Can QCD Axion explain the CMB Anisotropy ?

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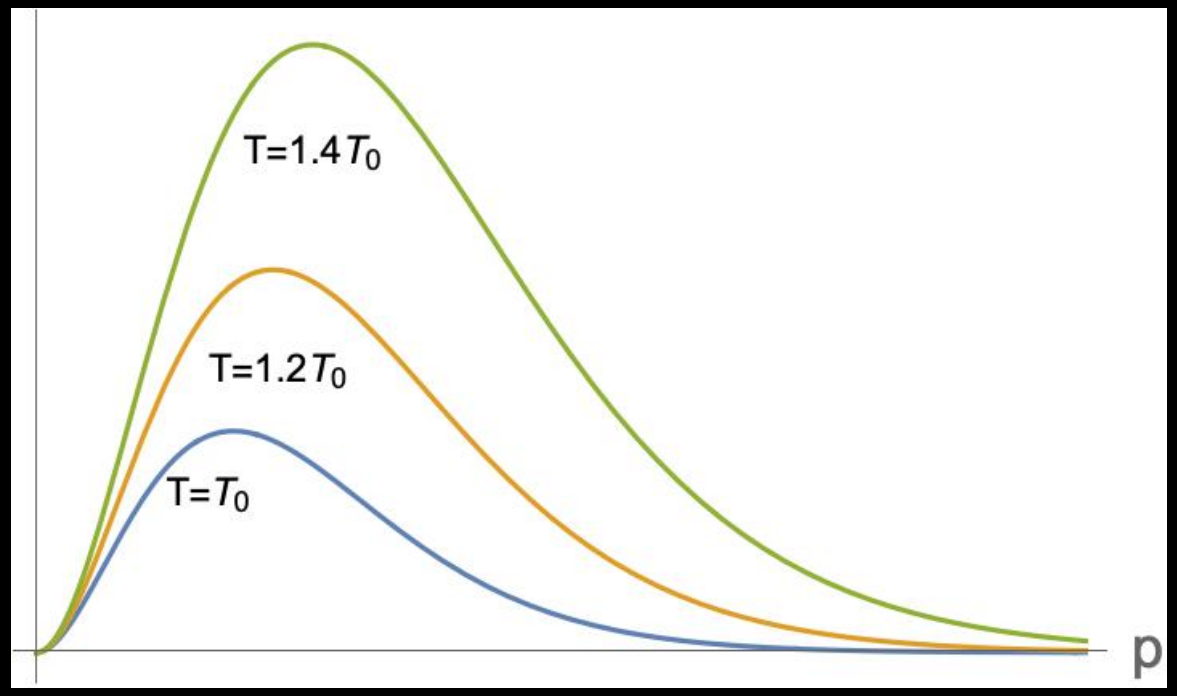
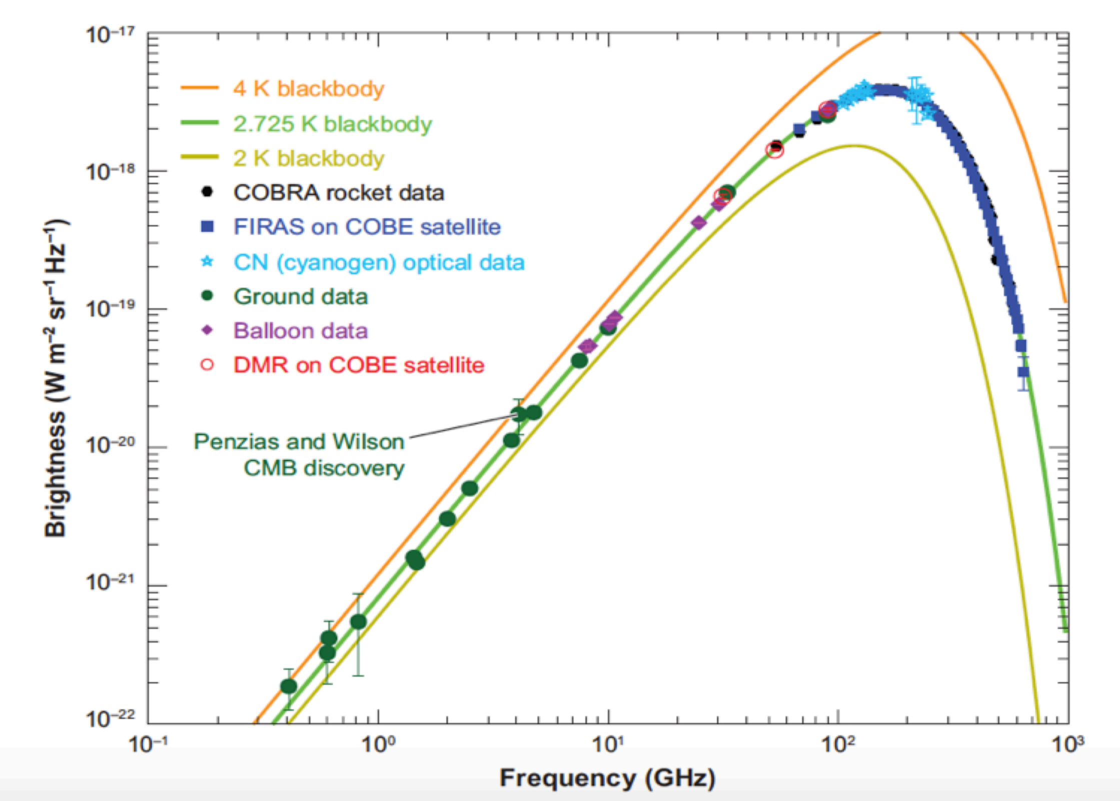
Based on Phys. Rev. D 102, 103513 (2020), 2007.06802 and 2105. 06803
Collaboration with Iso Satoshi (KEK) and Kengo Shimada (KEK)

Cosmic Microwave Backgrounds





- The Universe is filled with photons = Cosmic Microwave Backgrounds (CMB)
- Its energy spectrum is well fitted by Planck distribution with temperature T_0=2.73K

$$\frac{dn}{dp} = \frac{1}{2\pi^2}\frac{p^2}{e^{p/T_0}-1}$$

- The expansion of the Universe tells us that our universe was hotter and SM particles were in thermal equilibrium at early Universe = Big Bang
- Besides, the CMB temperature does not much depend on direction in the sky
  → One of the evidences of almost homogeneous and isotropic Universe

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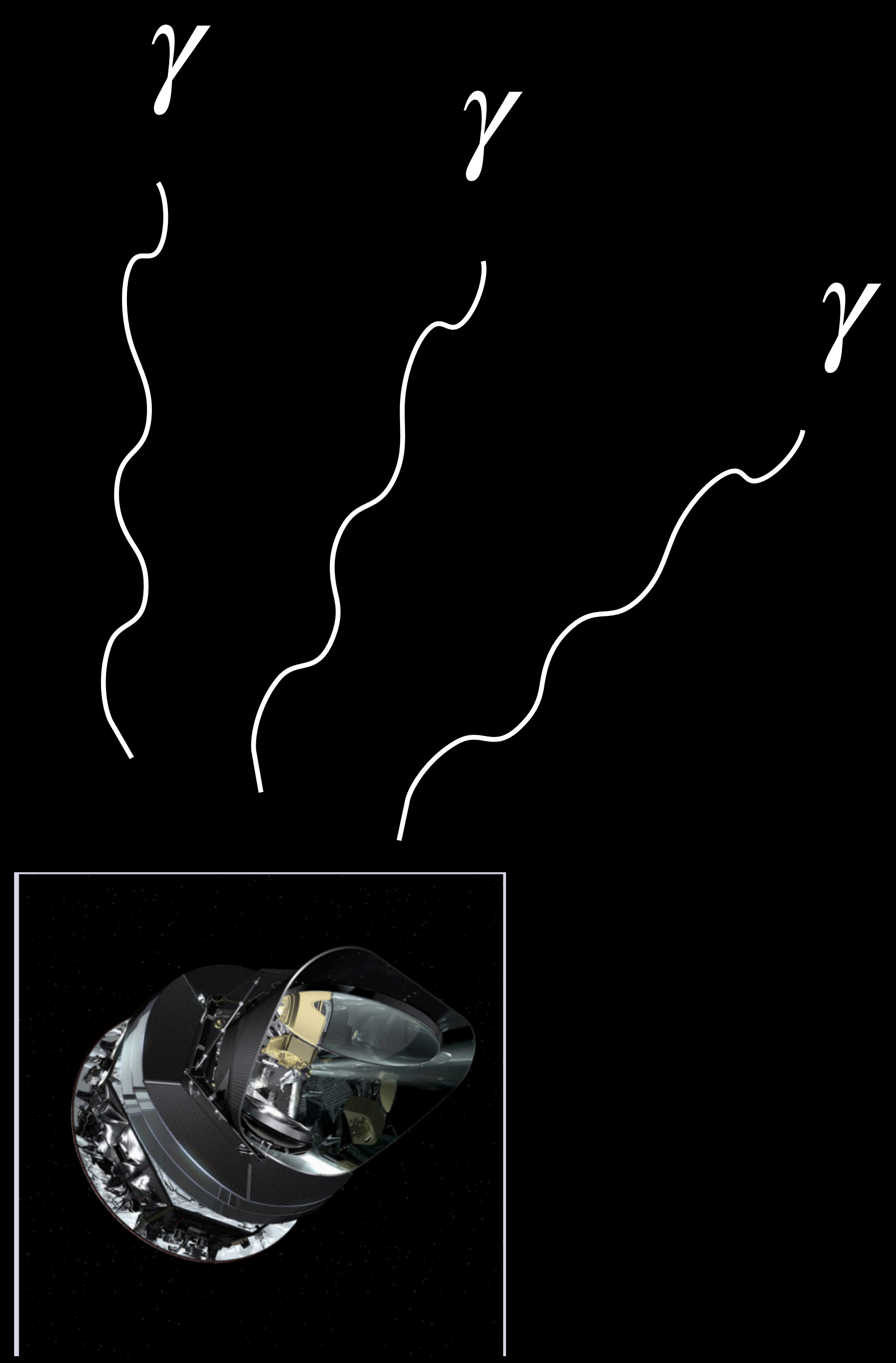
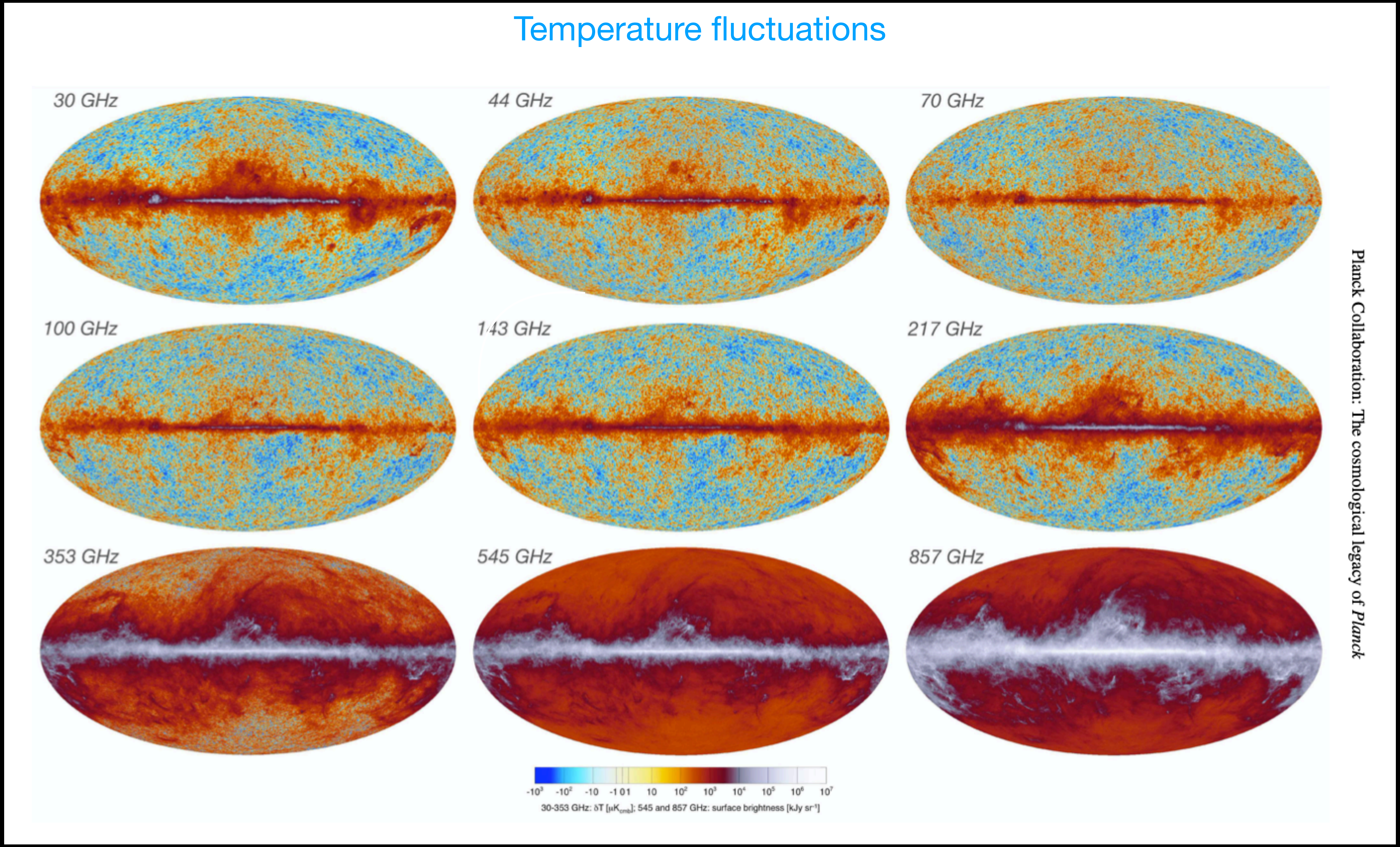
- However, small fluctuation ΔT does exist

