Higgs boson as gauge fields between discrete spaces

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Higgs boson as gauge fields between discrete spaces

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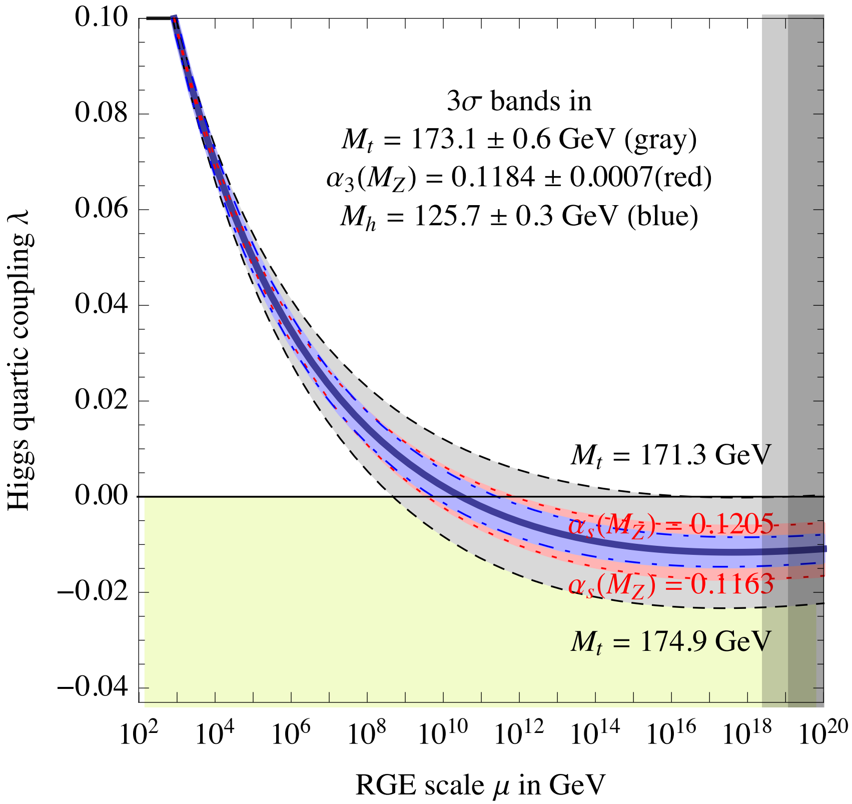
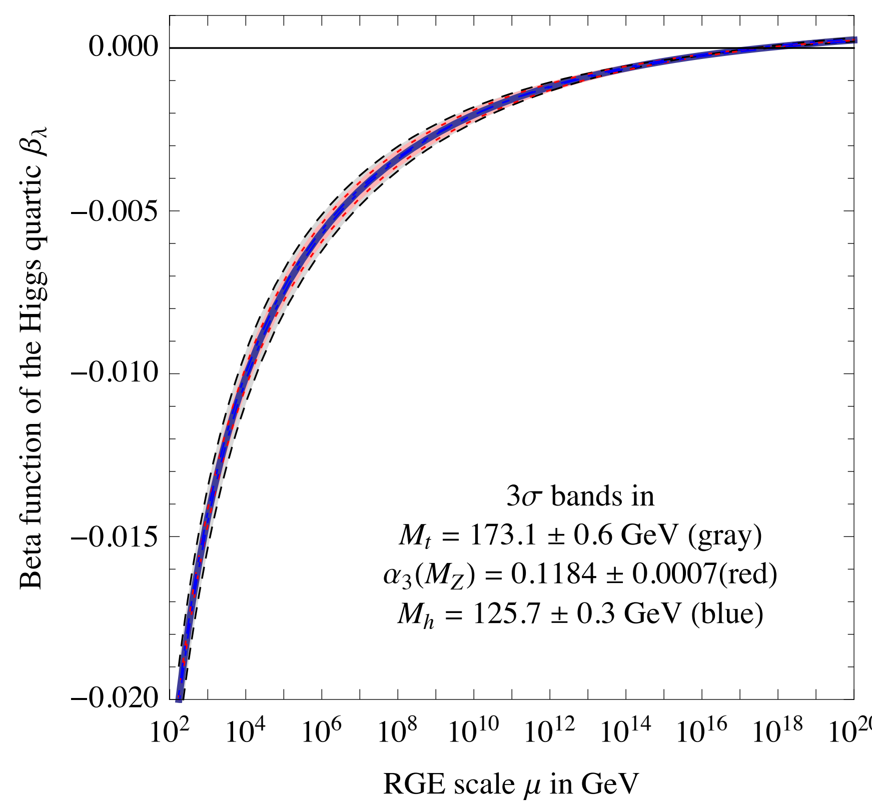
1. Introduction

Current condition of Higgs boson

$m_h = 125\text{GeV}$ & SM RGE

Buttazzo et al., 1307.3536, 380 cited





Current condition of Higgs boson

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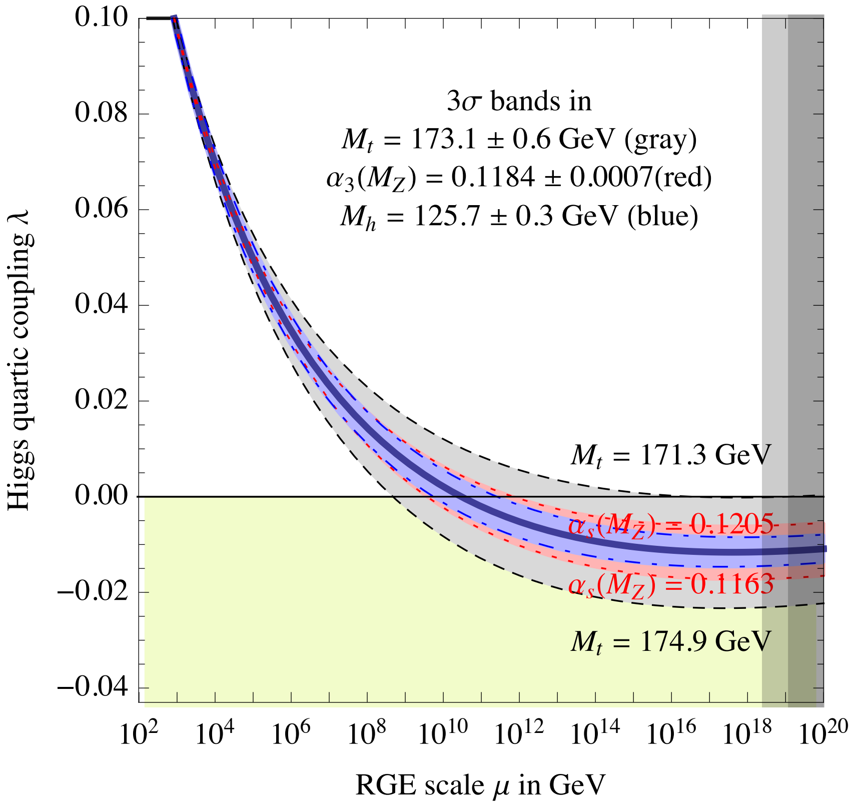

