Flux Compactification and Naturalness

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Based on papers with Takuya Hirose

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Problems in the Standard Model

- Hierarchy problem
- Unpredictable observables

  (masses of quarks, leptons & Higgs, CP phase, flavor mixing angles, etc)
- Dark matter, Dark energy
- Neutrino oscillation
- Unification
- Gravity
- Charge quantization
- Number of generations
- …

Extension of the SM required

Problems in the Standard Model

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Extension of the SM required

Hierarchy problem

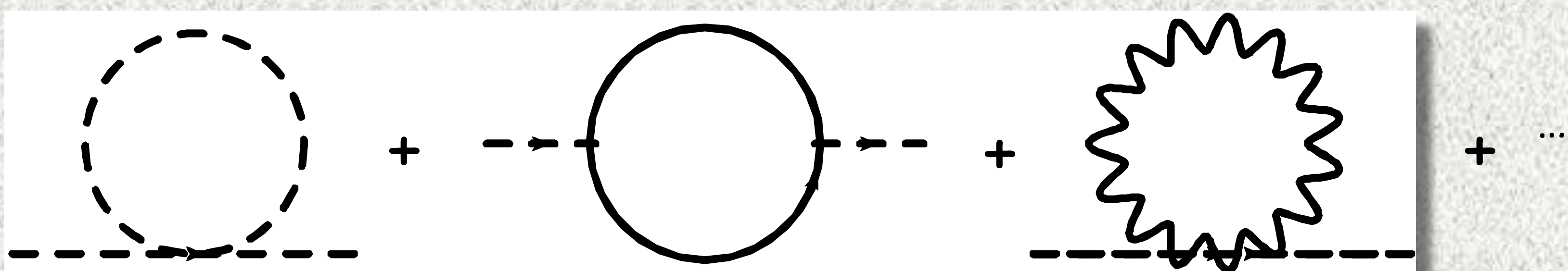
Unnatural fine-tuning of parameters
in the SM Higgs mass

$$m_H^2 = m_0^2 + \delta m^2 \approx \left(125 GeV\right)^2$$

Physical Higgs mass

classical

Quantum corrections



Higgs

Top quark

W、Z

$$m_H^2 = \mathcal{O}\left(M_P^2\right) - \mathcal{O}\left(M_P^2\right) \approx \left(125GeV\right)^2$$

$$\begin{array}{r} 1.0000000000000000000000000000000001 \\ -\ 1.0000000000000000000000000000000000 \end{array}$$

(32digits)

Unnatural fine-tuning!!

Can we explain naturally Higgs mass without unnatural fine-tuning??

$$m_H^2 = \mathcal{O}\left(M_P^2\right) - \mathcal{O}\left(M_P^2\right) \approx \left(125GeV\right)^2$$

1.00000000000000000000000000000001
− 1.00000000000000000000000000000000
(32digits)

New physics around 1TeV expected

$$m_H^2 = \mathcal{O}\left(1TeV^2\right) - \mathcal{O}\left(1TeV^2\right) \approx \left(125GeV\right)^2$$

However, no signature of new physics so far …

Good chance to reconsider??

SM might be correct at higher scale

- GUT $\Rightarrow$ up to 10^{16}GeV
- or up to Planck scale 10^{18}GeV

$m_{Higgs}^2 = 0 @ 10^{16}, 10^{18}$ GeV
including quantum corrections
Possible to generate $m_{Higgs}^2 = 125$GeV
by some mechanism??

example

Magnetic Flux Compactification

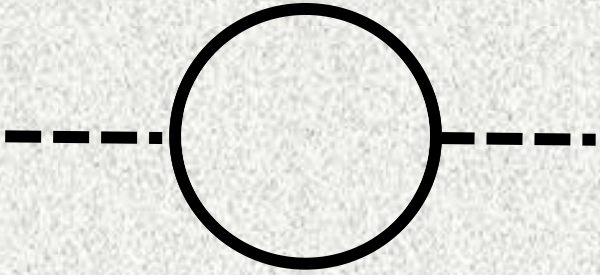
Motivations of Flux Compactification

- Chiral fermions
- 3 generations?
- Yukawa hierarchy
- SUSY breaking
- String phenomenology
- …

6D QED on T^2 with magnetic flux

Buchmuller, Dierigl and Dudas, JHEP04 (2018) 151 [hep-th:1804.07497]

Cancellation of 1-loop corrections to mass of 0 mode scalar $A_{5,6}$ (WL scalar)



= 0 @1-loop

0 mode scalar $A_{5,6}$
= NG boson of $x_{5,6}$ translation

We would like to identify scalar 0 modes $A_{5,6}$ with SM Higgs

Extension of Buchmuller et al's work to Non-abelian gauge theory is necessary

As a first step, extended to SU(2) YM

Hirose & Maru, JHEP1908 (2019) 054

[arXiv:1904.06028]

1. Introduction

2. Cancellation of 1-loop corrections to WL scalar mass in SU(2) Yang-Mills theory

Hirose & Maru, JHEP08 (2019) 054, J. phys. G48 (2021) 055005

3. Nonvanishing finite scalar masses in flux compactification

4. Summary

6D SU(2) Yang-Mills compactified on T^2 with flux

$$\mathcal{L}_6 = -\frac{1}{4} F^a_{MN} F^{aMN} - \frac{1}{2\xi}\left(D_\mu A^{a\mu} + \xi \mathcal{D}_m A^{am}\right)^2 - \bar{c}^a \left(D_\mu D^\mu + \xi D_m \mathcal{D}^m\right) c^a$$

$$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M - ig\left[A_M, A_N\right]^a$$

$$D_M A^a_N = \partial_M A^a_N - ig\left[A_M, A_N\right]^a, \ \mathcal{D}_m A^{am} = \partial_m A^{am} - ig\left[\langle A_m \rangle, A^m\right]^a$$

M=0,1,2,3,5,6
m=5,6, a=1,2,3

flux

$$D^m \langle F_{mn} \rangle = 0 \Rightarrow \langle A^1_5 \rangle = -\frac{1}{2} f x_6, \ \langle A^1_6 \rangle = \frac{1}{2} f x_5, \ \langle A^{2,3}_5 \rangle = \langle A^{2,3}_6 \rangle = 0$$

$$\Rightarrow \langle F^1_{56} \rangle = f, \ \frac{g}{2\pi} \int_{T^2} d^2x f = \frac{g}{2\pi} f L_5 L_6 = N \in \mathbb{Z}$$