Planck 2018





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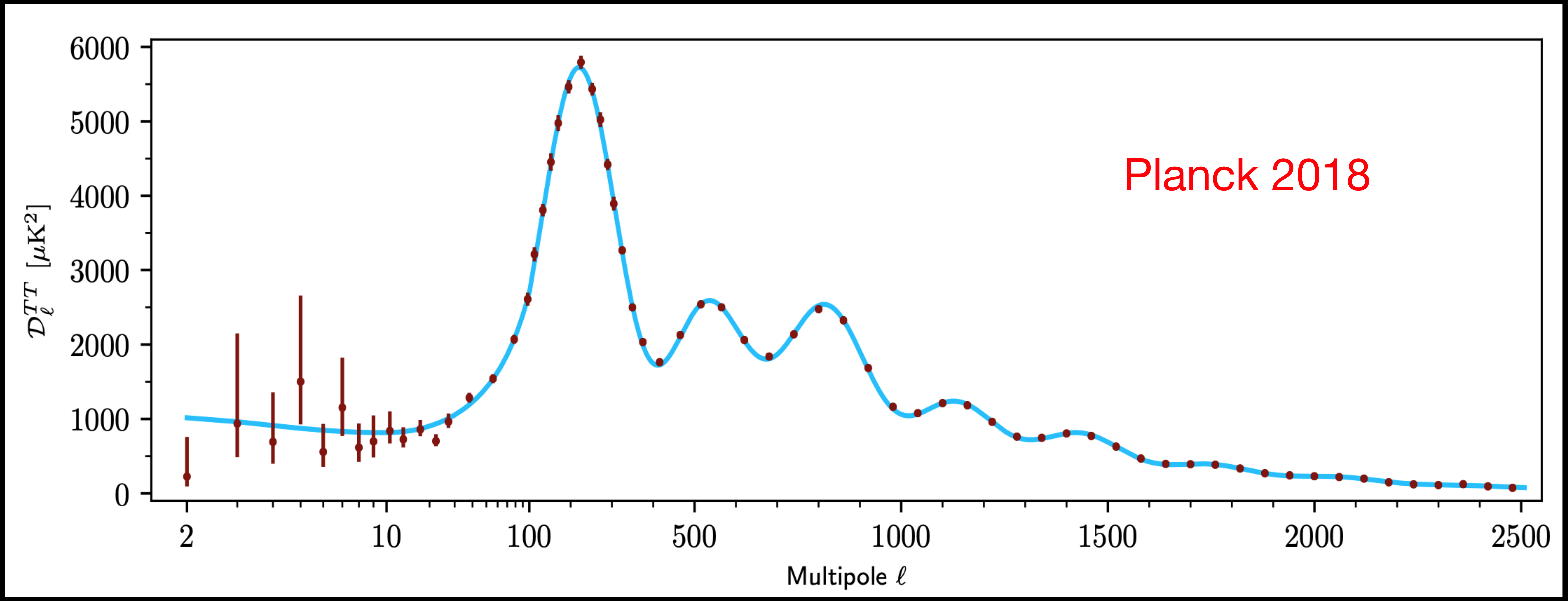
- The temperature fluctuation ΔT is usually measured by its angular (scale) dependences

$$C_l = \int d\Omega \int d\Omega' \frac{\langle \Delta T(\vec{n}) \Delta T(\vec{n}') \rangle}{T_0^2} P_l(\vec{n} \cdot \vec{n}')$$

$$P_l(\vec{n} \cdot \vec{n}') = \sum_{m=-l}^{l} \frac{4\pi}{2l+1} Y_l^m(\vec{n}) Y_l^{m*}(\vec{n}')$$

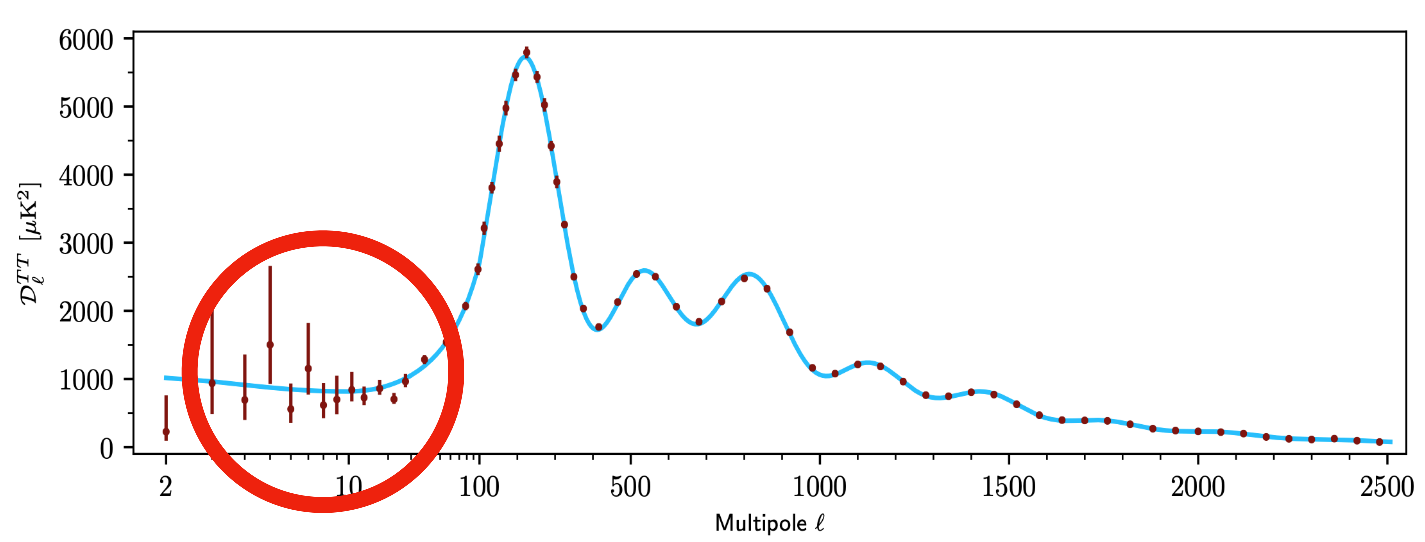
$$\mathcal{D}_l = l(l+1)C_l/(2\pi)$$



Roughly, $P_l \sim \cos(l\theta)$.
For fixed l, we are looking at $\theta \sim \pi/l$
Large l ↔ Small θ

1. What can we learn from this result ? (next slide)
2. How do we explain this fluctuations theoretically ?

Necessary Inputs for This talk



① Amplitude of Power Spectrum

- C_l is expressed as $$C_l = \frac{2}{\pi}\int_0^\infty (d\ln k)\ k^3 P_{\mathscr{R}}(k)\Delta_l(k)$$ where $\Delta_l(k)$: transfer function which depends on cosmological models
- For modes well outside horizon at recombination (l<<100), only gravitational red or blue shift matters (Sachs-Wolfe effects).

  → In this region, Δ_l is simply a spherical Bessel function

$$\Delta_l(k) = \frac{1}{3} j_l(kr_L), \quad r_L \text{ : comoving radius of the last scattering surface}$$

- Spherical Bessel function $j_l(x)$ has a peak around x~l, thus roughly

$$C_l \sim \frac{2}{3^2\pi} k^3 P_{\mathcal{R}}(k)\Big|_{k=l/r_L} \underbrace{\int_0^\infty (d\ln x) j_l(x)^2}_{\propto l^{-1}(l+1)^{-1}} \quad \therefore\ l(l+1)C_l \sim \frac{2}{9\pi} k^3 P_{\mathcal{R}}(k)\Big|_{k=l/r_L}$$

We can know $P_R(k)$ (primordial power spectrum) from C_l

Necessary Inputs for This talk

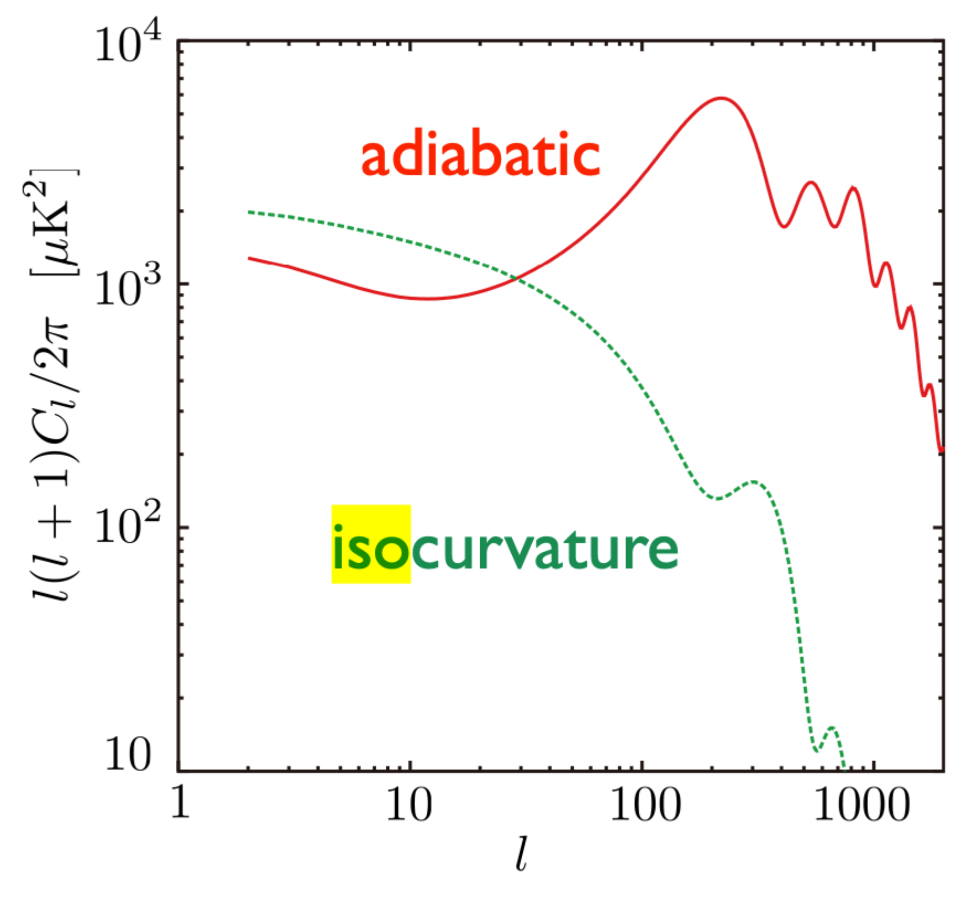
Cont'd

- Planck 2018 results

$$A_s(k) := \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k) = A_s(k_*) \left(\frac{k}{k_*}\right)^{n_s - 1}, \quad A_s(k_*) = 2.1^{+0.031}_{-0.034} \times 10^{-9}, \quad n_s = 0.965 \pm 0.004 \ (68\% \ \mathrm{CL})$$

amplitude of fluctuations — Small scale dependence

k_*=0.05 Mpc^{-1} (a reference scale)



② Isocurvature perturbation ← different type of fluctuation

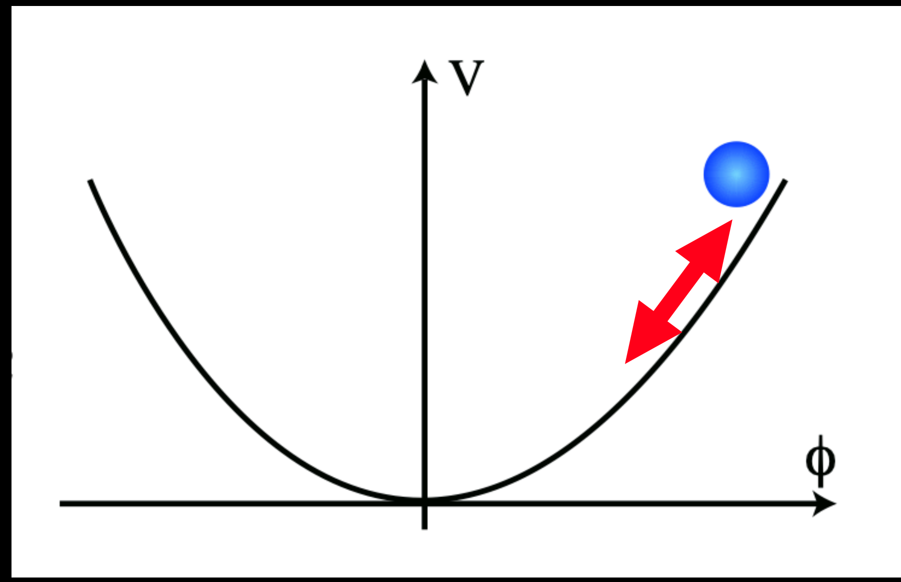
$$\beta_{\rm iso}(k_*) := \left.\frac{P_{\mathcal{I}}}{P_{\mathcal{R}} + P_{\mathcal{I}}}\right|_{k=k_*} < 0.00107 \ (95\,\% \ \mathrm{CL \ for \ fully \ anti-correlated \ case})$$

③ Non-Gaussianity constraints ← deviation from Gaussian fluctuations

$$f_{\rm NL}^{\rm local} = 4 \pm 20, \quad g_{\rm NL} = (-5.8 \pm 13) \times 10^4 \quad (95\% \mathrm{CL \ by \ Planck \ 2018})$$

＊ I will explain the details later

Inflation is an Unique Solution ?



- The most popular solution for the CMB anisotropy is (single field) inflation because it can solve a few cosmological problems simultaneously.

- Quantum fluctuation of inflaton $<\delta\phi^2>$ is the origin of the CMB anisotropy.

- In particular, single field inflation automatically predicts adiabatic perturbations which is favored by the CMB observations.

- This is simple and seems to be reasonable, but not unique

  e.g.) Any (massless) scalar field can have fluctuation during the inflation.
  After the reheating, it can convert its fluctuation into radiation = Curvaton scenario

D.H. Lyth and D. Wands, ('02)

Why QCD Axion ?

R.D. Peccei and H. R. Quinn ('77)
F. Wilczek ('78), S. Weinberg ('78),
M A. Shifman, A.I. Vainshtein, and V I. Zakharov (80)

- QCD axion is pseudo NG boson that appears as a result of SSB of some global U(1) symmetry = Peccei-Quinn symmetry
- Typically, it has interaction with fermions like $e^{iA/f_A}\overline{\psi}_L \psi_R + (h.c.)$. But, we can always redefine ψ_L or ψ_R to eliminate e^{iA/f_A}
- Due to the anomaly, the interaction between gauge fields appears instead

$$\frac{g^2}{32}\int d^4x \left(\theta + \frac{A}{f_A}\right) \mathrm{Tr}[\tilde{G}^{\mu\nu}G_{\mu\nu}] := \frac{g^2}{32}\int d^4x A'(x) \mathrm{Tr}[\tilde{G}^{\mu\nu}G_{\mu\nu}]$$

- θ is now dynamical quantity and fixed at the minimum of the axion potential $V_A(A)$
If $V_A(A)$ has a minimum at A=0, this can be a dynamical solution of strong CP problem

$$|\theta_{\mathrm{eff}}| \lesssim 10^{-10}$$

nEDM collaboration (2020)

Axion Potential

- Non-perturbative effects of QCD produce axion potential.

