Rethinking of the Axion Mechanism

KYOTO UNIVERSITY : Oct 4th 2017

Masahiro Ibe (ICRR)

H. Fukuda (IPMU), M.Suzuki(ICRR), T.T.Yanagida (IPMU)

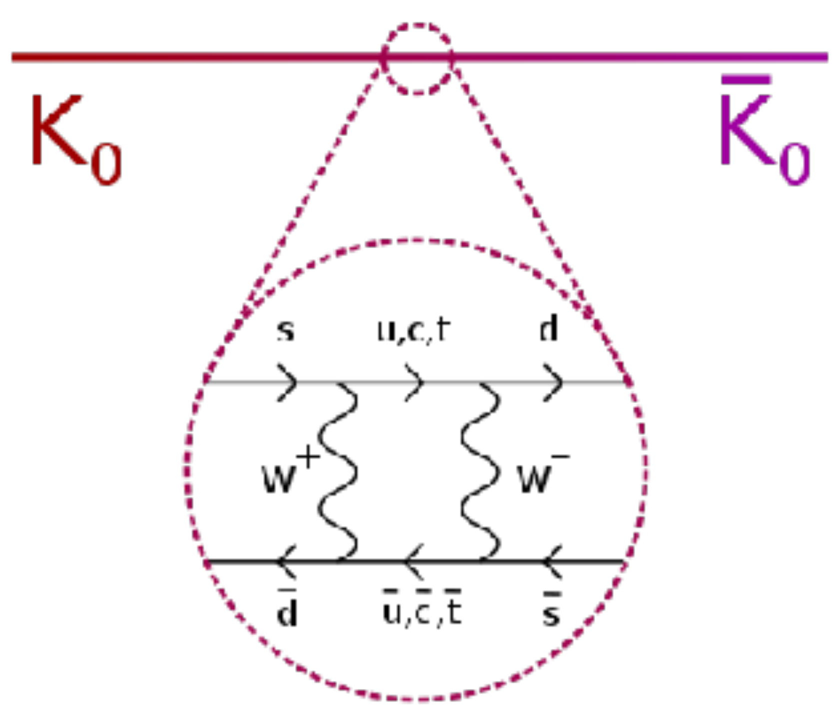
Phys.Rev.D95(2017),9,095017

Strong CP problem

Experimentally, ***QCD*** is known to preserve ***CP*** symmetry very well.

- ✓ Hadron spectrum respects ***CP*** symmetry very well.

- ✓ ***CP*** violating transitions in the ***SM*** are caused by ***CP*** violation in the weak interaction (i.e. by the ***CKM*** phase).



Picture from : https://en.wikipedia.org/wiki/Kaon

Strong CP problem

This feature is not automatically guaranteed in ***QCD*** .

- ***QCD*** has its own ***CP***-violating parameter : $\boldsymbol{\theta}$

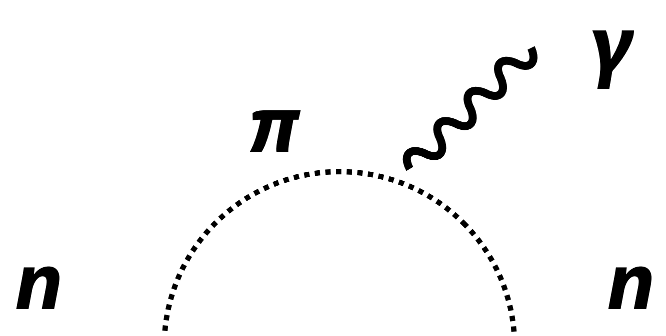
$$S_{\rm QCD} = \int d^4x \left(-\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} + \boxed{\frac{\theta}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}} + \sum_{i=1}^{N_f} \bar{q}_i (D - M) q_i \right)$$

(positive valued quark mass)

- $\boldsymbol{\theta}$ - term violates the ***P*** and ***CP*** symmetries

$$\int d^4x F_{\mu\nu}\tilde{F}^{\mu\nu} \to - \int d^4x F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- The $\boldsymbol{\theta}$ - term is highly constrained experimentally !



Null observation of the ***neutron EDM*** :

$\boldsymbol{d_n/e < 2.9 \times 10^{-26}}$ ***@ 90%CL***

[hep-ex/0602020]

$\rightarrow \boldsymbol{\theta < 10^{-11}}$

$\boldsymbol{d_n/e \sim 10^{-15}\,\theta}$ [1979 Crewther, Veccia, Veneziano, Witten]

Why so small ? = Strong CP Problem

Strong CP problem

This feature is not automatically guaranteed in ***QCD*** .

- ***QCD*** has its own ***CP***-violating parameter : ***θ***

$$S_{\rm QCD} = \int d^4x \left(-\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} + \boxed{\frac{\theta}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}} + \sum_{i=1}^{N_f} \bar{q}_i (D - M) q_i \right)$$

(positive valued quark mass)

- In the Standard Model, the quark mass matrix stems from the Yukawa couplings of ***3x3*** general complex matrices.

$$\boxed{\begin{array}{l} M_u \propto Y_u \ (\text{general complex}) \rightarrow (m_u, m_c, m_t) > 0 \\ M_d \propto Y_d \ (\text{general complex}) \rightarrow (m_d, m_s, m_b) > 0 \end{array}}$$

The phases of the Yukawa matrices also contribute to ***θ***.

Why the θ parameter and the phases of the Yukawa coupling conspire to be cancelling with each other ?

= Strong CP Problem

Weak θ phase ?

✔ Why don't we care the weak $\boldsymbol{\theta}$ phase ?

$$\frac{\theta_W}{32\pi^2} W_{\mu\nu} W^{\mu\nu}$$

The weak $\boldsymbol{\theta_W}$ shifts by

$$\boldsymbol{\theta_W \to \theta_W + 6\,\alpha}$$

under the baryon rotation

$$\boldsymbol{Q_L \to e^{i\alpha} Q_L}$$

$$\boldsymbol{u_R, d_R \to e^{-i\alpha} u_R, d_R}$$

while other parameters in the theory intact.

→ There is no weak $\boldsymbol{\theta_W}$ problem.

How Do We Explain a Small θ ?

- ✔ Spontaneous ***CP***-violation

[’84 Barr, ’84 Nelson, ’91 Bento et.al.]

- ✔ ***CP*** symmetry is an exact symmetry

  → ***θ*** - term is forbidden

$$\cancel{\int d^4x F_{\mu\nu} \tilde{F}^{\mu\nu}}$$

- ✔ At the same time, Yukawa couplings are required to be real valued…

$$\rightarrow \boldsymbol{\delta_{CKM} = 0}$$

How to generate the ***CKM*** phase ?

How Do We Explain a Small θ ?

✓ Spontaneous ***CP***-violation

[’84 Barr, ’84 Nelson, ’91 Bento et.al.]

How to generate ***CKM*** phase ?

(1) Spontaneously ***CP***-violation (easy)

$$cf\,)\;\; V = m^2\,(S^2 + S^{2*}) + \lambda(S^4 + S^{4*}) + \ldots$$

(S : complex scalar field, m, λ : Real)

(2) Naive generation of the complex Yukawa

$$L_{Yukawa} = (y_{ij} + y'_{ij}\, S/M_{PL})\, H\, Q\, \bar{d}_R + \ldots \qquad (\, y, y' : Real\,)$$

ends up with the strong ***CP*** problem…

$$L_{strong\,CP} = c\,(S/M_{PL} - S^*/M_{PL})\, F\tilde{F} \qquad (\, c : Real\,)$$

(too small S/M_{PL} does not explain δ_{CKM})

How Do We Explain a Small θ ?

✓ Spontaneous ***CP***-violation

[’84 Barr, ’84 Nelson, ’91 Bento et.al.]

How to generate ***CKM*** phase ?

(1) Spontaneously ***CP***-violation (easy)

$$cf)\ V = m^2 (S^2 + S^{2*}) + \lambda (S^4 + S^{4*}) + \ldots$$

(S : complex scalar field, m, λ : Real)

(2)’ Nelson-Barr Model

$$L_{Yukawa} = y_{ij} H Q \bar{d}_R + (f_i S + f_i' S^*) D_L \bar{d}_R + \mu D_L \bar{D}_R + h.c.$$

$D_{L,R}$ Extra vector-like quark

CP symmetry : parameters are real

Z_2 symmetry $D_{L,R}$, S : odd, others even

After CP & EW SSB

Z_2 forbidden

$$\mathcal{M}_d = \begin{pmatrix} m_d & 0 \\ M_D & \mu \end{pmatrix}_{4x4} \qquad M_{Di} = f_i \langle S \rangle + f_i' \langle S^* \rangle$$

How Do We Explain a Small θ ?

✓ Spontaneous ***CP***-violation

[’84 Barr, ’84 Nelson, ’91 Bento et.al.]

How to generate ***CKM*** phase ?

(1) Spontaneously ***CP***-violation (easy)

cf) $V = m^2 (S^2 + S^{2*}) + \lambda(S^4 + S^{4*}) + \ldots$

(S : complex scalar field, m, λ : Real)

(2)’ Nelson-Barr Model

After CP & EW SSB

Z_2 forbidden

$$\mathcal{M}_d = \begin{pmatrix} m_d & 0 \\ M_D & \mu \end{pmatrix}_{\mathbf{4x4}} \qquad M_{Di} = f_i\langle S\rangle + f_i'\langle S^*\rangle$$

Arg[det M_d] = 0

→ θ = 0 even after spontaneous CP breaking !

How Do We Explain a Small θ ?

✓ Spontaneous ***CP***-violation

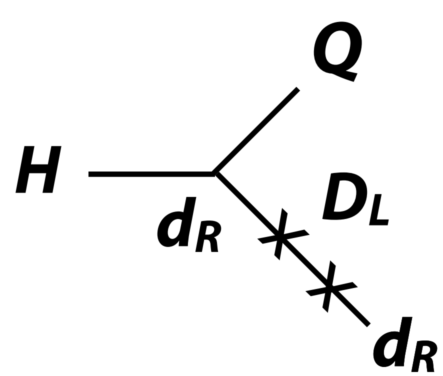
[’84 Barr, ’84 Nelson, ’91 Bento et.al.]

How to generate ***CKM*** phase ?

(1) Spontaneously ***CP***-violation (easy)

cf) $V = m^2 (S^2 + S^{2*}) + \lambda (S^4 + S^{4*}) + \ldots$

(2)’ Nelson-Barr Model



$$y^d_{\rm eff} \simeq y^d \left(1 - \frac{1}{2}\frac{M_D M_D^T}{\mu^2}\right)$$

Real Hermitian

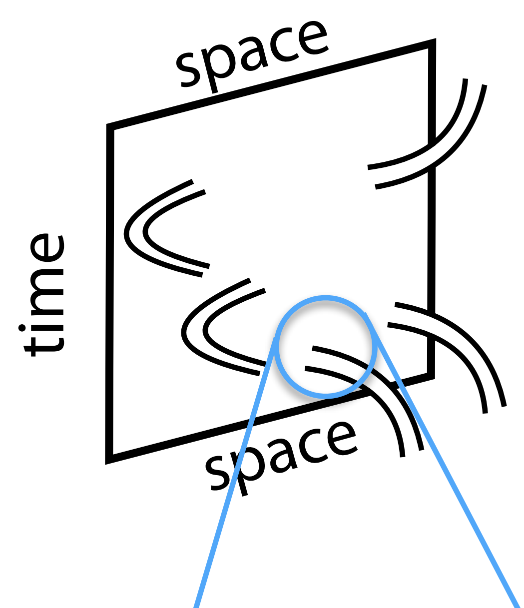
$\delta_{CKM} = O(1)$ ***is possible for*** $O(fS) \sim \mu$ **.**

Complete model?
It might be good time to rethink the spontaneous CP breaking…

How Do We Explain a Small θ ?

✓ Wormhole Solutions ?

