frac{e^{-ikt}}{\sqrt{2k}}$$

$$t \to +\infty \qquad u_k \to \frac{1}{\sqrt{2k}} 2 \exp\left(-\frac{\pi}{4}\frac{\Lambda_b^8}{\dot{\phi}^2\ddot{\phi}f^4}\right) \times \cos kt$$

1) $d\rho/dt$ from $u_k$

$$\frac{d\rho}{dt} = -\left(\frac{d}{dt}\frac{\dot{\phi}}{2f}\right) \times \frac{4\pi k^2}{(2\pi)^3}\left(\frac{1}{2}|\dot{u_k}|^2 + \frac{1}{2}k^2|u_k|^2\right)$$

$$= -\frac{\dot{\phi}^3\ddot{\phi}}{32\pi^2 f^4}\exp\left(-\frac{\pi}{2}\frac{\Lambda_b^8}{\dot{\phi}^2\ddot{\phi}f^4}\right)$$

2) $d\rho/dt$ from definition

$$\frac{d\rho}{dt} = \dot{\phi}\ddot{\phi}$$

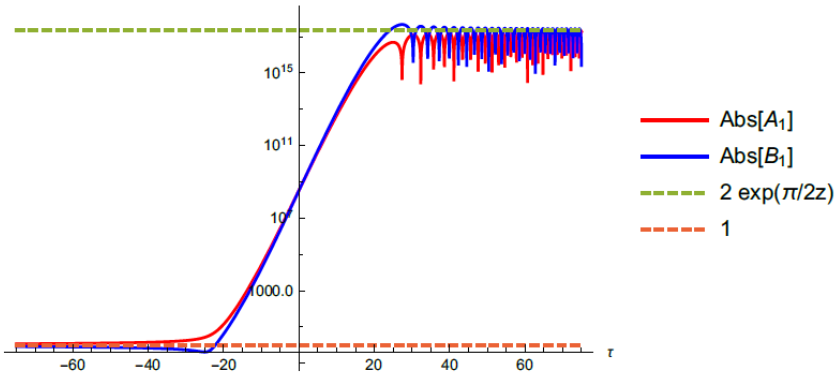
Consistency between 1) and 2) gives $\ddot{\phi} = -\frac{\pi\Lambda_b^8}{2\dot{\phi}^2 f}\left(\log\frac{32\pi^2 f^4}{\dot{\phi}^2}\right)^{-1}$

# "Modified" Mathieu equation.

$$\frac{d^2u_k}{dt^2} + \left(k^2 + m^2 \cos\frac{\bar{\phi}}{f}\right) u_k = 0 \qquad \textbf{with} \quad \dot{\bar{\phi}} = \dot{\bar{\phi}}_0 + \ddot{\bar{\phi}} t$$

Boundary condition at $t \to -\infty$ : $\quad u_k \to \frac{e^{-ikt}}{\sqrt{2k}}$

Behavior at $\dot{\bar{\phi}}/f \simeq 2k$ : $\quad u_k \simeq \frac{1}{\sqrt{2k}}\left(A(t)\cos\frac{\bar{\phi}}{2f} + B(t)\sin\frac{\bar{\phi}}{2f}\right)$




$$(1 + z\tau)A + \frac{dB}{d\tau} = 0 \qquad \tau = \frac{fm^2 t}{2\dot{\bar{\phi}}}$$

$$(1 - z\tau)B + \frac{dA}{d\tau} = 0 \qquad z = -\frac{2\dot{\bar{\phi}}_0 \ddot{\bar{\phi}}}{f^3 m^4}$$

Asymptotic behavior at $t \to +\infty$ : $\quad u_k \to \frac{1}{\sqrt{2k}} 2\exp\left(-\frac{\pi}{4}\frac{\Lambda_b^8}{\dot{\phi}_0^2 \ddot{\bar{\phi}} f^4}\right) \times \cos kt$

# d ^ 2 phi / dt ^ 2.

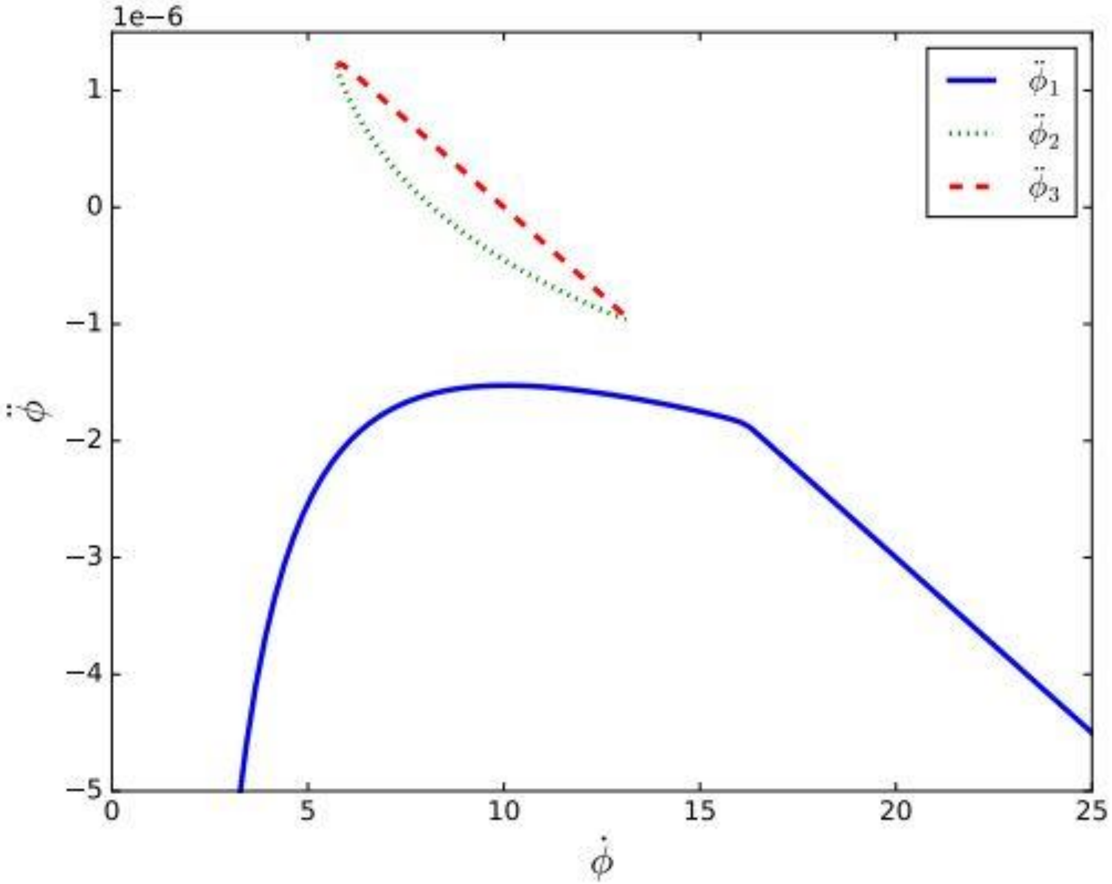



**Figure 23.** $\ddot{\phi}$ as a function of $\dot{\phi}$. Here we take $f = 10^3$, $H = 10^{-7}$, and $\mu^3 = 3 \times 10^{-6}$ in the unit of $\Lambda_b = 1$.

# Setup.

## > The original GKR (non-QCD) model [Graham, Kaplan, Rajendran (2015)]

$$V = -(\Lambda^2 - g'\Lambda\phi)H^2 + \lambda H^4 \quad + \quad g\Lambda^3\phi \quad + \quad \Lambda_b^4(H)\cos\frac{\phi}{f}$$

Higgs potential  slope  Wiggles

New strong dynamics gives wiggle

$$L_{eff} = m_N\bar{N}N + m_L\bar{L}L + yH\bar{N}L + \tilde{y}H^*\bar{L}N + \frac{\phi}{f}G'\widetilde{G'}$$

$$\Rightarrow \qquad V \simeq \frac{y\tilde{y}\Lambda_s^3}{m_L}|H|^2\cos\frac{\phi}{f}$$

$$(m_L > \Lambda_s > m_N)$$

# Parameter space.

$f$ is set to give correct EW scale : $$f \simeq \frac{2\pi\lambda\Lambda_b^8 v_{ew}}{g'\Lambda\dot{\phi}_0^4 \times O(10)}$$

We also impose the following consistency conditions:

- Wiggle makes local minima $$g\Lambda^3 < \sim \frac{\Lambda_b^4}{f}$$
- Potential stability against rad. corr. $$\Lambda_b < \sim v_{ew}$$
- Consistency of EFT $$f > \sim \Lambda$$
- Initial kinetic energy is large enough $$\frac{\dot{\phi}_0}{2} > \sim \Lambda_b^2$$
- Particle production is fast enough $$\Delta t_{pp} < \sim H^{-1}$$

