Electroweak Skyrmion as Asymmetric Dark Matter

Yu Hamada (KEK)

[arXiv:2108.12185]

in collaboration with

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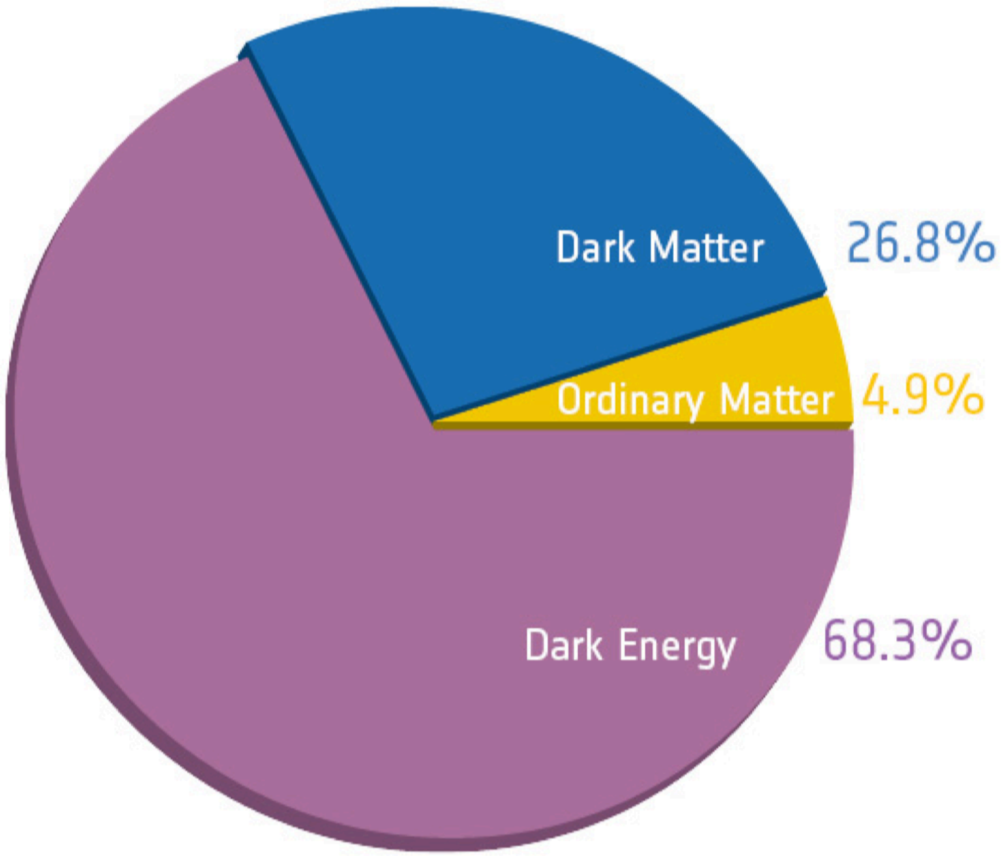
12th Novermber 2021, 京大素粒子論研究室セミナー

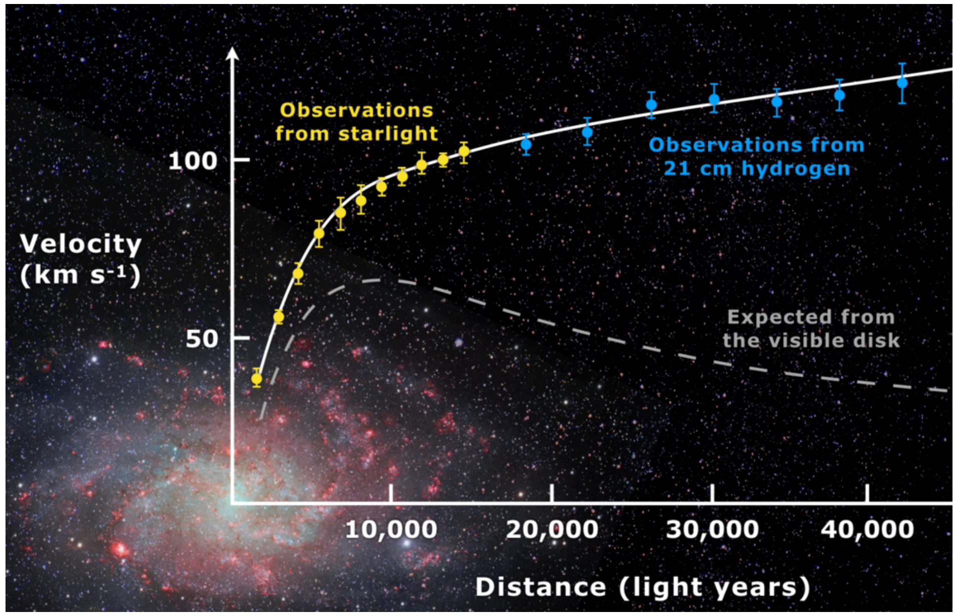
Plan of talk

- Introduction
- Electroweak Skyrmion
- Asymmetric darkmatter scenario
- Summary

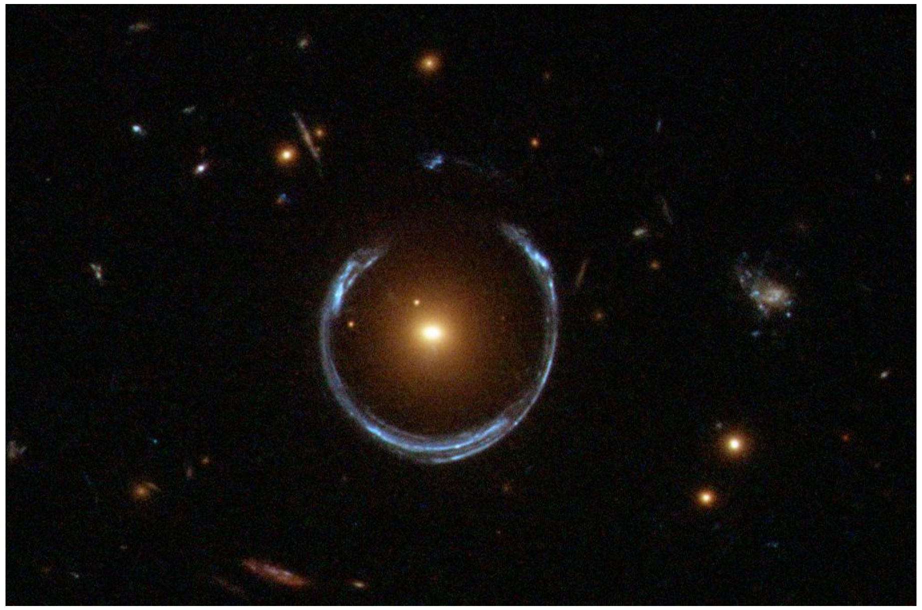
Dark matter

- **There are many evidences for dark matter**



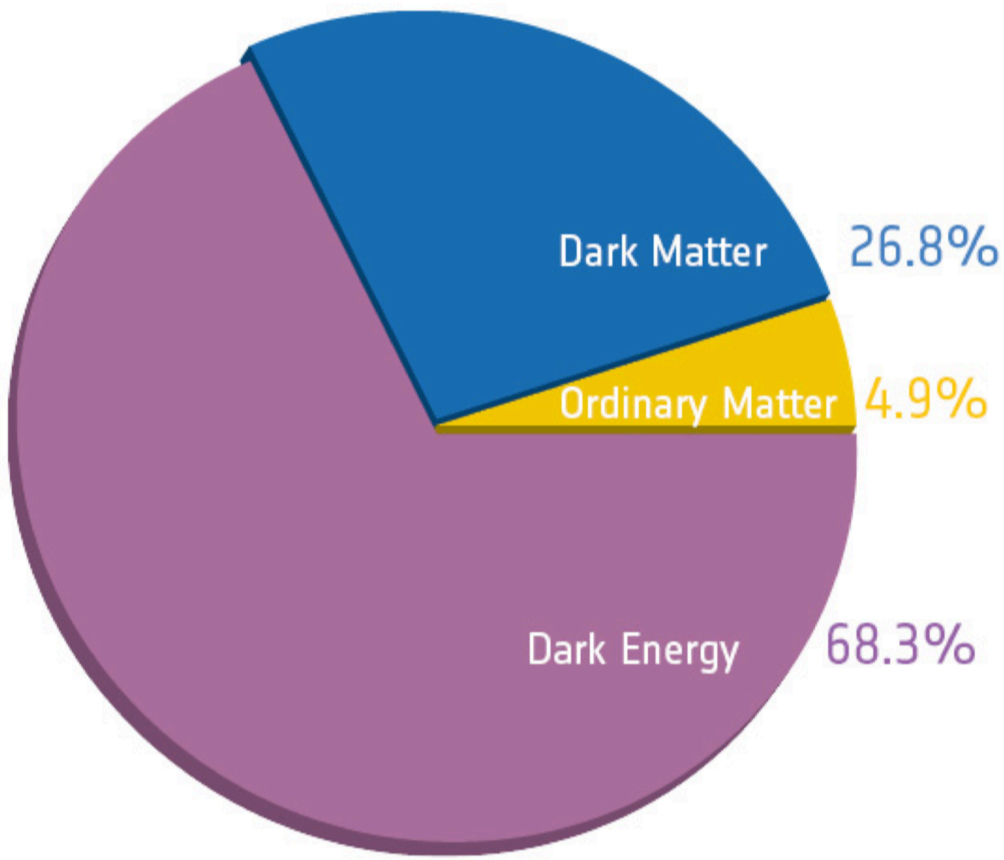


Galaxy rotation curve (from Wikipedia)



Gravitational lens (from Wikipedia)

Dark matter



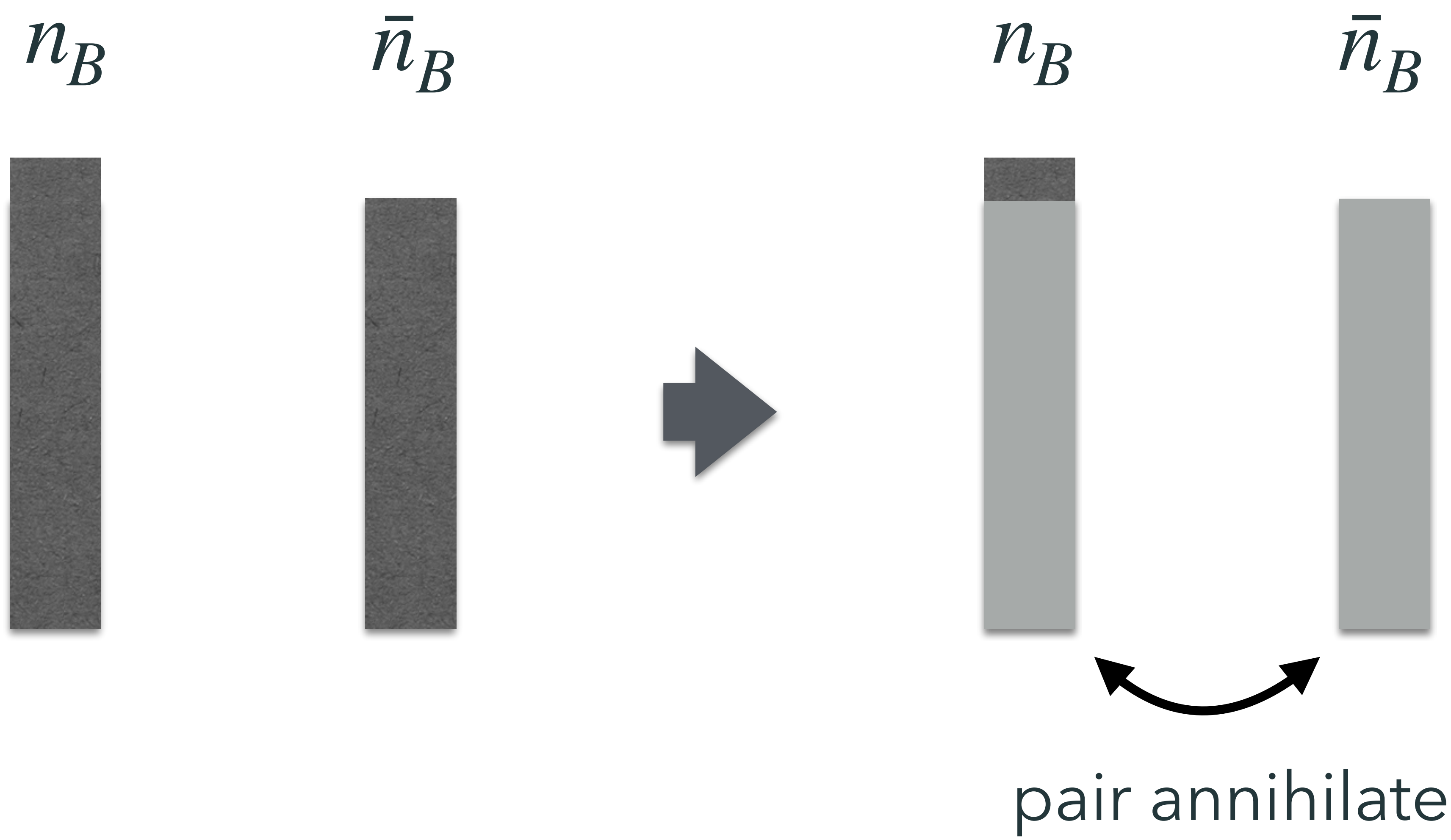
$$\rightarrow \Omega_{DM}/\Omega_B \simeq 5$$

Why are they close to each other?

(coincidence problem)

Common origin?