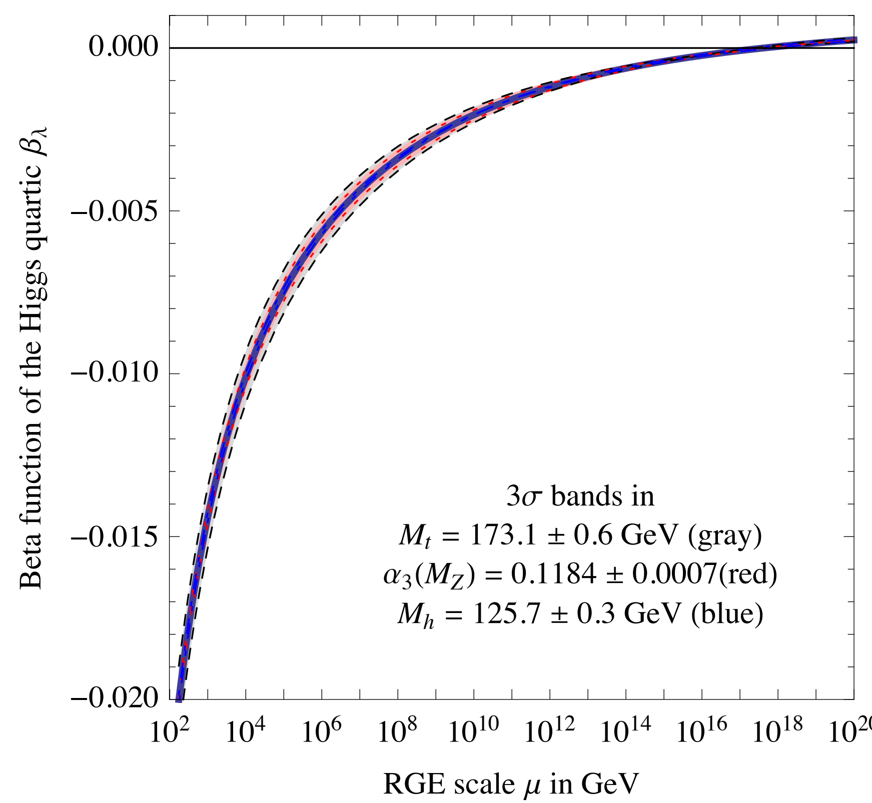

$$\boldsymbol{\lambda(M_{Pl}) \doteqdot 0 \ \& \ \beta_{\lambda}(M_{Pl}) \doteqdot 0}$$

What does it mean?

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- Multiple critical point principle Froggatt & Nielsen, '95, '01
  - $\lambda(M_{Pl}) = \beta_\lambda(M_{Pl}) = 0$
- Asymptotic safety of gravity Shaposhnikov, Wetterich, '10
- Extended SM w/ CCI Hempfling, '96
  - Recently linked with the Bardeen's argument

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Bardeen's argument

Bardeen '95

Alternative solutions to the Hierarchy problem

$$m^2(\mu) = m_0^2 + c\,\Lambda^2 + c'\log(\mu/\Lambda)$$

Bardeen’s argument

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Alternative solutions to the Hierarchy problem

(125 GeV)^2 (2×10^{18} GeV)^2

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RGE of the SM Higgs mass

$$\frac{dm^2}{dt} = \frac{m^2}{16\pi^2}\left(12\lambda_H + 6Y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g_Y^2\right)$$

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Classical conformal invariance (CCI) @ Λ, $m^2(\Lambda) = 0$

- Symmetry breaking by Coleman-Weinberg mechanism.
- Since $m_H < 10$ GeV in the SM, **some extension is required.**
- CCI is broken by other sector and transmitted to Higgs.

Extended SM w/ CCI

- There are several extensions of SM
  - SM + singlet scalar, 2HDM, LR model, etc....

- This study shows
  - YMH model in NCG (NonCommutative Geometry)

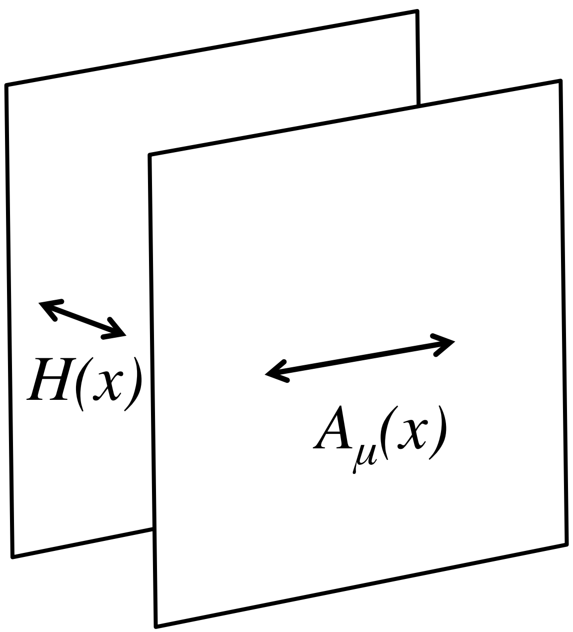
    $\Rightarrow$ SM w/ CCI
  - The possibility the hierarchy problem is solved in the context of NCG.