# Relic abundace of relaxion.

- Relaxion particle production

Energy density of relaxion $\dot{\phi}_0{}^2$

Typical energy of particle $\dot{\phi}_0/f$

⇨

$$n_\phi \sim \dot{\phi}_0{}^2 \times \left(\frac{\dot{\phi}_0}{f}\right)^{-1} = f\dot{\phi}_0,$$

$$m_\phi n_\phi \sim \Lambda_b^2 \dot{\phi}_0,$$

- Coherent oscillation (after particle production)

$$\rho \sim \frac{\Lambda_b^4}{f^2} \times f^2 = \Lambda_b^4,$$

If there is no inflation during/after relaxation,

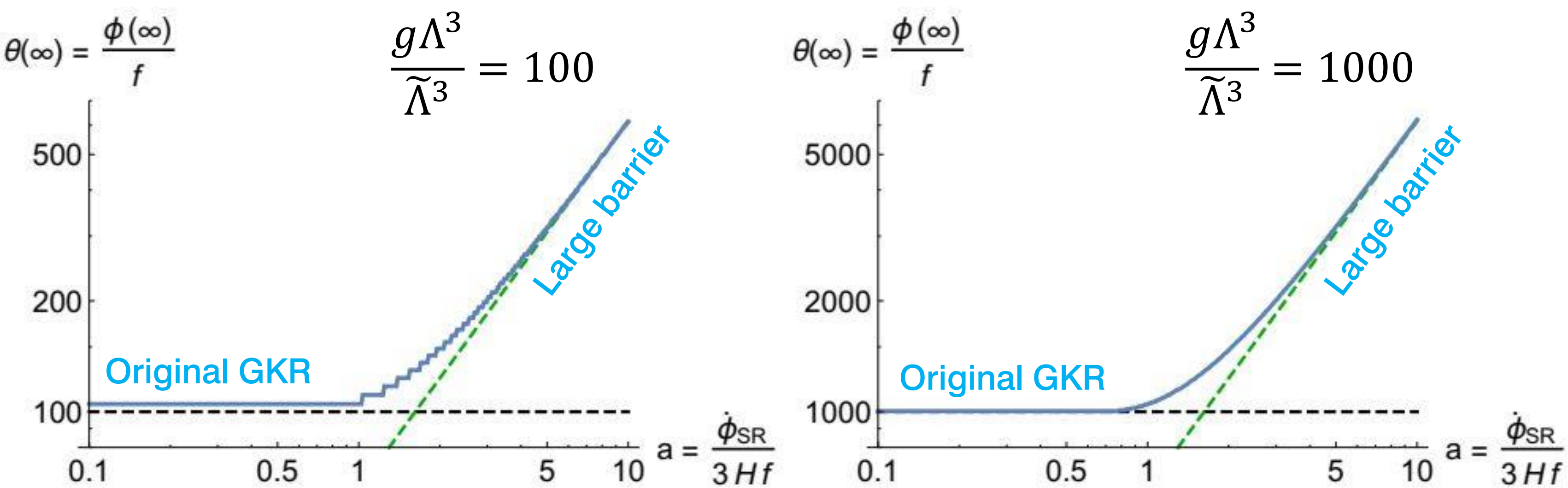
$$\Omega_\phi \sim \frac{g_0 T_0^3}{g_* T_*^3} \Lambda_b^2 \dot{\phi} \times \frac{1}{\rho_{cr}} \sim 10^5 \times \left(\frac{\Lambda}{10\ \mathrm{TeV}}\right)^2 \left(\frac{\Lambda_b}{10\ \mathrm{GeV}}\right)^2 \left(\frac{T_*}{10\ \mathrm{TeV}}\right)^{-3}$$

($T_{RH}$ is reheating temperature of SM plasma)

BBN constraint is very severe.

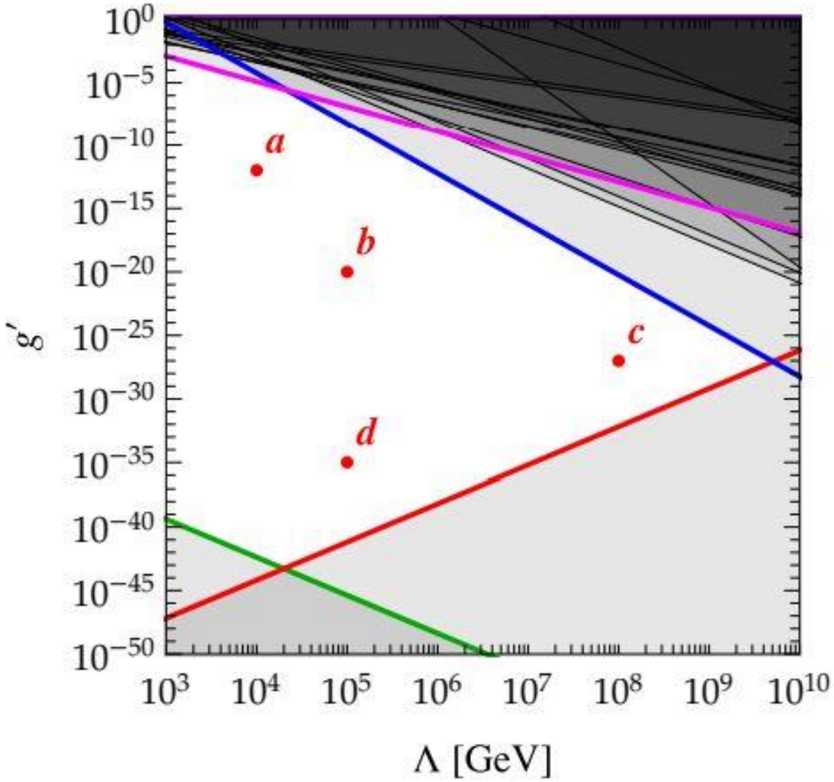
We need dilution by inflation!

# Stopping condition during inflation.



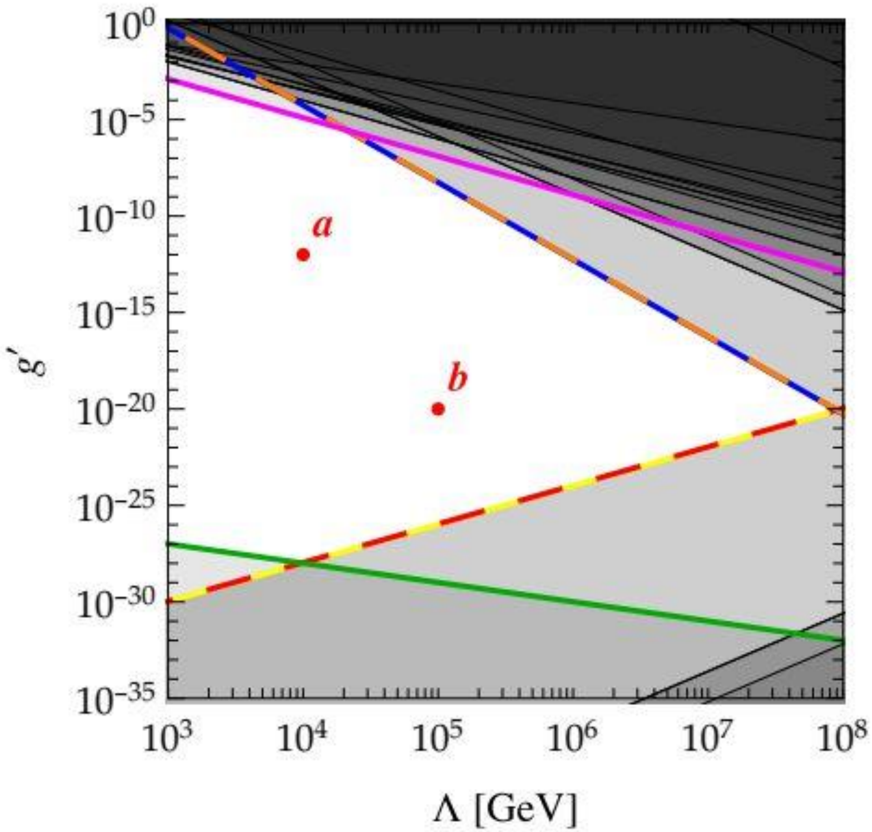

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{f}\widetilde{\Lambda}^3 \phi \sin\frac{\phi}{f} - g\Lambda^3 = 0$$

# GKR (Hubble friction).



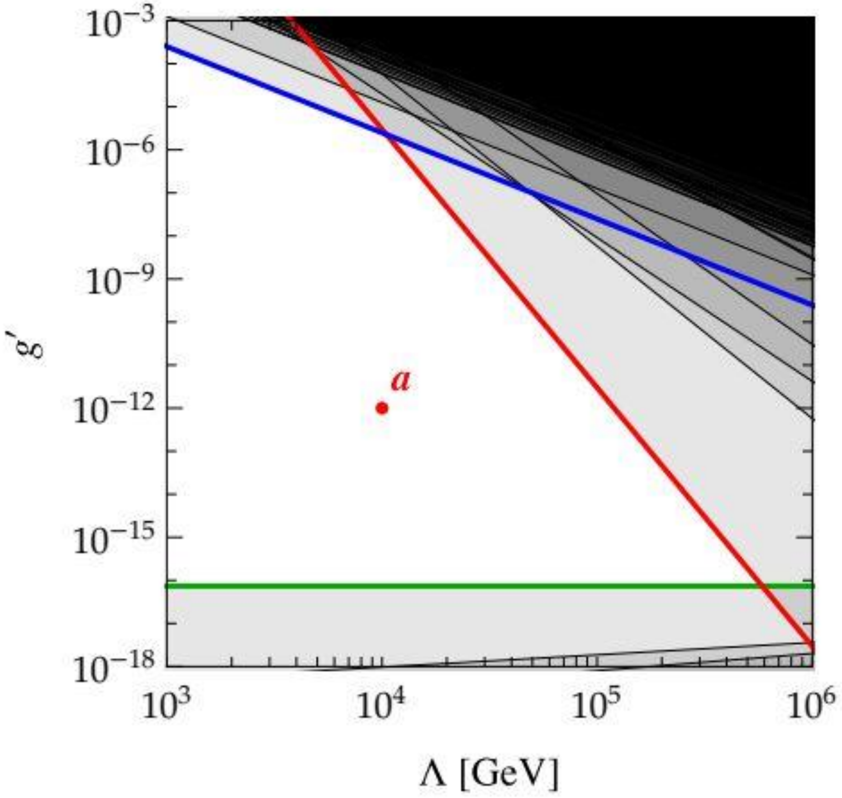

- Symmetry breaking pattern Eq. (3.5) and microscopic origin of the barriers Eq. (3.6)
- Classical rolling Eq. (3.16) and relaxion subdominant with respect to the inflaton Eq. (3.13)
- Reheating Eq. (3.9) and sub-Planckian decay constant Eq. (3.8)
- Eq. (3.5) and precision of the Higgs mass scanning Eq. (3.3)

