Axion mass in antiferromagnetic insulators

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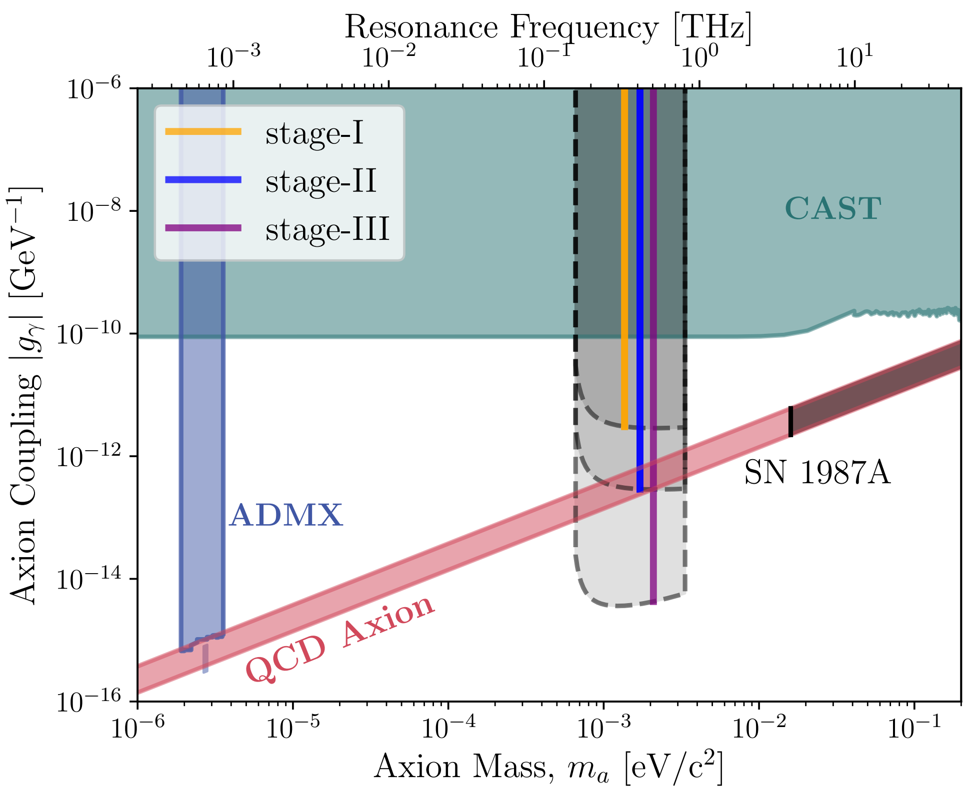
1. Introduction

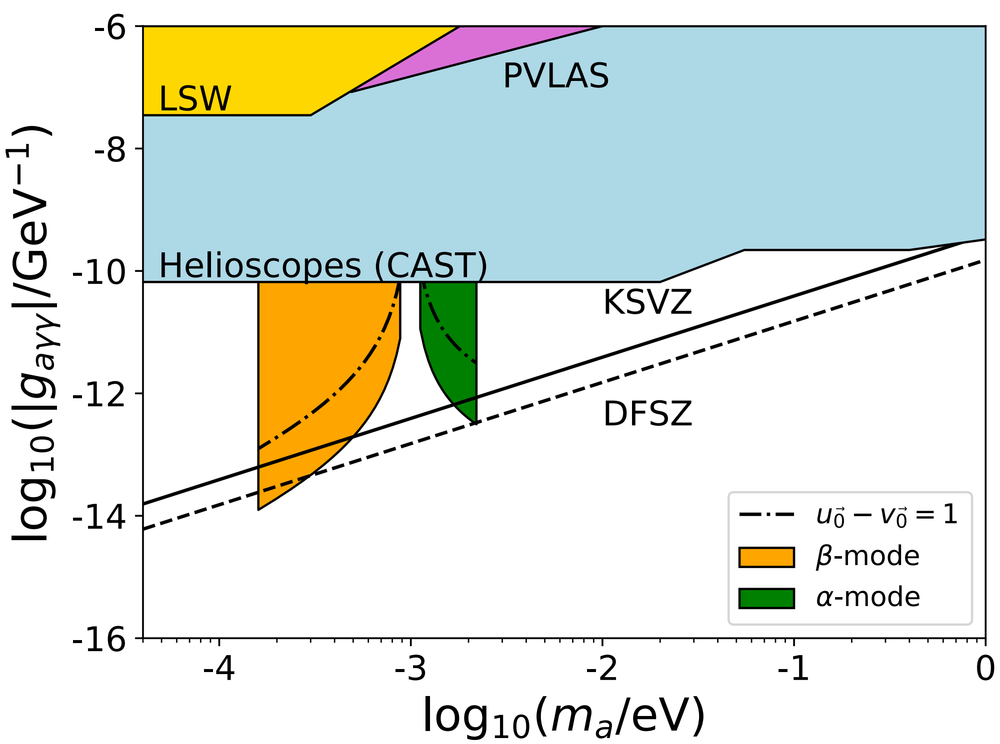
Axion in condensed matter physics

A hot topic and relates to

- Topological insulator
- Axion insulator
- Magnetoelectric effect
- Ferromagnetism / Antiferromagnetism
- Magnon
- It may be used in *particle* axion detection

Proposals to detect axion/axion-like particles (ALPs)





Marsh et al. '19

Chigusa et al. '21

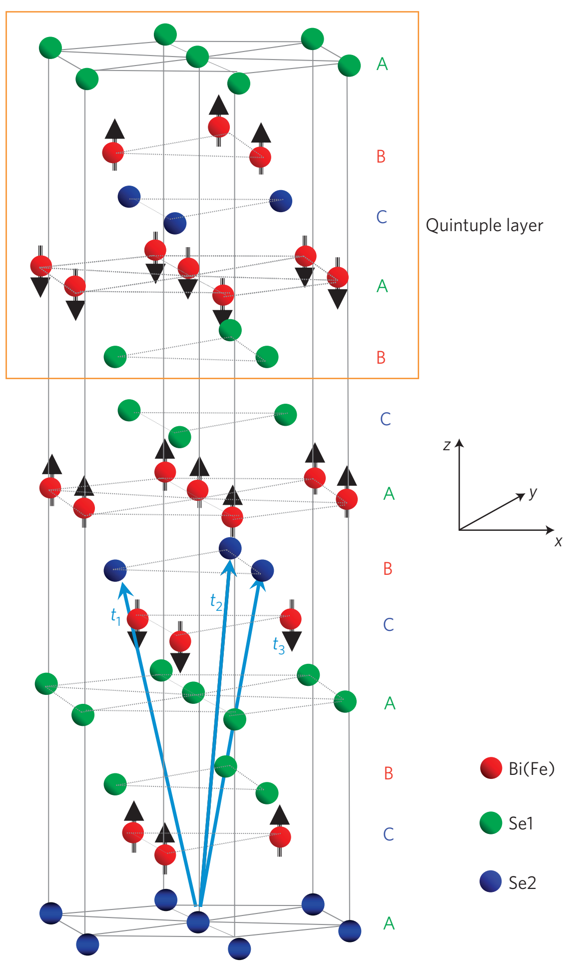
Topological magnetic insulators are used

Dynamical axion is predicted in topological magnetic insulators

ARTICLES
PUBLISHED ONLINE: 7 MARCH 2010 | DOI: 10.1038/NPHYS1534
nature physics

Dynamical axion field in topological magnetic insulators

Rundong Li[1], Jing Wang[1,2], Xiao-Liang Qi[1] and Shou-Cheng Zhang[1]★



Bi_2Se_3 ⟵ Topological insulator

$$\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\begin{aligned}\mathcal{S}_{\rm tot} &= \mathcal{S}_{\rm Maxwell} + \mathcal{S}_{\rm topo} + \mathcal{S}_{\rm axion}\\ &= \frac{1}{8\pi}\int d^3x\,dt\left(\epsilon\mathbf{E}^2 - \frac{1}{\mu}\mathbf{B}^2\right) + \frac{\alpha}{4\pi^2}\int d^3x\,dt\,(\theta_0+\delta\theta)\mathbf{E}\cdot\mathbf{B}\\ &\quad + g^2 J\int d^3x\,dt\,[(\partial_t\delta\theta)^2 - (v_i\partial_i\delta\theta)^2 - m^2\delta\theta^2] \qquad (4)\end{aligned}$$

Axion mass ~ $\mathcal{O}(\mathrm{meV})$

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Dynamical axion field in topological magnetic insulators

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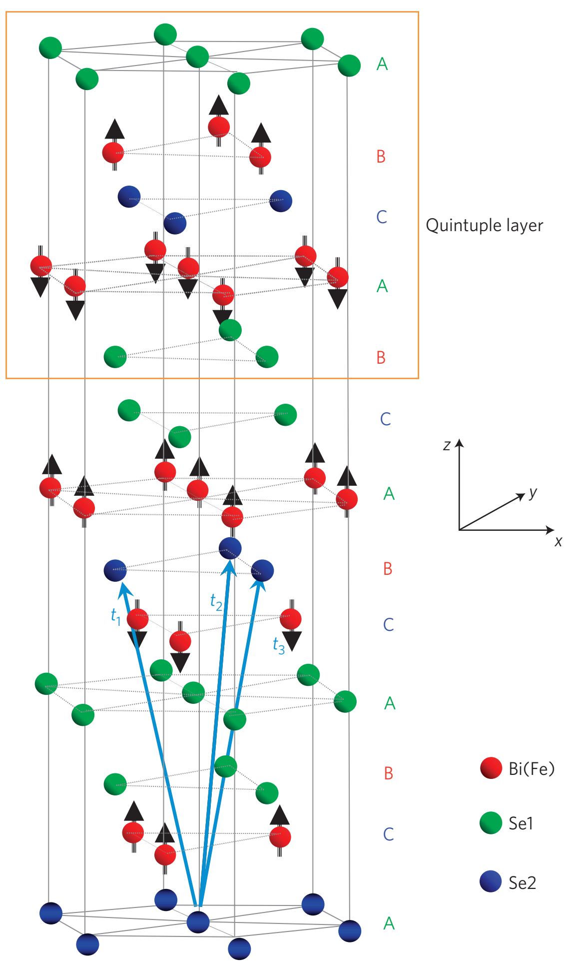
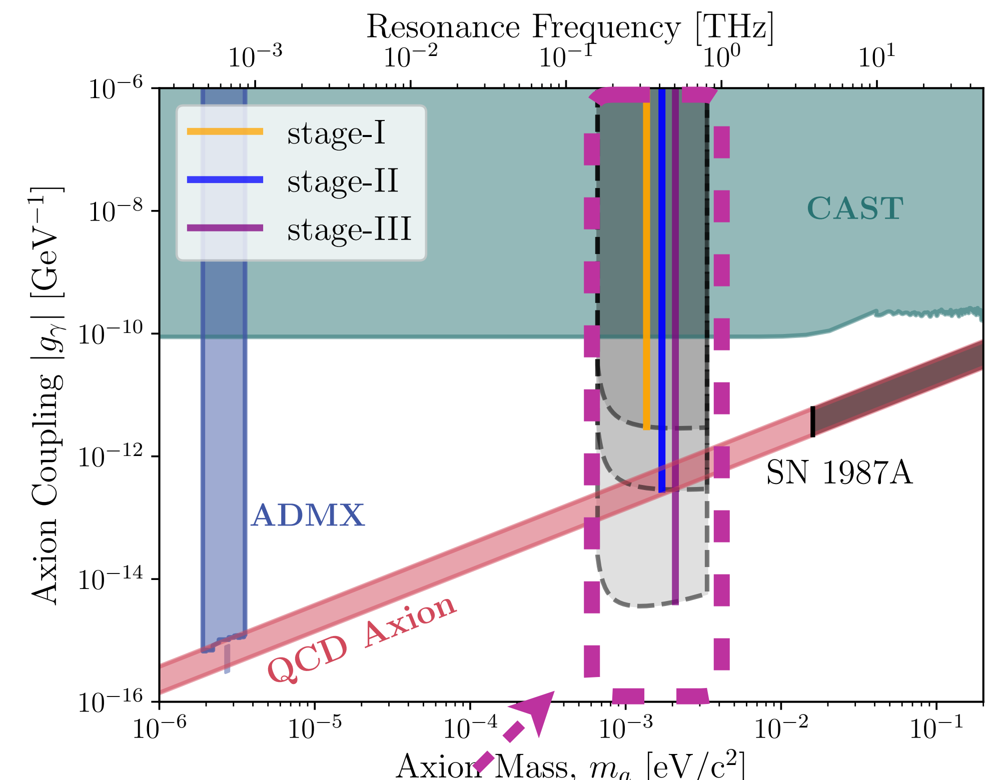




Marsh et al. '19

$\mathcal{O}(\mathrm{meV})$

Today, I would like to address

- What is topological insulator?
- How does antiferromagnetism play a role?
- How is axion in condensed matter described?

Method: Path integral (partition function)

Results: Axion mass depends on the state of insulators

Plan to talk

1. Introduction

2. Brief review of condensed matter physics (related to axion)

3. Axion in antiferromagnetic topological insulators

4. Conclusions and discussion

2. Brief review of condensed matter physics (related to axion)

Topics related to axion in condensed matter physics

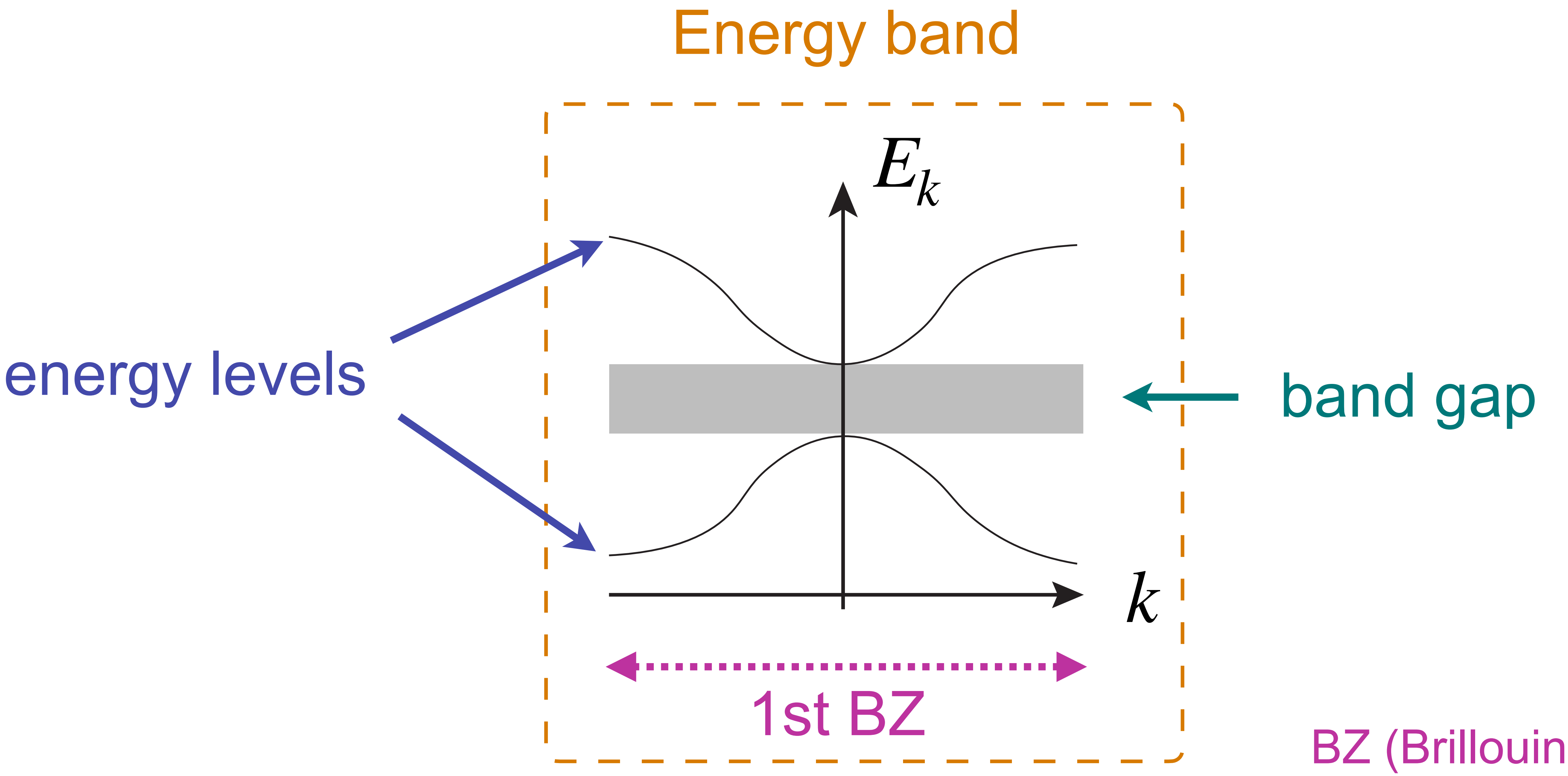
a). Insulators

b). Anomalous quantum Hall effect

c). Topological insulators

d). Magnetoelectric effect

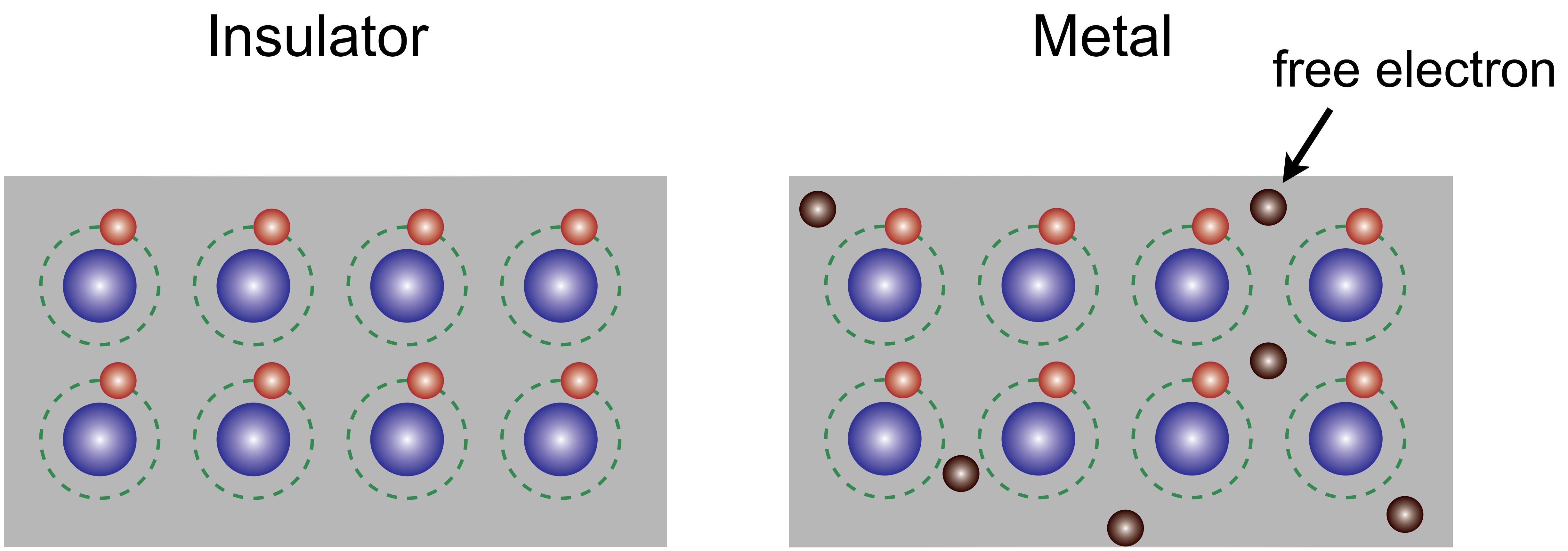
Minimum basics



BZ (Brillouin zone)

The region where there is no energy level is called band gap
(important for insulators)

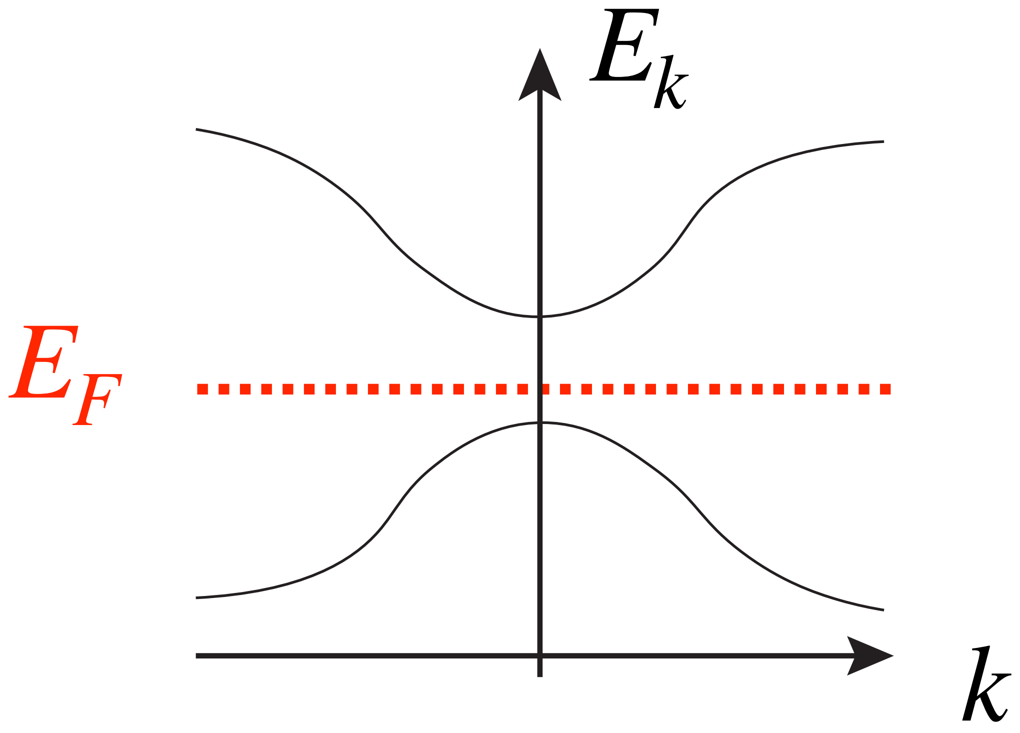
Insulators (and metals)



All electrons are tightly bound by nucleus

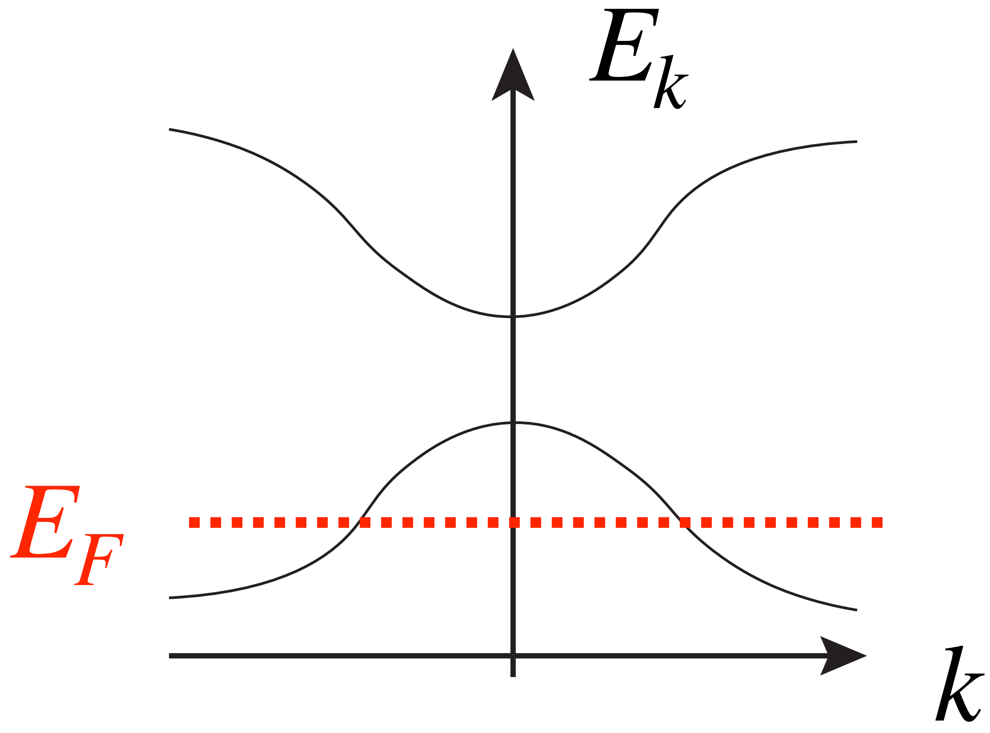
Free electrons exist

Insulator



Fermi energy (level) is in the band gap

Metal

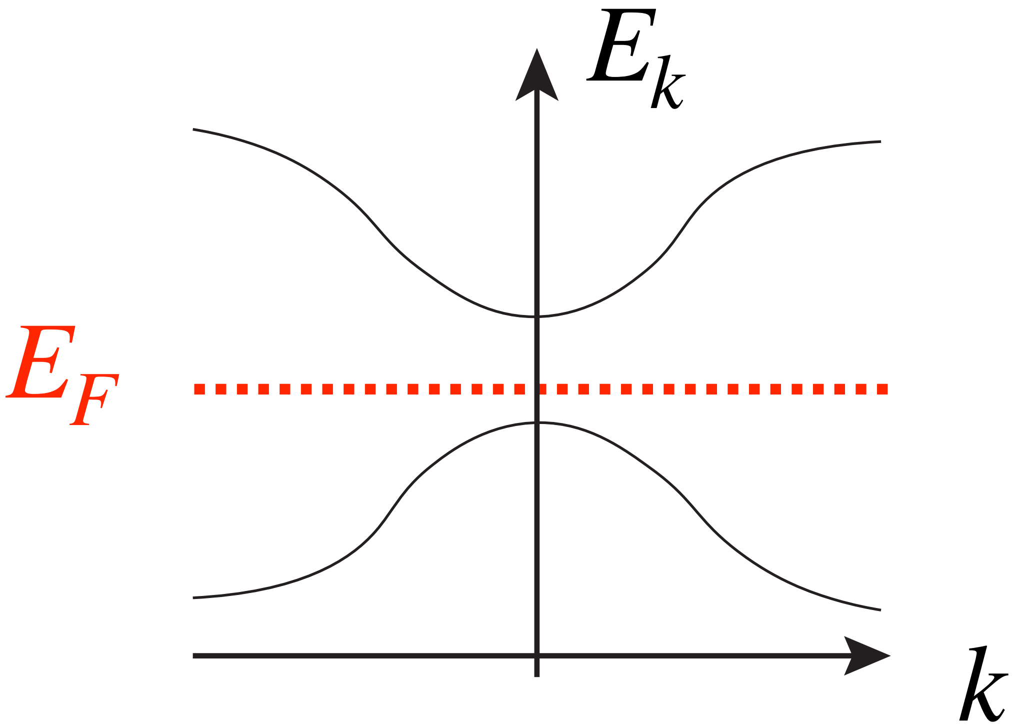


Fermi energy (level) is in the band

E_F : Fermi energy

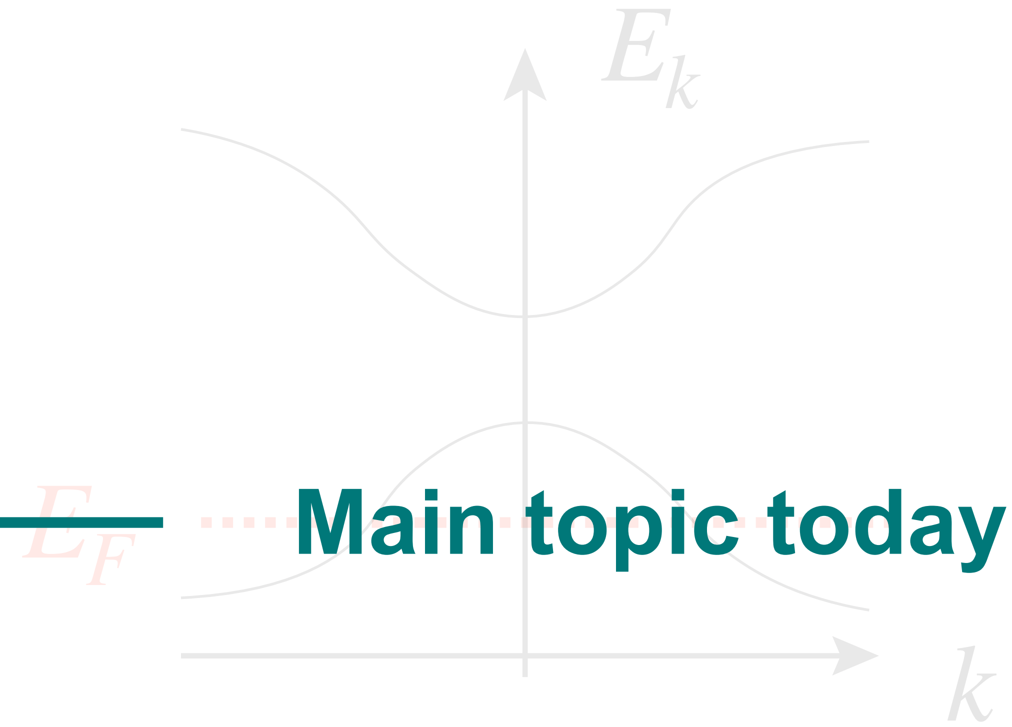
Electrons around the Fermi energy determines the features of the materials

