Nuclei in holographic QCD

Yoshinori Matsuo

Kyoto University

Based on arXiv:1902.07444 with Koji Hashimoto (Kyoto U.), Takeshi Morita (Shizuoka U.)

arXiv:2103.03563 with Koji Hashimoto (Kyoto U.)

April 28, 2021@Kyoto U.

Nuclei can be described by matrix model in holographic QCD

Nuclei = bound states of nucleons = bound states of D-branes

nucleons in QCD ⟺ D-branes (baryon vertex)

correspondence

⇓ Effective theory

Nuclear matrix model

Nuclei in holographic QCD appear as bound states in matrix model

Nuclei have following properties

- Saturation of nucleon number density
- Saturation of nuclear binding energy
- Nuclear magic number

These properties can be reproduced from nuclear matrix model

Plan of Talk

1. Important properties of nuclei

2. Holographic QCD

3. Nuclei in nuclear matrix model

4. Conclusion

1. Important properties of nuclei

1. Important properties of nuclei

Saturation of nucleon number density

Saturation of nuclear binding energy

Nuclear magic number

Saturation of nucleon number density: nucleon density is constant

Nucleon density can be read off from charge distribution

➡ almost constant $\rho \simeq A/V \simeq \text{const.}$

Constant is almost independent of nuclide for large mass number

Nuclear radius R

$$R \simeq 1.2 \times A^{1/3}\ \text{fm}$$

A: mass number (# of nucleons)

Saturation of nuclear binding energy: binding energy per nucleon is constant

Nuclear binding energy can be read off from mass defect

Binding energy per nucleon is almost constant in large A

Nuclear magic number:
of protons (neutrons) for stable nuclei

Magic Numbers: 2, 8, 20, 28, 50, 82, 126, ⋯

Nuclei at magic number are stable

Magic number is explained by nuclear shell model

Harmonic potential $V_H \propto r^2$

$$\psi \sim x^{n_x} y^{n_y} z^{n_z}$$

n	0	1	2
orbital state	1s	1p	2s+1d
# of states	1×2	3×2	(1+5)×2 =12

1s + 1p: 8

1s + 1p + (2s+1d): 20

1. Important properties of nuclei

Saturation of nucleon number density

Saturation of nuclear binding energy

Nuclear magic number

2. Holographic QCD

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Effective theory is obtained by holography

Sakai-Sugimoto model gives effective theory of QCD
(in confinement phase)

Baryons in holographic QCD = D-branes (baryon vertex)

Effective theory of baryon vertex = matrix model

Effective theory is obtained by holography

holography (AdS/CFT, gauge/gravity, fluid gravity, AdS/CMT)

CFT (gauge theory) ⟺ Classical solution in AdS

correspondence

Field theory on D-branes → CFT (gauge theory)