[1988 Coleman]



Wormhole

$$ds^2 = dr^2 + a^2(r)d\Omega_3$$

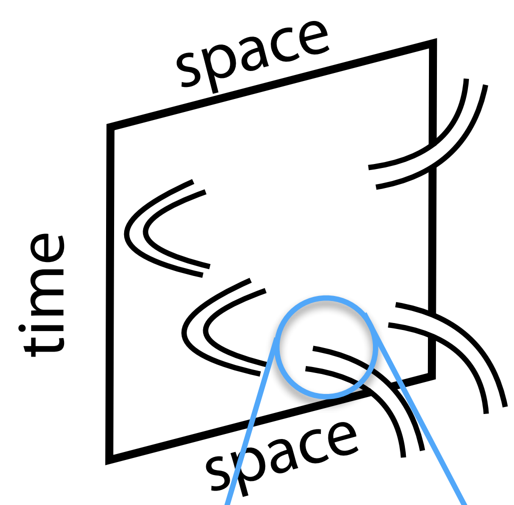
$$a(0) \neq 0$$

In quantum gravity, all the transition amplitudes are accompanied by spacetime transition via wormholes.

How Do We Explain a Small θ ?

✔ Wormhole Solutions ?

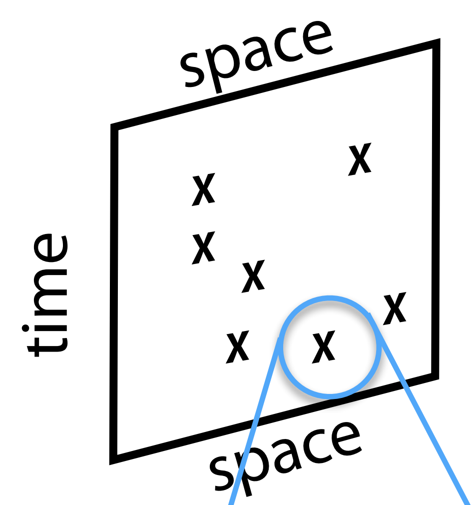
[1988 Coleman]



Wormhole

$$ds^2 = dr^2 + a^2(r)d\Omega_3$$
$$a(0) \neq 0$$

$$\simeq$$



Local operator

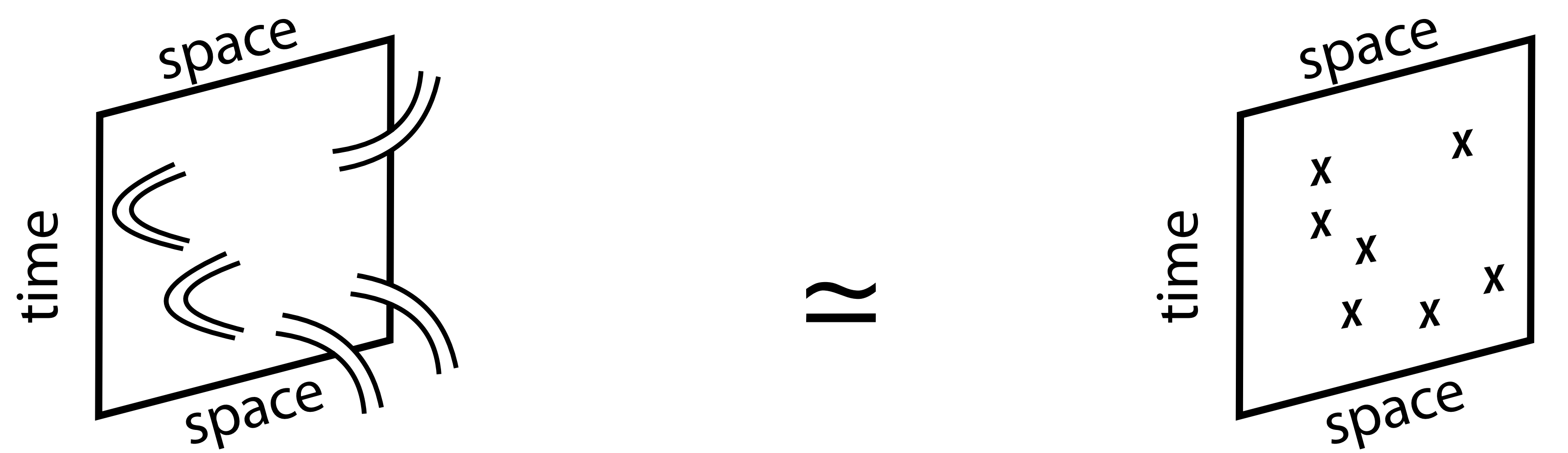
$$\text{e.g.)}\ L = \Phi(x)^4$$

The effects of the wormhole "gas" look like the insertions of the local operators.

How Do We Explain a Small θ ?

✔ Wormhole Solutions ?

[1988 Coleman]



The effective Lagrangian receives corrections from the wormhole gas effects :

$$\boldsymbol{L_{eff}(x) \sim L_0(x) + \Sigma_i \, L_i\,(x) \, \langle A_i \rangle}$$

($\boldsymbol{A_i}$: wormhole insertion operator)

[see also '15 Hamada, Kawai, Kawana for more elegant explanation]

How Do We Explain a Small θ ?

✔ Wormhole Solutions ?

When the Lagrangian parameters are dominated by the inserted part their values can be determined by the most probable values :

$$\int d\theta \int \mathcal{D}\phi \, e^{iS(\theta)} \sim \int \mathcal{D}\phi \, e^{iS(\theta_*)}$$

['89 Preskill, Trivedi Wise] ['89 Preskill, Trivedi Wise]

The effect of $\boldsymbol{\theta}$ on Newton constants and vacuum energy favors $\boldsymbol{\theta_* = \pi}$

['15 Hamada, Kawai, Kawana]

The effect of $\boldsymbol{\theta}$ on the ***QCD*** vacuum energy favors $\boldsymbol{\theta_* = 0}$ or $\boldsymbol{\pi}$.

→ interesting arguments but seem to be not conclusive yet…

($\boldsymbol{\theta_* = \pi}$ is disfavored by such as Meson spectrum

[1979 Crewther, Veccia, Veneziano, Witten])

Peccei-Quinn Mechanism ['77 Peccei, Quinn]

Two Higgs doublet Model ($\boldsymbol{H_u}$, $\boldsymbol{H_d}$)

$$\mathcal{L} = y_u H_u Q_L \bar{u}_R + y_d H_d Q_L \bar{d}_R - m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_u|^2 - \cdots$$

U(1) Peccei-Quinn symmetry (anomaly of $\boldsymbol{SU(3)_c}$)

$$\boldsymbol{H_{u,d} \to e^{i\alpha} H_{u,d}} \qquad \boldsymbol{u_R \to e^{-i\alpha} u_R} \qquad \boldsymbol{d_R \to e^{-i\alpha} d_R}$$

By the Peccei-Quinn rotation, $\boldsymbol{\theta}$ can be shifted away !

$$\boldsymbol{\theta \to \theta' = \theta - 2N_g\, \alpha} \qquad \boldsymbol{(N_g = 3)}$$

so that the $\boldsymbol{\theta}$ is unphysical (similar to $\boldsymbol{\theta_W}$).

Weinberg-Wilczek Axion ['78 Weinberg, '78 Wilczek]

$U(1)_{PQ}$ is spontaneously broken at the EWSB → ***axion*** = (***CP***-odd Higgs)

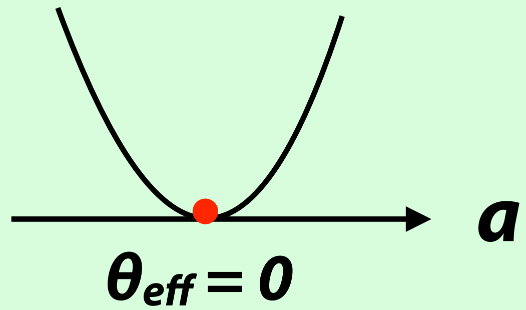
$$a = \frac{\sqrt{2}v_u v_d}{\sqrt{v_u^2 + v_d^2}}(\arg H_u + \arg H_d)$$

$$\mathcal{L}_{\rm eff} = \frac{g_s^2}{32\pi^2}\left(\theta - \frac{6a}{f_a}\right) G^{a\mu\nu}\tilde{G}^a_{\mu\nu} \qquad \left(f_a = 2\sqrt{2}v_u v_d/\sqrt{v_u^2 + v_d^2}\right)$$

Axion is massive due to the ***$SU(3)_c$*** anomaly

$$m_a = \frac{N_g\sqrt{m_u m_d}}{m_u + m_d}\frac{f_\pi}{f_a}m_\pi \quad \boldsymbol{\sim 100\ \mathrm{keV}}$$

In terms of the axion, the PQ mechanism can be interpreted as a dynamical tuning of the ***θ*** angle.

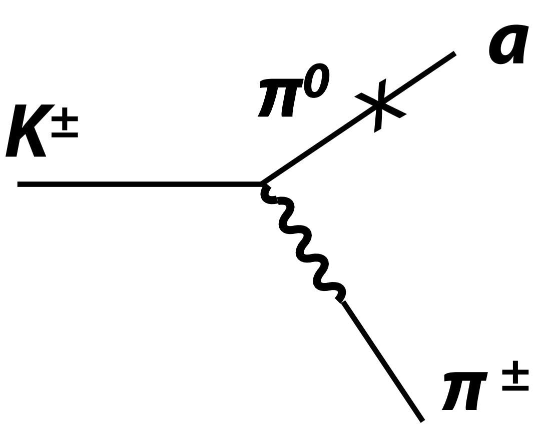


$$\mathcal{L} = \frac{1}{2}m_a^2 f_a^2 (a/f_a - \theta/6)^2 \longrightarrow \langle a/f_a\rangle = \theta/6$$

$$\theta_{\rm eff} = \theta - 6\langle a/f_a\rangle = 0$$

Weinberg-Wilczek Axion ['78 Weinberg, '78 Wilczek]

f_a is constrained by a meson decay rate into axion.



$$Br(K^\pm \to \pi^\pm + a \text{ (invisible)})$$
$$= O(f_\pi^2 / f_a^2) \times Br(K^\pm \to \pi^\pm + \pi^0)$$
$$< 5 \times 10^{-11}$$ [E787 hep-ex/0403034]

$f_a > O(1)\,\mathrm{TeV}$

[Axion decays into two photon but the lifetime is so long for $m_a \sim 100$ *keV.]*

Original PQ-mechanism has been excluded !

Invisible Axion : $f_a >> v_{EW}$ ['80 Zhitnitsky, '81 Dine, Fischler, Sredniki]

ZDFS axion : Two Higgs doublet Model (***H_u , H_d***) and a Singlet ***S***

$$\mathcal{L} = y_u H_u Q_L \bar{u}_R + y_d H_d Q_L \bar{d}_R - m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_u|^2 - \cdots$$
$$+ \boxed{\frac{1}{M_{PL}^{n-2}} S^n H_u H_d} + \cdots$$

U(1) Peccei-Quinn symmetry

$H_{u,d} \to e^{i\alpha} H_{u,d}$ $S \to e^{i2\alpha} S$ $u_R \to e^{-i\alpha} u_R$ $d_R \to e^{-i\alpha} d_R$

$U(1)_{PQ}$ is spontaneously broken by $\langle S \rangle = v_s \gg v$

$$a = \frac{f_a}{2} \arg S \qquad f_a = 2\sqrt{2}\langle S \rangle$$

$$m_a = \mathcal{O}(1)\text{meV} \times \left(\frac{10^9 \text{GeV}}{f_a} \right)$$

The axion evades constraints from the meson decay rates!

Invisible Axion : $f_a >> v_{EW}$ ['79 Kim, '80 Shifman, Vainshtein, Zakharov]

KSVZ axion : SM matter field are not ***$U(1)_{PQ}$*** neutral.

$$\mathcal{L} = \mathcal{L}_{\rm SM} + S q_L \bar{q}_R - \cdots$$

Singlet ***S*** Extra colored fermions ***q_L , q_R***

U(1) Peccei-Quinn symmetry (***$SU(3)_c$*** anomaly)

$S \rightarrow e^{i\alpha} S$ $q_{L,R} \rightarrow e^{-i\alpha/2} q_{L,R}$

$U(1)_{PQ}$ is spontaneously broken by $\langle S \rangle = v_s >> v$

$$a = f_a {\rm arg} S \qquad f_a = \sqrt{2}\langle S \rangle$$

$$m_a = \mathcal{O}(1){\rm meV} \times \left(\frac{10^9{\rm GeV}}{f_a}\right)$$

The axion evades constraints from the meson decay rates!

Invisible Axion : $f_a >> v_{EW}$

Invisible axion is very light :

$$m_a = \mathcal{O}(1)\mathrm{meV} \times \left(\frac{10^9\mathrm{GeV}}{f_a}\right)$$

→ axion is subject to constraints not only from the meson decays but also from astrophysics !