Insulator



Fermi energy (level) is in the band gap

Metal



← **Main topic today**

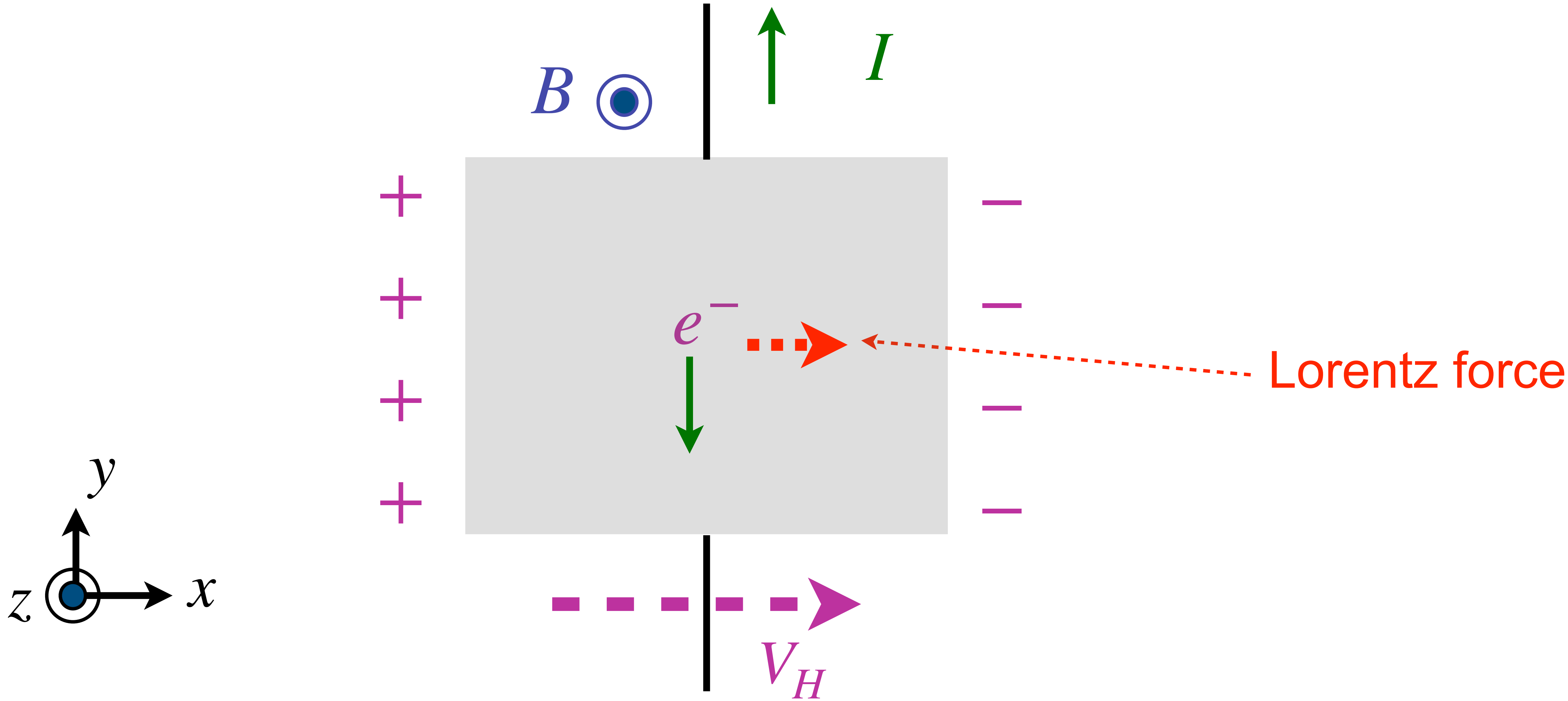
Fermi energy (level) is in the band

E_F : Fermi energy

Electrons around the Fermi energy determines the features of the materials

Hall effect

e.g., semiconductor



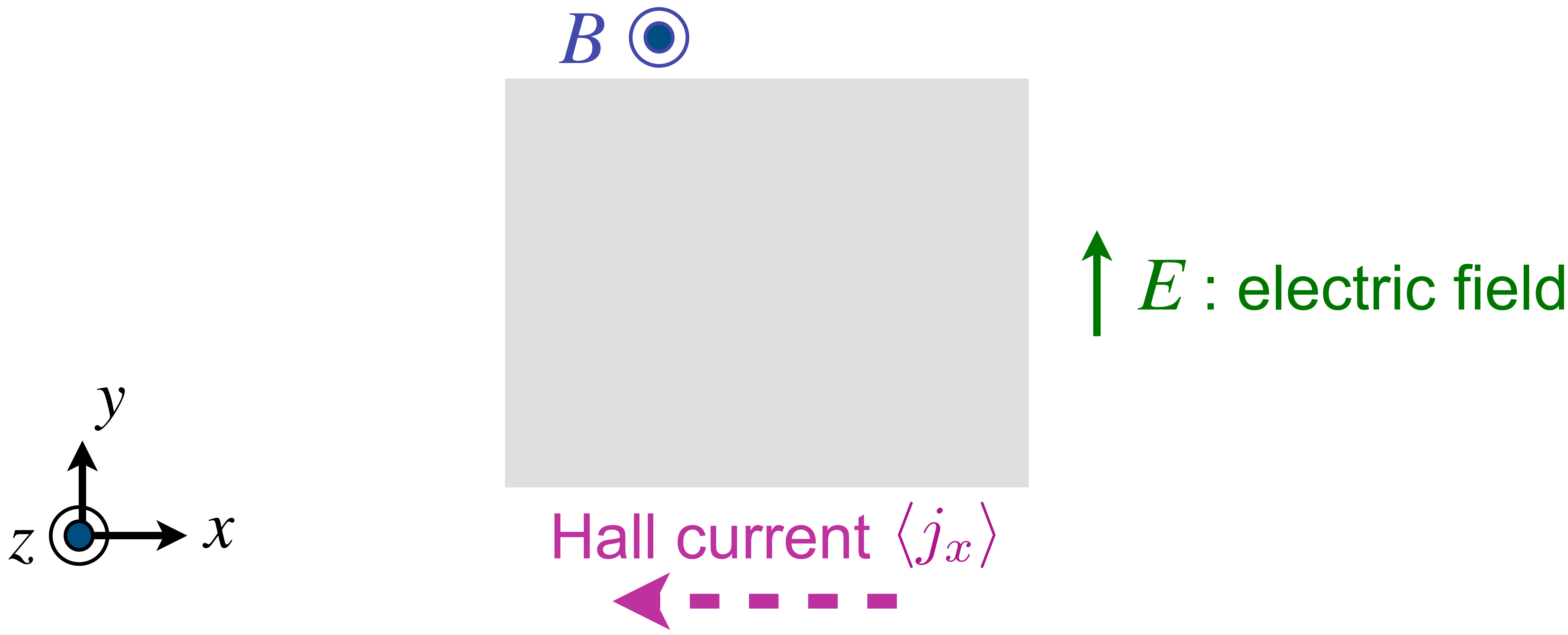
Electromotive force V_H is induced in the direction perpendicular to both the electric current and magnetic field

(In metal, V_H is too small to observe)

Quantum Hall (QH) effect

e.g., 2D insulator

QH effect appears in semiconductor too



Quantized electric current is induced in x direction

$\langle j_x \rangle$: current density in x direction

It seems weird but there was theoretical prediction:

Thouless, Kohmoto, Nightingale, den Nijs '82

Hall conductivity:

$$\sigma_{xy} \equiv \langle j_x \rangle / E$$
$$= \nu \frac{e^2}{h}$$

TKNN formula

$$\nu \equiv \sum_n \int_{\mathrm{BZ}} \frac{d^2k}{2\pi} [\boldsymbol{\nabla}_{\boldsymbol{k}} \times \boldsymbol{a}_n(\boldsymbol{k})]_z$$

$$\boldsymbol{a}_n(\boldsymbol{k}) \equiv -i \langle u_{n\boldsymbol{k}} | \frac{\partial}{\partial \boldsymbol{k}} | u_{n\boldsymbol{k}} \rangle$$

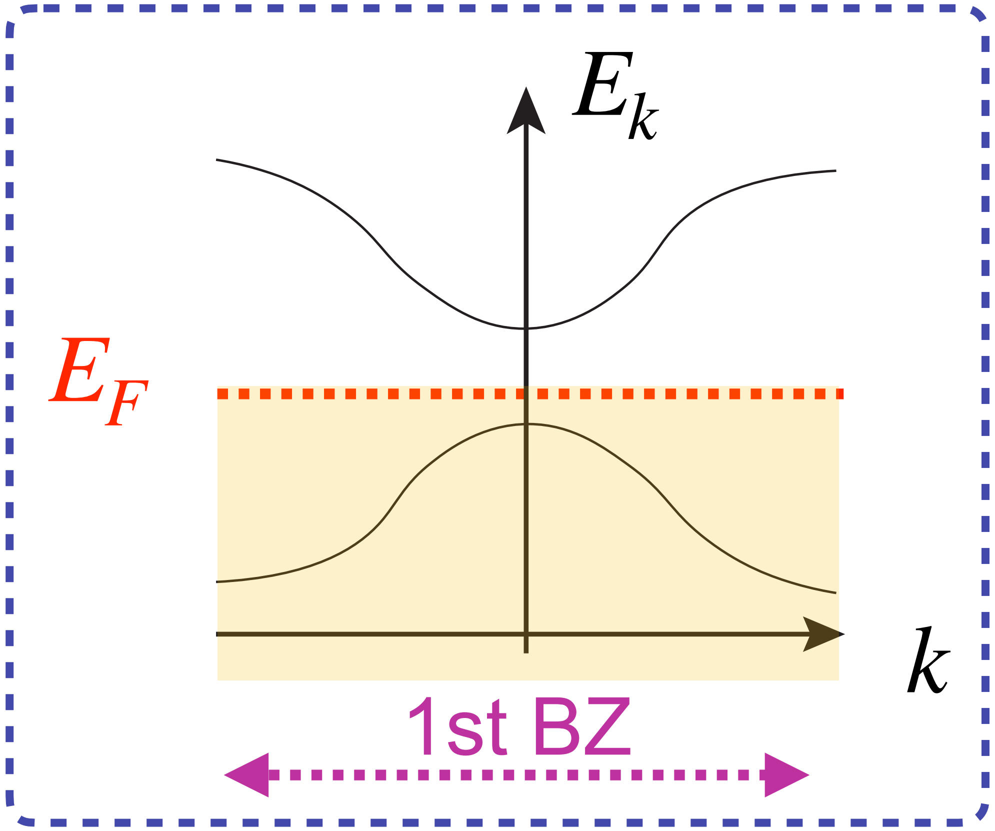
$|u_{n\boldsymbol{k}}\rangle$: Bloch state

n : label of band

For insulators

$$\nu \equiv \sum_n \int_{\mathrm{BZ}} \frac{d^2k}{2\pi} [\boldsymbol{\nabla}_{\boldsymbol{k}} \times \boldsymbol{a}_n(\boldsymbol{k})]_z$$

integral over 1st BZ



→ ν is given by an integer

"(Integer) QH effect"

e.g.,

$$H = \sum_{a=1}^{3} d^a \sigma^a$$

with

$$d^1 = \sin k_x$$
$$d^2 = \sin k_y$$
$$d^3 = m + 2 - \cos k_x - \cos k_y$$

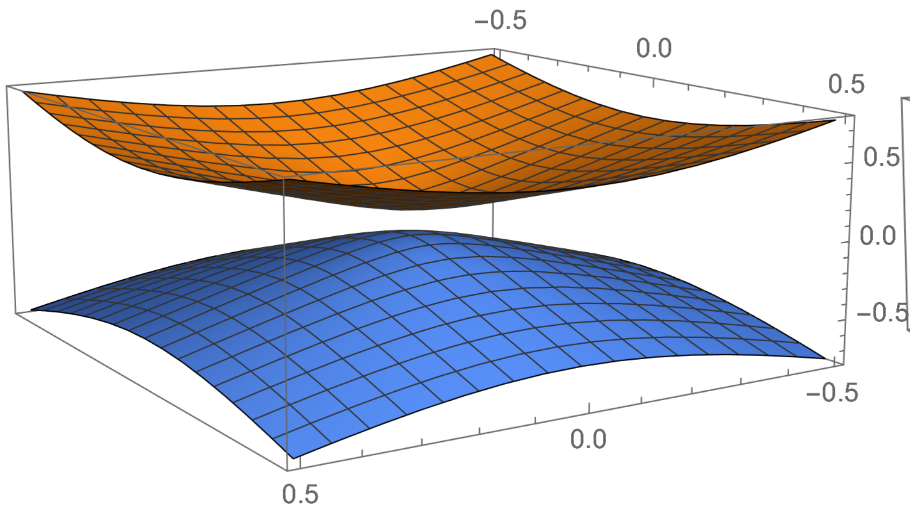
→ Two eigenvalues: $\pm\sqrt{\sum_{a=1}^{3} (d^a)^2}$

$$H \simeq \begin{pmatrix} m & k_x - ik_y \\ k_x + ik_y & -m \end{pmatrix}$$

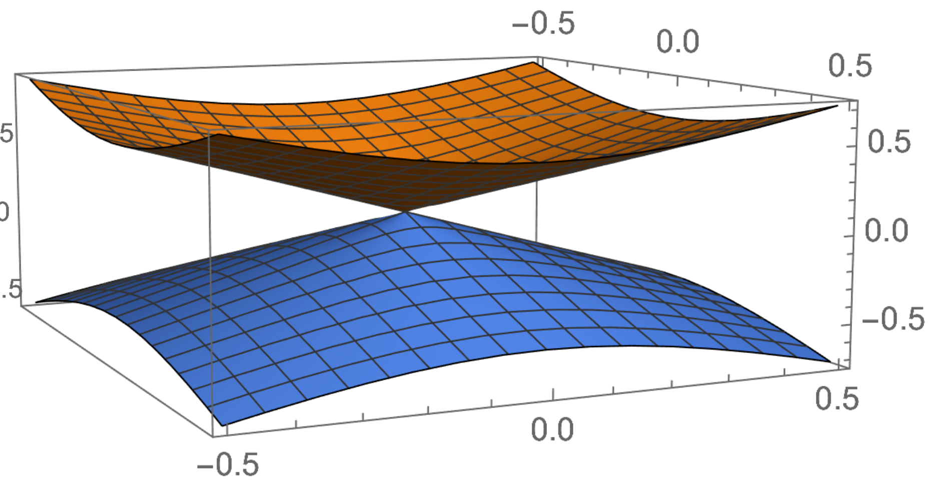
around $\boldsymbol{k} = 0$

The band structure

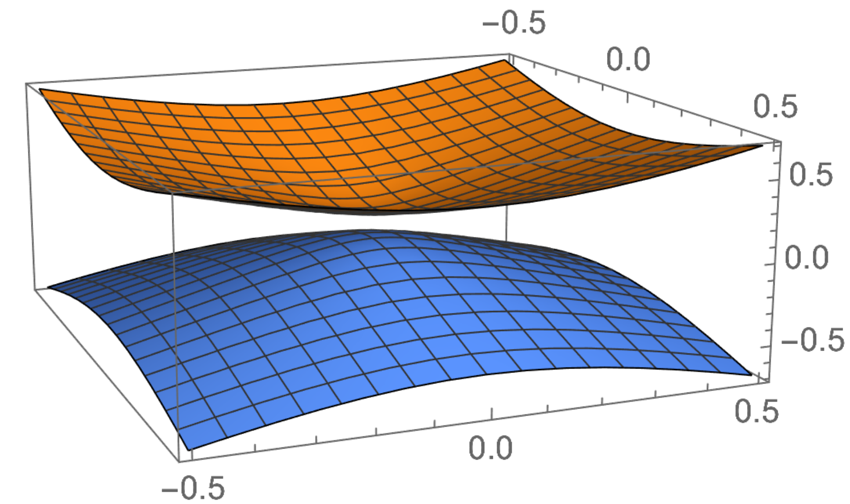
$m = 0.1$



$m = 0$

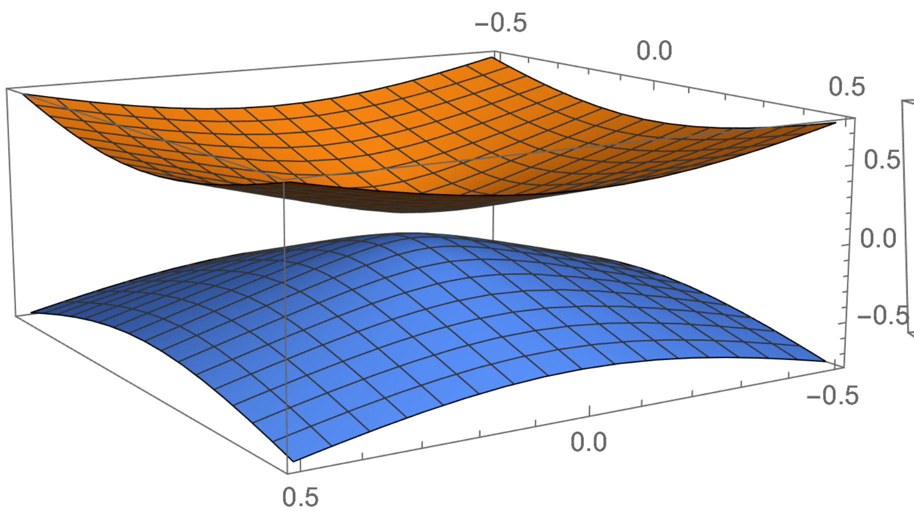


$m = -0.1$



The band structure

$m = 0.1$



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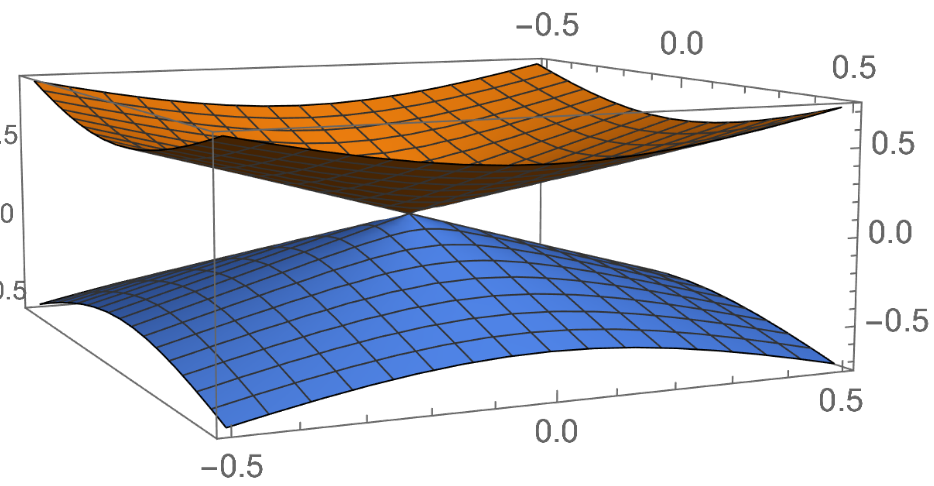

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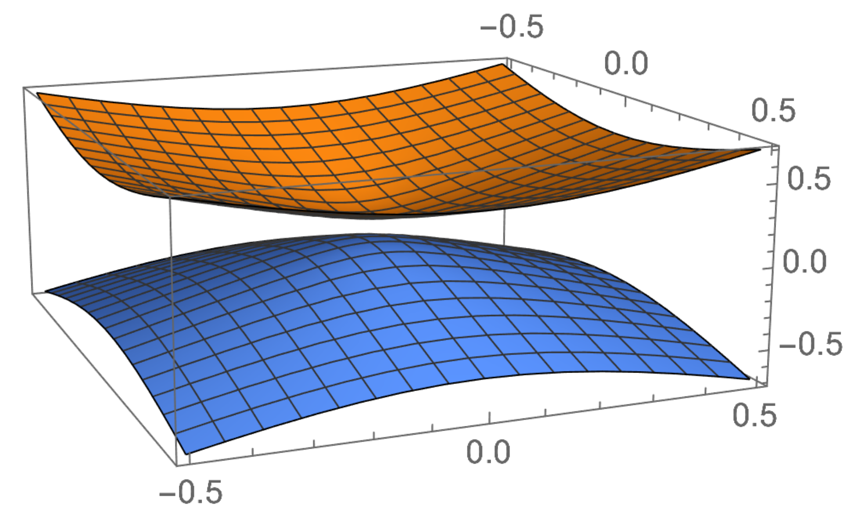


$\nu = 0$

Normal insulator

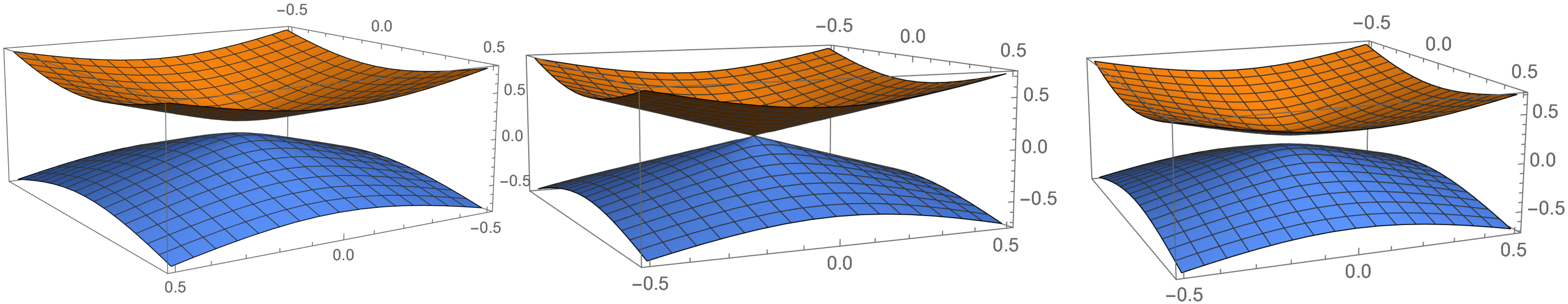
Band inversion around k=0

$\nu = 1$

QH insulator

The band structure

$m = 0.1$ $m = 0$ $m = -0.1$



$\nu = 0$ → $\nu = 1$

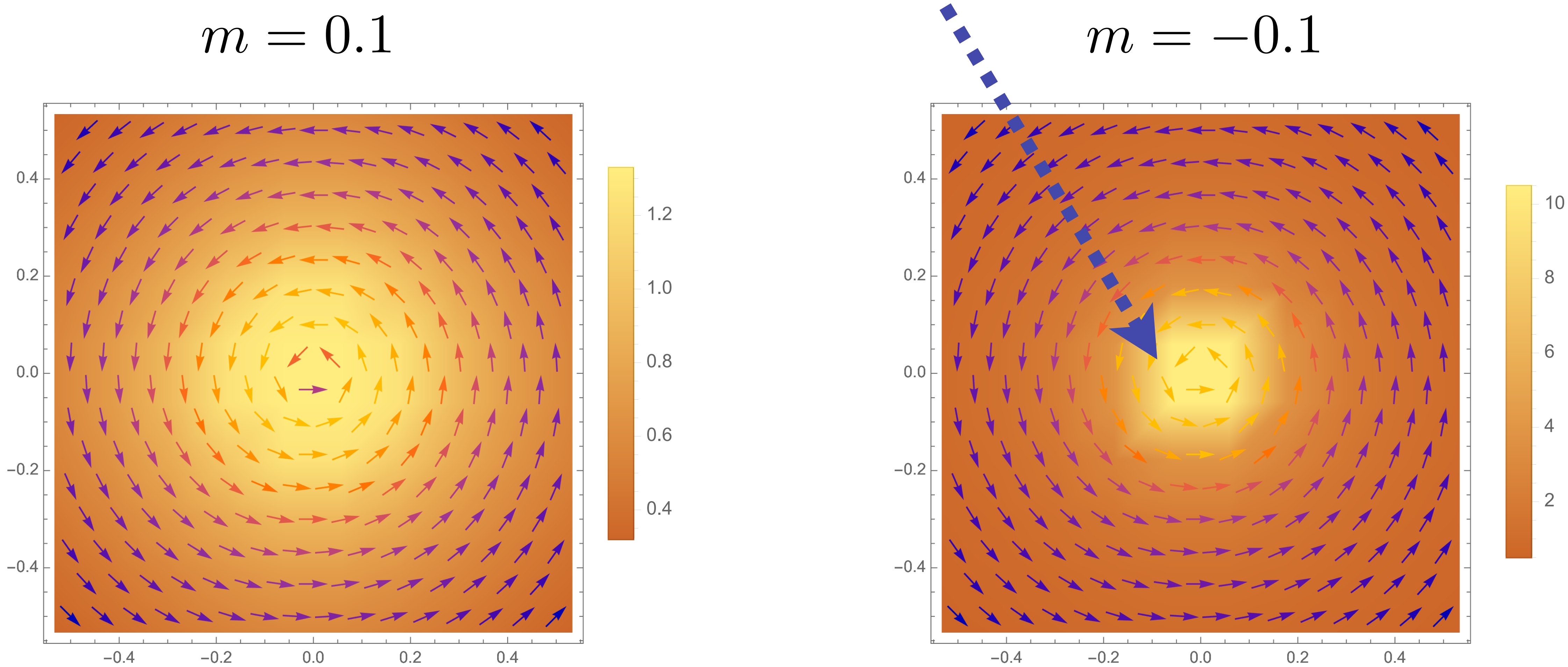
Normal insulator — Band inversion around k=0 — QH insulator