(4+2)-dim decomposition

$$\mathcal{L}_{total} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{2\xi}D_\mu A^{a\mu} D_\nu A^{a\nu} - \partial_\mu \bar{\phi}^a \partial^\mu \phi^a$$

$$-\frac{1}{2}\partial A^a_\mu \bar{\partial} A^{a\mu} + g^2\left[A_\mu, \phi\right]^a \left[A^\mu, \phi\right]^a - \frac{g}{\sqrt{2}}\left(-\partial A^a_\mu \left[A^\mu, \bar{\phi}\right]^a + \bar{\partial} A^a_\mu \left[A_\mu, \phi\right]^a\right)$$

$$+ig\left(\partial_\mu \phi^a \left[A^\mu, \bar{\phi}\right]^a + \partial^\mu \bar{\phi}^a \left[A_\mu, \phi\right]^a\right)$$

$$-\frac{1}{4}\left(D\bar{\phi}^a + \bar{D}\phi^a + \sqrt{2}g\left[\phi, \bar{\phi}\right]^a\right)^2 + \frac{\xi}{4}\left(\mathcal{D}\bar{\phi}^a - \bar{\mathcal{D}}\phi^a\right)^2$$

$$-\bar{c}^a\left(D_\mu D^\mu + \xi D_m \mathcal{D}^m\right)c^a$$

$$\partial \equiv \partial_z = \partial_5 - i\partial_6, \quad z \equiv \frac{1}{2}\left(x_5 + i x_6\right), \quad \phi = \frac{1}{\sqrt{2}}\left(A_6 + i A_5\right)$$

Gauge boson mass terms

$$\mathcal{L}_{A^2} = -\frac{1}{2}\partial A_\mu^a \bar{\partial} A^{a\mu} + g^2\left[A_\mu, \langle\phi\rangle\right]^a\left[A^\mu, \langle\bar{\phi}\rangle\right]^a$$

$$-\frac{g}{\sqrt{2}}\left(-\partial A_\mu^a\left[A^\mu, \langle\bar{\phi}\rangle\right]^a + \bar{\partial} A_\mu^a\left[A^\mu, \langle\phi\rangle\right]^a\right)$$

$$= -\frac{1}{2}A_\mu^a\left(-\mathcal{D}\bar{\mathcal{D}}\right)A^{a\mu}$$

$$\mathcal{D} = \begin{pmatrix} \partial & 0 & 0 \\ 0 & \partial & igf\bar{z} \\ 0 & -igf\bar{z} & \partial \end{pmatrix}$$

$$\bar{\mathcal{D}} = \begin{pmatrix} \bar{\partial} & 0 & 0 \\ 0 & \bar{\partial} & -igfz \\ 0 & igfz & \bar{\partial} \end{pmatrix}$$

$$\left[i\bar{\mathcal{D}}, i\mathcal{D}\right] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -2igf \\ 0 & 2igf & 0 \end{pmatrix} = 2igf\,\varepsilon^{a1c}$$

Gauge boson mass terms

$$\mathcal{L}_{A^2} = -\frac{1}{2}\partial A_\mu^a \bar{\partial} A^{a\mu} + g^2\left[A_\mu,\langle\phi\rangle\right]^a\left[A^\mu,\langle\bar{\phi}\rangle\right]^a$$
$$-\frac{g}{\sqrt{2}}\left(-\partial A_\mu^a\left[A^\mu,\langle\bar{\phi}\rangle\right]^a + \bar{\partial} A_\mu^a\left[A^\mu,\langle\phi\rangle\right]^a\right)$$
$$= -\frac{1}{2}A_\mu^a\left(-\mathcal{D}\bar{\mathcal{D}}\right)A^{a\mu}$$

$$\mathcal{D}_{diag} = \begin{pmatrix} \partial & 0 & 0 \\ 0 & \partial - gf\bar{z} & 0 \\ 0 & 0 & \partial + gf\bar{z} \end{pmatrix}$$

$$\bar{\mathcal{D}}_{diag} = \begin{pmatrix} \bar{\partial} & 0 & 0 \\ 0 & \bar{\partial} + gfz & 0 \\ 0 & 0 & \bar{\partial} - gfz \end{pmatrix}$$

$$\Rightarrow$$

$$a = \frac{1}{\sqrt{2gf}} i\bar{\mathcal{D}}, \quad a^\dagger = \frac{1}{\sqrt{2gf}} i\mathcal{D}$$

$$\left[a, a^\dagger\right] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

creation, annihilation operators

Gauge boson mass terms

$$\mathcal{L}_{A^2} = -\frac{1}{2}\partial A_\mu^a \bar{\partial} A^{a\mu} + g^2 \left[A_\mu, \langle \phi \rangle\right]^a \left[A^\mu, \langle \bar{\phi} \rangle\right]^a$$

$$-\frac{g}{\sqrt{2}}\left(-\partial A_\mu^a \left[A^\mu, \langle \bar{\phi} \rangle\right]^a + \bar{\partial} A_\mu^a \left[A^\mu, \langle \phi \rangle\right]^a\right)$$

$$= -\frac{1}{2} A_\mu^a \left(-\mathcal{D}\bar{\mathcal{D}}\right) A^{a\mu}$$

$$\begin{cases} a_1 = \dfrac{1}{\sqrt{2gf}} i\bar{\partial} \\ a_2 = \dfrac{1}{\sqrt{2gf}} i\left(\bar{\partial} + gfz\right) \\ a_3 = \dfrac{1}{\sqrt{2gf}} i\left(\bar{\partial} - gfz\right) \end{cases}, \quad \begin{cases} a_1^\dagger = \dfrac{1}{\sqrt{2gf}} i\partial \\ a_2^\dagger = \dfrac{1}{\sqrt{2gf}} i\left(\partial - gf\bar{z}\right) \\ a_3^\dagger = \dfrac{1}{\sqrt{2gf}} i\left(\partial + gf\bar{z}\right) \end{cases}$$

$$\left[a_1, a_1^\dagger\right] = 0$$

$$\left[a_2, a_2^\dagger\right] = 1$$

$$\left[a_3, a_3^\dagger\right] = -1$$

Gauge boson mass terms

$$\mathcal{L}_{A^2} = -\frac{1}{2}\partial A_\mu^a \bar{\partial} A^{a\mu} + g^2 \left[A_\mu, \langle\phi\rangle\right]^a \left[A^\mu, \langle\bar{\phi}\rangle\right]^a$$
$$-\frac{g}{\sqrt{2}}\left(-\partial A_\mu^a \left[A^\mu, \langle\bar{\phi}\rangle\right]^a + \bar{\partial} A_\mu^a \left[A^\mu, \langle\phi\rangle\right]^a\right)$$
$$= -\frac{1}{2} A_\mu^a \left(-\mathcal{D}\bar{\mathcal{D}}\right) A^{a\mu}$$

Diagonalized mass matrix

$$\left(m_A^2\right)_{diag} = -\mathcal{D}_{diag}\bar{\mathcal{D}}_{diag} = 2gf\begin{pmatrix} n_1 & 0 & 0 \\ 0 & n_2 & 0 \\ 0 & 0 & n_3+1 \end{pmatrix}$$