✓ Resultant constraint on the decay constant is

$f_a > 10^9$GeV

✓ Invisible axion is a good candidate for ***DM***

$$\Omega_a h^2 = 0.18\ \theta_1^2 \left(\frac{F_a}{10^{12}\mathrm{GeV}}\right)^{1.19} \left(\frac{\Lambda}{400\mathrm{MeV}}\right)$$

Initial angle
θ_1 = 0-2π

$$\Omega_{\mathrm{DM}} h^2 \simeq 0.12$$

[e.g. 1301.1123 Kawasaki, Nakayama]

What is the origin of the Peccei-Quinn symmetry ?

- The ***PQ*** symmetry cannot be an exact symmetry !
  - ***U(1) PQ*** symmetry is defined to be broken by the ***QCD*** anomaly.

- Why is the ***PQ*** symmetry broken only by the ***QCD*** anomaly?
  - Why is it not broken by at least higher dimensional term ?

If the physics at the Planck scale breaks ***PQ*** symmetry we would have

$$\Delta\mathcal{L} = \frac{S^m}{M_{\rm PL}^{m-4}} + h.c.$$

which distorts the axion potential

$$\mathcal{L} = \frac{1}{2} m_a^2 f_a^2 (a/f_a - \theta/6)^2 + \frac{f_a^m}{M_{\rm PL}^{m-4}} \frac{a}{f_a} + \cdots$$

The effective $\boldsymbol{\theta_{eff}}$-parameter is no more vanishing…

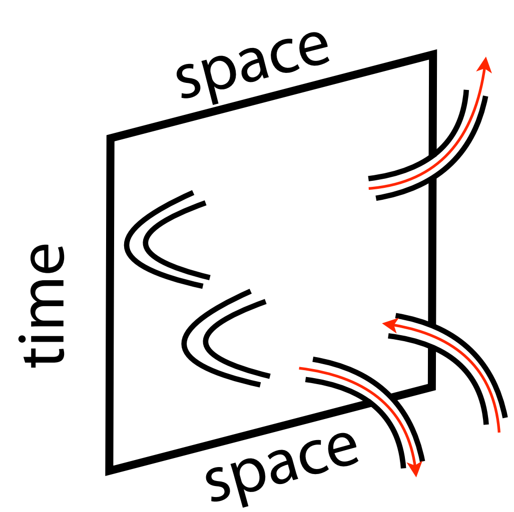
$$\Delta\theta_{\rm eff} = \frac{f_a^m}{f_\pi^2 m_\pi^2 M_{\rm PL}^{m-4}}$$

If we require $\boldsymbol{\theta_{eff} << 10^{-11}}$, no term with $\boldsymbol{m < 10}$ is allowed $\boldsymbol{f_a > 10^9}$***GeV***.

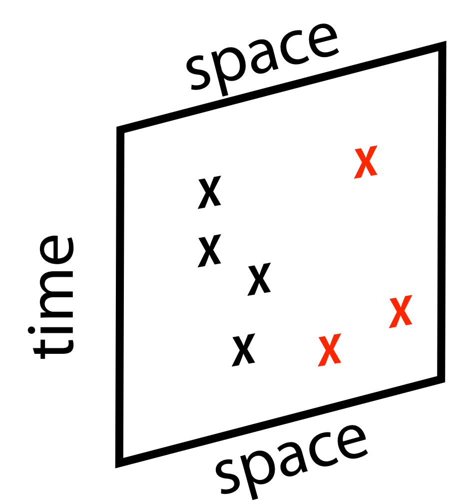
What is the origin of the Peccei-Quinn symmetry ?

- ✔ Why is the ***PQ*** symmetry broken only by the QCD anomaly?

The wormhole transitions may make things worse…



$\simeq$



The charged particle under global symmetries can go through the wormhole leaving symmetry breaking terms

$$L = g_n \Phi(x)^n + h.c.$$

$$g_n \sim (8\pi M_{PL})^{4-n} \quad \textit{(for a large n)}$$

[1989 Abott, Wise, 1995 Kallosh, Linde, Linda, Susskind]

The existence of the global symmetries is quite unnatural.

What is the origin of the Peccei-Quinn symmetry ?

Gauge symmetries do not suffer from explicit breaking…

Can we make the ***PQ*** symmetry a gauge symmetry ?

The ***PQ*** symmetry has an $\boldsymbol{SU(3)_c}$ anomaly…

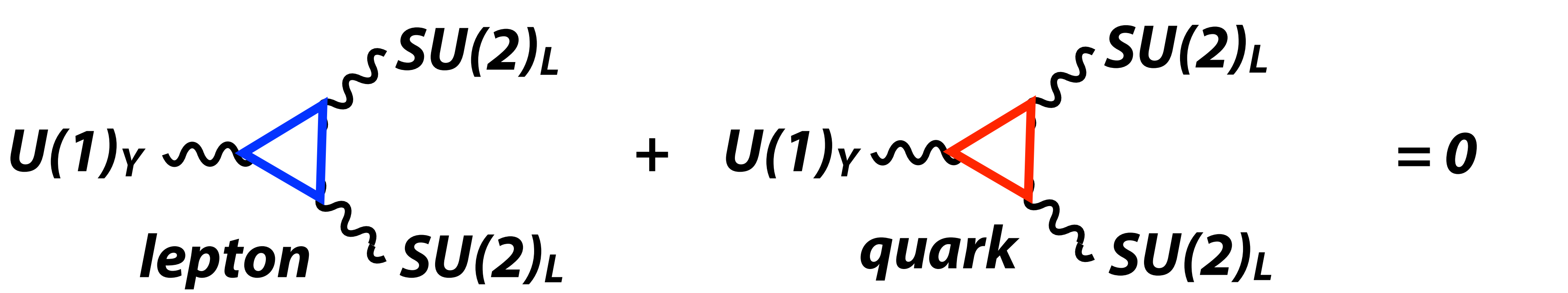
→ the PQ symmetry cannot be a gauge symmetry by itself.

✔ $\boldsymbol{U(1)_Y}$ in the Standard Model

$\boldsymbol{U(1)_Y}$ ***symmetry of the lepton sector*** has an $\boldsymbol{SU(2)_L}$ anomaly.

Cannot be a gauge symmetry ? Absolutely Yes !

The $\boldsymbol{SU(2)_L}$ anomaly of $\boldsymbol{U(1)_Y}$ ***of the lepton sector*** is cancelled by the $\boldsymbol{SU(2)_L}$ anomaly of $\boldsymbol{U(1)_Y}$ ***of the quark sector!***



What is the origin of the Peccei-Quinn symmetry ?

Gauge symmetries do not suffer from explicit breaking…

Can we make the ***PQ*** symmetry a gauge symmetry ?

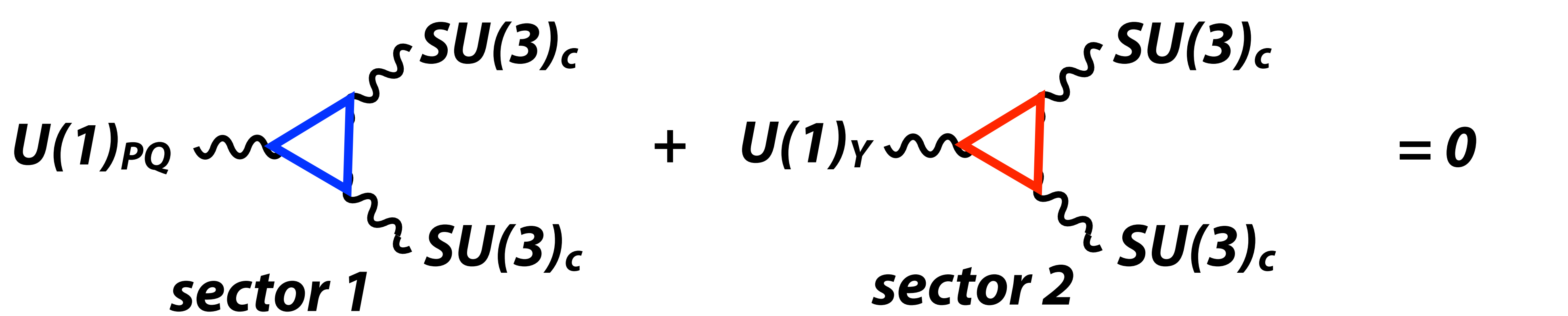
The ***PQ*** symmetry has an ***SU(3)$_c$*** anomaly…

→ the PQ symmetry cannot be a gauge symmetry by itself.

✔ ***Gauged U(1)$_{PQ}$***

We arrange the ***U(1)$_{PQ}$*** charges so that the total ***SU(3)$_c$*** anomaly is cancelled !

The ***SU(3)$_c$*** anomaly of ***U(1)$_{PQ}$ of sector 1*** is cancelled by the ***SU(3)$_c$*** anomaly of ***U(1)$_{PQ}$ of sector 2***



General Recipe to Construct gauged PQ models

Let us bring any "two" invisible axion models :

sector 1	sector 2
$U(1)_{PQ1}$	***$U(1)_{PQ2}$***
cf. DSFZ model	cf. DSFZ model
cf. KSVZ model	cf. KSVZ model

$$\partial j_{\mathrm{PQ}_1} = \frac{g_s^2}{32\pi^2} N_1 F^a \tilde{F}^a \qquad \partial j_{\mathrm{PQ}_2} = \frac{g_s^2}{32\pi^2} N_2 F^a \tilde{F}^a$$

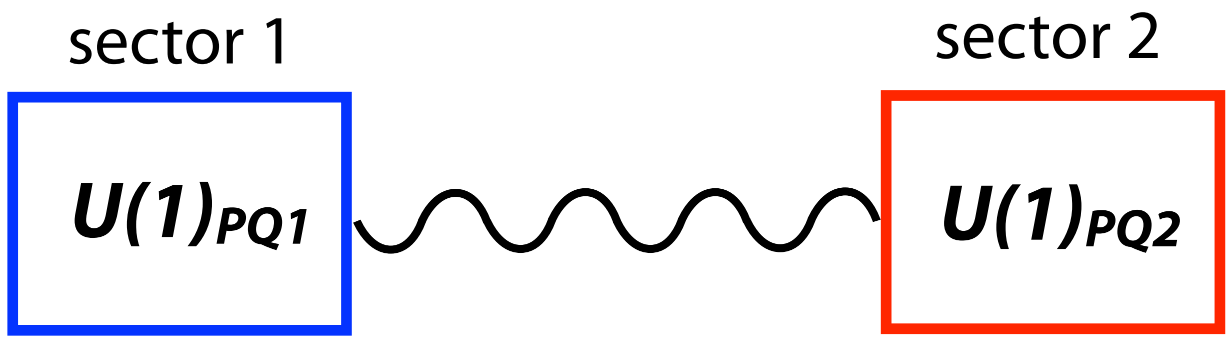
An anomaly free combination,

$$j^\mu_{PQ} \propto N_2 \, j^\mu_{PQ_1} - N_1 \, j^\mu_{PQ_2}$$

can be a gauge symmetry !

→ Gauged ***$U(1)_{PQ}$*** Symmetry !

General Recipe to Construct gauged PQ models



- No ***gauged*** $\boldsymbol{U(1)_{PQ}}$ breaking term is allowed since $\boldsymbol{U(1)_{PQ}}$ is an exact symmetry !