$$H \simeq \begin{pmatrix} m & k_x - ik_y \\ k_x + ik_y & -m \end{pmatrix}$$

around $\boldsymbol{k} = 0$

$\boldsymbol{a}(\boldsymbol{k})$

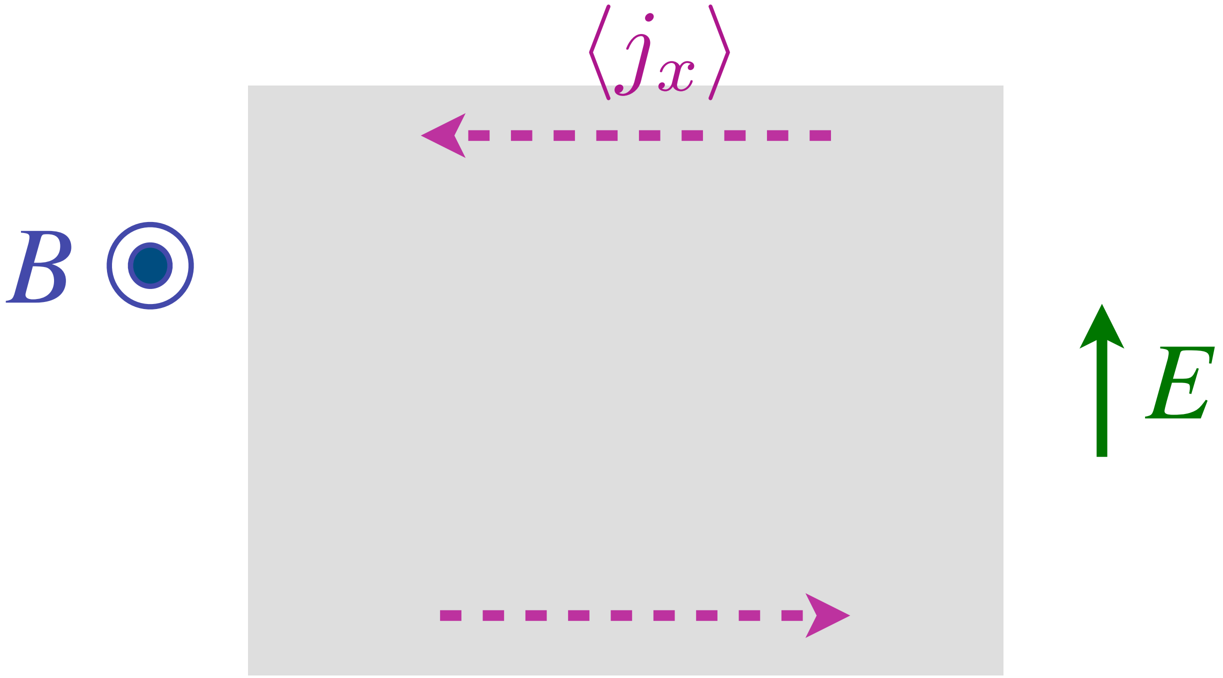
Singular at $\boldsymbol{k} = 0$



→ Integer ν

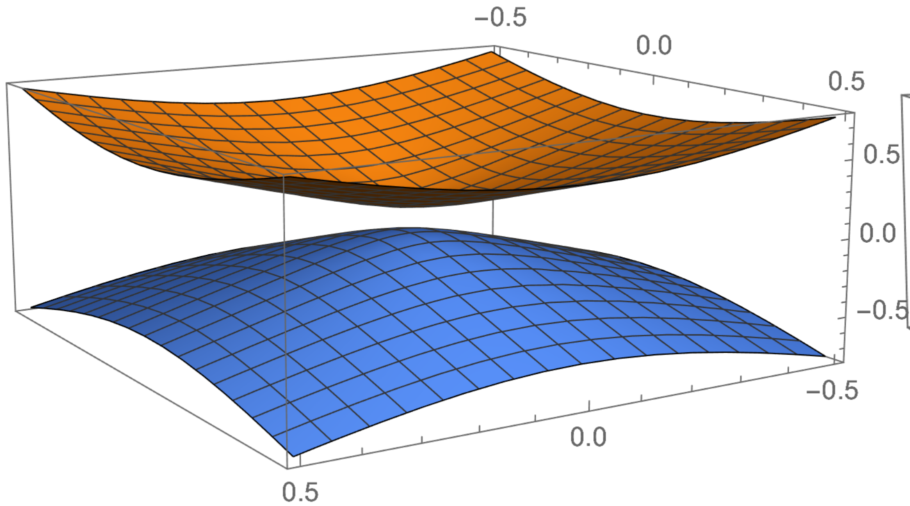
Note: ν is half-integer if the dispersion relation is Dirac type

The current flows at the edge

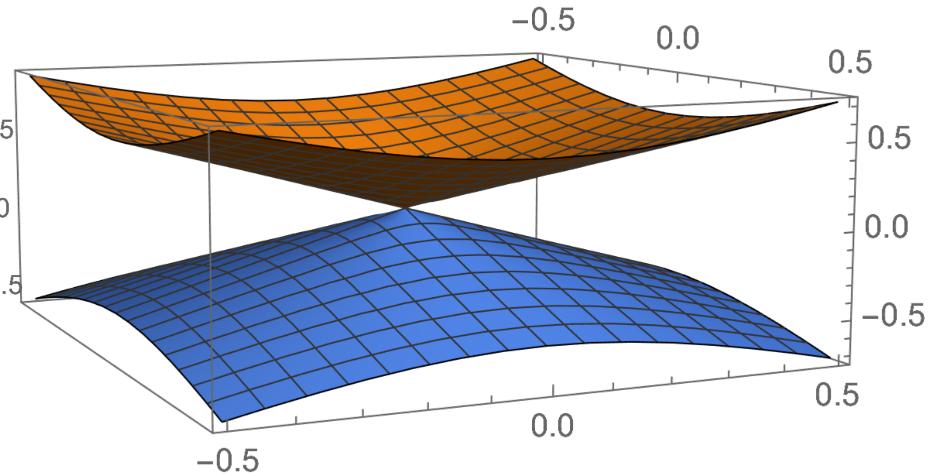


The band structure (bulk)

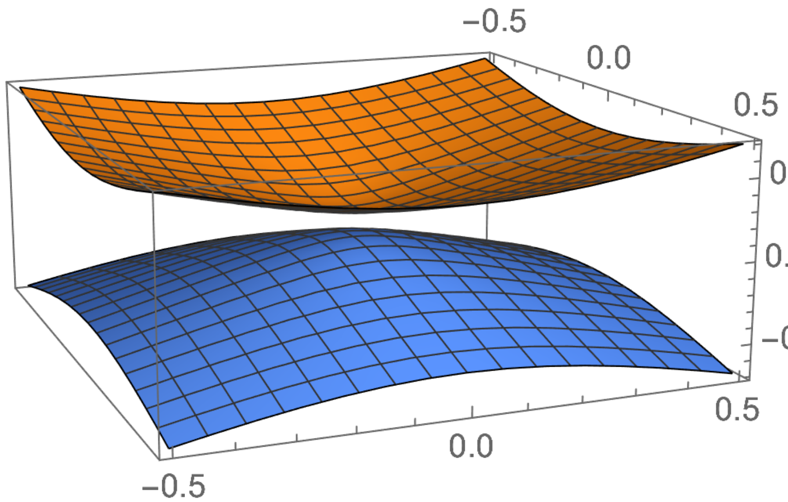
$m = 0.1$



$m = 0$



$m = -0.1$



$\nu = 0$

Normal insulator

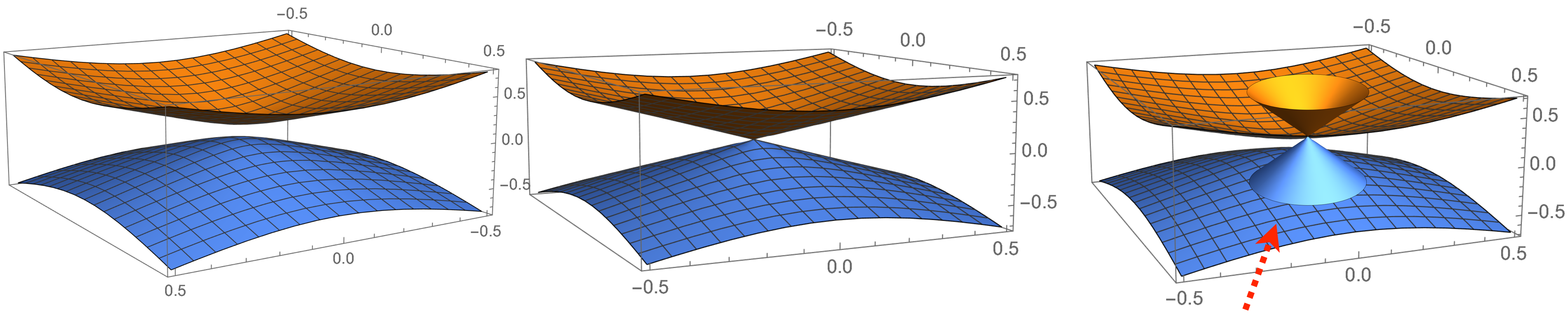
Band inversion
around k=0

$\nu = 1$

QH insulator

The band structure (bulk + edge)

$m = 0.1$ $m = 0$ $m = -0.1$



$\nu = 0$
Normal insulator

Band inversion
around k=0

$\nu = 1$
QH insulator

Gapless state at edge

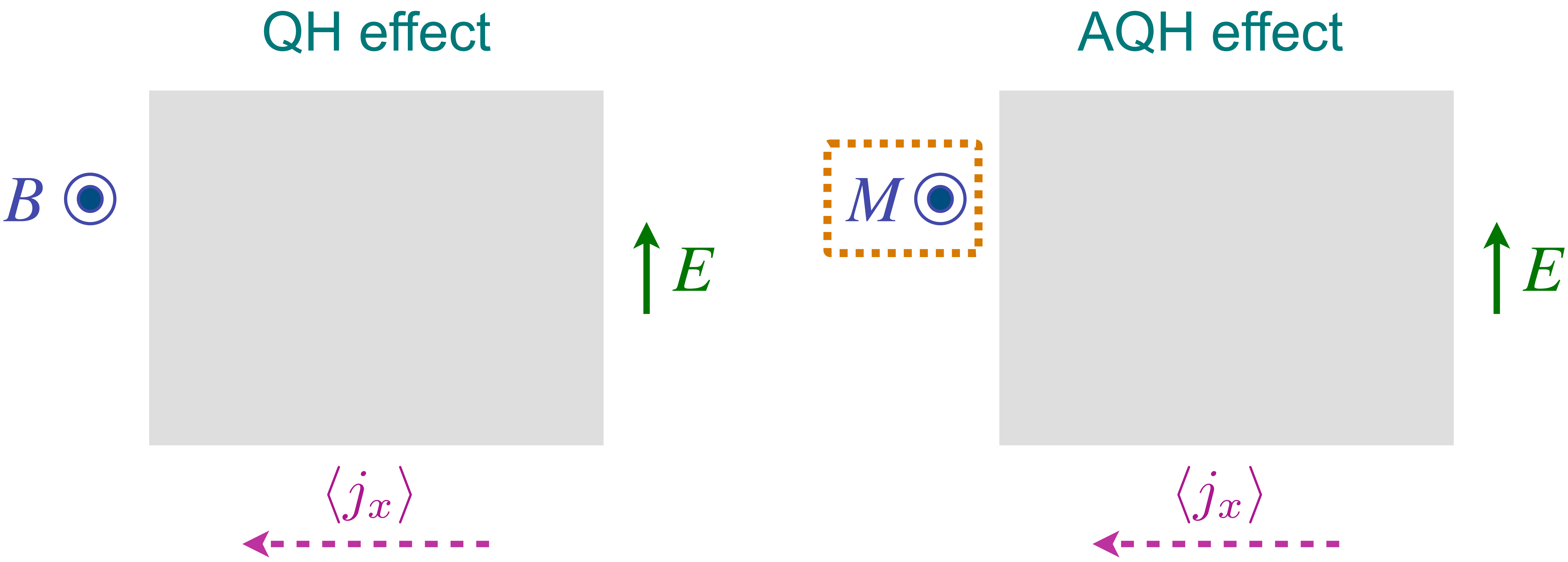
$E \sim \pm k$

"Dirac point"

Anomalous quantum Hall (AQH) effect

Same effect by magnetization M, not magnetic field

⟶ *Anomalous* QH effect



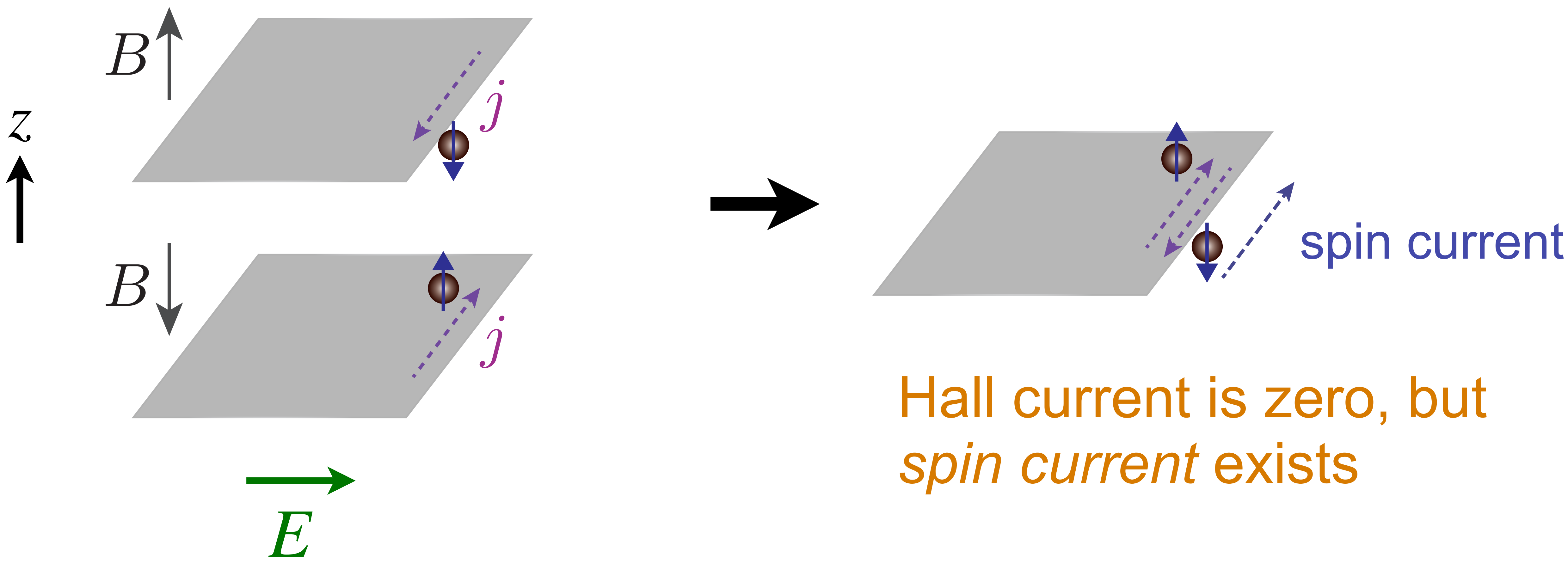
"AQH insulators"

"Chern insulators"

Topological insulators (TIs)

Kane, Mele '05

Idea: combination of two QH insulators



Hall current is zero, but
spin current exists

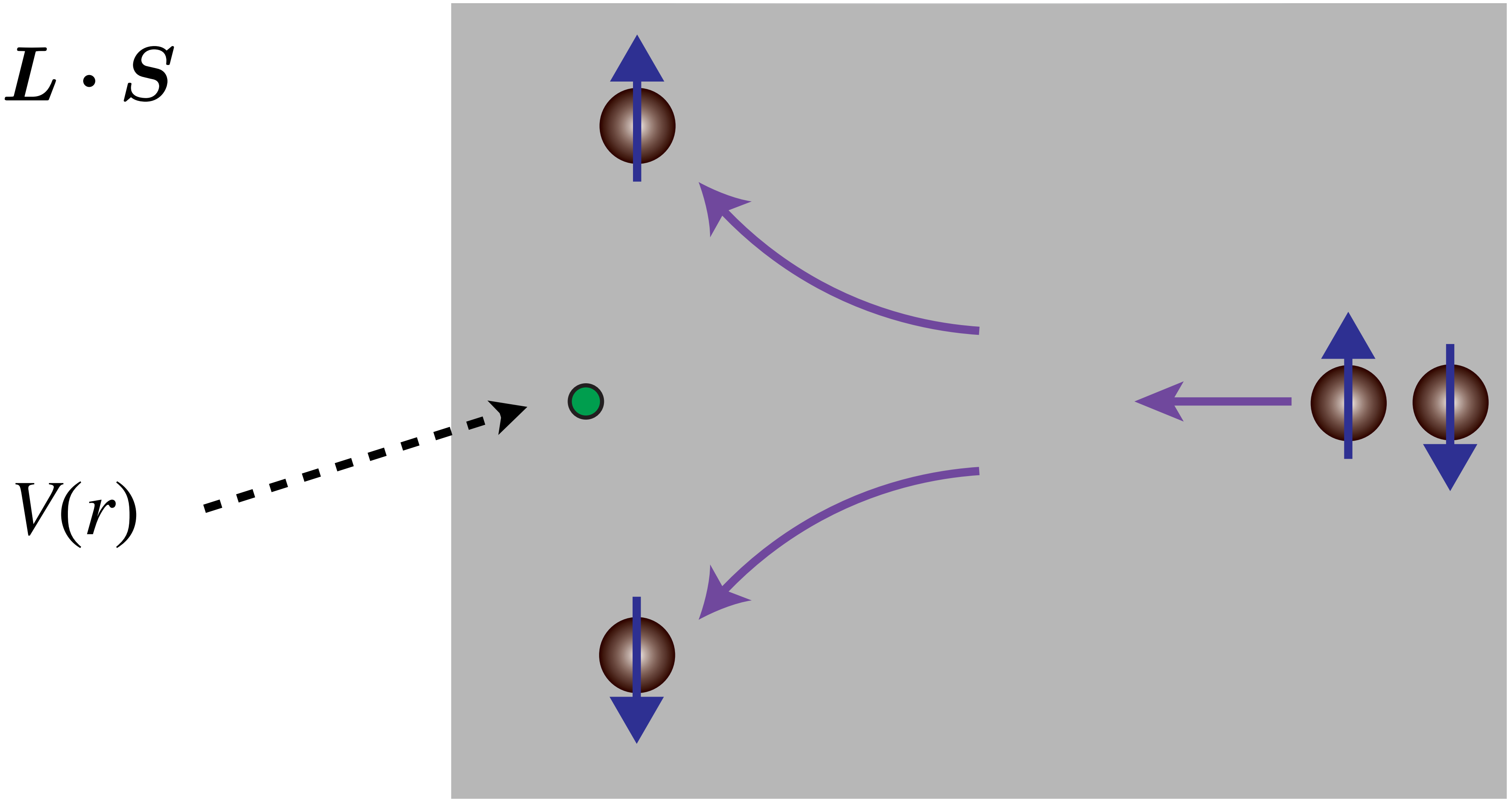
Such a system can be realized due to SOC
(without magnetic field)

SOC: spin orbit coupling

Spin-orbit coupling (SOC)

e.g.,

$$H_{\mathrm{SO}} = V(r)\boldsymbol{L}\cdot\boldsymbol{S}$$



Electrons with spin up or down are scattered off to the opposite directions

Keywords for topological insulators

- Time reversal invariance ($\mathcal{T}$)
- Strong spin-orbit coupling (SOC)

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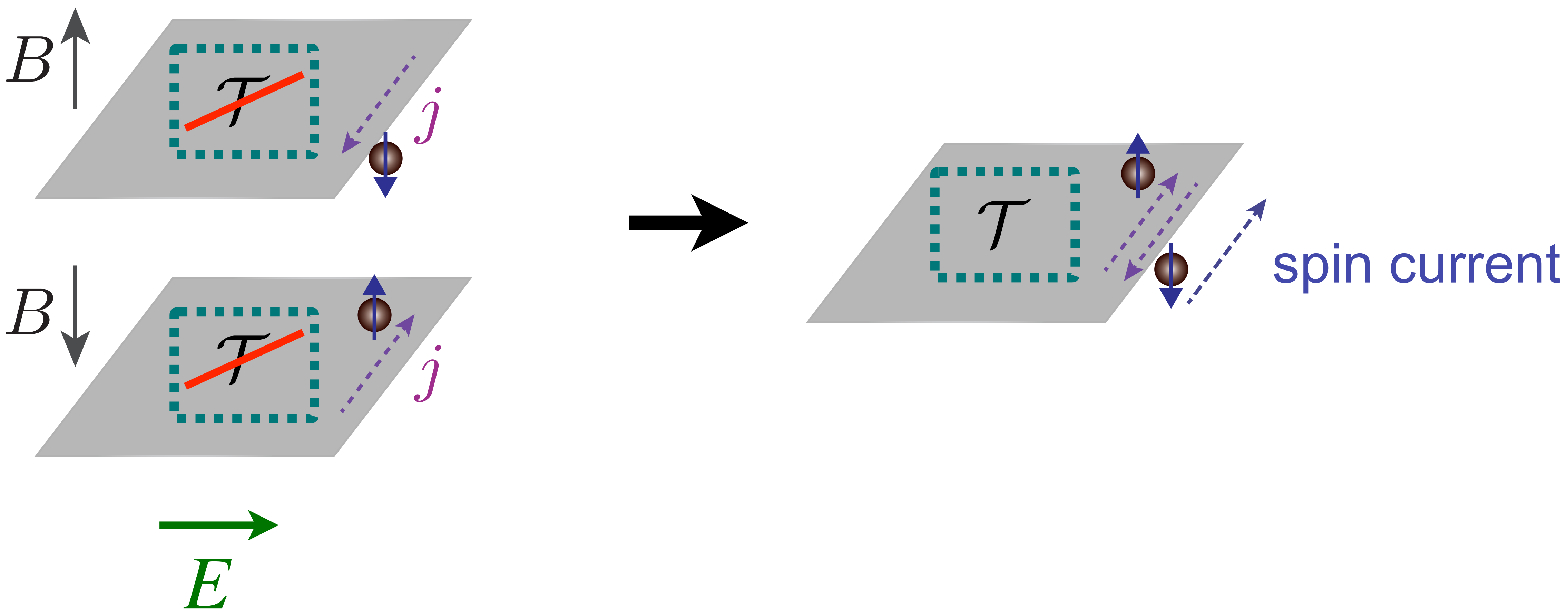
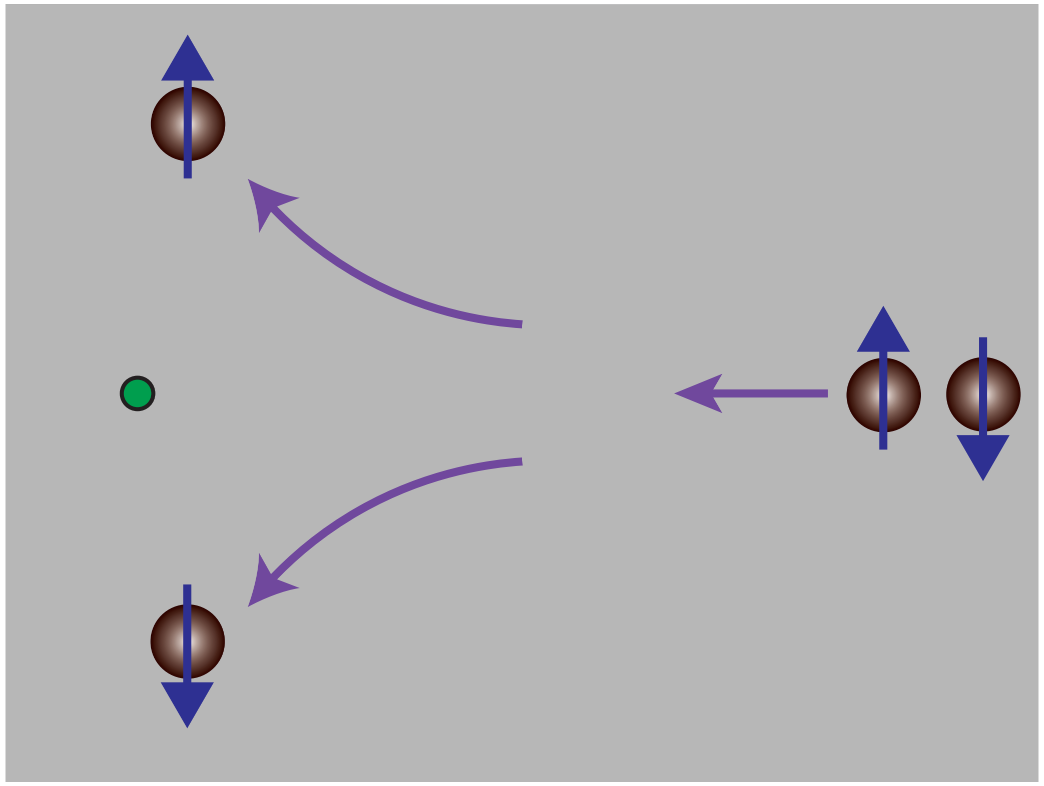
- Time reversal invariance ($\mathcal{T}$)
- Strong spin-orbit coupling (SOC)