# GKR (large barrier).



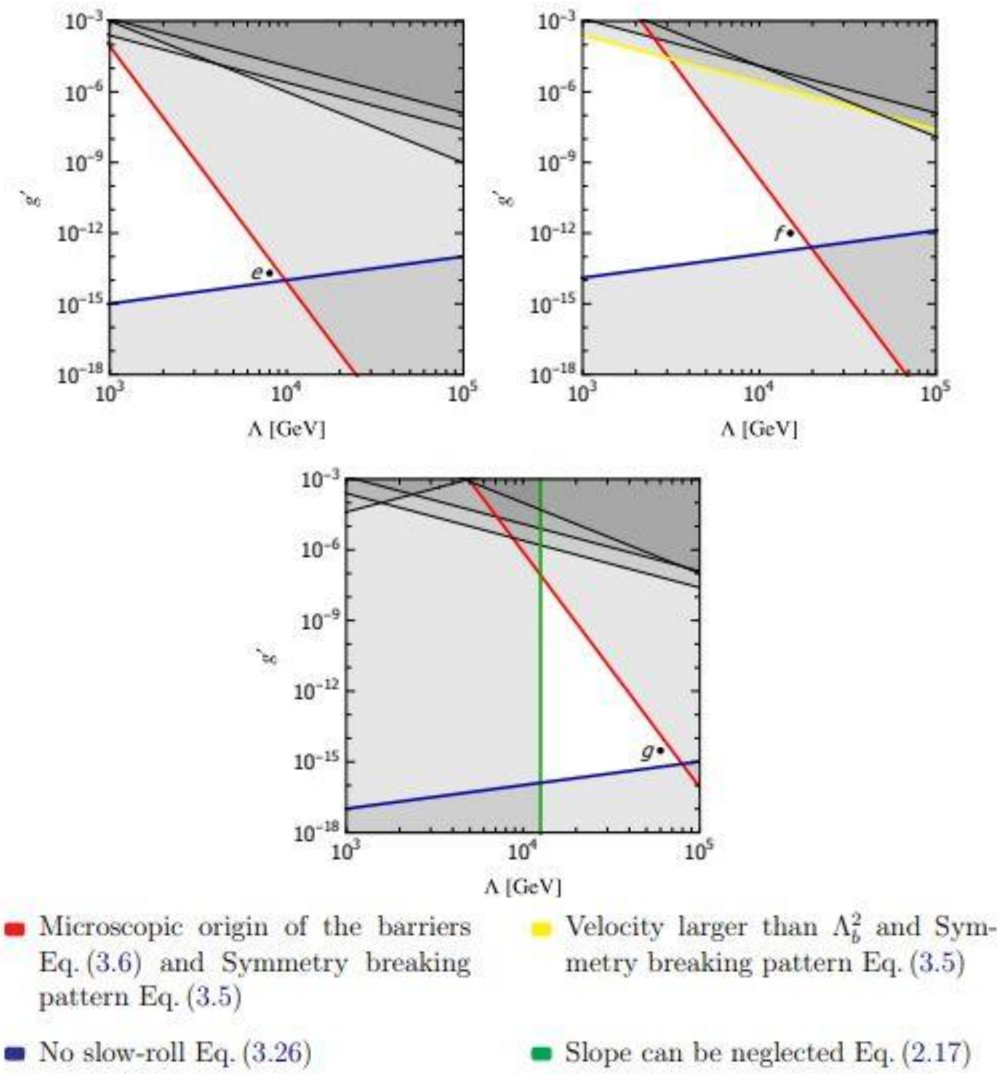

- Symmetry breaking pattern Eq. (3.5), going over 1 wiggle in less than 1 Hubble time Eq. (3.19) and microscopic origin of the barriers Eq. (3.6)
- Eq. (3.5), large barriers Eq. (3.4) and Eq. (3.6)
- Eq. (3.5), Eq. (3.19) and relaxion subdominant with respect to inflaton Eq. (3.13)
- Eq. (3.5), large barriers Eq. (3.4) and Eq. (3.13)
- Eq. (3.5) and precision of the Higgs mass scanning Eq. (3.3)
- Reheating Eq. (3.9) and Eq. (3.13)

# GKR (relaxion fragmentation w/ inflation).

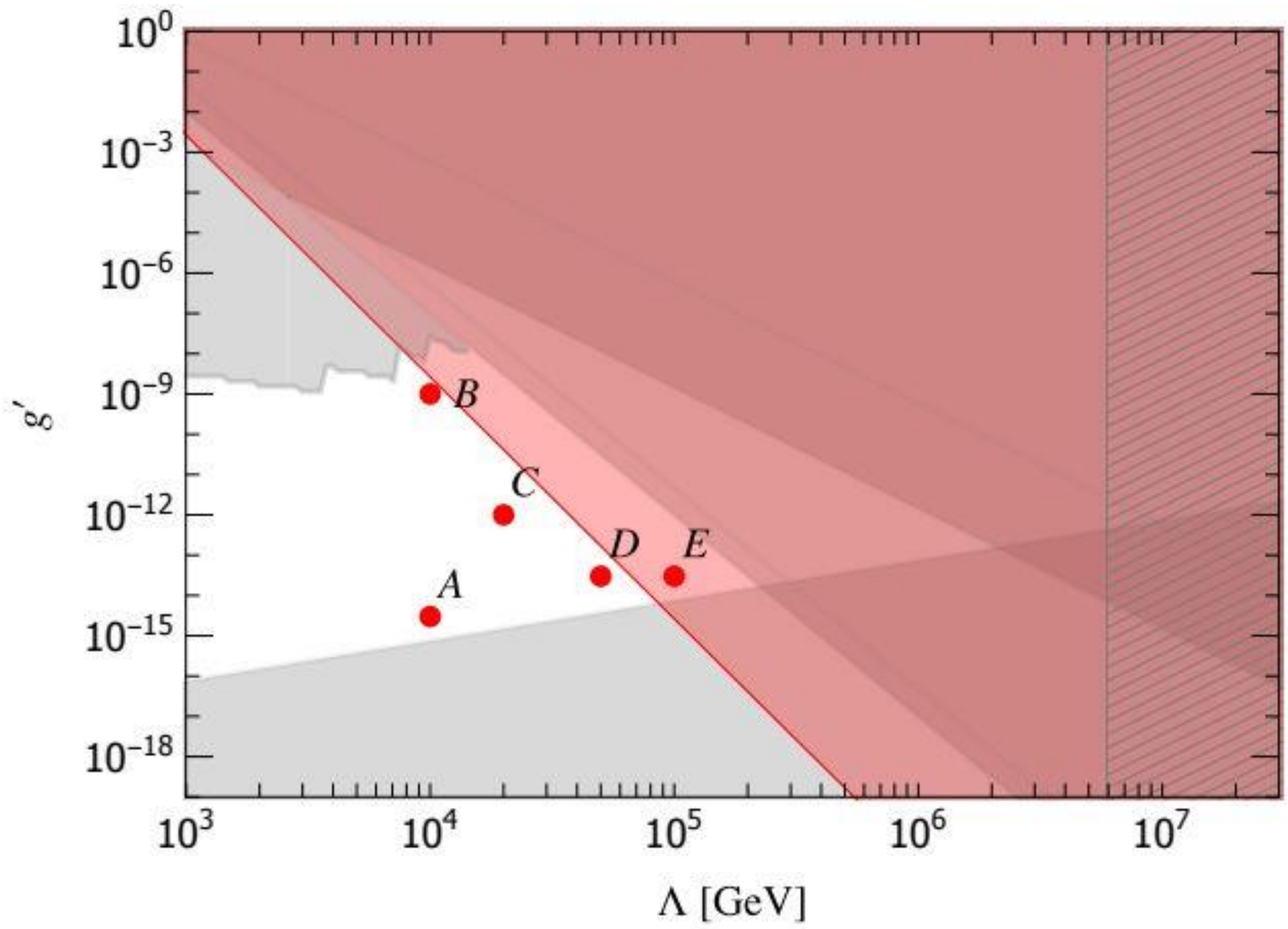



- Symmetry breaking pattern Eq. (3.5) and large velocity Eq. (3.22)
- Eq. (3.5), efficient fragmentation Eq. (2.16) and microscopic origin of the barriers Eq. (3.6)
- Relaxion subdominant with respect to inflaton Eq. (3.13) and Eq. (2.16)

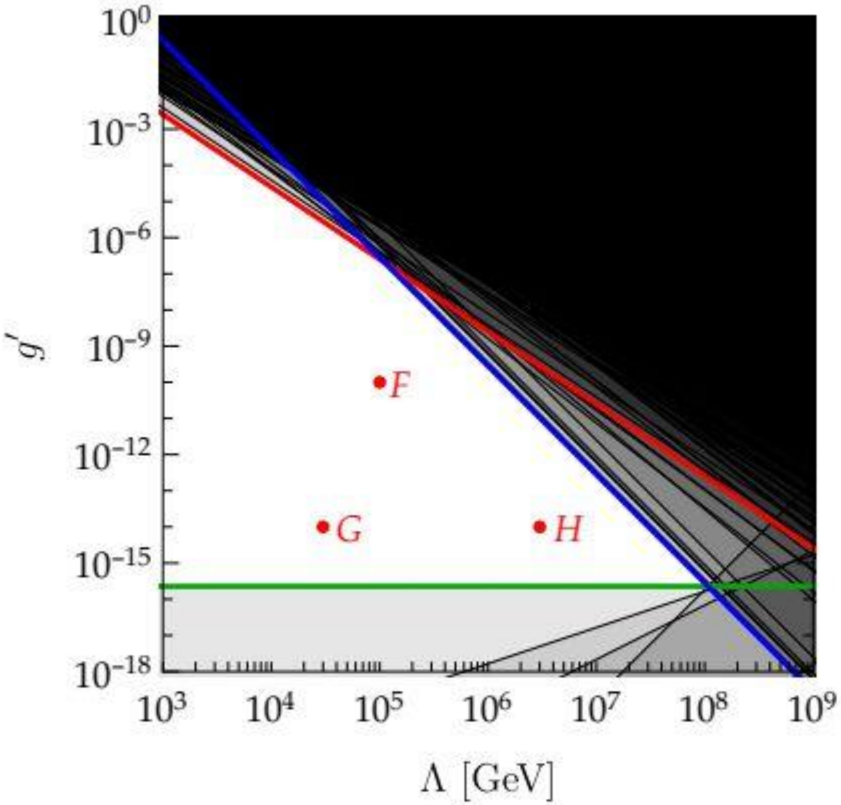
# GKR (relaxion fragmentation w/o inflation).