- No ***global*** $\boldsymbol{U(1)_{PQ1}}$ breaking term consisting of fields in the sector 1 due to the ***gauged*** $\boldsymbol{U(1)_{PQ}}$ symmetry.

$$U(1)_{PQ1} \text{ breaking term } = U(1)_{PQ} \text{ breaking term}$$

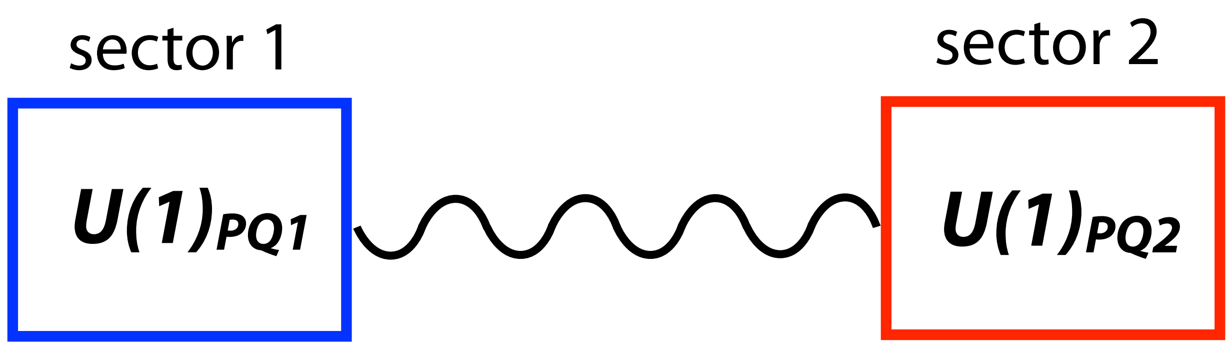
$$L = \Phi_1(x)^n + h.c.$$

- No ***gauged*** $\boldsymbol{U(1)_{PQ2}}$ breaking term consisting of fields in the sector 2 due to the ***global*** $\boldsymbol{U(1)_{PQ2}}$ symmetry.

$$U(1)_{PQ2} \text{ breaking term } = U(1)_{PQ} \text{ breaking term}$$

$$L = \Phi_2(x)^m + h.c.$$

General Recipe to Construct gauged PQ models



- No ***gauged*** $U(1)_{PQ}$ breaking term is allowed since $U(1)_{PQ}$ is an exact symmetry !

- Only dangerous operators to break $U(1)_{PQ1}$ and $U(1)_{PQ2}$ symmetries are

$$L = M_{PL}^{4-(\dim O1+\dim O2)}\, O_1\, O_2 + h.c.$$

$$U(1)_{PQ1}\ \text{of}\ O_1 \neq 0 \qquad U(1)_{PQ2}\ \text{of}\ O_2 \neq 0$$

$$\text{Gauged}\ U(1)_{PQ}\ \text{of}\ O_1 O_2 = 0$$

- If PQ_1 and PQ_2 breaking scales are $O(10^9)GeV$, the resultant breaking of either PQ_1 or PQ_2 is suppressed by arranging the charge assignment so that

$$\dim O_1 + \dim O_2 > 10$$

General Recipe to Construct gauged PQ models

✔ Example : Barr-Seckel Model

Bring two independent ***KSVZ*** axion models

$$L = y_1 S_1 q_{1L} \bar{q}_{1R} + y_2 S_2 q_{2L} \bar{q}_{2R} + h.c.$$

KSVZ fermions : N_1 flavor of q_1 , N_2 flavor of q_2

✔ $U(1)_{PQ1}$ symmetry

$S_1 \rightarrow e^{i2a} S_1 \quad q_{1L,R} \rightarrow e^{-ia} q_{1L,R}$

$$\partial j_{\mathrm{PQ}_1} = \frac{g_s^2}{32\pi^2} N_1 F^a \tilde{F}^a$$

✔ $U(1)_{PQ2}$ symmetry

$S_2 \rightarrow e^{i2a} S_2 \quad q_{2L,R} \rightarrow e^{-ia} q_{2L,R}$

$$\partial j_{\mathrm{PQ}_2} = \frac{g_s^2}{32\pi^2} N_2 F^a \tilde{F}^a$$

General Recipe to Construct gauged PQ models

✓ Example : Barr-Seckel Model

Gauged $U(1)_{PQ}$ symmetry

$S_1(q_1)$ $S_2(-q_2)$ $q_1 : q_2 = N_2 : -N_2$

$\rightarrow \partial j_{PQ} = 0$

$|q_1|$ and $|q_2|$ are taken to be relatively prime integers

✓ The lowest dimensional $U(1)_{PQ1,PQ2}$ breaking operators

$$L = M_{PL}^{4-(|q1|+|q2|)} S_1^{q1} S_2^{q2} + h.c.$$

To obtain high quality global PQ symmetry :

$$|q_1| + |q_2| > 10$$

ex) $N_1 = 1,\ N_2 = 9$

General Recipe to Construct gauged PQ models

- Decomposition of the ***gauged*** $U(1)_{PQ}$ and a ***global*** $U(1)_{PQ}$
  - Let us assume

$$\langle S_1 \rangle = f_1/\sqrt{2} \qquad \langle S_2 \rangle = f_2/\sqrt{2}$$

  - Invisible axion candidates a_1, a_2

$$S_1 = f_1/\sqrt{2}\, \mathrm{Exp}[\, i\, a_1/f_1\,]$$

$$S_2 = f_2/\sqrt{2}\, \mathrm{Exp}[\, i\, a_2/f_2\,]$$

  - Domains of the axial components

$$a_1/f_1 = [\,0, 2\pi\,) \qquad a_2/f_2 = [\,0, 2\pi\,)$$

  - Under the gauged $U(1)_{PQ}$

$$a_1(x)/f_1 \rightarrow a_1(x)/f_1 + q_1\, \alpha(x)$$

$$a_2(x)/f_2 \rightarrow a_2(x)/f_2 + q_2\, \alpha(x)$$

$$A_\mu(x) \rightarrow A_\mu(x) + g^{-1}\, \partial_\mu\, \alpha(x)$$

General Recipe to Construct gauged PQ models

✓ Decomposition of the ***gauged U(1)$_{PQ}$*** and a ***global U(1)$_{PQ}$***

$$L = |D_\mu S_1|^2 + |D_\mu S_2|^2$$

$$= (\partial_\mu a_1)/2 + (\partial_\mu a_2)/2 - g A_\mu (q_1 f_1 \partial^\mu a_1 + q_2 f_2 \partial^\mu a_2)$$

$$+ g^2 (q_1^2 f_1^2 + q_2^2 f_2^2) A_\mu A^\mu / 2$$

✓ Redefinition of the axial components

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}} \begin{pmatrix} q_2 f_2 & -q_1 f_1 \\ q_1 f_1 & q_2 f_2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\rightarrow \quad L = (\partial_\mu a)^2 / 2 + m_A^2 (A_\mu - \partial_\mu b / m_A)^2 / 2$$

$$[m_A^2 = g^2 (q_1^2 f_1^2 + q_2^2 f_2^2)]$$

a : axion ***b : would-be goldstone boson of gauged U(1)$_{PQ}$***

General Recipe to Construct gauged PQ models

✔ Decomposition of the gauged ***U(1)$_{PQ}$*** and a global ***U(1)$_{PQ}$***

The axion is gauge invariant !

$$a = \frac{f_1 f_2}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}} \left(\frac{q_2 a_1}{f_1} - \frac{q_1 a_2}{f_2} \right)$$

$a_1(x) \rightarrow a_1(x) + q_1 f_1 a(x)$ $\quad$ $a_2(x) \rightarrow a_2(x) + q_2 f_2 a(x)$

$\rightarrow \delta a(x) = 0$

The anomalous coupling :

$$\mathcal{L}_{QCD} = \frac{g_s^2}{32\pi^2} n_{GCD} \left(\frac{q_2 a_1}{f_1} - \frac{q_1 a_2}{f_2} \right) F^a \tilde{F}^a$$

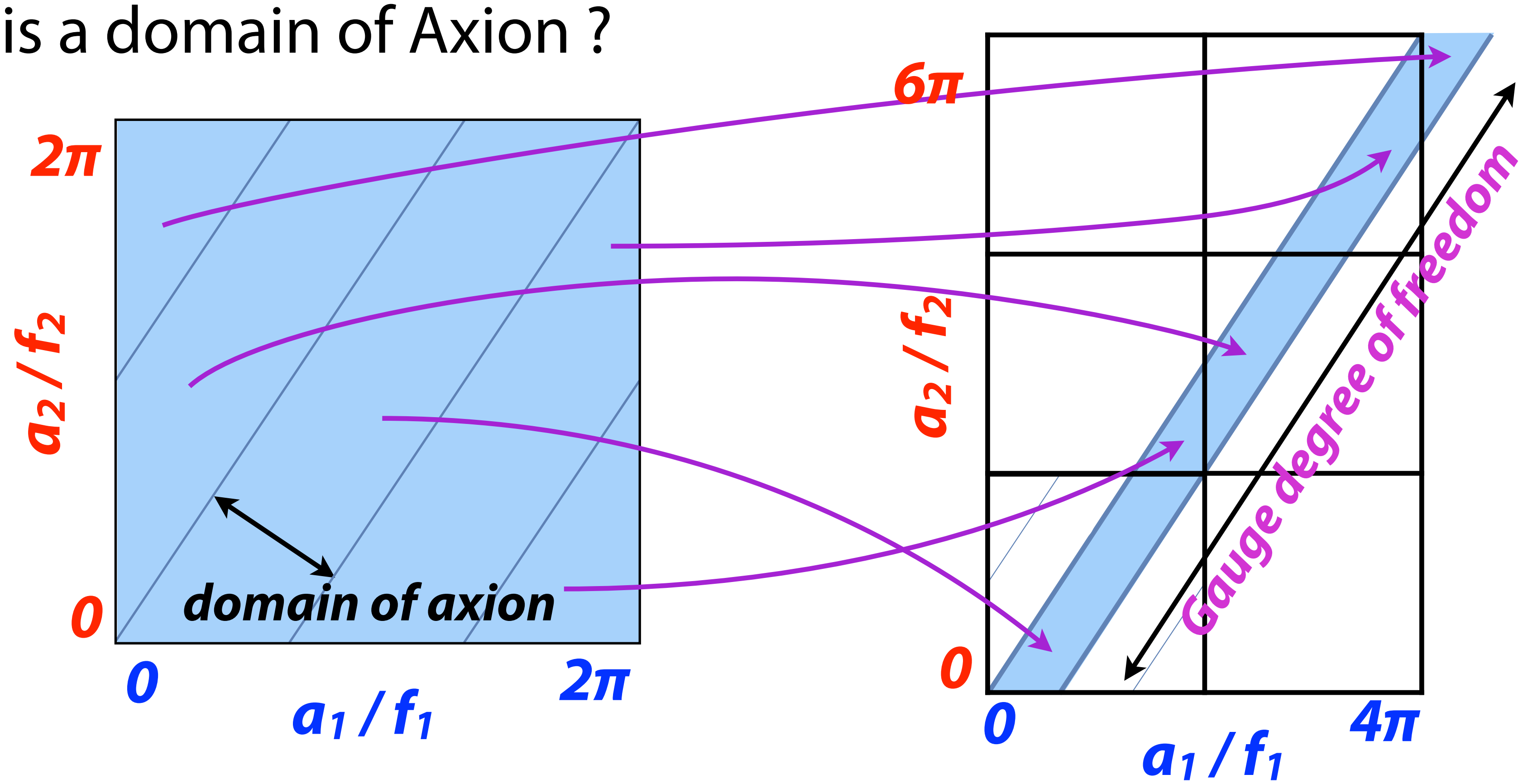
$$= \frac{g_s^2}{32\pi^2} n_{GCD} \frac{a}{F_a} F^a \tilde{F}^a$$

$$F_a = \frac{f_1 f_2}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}}$$

n_{GCD} : the greatest common divisor of N_1 and N_2 ($N_1 = n_{GCD}|q_2|$, $N2 = n_{GCD}|q_1|$)

General Recipe to Construct gauged PQ models

✔ What is a domain of Axion ?



ex) $N_1 = 2$, $N_2 = 3$

$(a_1/f_1, a_2/f_2) = 2\pi\,(i,j)$ $i, j \in Z$ are equivalent by definition.

The axion domain is given by the distance between the gauge orbit of ***(0,0)*** and the closest ***2π (i,j)*** point.

$$\frac{a}{F_a} = [0, 2\pi) \qquad F_a = \frac{f_1 f_2}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}}$$

(This expression is valid only when q_1 and q_2 are prime with each other.)

General Recipe to Construct gauged PQ models

✔ Example 2 : Application to the Composite Axion Model

SU(N_c) gauge theory [1985 Kim]

	$SU(N_c)$	$SU(3)$	$U(1)_{PQ}$
Q_L	N_c	3	1
$\bar{Q}_R$	$\bar{N}_c$	$\bar{3}$	1
q_L	N_c	1	-3
$\bar{q}_R$	$\bar{N}_c$	1	-3

$\subset SU(4)$

$U(1)_{PQ}$ is free from $SU(N_c)$ anomaly but is broken by ***QCD*** anomaly !

No quark mass terms

✔ Strong dynamics of $SU(N_c)$ causes the Chiral symmetry breaking.