D-branes in SUGRA → Classical solution in AdS

Holography is useful to calculate strongly coupled QCD

holographic QCD

Strongly coupled QCD ⟺ Classical solution in gravity

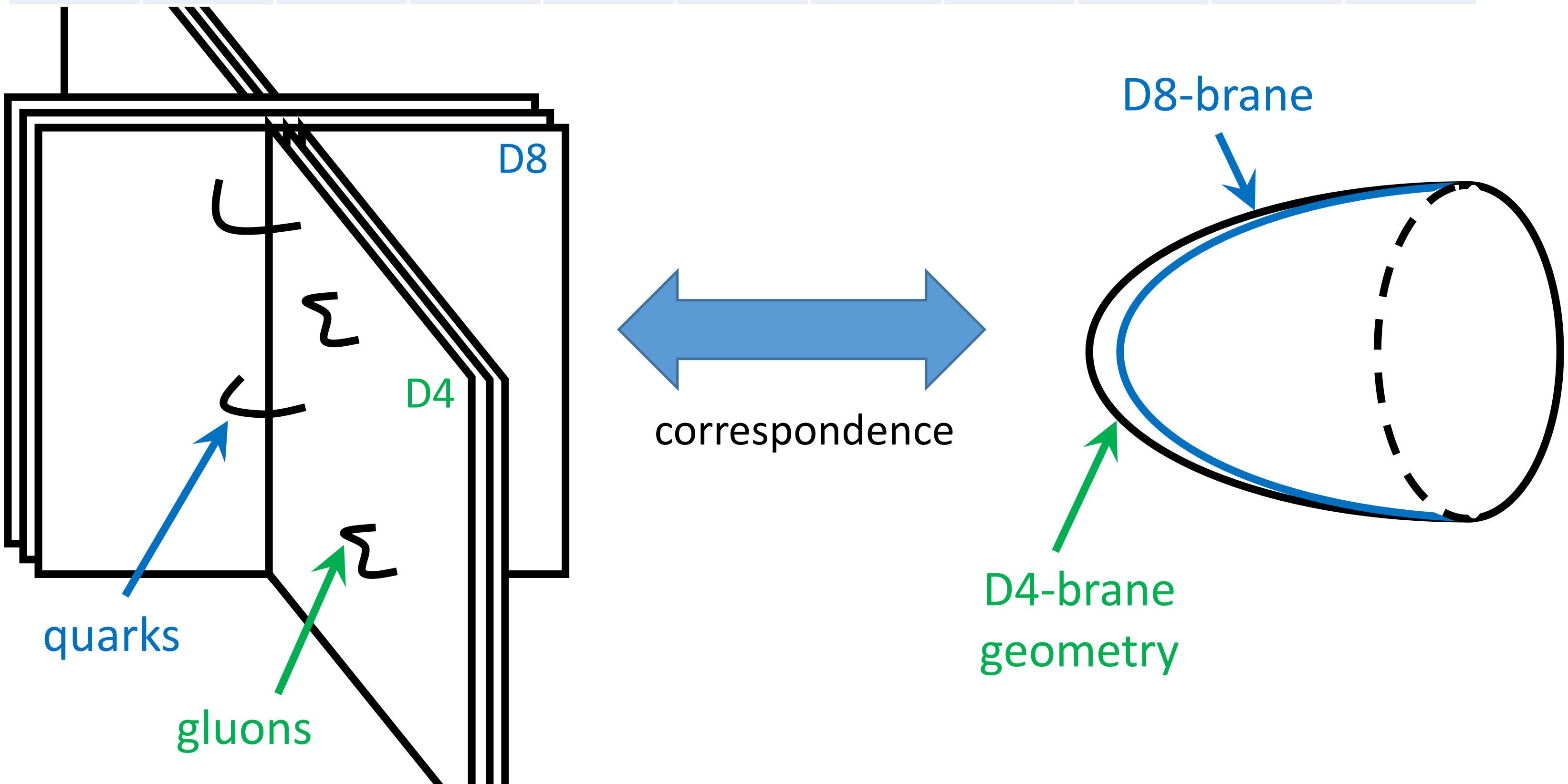
correspondence

Strongly coupled QCD → Result

Calculation → Classical solution in gravity

Sakai-Sugimoto model gives effective theory of QCD (in confinement phase)

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D4	✓	✓	✓	✓	✓					
D8	✓	✓	✓	✓		✓	✓	✓	✓	✓



Sakai-Sugimoto model gives effective theory of QCD (in confinement phase)