Baryon Asymmetry

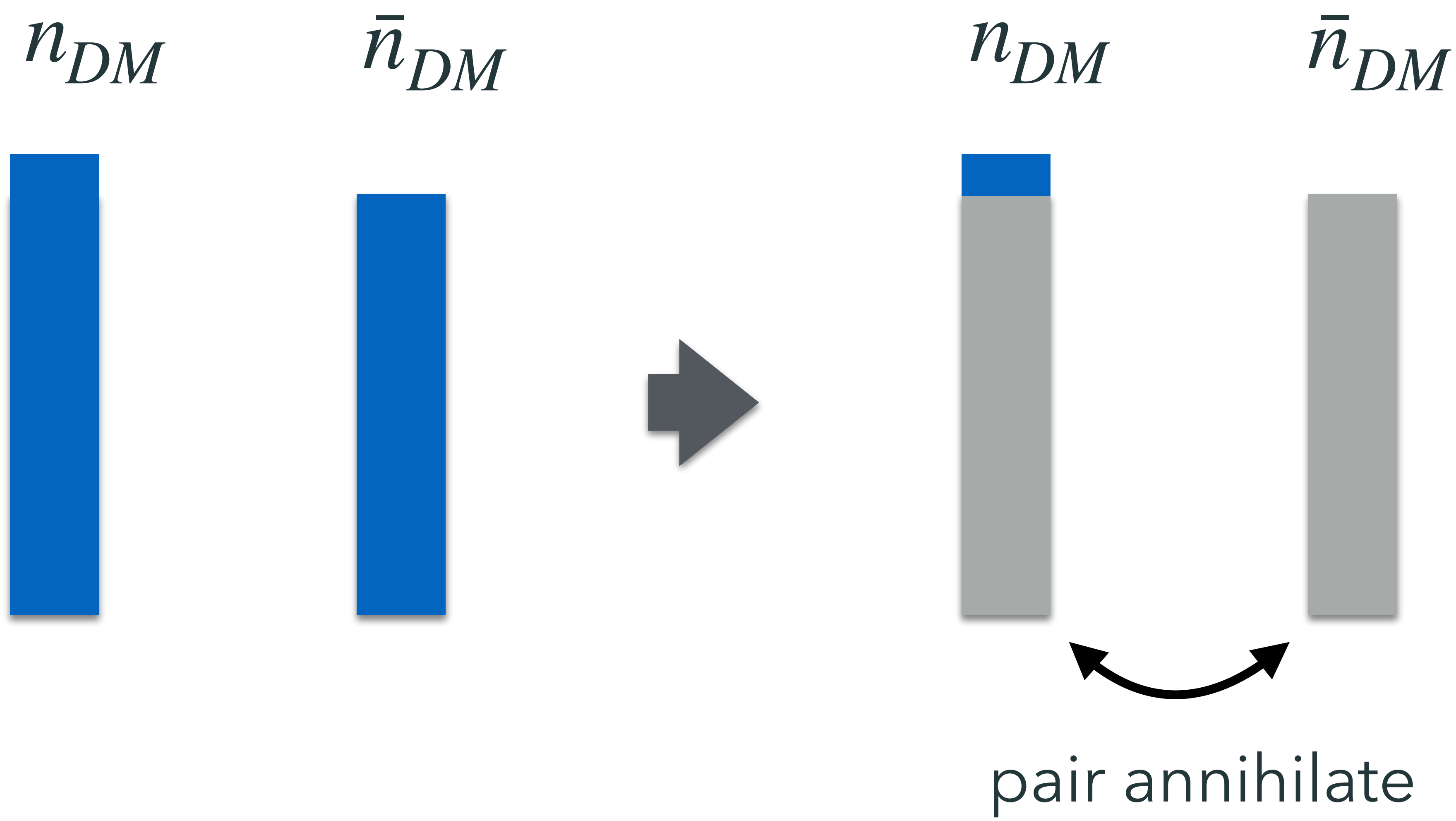


For baryons, symmetric part annihilates after QCD phase transition, and then asymmetric part remains:

$$\eta_B \equiv \frac{n_B - \bar{n}_B}{s} \simeq 10^{-10}$$

Asymmetric Dark Matter

[Barr, Chivukula, Farhi '90] [Kaplan '92]
[Kitano, Low '04] [Kaplan, Luty, Zurek '09]



Asymmetric DM hypothesis:

- **Similarly to baryons, DM relic abundance originates from asymmetric part of DM.**
- **DM asymmetry is related with that of baryons.**

Asymmetric Dark Matter

- A naive example of the realization is dark QCD…

QCD		dark QCD
baryon	$\longleftrightarrow$	dark baryon (DM)
meson	portal interaction	dark meson

But such a model is rather complicated…

(UV completion, dark radiation..)

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Soliton!

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Soliton! = non-perturbative object in field theory

Take-home Message

If SM Higgs Lagrangian is extended by $\mathcal{O}(p^4)$ terms, the theory contains asymmetric DM, which is a soliton made of Higgs and EW gauge fields!!

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Electroweak Skyrmion

Skyrme model (review)

[Skyrme '62]

$$\boxed{\mathscr{L}_{\text{Skyrme}} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right] + \frac{1}{32e^2}\mathrm{Tr}\left[\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]^2\right]}$$

$U(x) \in SU(2)$ $\qquad$ $U^\dagger U = 1$ $\qquad$ $U = \exp\left[i\underline{\pi^a(x)}\,\sigma^a\right]$

pion field (π^1, π^2, π^3)

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- Vacuum manifold (order parameter space):

$$\mathscr{M}_{\text{vac}} = \frac{SU(2)_L \times SU(2)_R}{SU(2)_V} \simeq S^3$$

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[Skyrme '62]

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non-trivial solution of EOM → **Skyrmion!** $\quad (r \equiv \sqrt{x_i x_i}, \quad \hat{x}^a \equiv x^a/r)$

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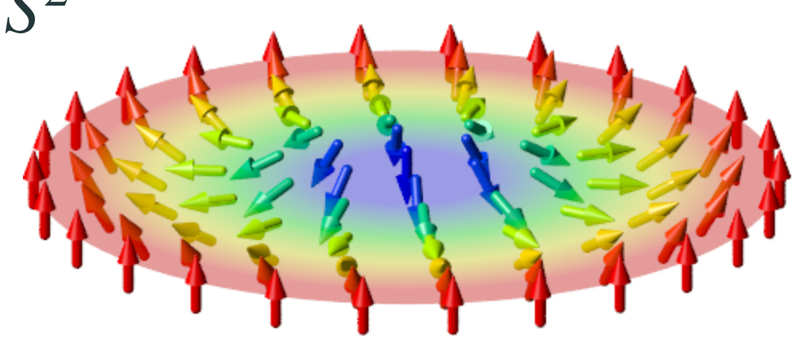
eg.) 2D → S^2



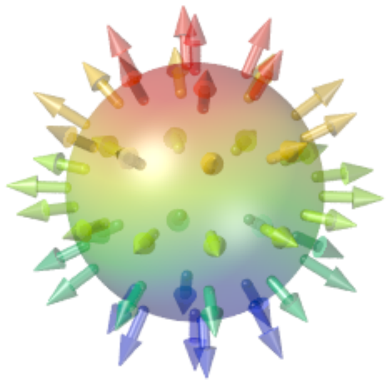


fig from arXiv:1405.0987

Skyrme term

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$e \to \infty \;\Rightarrow$ **shrinks into an infinitely small one:**

cf. Derrick's theorem

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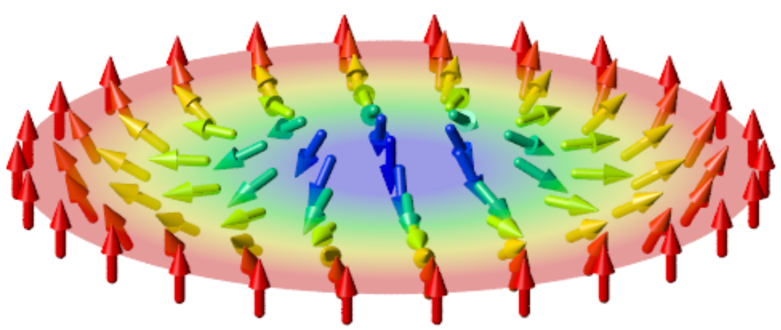

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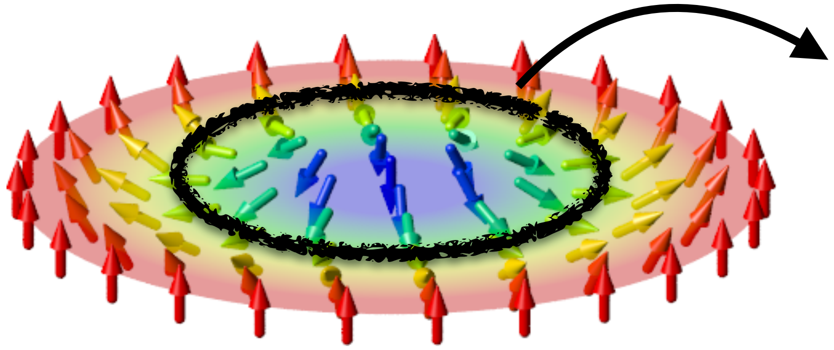

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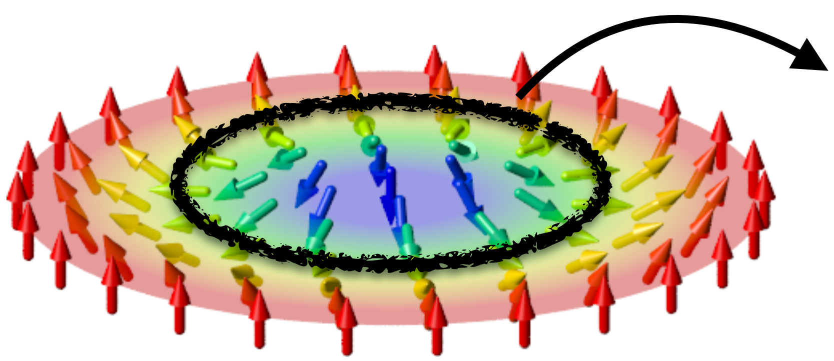

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- Let R be a typical size of Skyrmion.

$$E = -\int d^3x\, \mathscr{L} \simeq \frac{f_\pi^2}{4}R + \frac{1}{32e^2}R^{-1} \Rightarrow R \sim (ef_\pi)^{-1}$$

Symmetry in Higgs sector

- **SM Higgs sector has a similar symmetry breaking structure:**

$$\mathscr{L}_{higgs}\Big|_{g=g'=0} = |\partial_\mu \Phi|^2 - \lambda \left(|\Phi|^2 - \frac{v_{\mathrm{EW}}}{2} \right)^2$$

$$= \frac{1}{2}\mathrm{Tr}|\partial_\mu H|^2 - \frac{\lambda}{4}\left(\mathrm{Tr}|H|^2 - v_{\mathrm{EW}}\right)^2$$

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Higgs Lagrangian

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EW-Skyrmion Solution

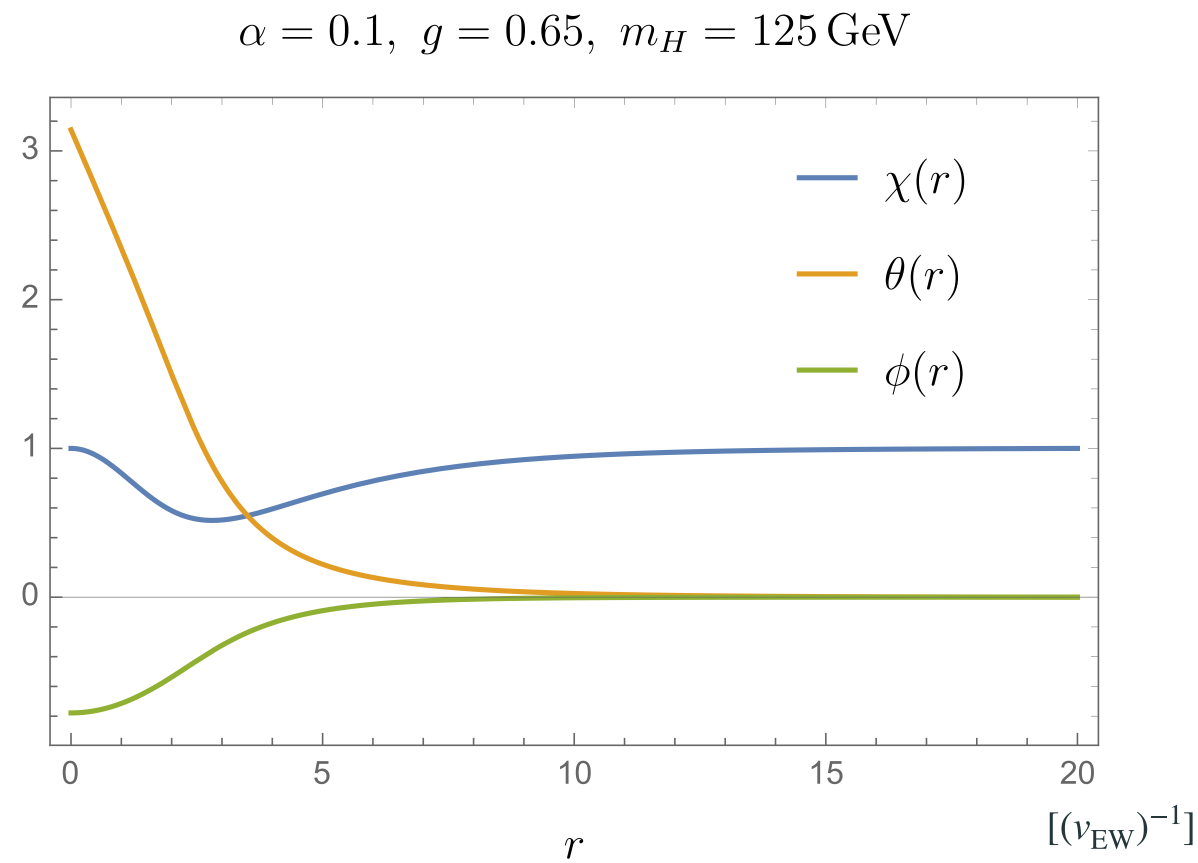
- Hedgehog ansatz:

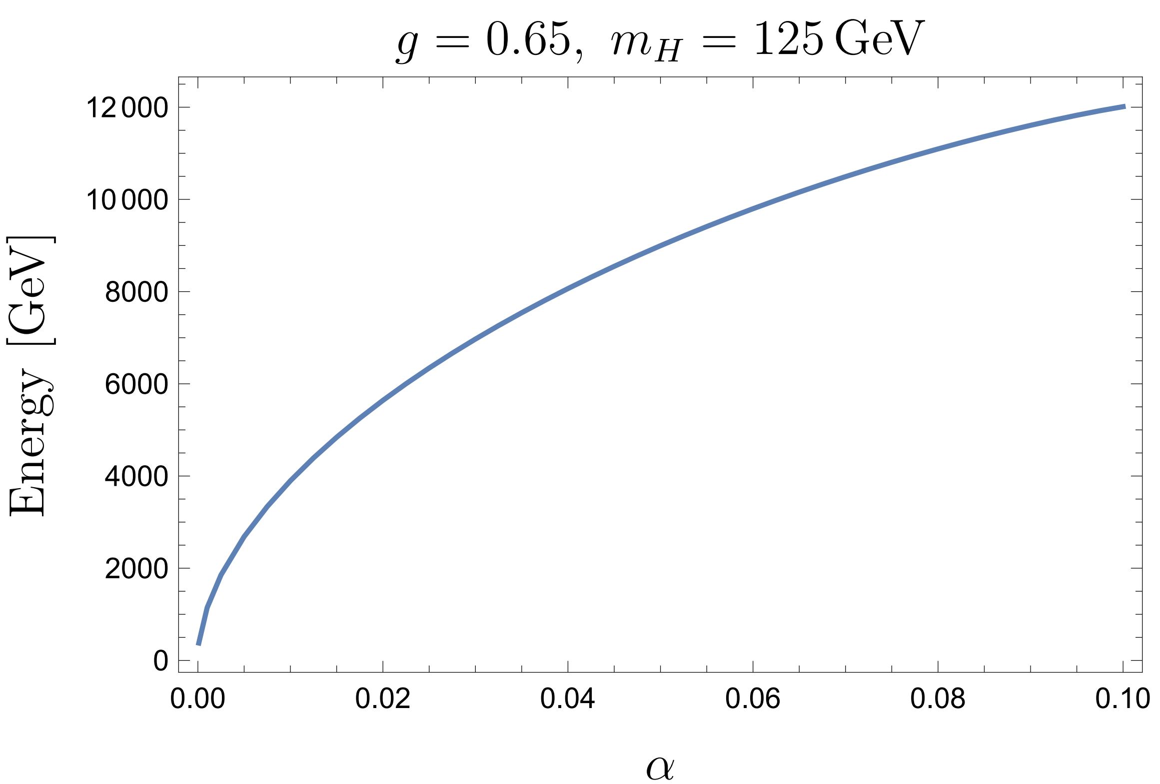
$$U = \exp\left[i\theta(r)\,\hat{x}^a\sigma^a\right] \qquad h(x) = \phi(r)/v_{\mathrm{EW}}$$

$$W_i^a(x) = \frac{\chi(r)-1}{r}\epsilon_{iab}\hat{x}_b - \xi(r)\hat{x}_i\hat{x}_a$$

auxiliary field
(explicitly solvable)

- non-trivial solution of EOM:





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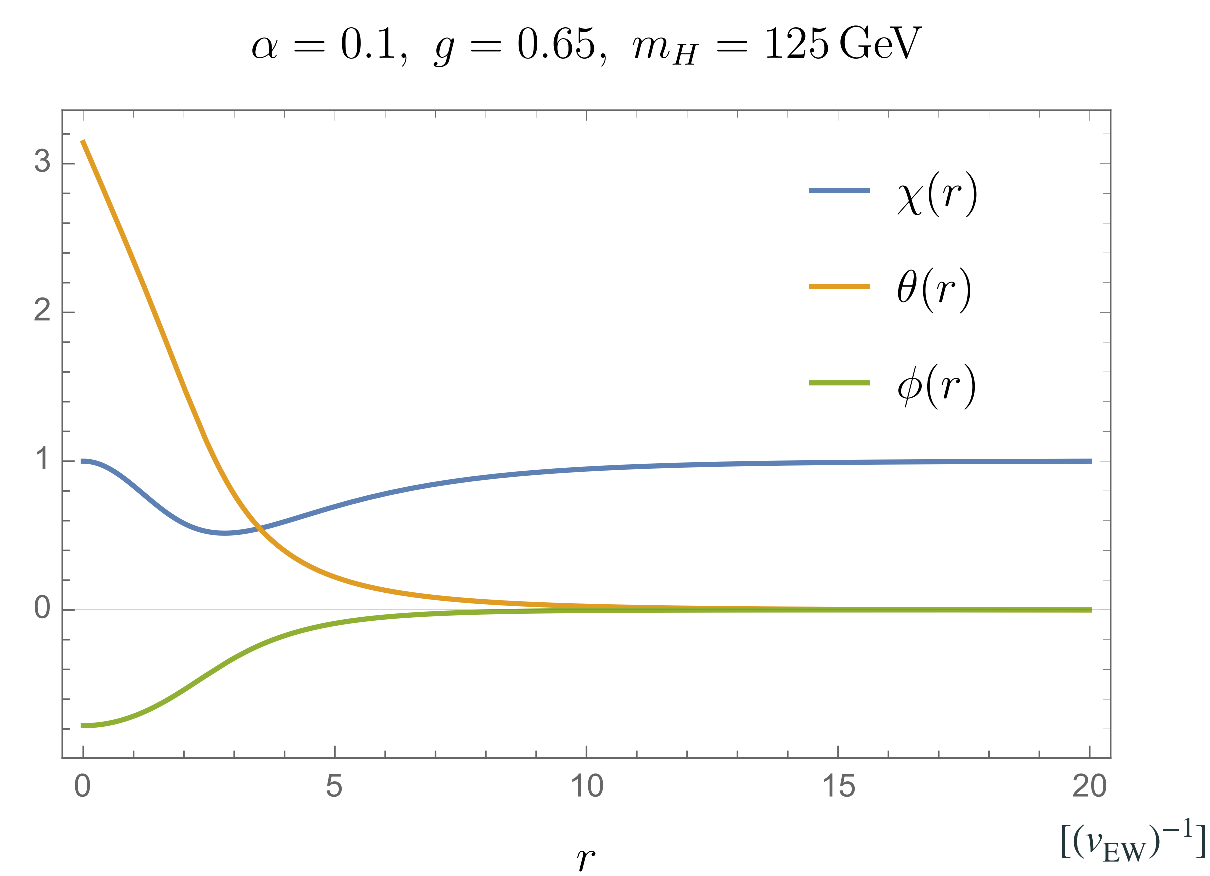
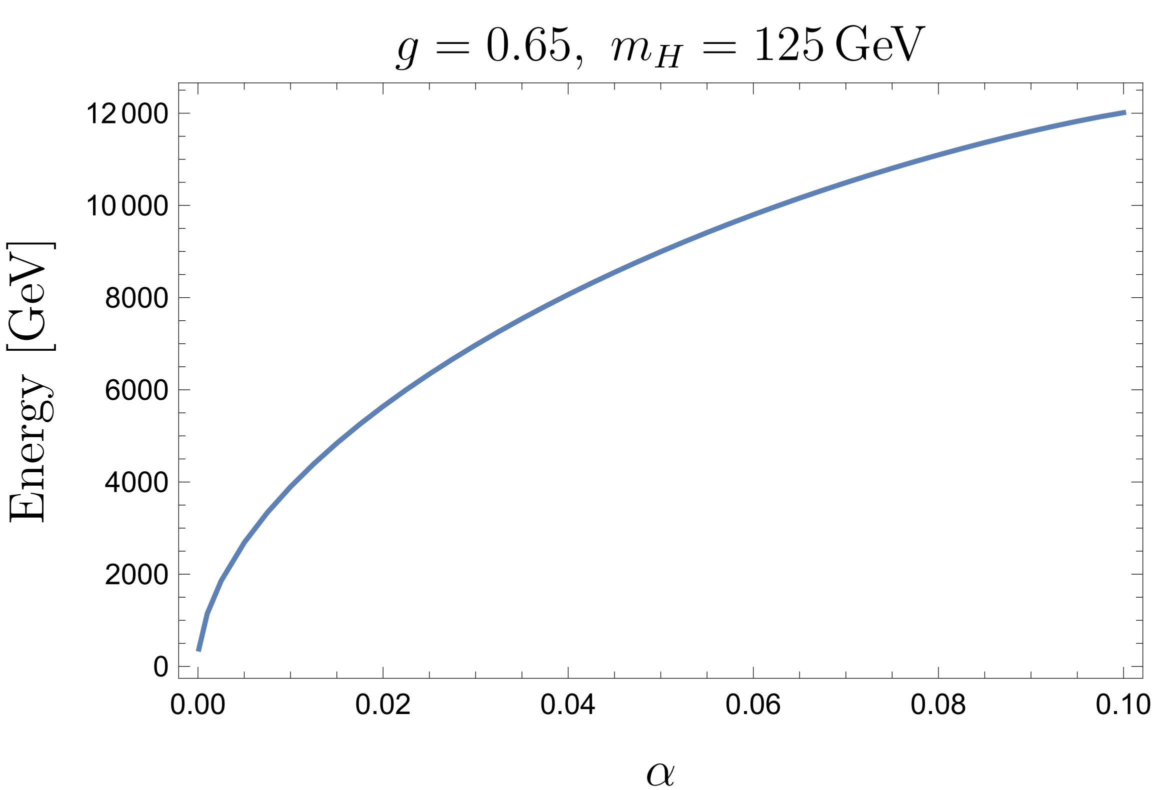


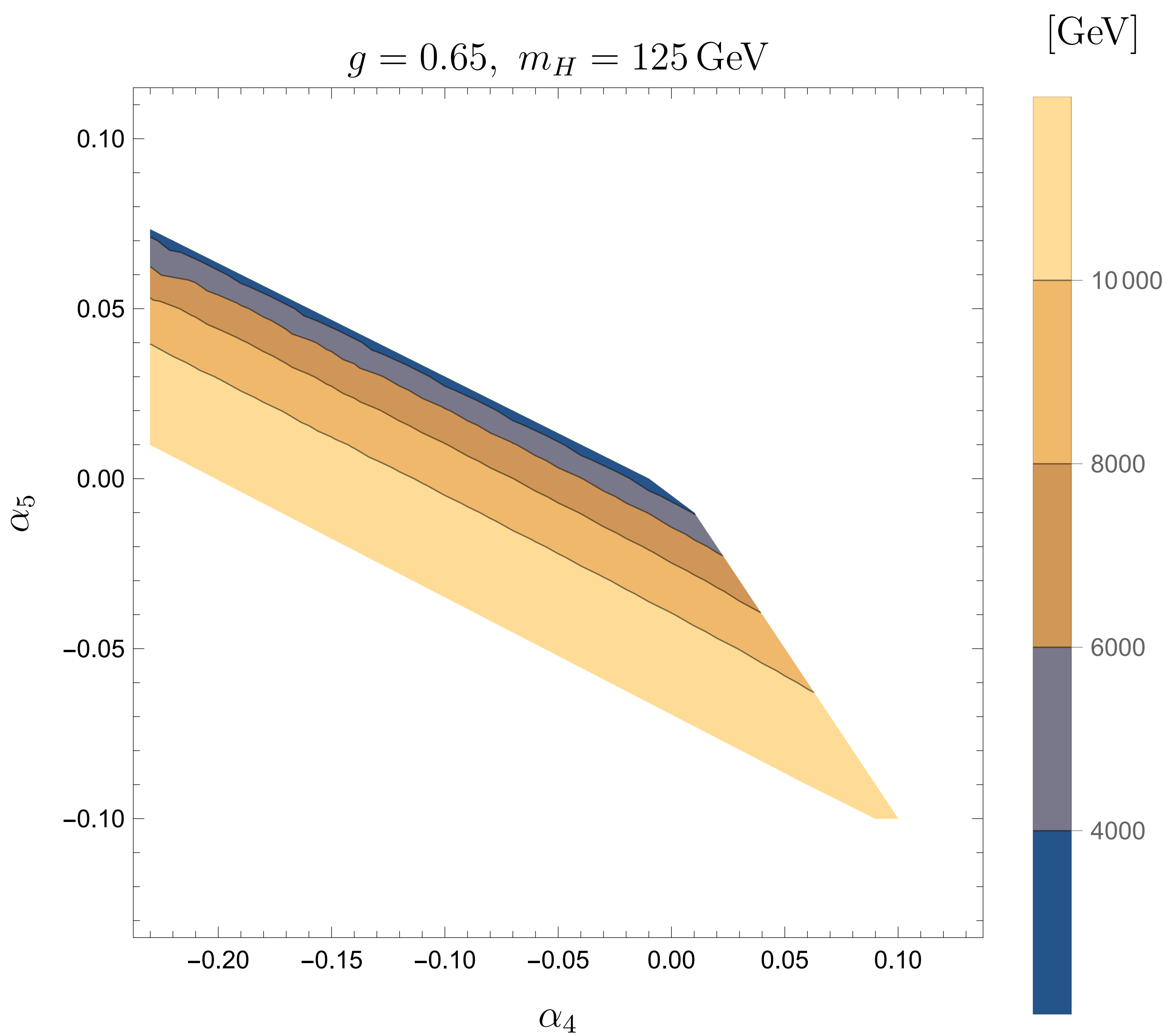


→EW-Skyrmion does exist!!

Energy of EW-Skyrmion

- Take two coefficients as independent parameters

$$\mathscr{L}_{p^4} = \alpha_4 \mathrm{Tr}\left[D_\mu U^\dagger D_\nu U\right] \mathrm{Tr}\left[D^\mu U^\dagger D^\nu U\right] + \alpha_5 \left(\mathrm{Tr}\left[D_\mu U^\dagger D^\mu U\right]\right)^2$$



Quartic Gauge Coupling

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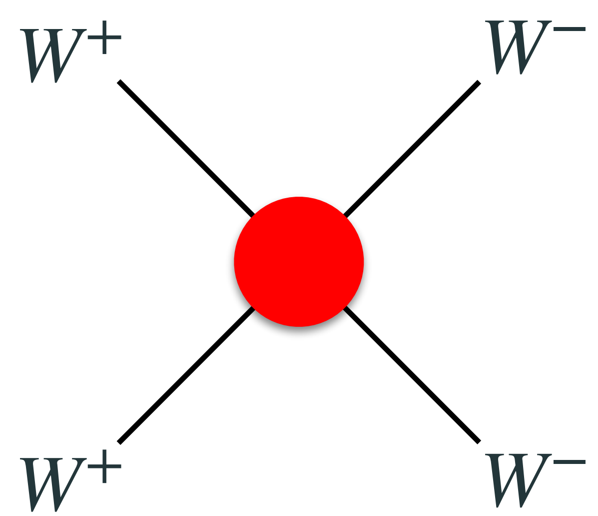
- α_4 and α_5 lead to anomalous quartic gauge coupling (aQGC)



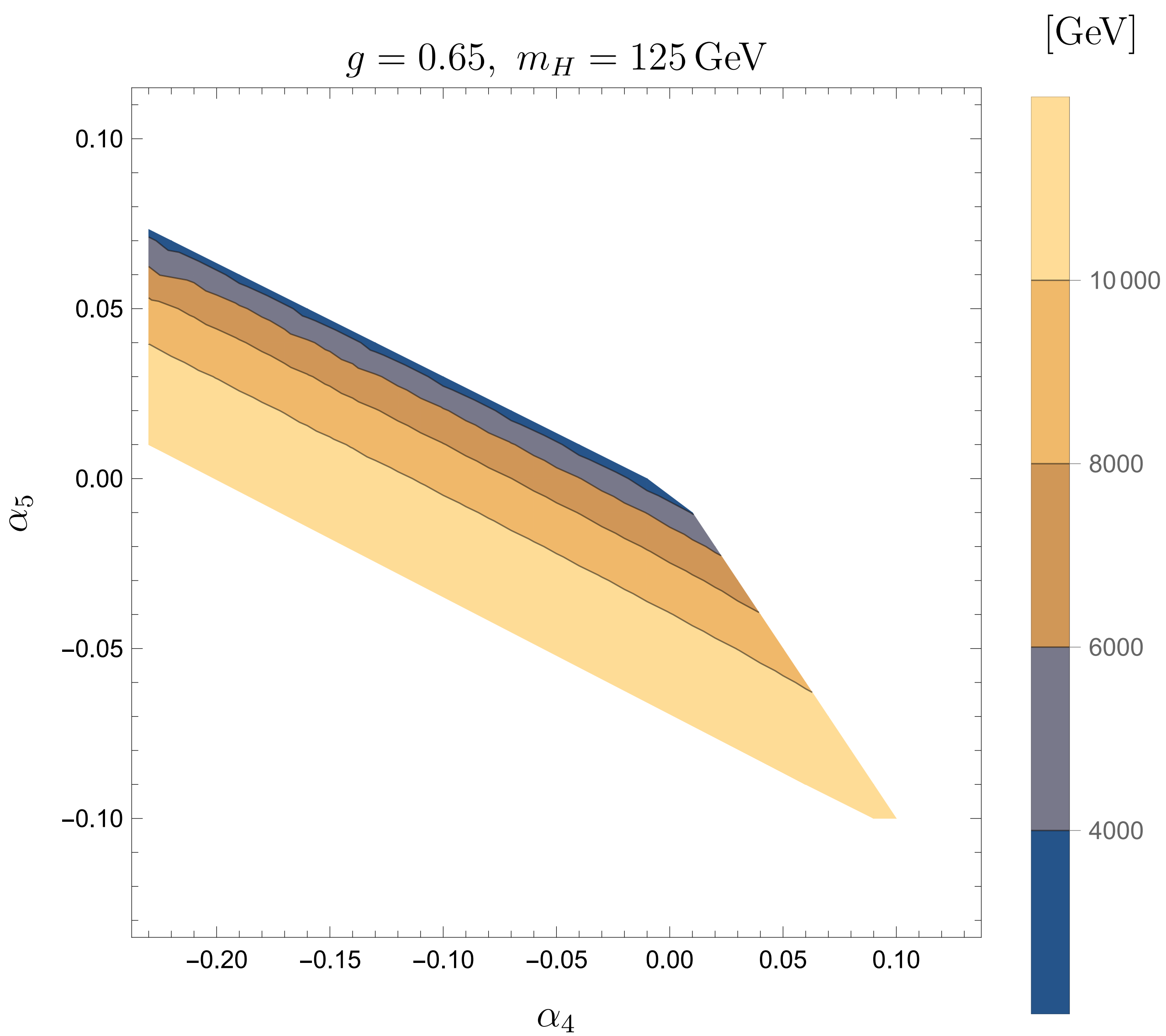
- They are measured by WW scattering process at LHC.