16 Goldstone Modes

$$\begin{cases} SU(3): \text{Octet} + 3 + \bar{3} = \text{Massive} \ (\sim g_s \Lambda_{Nc}) \\ U(1)_A: \text{singlet} = \text{Massive} \ (\sim \Lambda_{Nc}) \\ U(1)_{PQ}: \text{singlet} = \text{axion} \end{cases}$$

$$\mathcal{L}_{QCD} = \frac{g_s^2}{32\pi^2} N_c \frac{a}{f_a} F^a \tilde{F}^a$$

General Recipe to Construct gauged PQ models

✓ Example 2 : Application to the Composite Axion Model

$SU(N_c)$ gauge theory [1985 Kim]

	$SU(N_c)$	$SU(3)$	$U(1)_{PQ}$
Q_L	N_c	3	1
$\bar{Q}_R$	$\bar{N}_c$	$\bar{3}$	1
q_L	N_c	1	-3
$\bar{q}_R$	$\bar{N}_c$	1	-3

$\subset SU(4)$

$U(1)_{PQ}$ is free from $SU(N_c)$ anomaly but is broken by ***QCD*** anomaly !

No quark mass terms

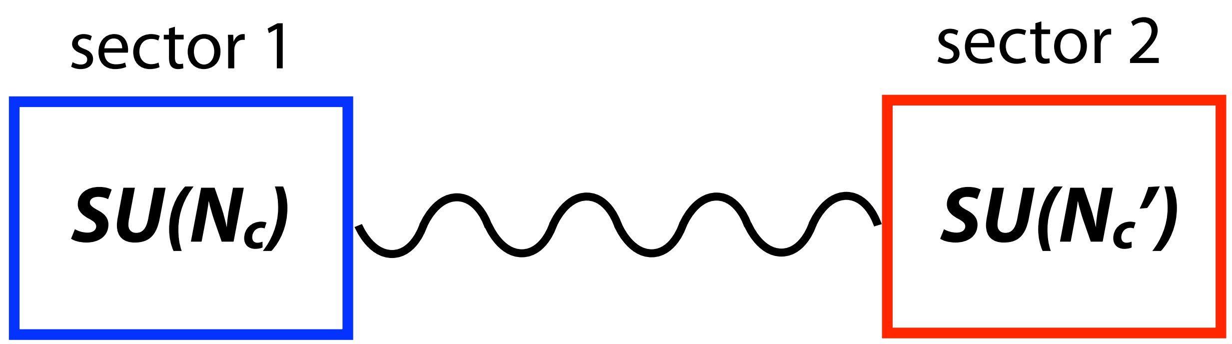
✓ ***PQ*** breaking operators…

$$L = m\,(Q_L\,\bar{Q}_R) + (Q_L\,\bar{Q}_R)^2/M_{PL}^2 + \dots$$

Gauged PQ mechanism suppresses those operators !

General Recipe to Construct gauged PQ models

✔ Example 2 : Application to the Composite Axion Model



Bring two composite axion models and consider gauged $U(1)_{PQ}$ with the charge normalization

$$Q_L(q)\,,\bar{Q}_R(q),\ Q_L'(q')\,,\bar{Q}_R'(q') \qquad q : q' = N_c' : -N_c$$

$$\rightarrow\ \partial j_{PQ} = 0$$

The lowest dimensional global ***PQ*** breaking operators

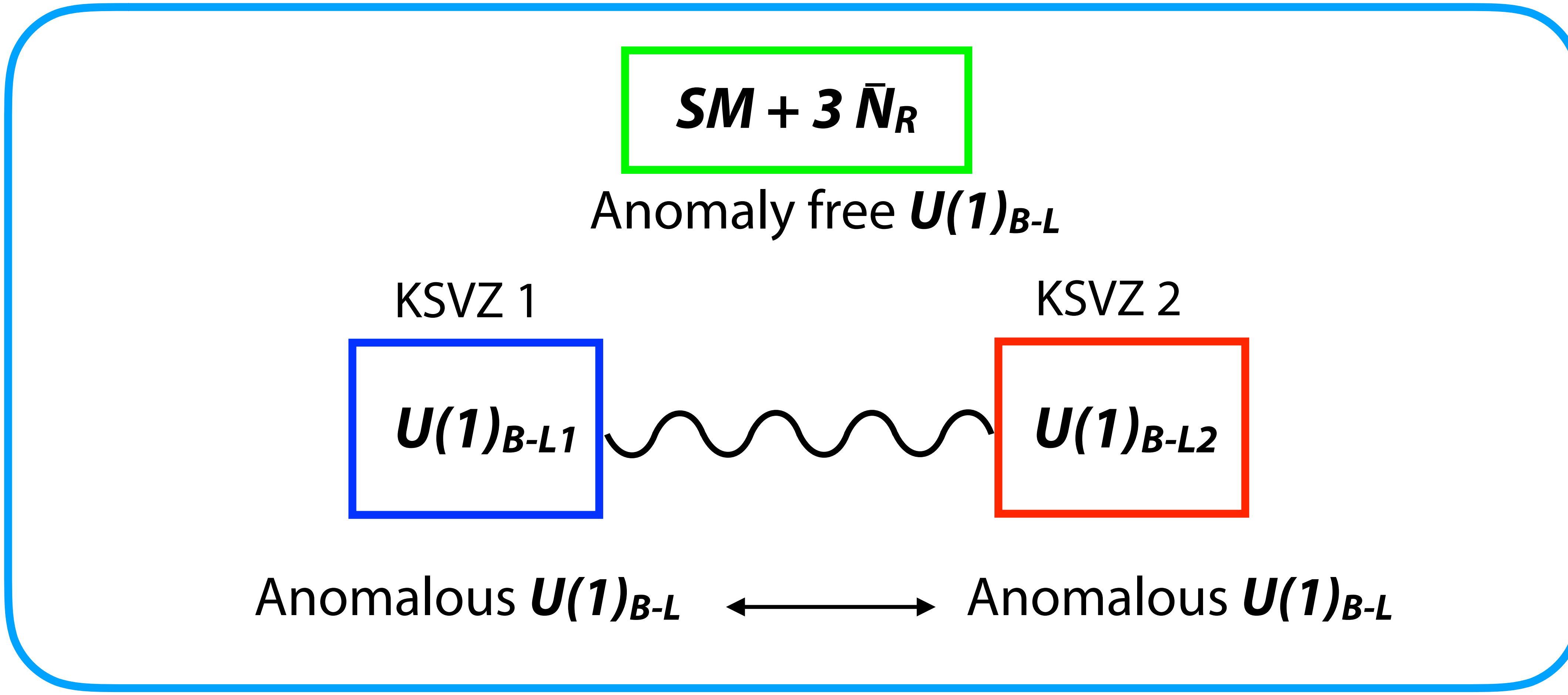
$$L = (Q_L\,\bar{Q}_R)^{|q'|}\,(Q'_L\,\bar{Q}'_R)^{|q|} / M_{PL}{}^{3|q| + 3|q'| - 4}$$

→ $N_c = 2,\ N_c' = 5$ model is good enough to obtain the high quality global PQ symmetry !

General Recipe to Construct gauged PQ models

✔ Example 3 : Can we identify the gauged $U(1)_{PQ}$ with gauged $U(1)_{B\text{-}L}$?

Answer : Yes



→ ***$U(1)_{B\text{-}L}$ solves the strong CP problem if it is accompanied by the KSVZ sectors !***

Compatible with Leptogenesis ? → Future work

General Recipe to Construct gauged PQ models

✔ Example 4 : Can we make a model with small charges ?

Sector 1 : Supersymmetric ***SU(2)*** dynamics

SU(2) fundamentals : $Q_1(1)$, $Q_2(1)$, $Q_3(-1)$, $Q_4(-1)$

SU(2) singlets : Z_1, Z_2, Z_3, Z_4

(We use $U(1)_{PQ} \subset SU(4)$ flavor symmetry)

$$W = \lambda Z_1 Q_1 Q_3 + \lambda Z_2 Q_1 Q_4 + \lambda Z_1 Q_2 Q_3 + \lambda Z_2 Q_2 Q_4$$

Below the dynamical scale :

✔ ***PQ*** neutral "Mesons"

$$M_1 = Q_1 Q_3 \qquad M_2 = Q_1 Q_4 \qquad M_3 = Q_2 Q_3 \qquad M_4 = Q_2 Q_4$$

✔ ***PQ*** charged "Baryons"

$$B_+(+2) = Q_1 Q_2 \qquad B_-(-2) = Q_3 Q_4$$

✔ Effective potential

$$W_{eff} = \lambda \Lambda Z_i M_i + X (M_i^2 + B_- B_+ - \Lambda^2)$$

deformed moduli constraint

→ Leading to ***PQ*** breaking $\langle B_{2\pm} \rangle \neq 0$

General Recipe to Construct gauged PQ models

- Example 4 : Can we make a model with small charges ?

Sector 2 : Supersymmetric ***SU(3)*** dynamics

SU(3) fundamentals : $Q'_i(1)$, $\bar{Q}'_i(-1)$

SU(3) singlets : 9 singlets Z'_{ij} (We use $U(1)_{PQ} = U(1)_B$ flavor symmetry)

$$W = \lambda Z'_{ij} Q'_i \bar{Q}'_j$$

Below the dynamical scale :

- ***PQ*** neutral "Mesons"

$$M_1 = Q'_i \bar{Q}'_j$$

- ***PQ*** charged "Baryons"

$$B'_+(+3) = Q'_1 Q'_2 Q'_3 \qquad B'_-(-3) = \bar{Q}'_1 \bar{Q}'_2 \bar{Q}'_3$$

- Effective potential

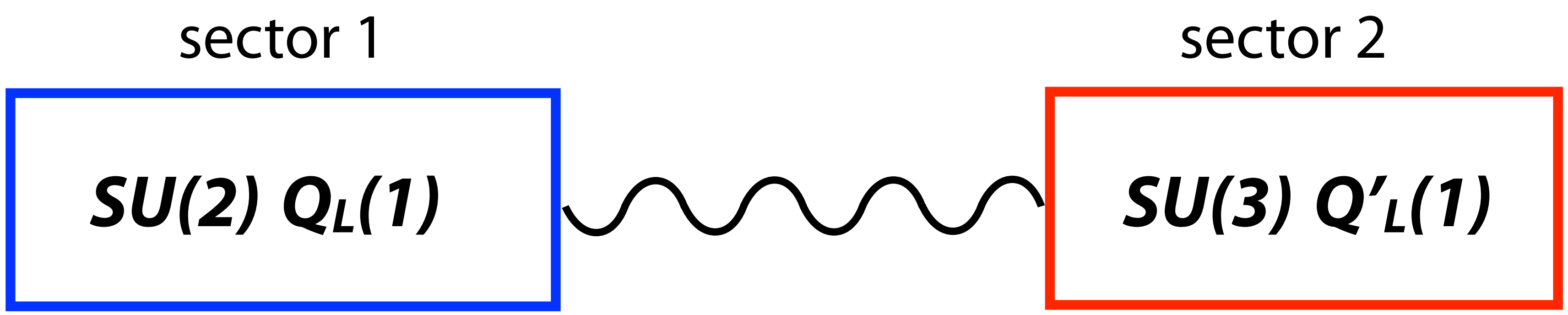
deformed moduli constraint

$$W_{eff} = \lambda\Lambda Z'_{ij} M'_{ij} + X' (\det[M] + \Lambda' B'_- B'_+ - \Lambda^3)$$

→ Leading to ***PQ*** breaking $\langle B'_\pm \rangle \neq 0$

General Recipe to Construct gauged PQ models

✓ Example 4 : Can we make a model with small charges ?



$$L = y_1\,(Q_1 Q_3)/M_{PL}^2\; q_L\,\bar{q}_R + y_2\,(\bar{Q}'_1\,\bar{Q}'_2\,\bar{Q}'_3)/M_{PL}^2\; q'_L\,\bar{q}'_R\; + h.c.$$

3-flavor of ***KSVZ*** quarks

2-flavor of ***KSVZ*** quarks

The lowest dimensional ***PQ***-breaking operators :

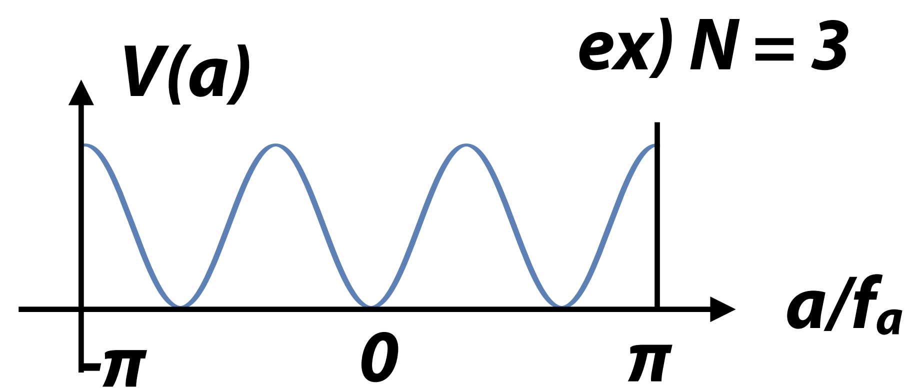
$$W = (Q_1 Q_3)^3 (\bar{Q}'_1\,\bar{Q}'_2\,\bar{Q}'_3)^2 / M_{PL}^9$$

→ Effects are small enough !