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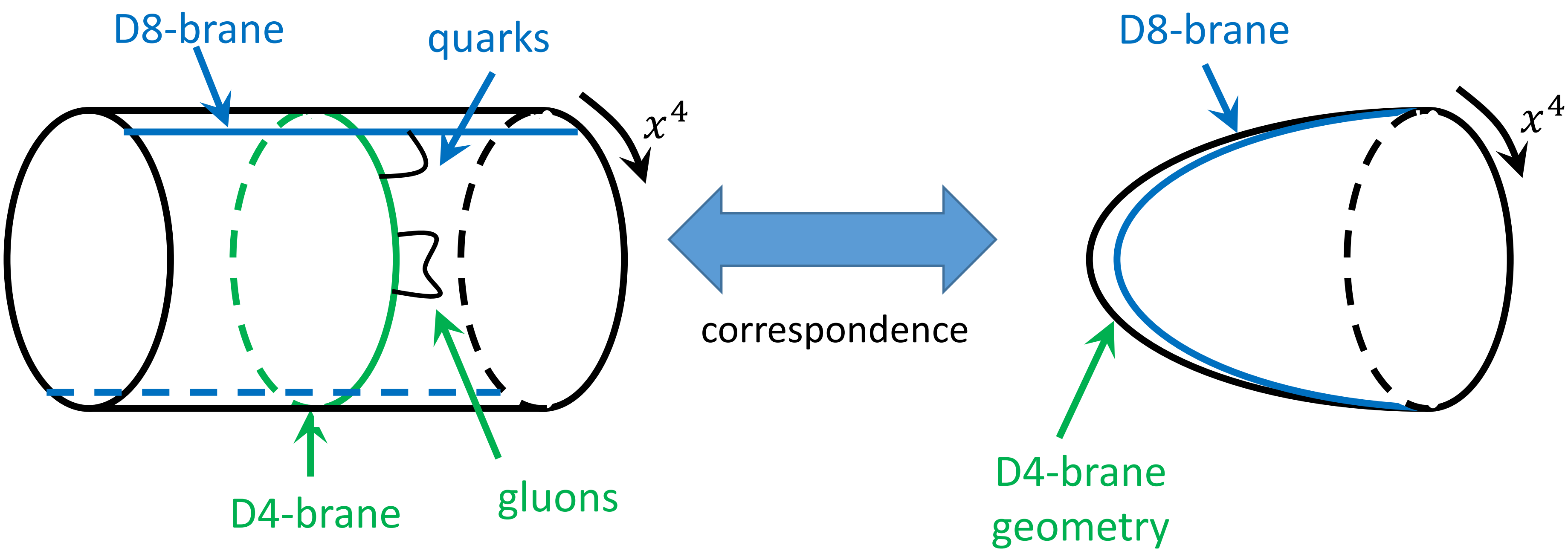
D8	✓	✓	✓	✓		✓	✓	✓	✓	✓



Sakai-Sugimoto model gives effective theory of QCD (in confinement phase)

D4: color
D8: flavor

Holographic QCD (Sakai-Sugimoto model)

Strongly coupled QCD ⟷ Classical solution in gravity

correspondence

QCD on D4-branes — D4-brane geometry

Effective theory of mesons — Field theory on D8-branes

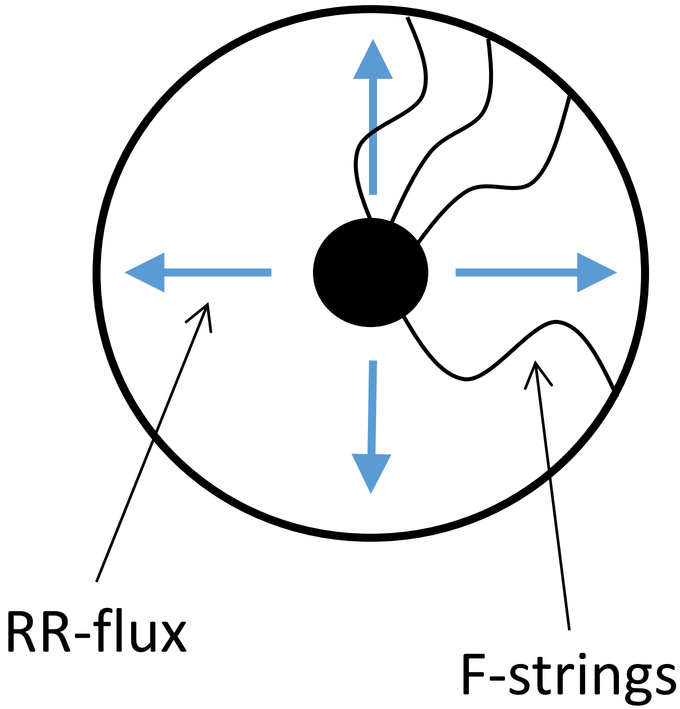
Sakai-Sugimoto model reproduces meson spectra