# HMT w/o inflation.

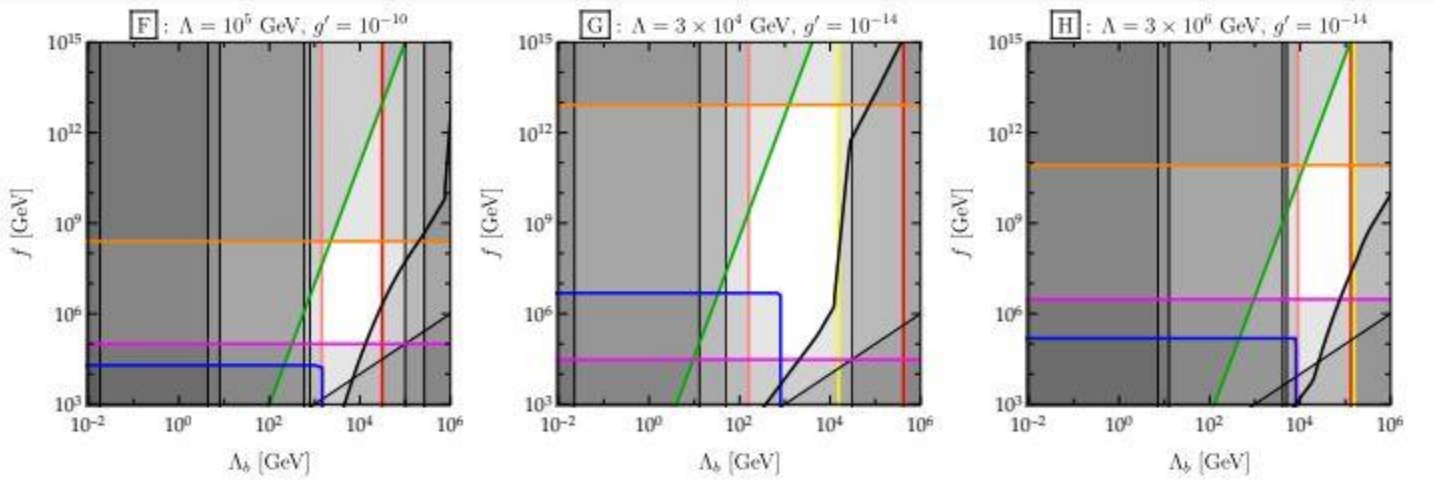


# HMT w/ inflation.



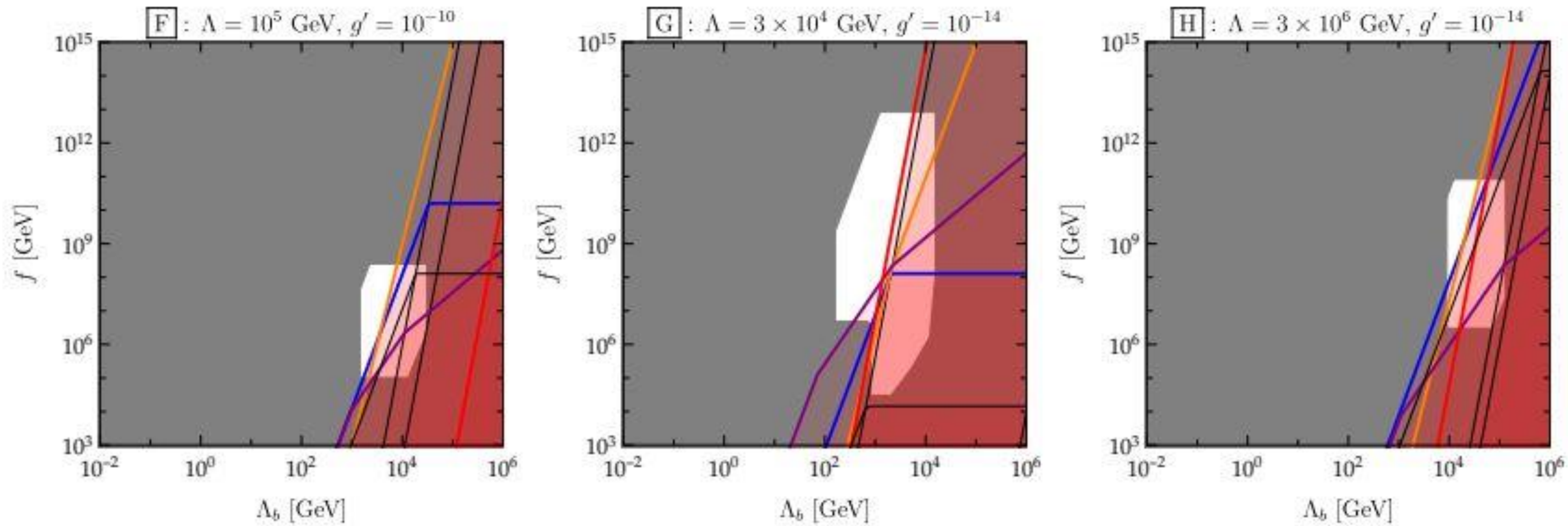

- Small variation of the Higgs mass Eq. (4.19), consistency of the EFT Eq. (4.24) and large velocity Eq. (4.15)
- Symmetry breaking pattern Eq. (4.28) and precision of the Higgs mass scanning Eq. (4.27)
- Relaxion subdominant with respect to inflaton Eq. (4.36) and Eq. (4.24)

# HMT w/ inflation.



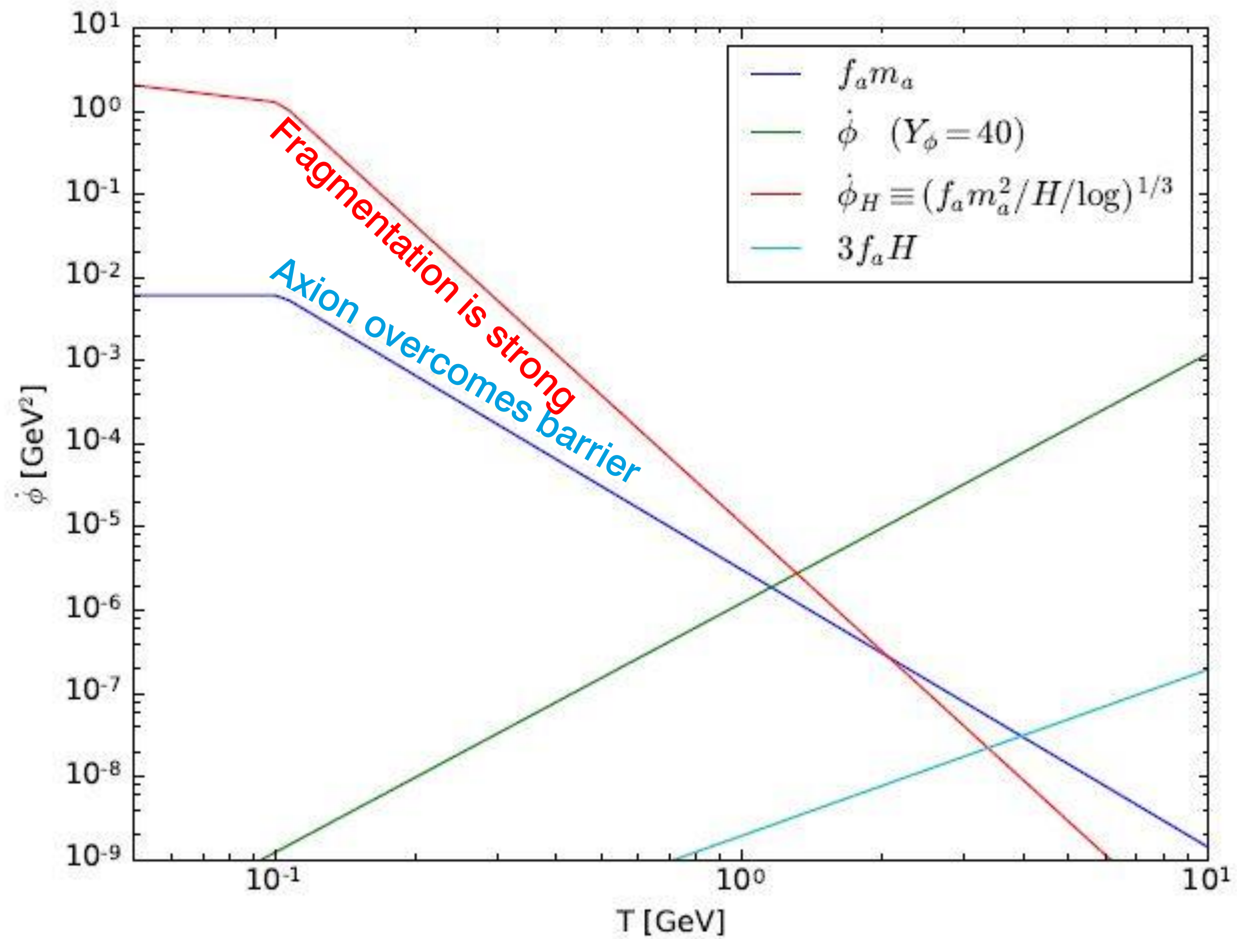

- Efficient energy dissipation Eq. (4.18) and consistency of the EFT Eq. (4.24)
- Small variation of the Higgs mass Eq. (4.19) and large velocity Eq. (4.15)
- Relaxion subdominant with respect to inflaton Eq. (4.36) and Eq. (4.15)
- Large barriers Eq. (4.26)
- Precision of the Higgs mass scanning Eq. (4.27)
- Symmetry breaking pattern Eq. (4.28)
- No restoration of the shift symmetry Eq. (4.21) and Eq. (4.24)
- Suppressed coupling to photons Eq. (4.23)