Domain Wall Problem in conventional PQ models

- In the conventional ***PQ*** model, $U(1)_{PQ}$ is explicitly broken down to Z_N symmetry by the ***QCD*** anomaly.

$$\mathcal{L}_{QCD} = \frac{g_s^2}{32\pi^2} N \frac{a}{f_a} F^a \tilde{F}^a$$



- Z_N is eventually broken spontaneously by the VEV of the axion .

→ Domain walls are formed when the axion gets VEV!

$$\rho_{DW} \sim \sigma \times H \propto T^2 \qquad (\sigma \sim f_a \Lambda_{QCD}^2)$$

[scaling solution 1990 Ryden, Press, Spergel]

Domain wall dominates over the energy density of the Universe for ***N>1*** !

What happens in the gauged ***PQ*** models ?

Domain Wall Problem in the conventional PQ model

Closer look at the domain wall problem :

(1) In the conventional ***PQ*** model, $U(1)_{PQ}$ is spontaneously broken at f_a.

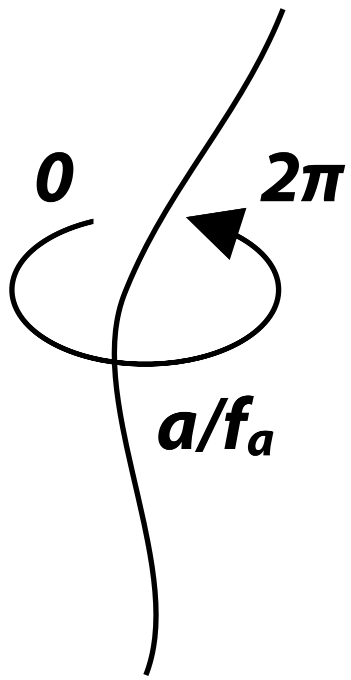
One global strings are formed in each Hubble volume in average.

Tension : $\mu^2 \sim 2\pi f_a^2 \log[f_a/H] \sim 2\pi f_a^2 \log[M_{PL}/f_a]$

Energy density of the strings do not cause problem due to its scaling nature:

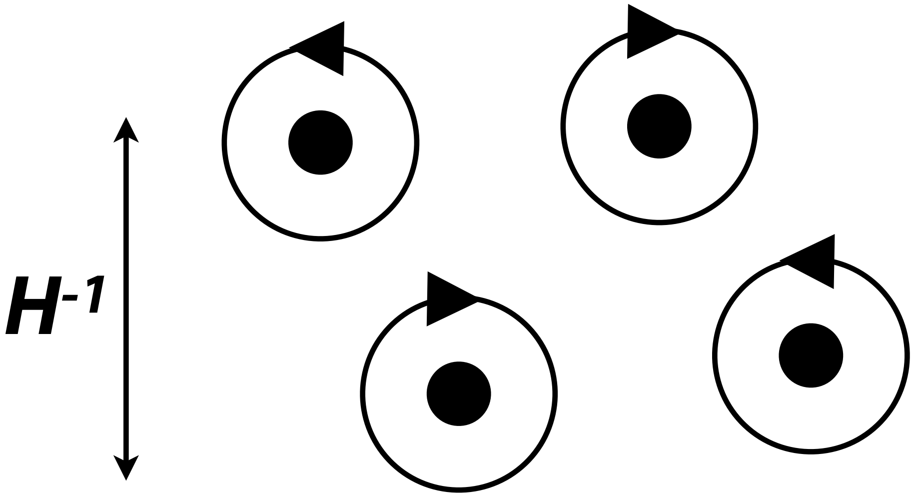
$\rho_{string} \sim \mu^2 H^2 \propto T^4$ [e.g. 1012.5502 Hiramatsu et.al.]

Around the cosmic strings, the axion field takes nontrivial configuration.



Winding number = 1

Top view of the strings

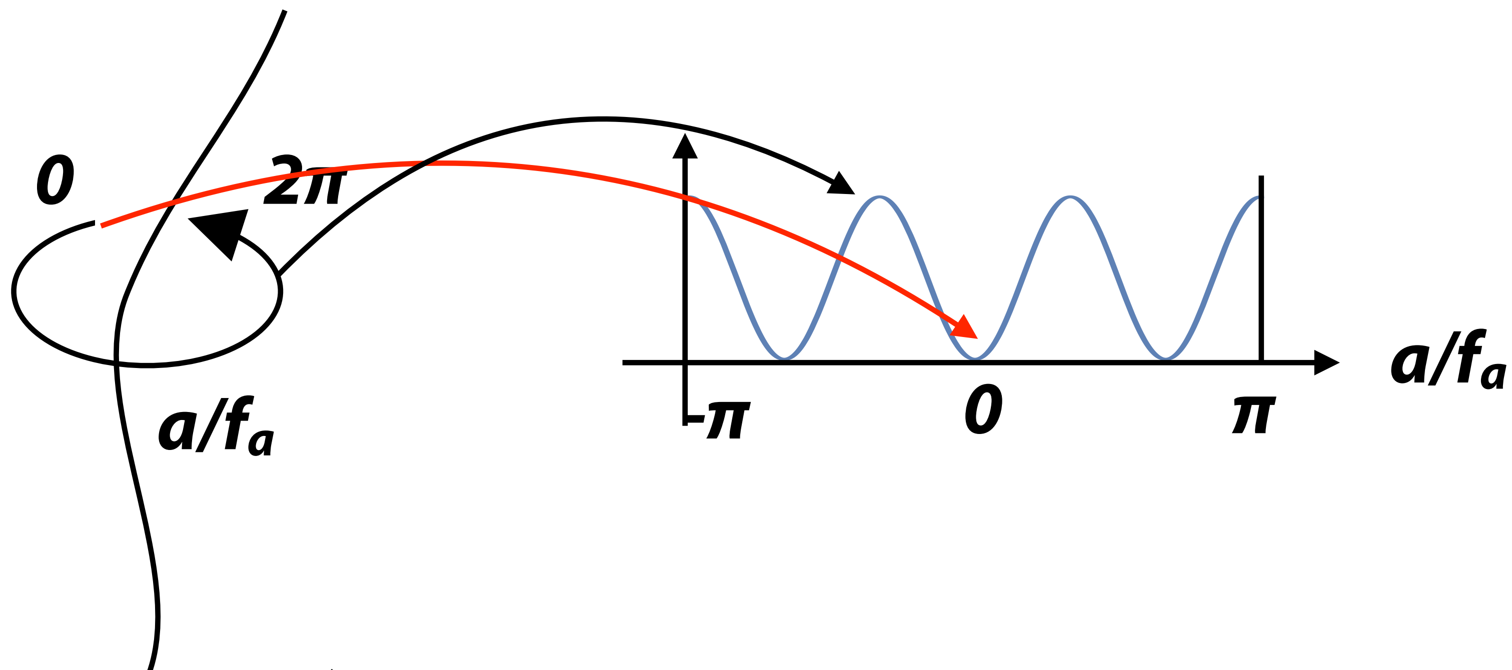


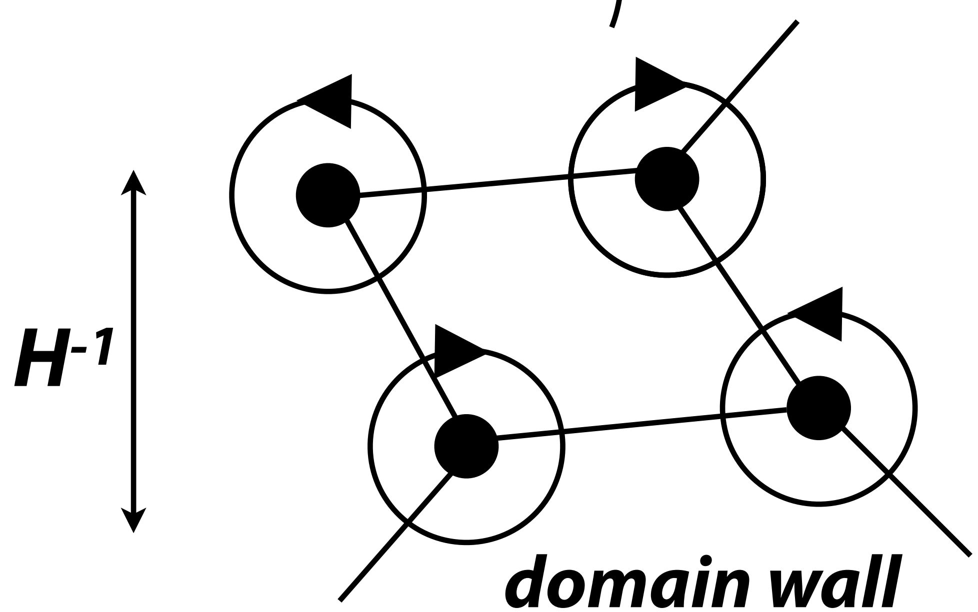
Domain Wall Problem in the conventional PQ model

Closer look at the domain wall problem :

(2) Below the QCD scale, the axion feels its axion potential.

Non-trivial axion field values around the strings causes non-uniformity of the energy density around the cosmic strings.





Non-uniform energy density is concentrated on the domain walls stretching between the cosmic strings

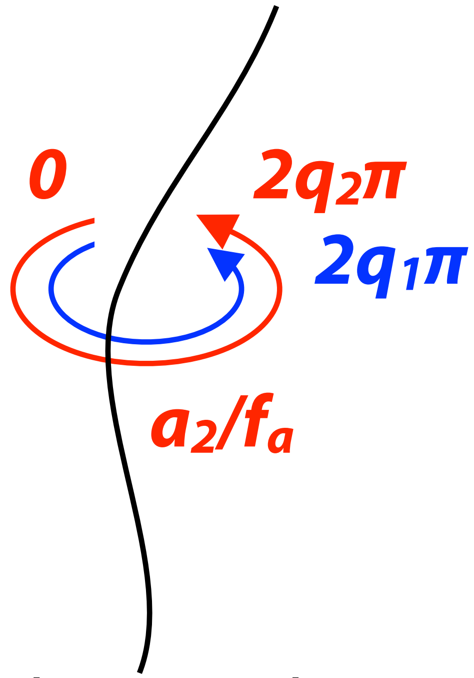
→ ***domain wall problems.***

[1012.4558 Hiramatsu et. al.]

Domain Wall Problem in the gauged PQ model

In the gauged ***PQ*** model, the genuine symmetry is not $\boldsymbol{Z_N}$ but $\boldsymbol{U(1)_{PQ}}$.

✔ Does it mean that there is no domain wall problem ?



Winding number
= (q_1, q_2)

Around the local string, only the would-be goldstone mode winds, and hence, the axion is trivial.