Baryons in Sakai-Sugimoto model

Baryons ⟷ Soliton on D8-branes ⟷ D4-branes

Baryons in AdS/CFT correspondence

Baryons = D-branes wrapping on color D-branes



RR-flux coupled with
gauge field on baryon vertex

$$\int_{R\times S^n} A \wedge G = N_c \int_R A$$

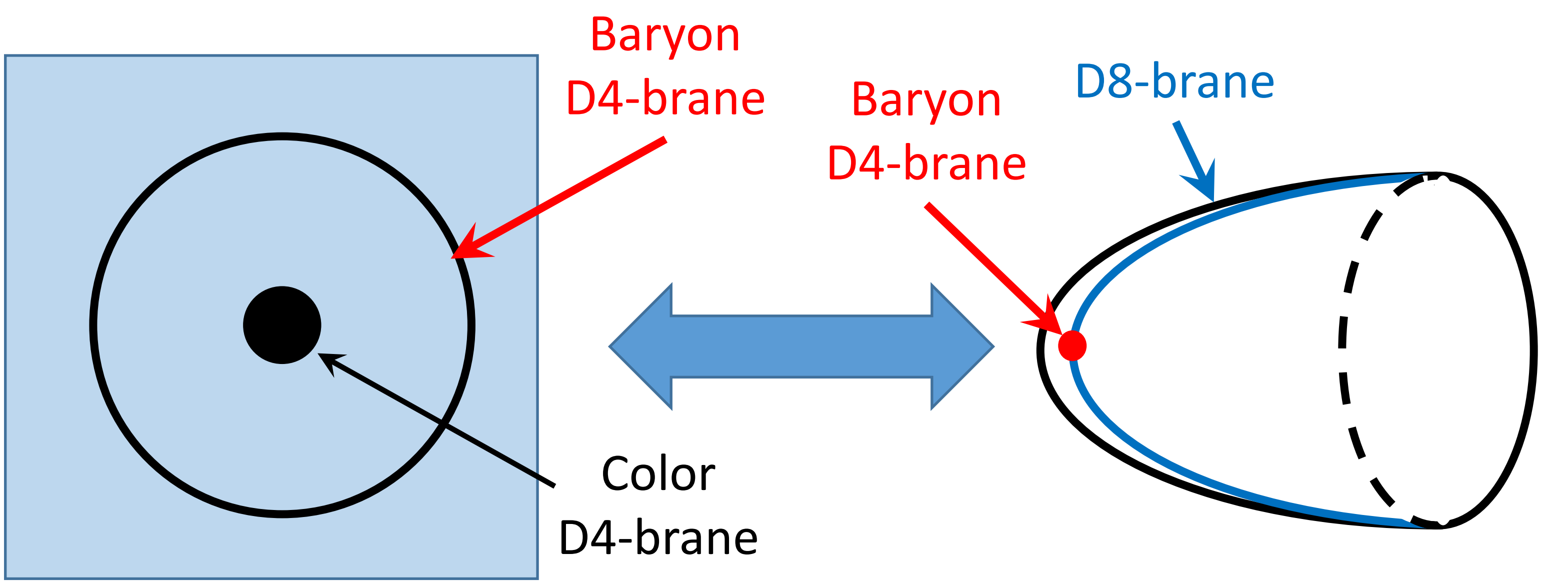
Gauge field　RR-flux

N_c endpoints of F-strings
on baryon vertex

Corresponds to baryons

Baryons in Sakai-Sugimoto model: D4-branes wrapping on color D4-branes

	x^0	x^1	x^2	x^3	x^4	U	θ^1	θ^2	θ^3	θ^4
D4	✓	✓	✓	✓	✓					
D8	✓	✓	✓	✓		✓	✓	✓	✓	✓
Baryon	✓						✓	✓	✓	✓