# HMT w/ inflation.




- Efficient energy dissipation Eq. (4.18) and inefficient fragmentation Eq. (4.33)
- No restoration of the shift symmetry Eq. (4.21) and Eq. (4.33)
- Small variation of the Higgs mass Eq. (4.19) and Eq. (4.33)
- Relaxion subdominant with respect to inflaton Eq. (4.36) and Eq. (4.33)

# QCD axion fragmentation.




- Impact on relic abundance might be not so large
- Fragmentation could change momentum distribution (work in progress)

# Technical difficulty

Computer power limits the size of the lattice:
O(100) points in one side is maximal value.

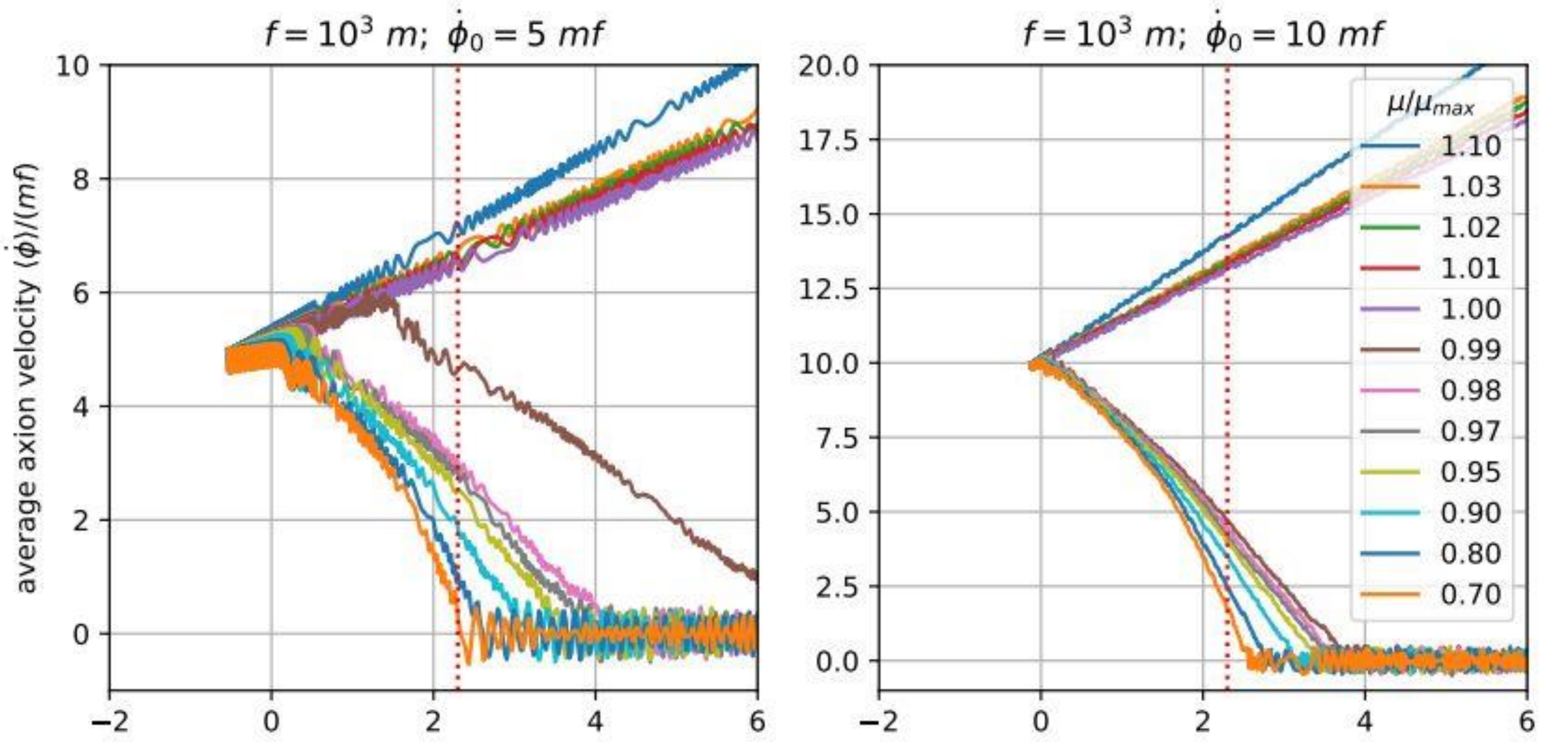
Lattice spacing should be $\ll f/\dot{\phi}$

Fragmentation time is $\sim f\dot{\phi}^3/\Lambda_b^8$

In order to estimate the size of long-wave length fluctuation,
we did multiple-run with given size of box L.

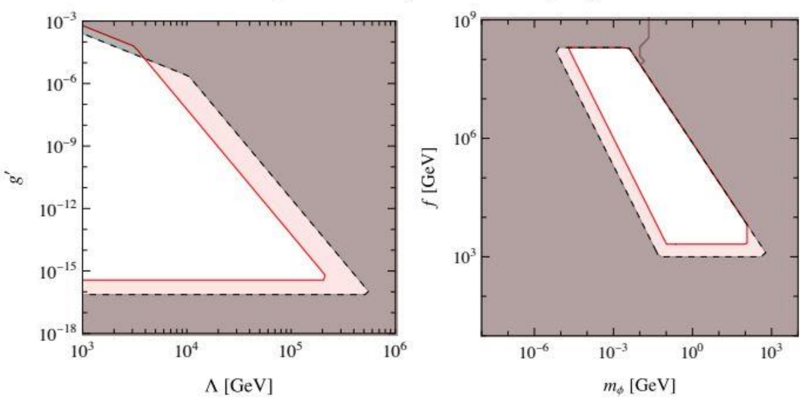
The variance depends on L.
The scaling of L tells us that long-wave length fluctuation

# Lattice calc. w/ slope term

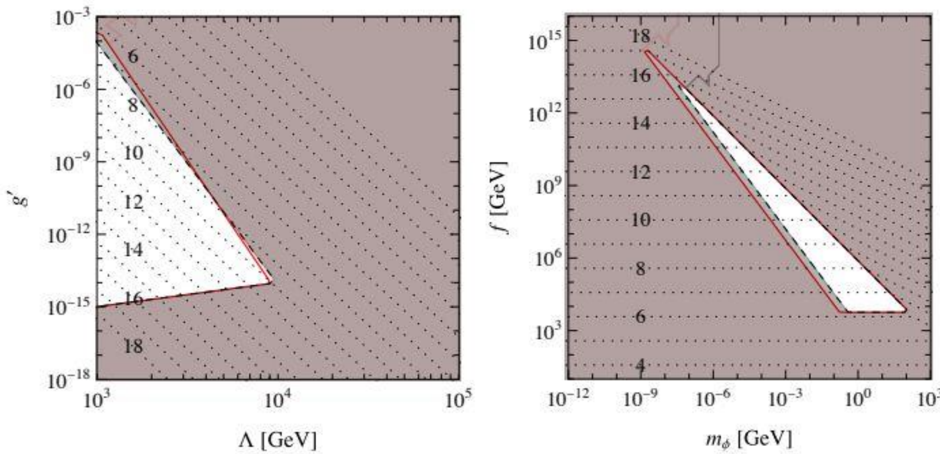


# How non-linear effect changes?

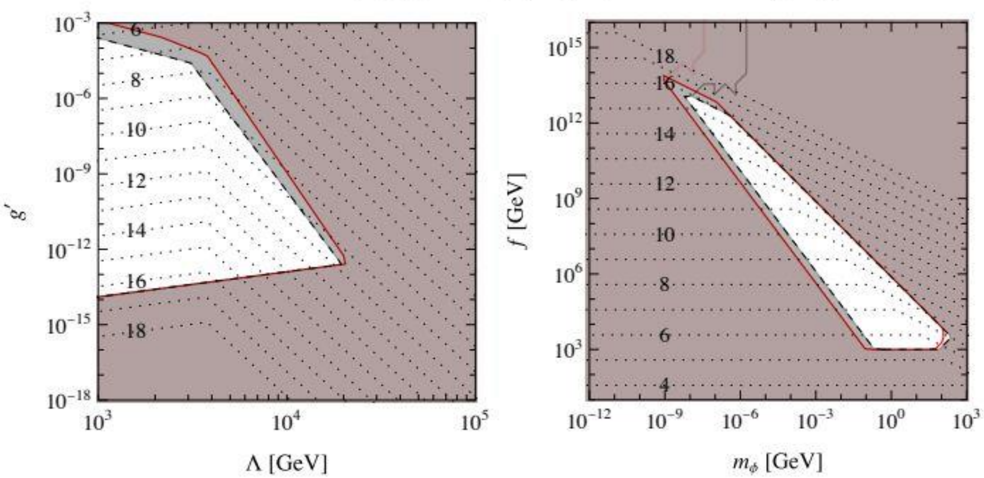
During inflation (Sec. 3.1 of [51])