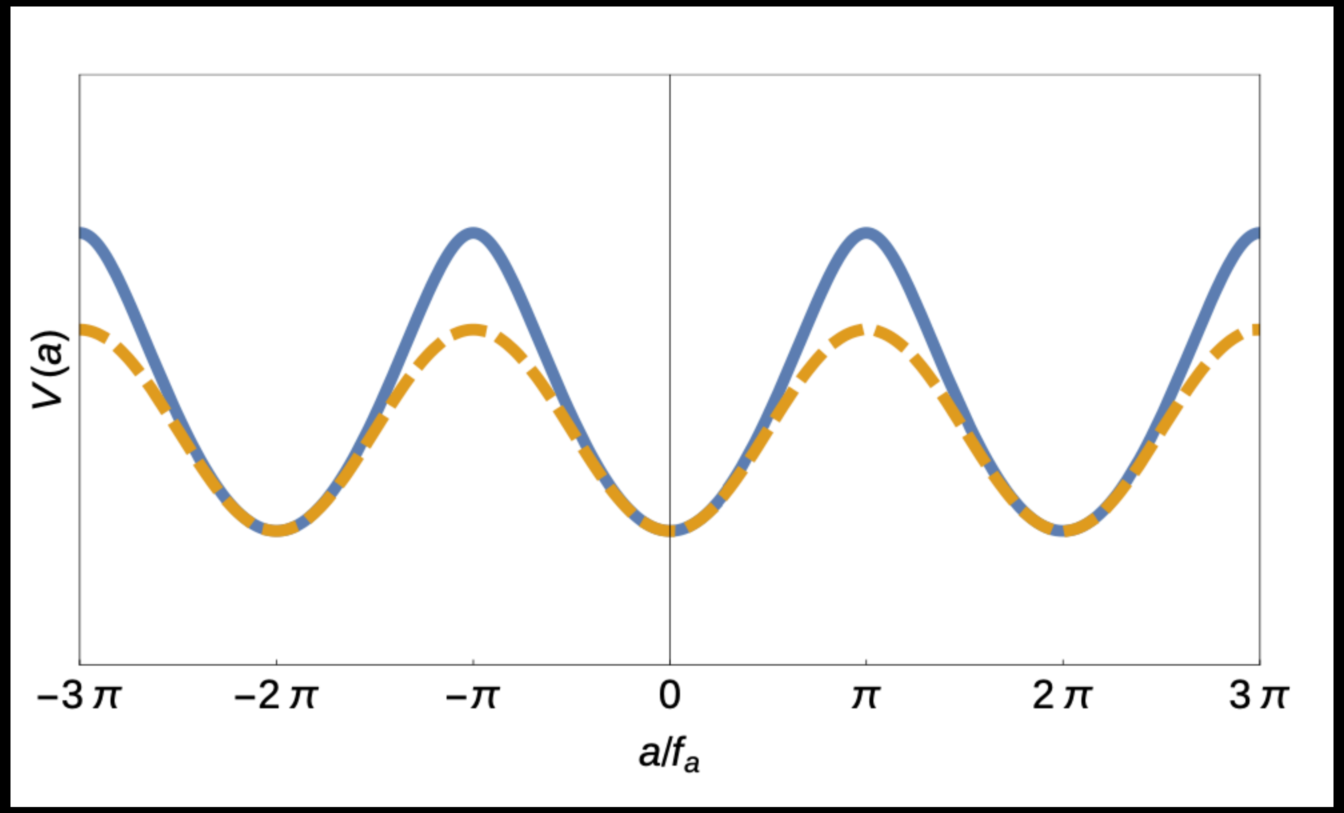
  Naive instanton calculation gives

$$V_A(A) = m_{A0}^2 f_A^2 [1 - \cos(A/f_A)], \quad m_{A0}^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_A^2}{f_A^2}$$

  C.f. Calculation based on chiral Lagrangian gives

$$V_A(A) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{A}{2 f_A}\right)}$$

V_A actually has minimum at A=0 !



Blue: instanton
Orange: based on chiral Lagrangian

- Finite temperature potential is also important to determine its abundance.

  Lattice calculation gives

b=1.02 for three light quarks

$$V_A = m_A(T)^2 f_A^2 [1 - \cos(A/f_A)], \text{ where } m_A(T) = m_{A0} \times \begin{cases} (T_{\rm QCD}/T)^{4b} & \text{for } T \geq T_{\rm QCD}, \\ 1 & \text{for } T \leq T_{\rm QCD}, \end{cases}$$

Temperature dependent mass

[Sz. Borsanyi et al. Nature, 539(7627):69–71, 2016.]

Short Summary of this talk

- Consider a possibility whether QCD axion can explain the CMB anisotropy
- The following situations are assumed:
    1. Fluctuation from the inflaton is too small to explain the CMB anisotropy (trivial)
    2. QCD axion is massless during the inflation, i.e. PQ symmetry is broken
       → Axion also gets primordial fluctuations as well as the inflaton $\langle \delta A^2 \rangle \sim \frac{H_{\rm inf}^3 t}{2\pi}$
- The axion fluctuation is transferred to SM radiation by the energy conversion at around the QCD phase transition, T_{QCD} ~150 MeV
- Constrain parameter space of QCD axion by the CMB observations
  In particular, we see that some dilution of QCD axion is necessary
  → As a simplest possibility, we study secondary (thermal) inflation

Talk Plan

1. Brief review of cosmological perturbations
2. Fluctuation of QCD Axion and its conversion to radiation
3. Isocurvature constraints and Non-Gaussianity
4. Secondary (thermal) Inflation scenario
5. Conclusion

$\ast$ In the following, background quantities are represented with bar, e.g. $\bar{a},\ \bar{\rho}$.

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$\ast$ In the following, background quantities are represented with bar like $\bar{a},\ \bar{\rho}$.

Curvature Perturbations ζ

[J.M. Bardeen et al. ('83)]

- We write the Friedmann-Robertson-Walker (FLRW) metric as

$$ds^2 = -dt^2 + \bar{a}(t)^2 e^{2\psi(x)} \delta_{ij} dx^i dx^j, \quad \psi(x) : \text{fluctuation of local e-folding number}$$

- We consider perturbations at linear order for simplicity

- The uniform density curvature perturbations ζ is defined by

$$\zeta := -\psi + \frac{\delta\rho}{3(\bar{\rho}+\bar{p})}, \quad \bar{\rho}, \bar{p} : \text{total energy density and pressure}$$

- ζ is gauge invariant and conserved on super horizon scale if total pressure p is a function of total energy density ρ, i.e. p=p(ρ) (barotropic)

$$\zeta = -\psi \quad \text{(for uniform density slice i.e. } \delta\rho = 0\text{)}$$

＊But, the universe generally consists of various components.

So ζ is not a conserved quantity in general.

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- We can also define a similar curvature perturbation for each component X

$$\zeta_X := -\psi + \frac{\delta\rho_X}{3(\bar{\rho}_X + \bar{p}_X)} = -\psi + \frac{\delta\rho_X}{3(1+\omega_X)\bar{\rho}_X}, \quad \omega_X = \begin{cases} 0 & \text{for matter} \\ 1/3 & \text{for radiation} \\ -1 & \text{for vacuum energy} \end{cases}$$

- This is also gauge invariant and conserved on super horizon scale if X does not interact with other components.

- The original ζ can be also rewritten as

$$\zeta = \frac{\sum_X (1+\omega_X)\Omega_X \zeta_X}{(1+\omega_{\text{tot}})}, \quad \text{where } \Omega_X = \frac{\bar{\rho}_X}{\bar{\rho}}, \ \omega_{\text{tot}} = \sum_X \Omega_X \omega_X, \qquad \cdots \ast$$

- A perturbation is called adiabatic if all ζ_X are the same, i.e. $\zeta_X = \zeta_Y = \zeta_{\text{adi}}$

  → In particular, ζ also coincides with ζ_{adi} from $\ast$

- Other perturbations are generally called non-adiabatic or entropic. Thus, it is characterized by the difference between ζ_X's

$$S_{Ar} := -3(\zeta_A - \zeta_r)$$

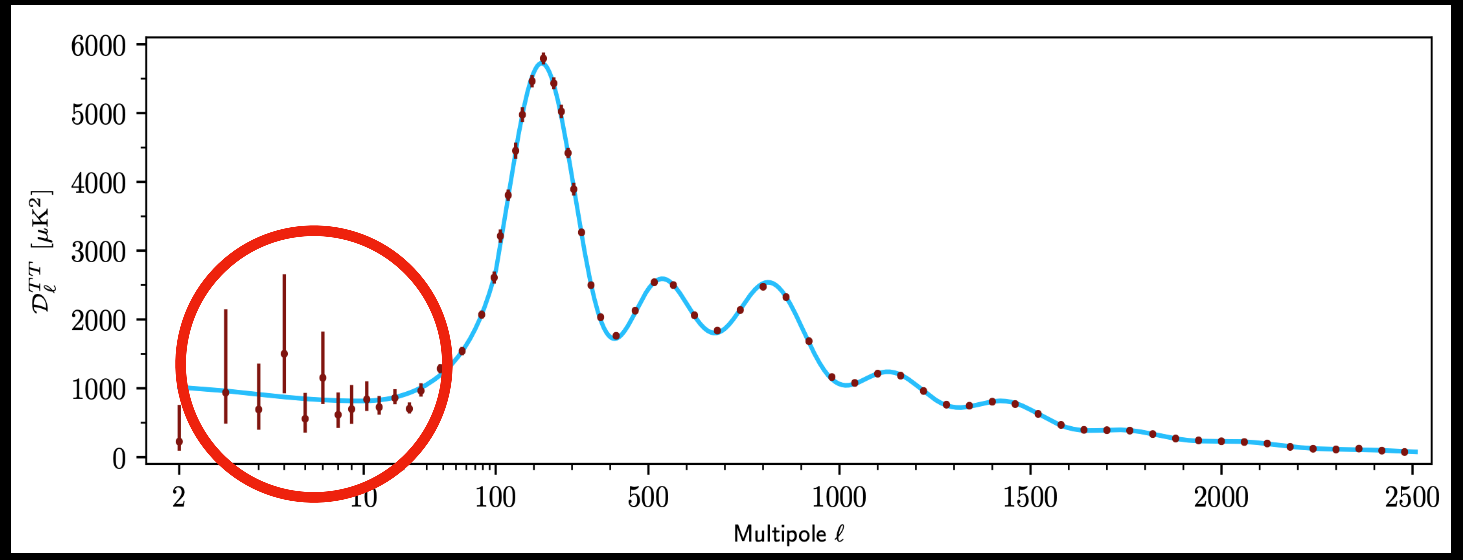
$\ast$ In particular, the fluctuation with $\delta\rho = 0$ is called isocurvature perturbation.

Relation between power spectrum and ζ

Cont'd

- On superhorizon scale (l<<100) at recombination, the effect of gravitational potential Φ is dominant = Sachs-Wolfe effect

$$\left.\frac{\delta T}{T} = \frac{1}{3}\Phi\right|_{\rm rec} \quad \text{(for super horizon)}$$



- In the matter dominated era, Φ is given by

$$\Phi = -\frac{3}{5}\zeta \sim -\frac{3}{5}\zeta_r \ \text{(neglecting isocurvature perturbation)}$$

- Thus, we obtain

$$\left.\frac{\delta T}{T} = -\frac{1}{5}\zeta_r\right|_{\rm rec} \quad \text{(for superhorizon scale)}$$

Recall

$$C_l = \frac{2}{\pi}\int_0^\infty (d\ln k)\ k^3 P_{\mathscr{R}}(k)\Delta_l(k)$$

$\therefore$ Power spectrum $P_{\mathscr{R}}(k)$ is determined by $\langle\zeta\zeta\rangle$

We have to calculate ζ theoretically.

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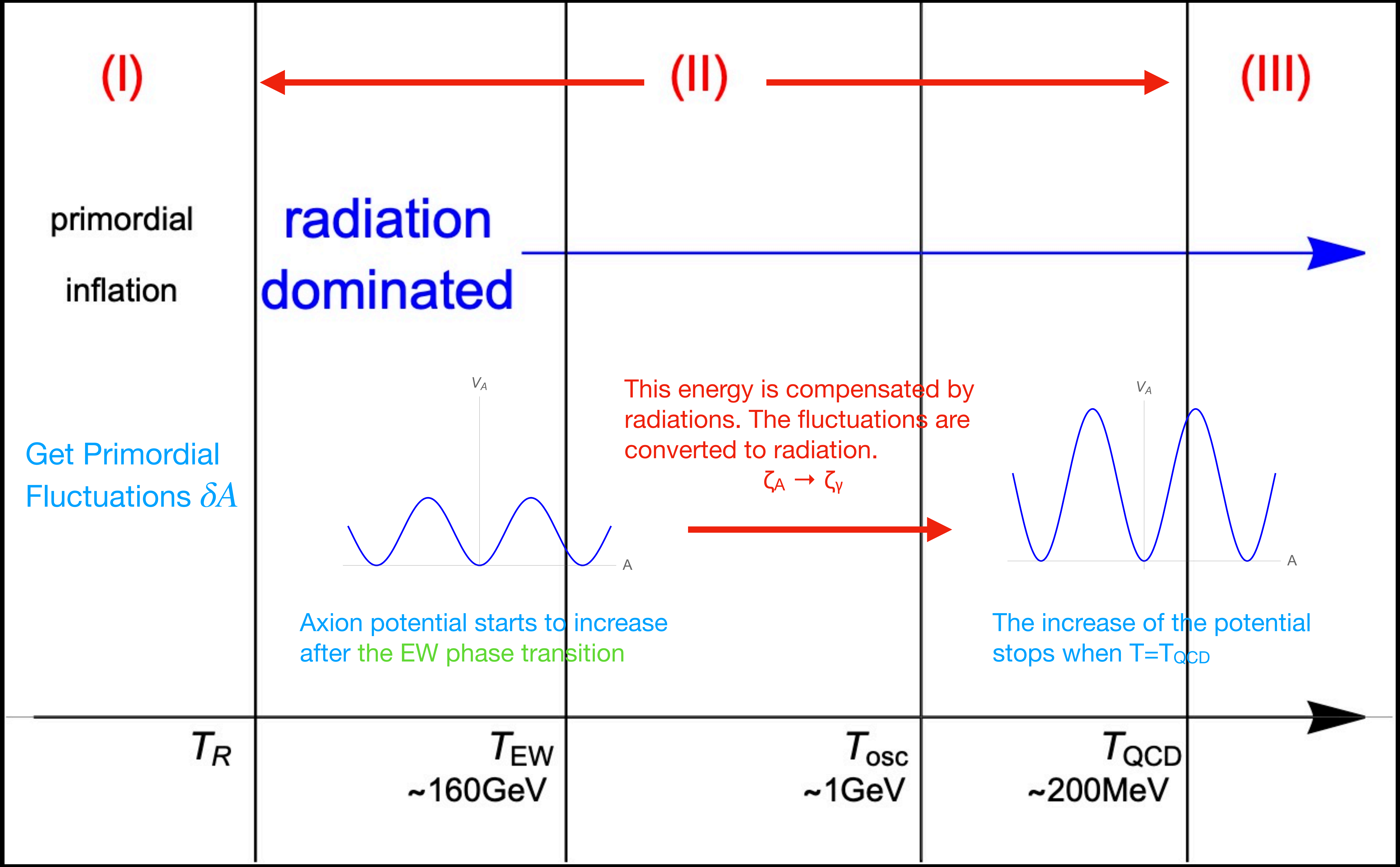
Before going into the details….