Effective theory of baryons = Matrix models

Action for baryons [Hashimoto-Iizuka-Yi,'10]

$$S = S_0 + N_c \int dt\, \mathrm{tr} A_t$$

$$S_0 = \int dt\, \mathrm{tr}\left[\frac{1}{2}(D_t X^I)^2 + \frac{1}{2}\left(D_t \bar{w}^{\dot{\alpha} i}\right)(D_t w_{\dot{\alpha} i}) - \frac{1}{2} M^2 \bar{w}^{\dot{\alpha} i} w_{\dot{\alpha} i} + \frac{1}{4\lambda}(D^I)^2 + D^I\left(2i\epsilon^{IJK} X^J X^K + \bar{w}^{\dot{\alpha} i}(\tau^I)_{\dot{\alpha}}{}^{\dot{\beta}} w_{\dot{\beta} i}\right)\right]$$

X^I: D4-D4 scalar ⇒ Position of baryon vertex

w ($\bar{w}$): D4-D8 scalar ⇒ Spin, flavor, baryon number

Nuclei = bound states of baryons

Nuclei appear in eigenstates of matrix quantum mechanics

2. Holographic QCD

Effective theory is obtained by holography

Sakai-Sugimoto model gives effective theory of QCD
(in confinement phase)

Baryons in holographic QCD = D-branes (baryon vertex)

Effective theory of baryon vertex = matrix model

3. Nuclei in nuclear matrix model

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Saturation of nuclear number density (Review of [Hashimoto-Morita,'11])

Eigenstates of nuclear matrix model have similar structure to nuclear shell model ➡ magic number

Saturation of nuclear binding energy

Saturation of nucleon number density

[Hashimoto-Morita,'11]

Potential is approximated by harmonic potential

$$S_X = \int dt\,\mathrm{tr}\left[\frac{1}{2}(D_t X^I)^2 + 2\lambda[X^I, X^J]^2\right] \simeq \int dt\,\mathrm{tr}\left[\frac{1}{2}(D_t X^I)^2 - \frac{1}{2}m^2 (X^I)^2\right]$$

$$m^2 \simeq \frac{16A\lambda}{3(A^2-1)}\langle X^2 \rangle \quad \Rightarrow \quad m^3 \simeq 8\lambda A$$

$$r_0 = \frac{2A}{m}$$

Nucleon density

$$\rho = \frac{1}{(2\pi)^3}\int d^3k\, e^{-ikx}\langle \mathrm{tr}\exp ikX\rangle \simeq \begin{cases} \dfrac{A}{\pi^2 r_0^2\sqrt{r_0^2 - r^2}} & (r < r_0) \\ 0 & (r > r_0) \end{cases}$$

Nuclear radius $E = 2\langle V\rangle = m^2\langle \mathrm{tr}\, X^2\rangle = \frac{3}{2}m\, A^2$

$$R^2 = \frac{1}{A}\langle \mathrm{tr}\, X^2\rangle \quad \Rightarrow \quad R \sim \sqrt{A/m} \propto A^{1/3}$$

Eigenstates of nuclear matrix model has similar structure to nuclear shell model

Action for baryons

X^I: Position of baryon vertex

$w^a_{\dot{\alpha}i}$: Spin, flavor, baryon number

$$S = S_0 + N_c \int dt\, \mathrm{tr} A_t$$

$$S_0 = \int dt\ \mathrm{tr}\left[\frac{1}{2}(D_t X^I)^2 + \frac{1}{2}\left(D_t \overline{w}^{\dot{\alpha}i}\right)(D_t w_{\dot{\alpha}i}) - \frac{1}{2}M^2 \overline{w}^{\dot{\alpha}i} w_{\dot{\alpha}i} + \frac{1}{4\lambda}(D^I)^2 + D^I\left(2i\epsilon^{IJK}X^J X^K + \overline{w}^{\dot{\alpha}i}(\tau^I)_{\dot{\alpha}}{}^{\dot{\beta}} w_{\dot{\beta}i}\right)\right]$$