After inflation, $g/g' = 1$ (Sec. 3.2 of [51])




After inflation, $g/g' = 1/(4\pi)^2$ (Sec. 3.2 of [51])



1. Introduction
2. Axion Fragmentation
3. Axion Fragmentation on the lattice
4. **Impact on Relaxion scenario**

# Relaxion scenarios

- Graham-Kaplan-Rajendran model (original relaxion model)

  Higgs-dependent wiggles : $V(\phi) = -g\Lambda^3\phi + \Lambda_b^4 \frac{h^2}{v^2} \cos\frac{\phi}{f}$

- Hook--Marques-Tavares model

  Higgs-independent wiggles : $V(\phi) = -g\Lambda^3\phi + \Lambda_b^4 \cos\frac{\phi}{f}$

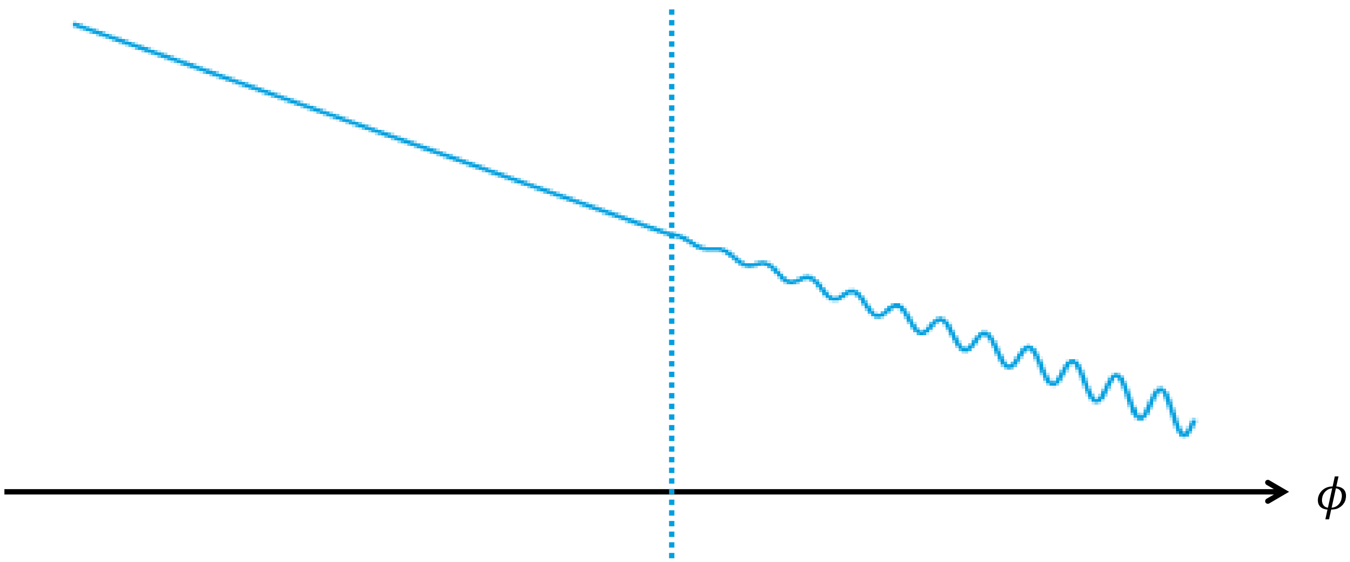
  EW gauge boson production gives friction ← $\frac{\phi}{4F} V_{\mu\nu}\tilde{V}^{\mu\nu}$

| | w/ inflation | w/o inflation |
|---|---|---|
| GKR model | New parameter region | New scenario |
| HMT model | constrained<br>but still available | killed |

# Graham-Kaplan-Rajendran model

$$V = -(\Lambda^2 - g'\Lambda\phi)H^2 + \lambda H^4 \quad + \quad g\Lambda^3\phi \quad + \quad \Lambda_b^4(H)\cos\frac{\phi}{f}$$

Higgs potential — slope — Wiggles



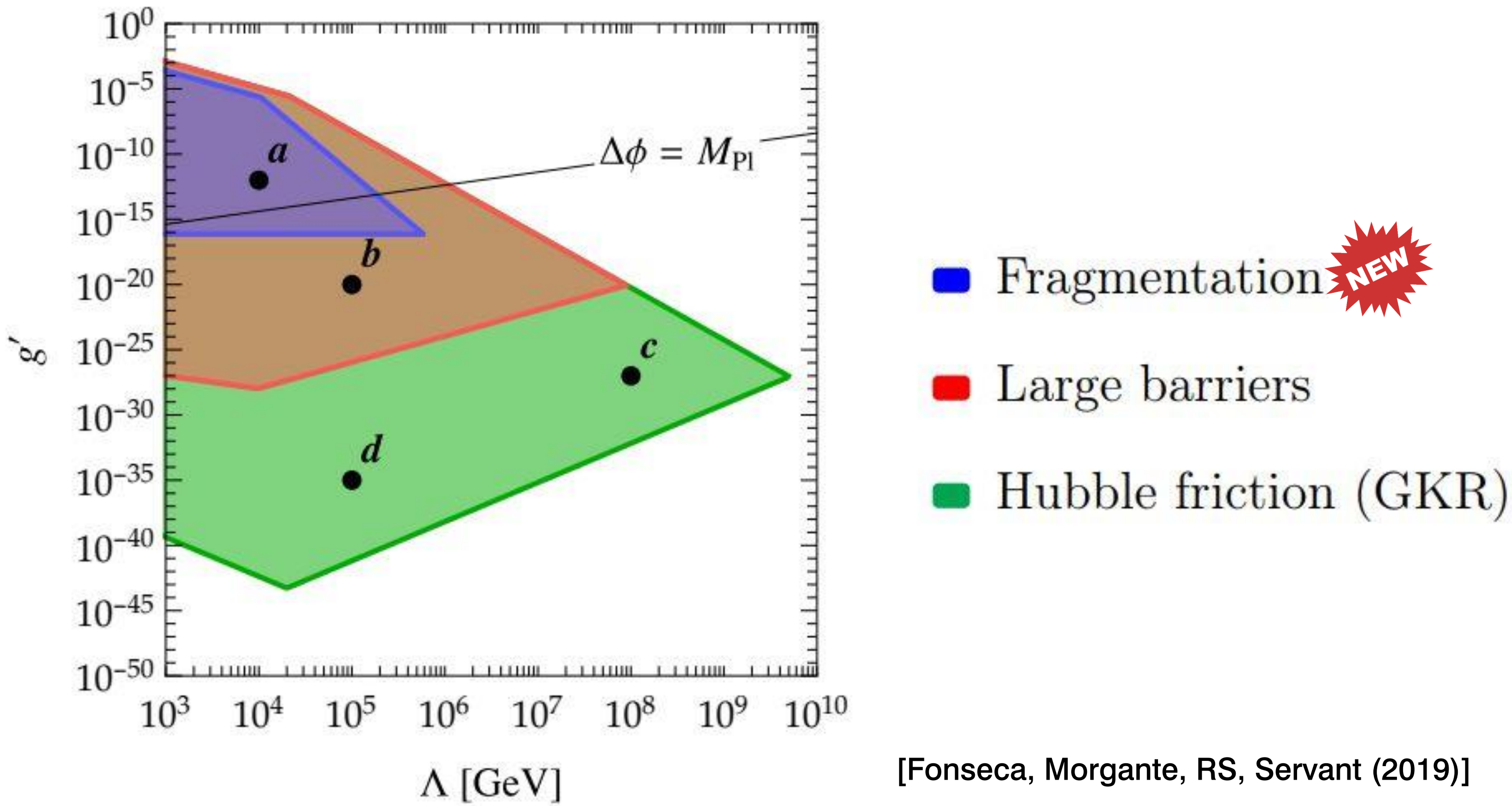

| | |
|---|---|
| $\Lambda^2 - g'\Lambda\phi < 0$ | $\Lambda^2 - g'\Lambda\phi > 0$ |
| $\Lambda_b(H) = 0$ | $\Lambda_b(H) > 0$ |
| $H = 0$ | $\lvert H\rvert > 0$ |

# Graham-Kaplan-Rajendran model

Friction is required to stop relaxion scanning (because of energy conservation)