B breaks $\mathcal{T}$ but the combination of B and $-B$ keeps $\mathcal{T}$

Keywords for topological insulators

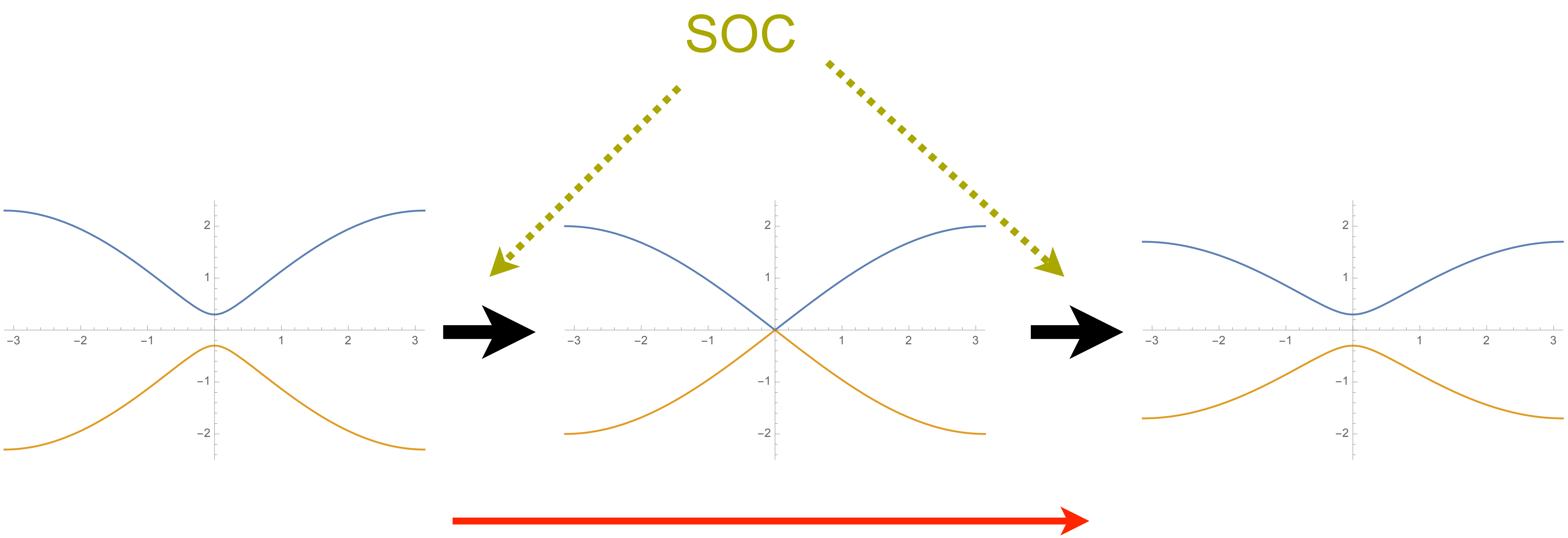
- Time reversal invariance ($\mathcal{T}$)
- Strong spin-orbit coupling (SOC)



Strong SOC is crucial for the realization

The band structure (bulk)

→ Same as (A)QH insulators



Band inversion
around k=0

Normal insulator
(NI)

Topological insulator
(TI)

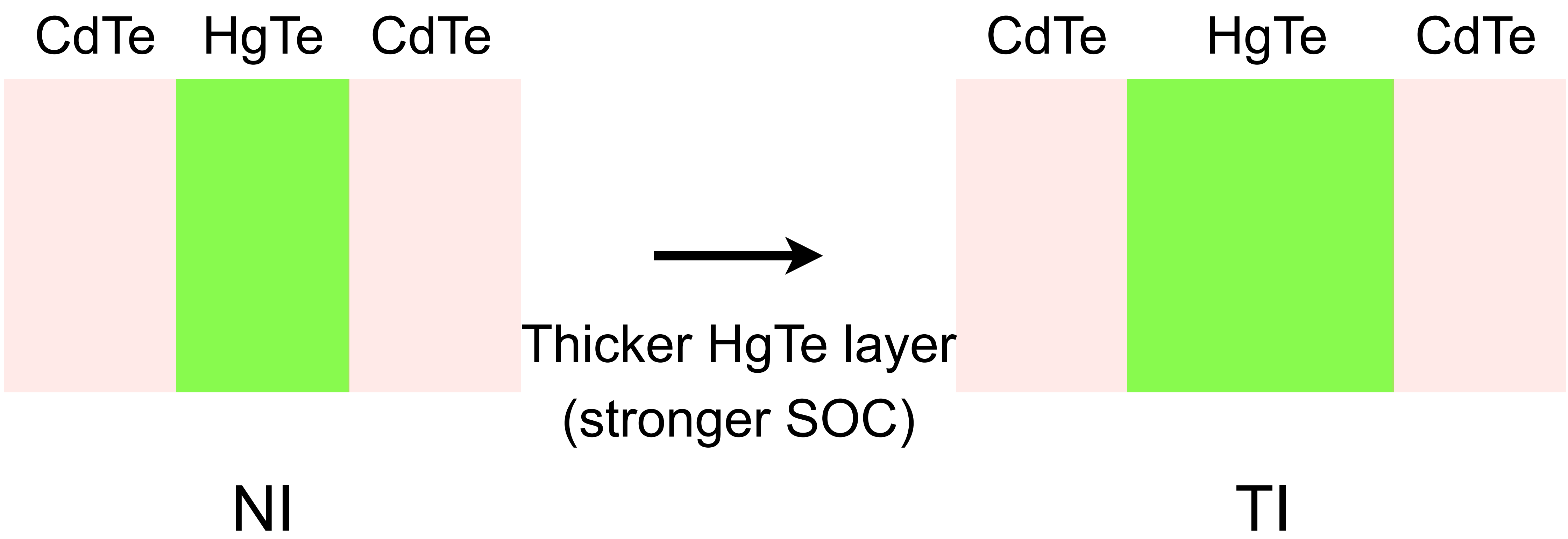
Topological insulators are classified by Z_2

The number of band inversions (Dirac points) is

- Even → "Normal insulator"
- Odd → "Topological insulator"

Example of 2D TI: HgTe/(Hg , Cd)

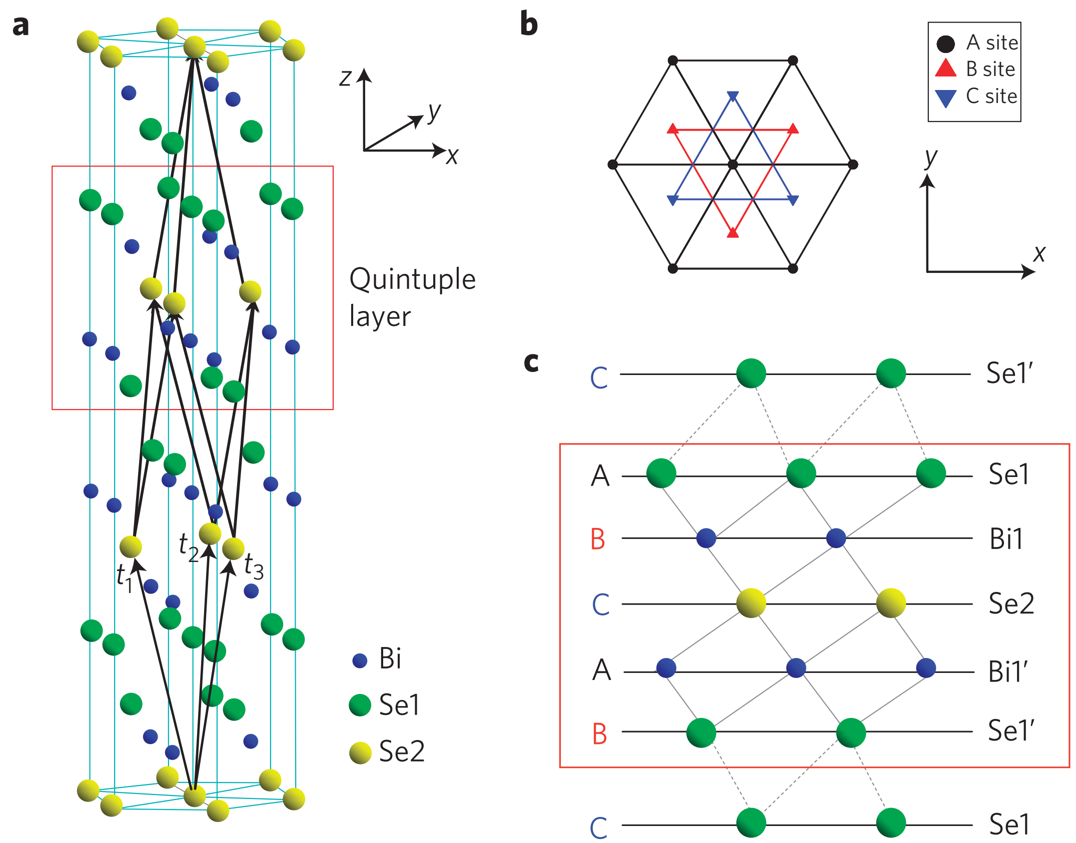
König et al. ’07



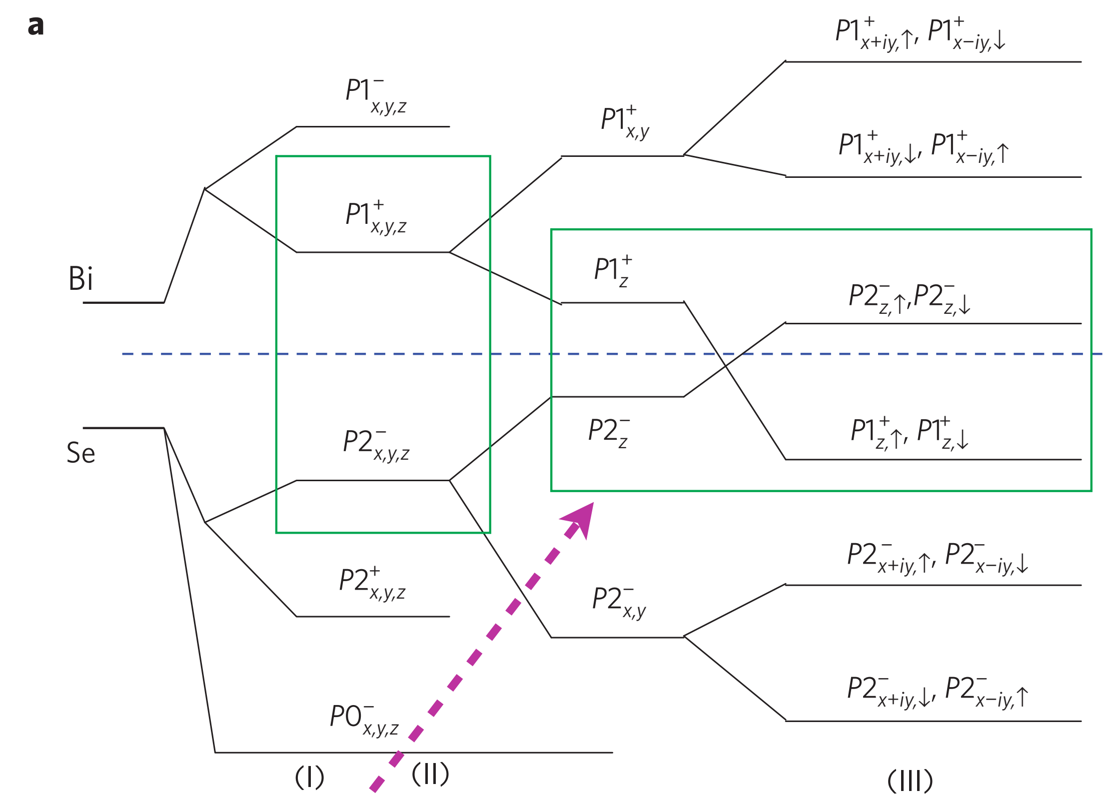
Band inversion happens in the energy band of HgTe

Example of 3D TI: Bi_2Se_3

Zhang et al. '09



Cristal structure



Energy levels

Band inversion due to strong SOC

Magnetoelectric (ME) effect

predicted by Landau&Lifshitz

discovered by Dzyaloshinskii '60

- Electric field ($\boldsymbol{E}$) induces magnetization $\boldsymbol{M}$
- Magnetic field ($\boldsymbol{B}$) induces electric polarization $\boldsymbol{P}$

$$M_j = \alpha_{ij} E_i$$
$$P_i = \alpha_{ij} B_j$$

←

$$F = -\frac{1}{\mu_0 c} \int d^3x \; \alpha_{ij} E_i B_j$$

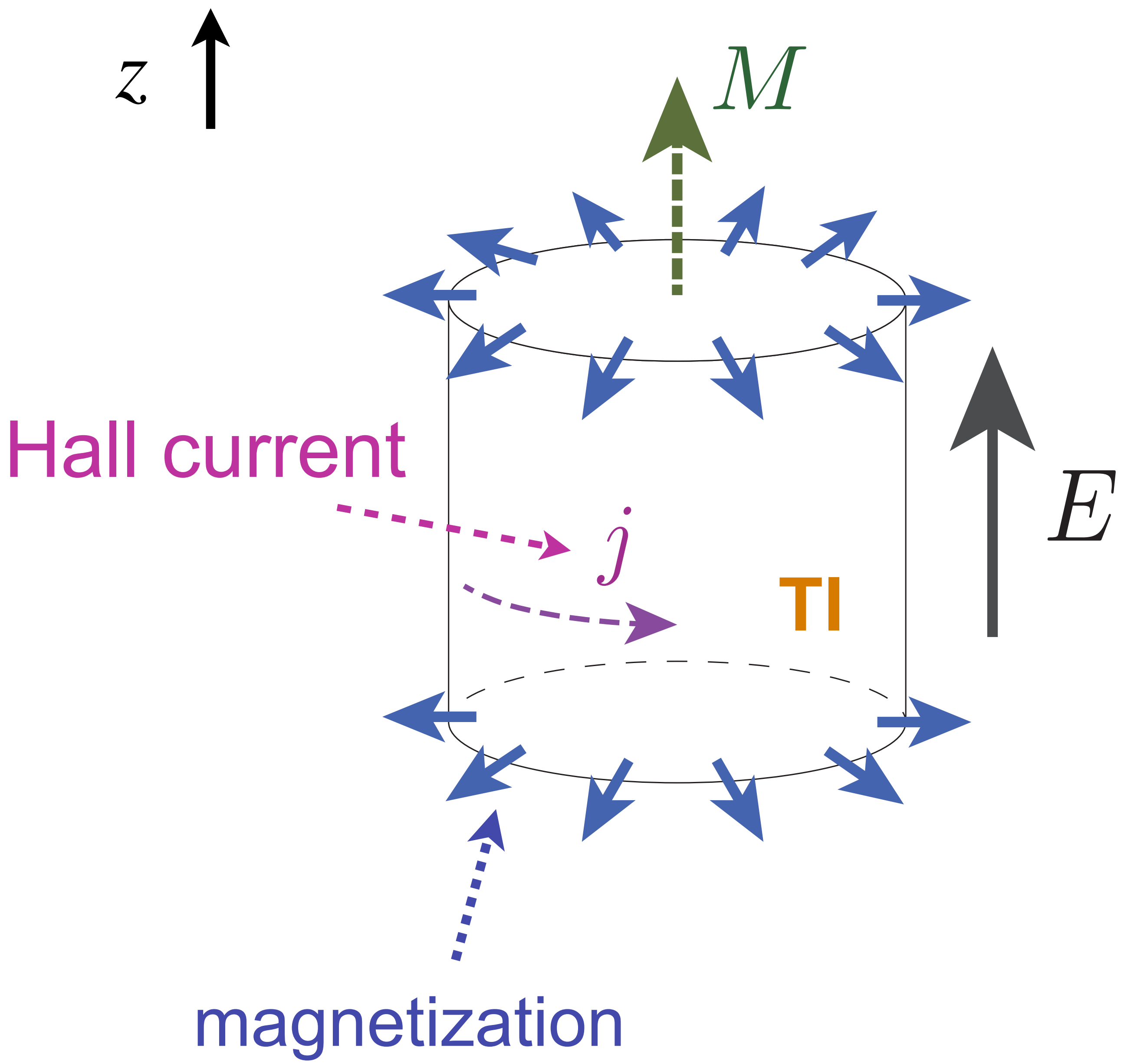
$$M_i = -\frac{1}{V} \left. \frac{\partial F}{\partial E_i} \right|_{B=0}$$

$$P_i = -\frac{1}{V} \left. \frac{\partial F}{\partial B_i} \right|_{E=0}$$

α_{ij} : constant

Let's consider

TI coated with magnetization directing outside



Half-integer AQH current

↓

The current induces magnetization in z direction

$$\boldsymbol{M} = \pm \frac{\alpha}{\mu_0 c} \boldsymbol{E}$$

α : fine-structure constant

This ME effect can be understood from a free energy:

$$F_\theta = -\frac{1}{\mu_0}\int d^3x \, \frac{\alpha}{c\pi}\theta \; \boldsymbol{E}\cdot\boldsymbol{B} \qquad \text{with} \quad \theta = \pm\pi$$

$$\longrightarrow \quad -\frac{\alpha}{4\pi}\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$\theta = \pm\,\pi$ is called static axion

($\theta = 0$ in NI)

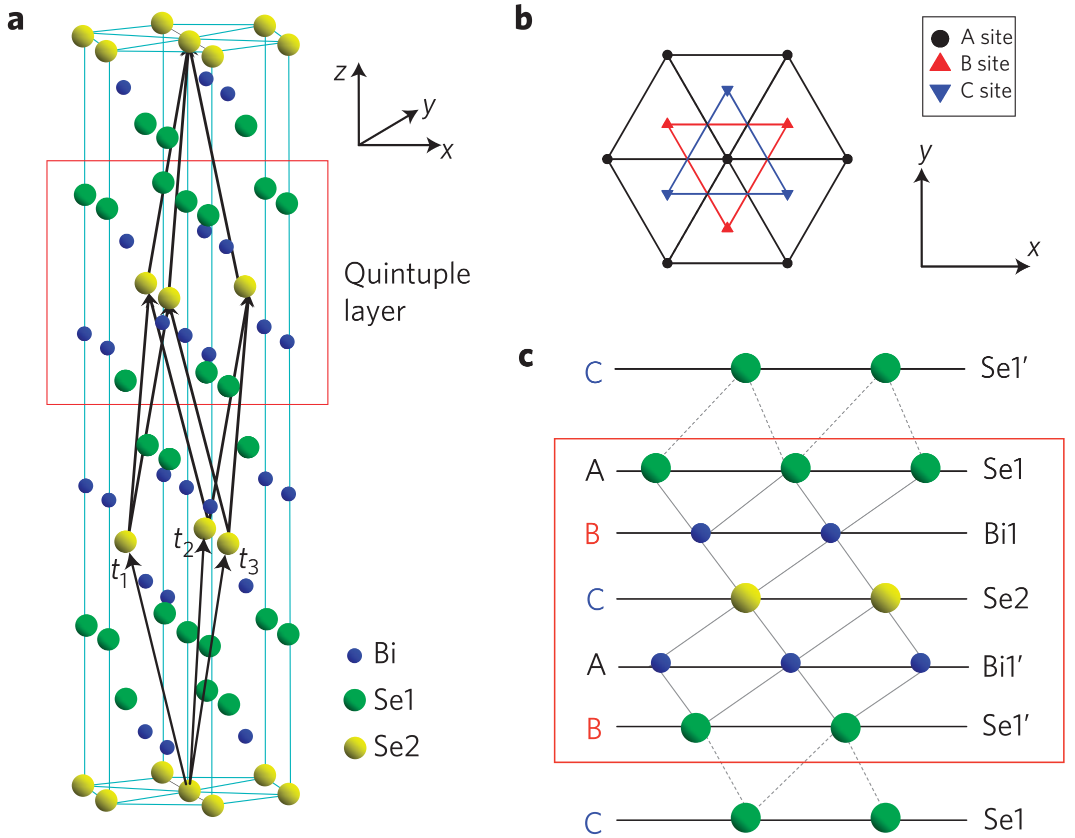
Quick summary

- Energy bands around the Fermi energy are crucial to determine the characteristics of the materials
- SOC and $\mathcal{T}$ are crucial for TI
- ME effect in TI is described by “static axion”

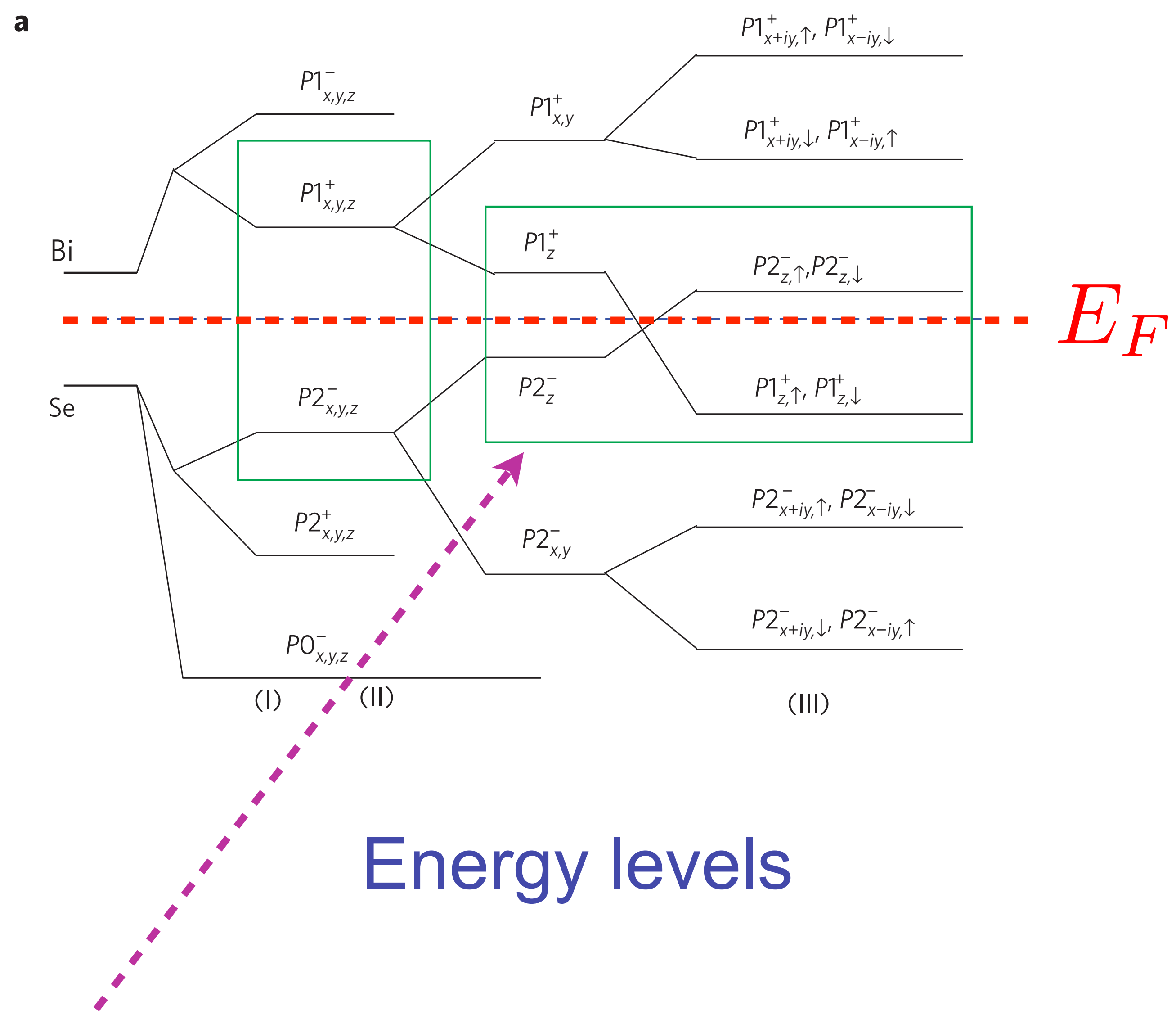
3. Axion in antiferromagnetic topological insulators

We consider 3D TI, Bi_2Se_3

H. Zhang et al. '09



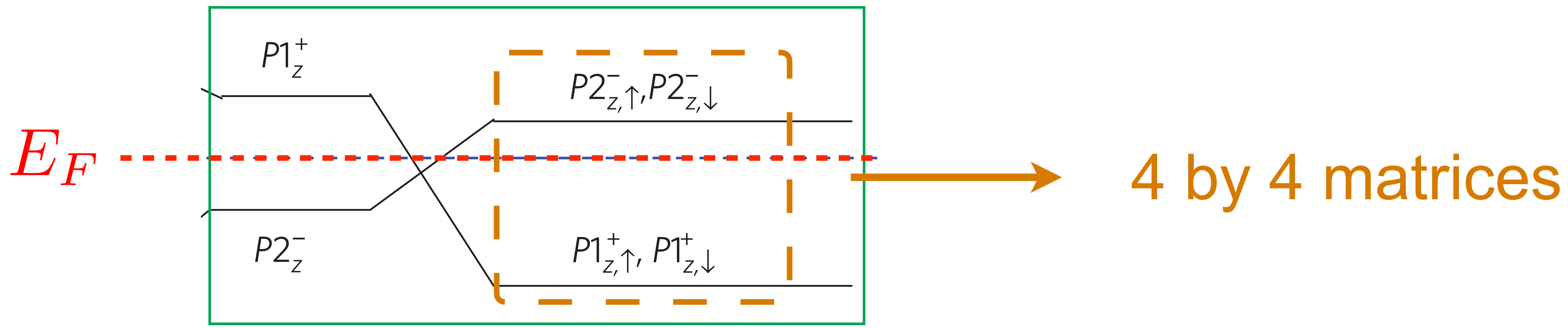
Cristal structure



Energy levels

Band inversion due to strong SOI

Two bands near the Fermi energy are important



$$H_0(\boldsymbol{k})$$

$$= \begin{pmatrix} \epsilon_0(\boldsymbol{k}) + \mathcal{M}(\boldsymbol{k}) & 0 & -iA_1 \sin k_z & A_2(\sin k_x - i \sin k_y) \\ 0 & \epsilon_0(\boldsymbol{k}) + \mathcal{M}(\boldsymbol{k}) & A_2(\sin k_x + i \sin k_y) & -iA_1 \sin k_z \\ iA_1 \sin k_z & A_2(\sin k_x - i \sin k_y) & \epsilon_0(\boldsymbol{k}) - \mathcal{M}(\boldsymbol{k}) & 0 \\ A_2(\sin k_x + i \sin k_y) & iA_1 \sin k_z & 0 & \epsilon_0(\boldsymbol{k}) - \mathcal{M}(\boldsymbol{k}) \end{pmatrix}$$

basis: $\left(\left|P1_z^+, \uparrow\right\rangle, \left|P1_z^+, \downarrow\right\rangle, \left|P2_z^-, \uparrow\right\rangle, \left|P2_z^-, \downarrow\right\rangle\right)$

"Effective Hamiltonian for 3D TI"

The effective Hamiltonian is written by Gamma matrices:

$$H_0(\boldsymbol{k}) = \epsilon_0(\boldsymbol{k})\mathbf{1}_{4\times 4} + \sum_{a=1}^{5} d^a(\boldsymbol{k})\Gamma^a$$

$$(d^1,\, d^2,\, d^3,\, d^4,\, d^5) = (A_2 \sin k_x,\; A_2 \sin k_y,\; A_1 \sin k_z,\; \mathcal{M}(\boldsymbol{k}), 0)$$

$$\mathcal{M}(\boldsymbol{k}) = M - 2B_1 - 4B_2 + 2B_1 \cos k_z + 2B_2(\cos k_x + \cos k_y)$$

$$\Gamma^1 = \begin{pmatrix} 0 & \sigma^x \\ \sigma^x & 0 \end{pmatrix} \qquad \Gamma^4 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \qquad \{\Gamma^a, \Gamma^b\} = 2\delta^{ab}\mathbf{1}_{4\times 4}$$