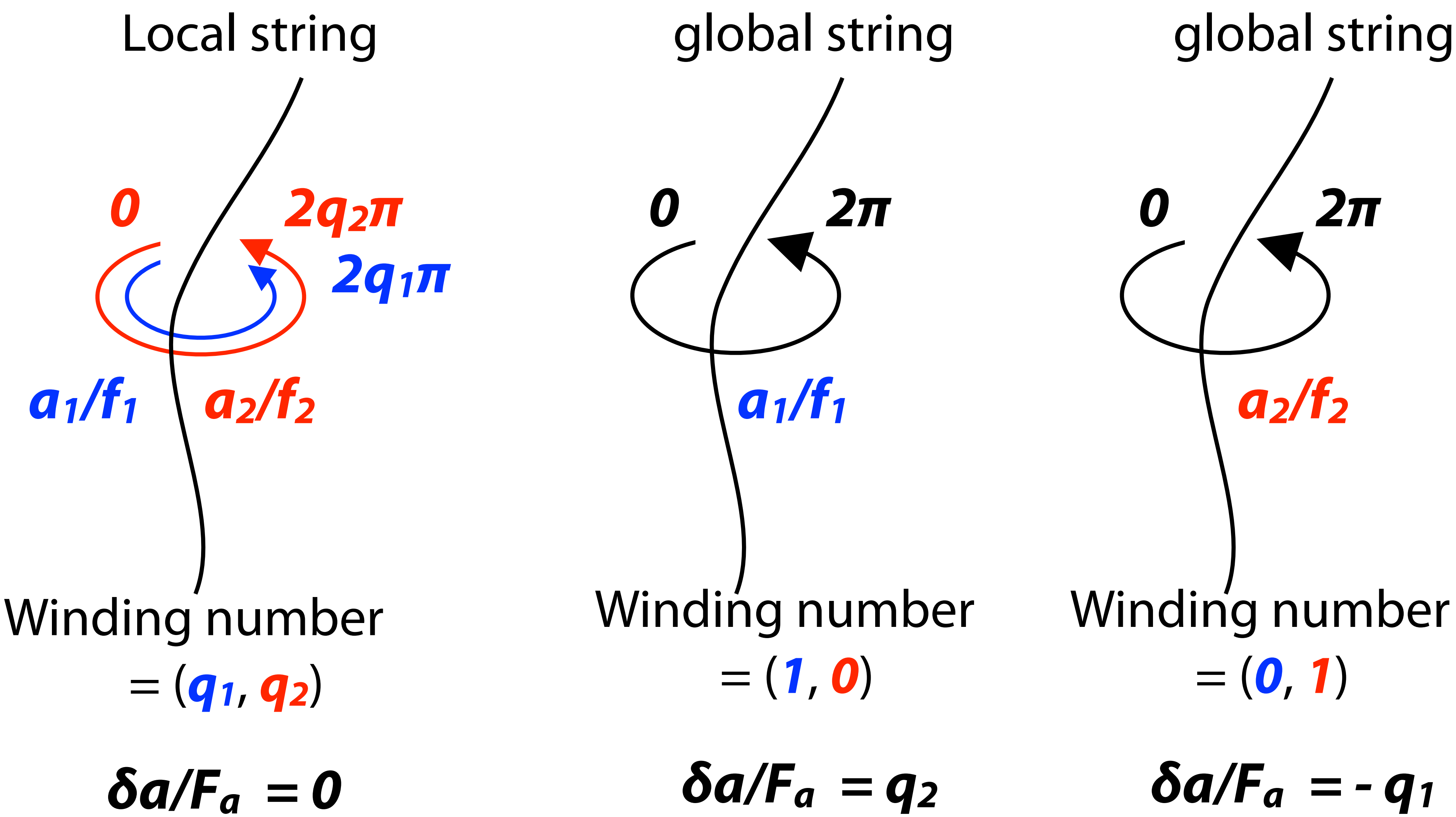
$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}} \begin{pmatrix} q_2 f_2 & -q_1 f_1 \\ q_1 f_1 & q_2 f_2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

→ Around the local string, ***no domain walls are formed*** even below the ***QCD*** scale !

Domain walls problems are solved … No Unfortunately…

Domain Wall Problem in the gauged PQ model

Even in the gauged PQ model, we could have global strings…



Once the global strings are formed in the universe, the domain walls are formed below the ***QCD*** scale unless global strings disappear which is unlikely due to the suppressed interaction between two sectors.

[The lifetime of the domain wall between the global strings are very long…]

Domain Wall Problem in the gauged PQ model

Cosmologically Safe Scenarios ?

(1) Trivial solution : ***PQ*** breaking before inflation.

- ✔ In this case, the axion field value is fixed to a single value and hence, no domain wall problem happens.

- ✔ Such scenarios predict the isocurvature fluctuation which is constrained by CMB observations.

$$\frac{\delta\rho_a}{\rho_a} \sim \frac{H_{\rm INF}}{\pi F_a} < 10^{-5} \times \frac{\Omega_{\rm DM} h^2}{\Omega_a h^2}$$ [isocurvature constraint]

$$\Omega_a h^2 = 0.18\ \theta_1^2 \left(\frac{F_a}{10^{12}{\rm GeV}}\right)^{1.19} \left(\frac{\Lambda}{400{\rm MeV}}\right)$$ Initial angle ***θ_1 = 0-2π***

$$\Omega_{\rm DM} h^2 \simeq 0.12$$ [e.g. 1301.1123 Kawasaki, Nakayama]

This scenario requires rather low scale inflation : e.g. $\boldsymbol{H_{INF} \sim 10^7 GeV}$ for $\boldsymbol{\Omega_a = \Omega_{DM}}$.

Domain Wall Problem in the gauged PQ model

Cosmologically Safe Scenarios ?

(2) Less trivial solution : ***PQ*** breaking before inflation.

Bring two independent ***KSVZ*** axion models

$$L = y_1 S_1 q_{1L} \bar{q}_{1R} + y_2 S_2 q_{2L} \bar{q}_{2R} + h.c.$$

KSVZ fermions : N_1 flavor of 1 , N_2 flavor of 9

Gauged $U(1)_{PQ}$ ***charge :*** $S_1(9)$, $S_2(-1)$

S_1-global string = axion winding number 1

S_2-global string = axion winding number 9

Arrange $\langle S_2 \rangle \gg T_R$ so that the gauged $U(1)_{PQ}$ is not restored after inflation while the global $U(1)_{PQ}$ breaking takes place after inflation.

- ✔ All the non-trivial axion winding strings are inflated away.
- ✔ No isocurvature fluctuation is generated since there is no massless mode during inflation.

No problems ! No clues…

Summary

- ***PQ*** axion models are one of the most successful solution to the strong ***CP*** problem.
- By definition, the origin of the ***PQ*** symmetry is quite puzzling…
- We propose to make the ***PQ*** symmetry a gauge symmetry by bringing multiple ***PQ*** sectors together.
  [generalization of the Barr-Seckel model]
- We can successfully construct models with high quality global ***PQ*** symmetry which is durable to the Planck suppressed ***PQ*** breaking operators !
- Even gauged ***B-L*** symmetry can solve the strong ***CP*** problem.
- Models with no domain wall problems and no isocurvature fluctuations are possible in the gauged ***PQ*** model.

Backup Slides

How about $\boldsymbol{\theta = \pi}$?

[1979 Crewther, Vecchia, Veneziano, Witten]

$$S_{\rm QCD} = \int d^4x \left(-\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} + \frac{\theta}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} + \sum_{i=1}^{N_f} \bar{q}_i (D - M) q_i \right)$$

✔ $\boldsymbol{\theta = \pi}$ can be eliminated by changing the sign of the "smallest quark mass".

(If the sign of larger masses are changed, some of the mesons get negative squared mass and the vacuum alignment is violated.)

$$\boldsymbol{m_u \to - m_u}$$

The change of the sign contradicts with the Mass relation :

$$\frac{m_d}{m_s} = \frac{m^2_{K_0} + m^2_{\pi^+} - m^2_{K^+}}{m^2_{K_0} + m^2_{K^+} - m^2_{\pi^+}} \simeq 0.053 \pm 0.002$$

$$\frac{m_u}{m_s} = \frac{2m^2_{\pi_0} - m^2_{K^0} - m^2_{\pi^+} + m^2_{K^+}}{m^2_{K_0} + m^2_{K^+} - m^2_{\pi^+}} \simeq 0.029 \pm 0.003$$

Prediction of neutron EDM

[1979 Crewther, Vecchia, Veneziano, Witten]

$$S_{\rm QCD} = \int d^4x \left(-\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} + \frac{\theta}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} + \sum_{i=1}^{N_f} \bar{q}_i (D - M) q_i \right)$$

A ***CP***-violating $\boldsymbol{\theta}$ is eliminated by an axial rotation without spoiling vacuum alignment

$$(u\,,d\,) \to \mathrm{Exp}[\,-\,i\,\theta\, Q_A\, \gamma_5]\,(u\,,d\,)$$

$$Q_A = \mathrm{Tr}[\,M^{-1}]^{-1}\, M^{-1}$$

$$\to\ L = m_u \bar{u}\, u + m_d \bar{d}\, d - i\,\theta\, \mathrm{Tr}[\,M^{-1}]^{-1}\,(\,\bar{u}\,\gamma_5 u + \bar{d}\gamma_5 d\,)$$

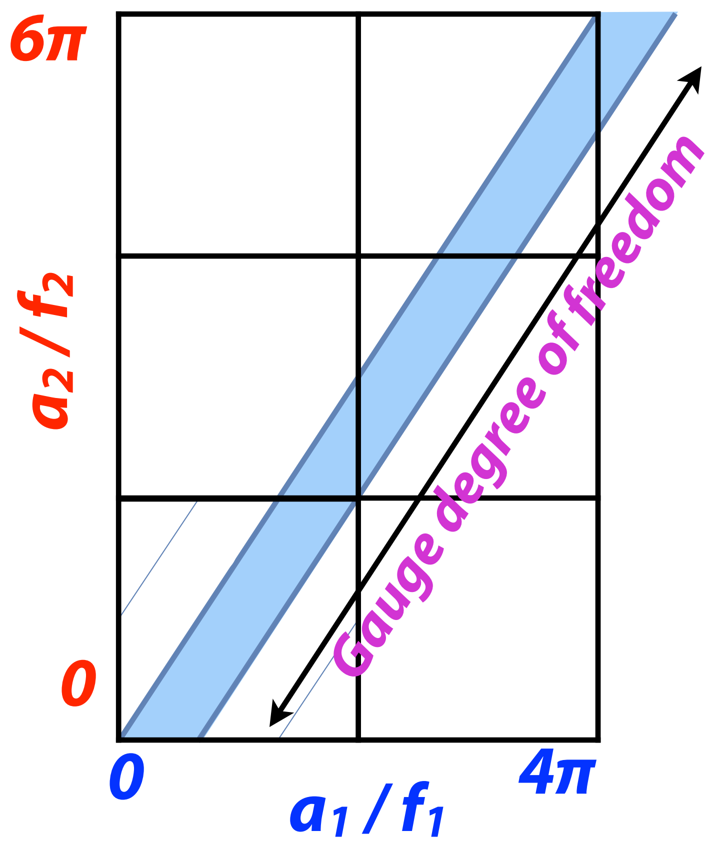
By using PCAC and baryon mass splitting

$$L = g\,\pi^a\, \bar{N} \tau^a \gamma_5 N + g'\,\pi^a\, \bar{N}\tau^a N$$

$$g' \simeq -\,\theta\, f_\pi^{-1}\, m_u\, m_d\,(m_u + m_d)^{-1}\, x\, 2(M_\Theta\, - M_\Sigma\,)/\,(2m_s - m_u - m_d\,)$$

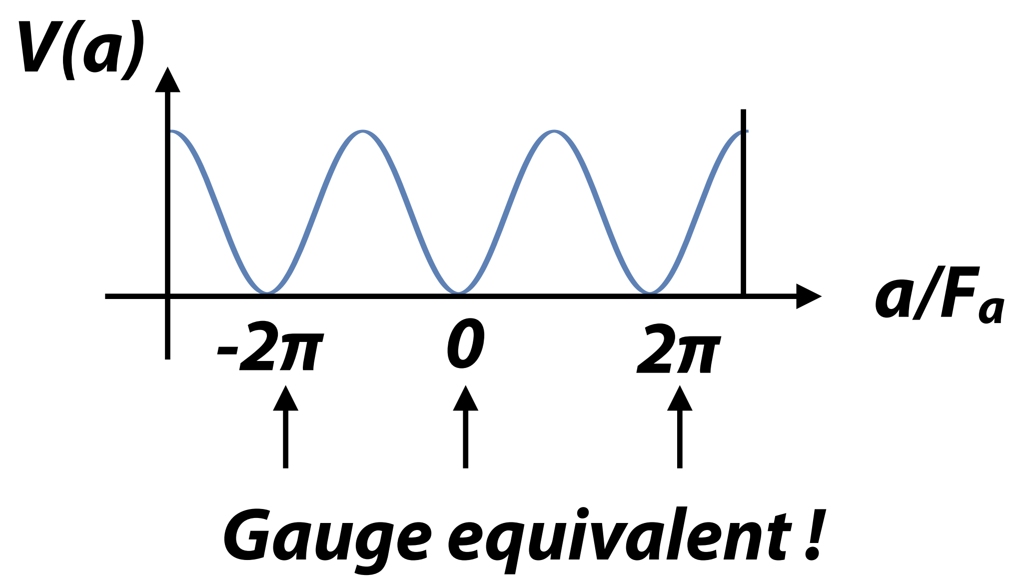
$$\simeq 0.036\theta$$

Domain Wall Problem in the gauged PQ model



For a model with $n_{GCD} = 1$, the bottoms of the axion potential are gauge equivalent !