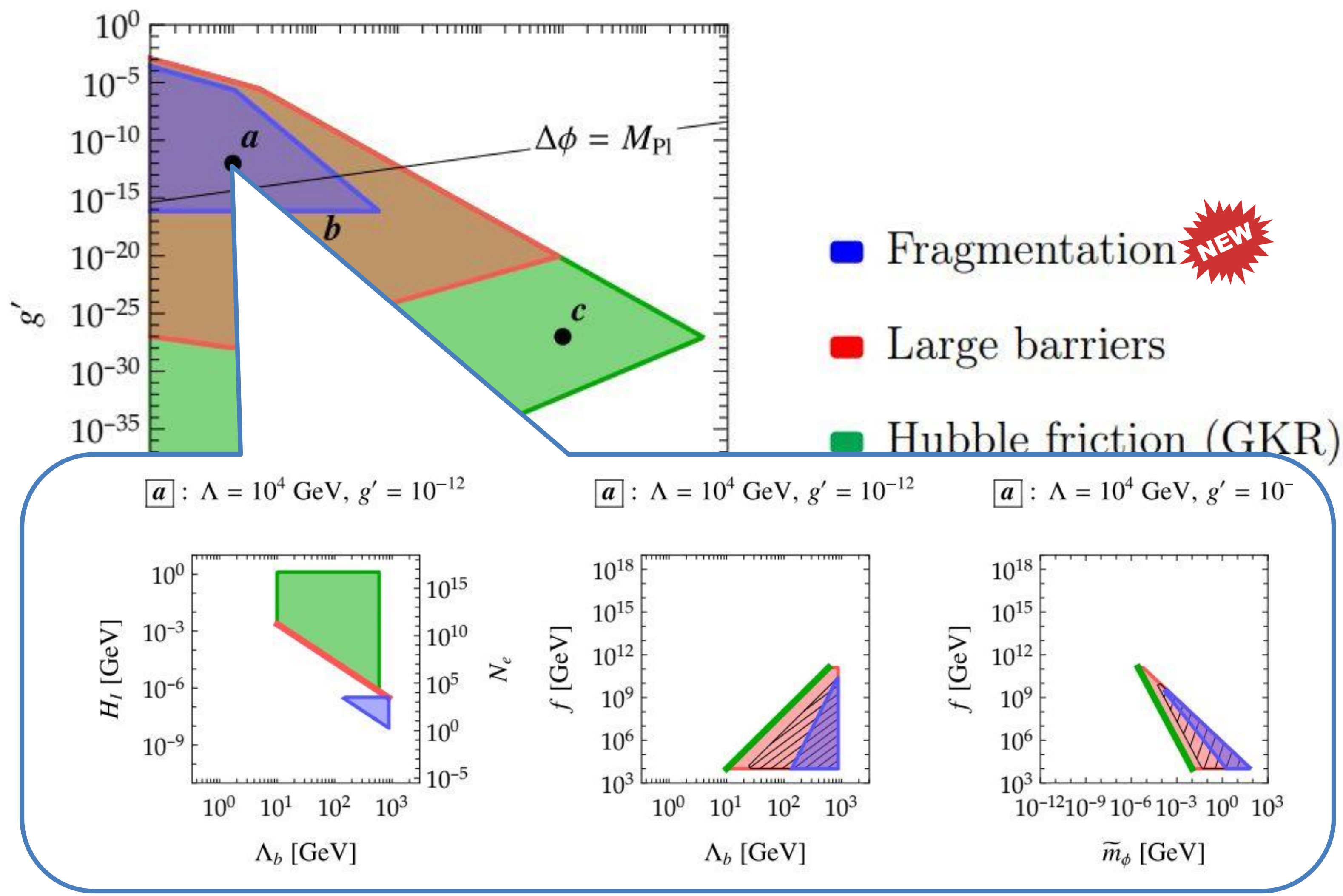
| Why relaxion stops? | when/where relaxion stops? |
|---|---|
| **Hubble friction** (original GKR model)<br>Velocity $\dot{\phi}$ always tracks $V'/3H$ | $g\Lambda^3 \sim \Lambda_b^4(\phi)/f$ |
| **Large barrier** (mild Hubble friction) NEW<br>Velocity $\dot{\phi}$ does not track $V'/3H$<br>But the average velocity is maintained as $\dot{\phi}_{SR} = g\Lambda^3/3H$ | $\dot{\phi}_{SR}^2 \sim \Lambda_b^4(\phi)$ |
| **Axion Fragmentation** NEW | $\Delta\phi_{frag} \sim f\dfrac{\dot{\phi}_0^4}{\Lambda_b^8}\log\dfrac{f^4}{\dot{\phi}_0^2}$ |

# GKR model w/ inflation

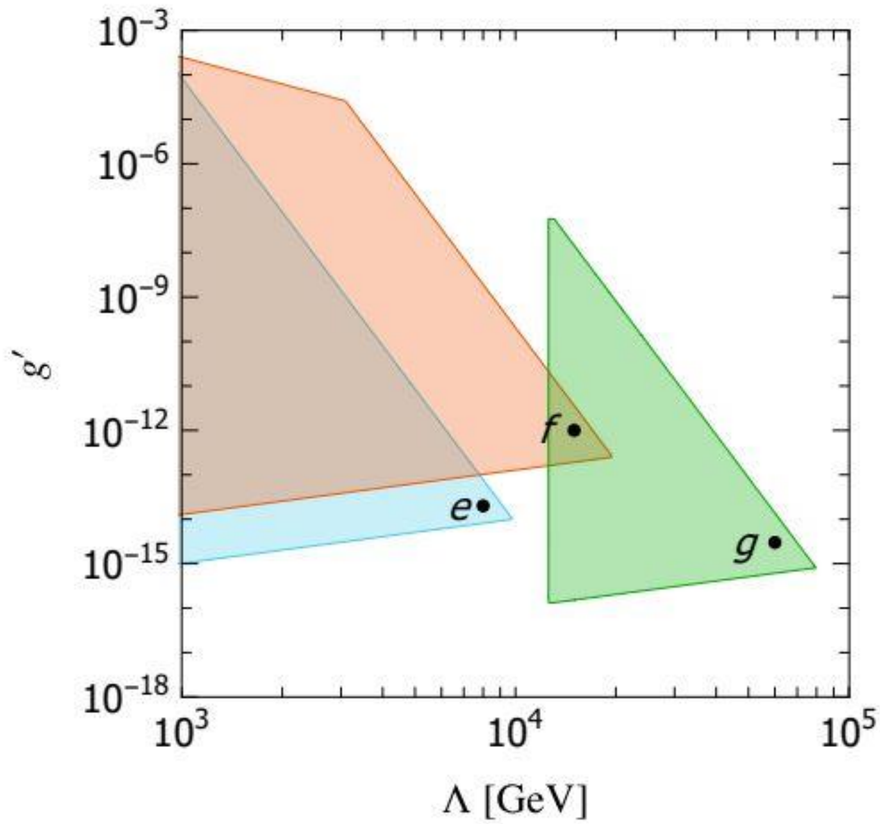



[Fonseca, Morgante, RS, Servant (2019)]

# GKR model w/ inflation



# GKR model w/o inflation


$\dot\phi = \sqrt{2\,g/g'}\,\Lambda^2,\ g/g' = 1$

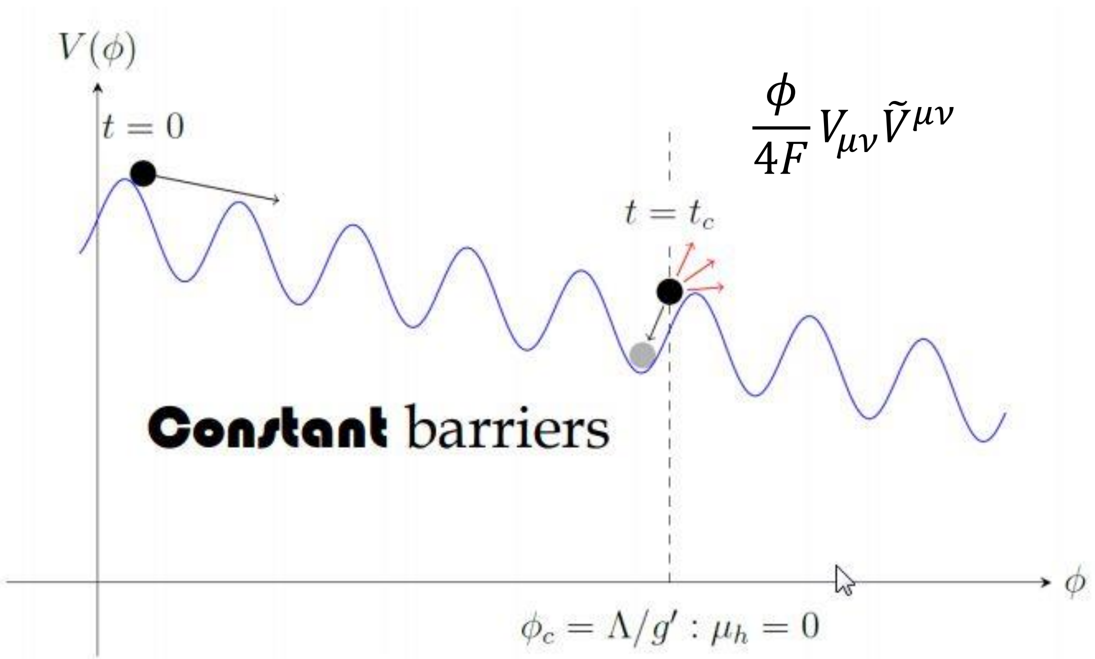
$\dot\phi = \sqrt{2\,g/g'}\,\Lambda^2,\ g/g' = 1/(4\pi)^2$

$\dot\phi = 10^{-2}\sqrt{2}\,\Lambda^2$

[Fonseca, Morgante, RS, Servant (2019)]