- Rough sketch of the cosmological history of QCD axion



We have to calculate curvature perturbations carefully for each epochs

(I) Primordial fluctuations by Inflation

Cont'd

- Fourier modes

Meaning of these creation-annihilation operators are explained later

$$\delta\hat{A}(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^d}\left(\delta\tilde{A}_{\mathbf{k}}(t)\hat{a}_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{x}} + \delta\tilde{A}_{\mathbf{k}}{}^*(t)\hat{a}^\dagger_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{x}}\right)$$

Their Heisenberg equations are

$$\ddot{\delta\tilde{A}}_{\mathbf{k}}(t) + 3H_{inf}\dot{\delta\tilde{A}}_{\mathbf{k}}(t) + \left(\frac{k^2}{a(t)^2} + m^2\right)\delta\tilde{A}_{\mathbf{k}}(t) = 0$$

- Or, by introducing conformal time $\eta = -(a(t)H_{inf})^{-1}$ and new function $\chi_k(t) \equiv (-\eta)^{-3/2}\delta\tilde{A}_k(t)$, it can be rewritten as

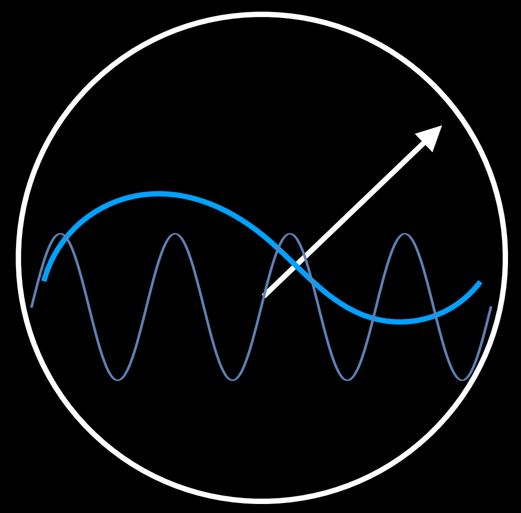
$$\frac{d^2\chi_k}{d(k\eta)^2} + \frac{1}{k\eta}\frac{d\chi_k}{d(k\eta)} + \left[1 - \underbrace{\left(\frac{9}{4} - \frac{m^2}{H_{inf}^2}\right)}_{\equiv \nu^2}\frac{1}{(k\eta)^2}\right]\chi_k = 0$$ ← Bessel differential equation

→ In general, there are two independent solutions $J_\nu(k\eta)$ and $Y_\nu(k\eta)$

Their arbitrary linear combination is also a solution

Bunch-Davies Vacuum

[T. S. Bunch and P. C. W. Davies ('78)]



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- When $a(\eta) \to 0$, (almost) all physical modes are within horizon i.e. $k_{phys} := k/a \gg H_{inf}$

  → Spacetime is essentially the same as flat Minkowski spacetime

- Natural choice of mode function is the plane wave when t → 0 (η =-∞).

$$\delta \hat{A}_H(x) \underset{t\to 0}{\to} \int \frac{d^3\mathbf{k}_{phys}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\mathbf{k}}(t)}} \left(\hat{a}_{\mathbf{k}} e^{-i\int^t ds\omega_{\mathbf{k}}(t) + i\mathbf{k}_{phys}\cdot\mathbf{x}_{phys}} + \hat{a}^\dagger_{\mathbf{k}} e^{i\int^t ds\omega_{\mathbf{k}}(t) - i\mathbf{k}_{phys}\cdot\mathbf{x}_{phys}} \right),$$

- Hankel function satisfies this condition: $\chi_{\mathbf{k}}(\eta) = H^{(1)}_\nu(-k\eta) := J_\nu(-k\eta) + iY_\nu(-k\eta)$

- Creation-annihilation operators $(\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}})$ define a vacuum state at initial time

$$\hat{a}_{\mathbf{k}} |0\rangle_{BD} = 0 \quad \text{for all } k$$ (Bunch-Davies (BD) Vacuum)

Axion Fluctuation at the end of Inflation

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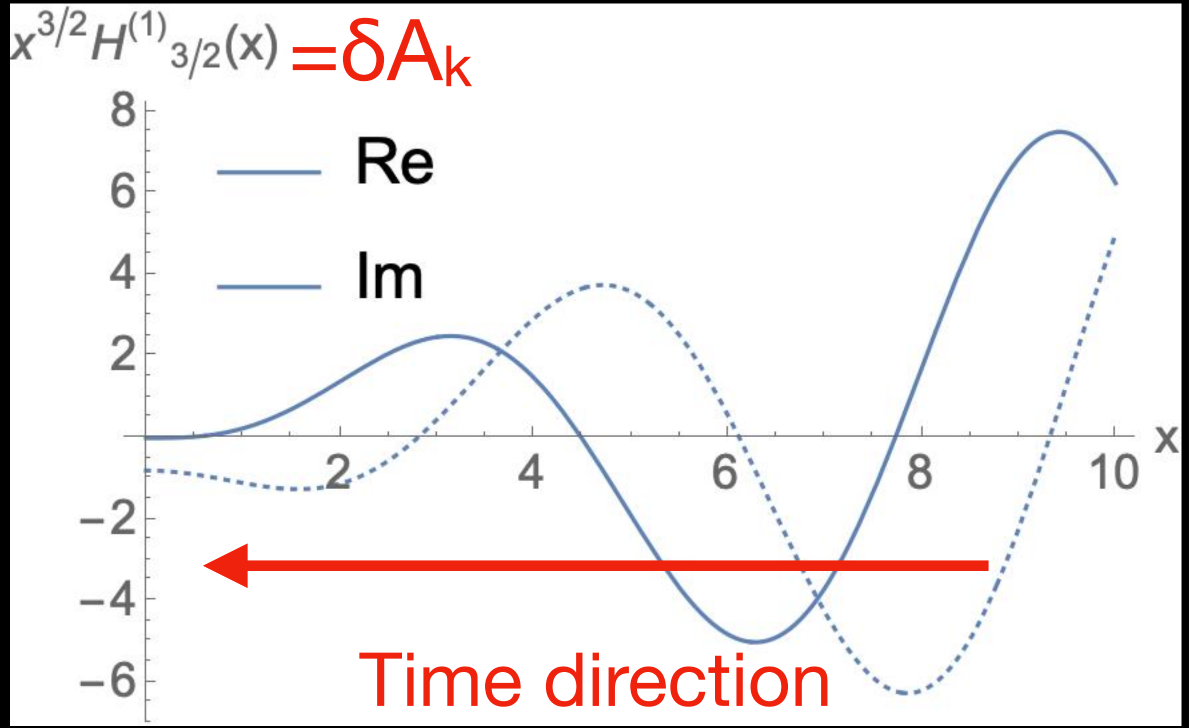
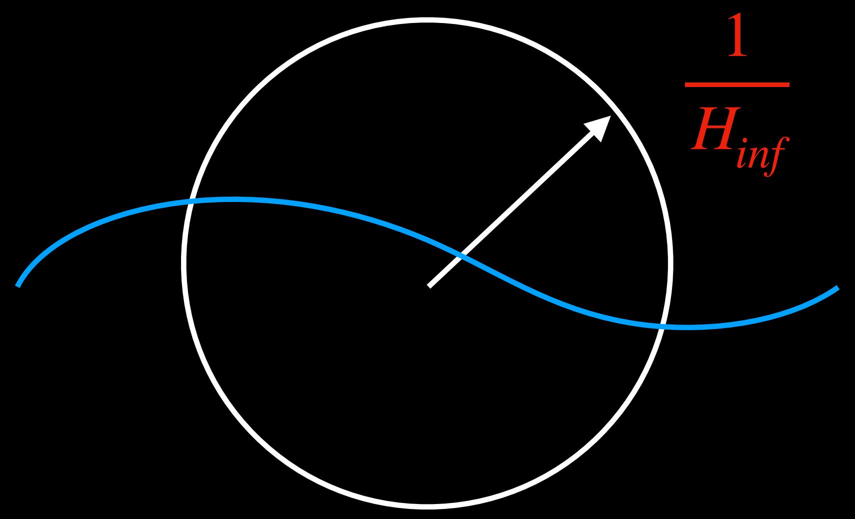


- However, as time goes, $H_\nu^{(1)}(-k\eta)$ gradually begins to deviate from the plane wave.
- In particular, on the superhorizon limit $\frac{k}{a(t)H_{inf}} \ll 1$, δA_k approaches constant value (equilibrium distribution):



Remember this

$$|\delta A_k|^2 \to \frac{H_{inf}^2}{2k^3}\left(\frac{k}{a(t_f)H}\right)^{\frac{2m^2}{3H_{inf}^2}} \xrightarrow[m=0]{} \frac{H_{inf}^2}{2k^3}$$

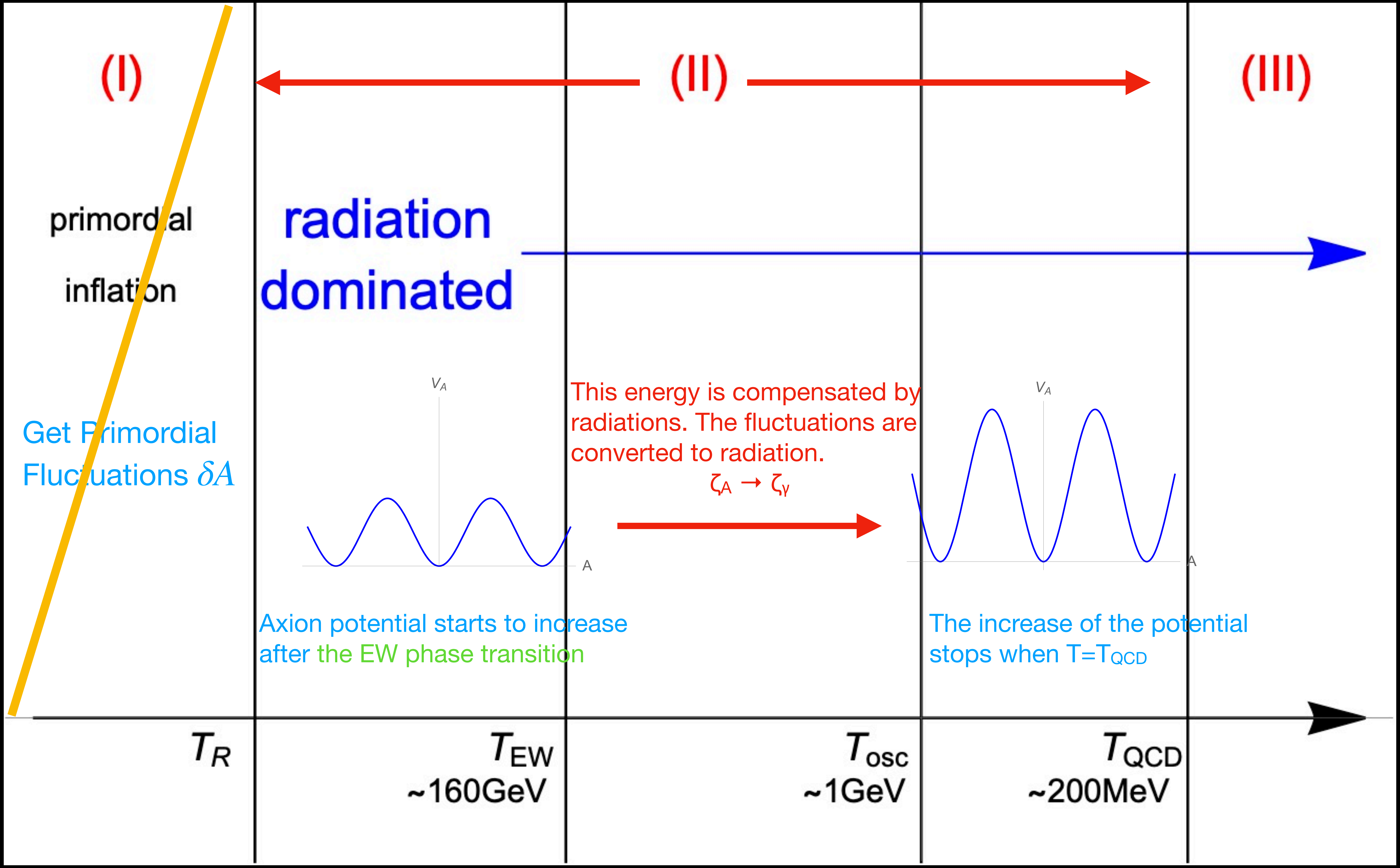
- As a result, the modes outside horizon accumulate, and produce a field fluctuation:

$$\langle \delta A(t_f)^2 \rangle \sim \int_0^{H_{inf}a_f} \frac{d^3\mathbf{k}}{(2\pi)^3} |\delta A_k(t_f)|^2 = \frac{H_{inf}^3}{4\pi^2} t \quad (\text{for } m = 0)$$ (Bunch-Davies distribution)

〜 same as Minkowski spacetime

Before going into the details….