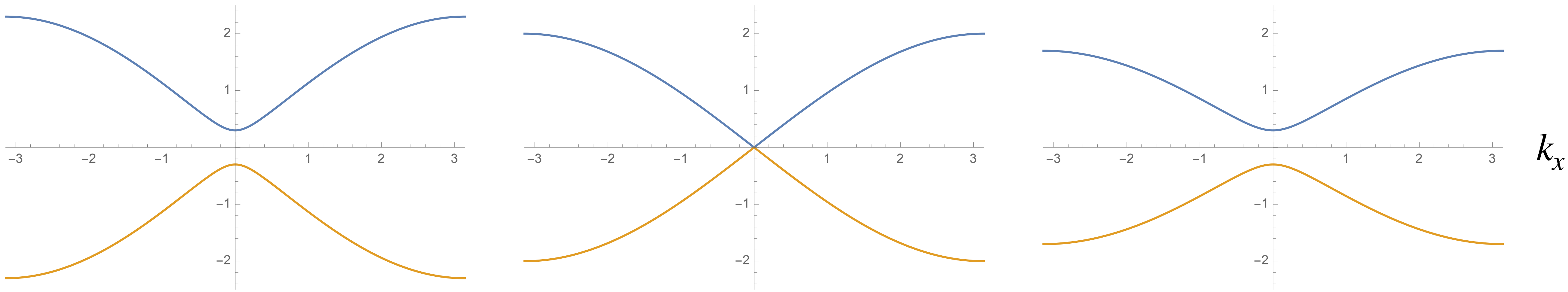
$$\mathrm{tr}(\Gamma^a\Gamma^b) = 4\delta^{ab}$$

$$\Gamma^2 = \begin{pmatrix} 0 & \sigma^y \\ \sigma^y & 0 \end{pmatrix} \qquad \Gamma^5 = \begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix} \qquad \mathrm{tr}(\Gamma^a) = 0$$

$$\Gamma^3 = \begin{pmatrix} 0 & -i\mathbf{1} \\ -i\mathbf{1} & 0 \end{pmatrix}$$

The band structure (bulk)

$$\epsilon_0 = 0$$
$$A_1 = A_2 = 1$$
$$B_1 = B_2 = -0.5$$
$$k_y = k_z = 0$$



$M = 0.3$ $M = 0$ $M = -0.3$

NI ⟷ TI

Phase transition

($M/B_1, M/B_2$ describe SOI)

Partition function (given by path integral)

$$S_0 = \int d^4x \, \psi^\dagger(x) \left[i\partial_t - H_0 \right] \psi(x) \qquad = \int dt \; \mathcal{H}_0$$

$$\longrightarrow \qquad Z_0 = \int \mathcal{D}\psi \mathcal{D}\psi^\dagger e^{iS_0}$$

$\psi(x)$: wavefunction of electron

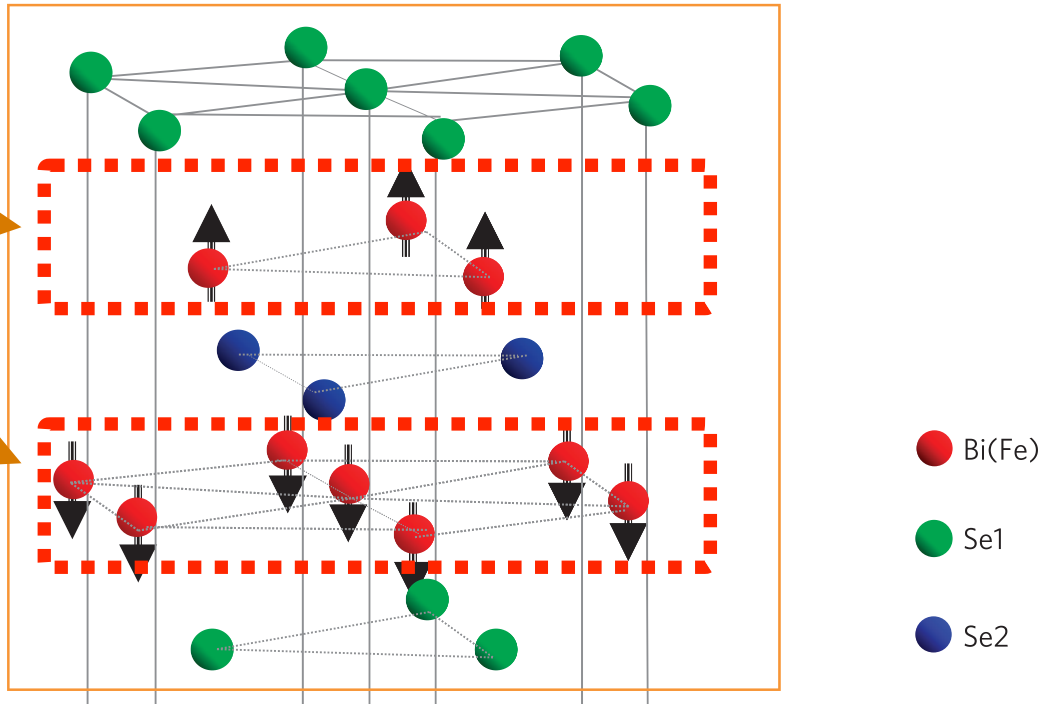
$$\sim \left(\left|P1_z^+, \uparrow\right\rangle, \left|P1_z^+, \downarrow\right\rangle, \left|P2_z^-, \uparrow\right\rangle, \left|P2_z^-, \downarrow\right\rangle \right)^T$$

Now we introduce *antiferromagnetism (AFM)* by hand

⟶ Explicit $\mathcal{T}$ breaking

R. Li et al. '10

Suppose electrons at Bi are AFM order



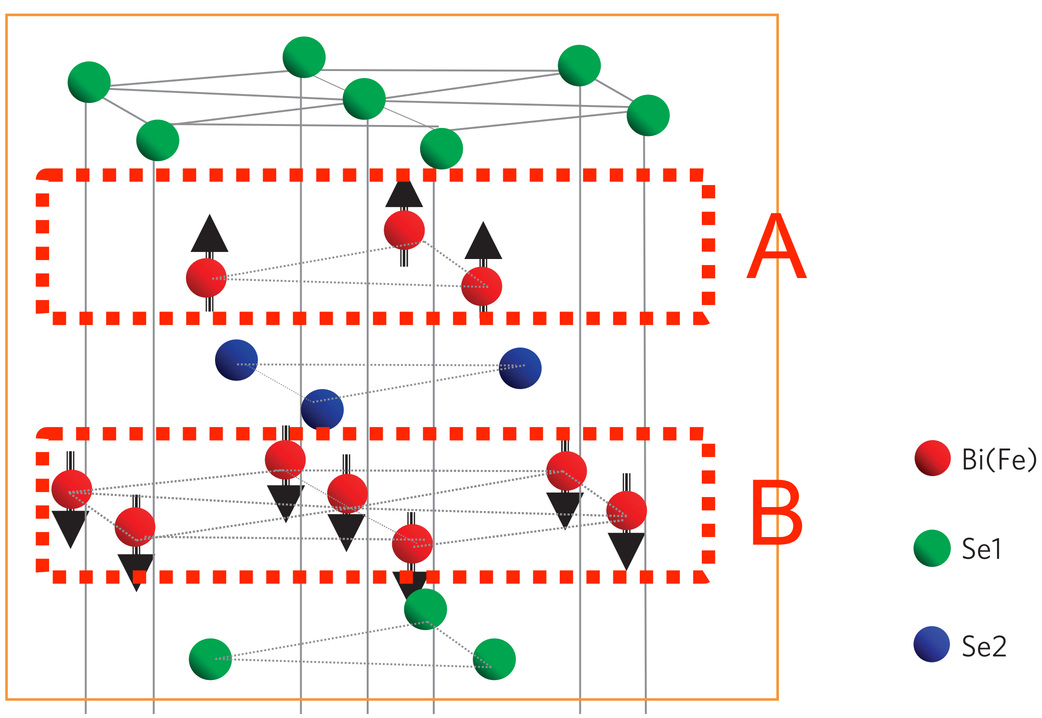
This corresponds to introducing the Hubbard terms:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\rm int}$$

where

$$\mathcal{H}_{\rm int} = \frac{UV}{N} \int d^3x \ (n_{A\uparrow} n_{A\downarrow} + n_{B\uparrow} n_{B\downarrow})$$

U : parameter to give AFM



V : volume

N : number of site

$$n_{A\sigma} = \psi^\dagger_{A\sigma} \psi_{A\sigma}$$

$$n_{B\sigma} = \psi^\dagger_{B\sigma} \psi_{B\sigma}$$

This corresponds to introducing Hubbard terms:

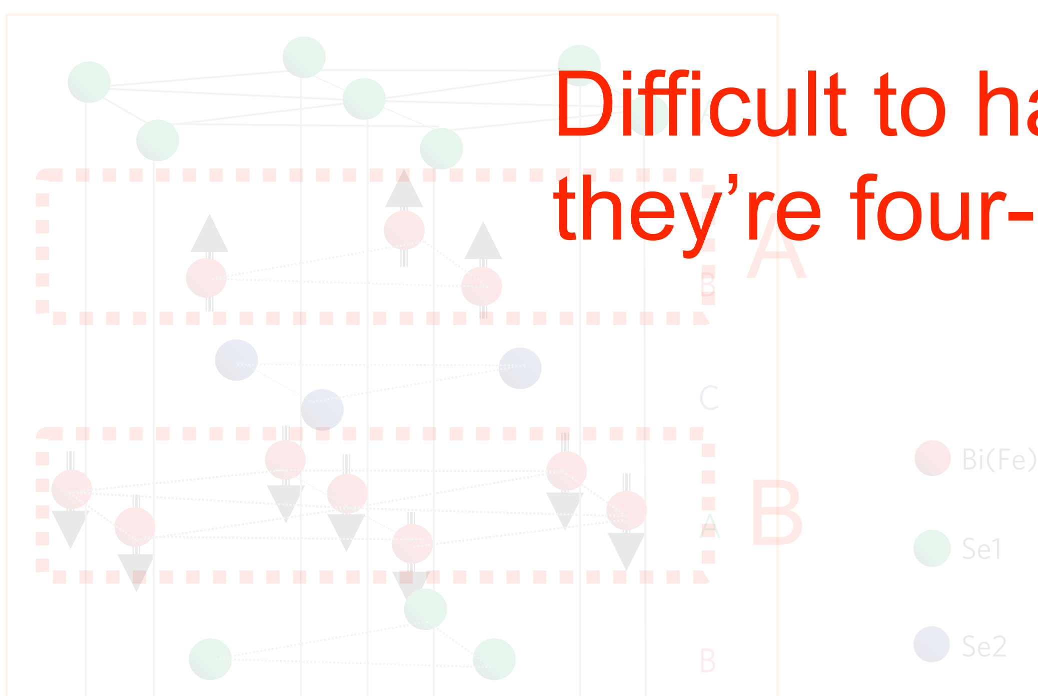
$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\rm int}$$

where

$$\mathcal{H}_{\rm int} = \frac{UV}{N} \int d^3x \, \left(n_{\rm A\uparrow} n_{\rm A\downarrow} + n_{\rm B\uparrow} n_{\rm B\downarrow} \right)$$

U : parameter to give AFM

Difficult to handle since they're four-body int.



V : volume

N : number of site

$$n_{\rm A\sigma} = \psi^\dagger_{\rm A\sigma} \psi_{\rm A\sigma}$$

$$n_{\rm B\sigma} = \psi^\dagger_{\rm B\sigma} \psi_{\rm B\sigma}$$

Hubbard-Stratonovich (HS) transformation

$$\mathcal{H}_{\mathrm{int}} = \frac{UV}{N} \int d^3x \, \left(n_{\mathrm{A}\uparrow} n_{\mathrm{A}\downarrow} + n_{\mathrm{B}\uparrow} n_{\mathrm{B}\downarrow} \right)$$

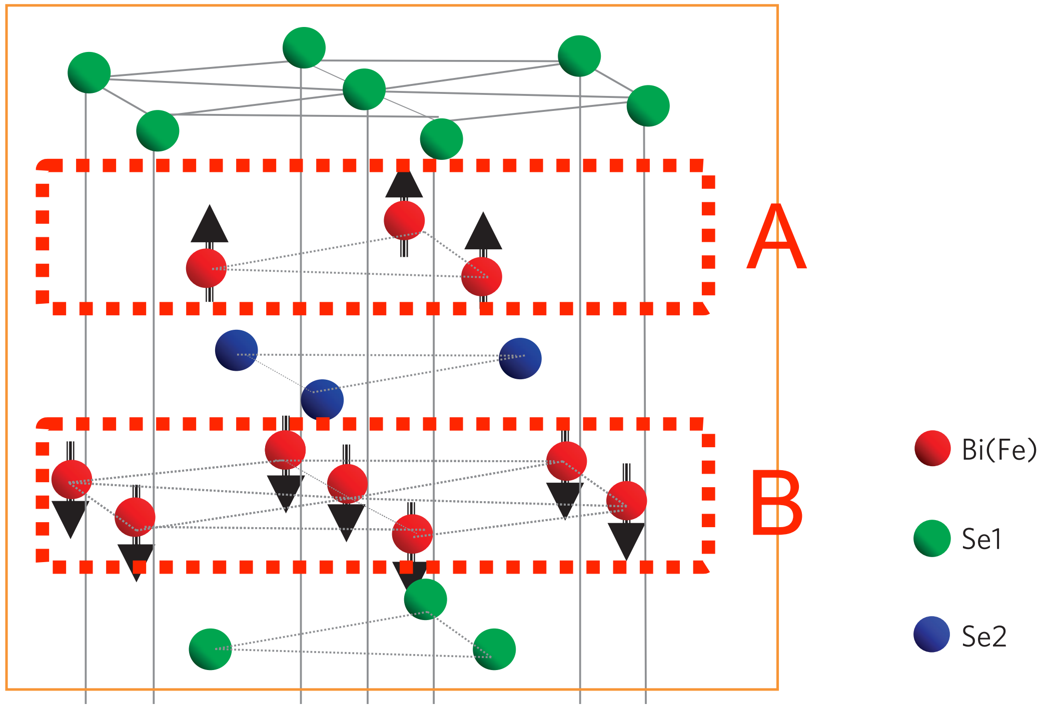
$\downarrow$ HS transformation

Hubbard-Stratonovich (HS) transformation

$$\mathcal{H}_{\mathrm{int}} = \frac{UV}{N}\int d^3x\ (n_{\mathrm{A}\uparrow}n_{\mathrm{A}\downarrow} + n_{\mathrm{B}\uparrow}n_{\mathrm{B}\downarrow})$$

↓ HS transformation

- A dynamical scalar ϕ that gives $\Gamma^5 d_5$ ($d_5 = \phi$)



$$\langle\phi\rangle \sim \langle S_A\rangle = -\langle S_B\rangle$$

VEV of ϕ is the order parameter of AFM

- $\langle\phi\rangle = 0$ PM order
- $\langle\phi\rangle \neq 0$ AFM order

PM (paramagnetic)

Hubbard-Stratonovich (HS) transformation

$$\mathcal{H}_{\rm int} = \frac{UV}{N} \int d^3x \; (n_{A\uparrow} n_{A\downarrow} + n_{B\uparrow} n_{B\downarrow})$$

↓ HS transformation

- A dynamical scalar ϕ that gives $\Gamma^5 d_5$ $(d_5 = \phi)$

Hubbard-Stratonovich (HS) transformation

$$\mathcal{H}_{\rm int} = \frac{UV}{N} \int d^3x \; (n_{\rm A\uparrow}n_{\rm A\downarrow} + n_{\rm B\uparrow}n_{\rm B\downarrow})$$

↓ HS transformation

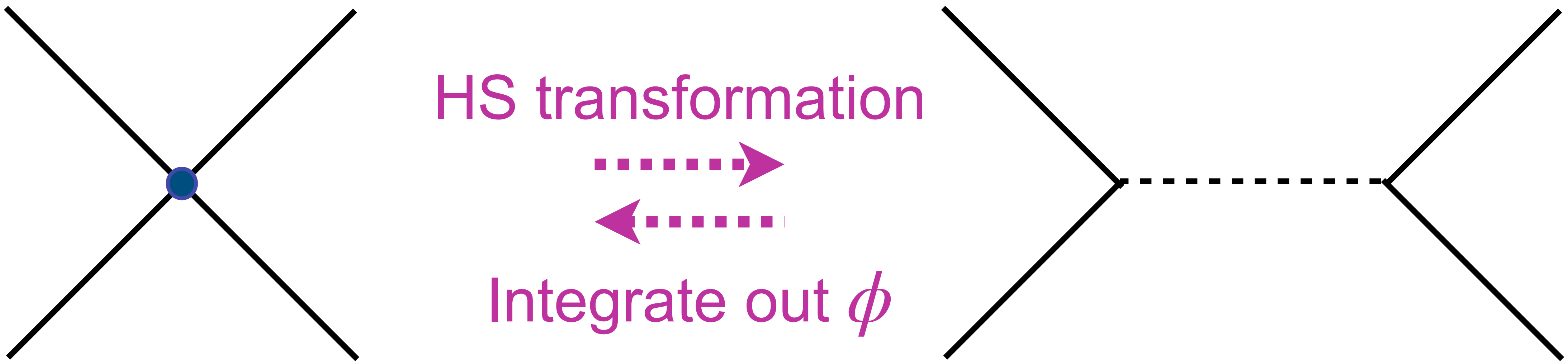
- A dynamical scalar ϕ that gives $\Gamma^5 d_5$ $(d_5 = \phi)$
- Mass term of ϕ ← missed in Sekine, Nomura '16, Sekine, Nomura '20, Schütte-Engel '21

(confirmed by private communication with Sekine-san)

This can be understood from

HS transformation

$\sim$ Inverse of integrating out a scalar



$n_{A\uparrow}n_{A\downarrow},\ n_{B\uparrow}n_{B\downarrow}$
$(\psi_\uparrow^\dagger\psi_\uparrow\psi_\downarrow^\dagger\psi_\downarrow)$

Four Fermi int.

$\phi\psi_\uparrow^\dagger\psi_\uparrow\ ,\ \phi\psi_\downarrow^\dagger\psi_\downarrow$

Yukawa int.

Thus the mass term should appear

Partition function (TI + AFM)

$$Z = \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \mathcal{D}\phi \; e^{iS + iS_\phi^{\text{mass}}}$$

$$\Gamma^5 \phi$$

$$S = \int d^4x \; \psi^\dagger(x) \left[i\partial_t - H \right] \psi(x)$$

$$H = H_0 + \delta H$$

$$S_\phi^{\text{mass}} = -\int d^4x \; M^2 \phi^2$$

$$M^2 = \int \frac{d^3k}{(2\pi)^3} \frac{2}{U}$$

Partition function (TI + AFM)

$$Z = \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \mathcal{D}\phi \, e^{iS + iS_\phi^{\mathrm{mass}}}$$

$\Gamma^5 \phi$

$$S = \int d^4x \, \psi^\dagger(x) \left[i\partial_t - H \right] \psi(x)$$

$$H = H_0 + \delta H$$

$$S_\phi^{\mathrm{mass}} = - \int d^4x \, M^2 \phi^2$$

$$M^2 = \int \frac{d^3k}{(2\pi)^3} \frac{2}{U}$$

Integrate out $\psi \, , \psi^\dagger$

Effective action for ϕ

Effective potential for ϕ

KI '21

$$V_\phi = -2\int \frac{d^3k}{(2\pi)^3}\left(\sqrt{|d_0|^2+\phi^2} - |d_0|\right) + M^2\phi^2$$

$$|d_0|^2 = \sum_{a=1}^{4} |d^a|^2$$

$$M^2 = \int \frac{d^3k}{(2\pi)^3}\frac{2}{U}$$

Effective potential for ϕ

KI '21

$$V_\phi = -2\int \frac{d^3k}{(2\pi)^3}\left(\sqrt{|d_0|^2+\phi^2} - |d_0|\right) + M^2\phi^2$$