Eigenstate of Hamiltonian

A: Number of baryons

$$|\psi_0\rangle = \left[\epsilon_{a_1\cdots a_A} w^{a_1}\cdots w^{a_{2N_f}}(X^I w)\cdots(X^J w)\cdots(X^K\cdots X^L w)^{a_A}\right] \times\cdots\times\left[\epsilon_{b_1\cdots b_A} w^{b_1}\cdots w^{b_{2N_f}}(Xw)\cdots(X\cdots Xw)^{b_A}\right]|0\rangle$$

N_c of $\left[\epsilon_{a_1\cdots a_A} w^{a_1}\cdots(X^I\cdots X^J)w^{a_A}\right]$

Eigenstates of nuclear matrix model has similar structure to nuclear shell model

Eigenstate of Hamiltonian

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$$|\psi_0\rangle = [\epsilon_{a_1\cdots a_A} w^{a_1} \cdots w^{a_{2N_f}} (X^I w) \cdots (X^J w) \cdots (X^K \cdots X^L w)^{a_A}]$$
$$\times \cdots \times [\epsilon_{b_1\cdots b_A} w^{b_1} \cdots w^{b_{2N_f}} (Xw) \cdots (X \cdots Xw)^{b_A}]|0\rangle$$

A_t: gauge field (baryon $U(A)$)

Non-dynamical field → EOM gives constraints

$$0 = \frac{\delta S}{\delta A_t} = \frac{\delta S_0}{\delta A_t} - N_c \mathbb{I} = Q_{U(A)} - N_c \mathbb{I}$$

We impose constraints to physical states

$Q_{SU(A)}|\psi\rangle = 0$ → Singlet state of baryon $SU(A)$

$Q_{U(1)_B}|\psi\rangle = N_c A|\psi\rangle$ → Baryon (quark) number is $N_c A$

Eigenstates of nuclear matrix model has similar structure to nuclear shell model

Eigenstate of Hamiltonian

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$$|\psi_0\rangle = \left[\epsilon_{a_1\cdots a_A} w^{a_1}\cdots w^{a_{2N_f}} (X^I w)\cdots(X^J w)\cdots(X^K\cdots X^L w)^{a_A}\right]$$
$$\times\cdots\times\left[\epsilon_{b_1\cdots b_A} w^{b_1}\cdots w^{b_{2N_f}} (Xw)\cdots(X\cdots Xw)^{b_A}\right]|0\rangle$$

N_c of $\left[\epsilon_{a_1\cdots a_A} w^{a_1}\cdots(X^I\cdots X^J)w^{a_A}\right]$

Anti-symmetric under exchange of baryon indices

⇓

Pauli exclusion principle

X^I has effective mass ⇒ Orbital energy levels (excited state)

magic number

Magic number for u (or d) quarks ($N_c = 1$ case for simplicity)

$w = u^{\dot{\alpha}}$ $\quad \dot{\alpha} = 1, 2 \quad (i = 1)$ ⇒ 2 of u (spin ↑ and ↓)

$A = 1 \qquad |\psi_0\rangle = \epsilon u_\uparrow |0\rangle$

Magic number → $A = 2 \qquad |\psi_0\rangle = \epsilon u_\uparrow u_\downarrow |0\rangle$

$A = 3 \qquad |\psi_0\rangle = \epsilon u_\uparrow u_\downarrow u_\uparrow |0\rangle = 0$ ⇒ $|\psi_0\rangle = \epsilon uu(Xu)|0\rangle$

Additional energy of X^I

$A = 4 \qquad |\psi_0\rangle = \epsilon uu(uX)(uX)|0\rangle$

⋮

Magic number → $A = 8 \qquad |\psi_0\rangle = \epsilon uu\underbrace{(Xu)\cdots(Xu)}_{6 \text{ of } Xu}|0\rangle$