2. Yang-Mills-Higgs model in noncommutative geometry

Yang-Mills-Higgs model in NCG

Connes & Lott, 1990, 250 cited **(Market size is so small !)**

Ex) $M_4 \times Z_2$ model



The Higgs boson

=

A Gauge boson

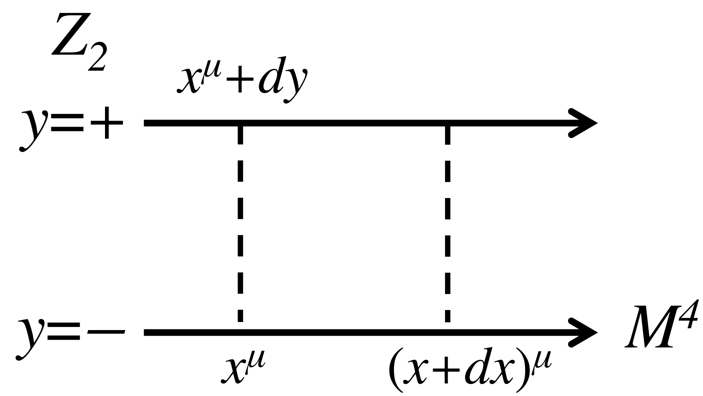
between the noncommutative
discrete extra dimension

The differential algebra

K. Morita and Y. Okumura, PTP. 91, 975 (1994),
Y. Okumura, PTP. 96, 1021 (1996),

- Generalized derivative

$$\boldsymbol{d}f \equiv df + d_5 f \equiv \partial_\mu f dx^\mu + [M, f]dy,$$



Ex) $M_4 \times Z_2$ model (matrix rep)

$$f = \begin{pmatrix} f_+ & 0 \\ 0 & f_- \end{pmatrix}, \quad M = \begin{pmatrix} 0 & M \\ M^\dagger & 0 \end{pmatrix},$$

$$\boldsymbol{d}f = \begin{pmatrix} \partial_\mu f_+ & 0 \\ 0 & \partial_\mu f_- \end{pmatrix} dx^\mu + \begin{pmatrix} 0 & Mf_- - f_+M \\ M^\dagger f_+ - f_- M^\dagger & 0 \end{pmatrix} \begin{pmatrix} dy_1 & 0 \\ 0 & dy_2 \end{pmatrix},$$

Wedge products

$$dx^\mu \wedge dx^\nu = -dx^\nu \wedge dx^\mu, \quad dx^\mu \wedge dy_n = -dy_n \wedge dx^\mu, \quad dy_m \wedge dy_n \neq dy_n \wedge dy_m \neq 0,$$

The differential algebra

- Generalized derivative

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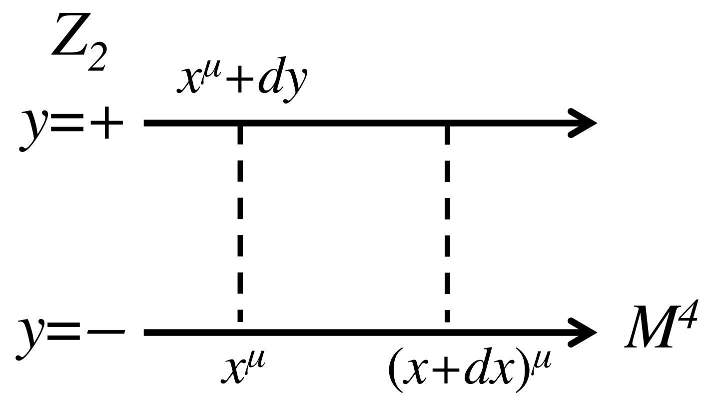

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"Difference" between two points

$$\boldsymbol{d}f = \begin{pmatrix} \partial_\mu f_+ & 0 \\ 0 & \partial_\mu f_- \end{pmatrix} dx^\mu + \begin{pmatrix} 0 & \underline{Mf_- - f_+M} \\ \underline{M^\dagger f_+ - f_-M^\dagger} & 0 \end{pmatrix} \begin{pmatrix} dy_1 & 0 \\ 0 & dy_2 \end{pmatrix},$$

Wedge products

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Composite / elemental scheme

Gauge and Higgs boson is treated in several ways

- Composite scheme (original) Connes & Lott, 1990, 250 cited

$$\boldsymbol{A}_{nm}(x) = \sum_i a_n^i(x) \boldsymbol{d}_{nm} b_m^i(x) \equiv A_n(x)\delta_{nm} + \Phi_{nm}(x)dy,$$

For the consistency between Higgs interpretation & NCG algebra.

- Elemental scheme Coquereaux, Esposito-Farese, Vaillant, 1991, 180 cited

$$\boldsymbol{A}(x) = \boldsymbol{A}_M(x)dx^M = A_\mu(x)dx^\mu + H(x)dy = A(x) + H(x)dy.$$

It works only in limited situation.

Comparison btw composite & elemental

Difference is in only $d_5A_5 \in F = dA + A \wedge A$.