Negative potential

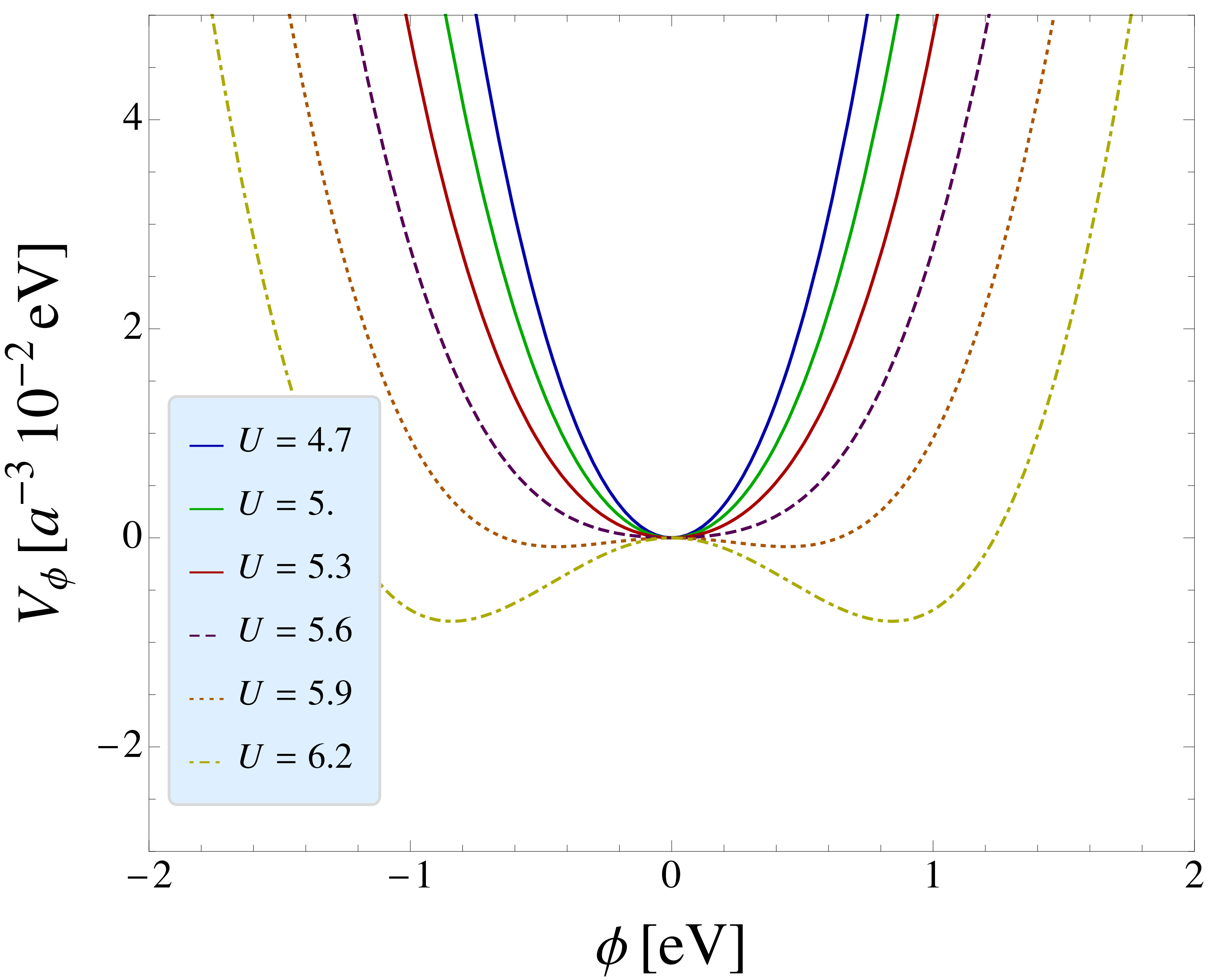
The mass term stabilizes the potential

$$|d_0|^2 = \sum_{a=1}^{4} |d^a|^2$$

$$M^2 = \int \frac{d^3k}{(2\pi)^3}\frac{2}{U}$$

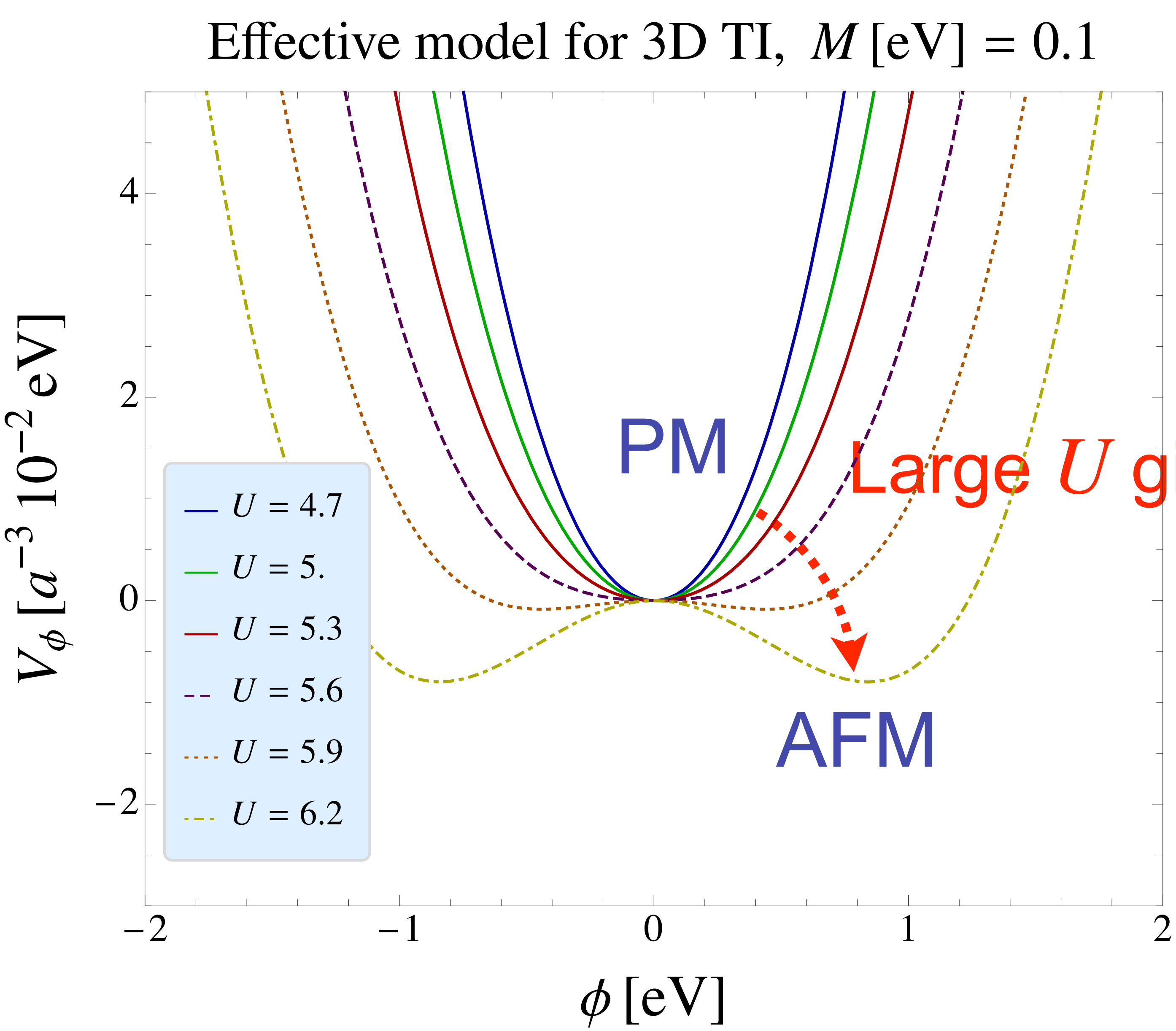
KI '21

Effective model for 3D TI, $M\,[\mathrm{eV}] = 0.1$



$A_1 = A_2 = 1$

$B_1 = B_2 = -0.5$



Large U gives VEV of ϕ

$A_1 = A_2 = 1$

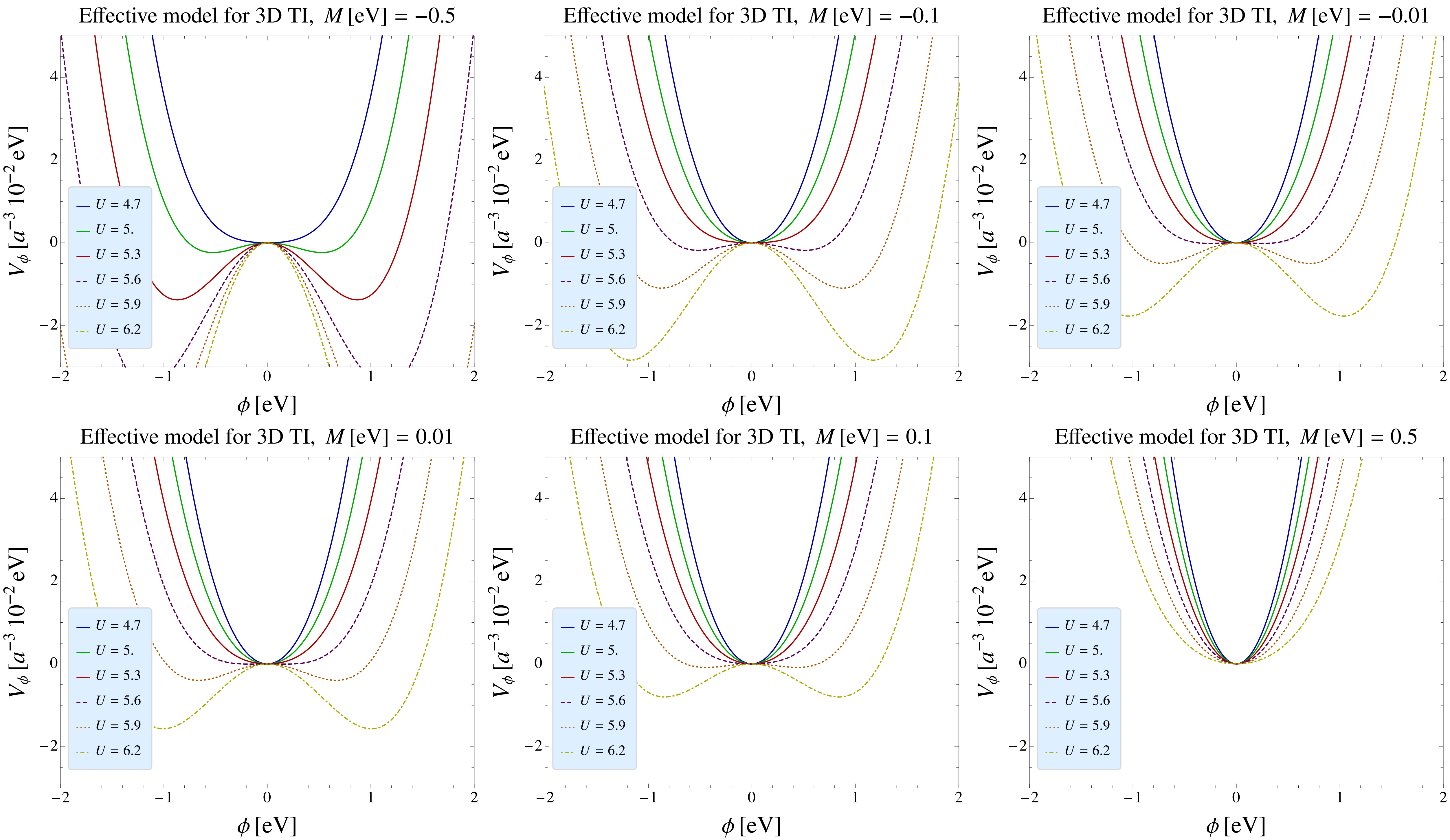
$B_1 = B_2 = -0.5$

Phase transition from PM to AFM

$$\because\ M^2 \propto 1/U$$

M dependence

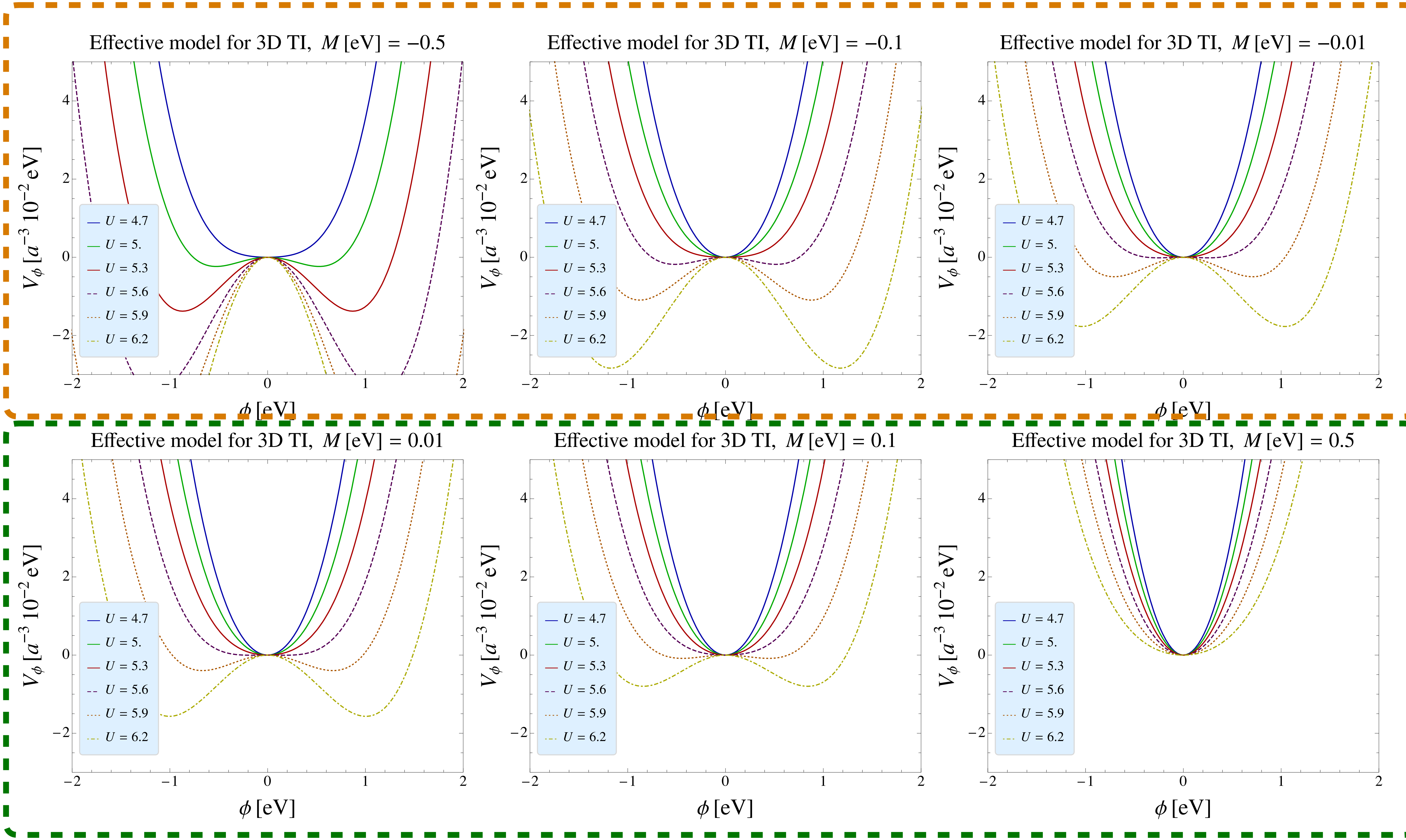
KI '21



M dependence

KI ’21

TI



NI

The difference between TI and NI is not clear

Recall ME effect in TI is described by

$$\mathcal{L}_\theta = -\frac{\alpha}{4\pi}\int d^4x \; \theta F_{\mu\nu}\tilde{F}^{\mu\nu} \qquad \text{with} \qquad \theta = \pm\pi$$

Namely,

- $\theta = \pm\pi$ for TI
- $\theta = 0$ for NI

→ θ is the order parameter of phase transition between TI and NI

We need potential for θ

θ can be computed from Hamiltonian:

- $$\theta = \frac{1}{4\pi} \int d^3k \frac{2|d| + d^4}{(|d| + d^4)^2 |d|^3} \epsilon^{ijkl} d^i \partial_{k_x} d^j \partial_{k_y} d^k \partial_{k_z} d^l$$

R. Li et al. '10

- Approximately given by chiral anomaly (Fujikawa method)

θ can be computed from Hamiltonian:

- $\theta = \frac{1}{4\pi} \int d^3k \frac{2|d| + d^4}{(|d| + d^4)^2 |d|^3} \epsilon^{ijkl} d^i \partial_{k_x} d^j \partial_{k_y} d^k \partial_{k_z} d^l$

R. Li et al. '10

- Approximately given by chiral anomaly (Fujikawa method)

Derivation as chiral anomaly

$$H(\boldsymbol{k}) = \sum_{a=1}^{5} d^a(\boldsymbol{k})\Gamma^a$$

$$(d^1,\, d^2,\, d^3,\, d^4,\, d^5) = (A_2 \sin k_x,\, A_2 \sin k_y,\, A_1 \sin k_z,\, \mathcal{M}(\boldsymbol{k}),\, \phi)$$

$$\mathcal{M}(\boldsymbol{k}) = M - 2B_1 - 4B_2 + 2B_1 \cos k_z + 2B_2(\cos k_x + \cos k_y)$$

Derivation as chiral anomaly

$$H(\boldsymbol{k}) = \sum_{a=1}^{5} d^a(\boldsymbol{k})\Gamma^a$$

$$(d^1,\, d^2,\, d^3,\, d^4,\, d^5) = (A_2 \sin k_x,\ A_2 \sin k_y,\ A_1 \sin k_z,\ \mathcal{M}(\boldsymbol{k}),\ \phi)$$

$$\mathcal{M}(\boldsymbol{k}) = M - 2B_1 - 4B_2 + 2B_1 \cos k_z + 2B_2(\cos k_x + \cos k_y)$$

- expand around $\boldsymbol{k} = 0$
- redefine $\boldsymbol{k}$

$$H(\boldsymbol{k}) = k_x\Gamma^1 + k_y\Gamma^2 + k_y\Gamma^3 + M\Gamma^4 + \phi\Gamma^5$$

"Dirac model"

$$H(\boldsymbol{k}) = k_x\Gamma^1 + k_y\Gamma^2 + k_y\Gamma^3 + M\Gamma^4 + \phi\Gamma^5$$

Unitary transformation of the basis

$$\tilde{U}H(\boldsymbol{k})\tilde{U}^\dagger = \beta(\boldsymbol{\gamma}\cdot\boldsymbol{k} + M + \phi\gamma_5)$$

$$S = \int d^4x\ \bar{\psi}[i\gamma^\mu(\partial_\mu - ieA_\mu) - M - i\phi\gamma_5]\psi$$

$\Gamma^5\phi$ reduces to $i\gamma^5\phi$

$i\gamma^5\phi$ term can be rotated away, which gives rise to θ term:

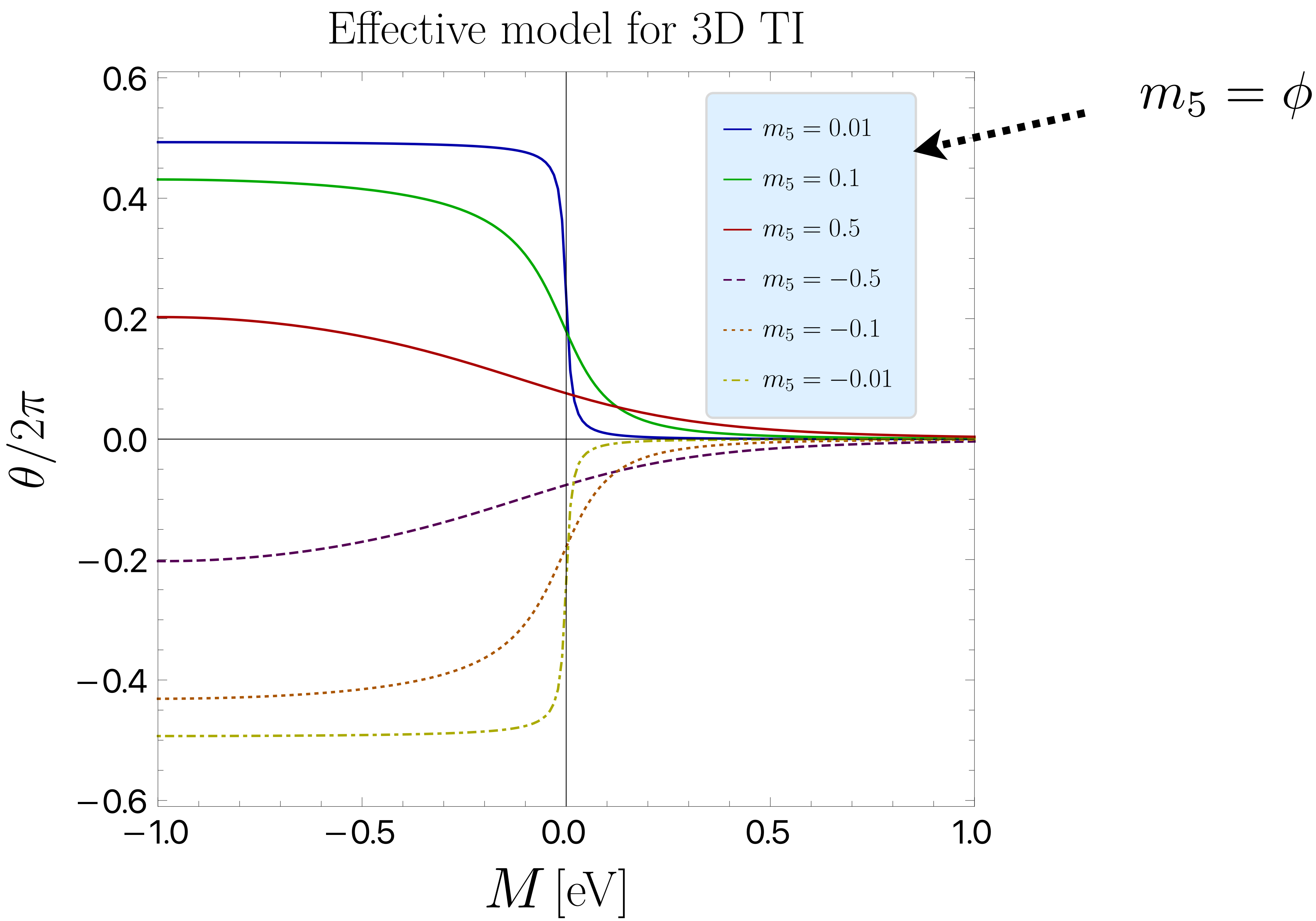
$$S_\Theta = -\frac{\alpha}{4\pi}\int \mathrm{d}^4x\,\Theta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\Theta = \frac{\pi}{2}[1-\mathrm{sgn}(M)]\mathrm{sgn}(\phi) + \tan^{-1}\frac{\phi}{M}$$

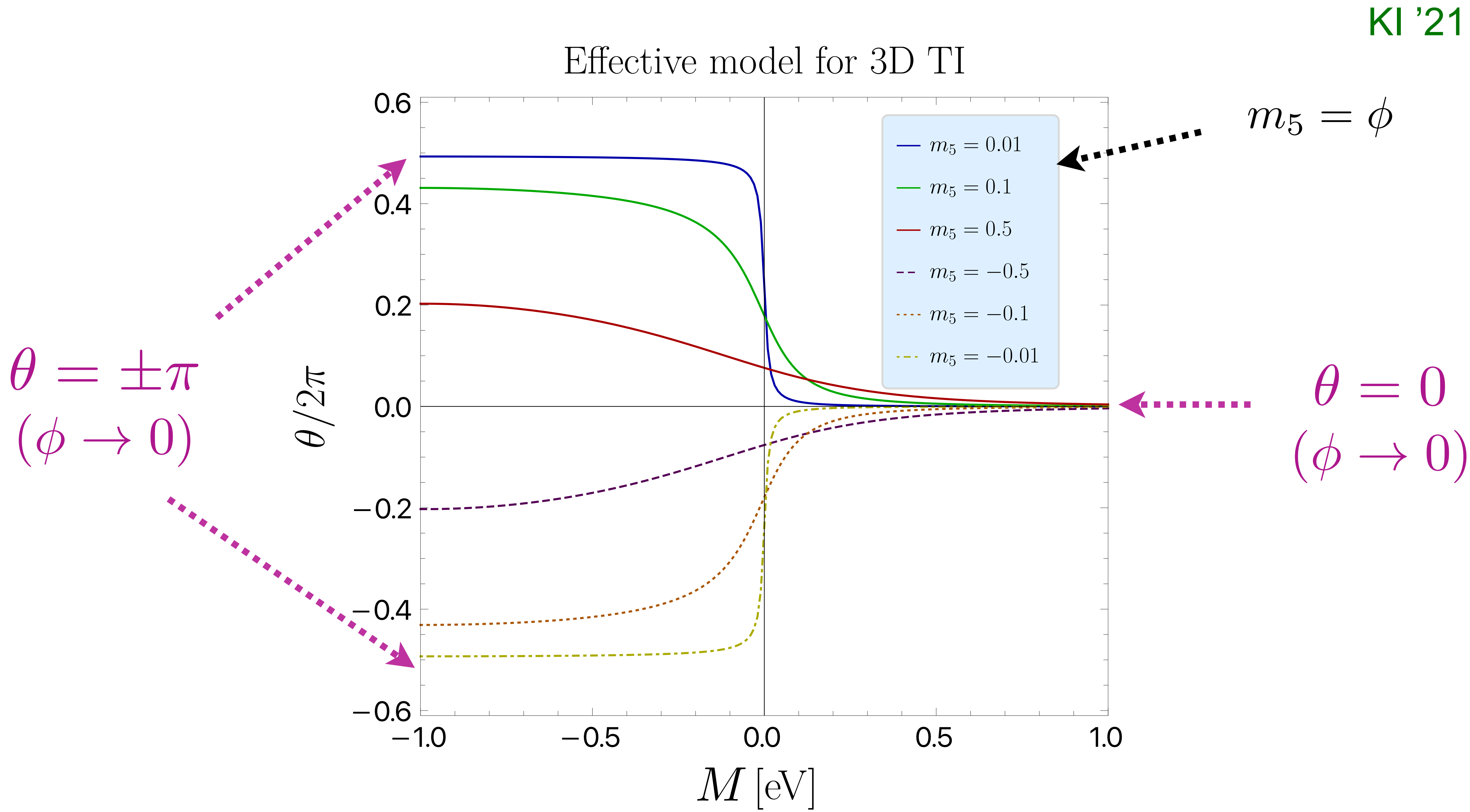
it is consistent with

$$\theta = \frac{1}{4\pi}\int d^3k \frac{2|d|+d^4}{(|d|+d^4)^2|d|^3}\epsilon^{ijkl}d^i\partial_{k_x}d^j\partial_{k_y}d^k\partial_{k_z}d^l$$

KI '21

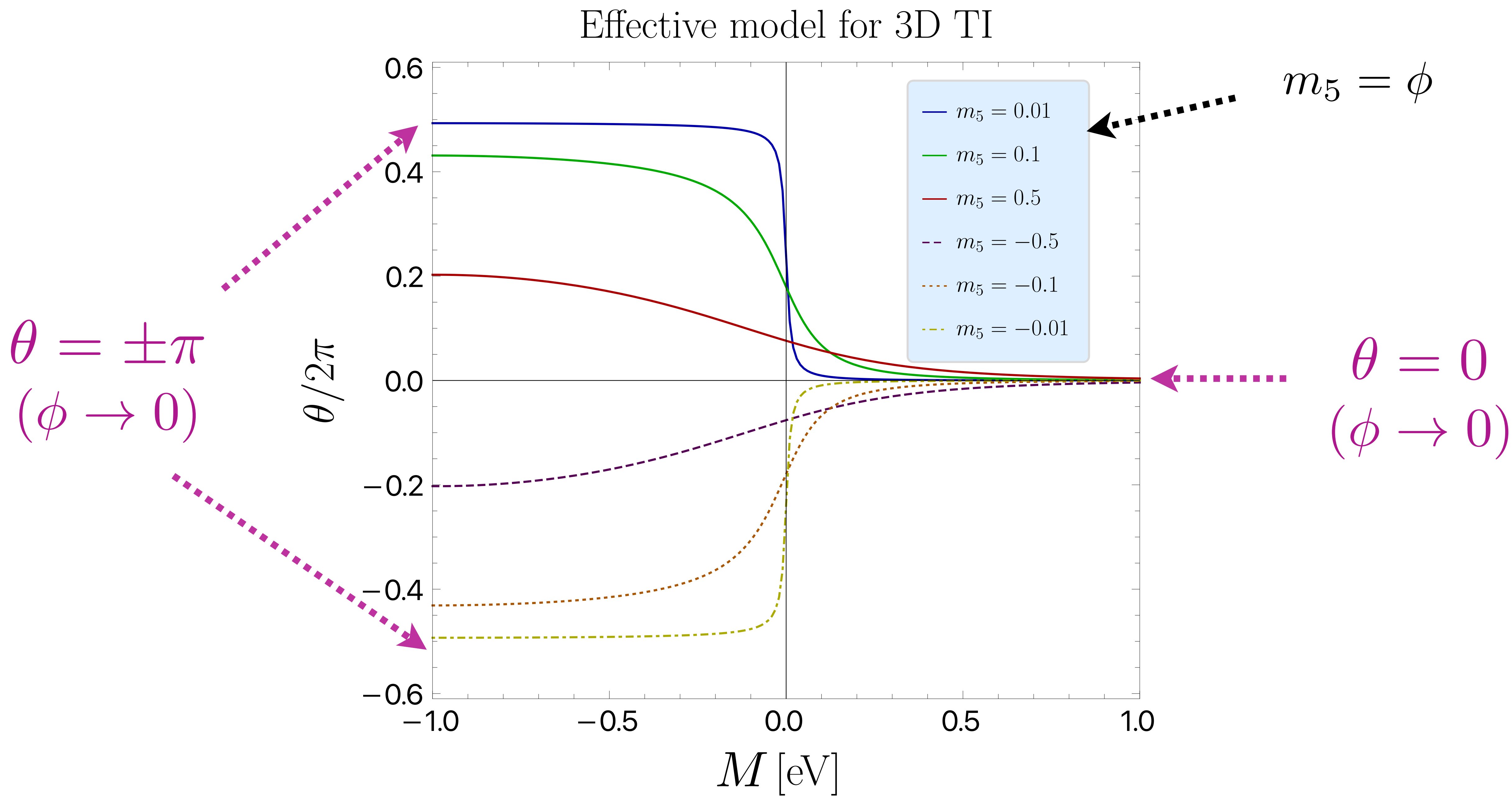


calculation for Dirac model is done by Zhang '19



KI '21
Effective model for 3D TI
$m_5 = \phi$
$m_5 = 0.01$
$m_5 = 0.1$
$m_5 = 0.5$
$m_5 = -0.5$
$m_5 = -0.1$
$m_5 = -0.01$
$\theta = \pm\pi$
$(\phi \to 0)$
$\theta = 0$
$(\phi \to 0)$
$\theta/2\pi$
M [eV]
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−1.0
−0.5
0.0
0.5
1.0

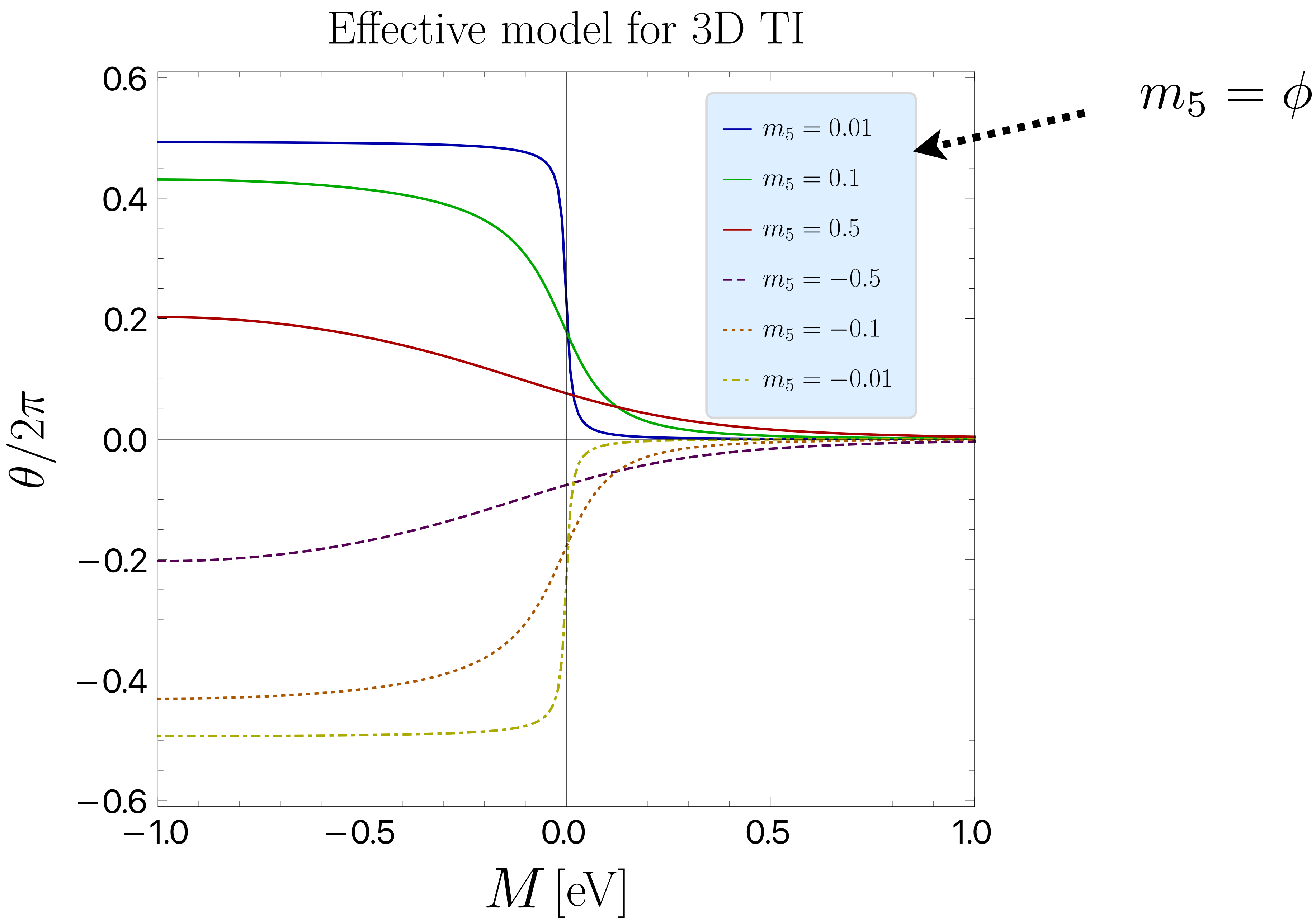
KI ’21



$\phi = 0$ ($\mathcal{T}$) ⟷ PM order ⟶

$\theta = \pm\pi$ $M < 0$ (TI)