- Rough sketch of the cosmological history of QCD axion



(II) (iii) Transfer of fluctuations from Axion to Radiation

- After the EW phase transition (T~160GeV), the axion potential starts to increase

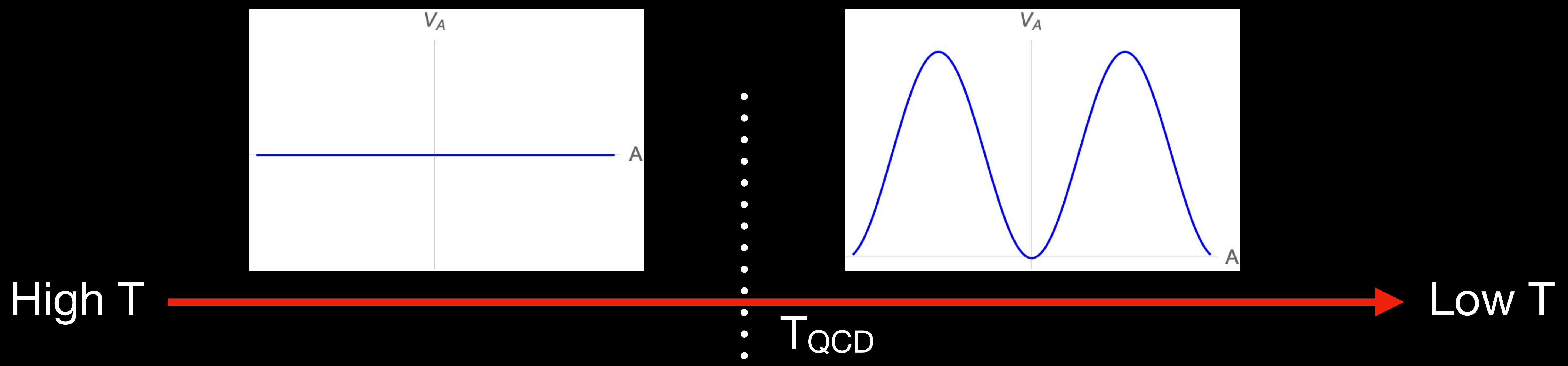
$$V_A = m_A(T)^2 f_A^2[1-\cos(A/f_A)], \ \text{where } m_A(T) = m_{A0} \times \begin{cases} (T_{\rm QCD}/T)^{4b} & \text{for } T \geq T_{\rm QCD}, \\ 1 & \text{for } T \leq T_{\rm QCD}, \end{cases}$$

b~1.02 by Lattice calculation
Sz. Borsanyi et al. (2016)

- This increase of energy is compensated by radiation

$$\delta\rho_A + \delta\rho_r = 0 \quad (\text{energy conservation})$$

- It is very hard to solve the energy conservation rigorously…
  → Let us assume that the energy transfer suddenly occurs at $T=T_{QCD}$ ~ 150MeV



Within this approximation, energy conservation only at $T=T_{QCD}$ matters !

(II) Fluctuation before T=T_{QCD}

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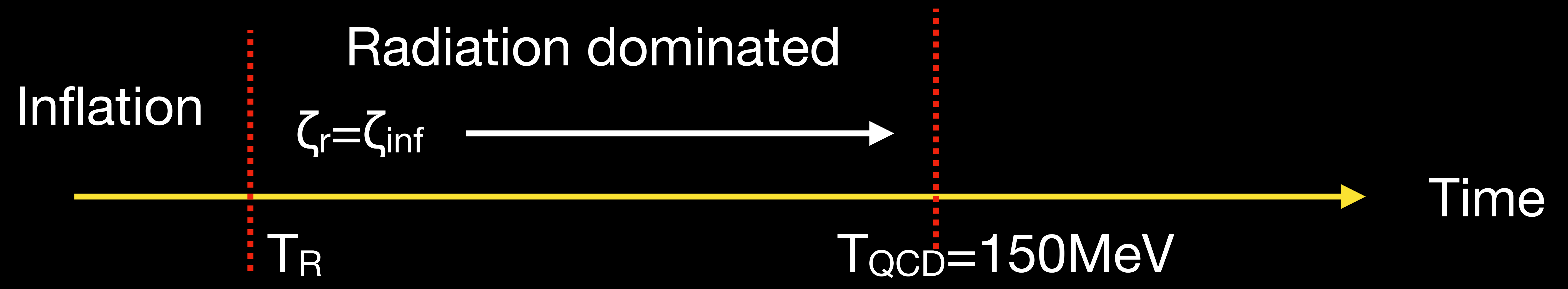

- After reheating, radiation obtains the fluctuation $\zeta_r = \zeta_{\rm inf}(\sim 0)$ from inflaton, and it is conserved until T=T_{QCD}
- To calculate fluctuations after T=T_{QCD}, we need to know ψ
- Right before T=T_{QCD}, only radiation energy exists. Therefore, it is natural to choose the uniform temperature slice i.e. $\delta\rho_r$=0.

$$\zeta_r \underset{\text{def}}{:=} -\psi + \frac{\delta\rho_r}{4\bar{\rho}_r} \underset{\text{In this slice}}{=} -\psi = \zeta_{\rm inf} \sim 0$$

Recall

$$ds^2 = -dt^2 + \bar{a}(t)^2 e^{\psi(x)} \delta_{ij} dx^i dx^j$$

∴ ψ is given by $\zeta_{\rm ini}$ at T=T_{QCD}

(III) Fluctuation after T=T_{QCD}

Cont'd

- Right after T=T_{QCD}, we have $\delta\rho = \delta\rho_A + \delta\rho_r = 0$ (energy conservation) ・・・①

- From the definition $\zeta_r = -\psi - \delta\rho_r/(4\bar{\rho}_r)$, $\delta\rho_r$ can be written as

$\psi = \zeta_{\rm inf}$

$$\delta\rho_r = -4\bar{\rho}_r(\zeta_r + \psi) = -4\Omega_r\bar{\rho}(\zeta_r - \zeta_{\rm inf}) \quad \cdots ②$$

- Energy fluctuation of axion is $\delta\rho_A = \delta V_A(A) = V_A'(\bar{A})\delta A$ ・・・③

- By substituting ②③ into ①, we can solve ζ_r as

$$\zeta_r = \cancel{\zeta_{\rm inf}} + \frac{R}{4}\frac{V_A'}{V_A}\bigg|_{A=\bar{A}}\delta A \sim \frac{R}{2}\frac{\delta A}{\bar{A}}, \quad \text{where } R := \frac{\Omega_A}{\Omega_r}\bigg|_{\text{right after } T_{\rm QCD}}$$

Negligible

determined by initial fluctuation

Ratio of energy densities after the energy transfer

Cont'd

Result $\zeta_r = \zeta_{\rm inf} + \frac{R}{4}\frac{V_A'}{V_A}\Big|_{A=\bar{A}} \delta A \sim \frac{R}{2}\frac{\delta A}{\bar{A}}, \quad \text{where } R := \frac{\Omega_A}{\Omega_r}\Big|_{\text{right after } T_{\rm QCD}}$

- Recall $|\delta A_k|^2 = \frac{H_{\rm inf}^2}{(2\pi)^3 k^3}$ from the primordial inflation
- Thus, the CMB scalar amplitude is given by

$$\sqrt{A_s(k_*)} := \left(\frac{k^3}{2\pi^2}|\zeta_r|^2\right)^{1/2} = \left(\frac{R^2}{2^2\pi^2}\frac{|\delta A_k|^2}{\bar{A}^2}\right)^{1/2} = R \times \frac{H_{\rm inf}}{4\pi f_A \bar{\theta}} \simeq 4.6 \times 10^{-5}$$ (Planck 2018)

f_A : axion decay constant, θ : initial angle

- As long as we consider radiation dominated era, R can not be larger than 1.

$$\frac{H_{\rm ini}}{f_A\bar{\theta}} = \frac{4\pi \times 4.6 \times 10^{-5}}{R} \geq 4.8 \times 10^{-4}$$

- Besides, we have assumed that PQ symmetry is broken during inflation, i.e. $\frac{H_{\rm inf}}{f_A} < 1$

Thus, we obtain

$$4.8 \times 10^{-4} < \frac{H_{\rm inf}}{f_A\bar{\theta}} < \bar{\theta}^{-1}$$

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Non-Gaussianity (NG)

- In the following, we simply assume adiabatic fluctuation, i.e. $\zeta = \zeta_X$
- If the fluctuation ζ is Gaussian, all observables are determined by two point function $\langle \zeta\zeta \rangle$
  We represent such a Gaussian fluctuation as

$$\hat{\zeta}(x) = \hat{\zeta}^G(x) = \int \frac{d^3k}{(2\pi)^3} \hat{\zeta}^G_k e^{-i\mathbf{k}\cdot\mathbf{x}} \quad \text{(Gaussian distribution)}$$

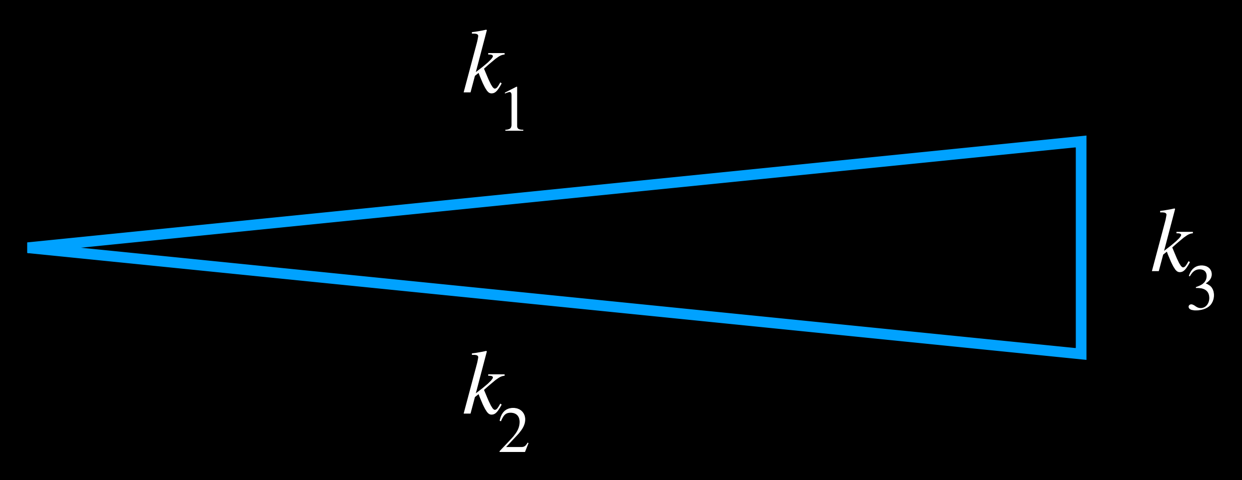
- The deviation from this is characterized by the three point function:

$$\langle \hat{\zeta}_{k_1} \hat{\zeta}_{k_2} \hat{\zeta}_{k_3} \rangle = (2\pi)^3 B(k_1, k_2, k_3) \delta^{(3)}(k_1 + k_2 + k_3)$$

- Functional form of $B(k_1,k_2,k_3)$ depends on the statistical properties of ζ
  → Three famous types:
  ① equilateral type ② orthogonal type ③ local type ←In axion scenario, this is produced

＊ Planck 2018 gives constraints for each types of NG

Local Type NG



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Recall

$$\langle \hat{\zeta}_{k_1} \hat{\zeta}_{k_2} \rangle := (2\pi)^3 P_\zeta(k_1) \delta^{(3)}(k_1 + k_2)$$

- Local type is defined by

$$B^{\mathrm{local}}(k_1, k_2, k_3) = -\frac{6}{5} f_{\mathrm{NL}}^{\mathrm{local}} \left[P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1) \right]$$

- This type is produced when full ζ has the following functional form:

$$\hat{\zeta} = \hat{\zeta}_G - \frac{3}{5} f_{\mathrm{NL}}^{\mathrm{local}} \hat{\zeta}_G^2 + \frac{9}{25} g_{\mathrm{NL}} \hat{\zeta}_G^3 + \cdots$$

∵ Just calculate <ζζζ> by using Wick's theorem

- Current observational bounds are

$$f_{\mathrm{NL}}^{\mathrm{local}} = 4 \pm 20, \quad g_{\mathrm{NL}} = (-5.8 \pm 13) \times 10^4 \quad (95\%\mathrm{CL\ by\ Planck\ 2018})$$

∗ As dimensionless parameters, those bounds themselves are not so strong.
But, they give (relatively) strong lower bounds on $R = \Omega_A / \Omega_r$ in our scenario.

NG in QCD Axion Scenario

Cont'd

- Because we are now interested in higher order terms, we have to solve the energy conservation low $\delta\rho_A + \delta\rho_r = 0$, nonlinearly.