# Hook—Marques-Tavares Model

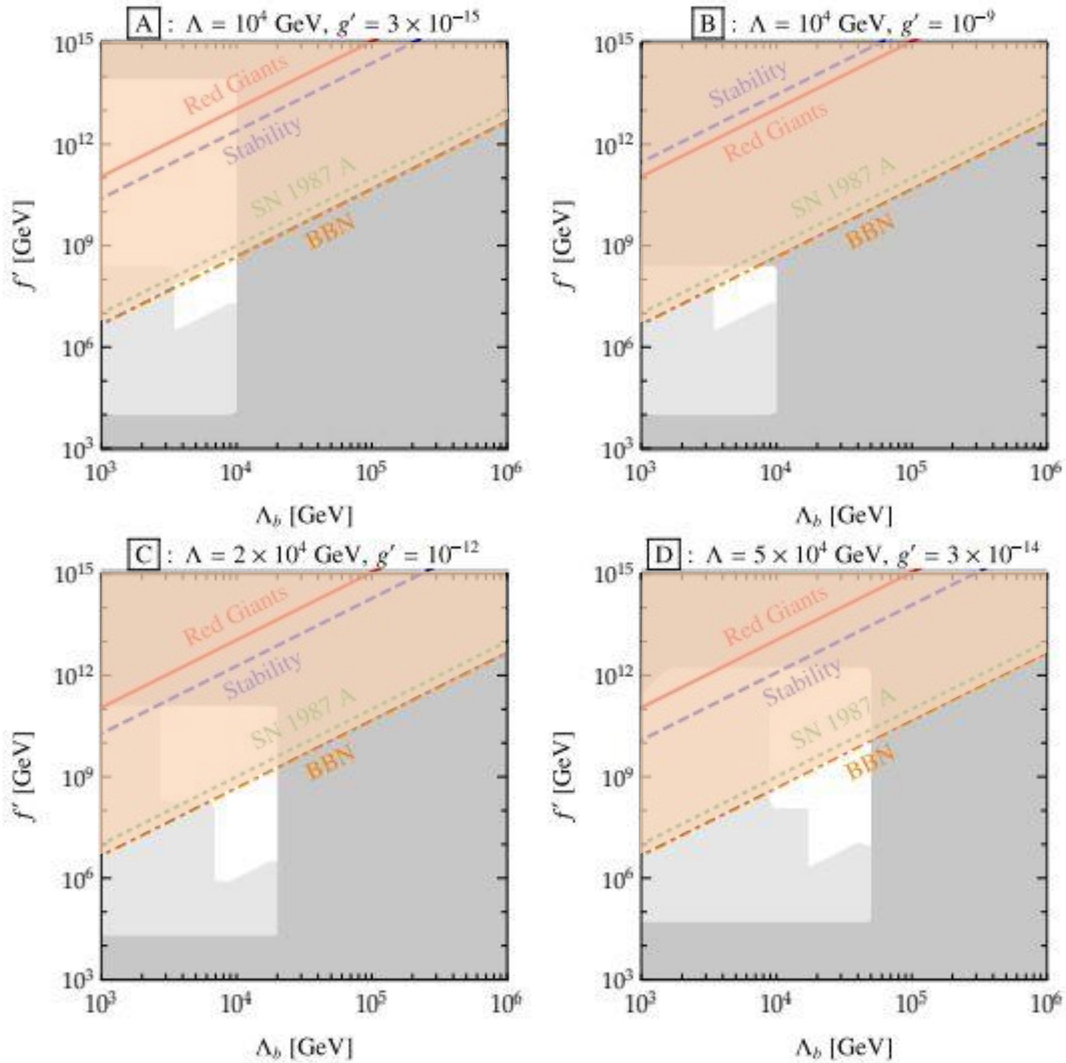
- EW gauge boson production gives friction
- Higgs independent barrier → axion fragmentation always works




[Fonseca, Berlin COST BSM workshop 2020]

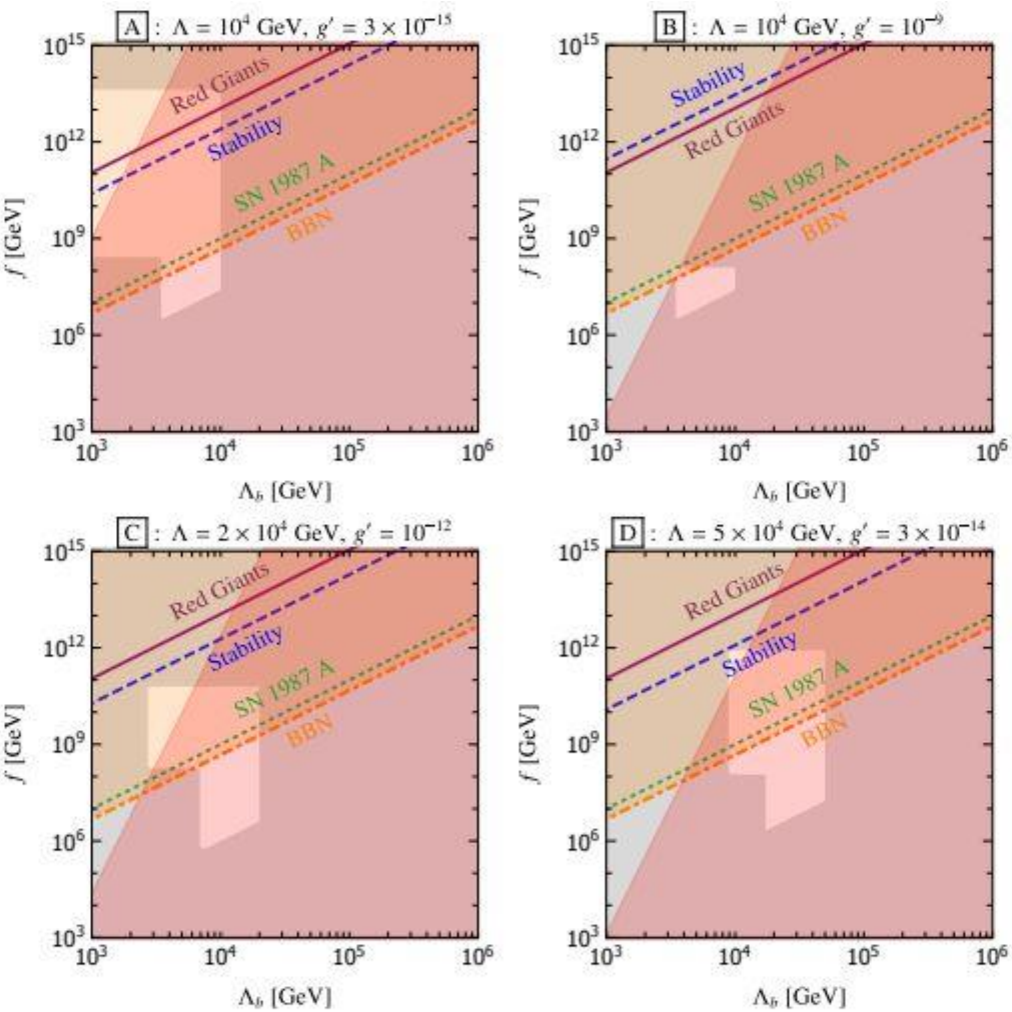
Relaxion rolling could be stopped before the right position.

# HMT model w/o inflation




[Fonseca, Morgante, Servant (2018)]

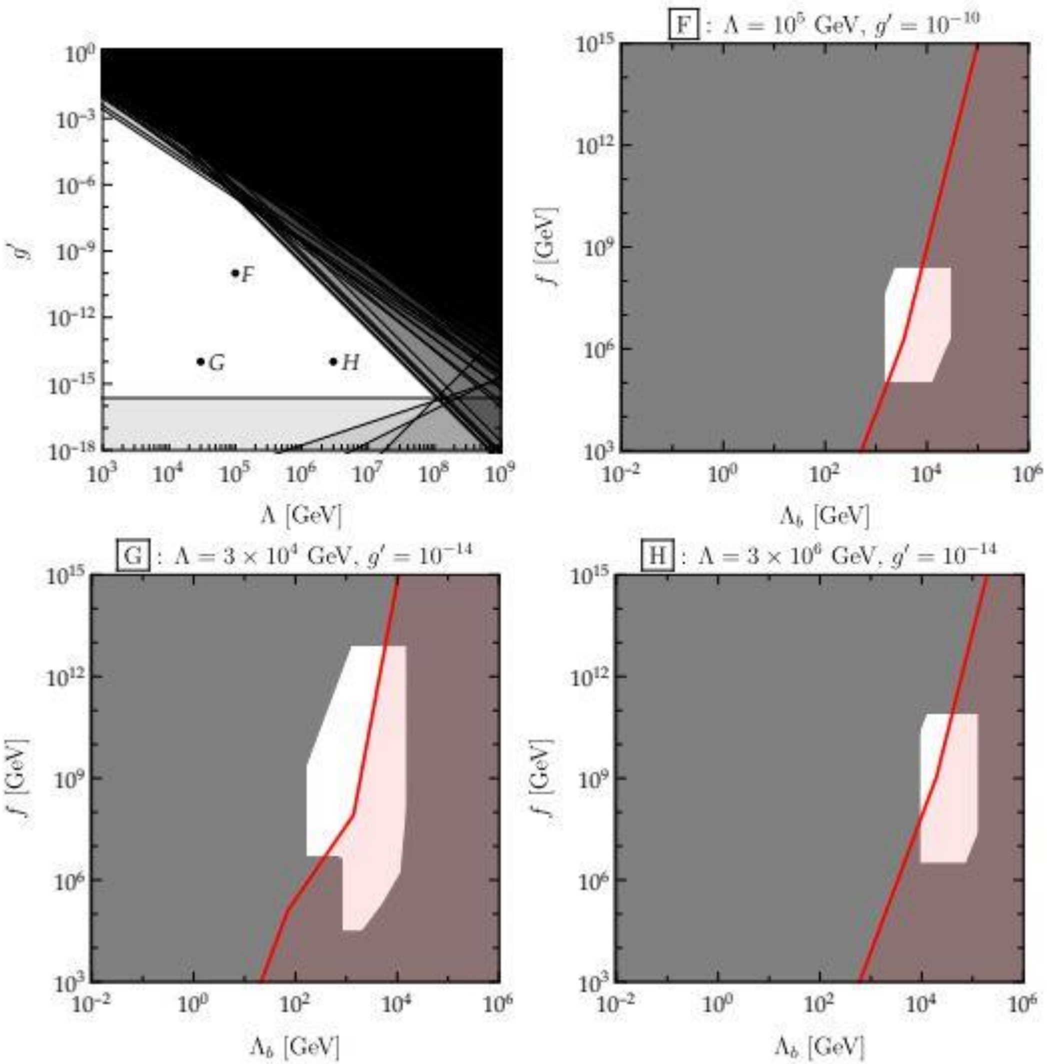
# HMT model w/o inflation




Fragmentation kills parameter space!

[Fonseca, Morgante, RS, Servant (2019)]

# HMT model w/ inflation




Still there is available parameter region

[Fonseca, Morgante, RS, Servant (2019)]