$n_{2,3}$: Landau level

Scalar mass & ghost mass

$$\mathcal{L}_{\varphi\varphi} = -\frac{1}{4}\left(\mathcal{D}\bar{\varphi}^a\mathcal{D}\bar{\varphi}^a + \mathcal{D}\bar{\varphi}^a\bar{\mathcal{D}}\varphi^a + \bar{\mathcal{D}}\varphi^a\mathcal{D}\bar{\varphi}^a + \bar{\mathcal{D}}\varphi^a\bar{\mathcal{D}}\varphi^a - 4gf\left[\varphi,\bar{\varphi}\right]^1\right)$$
$$-\frac{\xi}{4}\left(\mathcal{D}\bar{\varphi}^a\mathcal{D}\bar{\varphi}^a - \mathcal{D}\bar{\varphi}^a\bar{\mathcal{D}}\varphi^a - \bar{\mathcal{D}}\varphi^a\mathcal{D}\bar{\varphi}^a + \bar{\mathcal{D}}\varphi^a\bar{\mathcal{D}}\varphi^a\right)$$

$$\mathcal{L}_{cc} = -\bar{c}^a\xi\mathcal{D}_m\mathcal{D}^m c^a \qquad \phi^a = \left\langle\phi^a\right\rangle + \varphi^a$$

$$\left(m_\varphi^2\right)_{diag} = gf\begin{pmatrix} (1+\xi)n_1 & 0 & 0 \\ 0 & (1+\xi)n_2+1 & 0 \\ 0 & 0 & (1+\xi)n_3+\xi \end{pmatrix}$$

$$\left(m_c^2\right)_{diag} = 2gf\xi\begin{pmatrix} n_1 & 0 & 0 \\ 0 & n_2+1/2 & 0 \\ 0 & 0 & n_3+1/2 \end{pmatrix}$$

4D effective theory

$$\mathcal{L}_{total} = -\frac{1}{4}\tilde{F}^a_{\mu\nu}\tilde{F}^{a\mu\nu} - \partial_\mu \bar{\tilde{\varphi}}^a \partial^\mu \tilde{\varphi}^a - \bar{\tilde{c}}^a \mathcal{D}_\mu \mathcal{D}^\mu \tilde{c}^a \qquad \phi^a = \langle \phi^a \rangle + \varphi^a$$

$$-\frac{1}{2}\tilde{A}^a_\mu m_A^2 \tilde{A}^{a\mu} - \bar{\tilde{\varphi}}^a m_\varphi^2 \tilde{\varphi}^a - \bar{\tilde{c}}^a m_c^2 \tilde{c}^a$$

Interaction terms

$$+ig\left(\partial_\mu \varphi^a \left[A^\mu, \varphi\right]^a + \partial^\mu \bar{\varphi}^a \left[A_\mu, \varphi\right]^a\right) + g^2 \left[A_\mu, \varphi\right]^a \left[A^\mu, \varphi\right]^a$$

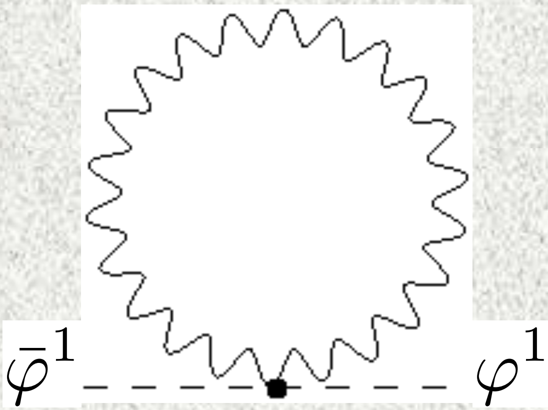
$$+\frac{g}{\sqrt{2}}\left(\mathcal{D}\bar{\varphi}^a - \bar{\mathcal{D}}\varphi^a\right)[\varphi, \bar{\varphi}]^a - \frac{1}{2}g^2 [\varphi, \bar{\varphi}]^a [\varphi, \bar{\varphi}]^a$$

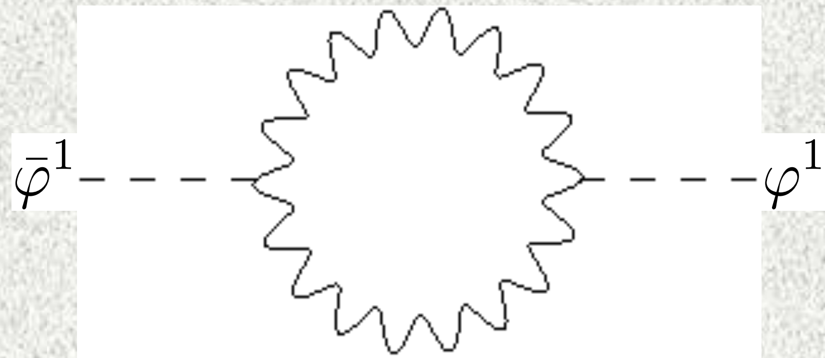
$$-\frac{g\xi}{\sqrt{2}}\left([\varphi, \bar{c}]^a \bar{\partial} c^a - [\bar{\varphi}, \bar{c}]^a \partial c^a\right)$$

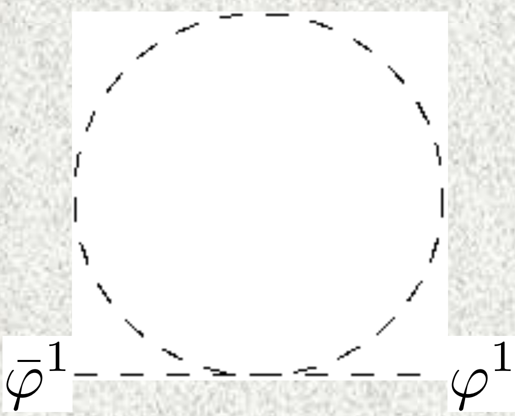
$$\tilde{A}^a_\mu = U A^a_\mu, \quad \tilde{A}^{a\mu} = U^{-1} A^{a\mu},$$

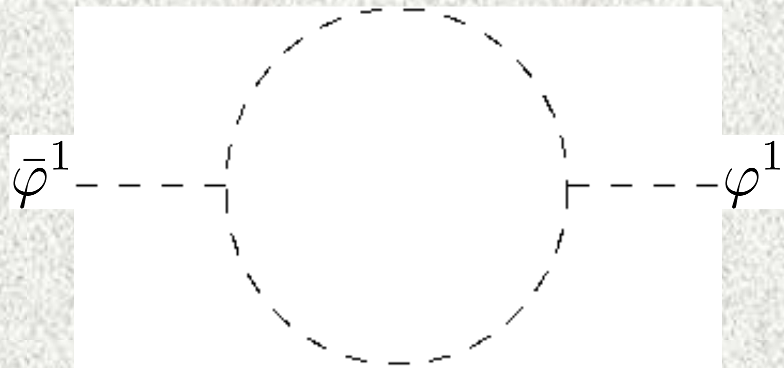
$$\tilde{\varphi} = U^{-1}\varphi, \quad \bar{\tilde{\varphi}} = U\bar{\varphi}, \quad \bar{\tilde{c}}^a = U\bar{c}^a, \quad \tilde{c}^a = U^{-1} c^a \qquad U = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & i \\ 0 & i & 1 \end{pmatrix}$$