$X^I \quad I = 1, 2, 3$ ⇓ $2 \times 3 = 6$ of Xu

$A = 9 \qquad |\psi_0\rangle = \epsilon uu\underbrace{(Xu)\cdots(Xu)}_{6 \text{ of } Xu}(Xu)|0\rangle = 0$

⇓

$A = 9 \qquad |\psi_0\rangle = \epsilon uu\underbrace{(Xu)\cdots(Xu)}_{6 \text{ of } Xu}(XXu)|0\rangle$

No correction at leading order of large A

Potential is approximated by harmonic potential

$$S_X = \int dt\ \mathrm{tr}\left[\frac{1}{2}(D_t X^I)^2 + 2\lambda[X^I, X^J]^2\right] \simeq \int dt\ \mathrm{tr}\left[\frac{1}{2}(D_t X^I)^2 - \frac{1}{2}m^2(X^I)^2\right]$$

$$m^2 \simeq \frac{16A\lambda_r}{3(A^2-1)}\langle \mathrm{tr}\, X^2\rangle \qquad E = 2\langle V\rangle = m^2\langle \mathrm{tr}\, X^2\rangle$$

Harmonic oscillator with N_X excitations

$$E = m\left(N_X + \frac{3}{2}(A^2-1)\right) \qquad N_X \simeq \left(\frac{3}{2}\right)^{7/3} A^{4/3}$$

$$\Rightarrow \quad m^3 = 8\lambda_r A + 2^{5/3}3^{4/3}\lambda_r A^{1/3}$$

Binding energy of fermions can be read off from density distribution

Multi-fermion ground state in given potential $V(r)$

WKB approximation

$$\psi = \frac{C}{r} p^{-1/2}\, e^{i \int dr\, p(r)} Y_{lm} \qquad p(r) = \sqrt{E - V(r) - \frac{l(l+1)}{r^2}}$$