→**We can put a bound on mass of EW-Skyrmion!**

Energy of EW-Skyrmion

[CMS collaboration, 1905.07445]

Shaded region is excluded by aQGC measurement



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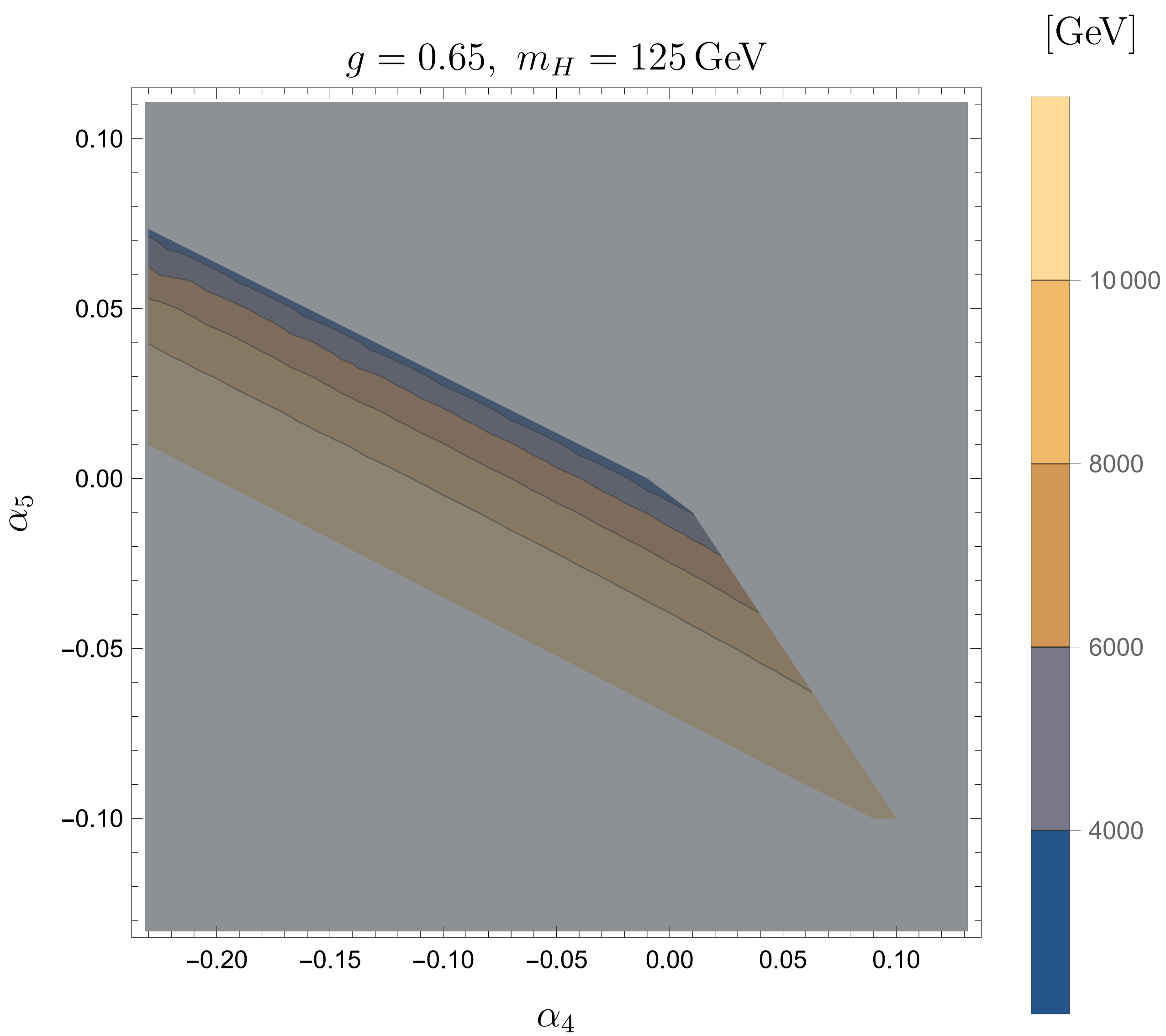

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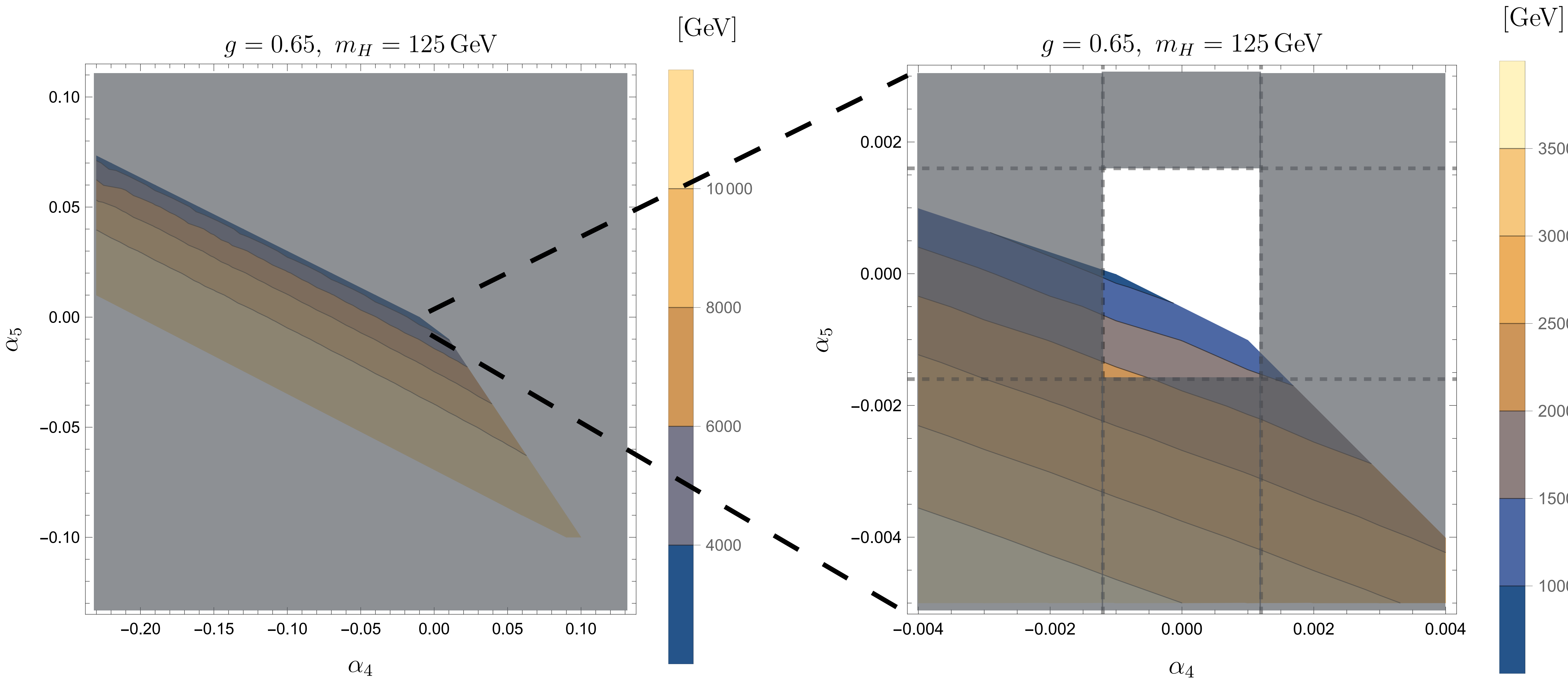

Energy of EW-Skyrmion

[CMS collaboration, 1905.07445]

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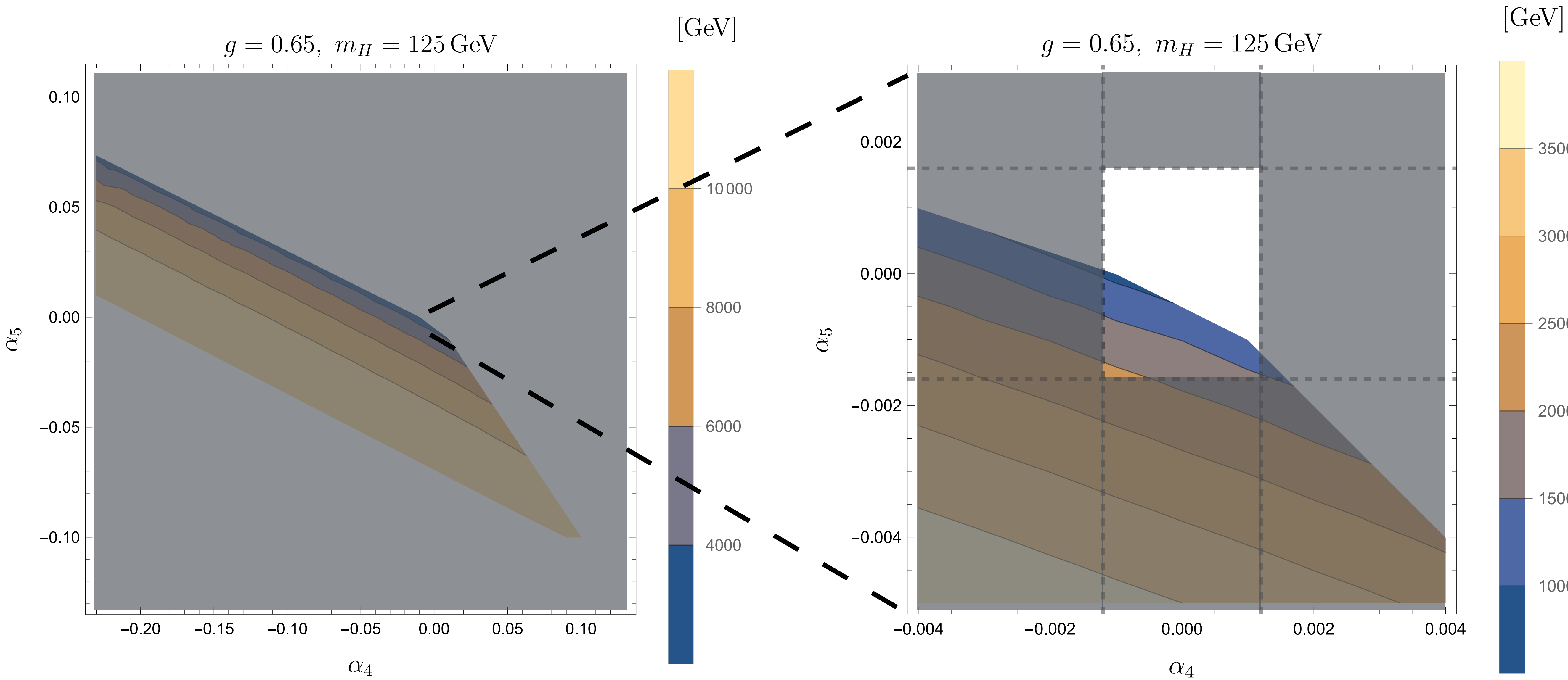

Mass of EW-Skyrmion is bounded as

$$\boxed{m_{Sk.} \lesssim 2.2\,\mathrm{TeV}}$$

Decay of EW-Skyrmion

- Actually, EW-Skyrmion can decay because winding #

$$N_H = \frac{-1}{24\pi^2}\int d^3x\, \epsilon^{ijk}\, \mathrm{Tr}\left[V_i V_j V_k\right] \qquad V_i \equiv (\partial_i U)U^\dagger$$

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But this is **not topological quantity!!**

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EW-Skyrmion		EW-Skyrmion		Vacuum
$N_H = 1$ $N_{CS} = \epsilon$ $Q = 1+\epsilon$	$\longrightarrow$ large gauge trsf.	$N_H = 0$ $N_{CS} = 1+\epsilon$ $Q = 1+\epsilon$	$\longrightarrow$ continuous deformation	$N_H = 0$ $N_{CS} = 0$ $Q = 0$

Decay of EW-Skyrmion

[**YH**-Kitano-Kurachi '21]

- Actually, EW-Skyrmion can decay because winding #

  is not ga

- A gauge

  But this is

This decay process generates Baryon # due to the chiral anomaly:

$$\Delta B = 3\Delta N_{CS}$$

One can predict the relation btw amounts of baryons and Skyrmions in the universe!

EW-Skyrmion		EW-Skyrmion		Vacuum
$N_H = 1$	$\longrightarrow$	$N_H = 0$	$\longrightarrow$	$N_H = 0$
$N_{CS} = \epsilon$	large gauge trsf.	$N_{CS} = 1 + \epsilon$	continuous deformation	$N_{CS} = 0$
$Q = 1 + \epsilon$		$Q = 1 + \epsilon$		$Q = 0$

Plan of talk

- Introduction
- Electroweak Skyrmion
- Asymmetric darkmatter scenario
- Summary

Asymmetric darkmatter scenario

Quantum vs Thermal Decay

- There are two types of decay of EW Skyrmion:

  - **Quantum tunneling at $T = 0$**

$$\Gamma \propto \exp\left(-\frac{8\pi^2}{g^2}\right)$$ → **sufficiently long-lived**

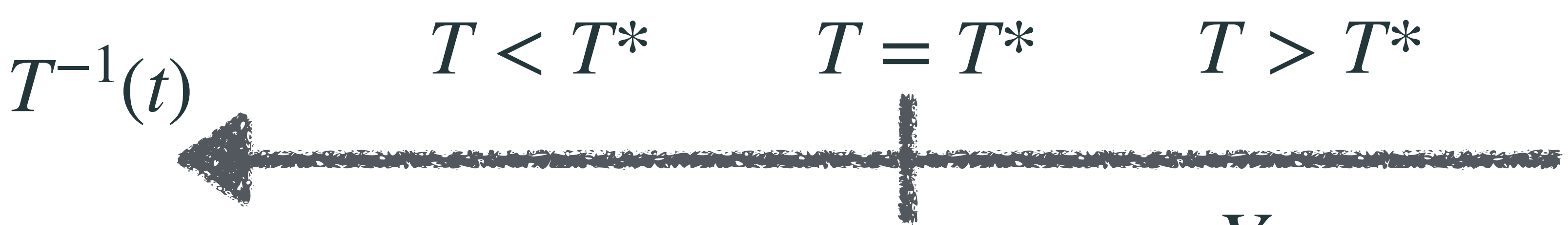
  - **Thermal sphaleron-like process at $T \neq 0$**

$\Gamma(T) \gtrsim$ **Hubble only when $T \gtrsim T^*$**

T^* is expected to be 10^{1-2} GeV

(cf. EW sphaleron process)

Thermal History



$$Y_{DM} \equiv \frac{n_{Sk.} - \bar{n}_{Sk.}}{s}$$

Y_B

$\updownarrow$ **equilibrium**

Y_{DM}

(inverse) decay process:

DM $\leftrightarrow (qqql)^3$

Thermal History

$T^{-1}(t)$ $\quad T < T^* \quad T = T^* \quad T > T^*$

$$Y_{DM} \equiv \frac{n_{Sk.} - \bar{n}_{Sk.}}{s}$$

Y_B $\qquad\qquad$ Y_B

$\cancel{\updownarrow}$ $\qquad\qquad$ $\updownarrow$ **equilibrium**

frozen ← Y_{DM} $\qquad\qquad$ Y_{DM}

(inverse) decay process:

DM ↔ $(qqql)^3$

Thermal History

$T^{-1}(t)$ $\quad T < T^*$ $\quad T = T^*$ $\quad T > T^*$

$$Y_{DM} \equiv \frac{n_{Sk.} - \bar{n}_{Sk.}}{s}$$

$T < T^*$: $Y_B \not\leftrightarrow Y_{DM}$

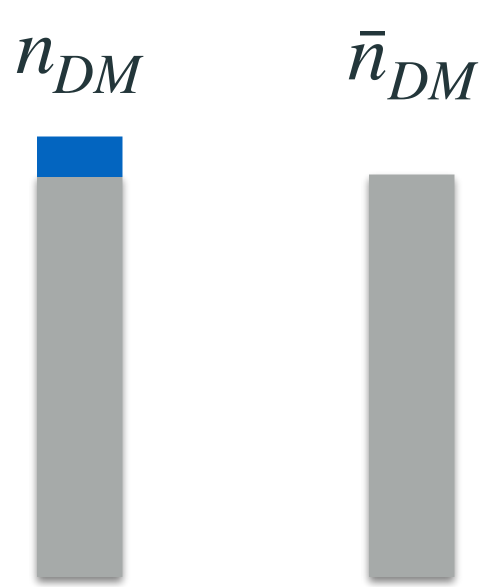
frozen $\leftarrow$ Y_{DM}

$T > T^*$: $Y_B \leftrightarrow Y_{DM}$ equilibrium

(inverse) decay process:

DM $\leftrightarrow (qqql)^3$

Symmetric part annihilates



Thermal History

$T^{-1}(t)$ $\quad T < T^*$ $\quad T = T^*$ $\quad T > T^*$

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Y_B $\qquad\qquad Y_B$

equilibrium

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Assuming thermal equilibrium at $T = T^*$, we obtain

$$\boxed{\frac{\Omega_{DM}}{\Omega_B} \simeq K \left(\frac{m^*_{DM}}{T^*}\right)^{3/2} \exp\left(-\frac{m^*_{DM}}{T^*}\right)}$$

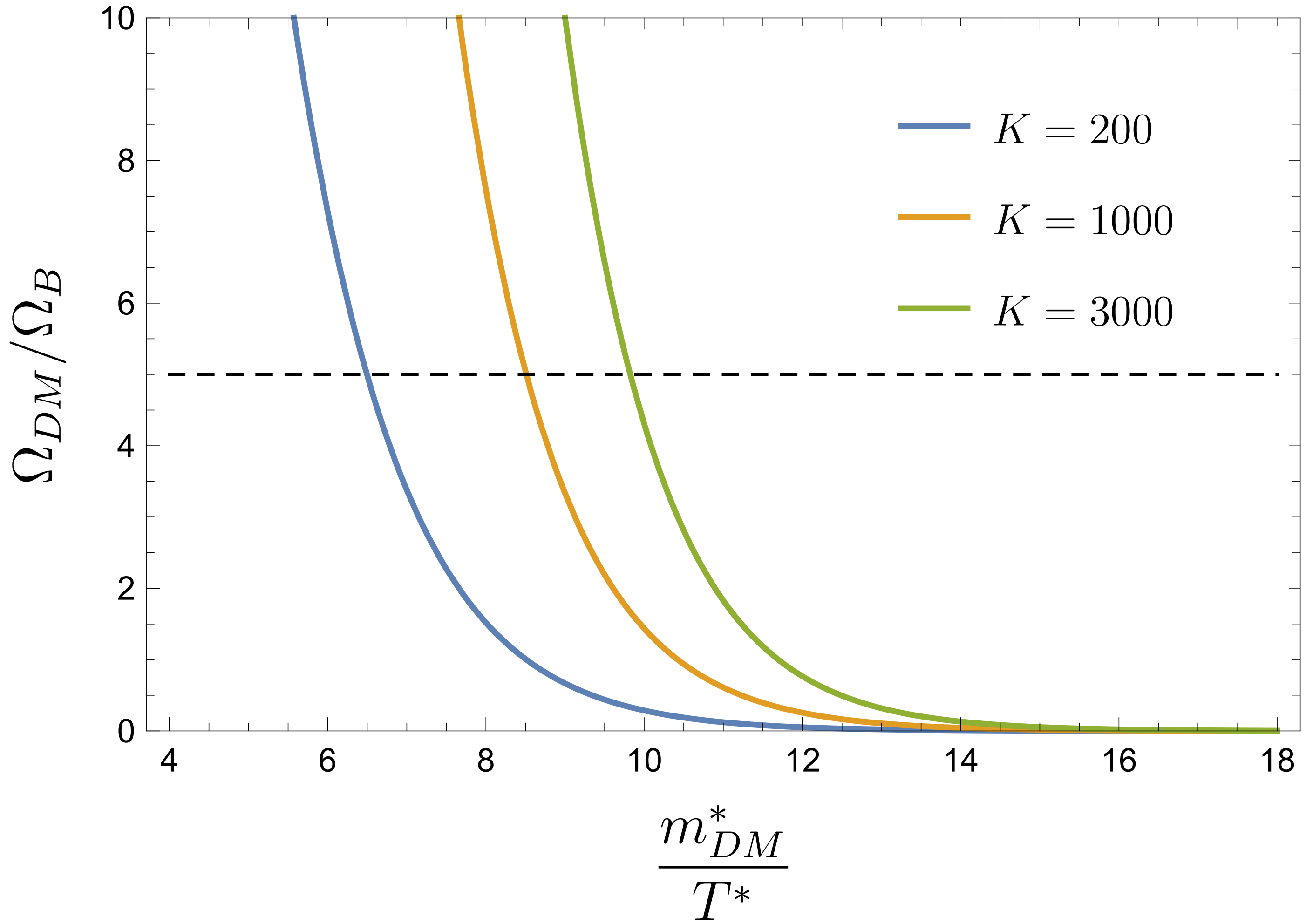
m^*_{DM} : DM mass at $T = T^*$

$$K \equiv \frac{6}{\sqrt{2}\pi^{3/2}} \left[\frac{25}{34} + \frac{6}{17}\frac{n_L}{n_B}\right] \frac{m_{DM}}{m_p} \sim 10^3 \quad \text{for } n_L/n_B = \mathcal{O}(1)$$

K depends on baryo/lepto-genesis scenarios

Ratio of relic abundance

$$\frac{\Omega_{DM}}{\Omega_B} \simeq K\left(\frac{m^*_{DM}}{T^*}\right)^{3/2} \exp\left(-\frac{m^*_{DM}}{T^*}\right)$$



Typically, $m^*_{DM}/T^* \simeq 10$ **can explain** $\Omega_{DM}/\Omega_B \simeq 5$ **!**

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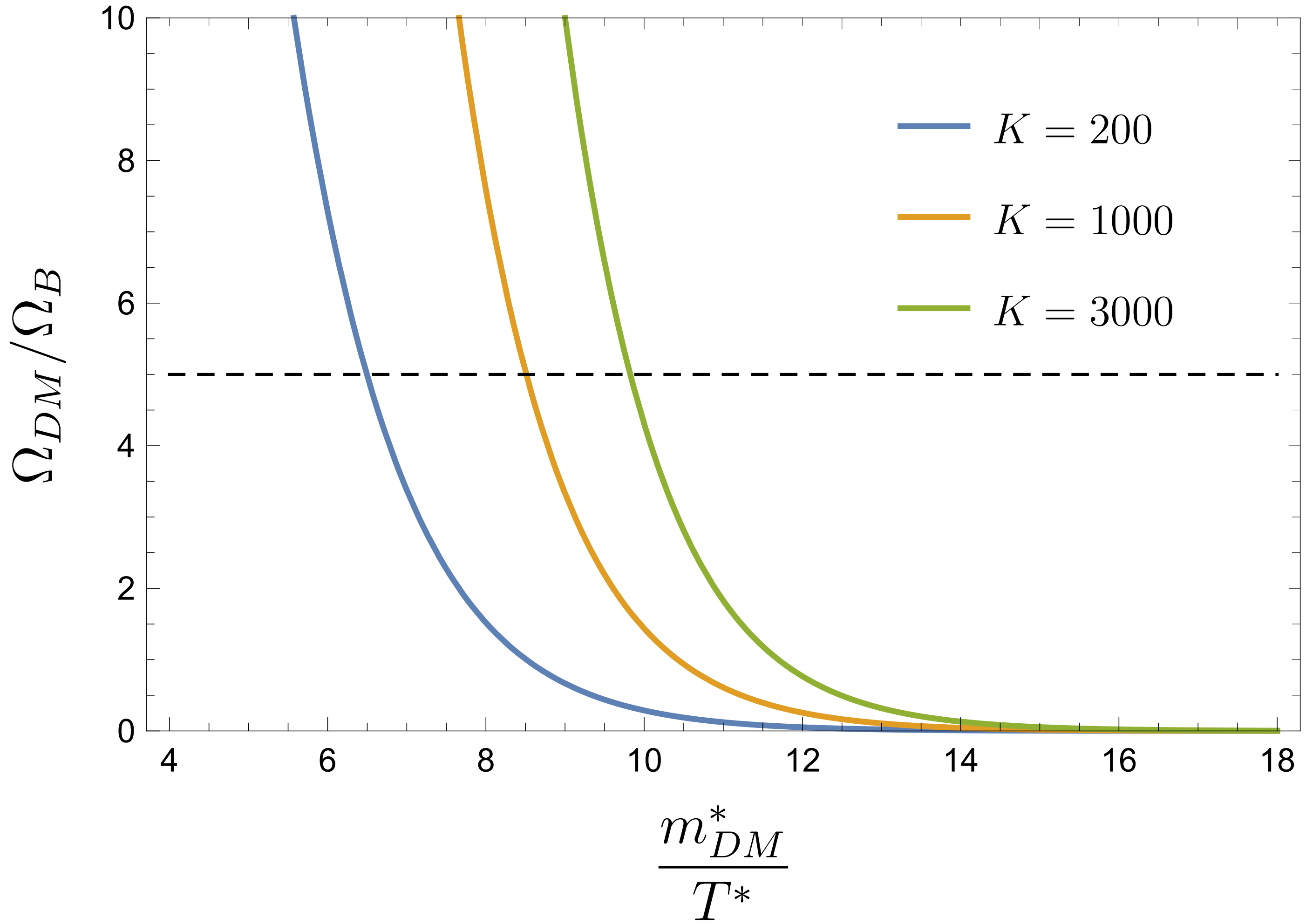

Typically, $m^*_{DM}/T^* \simeq 10$ **can explain** $\Omega_{DM}/\Omega_B \simeq 5$ **!**

very natural because $m_{DM} = \mathcal{O}(1)\mathrm{TeV}$, $T^* = 10^{1-2}\,\mathrm{GeV}$

Implication for baryo/lepto-genesis

- rewrite by two conserved quantities: $Y_{DM-B/3}$ and Y_{B-L},

$$\boxed{\frac{\Omega_{DM}}{\Omega_B} = X \frac{111 Y_{DM-B/3} + 12 Y_{B-L}}{-102 Y_{DM-B/3} + 36 X Y_{B-L}} \frac{m_{DM}}{m_p}}$$

$$Y_A \equiv \frac{n_A}{s}$$

$$X = \frac{12}{(2\pi)^{3/2}} \left(\frac{m_{DM}^*}{T^*} \right)^{3/2} \exp\left(-\frac{m_{DM}^*}{T^*} \right) \sim 10^{-3}$$

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- For leptogenesis, $Y_{DM-B/3} = 0$

$$\Rightarrow \frac{\Omega_{DM}}{\Omega_B} = \frac{1}{3} \frac{m_{DM}}{m_p} \qquad \therefore m_{DM} \simeq 15\,\mathrm{GeV}$$

this is very unlikely

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Our scenario seems inconsistent with leptogenesis.

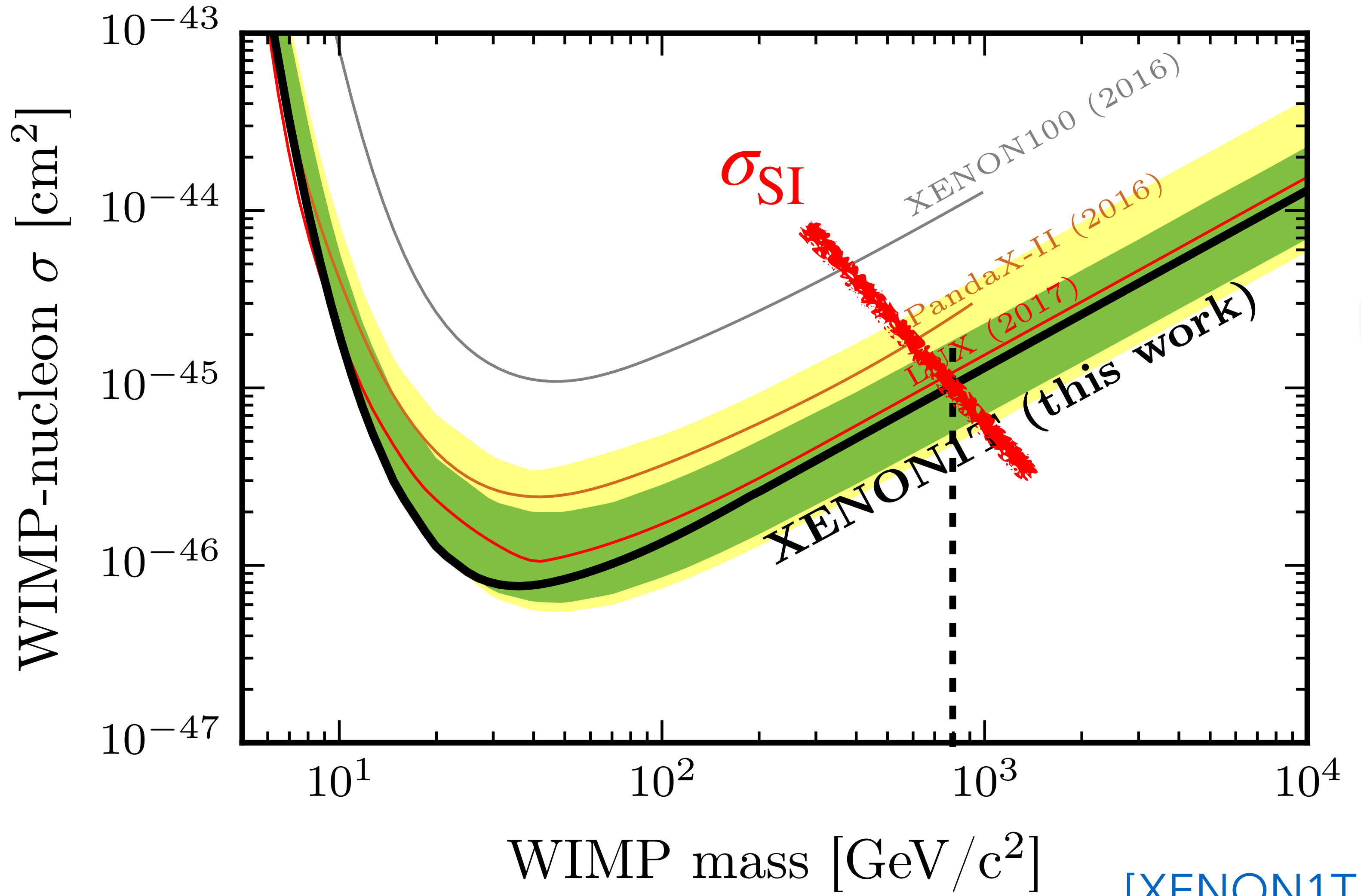
requires generation of either DM or B asymmetry

Direct detection experiment

- Assume effective coupling btw EW-Skyrmion and Higgs:

$$\mathscr{L}_{eff.} = -\kappa |S|^2 |H|^2$$ benchmark value: $\kappa = 0.1$

- Bound for spin-independent cross section with nucleon:



$m_{DM} \gtrsim 0.8\,\text{TeV}$

[XENON1T, 1705.06655]

Summary

- EW-Skyrmion = soliton made of Higgs and EW gauge fields
  - naturally arises by $\mathcal{O}(p^4)$ extension of Higgs Lagrangian
  - plays a role of an asymmetric DM

- $\Omega_{DM}/\Omega_B \simeq 5$ is realized for $m_{DM} = \mathcal{O}(1)\,\mathrm{TeV}$ and $T^* = \mathcal{O}(10^2)\,\mathrm{GeV}$.