- Composite $$d_5 A_5 = \sum_i d_5 a_n^{i\dagger}\, d_5 a_n^i$$
$$= \sum_i (M_{nm} a_m^{i\dagger} - a_n^{i\dagger} M_{nm})\,(M_{ml} a_l^i - a_m^i M_{ml}) dy_m \wedge dy_l$$
$$= \sum_{m \neq n, l \neq m} (M_{nm}\Phi_{ml} \underline{+} \Phi_{nm} M_{ml} + X_{nml}) dy_m \wedge dy_l,$$

- Elemental $$d_5 A_5 = d_5 \Phi_{nm} dy_m = [M_{nm}\Phi_{ml} \underline{-} \Phi_{nl} M_{lm}] dy_m \wedge dy_l,$$

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- Elemental $d_5 A_5 = d_5 \Phi_{nm} dy_m = [M_{nm}\Phi_{ml} \underline{-} \Phi_{nl} M_{lm}] dy_m \wedge dy_l,$

$$\boldsymbol{dA} + \boldsymbol{A} \wedge \boldsymbol{A} \ni (M_{nm}\Phi_{ml} - \Phi_{nl} M_{lm} + \Phi_{nm}\Phi_{ml}) dy_m \wedge dy_l$$

$$= [MM - \underline{(M+\Phi)(M-\Phi)}] dy_m \wedge dy_l,$$

It might not be interpreted as Higgs potential…

The composite scheme

- Generalized derivative

K. Morita and Y. Okumura, PTP. 91, 975 (1994),
Y. Okumura, PTP. 96, 1021 (1996),

$$\boldsymbol{d} f \equiv df + d_5 f \equiv \partial_\mu f dx^\mu + [M, f] dy,$$

Connection

$$\boldsymbol{A}_{nm}(x) = \sum_i a_n^i(x) \boldsymbol{d}_{nm} b_m^i(x)$$

$$A_n(x) = \sum_i a_n^i(x) d b_n^i(x), \quad \Phi_{nm}(x) = \sum_i a_n^i(x) M_{nm} b_m^i(x) - M_{nm}.$$

Higgs field w/ vev $\quad H_{nm}(x) = \Phi_{nm}(x) + M_{nm}$

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Higgs field w/ vev $\quad H_{nm}(x) \cancel{= \Phi_{nm}(x) + M_{nm}} = 0$

We cannot construct Higgs theory without vevs!

The elemental scheme

- Generalized derivative

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$$\boldsymbol{d}f \equiv df + d_5 f \equiv \partial_\mu f dx^\mu + [M, f]dy,$$

Only when $M = 0,$ the elemental scheme works well

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$$\boldsymbol{A} = \begin{pmatrix} A_1 & H_{12} dy_2 & \cdots & H_{1N} dy_N \\ H_{21} dy_1 & A_2 & \cdots & H_{2N} dy_N \\ \vdots & \vdots & \ddots & \vdots \\ H_{N1} dy_1 & H_{N2} dy_2 & \cdots & A_N \end{pmatrix},$$

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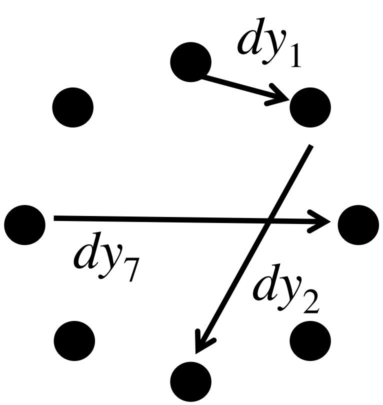

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And N→∞, It result in a M4 × S1 by **(de) construction**.

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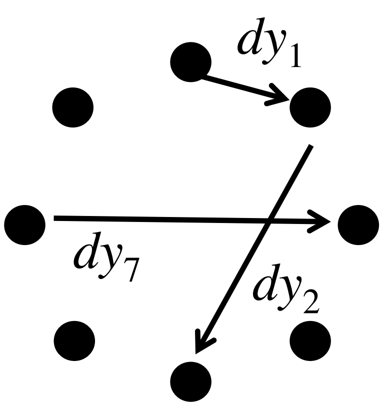

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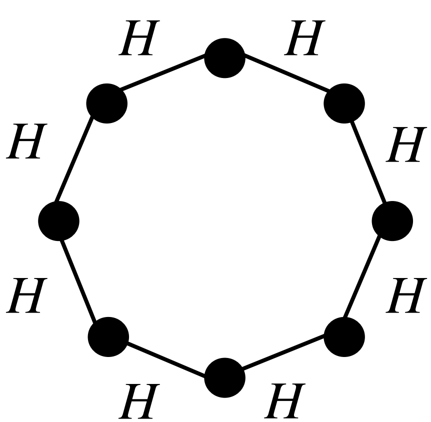

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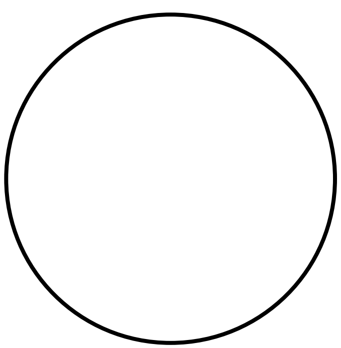

$N \rightarrow \infty$,
$M_4 \times S_1$!!

Mohsen Alishahiha,
PLB517 (2001)

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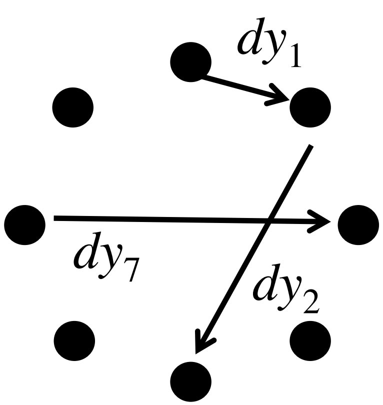
If we constrain Higgs field only **the nearest neighbor,**
And N→∞, It result in a M4 × S1 by **(de) construction**.
It can be interpreted as generalized **(de) construction**.