Diagrams to be calculated

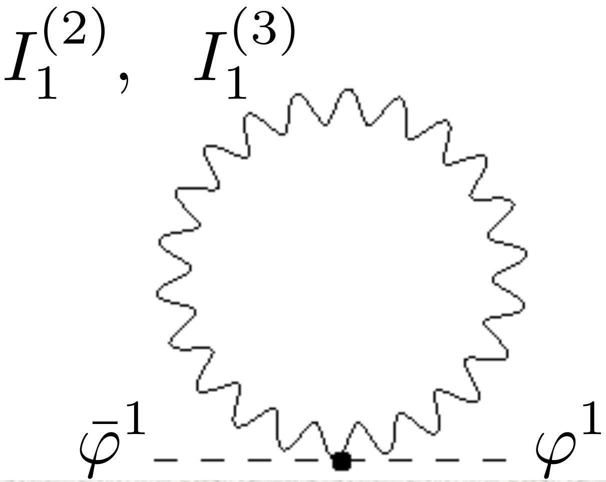


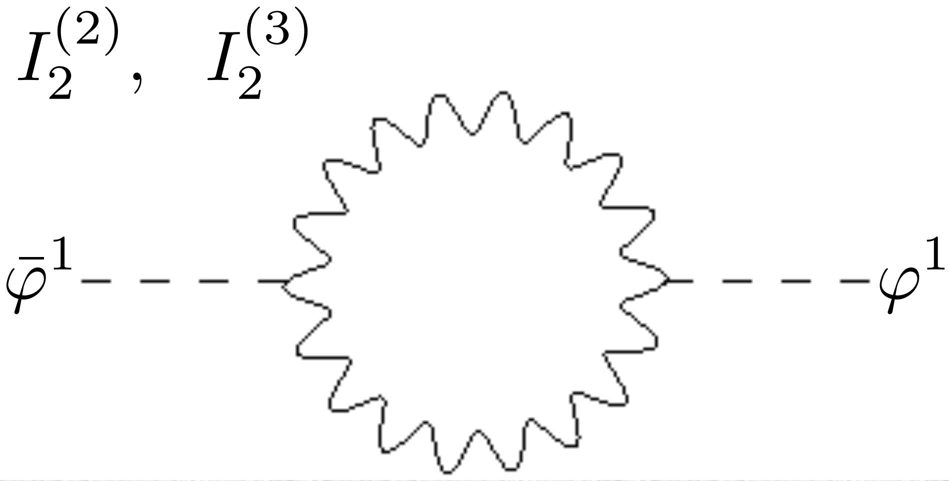






Gauge boson loop





$$I_1^{(2)} + I_2^{(2)} = -6ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\alpha n} - \frac{\alpha(n+1)}{(p^2+\alpha n)(p^2+\alpha(n+1))}\right)$$

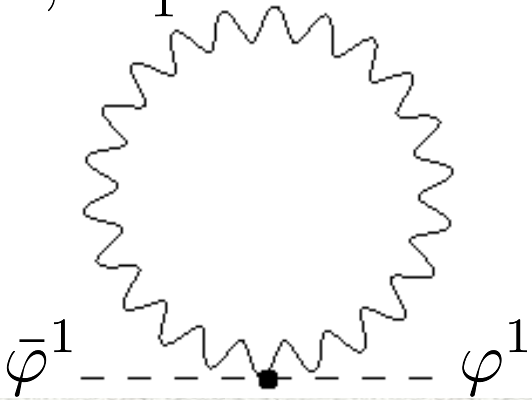
$$-2ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{\xi}{p^2+\alpha n} - \frac{\alpha(n+1)\xi^2}{(p^2+\alpha n\xi)(p^2+\alpha(n+1)\xi)}\right)$$

$$I_1^{(3)} + I_2^{(3)} = -6ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\alpha(n+1)} - \frac{\alpha(n+1)}{(p^2+\alpha(n+1))(p^2+\alpha(n+2))}\right)$$

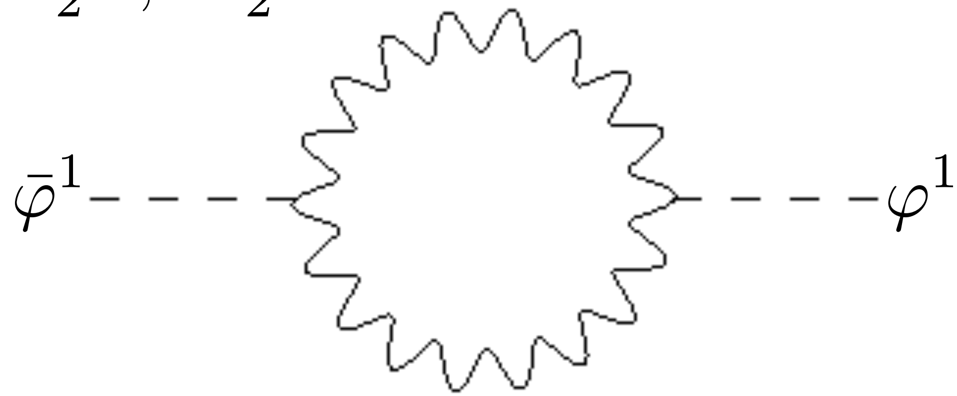
$$-2ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{\xi}{p^2+\alpha(n+1)} - \frac{\alpha(n+1)\xi^2}{(p^2+\alpha(n+1)\xi)(p^2+\alpha(n+2)\xi)}\right)$$

Gauge boson loop

$I_1^{(2)},\quad I_1^{(3)}$



$I_2^{(2)},\quad I_2^{(3)}$



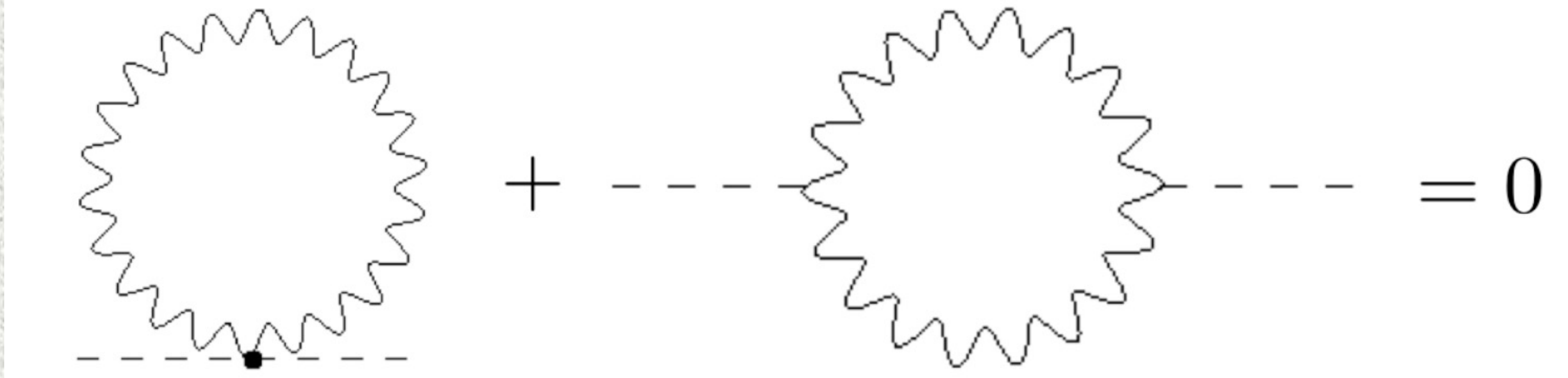
$$I_1^{(2)}+I_2^{(2)}=-6ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\alpha n}-\frac{\alpha(n+1)}{\left(p^2+\alpha n\right)\left(p^2+\alpha(n+1)\right)}\right)$$