  → As a result, ζ_r is given by a nonlinear function of δA (Gaussian)

- The calculation is straightforward but tedious… So, I just show the results:

$$f_{\rm NL}^{\rm local} = -\frac{10}{3R}\left[\frac{V_A'' V_A}{V_A'^{\,2}} + R\right]\Bigg|_{A=\bar{A}}, \quad g_{\rm NL} = \frac{1}{6}\left(\frac{20}{3R}\right)^2 \frac{V_A''' V_A^2}{V_A'^{\,3}}\Bigg|_{A=\bar{A}} + \frac{20}{3} f_{\rm NL}^{\rm local} - \frac{1}{6}\left(\frac{20}{3}\right)^2$$

- In the case of sinusoidal potential, i.e. $V_A = V_0(1 - \cos(A/f_A))$, they become

$$f_{\rm NL} = -\frac{10}{3R}\frac{\cos\bar{\theta}\left(1-\cos\bar{\theta}\right)}{\sin^2\bar{\theta}} - \frac{10}{3},$$

$$g_{\rm NL} = -\frac{1}{6}\left(\frac{20}{3R}\right)^2 \tan^2\left(\frac{\bar{\theta}}{2}\right) + \frac{20}{3} f_{\rm NL} - \frac{1}{6}\left(\frac{20}{3}\right)^2.$$

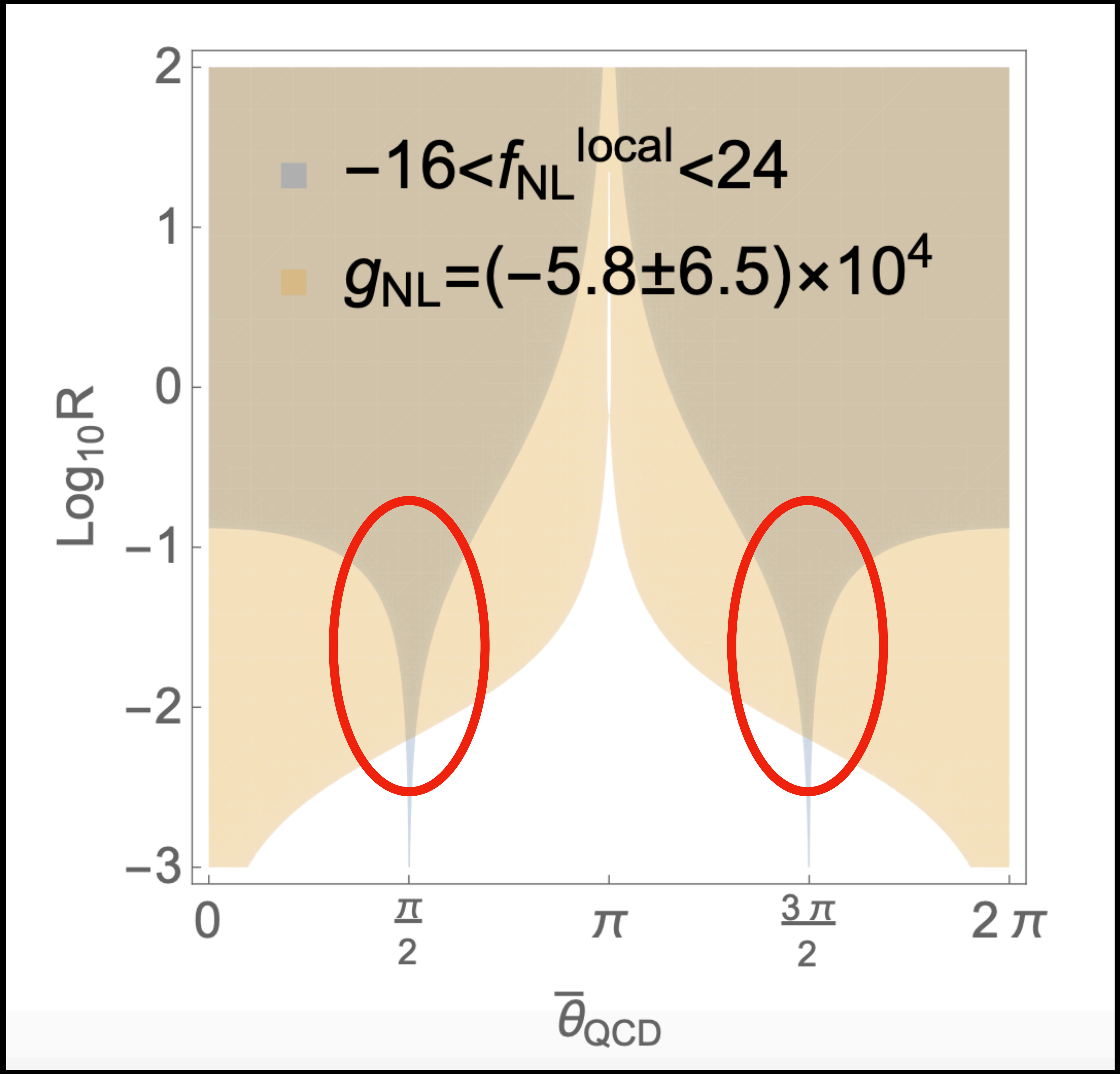
can become sizable when R<0.1 ↔ small R is not allowed

Allowed Region

Cont'd

$$f_{\rm NL} = -\frac{10}{3R}\frac{\cos\bar\theta\,(1-\cos\bar\theta)}{\sin^2\bar\theta} - \frac{10}{3}\,,$$

$$g_{\rm NL} = -\frac{1}{6}\left(\frac{20}{3R}\right)^2 \tan^2\left(\frac{\bar\theta}{2}\right) + \frac{20}{3}f_{\rm NL} - \frac{1}{6}\left(\frac{20}{3}\right)^2 .$$



- Dark blue region represents the allowed region by f_{NL}^{local}
- Orange region represents the allowed region by g_{NL}

Typically, R has to be $\gtrsim$ 0.1. But, smaller values are allowed around θ=π/2, 3π/2 !

where V''_A vanishes

Isocurvature Constraint

- Recall that isocurvature perturbations is measured by the difference of curvature perturbations:

$$S_{Ar} = -3(\zeta_A - \zeta_r)$$

- After the QCD phase transition, S_{Ar} is conserved because there is no energy transfer between axions and photons → ζ_A and ζ_r are both conserved

- Besides, recall also that S_{Ar} is gauge invariant

→ We can choose any gauge slice to calculate this

Step1) The easiest choice is the uniform temperature slice, i.e. $\delta\rho_\gamma=0$

$$\zeta_r \underset{\text{def}}{=} -\psi + \frac{\delta\rho_r}{4\bar{\rho}_r} = -\psi \ (\text{uniform temperature slice})$$

Step2) From the definition of ζ_A, we have

$$\zeta_A \underset{\text{def}}{=} -\psi + \frac{\delta\rho_A}{3\bar{\rho}_A} = \zeta_r + \frac{\delta\rho_A}{3\bar{\rho}_A} \quad \therefore \ S_{Ar} = -\frac{\delta\rho_A}{\bar{\rho}_A} = -\frac{V_A'}{V_A}\bigg|_{A=\bar{A}} \delta A$$

← well known result

Cont'd

- The isocurvature perturbation is constrained by the combination

$$\mathscr{I} := r_A S_{A\gamma}, \quad r_A = \Omega_A/\Omega_{\rm DM}|_{\rm today}$$

- Now, isocurvature power spectrum can be calculated as

$$\langle \mathcal{I}_k \mathcal{I}_q \rangle = \left(r_A \frac{V_A'}{V_A} \right)^2 \langle \delta A_k \delta A_q \rangle = (2\pi)^3 \delta^{(3)}(k+q) \left(r_A \frac{V_A'}{V_A} \right)^2 \frac{H_{\rm inf}^2}{2k^3} := \frac{2^4 \pi^5}{k^3} \mathcal{P}_{\mathcal{I}}$$

where $\mathcal{P}_{\mathcal{I}} = \left(\frac{r_A}{2\pi} \frac{V_A'}{V_A} H_{\rm inf} \right)^2 \simeq \left(\frac{r_A H_{\rm inf}}{\pi f_A \bar{\theta}} \right)^2$ Use $V_A \sim m_A^2 A^2/2$

- The current observational bound

$$\beta_{\rm iso}(k_*) := \left. \frac{\mathcal{P}_{\mathcal{I}}}{\mathcal{P}_{\mathcal{R}} + \mathcal{P}_{\mathcal{I}}} \right|_{k=k_*} \simeq \frac{\mathcal{P}_{\mathcal{I}}}{2.1 \times 10^{-9}} < 0.0011 \ (95\% \ \mathrm{CL}),$$

$$\therefore \quad \frac{r_A H_{\rm inf}}{\pi f_A \bar{\theta}} < 1.5 \times 10^{-6}$$

If axion is DM i.e. $r_A=1$ and $\theta=O(1)$, $H_{\rm inf}/f_A$ has to be very small

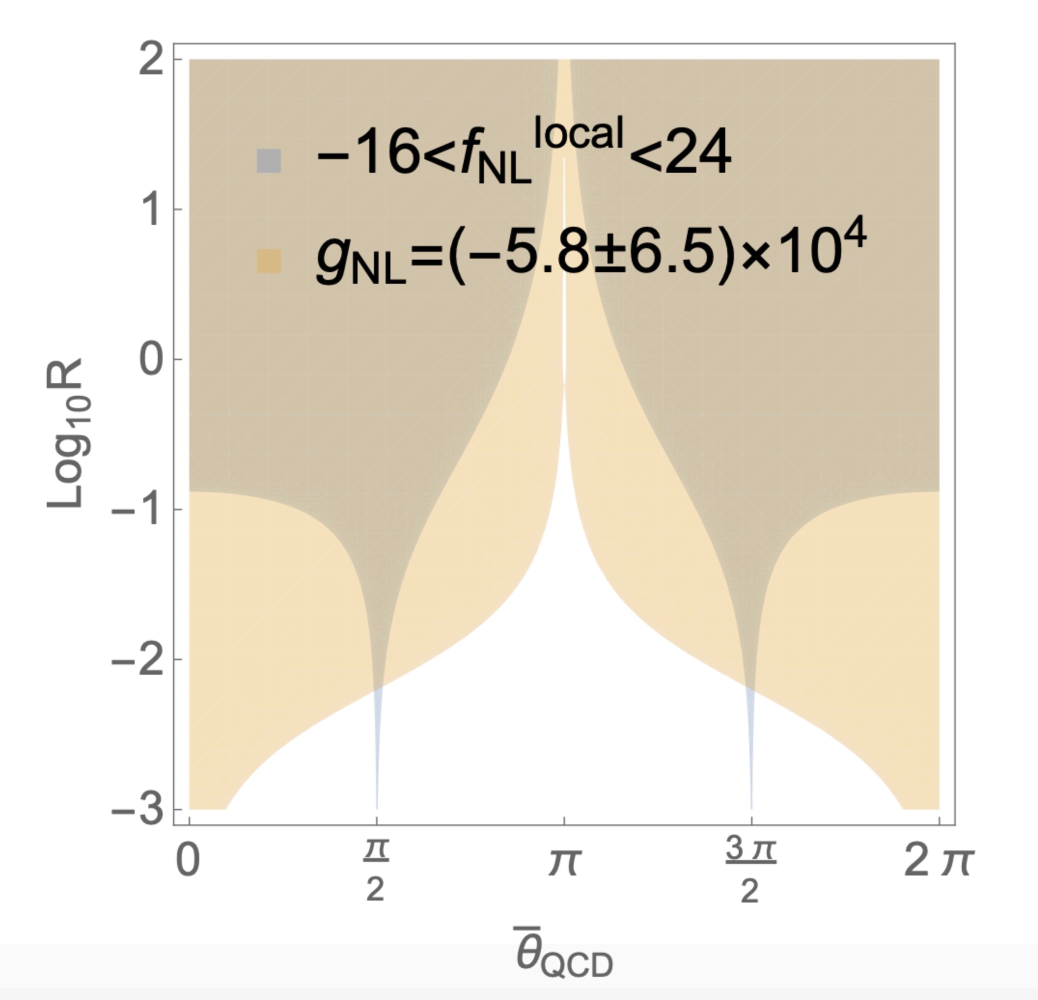
All Results

1. Amplitude of the CMB power spectrum: $\sqrt{A_s} = \dfrac{RH_{\rm inf}}{4\pi f_A \bar{\theta}} = 4.6 \times 10^{-5}$

2. Isocurvature perturbations: $\dfrac{r_A H_{\rm inf}}{\pi f_A \bar{\theta}} < 1.5 \times 10^{-6}$

We can eliminate $H_{\rm inf}/f_A\theta$

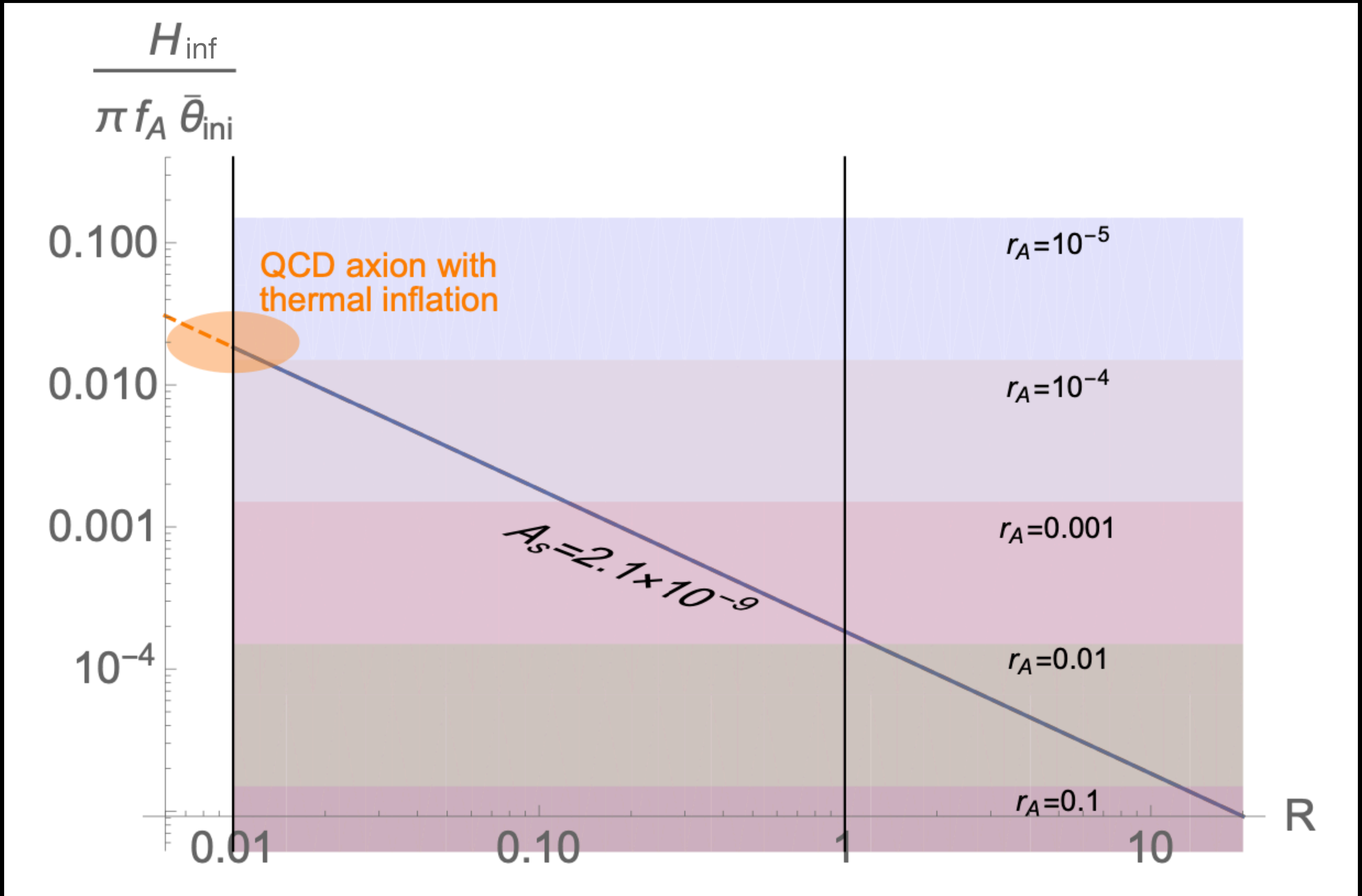
3. Non-Gaussianity:



$$\therefore \quad r_A < 8.2 \times 10^{-3} R \leq 8.2 \times 10^{-3}$$

Unfortunately, QCD axion can not become DM in this scenario….