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Covariant derivative $\quad \boldsymbol{D} = \boldsymbol{d} + \boldsymbol{A} = (\partial_\mu + A_{n\,\mu})dx^\mu + Hdy,$

Gauge trf. (requiring $\boldsymbol{D}'G = G\boldsymbol{D}$)

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Gauge trf. (requiring $\boldsymbol{D}'G = G\boldsymbol{D}$)

Just bi-fundamental scalars !!

$$A'_{n\,\mu} = G_n A_{n\,\mu} G_n^{-1} - (\partial_\mu G_n)G_n^{-1}, \quad \underline{H'_{nm} = G_{nk} H_{kl} G_{lm}^{-1}.}$$

The gauge theory on $M_4 \times Z_N$

K. Morita and Y. Okumura, PTP. 91, 975 (1994),
Y. Okumura, PTP. 96, 1021 (1996),

The field strength

$$\begin{aligned}\boldsymbol{F} &= \boldsymbol{dA} + \boldsymbol{A} \wedge \boldsymbol{A} = (dA + dHdy) + (A + Hdy) \wedge (A + Hdy) \\ &= dA + A \wedge A + (\partial_\mu H + A_\mu H - HA_\mu)\, dx^\mu \wedge dy + Hdy \wedge Hdy,\end{aligned}$$

The bosonic Lagrangian

$$\mathcal{L}_B = -\mathrm{Tr}\langle \boldsymbol{F}^\dagger, S\boldsymbol{F}\rangle = -\sum_{n,m} \langle \boldsymbol{F}^\dagger_{nm}, S_n \boldsymbol{F}_{nm}\rangle,$$

where $S = \mathrm{diag}(g_1^{-2}E_1, g_2^{-2}E_2, \cdots g_N^{-2}E_N)$, $\langle dx^\mu \wedge dy_n, dx^\nu \wedge dy_m\rangle = -\alpha^2 \delta_{nm} g^{\mu\nu}$

$$= \sum_n \frac{1}{g_n^2} \mathrm{tr}\left[-\frac{1}{2} F^\dagger_{n\,\mu\nu} F_n^{\mu\nu} + \alpha^2 \sum_m |D_\mu H_{nm}|^2 - \alpha^4 \sum_l |H_{nl} H_{lm}|^2 \right],$$

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Just a Yang-Mills-Higgs Lagrangian (w/o mass scale) !!

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I study this system, because it is elegant !

The fermionic sector

Constructed from the generalized covariant derivative

$$\boldsymbol{D} = \boldsymbol{d} + \boldsymbol{A} = (\partial_\mu + A_{n\,\mu})dx^\mu + H dy,$$
$$\not{\boldsymbol{D}} = D_M \Gamma^M = (\partial_\mu + A_{n\,\mu})\gamma^\mu + H i\gamma^5,$$

where $\Gamma^M = (\gamma^\mu, i\gamma^5)$ satisfies the Clifford algebra $\{\Gamma^M, \Gamma^N\} = 2g^{MN}$.

The fermion fields $\Psi = (\psi_1, \psi_2, \cdots, \psi_N)^T, \quad \bar{\Psi} = (\bar{\psi}_1, \bar{\psi}_2, \cdots, \bar{\psi}_N),$

The fermionic Lagrangian

$$\boxed{\mathcal{L}_F = \bar{\Psi} i \not{\boldsymbol{D}} \Psi = \sum \bar{\psi}_n i[(\partial_\mu + A_{n\,\mu})\delta_{nm}\gamma^\mu + H_{nm} i\gamma^5]\psi_m,}$$

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In the original context by Connes,
Yukawa interactions are introduced **by hand** $dy \to Y_{u,d,e} dy$

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This is the basic formalization of the YMH model in NCG.

Worries, concerns, issues

- Why they became forgotten theories ...??

1. **Higgs fields as Yang-Mills fields and discrete symmetries**
Robert Coquereaux, Gilles Esposito-Farese, G. Vaillant (Marseille, CPT). Jun 1990. 18 pp.
Published in **Nucl.Phys. B353 (1991) 689-706**
CPT-90/P-2407
DOI: 10.1016/0550-3213(91)90323-P
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
CERN Document Server ; CERN Library Record
レコードの詳細 - Cited by 181 records 100+

One of the first paper of the Matrix formalism,

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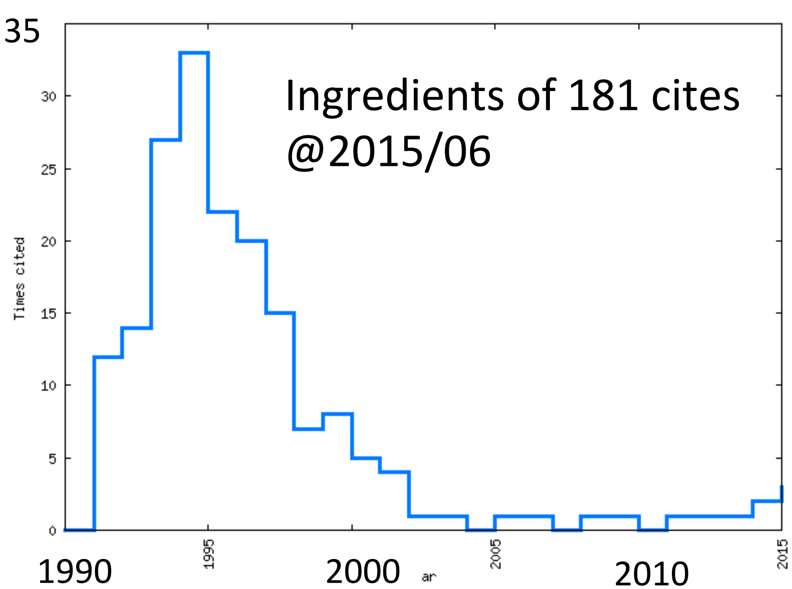

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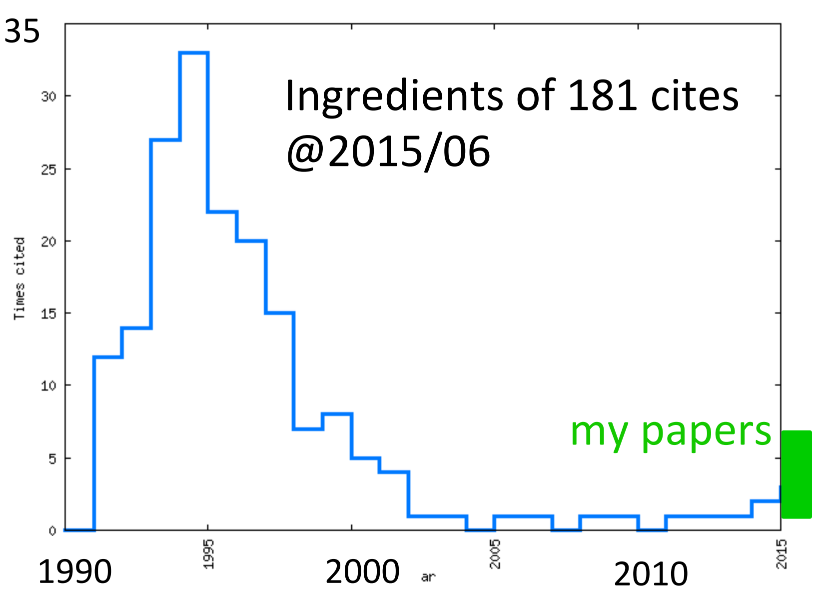