- DM direct detection experiments and measurements of aQGC put stringent window:

$$\boxed{0.8\,\mathrm{TeV} \lesssim m_{DM} \lesssim 2.2\,\mathrm{TeV}}$$

Backup

EW-Skyrmion Solution

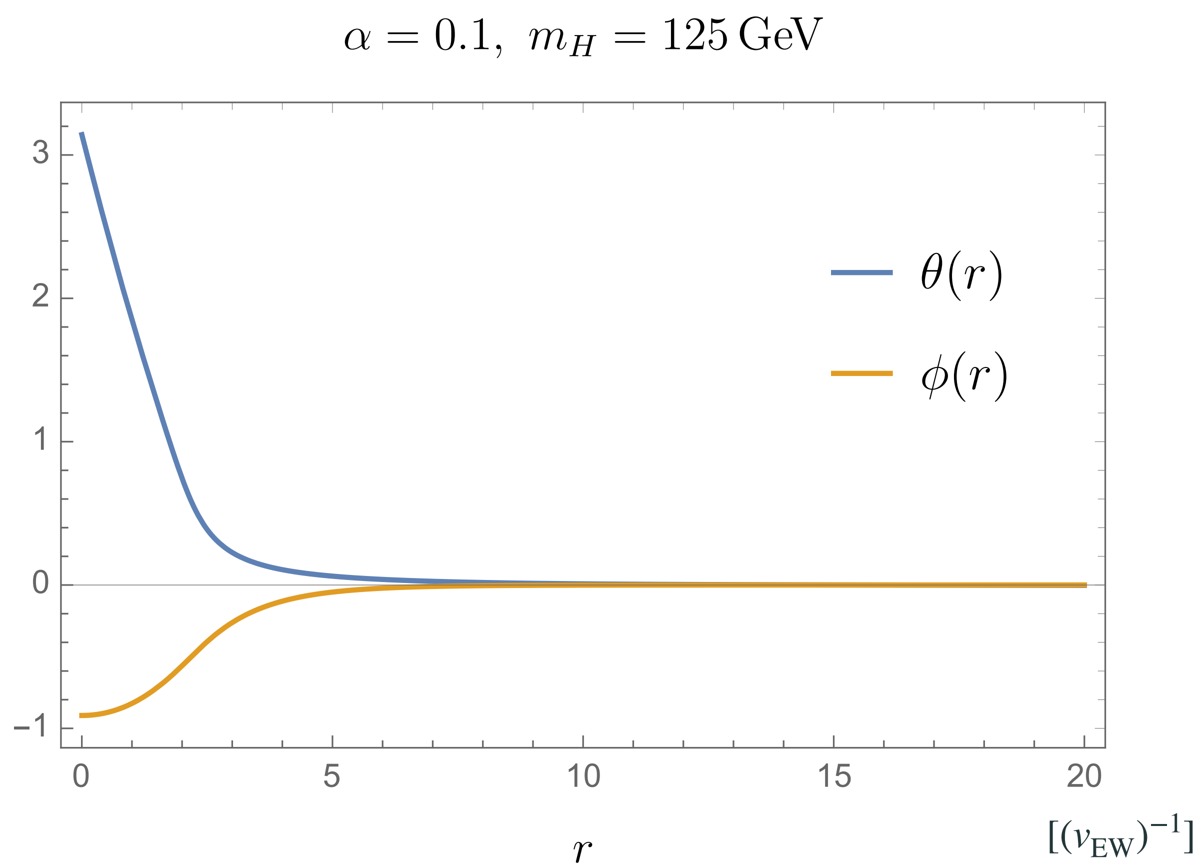
[Kitano, Kurachi '16, '17]

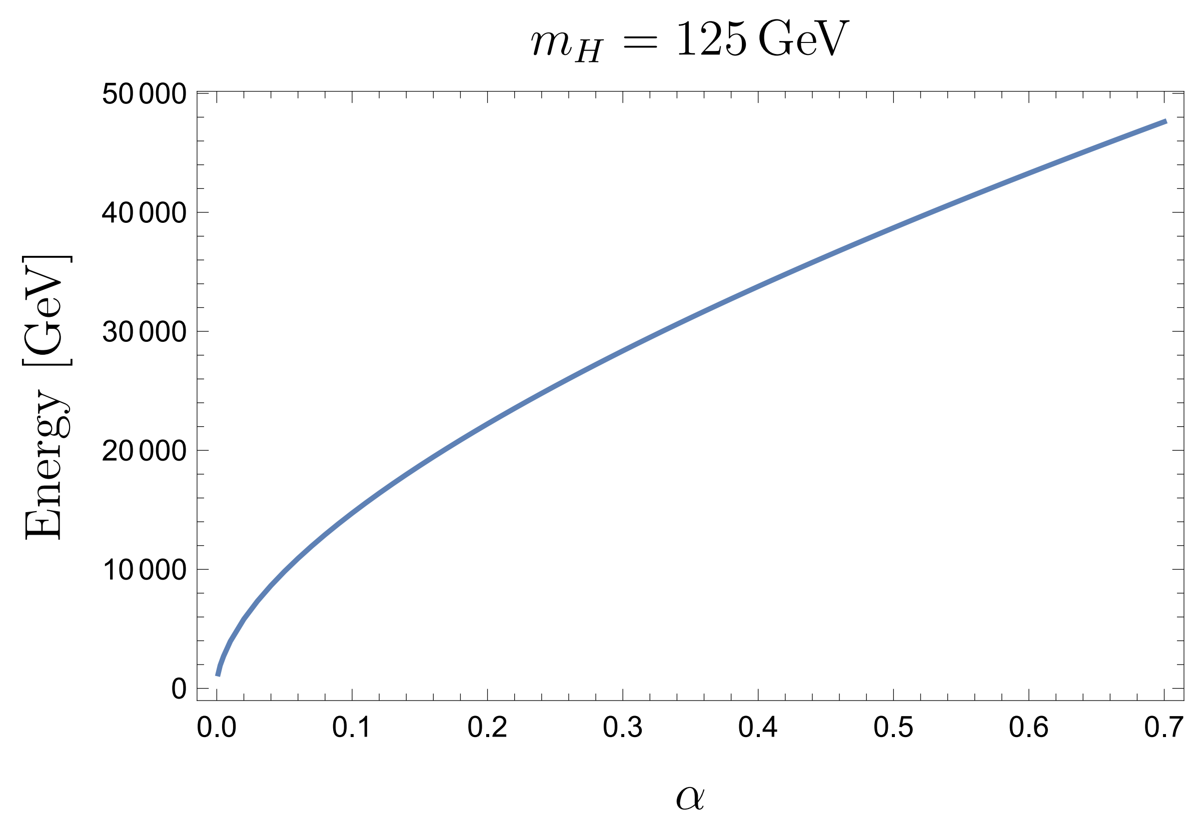
- Firstly, we take $g = g' = 0$

→ The only difference from Skyrme model is the existence of $h(x)$.

- Hedgehog ansatz:

$$U = \exp\left[i\theta(r)\,\hat{x}^a\sigma^a\right] \qquad h(x) = \phi(r)/v_{\mathrm{EW}}$$





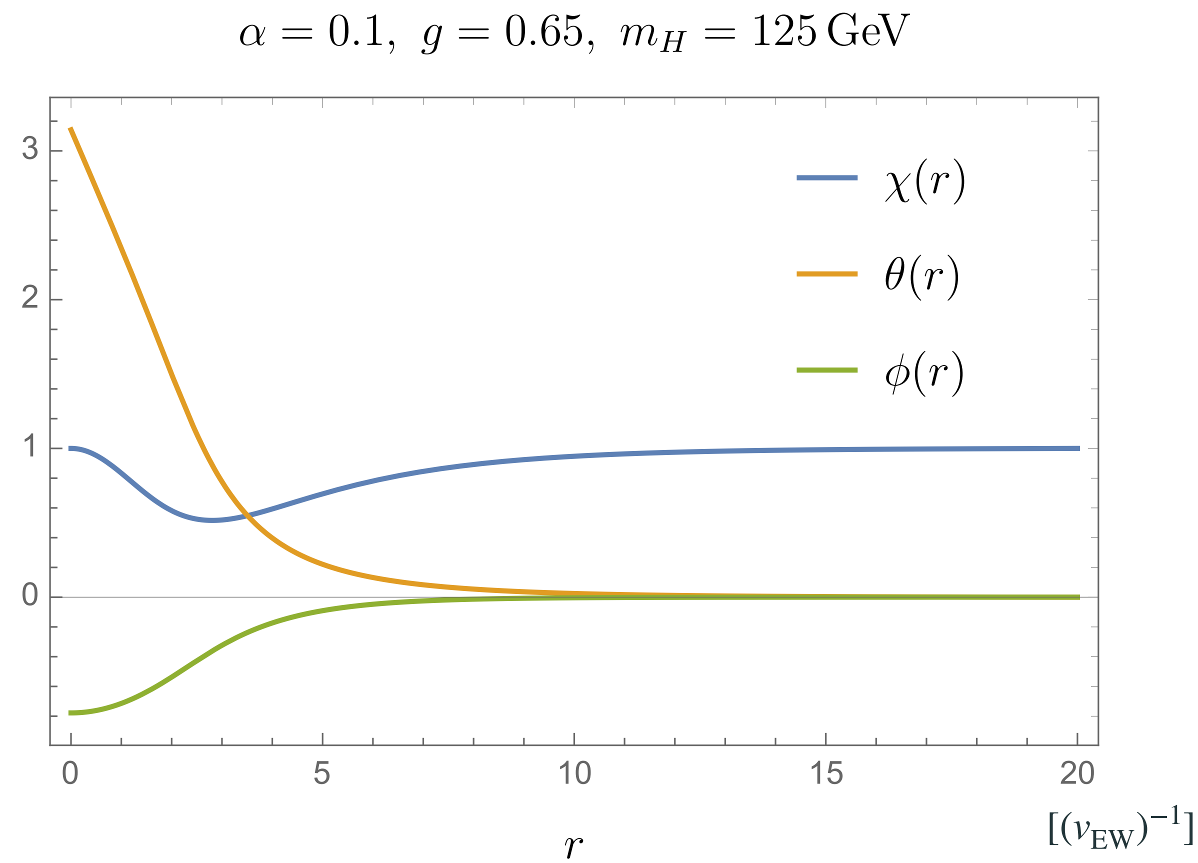
EW-Skyrmion Solution

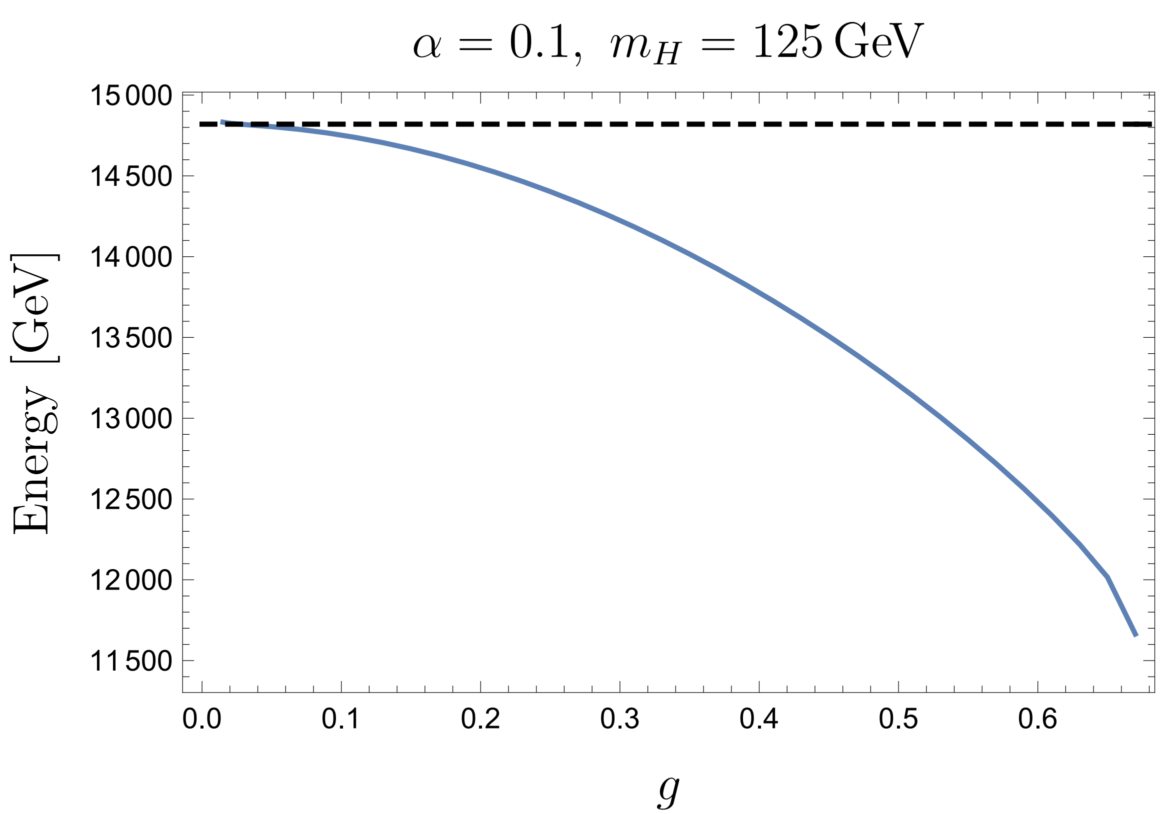
- Then, we set $g \simeq 0.65$ (keeping $g' = 0$)
- Hedgehog ansatz:

$$U = \exp\left[i\theta(r)\,\hat{x}^a\sigma^a\right] \qquad h(x) = \phi(r)/v_{\rm EW}$$

$$W_i^a(x) = \frac{\chi(r)-1}{r}\epsilon_{iab}\hat{x}_b - \xi(r)\hat{x}_i\hat{x}_a$$

auxiliary field
(explicitly solvable)





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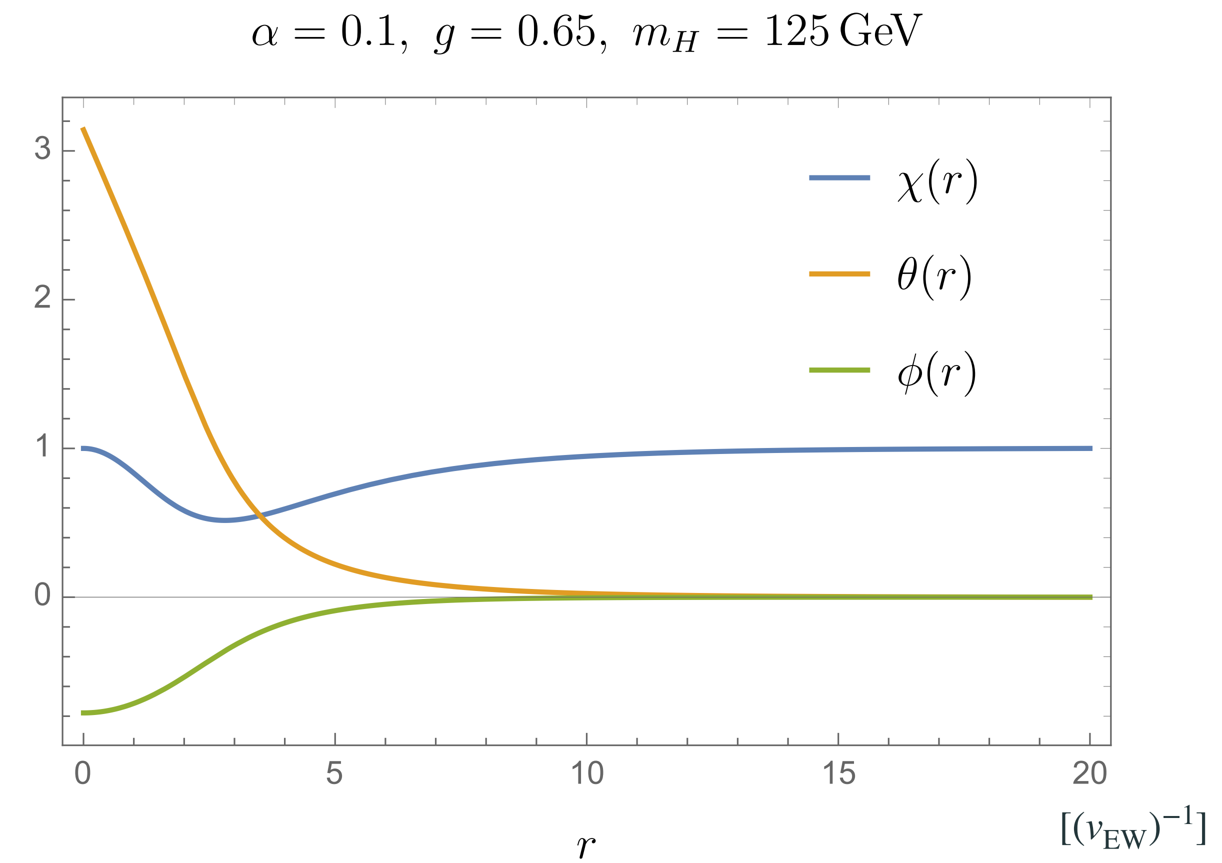