Around θ=π/2,3π/2, R ~0.01 is allowed.
In such regions, r_A has to be more suppressed



Cont'd

- The parameters has to lie on the blue contour to explain the CMB amplitude A_s

$$\sqrt{A_s} = \frac{RH_{\rm inf}}{4\pi f_A \bar{\theta}} = 4.6 \times 10^{-5}$$

- When we fix $H_{\rm inf}/f_A\theta$, there is an upper bound of r_A by isocurvature constraint

$$\frac{r_A H_{\rm inf}}{\pi f_A \bar{\theta}} < 1.5 \times 10^{-6}$$

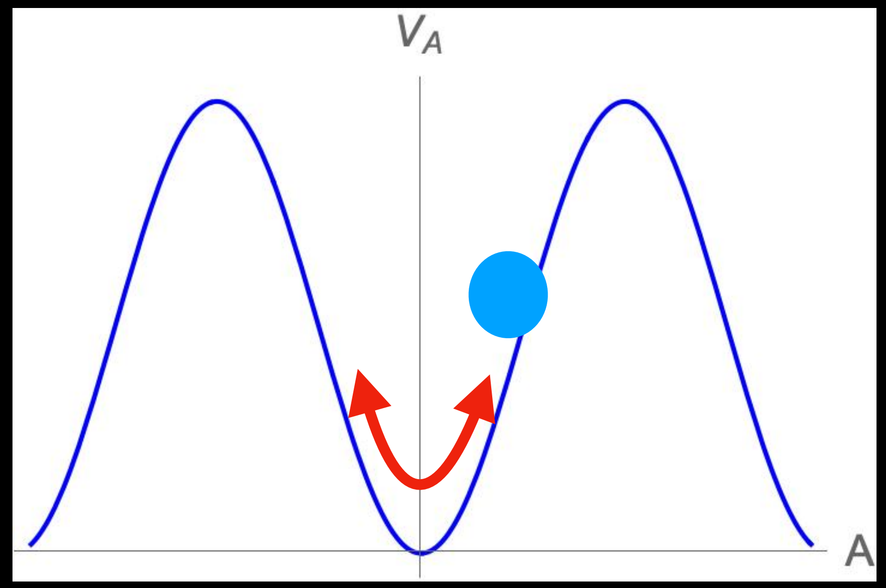
r_A needs to be very small << 0.01.

Thus, we need some dilution of axions→ Thermal (Secondary) Inflation (next)

Talk Plan

1. Brief review of cosmological perturbations
2. Fluctuation of QCD Axion and its conversion to photons
3. Constraints from Non-Gaussianity and Isocurvature
4. Secondary (thermal) inflation model
5. Conclusion

Axion Abundance



- As is well known, QCD axion can become DM by misalignment mechanism

$$\Omega_A \sim 0.3\bar{\theta}^2 \left(\frac{f_A}{10^{12}\mathrm{GeV}}\right)^{1.16} \left(\frac{150\mathrm{MeV}}{T_{\mathrm{QCD}}}\right)^{0.67}$$ ∴ $r_A \sim 1$ in the standard cosmology of QCD axion

- We need a dilution mechanism to obtain $r_A \lesssim 10^{-4}$
- Secondary (thermal) inflation is the simplest possibility

e.g. GUT model, [D. H. Lyth and D.Stewart ('95)]
Curvaton model, [Gong, Kitashima and Terada ('17)
Classically Conformal Models, [Iso, Shimada ('17)

- Various new physics models predict secondary inflation

→ We are not interested in model constructions
→ Let's proceed our calculations based on a few (reasonable) assumptions (next slices)

Assumptions

We call scalar field ϕ which causes the secondary inflation as flaton

1. QCD axion does not have any interactions with flaton
   → Only exponential expansion affects the axion abundance

2. Secondary inflation starts from $T_{inf} \gtrsim T_{QCD}$ ~150MeV (this is not crucial)

3. Secondary inflation ends at very low temperature $T_{end} << T_{QCD}$ ~150MeV

4. The secondary reheat temperature T_R is higher than T_{osc}~1GeV
   → This guarantees that QCD axion is following the standard thermal history after the secondary reheating.

＊Based on these assumptions, only the time evolution of axion field $\overline{A}(t)$ affects the abundance !

Axion Evolution during Secondary Inflation

- Axion potential $V_A = m_A(T)^2 f_A^2 \left[1 - \cos(A)\right], \quad m_A(T) = m_{A0} \times \begin{cases} (T_{\rm QCD}/T)^{4b} & \text{for } T \geq T_{\rm QCD} \\ 1 & \text{for } T < T_{\rm QCD} \end{cases}$

When $T \gtrsim T_{QCD}$, V_A is strongly suppressed $V_A \propto (T_{\rm QCD}/T)^8$

→ Axion field does not much evolve until $T = T_{QCD}$

- More precisely, the EOM of axion is

$$\ddot{A} + 3H_{\rm TI}\dot{A} + m_{A0}^2 f_A (T_{\rm QCD}/T)^8 \sin(A/f_A) = 0 \;\rightarrow\; \frac{d^2\theta}{dN^2} + 3\frac{d\theta}{dN} + 3\eta e^{8N}\sin(\theta) = 0$$

where $N := \int_{t_{\rm QCD}}^{t} dt H_{\rm TI}$ $\quad \eta := \frac{1}{3}\left(\frac{m_{A0}}{H_{\rm TI}}\right)^2 = 6.6 \times 10^{-6}\left(\frac{100}{g_{\rm TI}}\right)\left(\frac{10^{12}{\rm GeV}}{f_A}\right)^2\left(\frac{1{\rm TeV}}{T_{\rm TI}}\right)^4$

e-folding from $t = t_{QCD}$ $\quad$ Ratio between axion mass and Hubble of secondary inflation

- Because of the temperature suppression e^{8N}, slow roll approximation is good

$$\frac{d\theta}{dN} \simeq -\eta e^{8N}\sin(\theta)$$

Cont'd

- At t=t_{QCD} (i.e.T=T_{QCD}), we have

This is small even if η~O(1)

$$\tan(\theta_{\rm QCD}/2) = \tan(\theta_{\rm ini}/2)\exp\left[-\frac{\eta}{8}(1-e^{-8|N_{\rm EW}|})\right] \sim \tan(\theta_{\rm ini}/2)\underline{e^{-\eta/8}} \quad \therefore\ \theta_{\rm QCD} \sim \theta_{\rm ini} - \frac{\eta}{8}\sin\theta_{\rm ini}$$

∴ Axion field does not actually evolve before T=T_{QCD}

- After T=T_{QCD}, however, the axion field does evolve because there is no temperature suppression

$$\mathrm{EOM}:\ \frac{d^2\theta}{dN^2} + 3\frac{d\theta}{dN} + 3\eta\sin(\theta) = 0 \ \rightarrow\ \frac{d\theta}{dN} + \eta\sin(\theta) \sim 0\ \text{(slow roll)}$$

$$\therefore\ \theta_{\rm end} = 2\arctan\left[\tan(\theta_{\rm QCD}/2)e^{-\eta N_{\rm end}}\right] \sim 2\tan(\theta_{\rm QCD}/2)\underline{e^{-\eta N_{\rm end}}},\quad N_{\rm end} := \log\frac{T_{\rm QCD}}{T_{\rm end}}$$

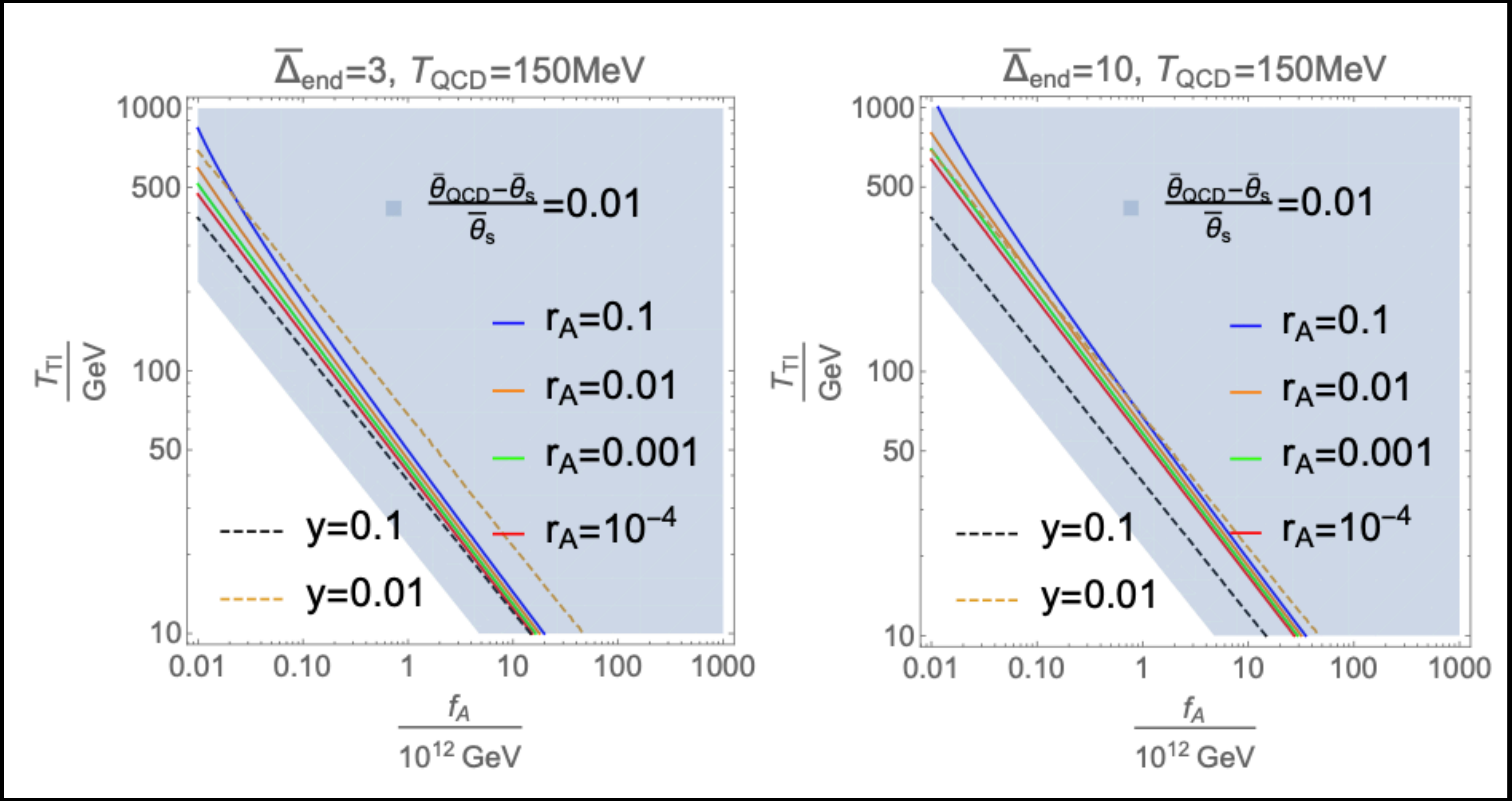
$$\Omega_A \sim 0.3\overline{\theta_{\rm ini}}^{-2}\left(\frac{f_A}{10^{12}\mathrm{GeV}}\right)^{1.16}\left(\frac{150\mathrm{MeV}}{T_{\rm QCD}}\right)^{0.67} \Rightarrow \Omega_A \sim 0.3\overline{\theta_{\rm ini}}^{-2}\left(\frac{f_A}{10^{12}\mathrm{GeV}}\right)^{1.16}\left(\frac{150\mathrm{MeV}}{T_{\rm QCD}}\right)^{0.67} \times e^{-2\eta N_{\rm end}}$$

Conventional result

Results

Cont'd

$N_{end}=3$ $N_{end}=10$



- Solid lines correspond to r_A=const

Blue regions are allowed by Non-Gaussianity constraints

→ Secondary Inflation can actually realize $r_A \sim 10^{-4}$ while explaining CMB anisotropy !

Summary

- We discussed a possibly whether QCD axion can explain the CMB fluctuation
- In general, this scenario has to satisfy the following conditions (at least)

  (i) the CMB amplitude (ii) Isocurvature, and (iii) Non-Gaussianity

- We found that QCD axion can explain the anisotropy if the current abundance is sufficiently small ($\sim 10^{-4}$).

  This requires additional dilution in the early universe, such as secondary inflation

- Future works:
  - Is it possible to construct another scenario that is compatible with axion DM ?
  - Construct a concrete model of secondary inflation [K.K, S. Iso, K. Shimada, arXiv:2105.06803]
  - Dark Sector, string axion, and more

Thank you for your attention !