$$-2ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{\xi}{p^2+\alpha n\xi}-\frac{\alpha(n+1)\xi^2}{\left(p^2+\alpha n\xi\right)\left(p^2+\alpha(n+1)\xi\right)}\right)$$

$$\sum_{n=0}^{\infty}\left(\frac{1}{p^2+\alpha n}-\frac{\alpha(n+1)}{\left(p^2+\alpha n\right)\left(p^2+\alpha(n+1)\right)}\right)=\sum_{n=0}^{\infty}\left(-\frac{n}{p^2+\alpha n}+\frac{n+1}{p^2+\alpha(n+1)}\right)=0$$

2nd line is also 0 after $p^2\to p^2\xi$ $I_1^{(3)}+I_2^{(3)}$ is applied, too

Gauge boson loop

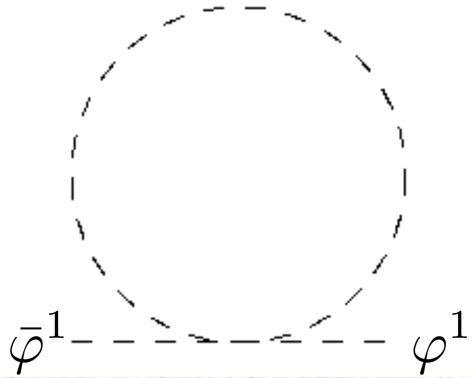


$$I_1^{(2)} + I_2^{(2)} = 0, \quad I_1^{(3)} + I_2^{(3)} = 0$$

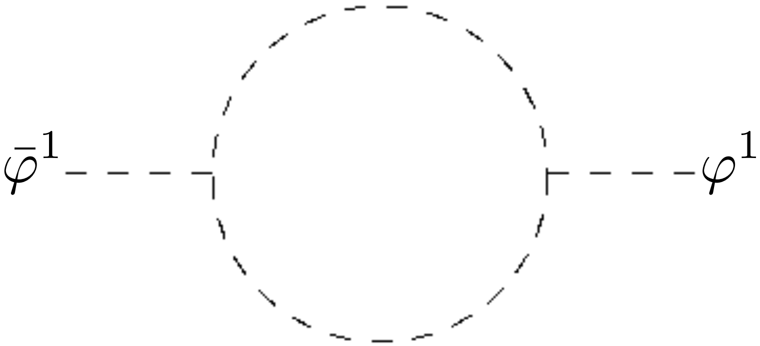
Cancelled in arbitrary gauge ξ !!

Scalar loop

$I_3^{(2)}, \quad I_3^{(3)}$



$I_4^{(2)}, \quad I_4^{(3)}$

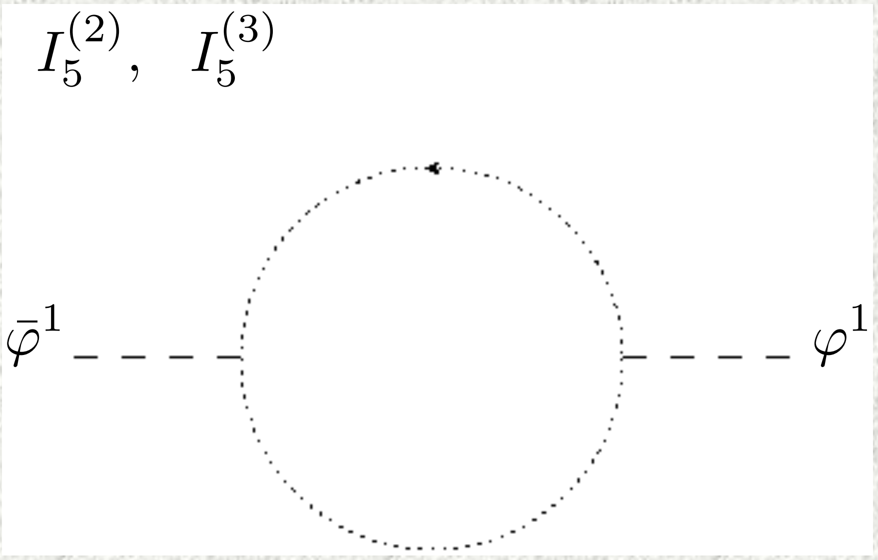


$$I_3^{(2)} = -ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+\frac{\alpha}{2}\left((1+\xi)n+1\right)}, \; I_3^{(3)} = -ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+\frac{\alpha}{2}\left((1+\xi)n+\xi\right)}$$

$$I_4^{(2)} = \frac{ig^2|N|}{2}\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)}{\left(p^2+\frac{\alpha}{2}\left((1+\xi)n+1\right)\right)\left(p^2+\frac{\alpha}{2}\left((1+\xi)(n+1)+1\right)\right)}$$

$$I_4^{(3)} = \frac{ig^2|N|}{2}\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)}{\left(p^2+\frac{\alpha}{2}\left((1+\xi)n+\xi\right)\right)\left(p^2+\frac{\alpha}{2}\left((1+\xi)(n+1)+\xi\right)\right)}$$

Ghost loop



$$I_5^{(2)} = I_5^{(3)} = \frac{ig^2|N|\xi^2}{2}\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)}{\left(p^2+\alpha(n+1/2)\right)\left(p^2+\alpha(n+3/2)\right)}$$