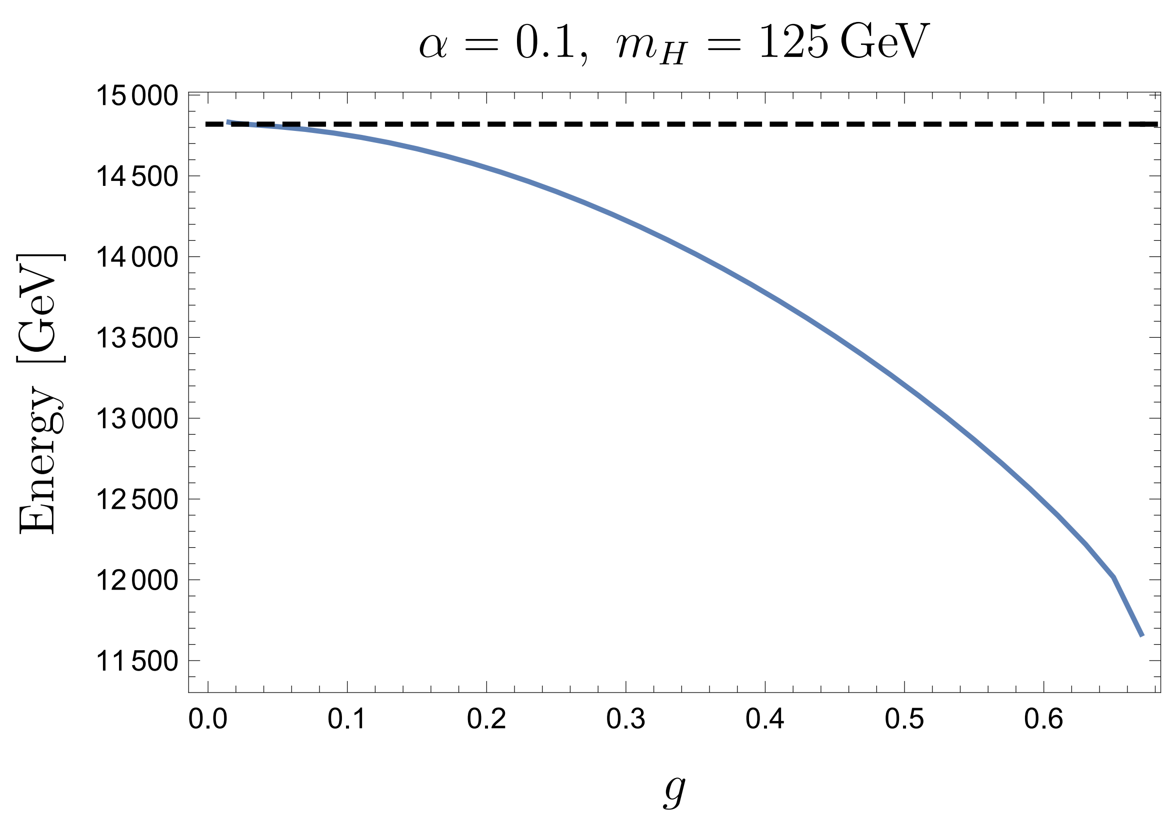

→EW-Skyrmion does exist!!

Thermal History

$$\mathrm{GeV}^{-2} = 0.04 \times 10^{-26}\,\mathrm{cm}^2$$

$$c \times \mathrm{GeV}^{-2} = 0.12 \times 10^{-16}\,\mathrm{cm}^3/\mathrm{s}$$

Solving Boltzman eq., late-time ratio is given by

$$r = \frac{\bar{n}_{DM}}{n_{DM}} \simeq \exp\left(-2\sigma_{ann}/\sigma_{\mathrm{WIMP}}\right)$$ [Graesser+, 1103.2771]

cf. $\langle \sigma v \rangle_{\mathrm{WIMP}} \sim 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$

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$$\sigma_{ann} \sim \pi R^2 \sim \pi\alpha(v_{\mathrm{EW}})^{-2} \sim \left(\frac{\alpha}{10^{-3}}\right) \times 10^{-23}\,\mathrm{cm}^3/\mathrm{s}$$

cf. $\langle \sigma v \rangle_{\mathrm{WIMP}} \sim 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$

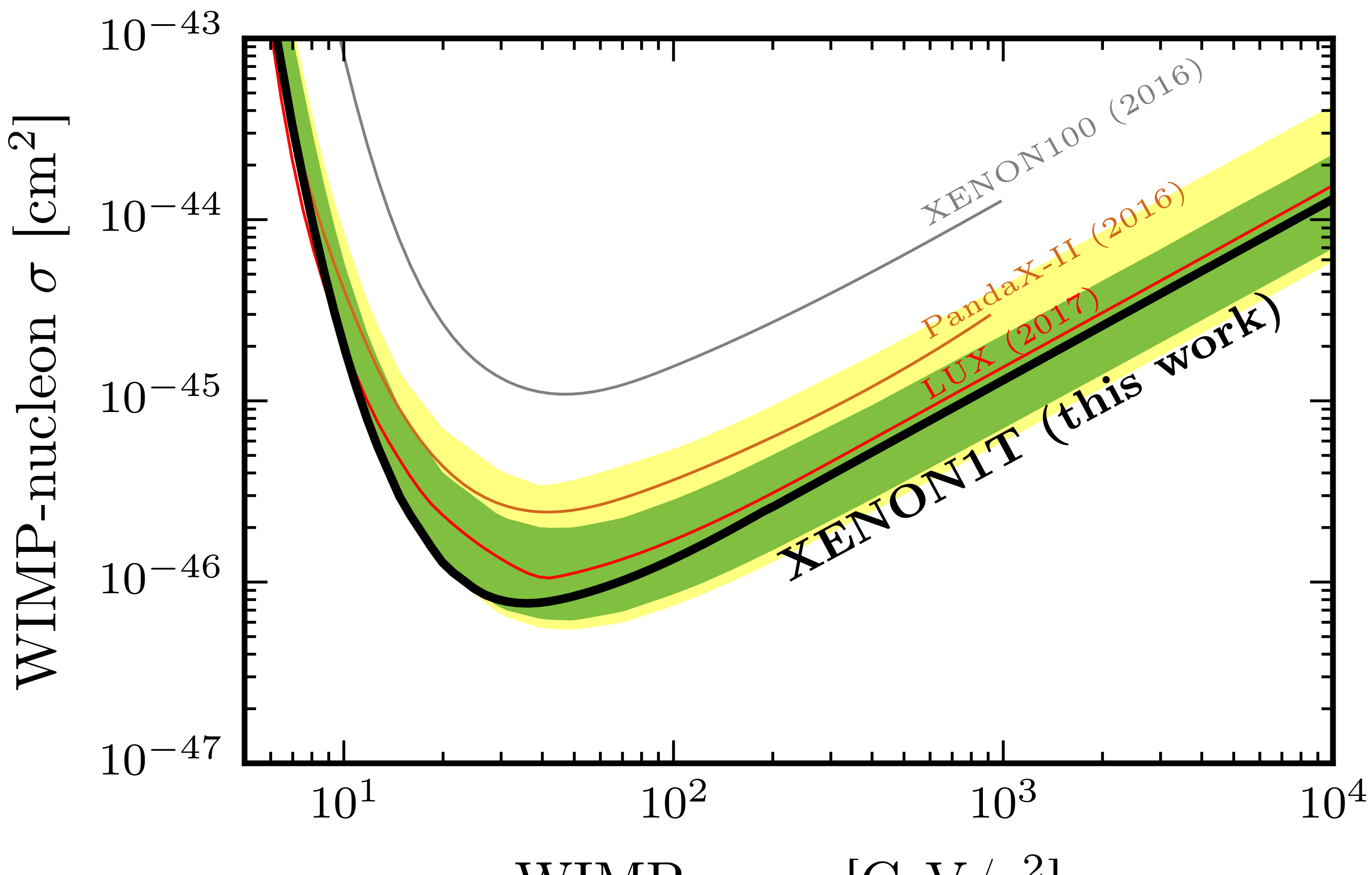
Direct detection experiment

- Assume effective coupling btw EW-Skyrmion and Higgs:

$$\mathscr{L}_{eff.} = -\kappa |S|^2 |H|^2$$

- Spin-independent cross section with nucleon:

$$\sigma_{\mathrm{SI}} \simeq \left(\frac{\kappa}{0.1}\right)^2 \left(\frac{1\,\mathrm{TeV}}{m_{DM}}\right)^2 \left(\frac{f}{0.3}\right)^2 \times 3.6 \times 10^{-46}\,\mathrm{cm}^2$$



Non-integer $B + L$

EW-Skyrmion itself has non-integer $B = 3\epsilon$

This is because fermionic vacuum (Dirac sea) carries non-integer number for anomalous charge in the non-trivial background.

$$\hat{Q} = :\hat{Q}: + \hat{Q}_{\rm vac}(A)$$

$\hat{Q}$: anomalous charge

- number operator (integer), $:\hat{Q}: = \hat{b}^\dagger \hat{b} + \cdots$
- vacuum contribution (non-integer), $\hat{Q}_{\rm vac}(A)$

EW-Skyrmion		Vacuum
$N_H = 0$		$N_H = 0$
$N_{CS} = 1 + \epsilon$	$\longrightarrow$ continuous deformation	$N_{CS} = 0$
$Q = 1 + \epsilon$		$Q = 0$

$$\underbrace{\Delta B_{\rm fer.}}_{3} + \underbrace{\Delta B_{\rm Sky.}}_{3\epsilon} = 3(1+\epsilon) = 3\Delta N_{CS}$$

Boson vs fermion

- Statistics of Skyrmion depends on the underlying UV theory.
- Wess-Zumino-Witten term

  When UV theory is (strongly coupled) $SU(N_C)$ gauge theory with $N_c \geq 3$, it is given by

$$\Gamma_{WZW} = -\frac{iN_c}{240\pi^2}\int_{\mathscr{M}_5} d^5x\,\epsilon^{\mu\nu\rho\sigma\tau}\,\mathrm{Tr}\left[U^\dagger\partial_\mu U\partial_\nu U^\dagger\partial_\rho U\partial_\sigma U^\dagger\partial_\tau U\right]$$

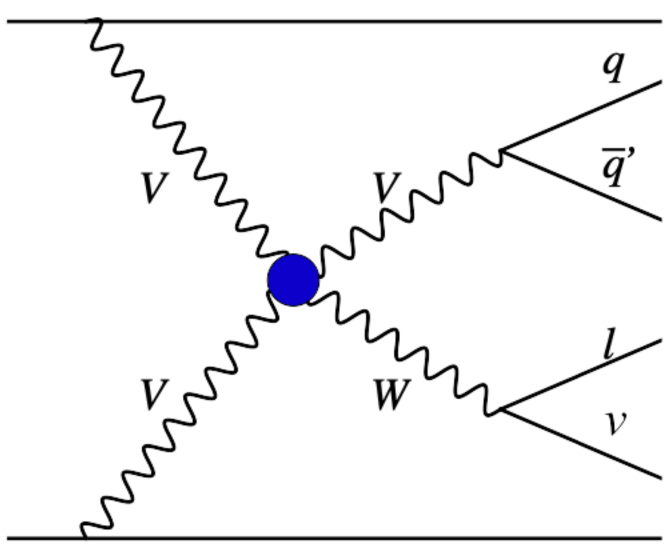
- N_c even → boson,   odd → fermion
- Electric charge also depends on Γ_{WZW} (cf.Witten effect in QED)
- In our work, we simply put $\Gamma_{WZW} = 0$, leading to electrically neutral and bosonic Skyrmion.

aQGC by ATLAS

[ATLAS, 1609.05122]

- using custodial **symmetric** operators in **non-linear rep.**

$$\mathscr{L}_4 = \alpha_4 \mathrm{Tr}\left[D_\mu U^\dagger D_\nu U\right] \mathrm{Tr}\left[D^\mu U^\dagger D^\nu U\right] \qquad \mathscr{L}_5 = \alpha_5 \mathrm{Tr}\left[D_\mu U^\dagger D^\mu U\right] \mathrm{Tr}\left[D_\nu U^\dagger D^\nu U\right]$$



final states: W(->leptons) V(->hadrons) + forward dijet

$$0.024 \le \alpha_4 \le 0.030 \qquad 0.028 \le \alpha_5 \le 0.033$$

aQGC by CMS

[CMS,1905.07445]

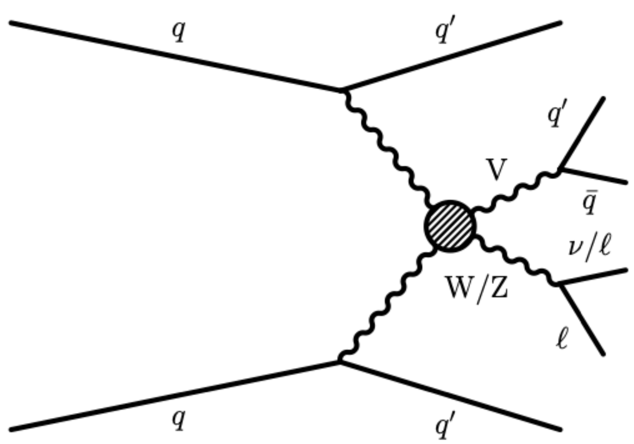


Figure 1: The Feynman diagram of a VBS process contributing to the EW-induced production of events containing a hadronically decaying gauge boson (V), a $W^{\pm}/Z$ boson decaying to leptons, and two forward jets. New physics (represented by a black circle) in the EW sector can modify the quartic gauge couplings.

final states:

W/Z(->leptons) V(->hadrons)

+ forward dijet

- using custodial **non-symmetric** operators in **linear rep.**

$$\mathscr{L}_{S,0} = \frac{f_0}{\Lambda^4}\left[(D_\mu\Phi)^\dagger D_\nu\Phi\right]\left[(D^\mu\Phi)^\dagger D^\nu\Phi\right] \qquad \mathscr{L}_{S,1} = \frac{f_1}{\Lambda^4}\left[(D_\mu\Phi)^\dagger D^\mu\Phi\right]\left[(D^\nu\Phi)^\dagger D^\nu\Phi\right]$$

They do not correspond to non-linear ones...

$$\mathscr{L}_{S,0} + \mathscr{L}_{S,1} = \mathscr{L}_4 + \mathscr{L}_5 + \cdots$$

But anyway, one can translate their constraints into non-linear ones..

$$\left|\frac{f_0}{\Lambda^4}\right| \leq 2.7\,\mathrm{TeV}^{-4} \qquad \left|\frac{f_1}{\Lambda^4}\right| \leq 3.3\,\mathrm{TeV}^{-4}$$

[Eboli+, hep-ph/0606118]

$$\Rightarrow \quad |\alpha_4| \leq 0.0012 \qquad |\alpha_5| \leq 0.0016$$

Example of Asymmetric DM

[Ibe, Kamada, Kobayashi, Nakano 1805.06876]

$B-L$ charge → dark baryon

$N_c = 3,\ N_f = 2$

$$\mathcal{O}_{\rm portal} = \frac{1}{\Lambda^3}\bar{D}\bar{U}\bar{U}LH$$

Symmetric part of dark baryon decays into dark radiations

dark radiations must decay into SM radiation (photon) via

$$\mathcal{L}_{mix} = \frac{\epsilon}{2}F_{\mu\nu}F_D^{\mu\nu}$$

$$\mathcal{L}_{A_D} \supset \frac{m_D^2}{2}A_{D\mu}A_D^{\mu}$$

EW-Skyrmion Solution

- Actually, EW-Skyrmion can decay because

$$N_H = \frac{-1}{24\pi^2} \int d^3x \, \epsilon^{ijk} \, \mathrm{Tr} \left[V_i V_j V_k \right]$$

is not gauge invariant.