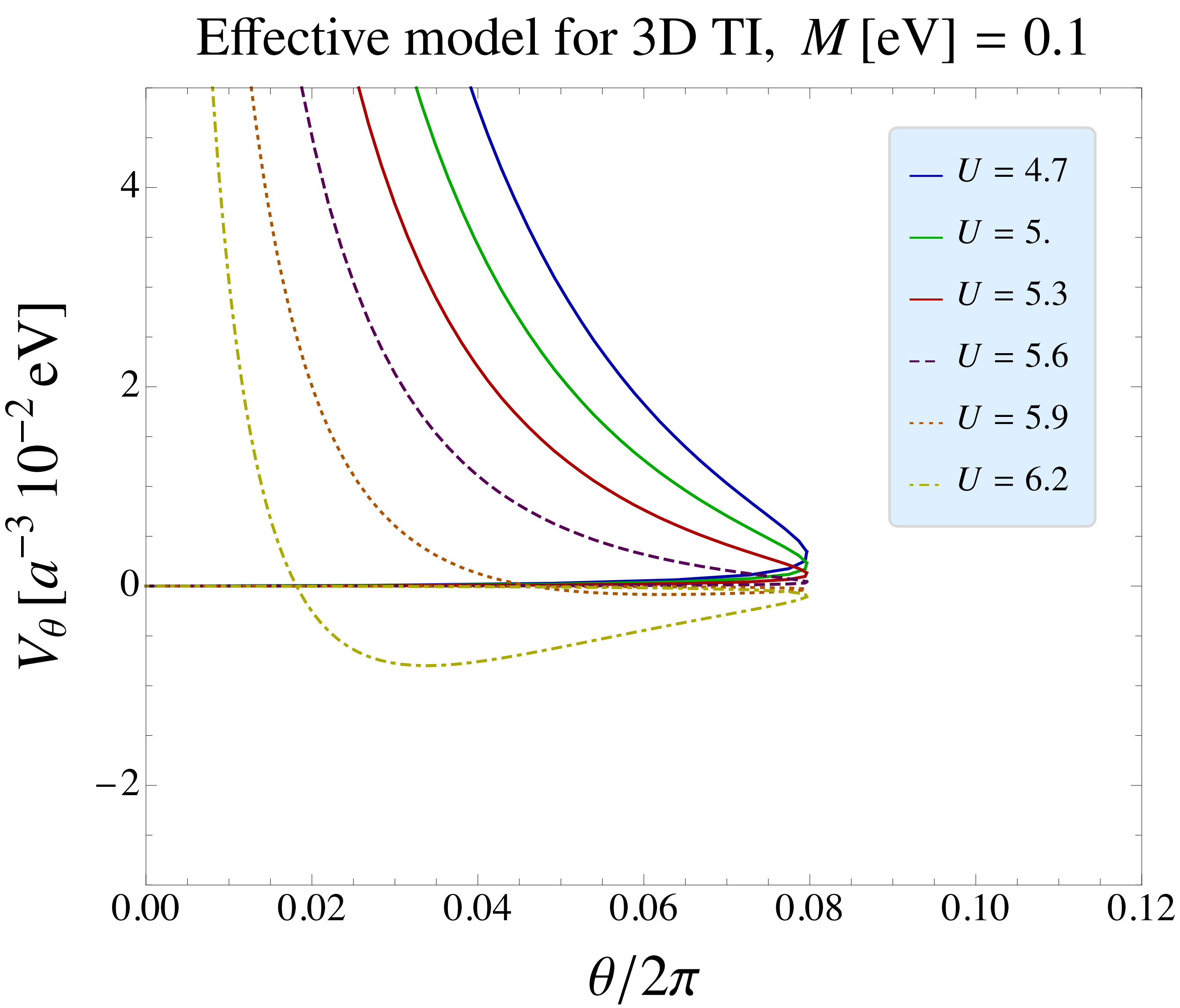
$\theta = 0$ $M > 0$ (NI)



$\phi \neq 0$ ($\not{\mathcal{T}}$) ⟷ AFM order ⟶ θ takes continuum variable

Effective potential in terms of θ

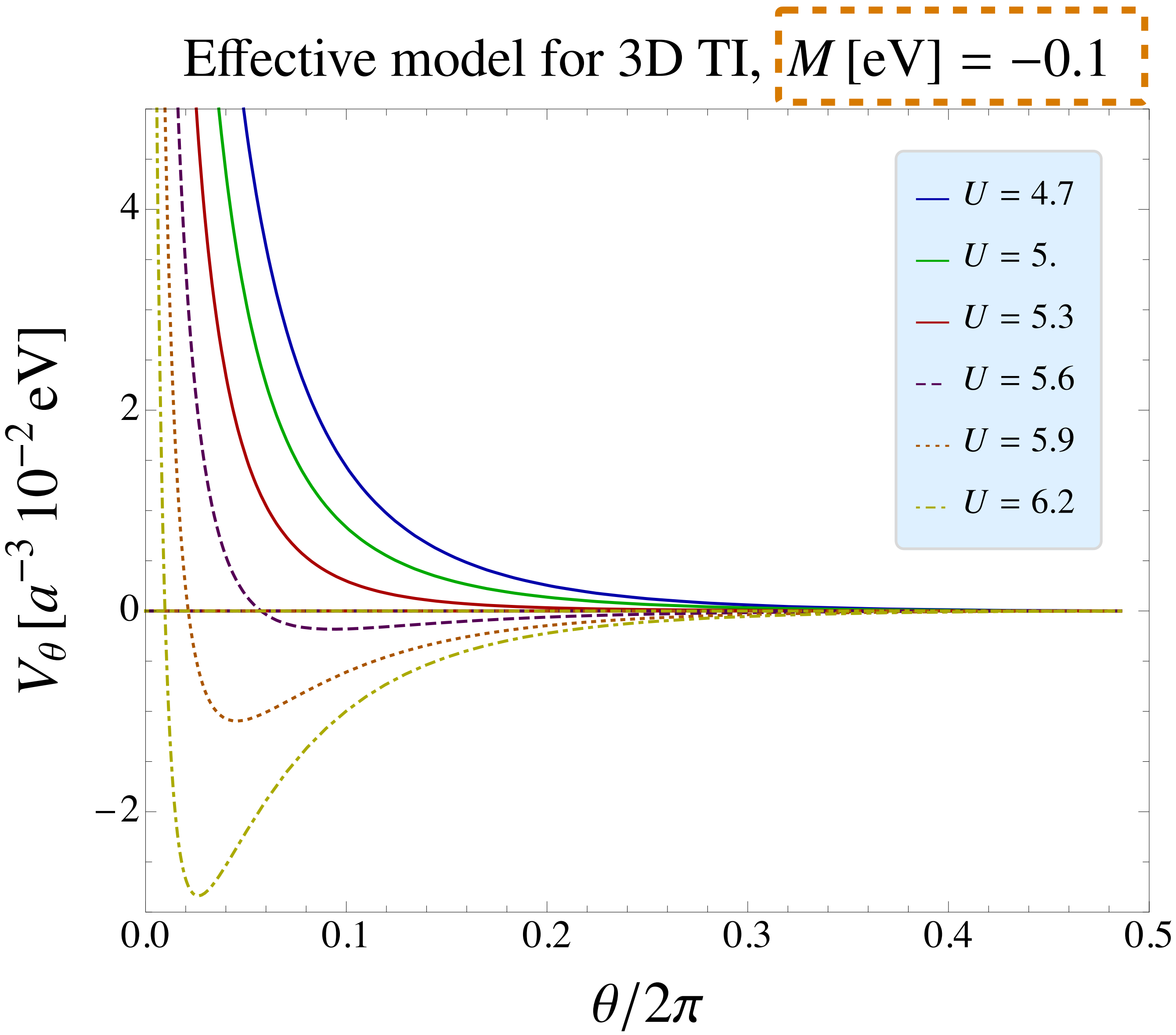
NI phase



Potential minimum:

- $\theta = 0$ (small U , i.e., PM)
- $\theta \neq 0$ (large U , i.e., AFM)

Effective potential in terms of θ



TI phase

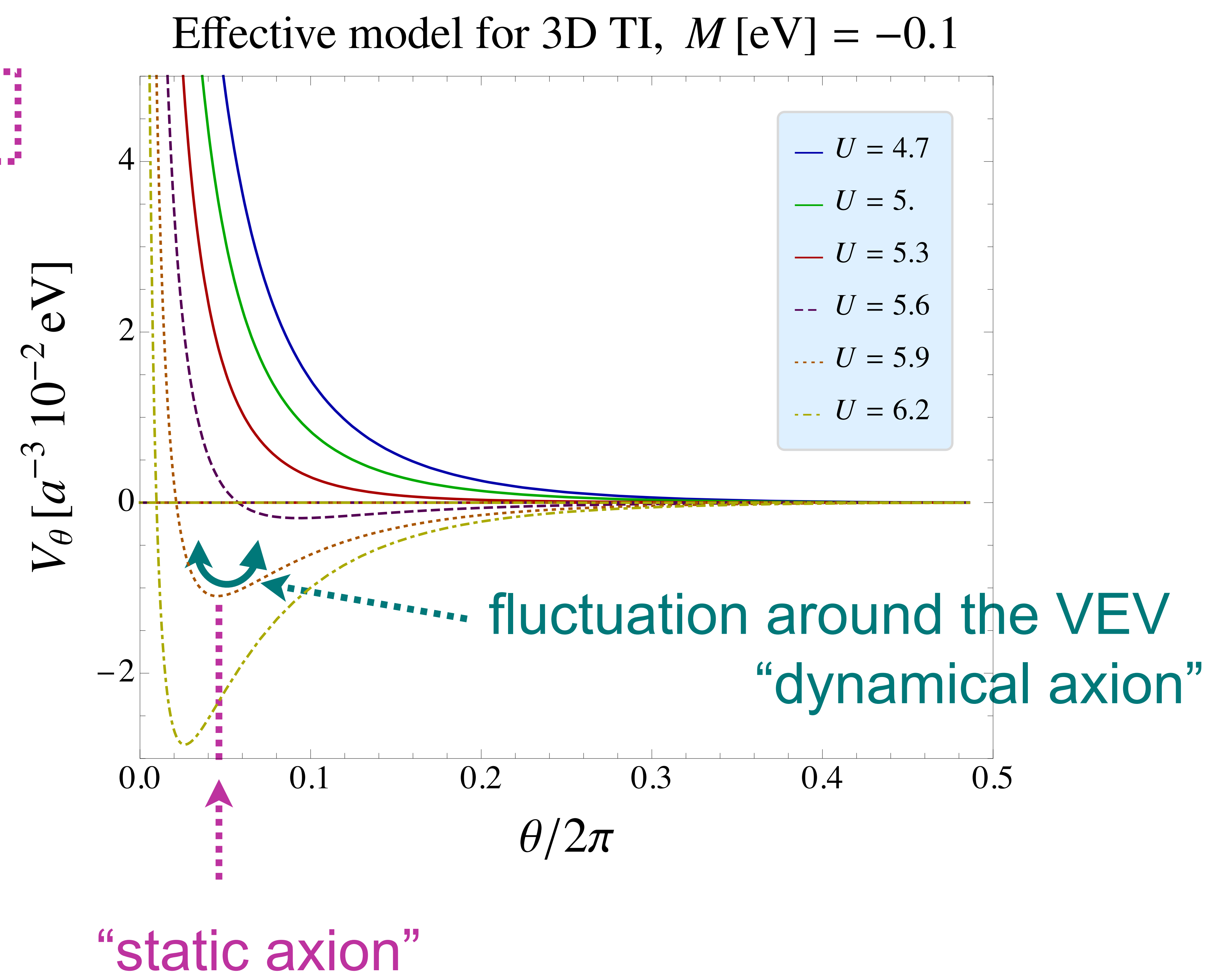
Potential minimum:

- $\theta = \pi$ (small U, i.e., PM)
- $\theta \neq 0$ (large U, i.e., AFM)

Axion in condensed matter

- static axion
- dynamical axion

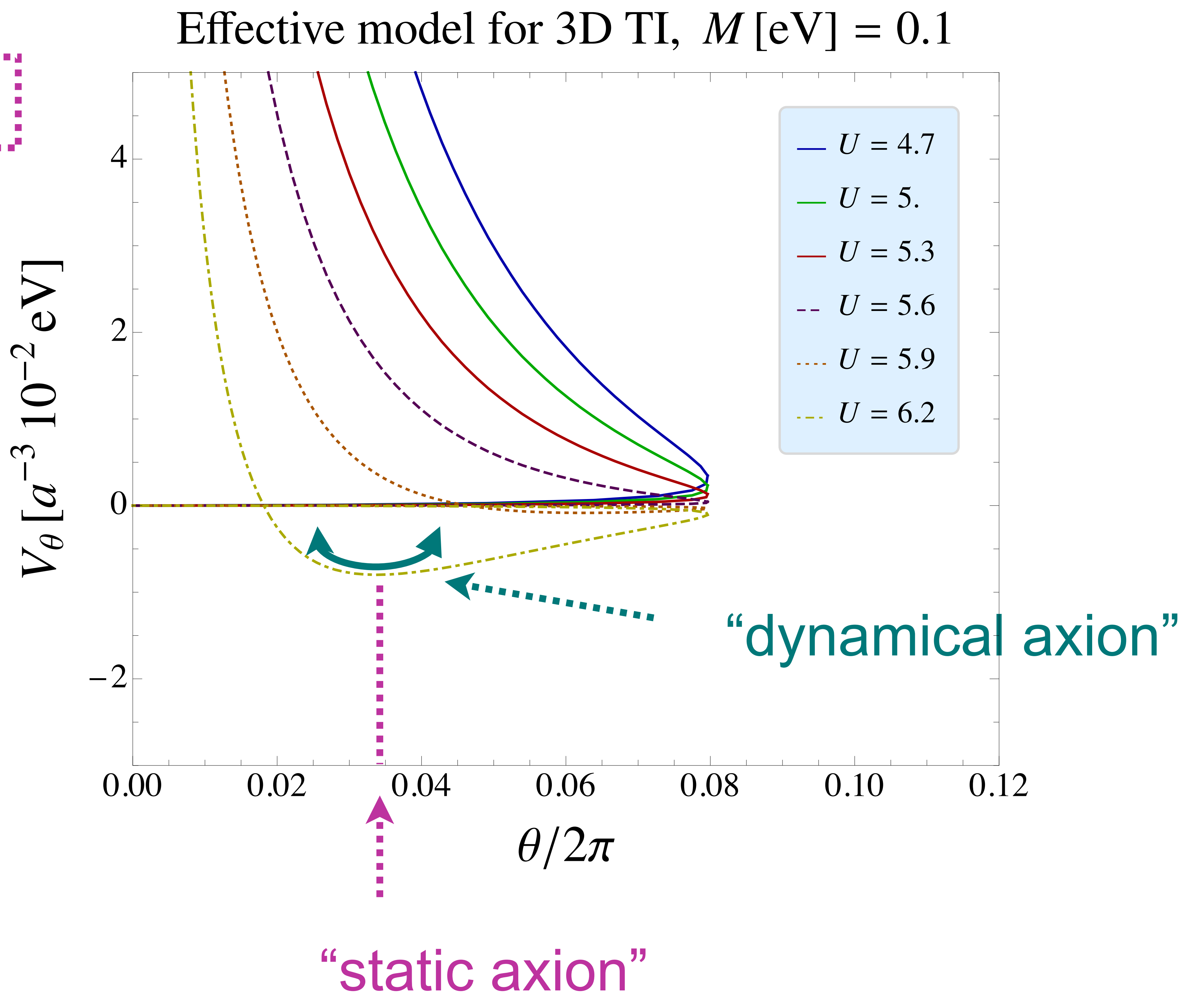
TI phase



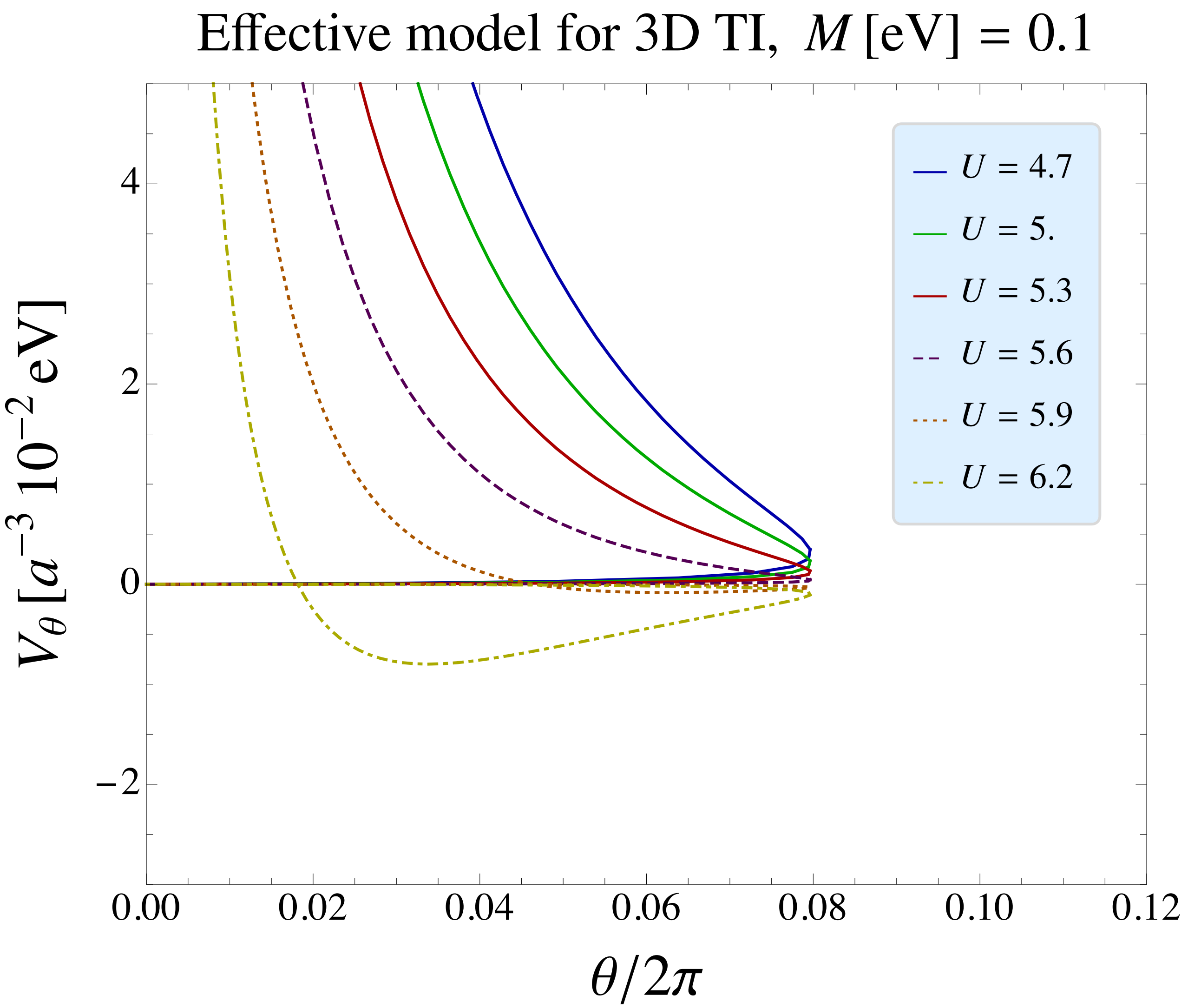
Axion in condensed matter

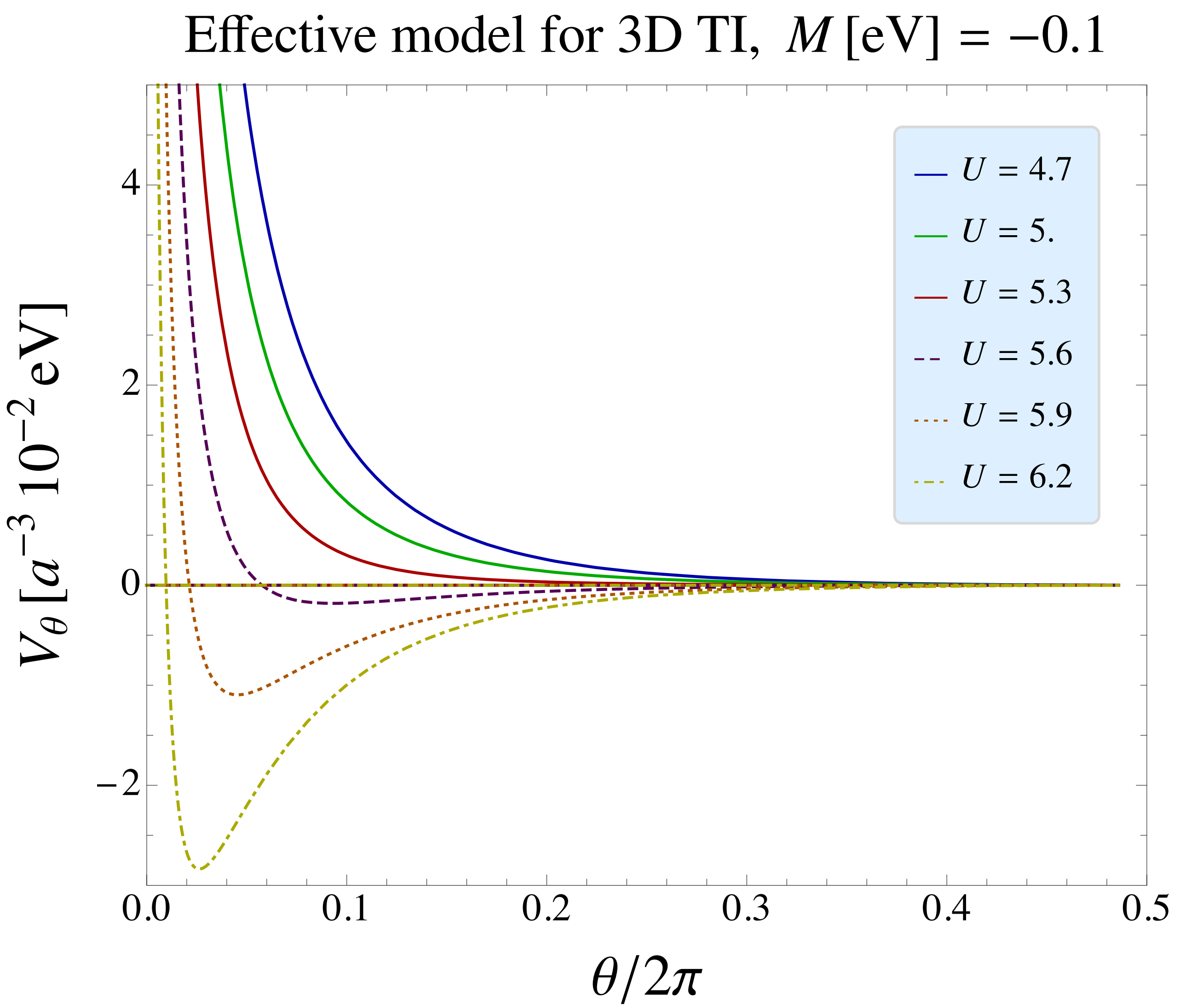
- static axion
- dynamical axion

NI phase



Axion in condensed matter





Dynamical axion exits in both TI and NI phases

Axion in condensed matter

"dynamical axion"

$$\phi = \langle \phi \rangle + \delta\phi$$

"static axion"

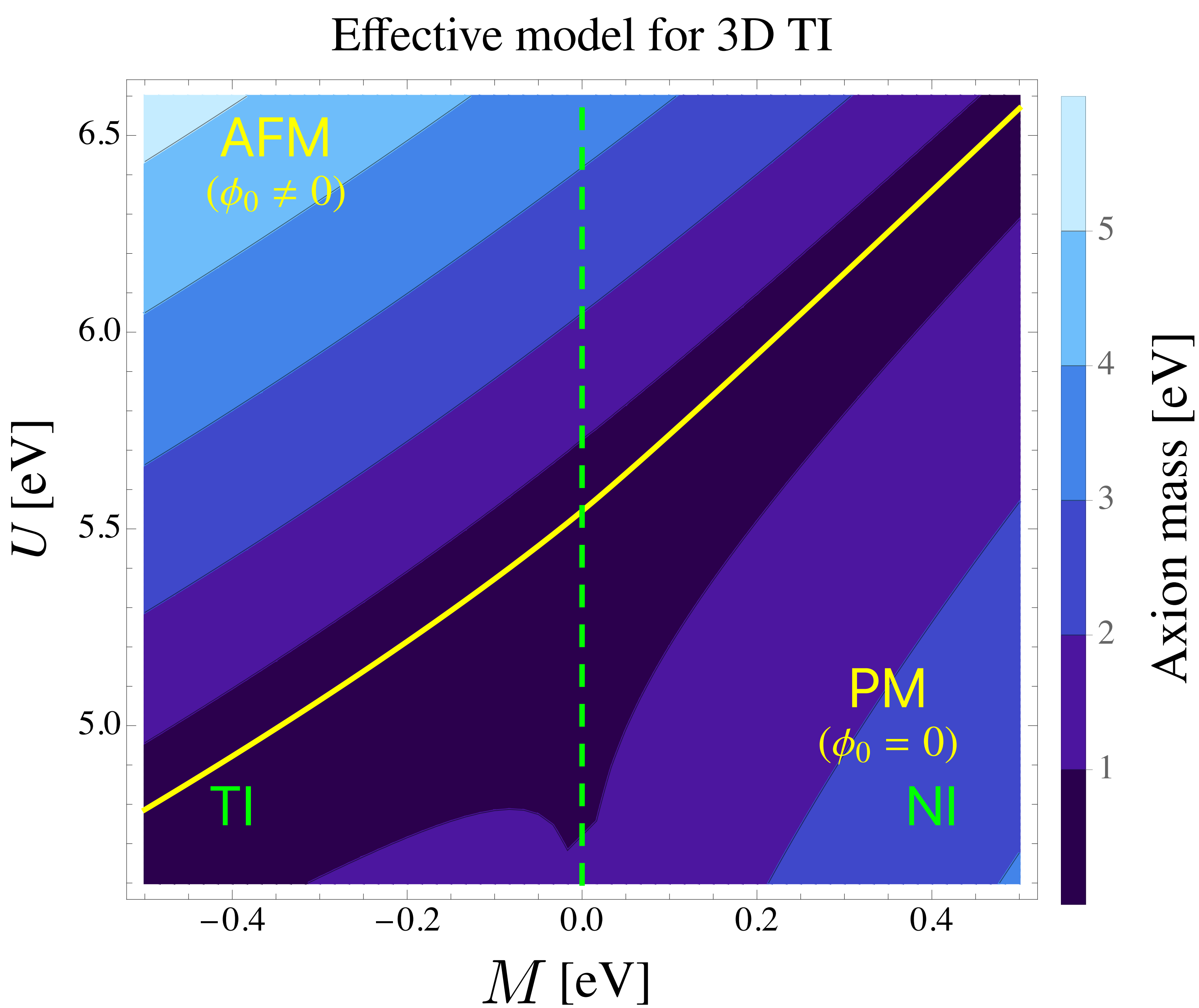
"dynamical axion" = amplitude mode of magnon

magnon: quantum of magnetization

Note: magnon can be dynamical axion

see Chigusa, Nakayama, Moroi '21

Axion mass



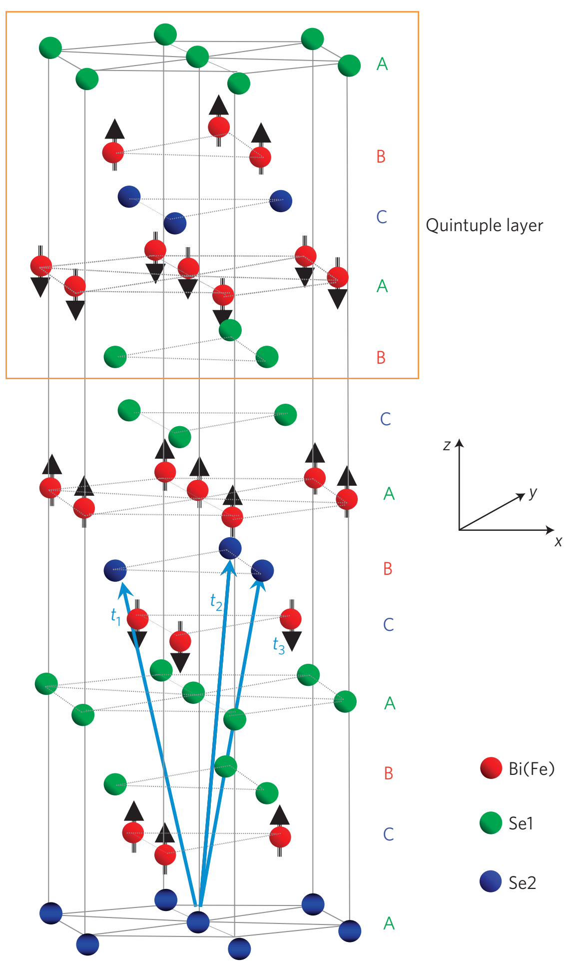
Axion mass is $\mathcal{O}(\mathrm{eV})$