Cancellation between scalar loop & ghost loop

Landau gauge: ξ=0 ⇒ ghost loop trivially zero

Scalar loop

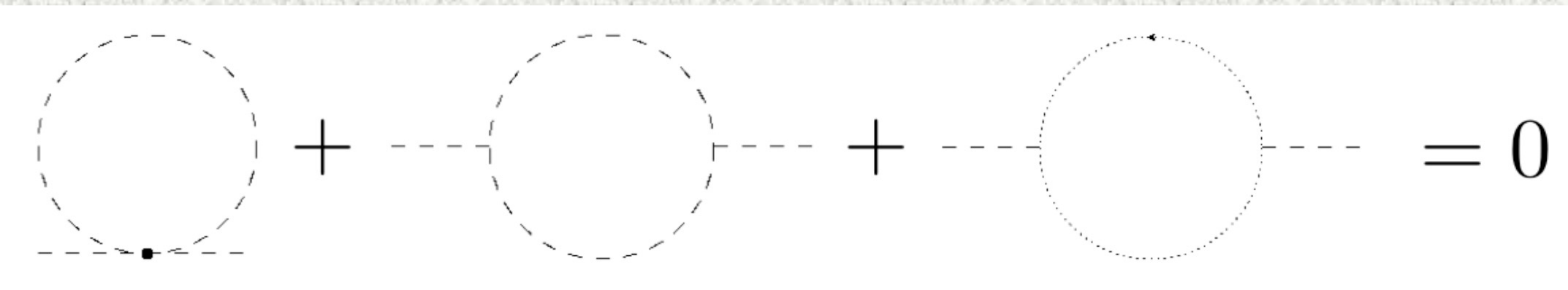
$$I_3^{(2)}+I_4^{(2)}=-ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\frac{\alpha}{2}(n+1)}-\frac{\frac{\alpha}{2}(n+1)}{\left(p^2+\frac{\alpha}{2}(n+1)\right)\left(p^2+\frac{\alpha}{2}(n+2)\right)}\right)$$

$$=-ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(-\frac{n}{p^2+\frac{\alpha}{2}(n+1)}+\frac{n+1}{p^2+\frac{\alpha}{2}(n+2)}\right)=0$$

$$I_3^{(3)}+I_4^{(3)}=-ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\frac{\alpha}{2}n}-\frac{\frac{\alpha}{2}(n+1)}{\left(p^2+\frac{\alpha}{2}n\right)\left(p^2+\frac{\alpha}{2}(n+1)\right)}\right)$$

$$=-ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(-\frac{n}{p^2+\frac{\alpha}{2}n}+\frac{n+1}{p^2+\frac{\alpha}{2}(n+1)}\right)=0$$

Feynman gauge: ξ=1



Scalar loop

Ghost loop

$$I_3^{(2)}+I_4^{(2)}+I_5^{(2)}=I_3^{(3)}+I_4^{(3)}+I_5^{(3)}$$

$$=-ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\alpha\left(n+\frac{1}{2}\right)}-\frac{\frac{\alpha}{2}(n+1)}{\left(p^2+\alpha\left(n+\frac{1}{2}\right)\right)\left(p^2+\alpha\left(n+\frac{3}{2}\right)\right)}-\frac{\frac{\alpha}{2}(n+1)}{\left(p^2+\alpha\left(n+\frac{1}{2}\right)\right)\left(p^2+\alpha\left(n+\frac{3}{2}\right)\right)}\right)$$

$$=-ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\alpha\left(n+\frac{1}{2}\right)}-\frac{\alpha(n+1)}{\left(p^2+\alpha\left(n+\frac{1}{2}\right)\right)\left(p^2+\alpha\left(n+\frac{3}{2}\right)\right)}\right)$$

$$=-ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(-\frac{n}{p^2+\alpha\left(n+\frac{1}{2}\right)}+\frac{n+1}{p^2+\alpha\left(n+\frac{3}{2}\right)}\right)=0$$

Physical reason of cancellation

Translations in compactified space

$$\delta_T A_5^a = \left(\varepsilon_5 \partial_5 + \varepsilon_6 \partial_6\right) \tilde{A}_5^a - \frac{f}{2} \varepsilon_6 \delta^{a1}$$

$$\delta_T A_6^a = \left(\varepsilon_5 \partial_5 + \varepsilon_6 \partial_6\right) \tilde{A}_6^a + \frac{f}{2} \varepsilon_5 \delta^{a1}$$

$$\delta_T \phi^a = \left(\varepsilon \partial + \bar{\varepsilon} \bar{\partial}\right) \varphi^a + \frac{f}{\sqrt{2}} \bar{\varepsilon} \delta^{a1}$$

a=1
0 mode

$$\delta_T \phi^1 = \frac{f}{\sqrt{2}} \bar{\varepsilon}$$

$$\partial \equiv \partial_5 - i \partial_6, \phi^a = \frac{1}{\sqrt{2}}\left(A_6^a + i A_5^a\right) = \frac{f}{\sqrt{2}} \bar{z} \delta^{a1} + \varphi^a$$

constant shift

$$\delta_T \phi^1 = \frac{f}{\sqrt{2}} \bar{\varepsilon}$$

constant shift

⇒ φ^1 NG boson of spontaneously broken translational symmetry in 5,6 directions

⇒ Only derivative interactions are allowed

(Not only mass terms, but also potential forbidden)

Analogy: π meson

NG boson of spontaneously broken chiral symmetry ⇒ chiral Lagrangian

We have also shown a cancellation
in a presence of higher dimensional operators

Hirose & Maru, J.Phys.G 48 (2021) 5 [2012.03494]

$$\mathcal{L} = -\frac{1}{4}F_{MN}F^{MN} + \frac{1}{\Lambda^2}\mathcal{O}_1(D,F) + \frac{1}{\Lambda^4}\mathcal{O}_2(D,F) + \frac{1}{\Lambda^6}\mathcal{O}_3(D,F) + \cdots$$

Only dim-6 operators are considered in our paper

$$\mathcal{O}_1 = Tr\left[D_L D^L D_M D_N F^{MN}\right] + 2Tr\left[D_L F_{MN} D^L F^{MN}\right] + \varepsilon^{M_1N_1M_2N_2M_3N_3} Tr\left[F_{M_1N_1}F_{M_2N_2}F_{M_3N_3}\right]$$

Reason of cancellation is very simple:
WL scalar appears in a commutator

$$[\quad ,\varphi]$$ invariant under a constant shift

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   Hirose & Maru, JHEP06 (2021) 159
4. Summary

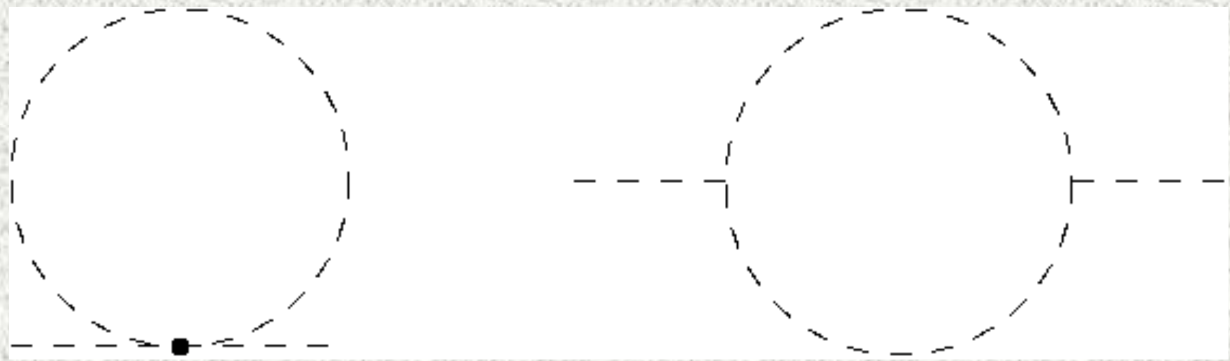
Pion has a mass since it is a pseudo NG boson of explicitly broken chiral symmetry by quark mass terms

To obtain WL scalar mass, it has to be a pseudo NG boson of explicitly broken translational symmetry in compactified space, too

↓

What are the explicit breaking terms, corresponding to quark mass terms in pion case ??