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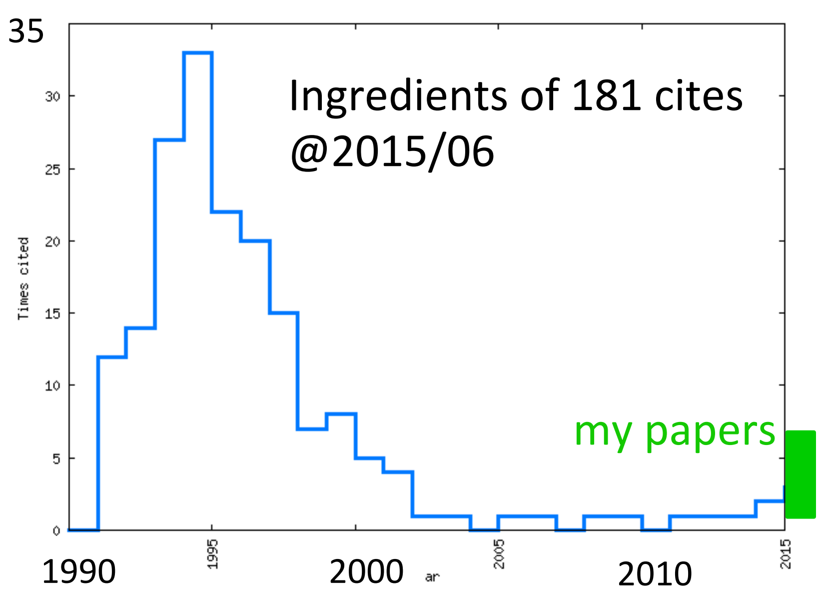

Something wrong proved ?

- Yukawa interactions
- Quantum corrections
- Higgs quadratic divergence

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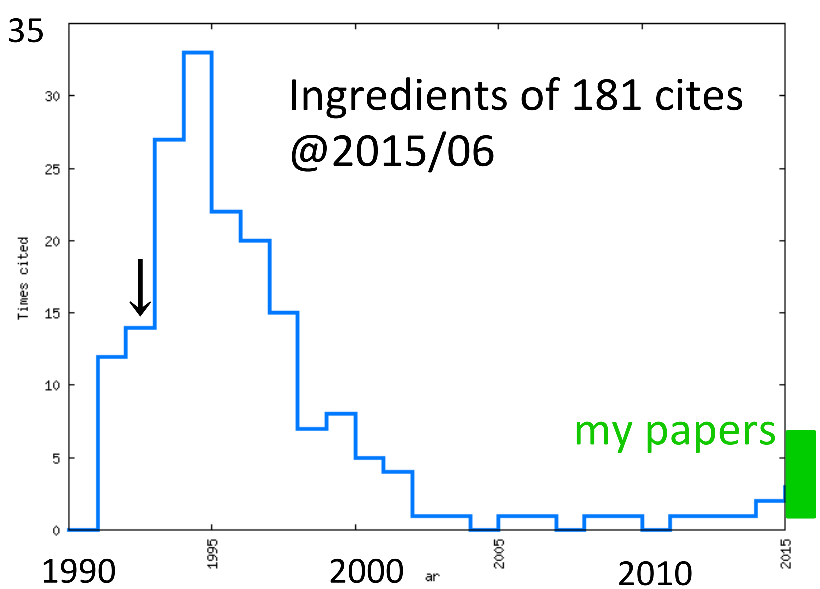

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Perhaps, one reason is that
The mass relation is found not to be RGE inv.

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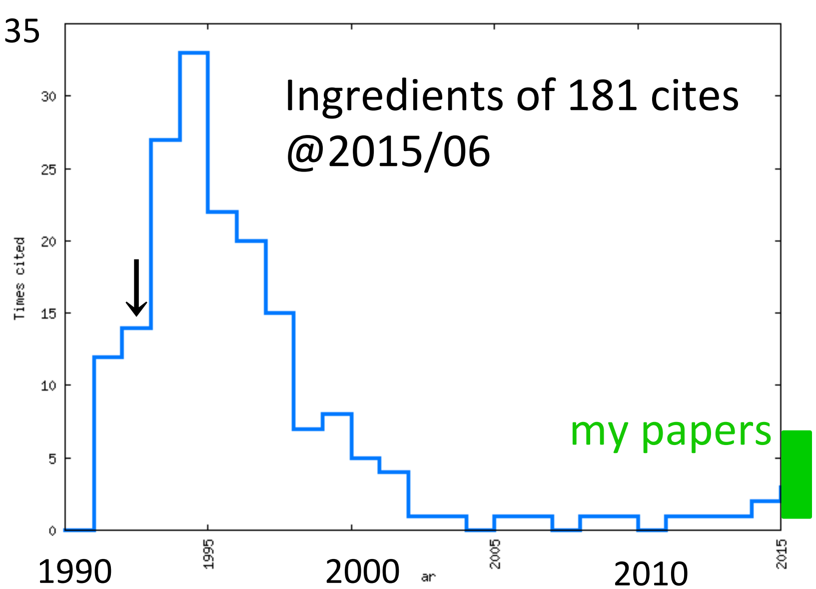

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Renormalization Group Equation analysis

Y. Okumura, hep-ph/9707350

- We can search unification scale from observed parameter (g_i, $m_{Z,W,t}$) and SM RGEs

Turn back the scale

$$\lambda(\Lambda) = \frac{g_2^2(\Lambda)}{4} = \frac{g_1^2(\Lambda)}{4}, \quad \Rightarrow \quad m_h = \sqrt{2\lambda(m_h)}v(m_h).$$