Dynamical axion is predicted in topological magnetic insulators

ARTICLES
PUBLISHED ONLINE: 7 MARCH 2010 | DOI: 10.1038/NPHYS1534
nature physics

Dynamical axion field in topological magnetic insulators

Rundong Li[1], Jing Wang[1,2], Xiao-Liang Qi[1] and Shou-Cheng Zhang[1]*



Bi_2Se_3

$$\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\begin{aligned}\mathcal{S}_{\text{tot}} &= \mathcal{S}_{\text{Maxwell}}+\mathcal{S}_{\text{topo}}+\mathcal{S}_{\text{axion}}\\ &= \frac{1}{8\pi}\int d^3x\,dt\left(\epsilon\mathbf{E}^2-\frac{1}{\mu}\mathbf{B}^2\right)+\frac{\alpha}{4\pi^2}\int d^3x\,dt\,(\theta_0+\delta\theta)\,\mathbf{E}\cdot\mathbf{B}\\ &\quad +g^2J\int d^3x\,dt\,[(\partial_t\delta\theta)^2-(v_i\partial_i\delta\theta)^2-m^2\delta\theta^2] \qquad (4)\end{aligned}$$

Axion mass ~ $\mathcal{O}(\text{meV})$

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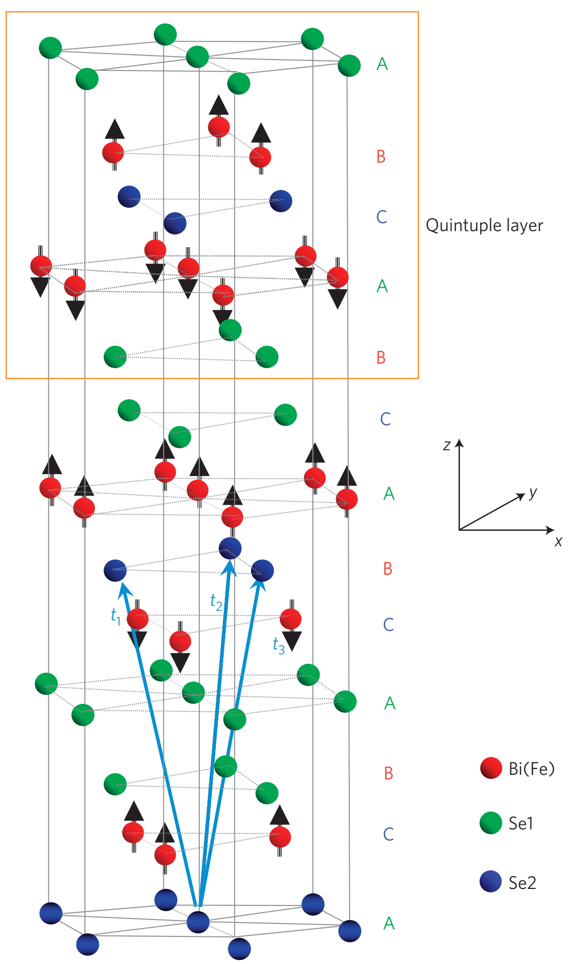


Bi_2Se_3

$$\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\begin{aligned}\mathcal{S}_{\rm tot} &= \mathcal{S}_{\rm Maxwell} + \mathcal{S}_{\rm topo} + \mathcal{S}_{\rm axion} \\ &= \frac{1}{8\pi}\int d^3x dt \left(\epsilon \mathbf{E}^2 - \frac{1}{\mu}\mathbf{B}^2\right) + \frac{\alpha}{4\pi^2}\int d^3x dt (\theta_0 + \delta\theta)\mathbf{E}\cdot\mathbf{B} \\ &\quad + g^2 J \int d^3x dt [(\partial_t \delta\theta)^2 - (v_i \partial_i \delta\theta)^2 - m^2 \delta\theta^2] \qquad (4)\end{aligned}$$

Axion mass ~ $\mathcal{O}(\mathrm{meV})$

?

The thing is …

R. Li et al. ’10

- $\langle \phi \rangle \ (= m_5) \ = 1 \ \mathrm{meV}$ is taken

  (i.e., $\langle \phi \rangle$ is considered to be a free parameter)

- AFM order is *assumed*

The thing is …

R. Li et al. ’10

- $\langle \phi \rangle \; (= m_5) \; = 1 \; \mathrm{meV}$ is taken

  (i.e., $\langle \phi \rangle$ is considered to be a free parameter)

  → Axion mass ~ $\mathcal{O}(\mathrm{meV})$ $\quad (\because \; m_a^2 \propto m_5^2)$

- AFM order is *assumed*

The thing is …

R. Li et al. '10

- $\langle\phi\rangle\ (= m_5)\ = 1\ \mathrm{meV}$ is taken

  (i.e., $\langle\phi\rangle$ is considered to be a free parameter)

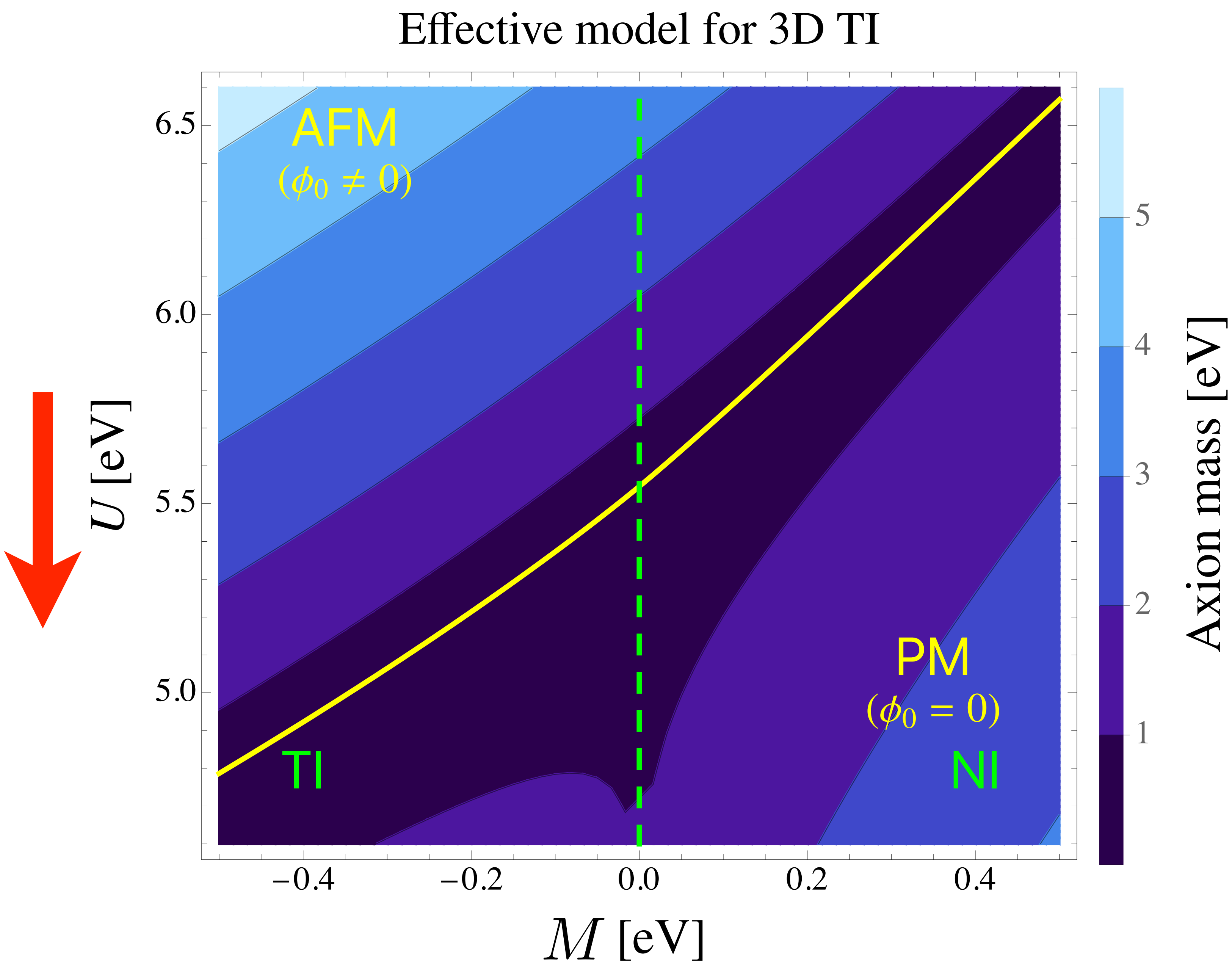
  ⟶ Axion mass ~ $\mathcal{O}(\mathrm{meV})$ $(\because\ m_a^2 \propto m_5^2)$

But this is not naively possible since

- AFM order is *assumed*

$m_5 \sim U \sim \mathrm{eV}$ (in AFM order)

Axion mass



Suppressed U → No AFM

The thing is …

R. Li et al. ’10

- $\langle\phi\rangle \ (= m_5) \ = 1 \ \mathrm{meV}$ is taken

  (i.e., $\langle\phi\rangle$ is considered to be a free parameter)

→ Axion mass ~ $\mathcal{O}(\mathrm{meV})$ $(\because \ m_a^2 \propto m_5^2)$

But this is not naively possible since

- AFM order is *assumed* $m_5 \sim U \sim \mathrm{eV}$ (in AFM order)

They may have misunderstood the amplitude mode as magnon?

The thing is …

R. Li et al. '10

- $\langle \phi \rangle \ (= m_5) \ = 1 \ \mathrm{meV}$ is taken

  (i.e., $\langle \phi \rangle$ is considered to be a free parameter)

  ⟶ Axion mass ~ $\mathcal{O}(\mathrm{meV})$ $(\because \ m_a^2 \propto m_5^2)$

- AFM order is *assumed*

The thing is …

R. Li et al. '10

- $\langle \phi \rangle \ (= m_5) \ = 1 \ \mathrm{meV}$ is taken

  (i.e., $\langle \phi \rangle$ is considered to be a free parameter)

  ⟶ Axion mass ~ $\mathcal{O}(\mathrm{meV})$ $\quad (\because \ m_a^2 \propto m_5^2)$

- AFM order is *assumed*

  No AFM in TI in the first place

  ⟶ Fe -doped $\mathrm{Bi_2Te_3}$ is proposed

- Fe-doped Bi_2Te_3

  "likely to be AFM" J.M. Zhang et al. '13

  (by first-principles calculation)

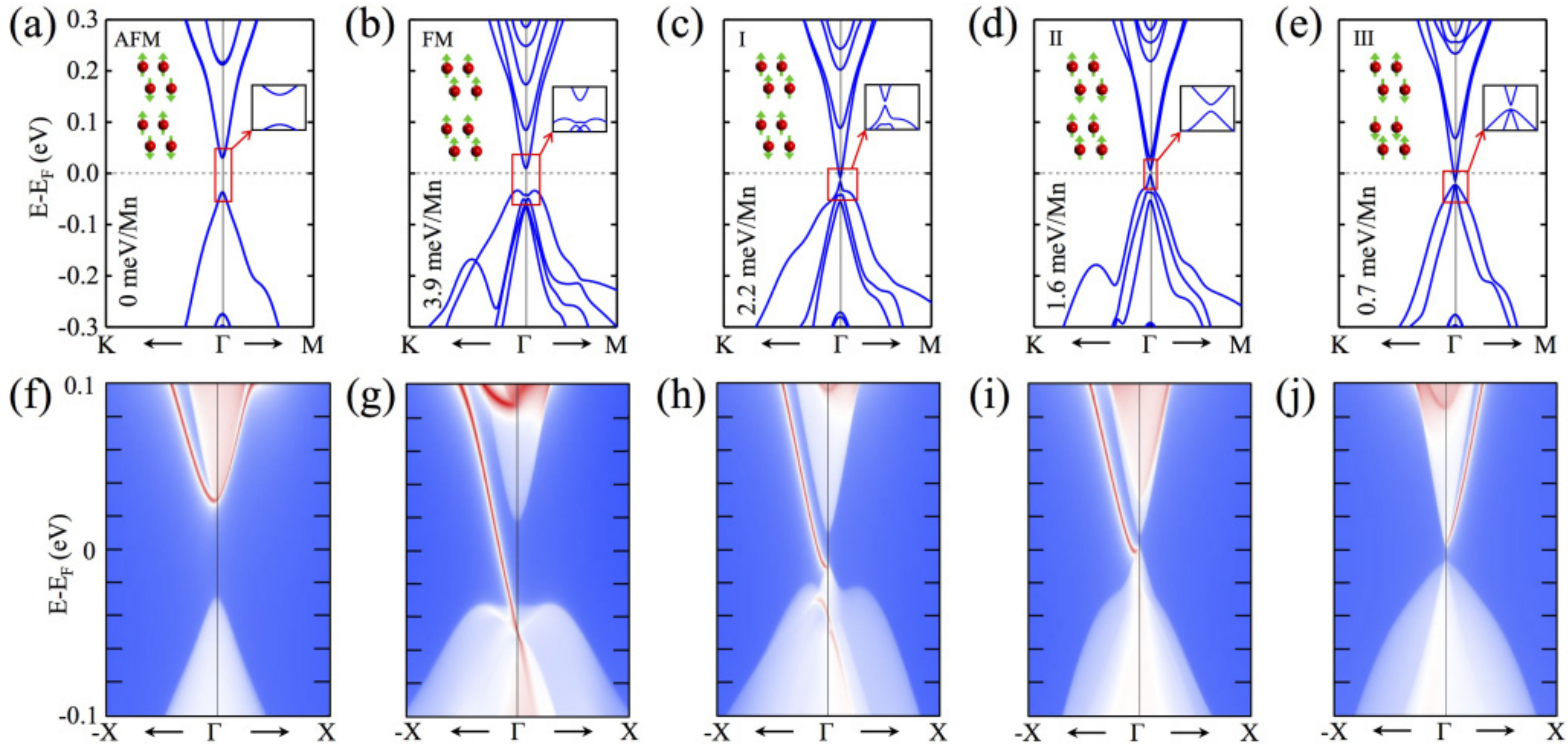
  → it looks unlikely to be realized …

- $Mn_2Bi_2Te_5$ J. Zhang et al. '19

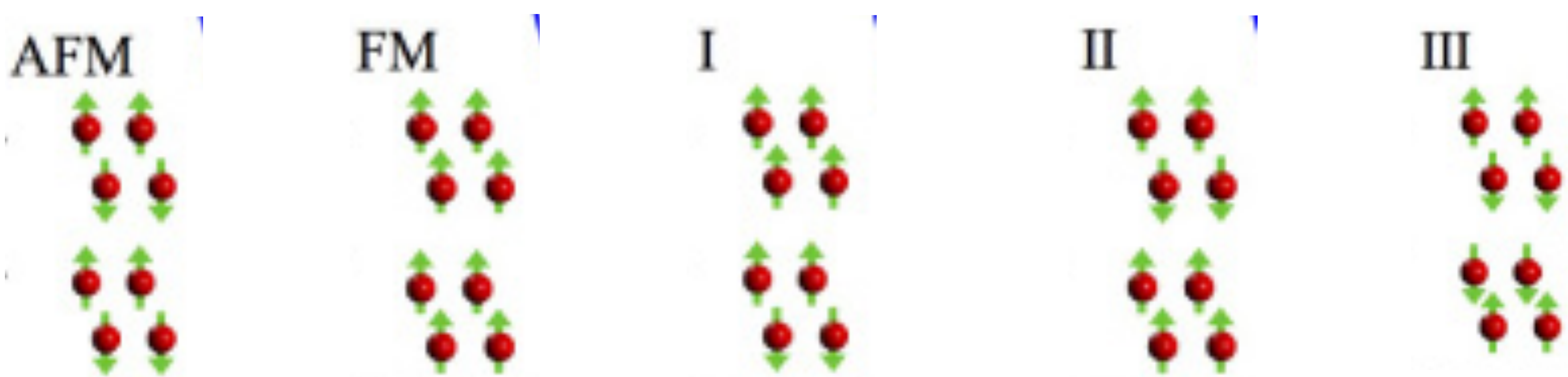
  "rich magnetic topological quantum states" Y. Li et al. '20

  (by first-principles calculation)

Y. Li et al. ’20

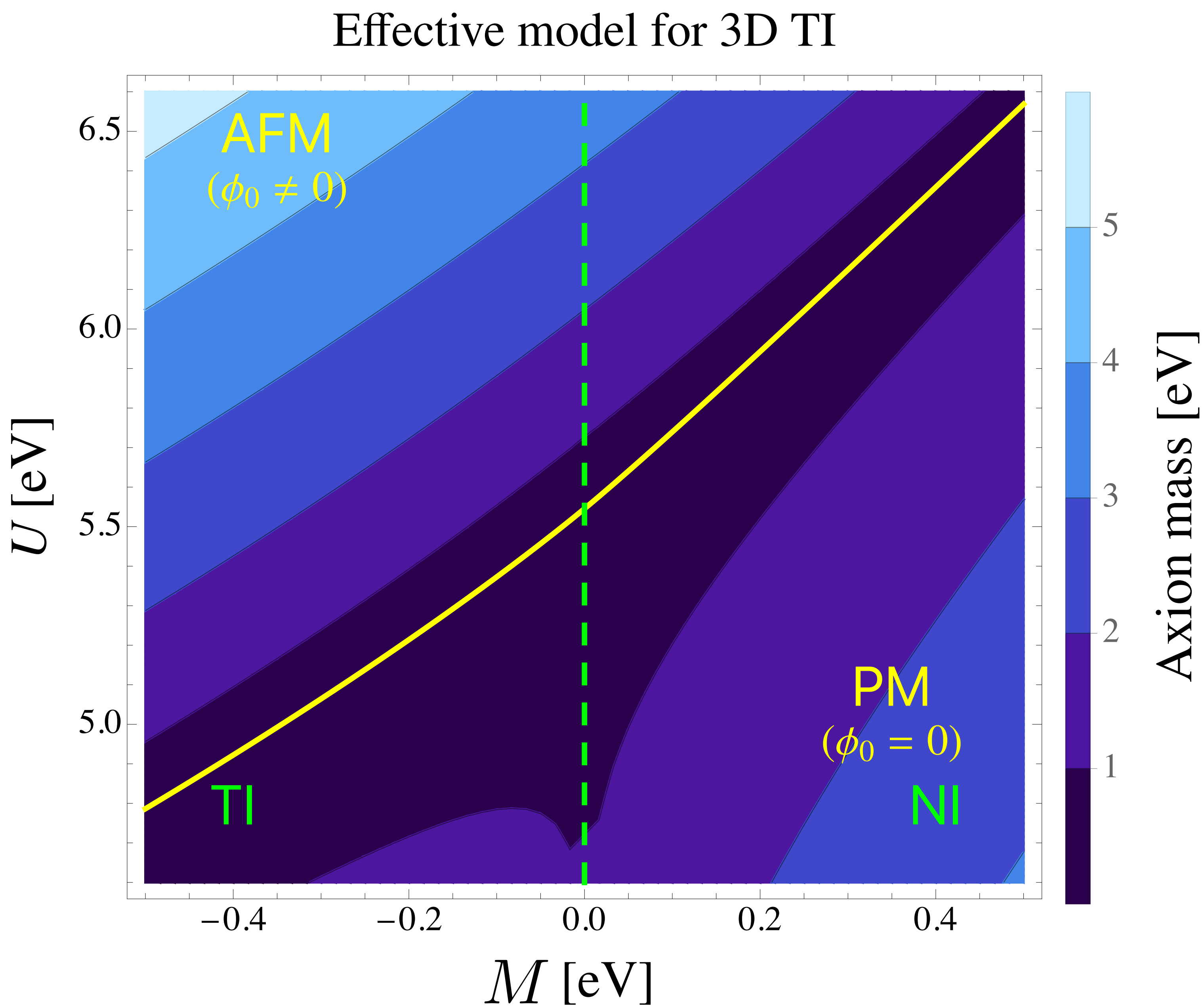


(a)
(b)
(c)
(d)
(e)
AFM
FM
I
II
III
0 meV/Mn
3.9 meV/Mn
2.2 meV/Mn
1.6 meV/Mn
0.7 meV/Mn
E-E$_F$ (eV)
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
K ← Γ → M
(f)
(g)
(h)
(i)
(j)
0.1
0
-0.1
-X ← Γ → X



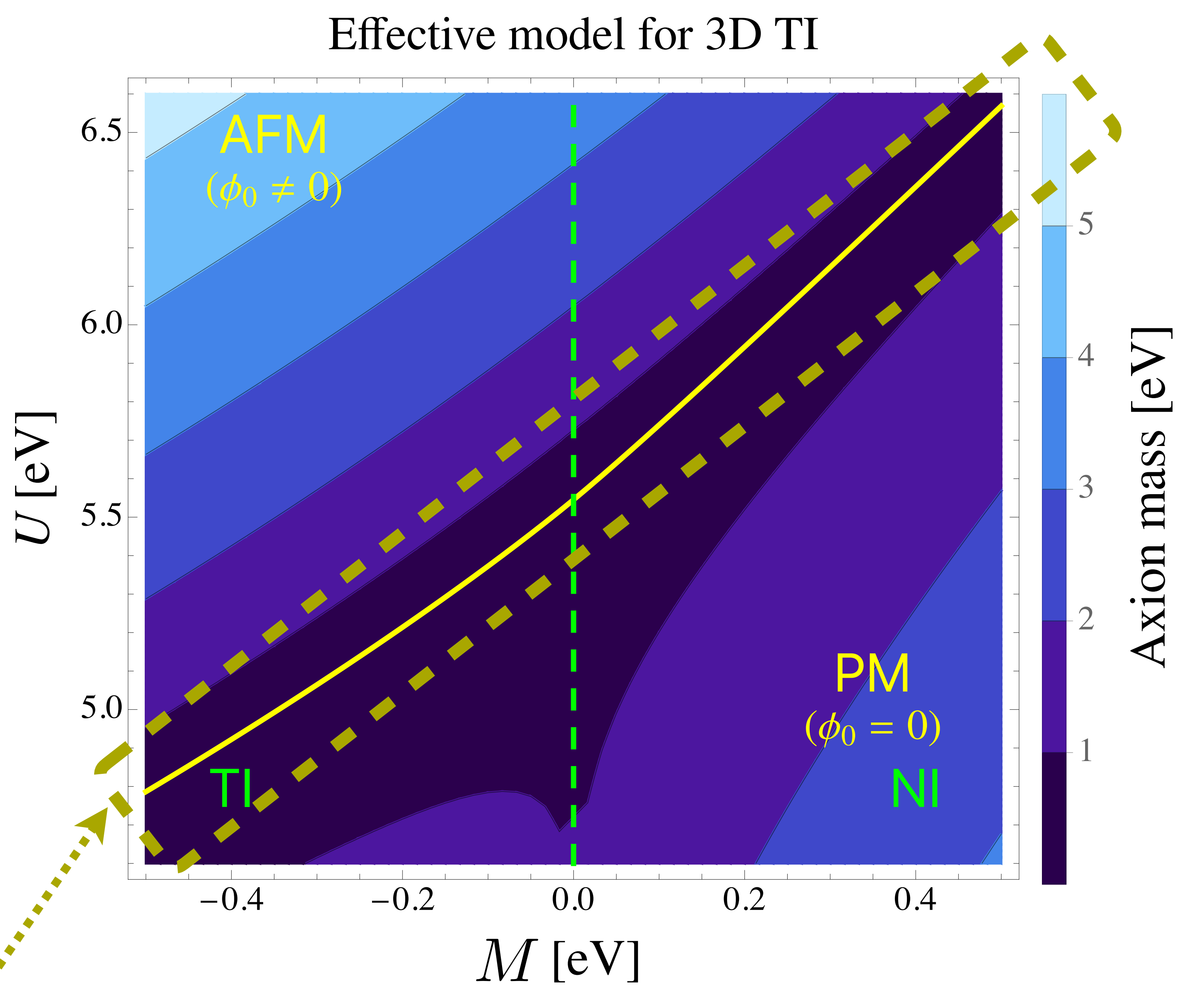
AFM
FM
I
II
III

Axion mass



Effective model for 3D TI
AFM
($\phi_0 \neq 0$)
PM
($\phi_0 = 0$)
TI
NI
6.5
6.0
5.5
5.0
U [eV]
−0.4
−0.2
0.0
0.2
0.4
M [eV]
1
2
3
4
5
Axion mass [eV]

Axion mass



It can be suppressed near the phase boundary

Rich magnetic topological states in that region?

4. Conclusions and discussion

We have static and dynamical axions in AFM TI consistently by using path integral

- Nonzero $\langle \phi \rangle$ is obtained from the effective potential, which gives rise to AFM order and breaks $\mathcal{T}$
- Dynamical axion appears both in TI and NI
- Axion mass is $\lesssim \mathcal{O}(\mathrm{eV})$

Discussion

- How do we describe axion in $\mathrm{Mn_2Bi_2Te_5}$?
- What about axion in NI ?
- Dynamical axion in ferromagnetic state or other magnetic states?