Divergence structure of the loop integral & mode sum



$$I(x;a,b)=\sum_{n=0}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{\left(k^2\right)^a}{\left(k^2+\alpha(n+x)\right)^b}$$

$$=\frac{1}{\alpha^{b-a}}\left(\frac{4\pi}{\alpha}\right)^{\varepsilon-2}\frac{\Gamma(a+2-\varepsilon)\Gamma(\varepsilon+b-a-2)}{\Gamma(b)\Gamma(2-\varepsilon)}\zeta[\varepsilon+b-a-2,x]$$

x: parameter specifying KK mass spectrum

(x=0(KK gauge), ½(KK scalar), 1(KK fermion))

2a: number of derivatives

b: number of propagators (only b=1,2 cases considered)

$$J(x;a,b)=\frac{\Gamma(a+2-\varepsilon)\Gamma(\varepsilon+b-a-2)}{\Gamma(b)\Gamma(2-\varepsilon)}\zeta[\varepsilon+b-a-2,x]$$

$$=\begin{cases}(-1)^a\,\Gamma(\varepsilon-1)\zeta[\varepsilon-a-1,x](b=1)\\(-1)^a(\varepsilon-a-1)\Gamma(\varepsilon-1)\zeta[\varepsilon-a,x](b=2)\end{cases}$$

$$\Gamma(\varepsilon-1)=\frac{\Gamma(\varepsilon)}{\varepsilon-1}=-\left(\frac{1}{\varepsilon}-\gamma_E+1+\mathcal{O}(\varepsilon)\right)$$

$$\zeta[\varepsilon-p,x]=\zeta[-p,x]+\frac{\partial\zeta[-p,x]}{\partial(-p)}\varepsilon+\mathcal{O}(\varepsilon^2)$$

$$\zeta[-2n,1]=\zeta[-2n,1/2]=\zeta[-2n,0]=0$$

$$\Gamma(\varepsilon-1)\zeta(\varepsilon-p,x)=\mathit{finite}(p=even)$$

J(x;a,1) for odd a, J(x;a,2) for even a are finite

1-loop finite corrections by 4-point interaction

Scalar loop

$$J\left(1/2;a,1\right)\rightarrow\bar{\varphi}\varphi\,\partial_{\mu_1}\cdots\partial_{\mu_a}\bar{\Phi}\,\partial^{\mu_1}\cdots\partial^{\mu_a}\Phi$$

Fermion loop

$$J\left(1;a,1\right)\rightarrow\bar{\varphi}\varphi\bar{\psi}\left(\gamma^{\mu}\partial_{\mu}\right)^{2a-1}\psi$$

SU(2) gauge loop

$$J\left(1/2;a,1\right)\rightarrow\bar{\varphi}\varphi\,\partial_{\mu_1}\cdots\partial_{\mu_a}A_{\nu}^{a}\,\partial^{\mu_1}\cdots\partial^{\mu_a}A^{a\nu}$$

1-loop finite corrections by 3-point interactions

Scalar loop

$$J\left(1/2;0,2\right)\rightarrow \bar{\varphi}\bar{\Phi}\Phi+\varphi\bar{\Phi}\Phi$$

$$J\left(1/2;a,2\right)\rightarrow \bar{\varphi}\,\partial_{\mu_1}\cdots\partial_{\mu_{a/2}}\bar{\Phi}\,\partial^{\mu_1}\cdots\partial^{\mu_{a/2}}\Phi+\varphi\,\partial_{\mu_1}\cdots\partial_{\mu_{a/2}}\bar{\Phi}\,\partial^{\mu_1}\cdots\partial^{\mu_{a/2}}\Phi$$

Fermion loop

$$J\left(1;a,2\right)\rightarrow \bar{\varphi}\bar{\psi}\left(\gamma^{\mu}\partial_{\mu}\right)^{a-1}\psi+\varphi\bar{\psi}\left(\gamma^{\mu}\partial_{\mu}\right)^{a-1}\psi$$

SU(2) gauge loop

$$J\left(1/2;a,2\right)\rightarrow \bar{\varphi}\,\partial_{\mu_1}\cdots\partial_{\mu_{a/2}}A^{a}_{\nu}\,\partial^{\mu_1}\cdots\partial^{\mu_{a/2}}A^{a\nu}+\varphi\,\partial_{\mu_1}\cdots\partial_{\mu_{a/2}}A^{a}_{\nu}\,\partial^{\mu_1}\cdots\partial^{\mu_{a/2}}A^{a\nu}$$

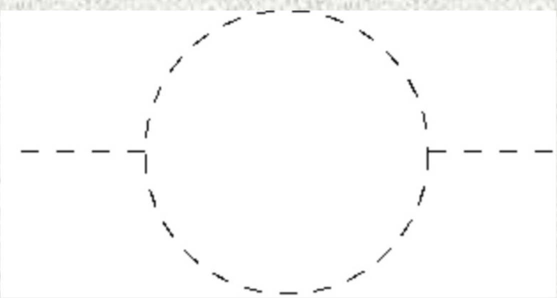
Illustration of finite WL scalar mass

6D scalar QED on T^2 with flux

$$\mathcal{L} = -\frac{1}{4} F_{MN} F^{MN} - D_M \bar{\Phi} D^M \Phi + \kappa \left(\phi \bar{\Phi} \Phi + \overline{\phi \Phi \Phi} \right)$$

$$\phi = \langle \phi \rangle + \varphi$$

New contribution from κ interactions



$$\delta m^2 = \frac{|N| \ln 2}{32\pi^2} \frac{\kappa^2}{L^2}$$

Finite as expected!!

- $\kappa = 0 \Rightarrow \delta m^2 = 0$
- same result is derived from V_{eff}@1-loop

If we regard $$\delta m^2 = \frac{|N| \ln 2}{32\pi^2} \frac{\kappa^2}{L^2}$$ as M_{Higgs}^2

$$\Rightarrow \quad \frac{\kappa}{L} \approx TeV$$

Even if 1/L is Planck scale,

$\kappa\left(\phi\bar{\Phi}\Phi + \overline{\phi\Phi}\Phi\right)$ is generated as $\kappa \sim$ TeV/Planck

by some dynamics, M_{Higgs}^2 is obtained

On the gauge invariance of interaction $\varphi\bar{\Phi}\Phi$

Gauge invariance?? $\because \quad \delta\varphi \propto \partial\Lambda\left(z,\bar{z}\right)$

$\Rightarrow \quad \varphi,\bar{\varphi}$ should be expressed by
a gauge invariant non-local Wilson line operator

$$U_5 = \exp\left[ig\oint dx^5 A_5\right] = \exp\left[\frac{g}{\sqrt{2}}\oint\left(\varphi dz + \varphi d\bar{z} - \bar{\varphi} dz - \bar{\varphi} d\bar{z}\right)\right]$$

$$U_6 = \exp\left[ig\oint dx^6 A_6\right] = \exp\left[\frac{g}{\sqrt{2}}\oint\left(\varphi dz - \varphi d\bar{z} + \bar{\varphi} dz - \bar{\varphi} d\bar{z}\right)\right]$$