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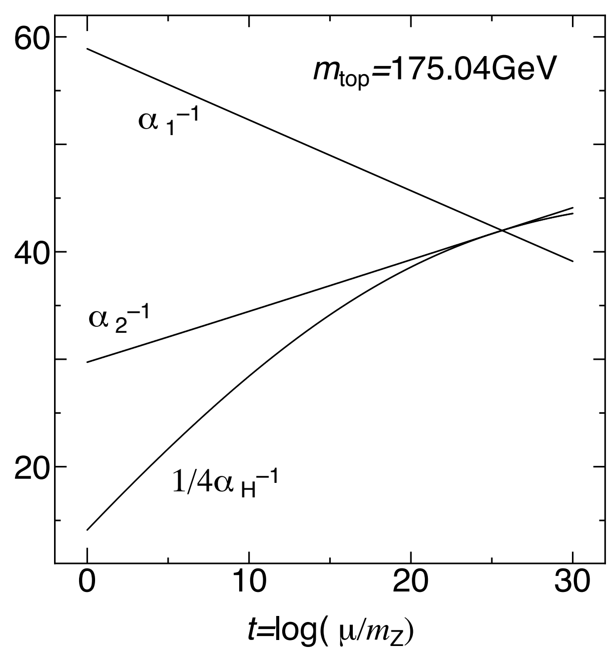

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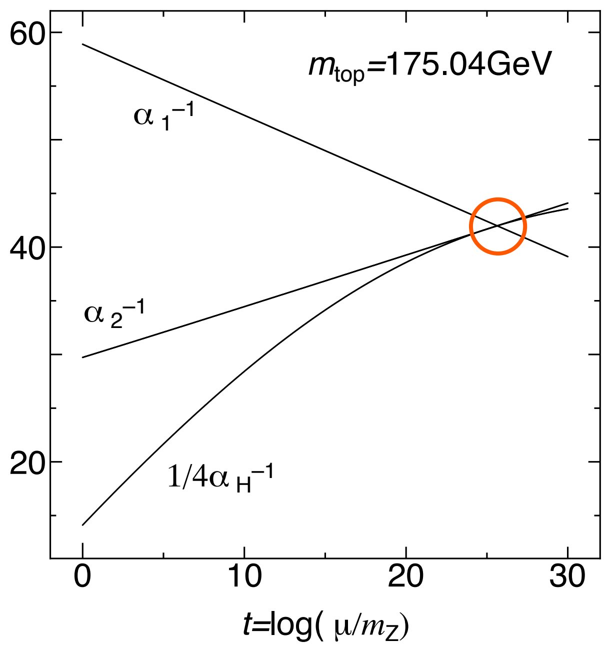

With the 2-loop SM RGEs,

$$\mu = 3.37 \times 10^{13}\mathrm{GeV}.$$

$$m_{\mathrm{H}} = 158.18\mathrm{GeV} \text{ for } m_{\mathrm{top}} = 175.04\mathrm{GeV}$$

The Spectral Action Principle

Ali H. Chamseddine, Alain Connes,
Commun.Math.Phys. 186 (1997) 731-750, hep-th/9606001 **370 times cited**

The current most popular approach in this context,
the constrcution of $\mathcal{L}$ including Gravity and Higgs boson

- Yukawa interactions $\Rightarrow$ those of SMs
- Quantum corrections $\Rightarrow$ SM RGEs
- Quadratic Divergence $\Rightarrow$ still does not solved.

Connes said on 1004.0464 on 1208.1030

$$m_h \sim 170\mathrm{GeV} \quad \Rightarrow \quad 125\mathrm{GeV}$$

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However, 5 dimensional quantum correction is not discussed!
(perhaps they somewhat gave up the 5 dim. picture.)

170 GeV → 126 GeV, 1208.1030

From Pierre Martinetti

The new field σ and the Higgs mass

Spectral action requires a unique unification scale. With $\Lambda = 10^{17}$GeV, the running of the Higgs quartic selfcoupling λ_H under the big desert hypothesis yields

$$\lambda_0(\Lambda) = \frac{16}{3}\pi\alpha_3(\Lambda) = 0.356 \quad \Rightarrow \quad m_H \simeq 170 \text{ GeV}.$$

RGE running

Generally NCG Higgs model predicts $\lambda \sim g^2$ @ Λ,

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RGE running

A new scalar field σ - that lives at high energy and gives mass to the neutrinos - has been introduced by phenomenologists† to solve some instability due to the low mass of the Higgs (radiative corrections may drive λ_H negative and destabilize the electroweak vacuum):

$$V(H,\sigma) = \frac{1}{4}(\lambda_H H^4 + \lambda_\sigma \sigma^4 + 2\lambda_{H\sigma} H^2\sigma^2).$$

As a bonus, it pulls m_H back to 126 GeV.

Resilience of the spectral SM, Chamseddine, Connes 2012

Is σ natural in NCG, or is it just an artifact for solving the model ?

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$$V(H,\sigma) = \frac{1}{4}(\lambda_H H^4 + \lambda_\sigma \ldots$$

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Resilience of the spectral SM, Chamseddine, Connes 2012

Actually, even this result Depends several assumptions

Is σ natural in NCG, or is it just an artifact for solving the model ?

3. Reconstruction of the SM with classical confomal invariance in NCG

arXiv:1510.04783   PTEP 2016 ******

Masaki J. S. Yang

Reconstruction of SM

Here, we consider **only one generation** for simplicity.

CAUTION : The following construction contains many **by hand** procedure.

(However, there are not much difference from the original one.)