- Gauge invariant quantity is

$$Q = N_H + N_{CS}$$

EW-Skyrmion

$$\boxed{\begin{gathered} N_H = 1 \\ N_{CS} = 0 \end{gathered}}$$

$$N_{CS} = \frac{g^2}{16\pi^2} \int d^3x \, \epsilon^{ijk} \, \mathrm{Tr} \left[W_{ij} W_k + \frac{2ig}{3} W_i W_j W_k \right]$$

Thermal History

$T^{-1}(t)$ ← $T < T^*$ | $T = T^*$ | $T > T^*$

$$Y_{DM} \equiv \frac{n_{Sk.} - \bar{n}_{Sk.}}{s}$$

Y_B $\cancel{\updownarrow}$ Y_{DM} frozen ←

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Assuming thermal equilibrium at $T = T^*$, we obtain

$$\frac{\Omega_{DM}}{\Omega_B} = X \frac{111 Y_{DM-B/3} + 12 Y_{B-L}}{-102 Y_{DM-B/3} + 36 X Y_{B-L}} \frac{m_{DM}}{m_p}$$

$$X \equiv 6 \times f(m^*_{DM}/T^*) = \frac{12}{(2\pi)^{3/2}} \left(\frac{m^*_{DM}}{T^*} \right)^{3/2} \exp\left(-\frac{m^*_{DM}}{T^*} \right)$$

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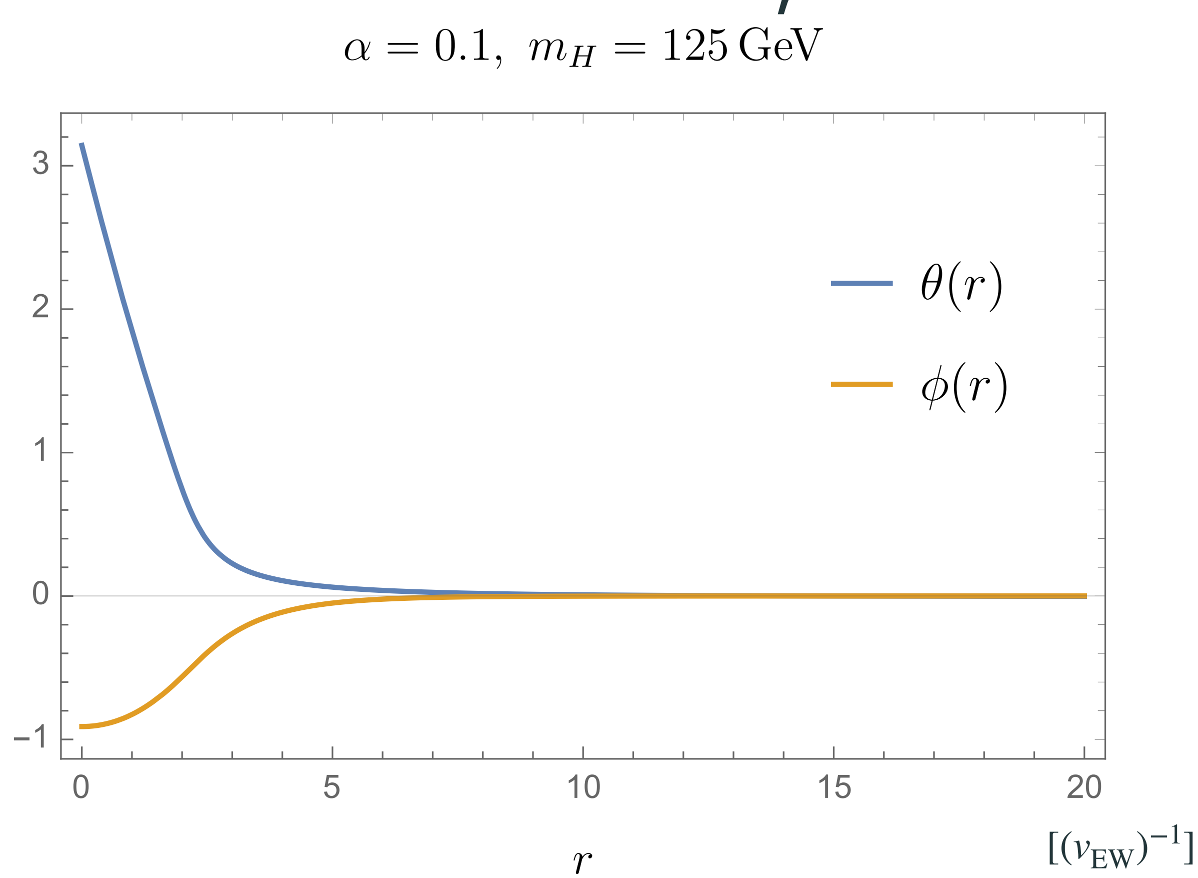
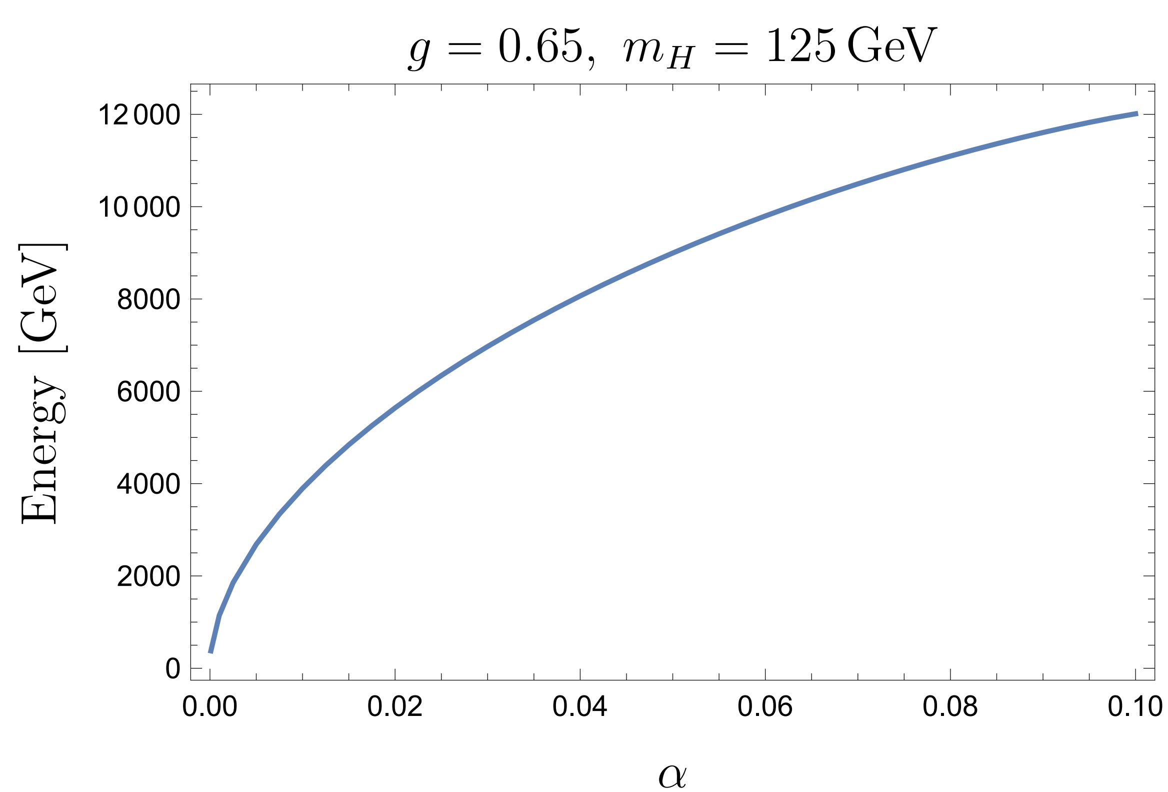
$$W_i^a(x) = \frac{\mathrm{Re}\,\chi(r) - 1}{r}\epsilon_{iab}\hat{x}_b - \frac{\mathrm{Im}\,\chi(r)}{r}(\delta_{ia} - \hat{x}_i\hat{x}_a) - \xi(r)\hat{x}_i\hat{x}_a$$





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Y_B

equilibrium

Y_{DM}

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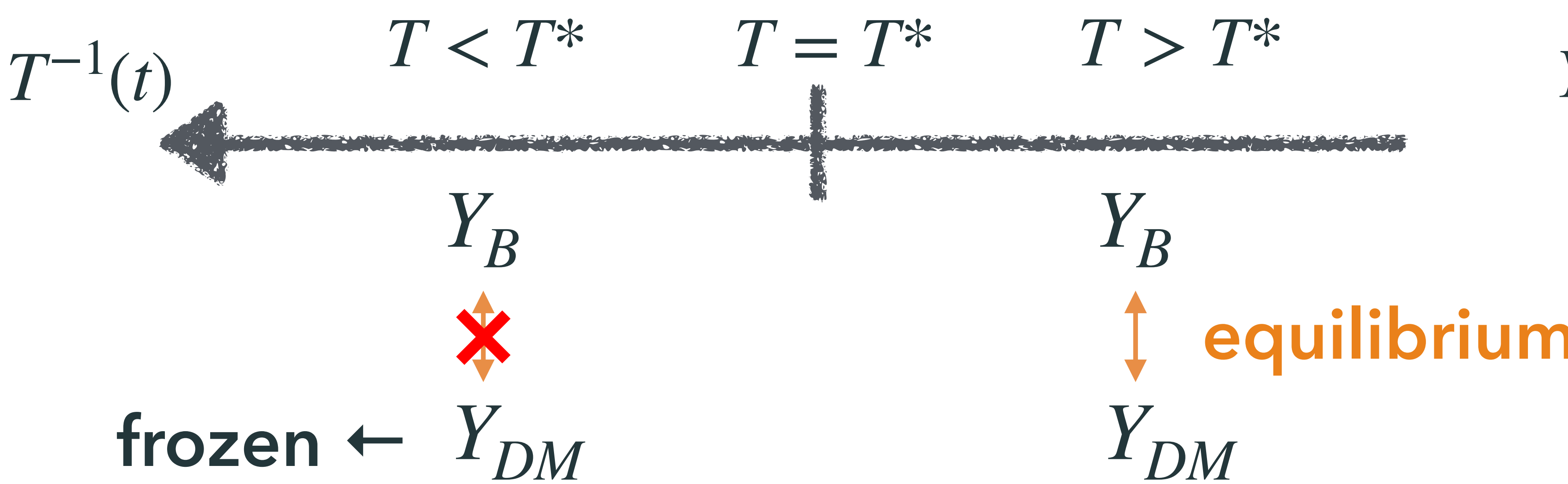

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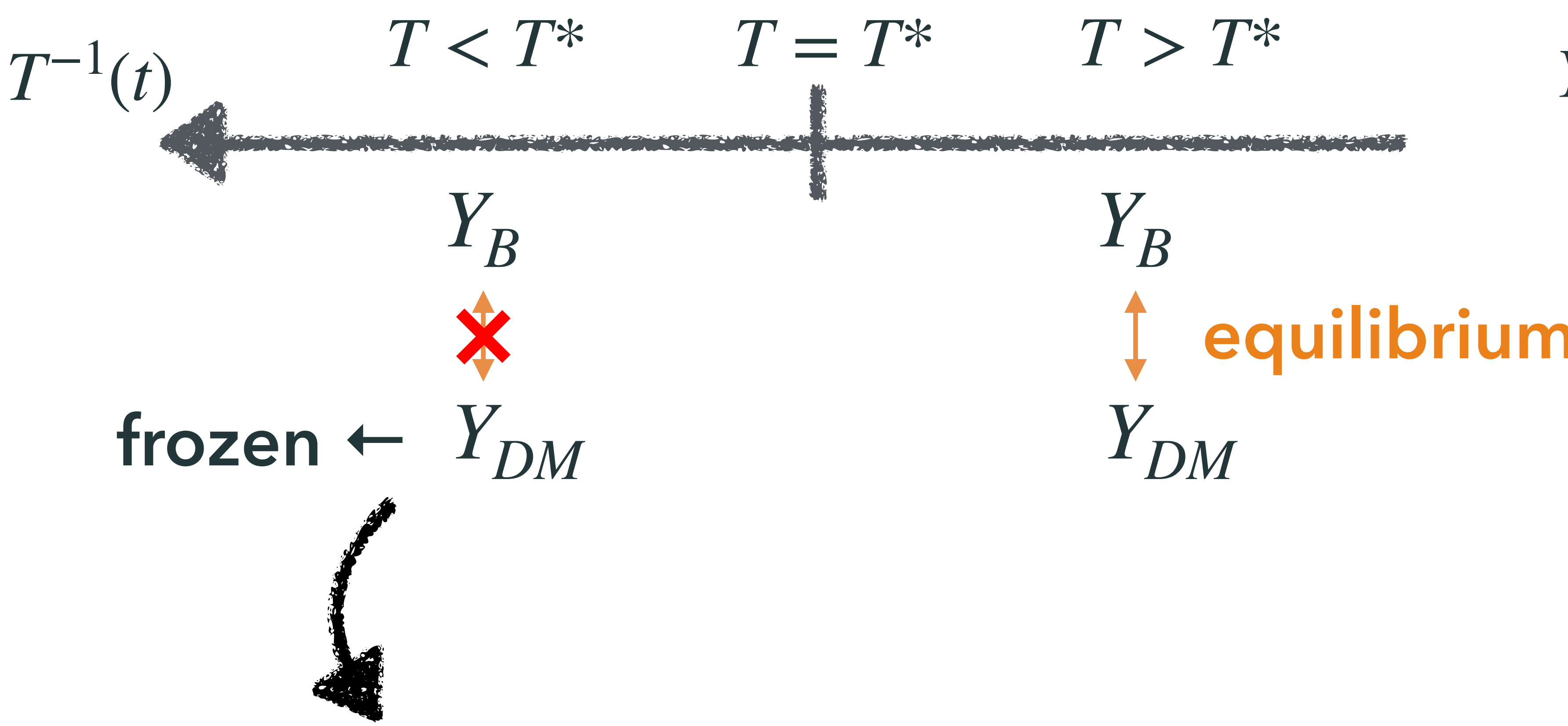
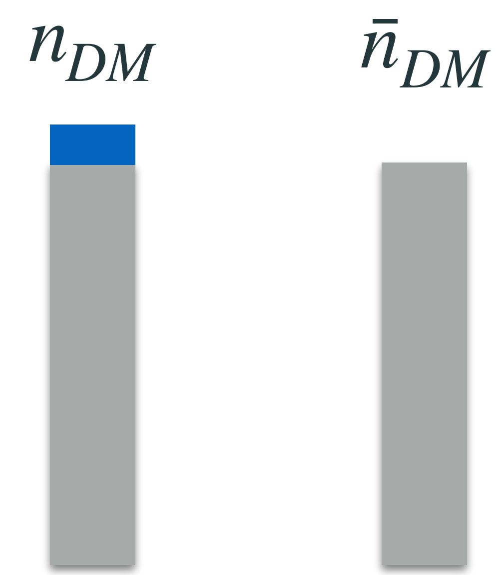
$$Y_{DM} \equiv \frac{n_{Sk.} - \bar{n}_{Sk.}}{s}$$





Symmetric part annihilates because of a quite large cross section:

$$\sigma_{ann} \sim \pi R^2 \sim \pi\alpha(v_{\mathrm{EW}})^{-2} \sim \left(\frac{\alpha}{10^{-3}}\right) \times 10^{-23}\,\mathrm{cm}^3/\mathrm{s}$$

cf. $\langle \sigma v \rangle_{\mathrm{WIMP}} \sim 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$