$\varphi\bar{\Phi}\Phi, \bar{\varphi}\bar{\Phi}\Phi$ can be obtained by expansion

$$i\left(U_5 - U_5^\dagger\right)\bar{\Phi}\Phi - i\left(U_6 - U_6^\dagger\right)\bar{\Phi}\Phi \supset ig_4\varphi\bar{\Phi}\Phi - ig_4\bar{\varphi}\bar{\Phi}\Phi$$

$\varphi\bar{\Phi}\Phi, \bar{\varphi}\bar{\Phi}\Phi$ can be obtained by expansion

$$i\left(U_5 - U_5^\dagger\right)\bar{\Phi}\Phi - i\left(U_6 - U_6^\dagger\right)\bar{\Phi}\Phi \supset ig_4\varphi\bar{\Phi}\Phi - ig_4\bar{\varphi}\bar{\Phi}\Phi$$

$$U_5 - U_5^\dagger = 2i\sin\left[g\oint dx^5 A_5\right], U_6 - U_6^\dagger = 2i\sin\left[g\oint dx^6 A_6\right]$$

are not obviously invariant

under the constant shift symmetry

$$A_5 \to A_5 - f\varepsilon_6/2,\ A_6 \to A_6 + f\varepsilon_5/2$$

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Summary

- No signature of New physics
  → SM might be correct up to Planck scale
  → reconsider the hierarchy problem??
- Scalar mass = 0@M_P is favorable
  → flux compactification
- In 6D SU(2)YM with flux compactification,
  WL scalar mass@1-loop=0 was shown
- Gauge loop → in arbitrary gauge
  Scalar loop + ghost loop →
  shown in Landau & Feynman gauge

Summary

- Masslessness
  $\Rightarrow$ WL scalar = NG boson of translation
  in compactified space
- To identify WL scalar with SM Higgs
  $\Rightarrow$ WL scalar should be a pseudo NG boson
  $\Rightarrow$ mass & potential are generated
- Classification of explicit breaking terms
  of translational symmetry providing
  finite WL scalar mass@ 1-loop

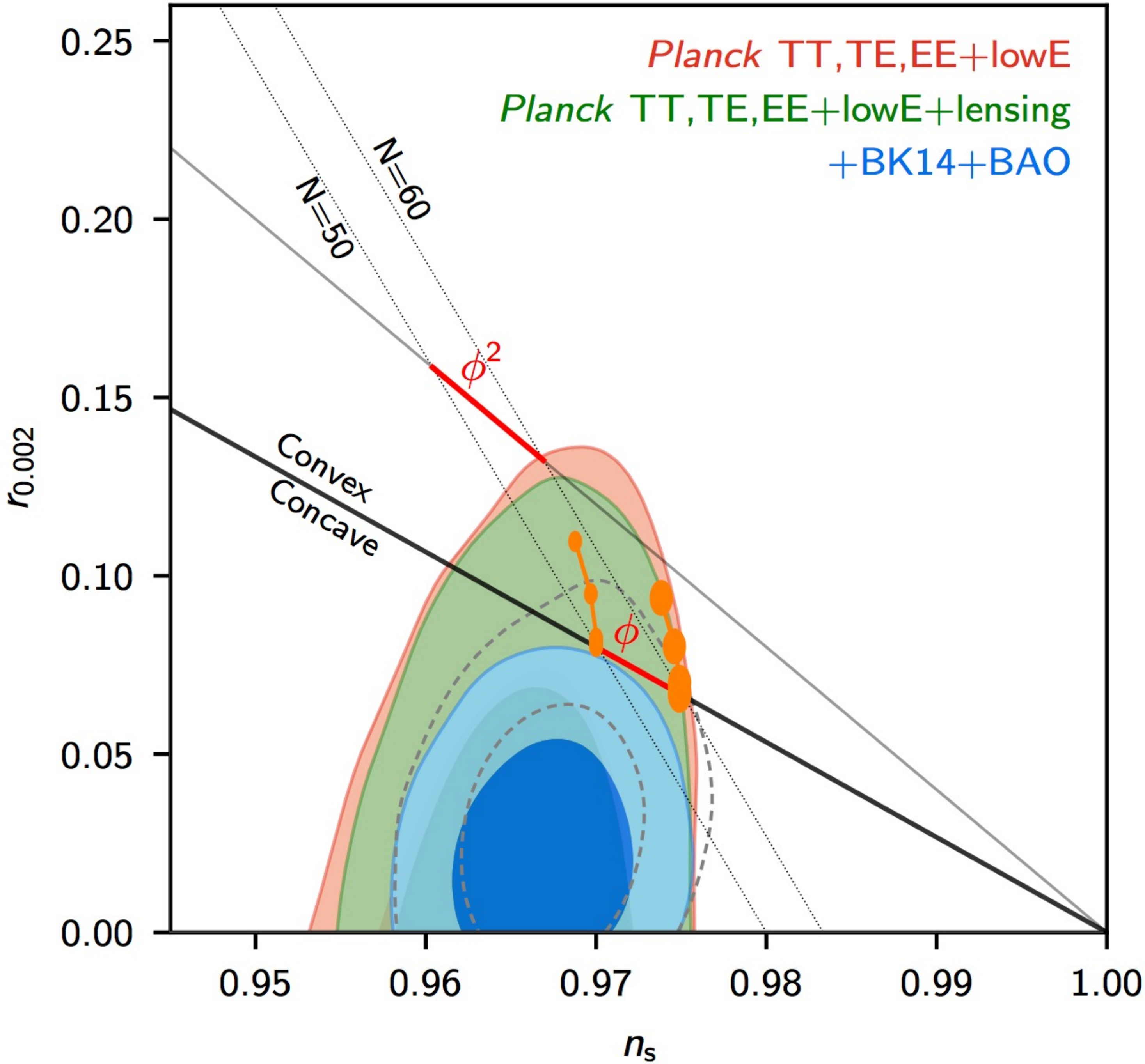
Summary

- Finite WL scalar mass@1-loop
  was obtained in 6D scalar QED
- Origin of explicit breaking terms??

  Quantum gravity effects?
  Nontrivial background as vortex?
  Dynamical SUSY breaking??
- Our analysis can be applicable to
  other scalar dynamics

  $\Rightarrow$ WL scalar as inflaton

  Hirose & Maru, 2105.11782[hep-th]



Planck TT,TE,EE+lowE
Planck TT,TE,EE+lowE+lensing
+BK14+BAO
N=50
N=60
ϕ^2
ϕ
Convex
Concave
$r_{0.002}$
n_s
0.00
0.05
0.10
0.15
0.20
0.25
0.95
0.96
0.97
0.98
0.99
1.00

Thank you!!