Backup Slides

Gauge slice (choice)

- We always have gauge degrees of freedom: $x'^{\mu} = f(x)$
- Correspondingly, operators change by their Lie derivative at the same coordinate point x

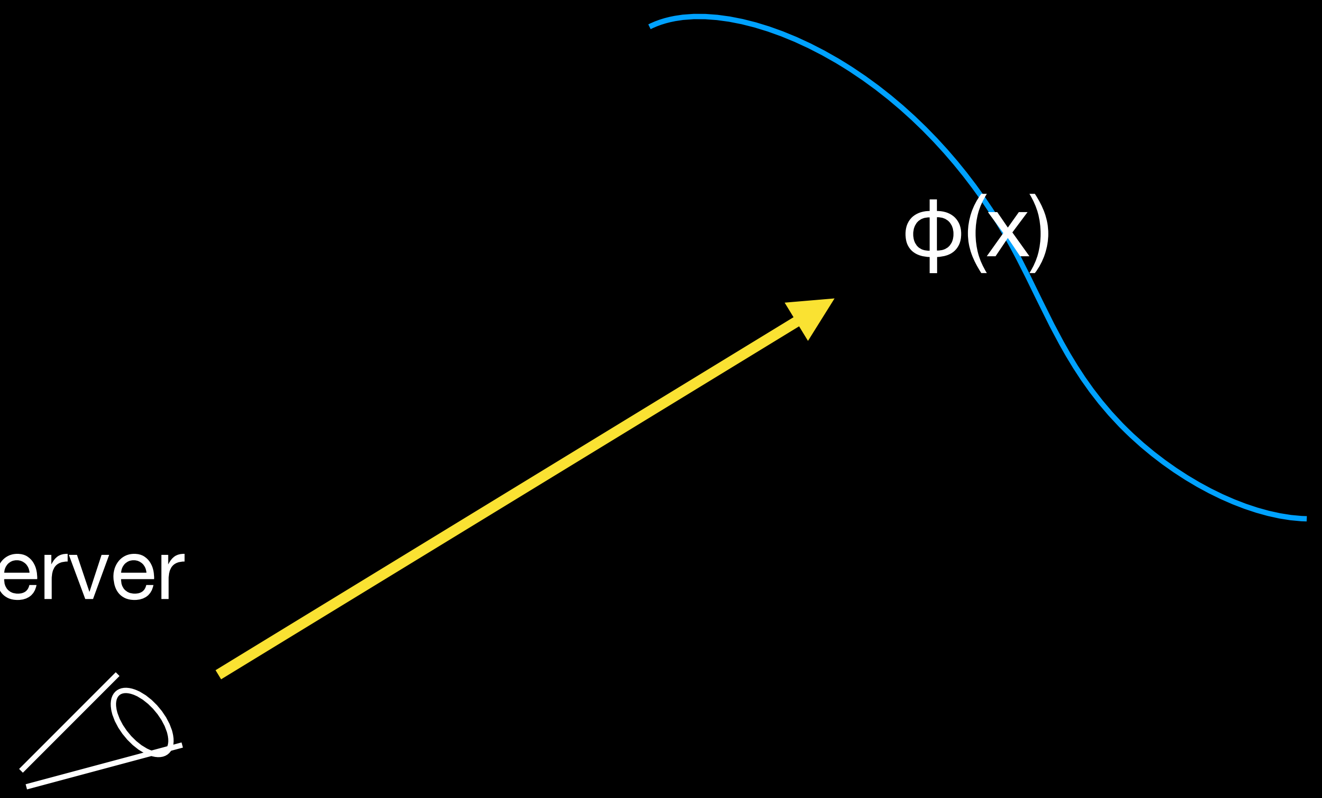
$$Q'(x) - Q(x) := \delta Q = \mathscr{L}_{\delta x} Q, \quad e.g.\ \delta\phi = -\delta x^{\mu}\partial_{\mu}\phi \ \text{(for scalar quantity)}$$

- Typically, a scalar quantity is set to zero by using gauge dof

  e.g.

  1. Uniform density slice ↔ total energy density has no fluctuation $\delta\rho=0$
  2. Spatially flat gauge ↔ no fluctuation of scale factor, $\psi=0$
  3. Comoving slicing (during inflation) ↔ no fluctuation of inflaton $\delta\phi=0$

Sachs-Wolfe Effects



- There are two contributions to temperature fluctuation
- One is the gravitational blue(red)-shift $\left.\frac{\delta T}{T}\right|_{rec} = \Phi(\vec{n} r_L)\cdots(1)$
- The other is the change of scale factor (time dilation)

$$ds^2 = -(1+2\Phi)dt^2 + \cdots$$

$$\therefore\ \delta t = \Phi t$$

$$\left.\frac{\delta T}{T}\right|_{\mathrm{rec}} = -\left.\frac{\delta a}{a}\right|_{\mathrm{rec}} = -H\delta t = -\underline{\frac{2}{3t}} \times \left. t\Phi(\vec{n} r_L)\right|_{\mathrm{rec}} = -\frac{2}{3}\Phi(\vec{n} r_L)\cdots(2)$$

Matter dominated

$$\therefore\ (1) + (2) = \frac{1}{3}\Phi(\vec{n} r_L)$$

(Sachs-Wolfe Effects)

Gravitational Potential and Curvature Perturbation

- Define $\delta_D := \delta\rho_{\rm DM}/\bar{\rho}_{\rm DM}$
- Linear order solution during matter dominated era (in Synchronous gauge)

$$\delta_{\mathrm{D}q} = \frac{9q^2t^2}{10a^2}\zeta_q$$

- On the other hand, from the Poisson equation

$$\nabla^2\Phi = 4\pi G\delta\rho_{\rm DM} \to \Phi_q = -(a/q)^2 4\pi G\delta\rho_{\mathrm{D}q} = -(a/q)^2 4\pi G\bar{\rho}_{\rm DM}\delta_{\mathrm{D}q}$$

- Using Friedman equation $H^2 = (2/3t)^2 = 8\pi G\bar{\rho}_{\rm DM}/3$

$$\therefore\ \Phi_q = -\frac{3}{5}\zeta_q\ \text{(matter dominated era)}$$

Conservation of ζ_X

- Consider super horizon mode (i.e. neglect space dependence), and take the linear order perturbation of energy conservation low

$$\dot{\rho}_X + 3H(\rho_X + p_X) = 0 \ \rightarrow \ \delta\dot{\rho}_X + 3\bar{H}(\delta\rho_X + \delta p_X) + 3\dot{\psi}(\bar{\rho}_X + \bar{p}_X) = 0$$

- Taking time derivative of ζ_X and using the above equation, we have

$$\frac{d\zeta_X}{dt} = \frac{d}{dt}\left[-\psi - \frac{\delta\rho_X}{3(\bar{\rho}+\bar{p})}\right] = \frac{\bar{H}}{\bar{\rho}_X + \bar{p}_X}\left[\delta p_X - \frac{\dot{\bar{p}}_X}{\dot{\bar{\rho}}_X}\delta\rho_X\right] \quad \text{(for superhorizon mode)}$$

- When X is a barotropic fluid i.e. p=p(ρ), we have

$$\frac{\dot{\bar{p}}_X}{\dot{\bar{\rho}}_X} = \frac{\delta p_X}{\delta\rho_X}\cancel{\frac{\dot{\bar{\rho}}_X}{\dot{\bar{\rho}}_X}} = \frac{\delta p_X}{\delta\rho_X} \ \rightarrow \ \therefore \ \frac{d\zeta_X}{dt} = 0 \quad \text{(for superhorizon modes)}$$

- This was also proven non-linearly by Lyth, Malik and Sasaki ('05)

Isocurvature contributions

- At recombination, the uniform density curvature perturbation is

$$\zeta = \frac{\sum_X (1+\omega_X)\Omega_X\zeta_X}{1+\omega_{\rm tot}} = \frac{(4\Omega_r + 3\Omega_m + 3\Omega_b)\zeta_r + 3\Omega_A\zeta_A}{4\Omega_r + 3\Omega_m + 3\Omega_b + 3\Omega_A} = \zeta_r + \frac{3\Omega_A}{4\Omega_r + 3\Omega_m + 3\Omega_b + 3\Omega_A}(\zeta_A - \zeta_r) \sim \zeta_r + \frac{\Omega_A}{\Omega_m + \Omega_A}(\zeta_A - \zeta_r)$$

$$\sim -\frac{r_A}{3}S_{A\gamma}$$

Add and subtract $3\Omega_A\zeta_r$ in numerator

Matter dominated

- Thus, the temperature fluctuation on superhorizon scale is

$$\frac{\delta T}{T} \sim -\zeta_r - 2\Phi = -\zeta_r + \frac{6}{5}\zeta = \frac{1}{5}\zeta_r - \frac{2r_A}{5}S_{A\gamma} \quad \text{(for superhorizon scale)}$$

∴ Isocurvature is constrained by the combination $\mathscr{I} := r_A S_{A\gamma}$

Consistency check

- Inflation gives $$A_s^{\rm inf} = \frac{V}{24\pi^2 M_{pl}^4 \epsilon} \sim \frac{H_{\rm inf}^2}{8\pi^2 \epsilon M_{pl}^2}$$ ϵ : slow roll parameter

- Our scenario gives $$A_s^{\rm QCD} \sim \frac{R^2 H_{\rm inf}^2}{16\pi^2 f_A^2 \bar{\theta}^2}$$

We can easily evade this

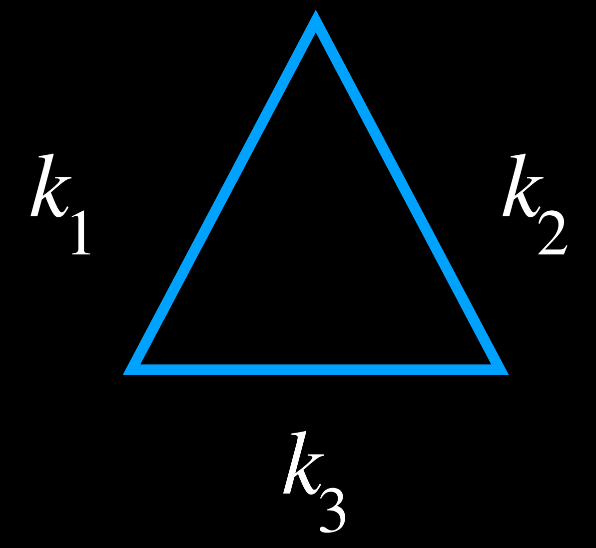
$$\therefore \quad A_s^{\rm inf} \ll A_s^{\rm QCD} \quad \rightarrow \quad \frac{1}{2}\left(\frac{f_A \bar{\theta}}{R M_{pl}}\right)^2 \ll \epsilon$$

For example, if we choose $f_A=10^{12}$GeV, θ=1, R=0.01, this becomes $8.5 \times 10^{-10} \ll \epsilon$

On the other hand, the current upper bound is $$\epsilon \lesssim \frac{0.05}{16} \sim 0.003$$

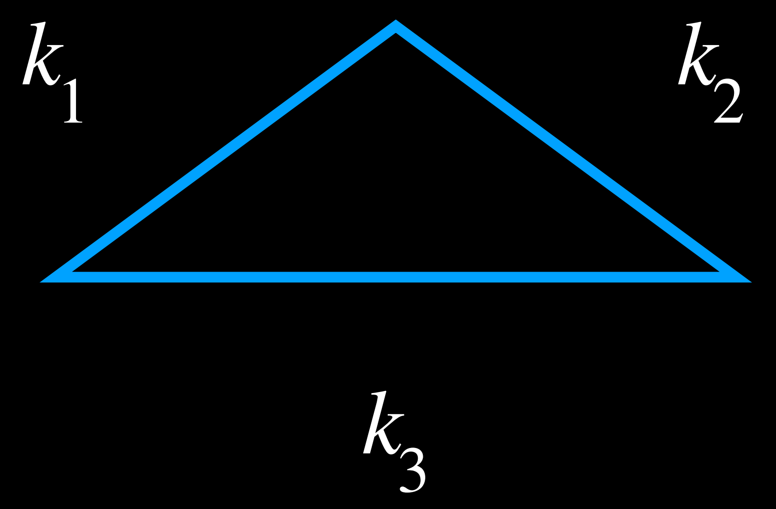
Other Non-Gaussianities

- Equilateral type



$$B_\Phi^{\mathrm{equil}}(k_1,k_2,k_3) = 6A^2 f_{\mathrm{NL}}^{\mathrm{equil}} \times \left\{ -\frac{1}{k_1^{4-n_s}k_2^{4-n_s}} - \frac{1}{k_2^{4-n_s}k_3^{4-n_s}} - \frac{1}{k_3^{4-n_s}k_1^{4-n_s}} - \frac{2}{(k_1k_2k_3)^{2(4-n_s)/3}} + \left[\frac{1}{k_1^{(4-n_s)/3}k_2^{2(4-n_s)/3}k_3^{4-n_s}} + 5\ \text{perms.} \right] \right\}, \quad (1)$$

- Orthogonal type



$$B_\Phi^{\mathrm{ortho}}(k_1,k_2,k_3) = 6A^2 f_{\mathrm{NL}}^{\mathrm{ortho}} \times \left\{ -\frac{3}{k_1^{4-n_s}k_2^{4-n_s}} - \frac{3}{k_2^{4-n_s}k_3^{4-n_s}} - \frac{3}{k_3^{4-n_s}k_1^{4-n_s}} - \frac{8}{(k_1k_2k_3)^{2(4-n_s)/3}} + \left[\frac{3}{k_1^{(4-n_s)/3}k_2^{2(4-n_s)/3}k_3^{4-n_s}} + 5\ \text{perms.} \right] \right\}. \quad (2)$$

Calculations of Non-Gaussianity

$$
\begin{aligned}
\zeta_r = {} & \frac{R}{4}\frac{\bar{V}_A'}{\bar{V}_A}\bigg|_{\mathrm{QCD}} \delta A_{\mathrm{QCD}} + \frac{R}{8}\left[\frac{\bar{V}_A''}{\bar{V}_A} + R\left(\frac{\bar{V}_A'}{\bar{V}_A}\right)^2\right]_{\mathrm{QCD}} (\delta A_{\mathrm{QCD}})^2 \\
& + \frac{R}{12}\left[\frac{\bar{V}_A'''}{2\bar{V}_A} + \frac{3R}{2}\frac{\bar{V}_A'\bar{V}_A''}{\bar{V}_A^2} + R^2\left(\frac{\bar{V}_A'}{\bar{V}_A}\right)^3\right]_{\mathrm{QCD}} (\delta A_{\mathrm{QCD}})^3 + \cdots \qquad (34) \\
= {} & \zeta_{r\mathrm{G}} + \frac{2}{R}\left[\frac{\bar{V}_A''\bar{V}_A}{\bar{V}_A'^2} + R\right]_{\mathrm{QCD}} \zeta_{r\mathrm{G}}^2 + \frac{16}{3R^2}\left[\frac{\bar{V}_A'''\bar{V}_A^2}{2\bar{V}_A'^3} + \frac{3R}{2}\frac{\bar{V}_A''\bar{V}_A}{\bar{V}_A'^2} + R^2\right]_{\mathrm{QCD}} \zeta_{r\mathrm{G}}^3 + \cdots .
\end{aligned}
$$

Gaussian part $\zeta_r \sim \frac{R}{4}\frac{V_A'}{V_A}\bigg|_{A=\bar{A}} \delta A$