$a/F_a = 0 - 2\pi$



There is no absolutely stable domain wall !

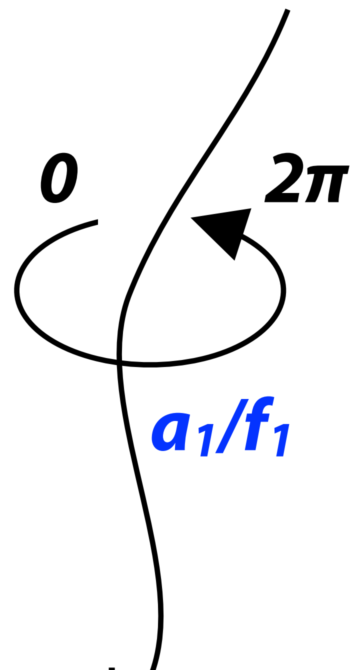
Practically, however, we have domain wall problem…

Domain Wall Problem in the gauged PQ model

ex) ***Gauged $U(1)_{PQ}$ charge : $S_1(3)$, $S_2(-1)$***

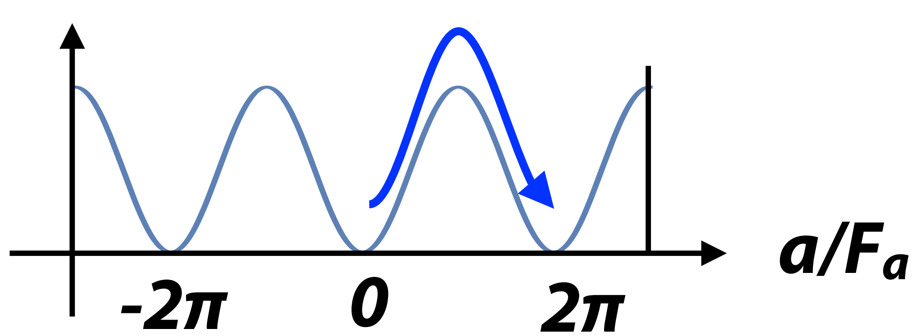
S_1-global string = axion winding number 1

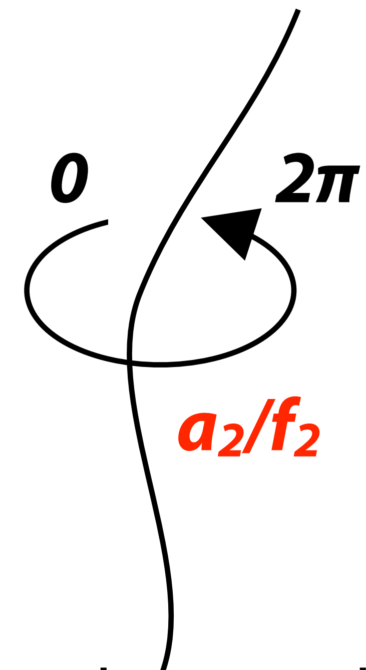
S_2-global string = axion winding number 3



Winding number
= (*1*, *0*)

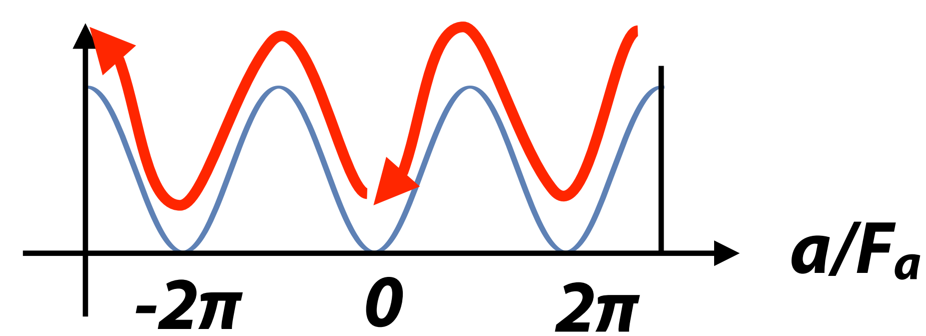
$\delta a/F_a = 1$





Winding number
= (*0*, *1*)

$\delta a/F_a = -3$



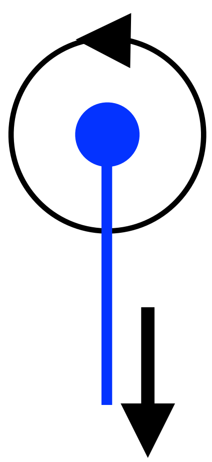
Domain Wall Problem in the gauged PQ model

ex) ***Gauged $U(1)_{PQ}$ charge : $S_1(3)$, $S_2(-1)$***

Below the ***QCD*** scale, the non-uniformity of the energy density leads to the domain wall formation.

S_1-domain wall

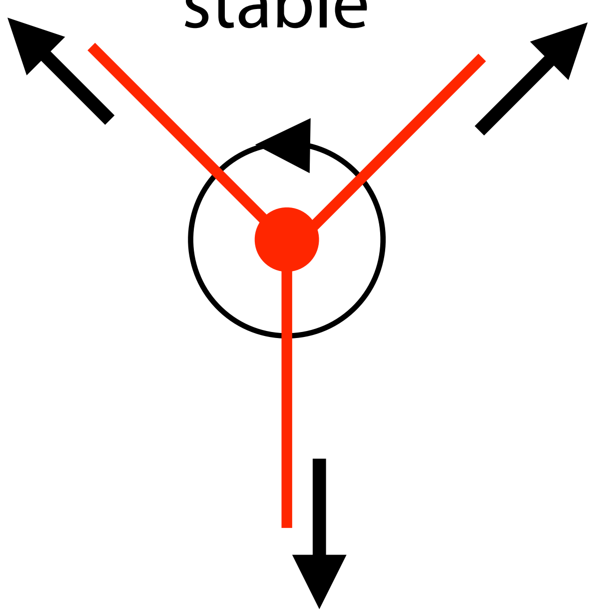
unstable



Pulled by other strings
in another Hubble patch

S_2-domain wall

stable



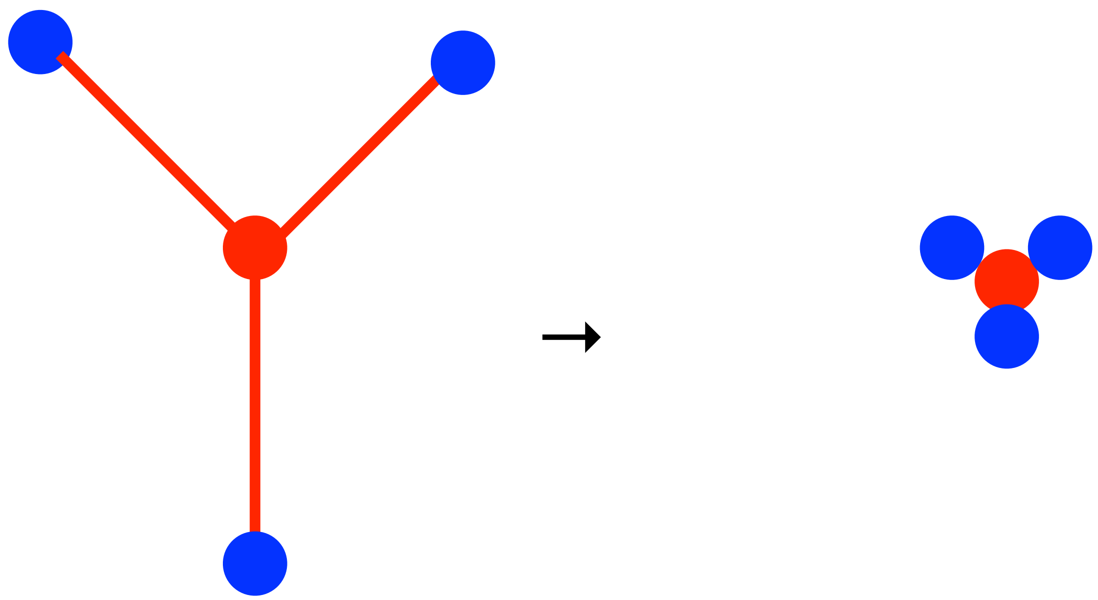
Pulled by other strings
in another Hubble patch

Since the domain wall formation is at very low energy, the walls do not know whether there were gauge bosons!

Domain Wall Problem in the gauged PQ model

ex) ***Gauged $U(1)_{PQ}$ charge : $S_1(3)$, $S_2(-1)$***

Some lucky domain walls can annihilate into composite strings

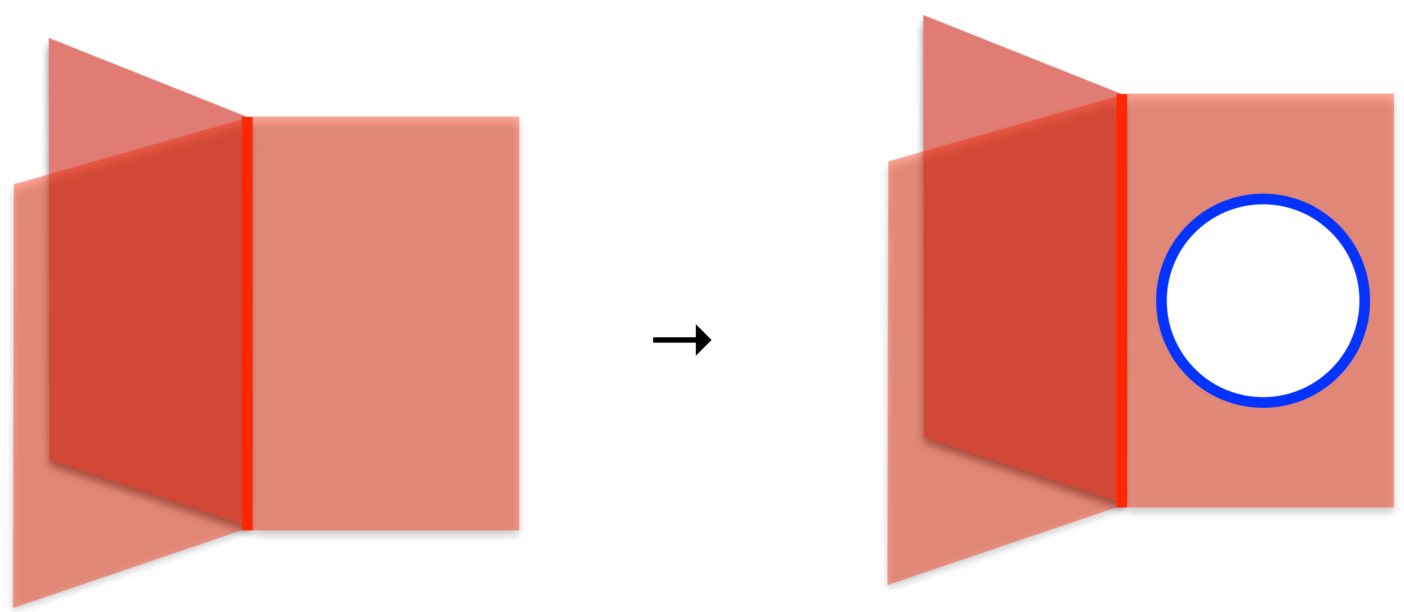


It is however difficult to imagine that all the walls annihilates away successfully, since the strings are typically separated by the Hubble length…

Domain Wall Problem in the gauged PQ model

ex) ***Gauged $U(1)_{PQ}$ charge : $S_1(3)$, $S_2(-1)$***

S_2 domain wall can be pierced by the S_1 string.



The tunneling rate is quite low…

$$\Gamma \propto Exp[\, - F_a^3 / \Lambda_{QCD}^2\, T\,] \sim Exp[- 10^{10}]$$

[1982, Kibble, Lazarides, Shafi]

The domain walls are almost stable…