Quantization condition

$$\int dr\, p(r) = \pi n$$

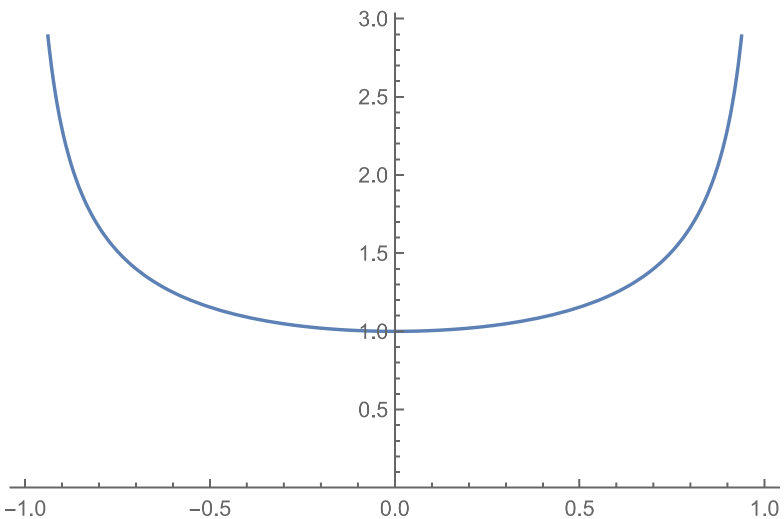
Fermion density

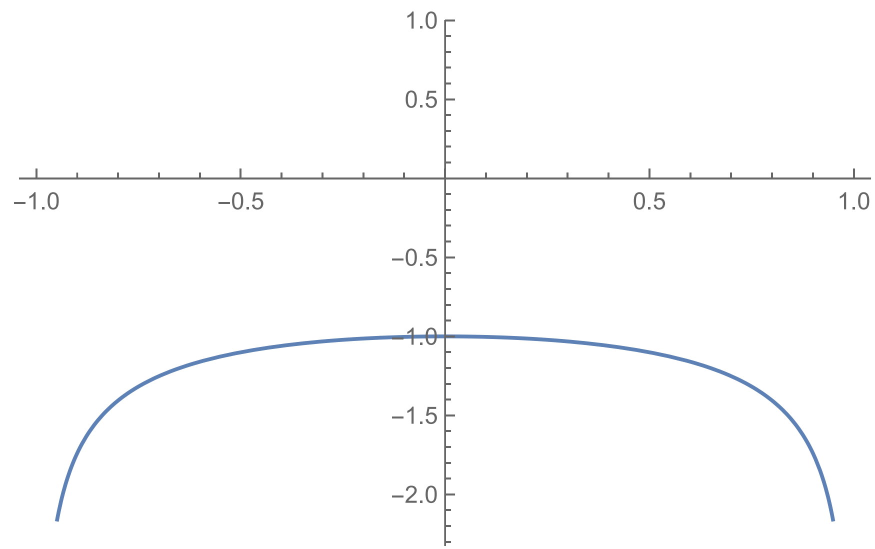
$$\rho(r) = 4 \sum_{n,l,m} |\psi(r)|^2 \;\Rightarrow\; \rho(r) = \frac{2}{3\pi^2}\left(E_f - V(r)\right)^{3/2}$$

Saturation of nuclear binding energy

Effective potential from nucleon density

$$\rho(r) = \frac{A}{\pi^2 r_0^2 \sqrt{r_0^2 - r^2}} \Rightarrow V(r) = E_f - \frac{3^{2/3} A^{2/3}}{2^{2/3} r_0^{4/3} (r_0^2 - r^2)^{1/3}}$$





Saturation of nuclear binding energy

Effective potential from nucleon density

$$\rho(r) = \frac{A}{\pi^2 r_0^2 \sqrt{r_0^2 - r^2}} \quad \Rightarrow \quad V(r) = E_f - \frac{3^{2/3} A^{2/3}}{2^{2/3} r_0^{4/3} (r_0^2 - r^2)^{1/3}}$$

Binding energy (we take $E_f = 0$)

$$B = 4\pi \int r^2 \rho(r) V(r) dr + 2\pi \int r^3 V'(r) dr$$

Potential energy — Kinetic term from virial theorem

Saturation of binding energy

$$\frac{B}{A} = \left[\frac{3^{8/3} \Gamma\left(\frac{7}{6}\right)}{2^{2/3} \sqrt{\pi} \Gamma\left(\frac{2}{3}\right)} - \frac{3^{8/3} \sqrt{\pi}}{2^{2/3} 5 \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{5}{6}\right)} \right] \lambda_r^{1/3}$$

Numerical estimation

Mass of nucleons and Δ ⇐ Energy of w excitations

$$E = (N_c + 2N_f)M + \frac{4\lambda}{M^2} I(I+1)$$

Harmonic oscillator First order perturbation $\lambda\langle w^4 \rangle$

Input $M_N = 939\ \mathrm{MeV}$ $M_\Delta = 1232\ \mathrm{MeV}$

Nuclear radius

$$R = 2.4 \times A^{1/3}\ \mathrm{fm}$$

Binding energy per nucleon

$$\frac{B}{A} = 9.7\ \mathrm{MeV}$$

Experiments

$$R = 1.2 \times A^{1/3}\ \mathrm{fm}$$

$$\frac{B}{A} = 8\ \mathrm{MeV}$$

3. Nuclei in nuclear matrix model

Saturation of nuclear number density (Review of [Hashimoto-Morita,'11])

Eigenstates of nuclear matrix model have similar structure to nuclear shell model ⇒ magic number

Saturation of nuclear binding energy

4. Conclusion

Nuclei can be described by matrix model in holographic QCD

Nuclei = bound states of nucleons = bound states of D-branes

nucleons in QCD ⟺ D-branes (baryon vertex)

correspondence

Effective theory

Nuclear matrix model

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Thank you