Reconstruction of SM

- The extended connection on $M^4 \times Z_2$

$$\begin{array}{ccc} \mathrm{SU}(3) & = & \mathrm{SU}(3) \\ \mathrm{SU}(2)\times\mathrm{U}(1)_{\mathrm{B\text{-}L}} & & \mathrm{U}(1)_{\mathrm{B\text{-}L+2I_R{}^3}} \\ \bullet & \xleftarrow[H]{\qquad\qquad} & \bullet \\ q_L, l_L & & q_R, l_R \end{array}$$

where

$$\boldsymbol{A}(x) = A_\mu(x)dx^\mu + A_5(x)dy = \begin{array}{cl} \begin{pmatrix} A_L & H dy_R \\ H^\dagger dy_L & A_R \end{pmatrix} & \begin{array}{l}]\ 8 \\ ]\ 8 \end{array} \\ \quad\ \underbrace{}_{8} \qquad\quad \underbrace{}_{8} & \end{array}$$

Gauge trf.

$$\boldsymbol{A}' = \begin{pmatrix} G_L & 0 \\ 0 & G_R \end{pmatrix} \begin{pmatrix} A_L & H dy_R \\ H^\dagger dy_L & A_R \end{pmatrix} \begin{pmatrix} G_L^{-1} & 0 \\ 0 & G_R^{-1} \end{pmatrix} + \begin{pmatrix} dG_L \cdot G_L^{-1} & 0 \\ 0 & dG_R \cdot G_R^{-1} \end{pmatrix}$$

Reconstruction of SM

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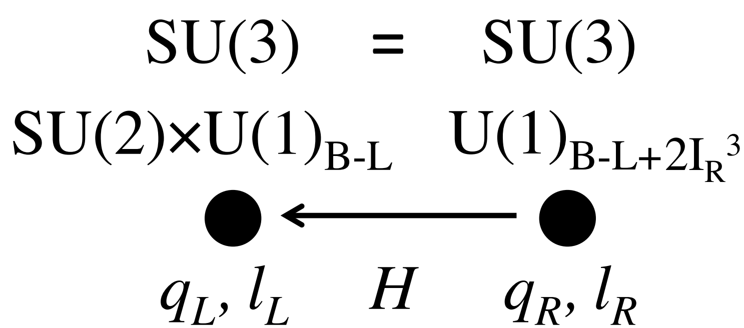

where

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Gauge trf.

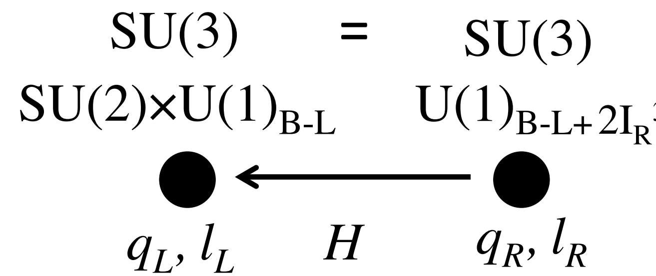
Higgs scalar transforms as a bi-fundamental

$$\boldsymbol{A}' = \begin{pmatrix} G_L & 0 \\ 0 & G_R \end{pmatrix} \begin{pmatrix} A_L & H dy_R \\ H^\dagger dy_L & A_R \end{pmatrix} \begin{pmatrix} G_L^{-1} & 0 \\ 0 & G_R^{-1} \end{pmatrix} + \begin{pmatrix} dG_L \cdot G_L^{-1} & 0 \\ 0 & dG_R \cdot G_R^{-1} \end{pmatrix}$$

The property of the Higgs

- Gauge trf. of Higgs

$$H' = G_L H G_R^{-1} \quad \sim \; (\mathbf{1}+\mathbf{8}, \mathbf{2}, \pm 1/2)$$



Although It is interesting possibility, A. Manohar, M. Wise, ph/0606172,
we omit it.

Imposing $H = h \otimes 1_4$, Color octet doublet Higgs is removed.

Then, $$H = \begin{pmatrix} H_u^0 & H_d^+ \\ H_u^- & H_d^0 \end{pmatrix} \otimes 1_4 \equiv (H_u, H_d) \otimes 1_4.$$ 2HDM

If we further impose $H_u = \tilde{H}_d, \text{ where } \tilde{H} = i\sigma^2 H^*$ 1HDM (SM)

Reconstruction of SM

- The bosonic Lagrangian

$$\boldsymbol{F} = \boldsymbol{dA} + \boldsymbol{A} \wedge \boldsymbol{A} = \begin{pmatrix} F_L + HH^\dagger \, dy_R \wedge dy_L & D_\mu H \, dx^\mu \wedge dy_R \\ D_\mu H^\dagger \, dx^\mu \wedge dy_L & F_R + H^\dagger H \, dy_L \wedge dy_R \end{pmatrix}$$

$$\begin{aligned}\mathcal{L}_B = -\mathrm{tr}\langle \boldsymbol{F}^\dagger, S\boldsymbol{F} \rangle = &-\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} G^\alpha_{\mu\nu} G^{\alpha\mu\nu} \\ &+ \mathrm{tr}\,(D_\mu H)^\dagger (D_\mu H) - \lambda\, \mathrm{tr}|H^\dagger H|^2,\end{aligned}$$

The gauge couplings S is chosen as commutative to SU(3)×SU(2)×U(1)

$$S = \mathrm{diag}(S_L, S_R), \qquad S_{L,R} = 1_2 \otimes (\frac{1}{3} a_{L,R}, \frac{1}{3} a_{L,R}, \frac{1}{3} a_{L,R}, b_{L,R}).$$

In this case, the relation holds

$$\boxed{\frac{4}{9} \frac{1}{g_c^2} + \frac{1}{g^2} + \frac{1}{g'^2} = \frac{2}{\lambda} \,.}$$

However, $\lambda = 0$ if we impose $dy \wedge dy = 0$ **(usual Exdim??)**.

Reconstruction of SM

- The fermionic Lagrangian

$$\psi(x,L) = \begin{pmatrix} u_L \\ \nu_L \\ d_L \\ e_L \end{pmatrix}, \quad \psi(x,R) = \begin{pmatrix} u_R \\ 0 \\ d_R \\ e_R \end{pmatrix}.$$

$$\begin{aligned} \boldsymbol{D} &= \boldsymbol{d} + \boldsymbol{A} = (\partial_\mu + A_{n\,\mu})dx^\mu + H dy, \\ \not{\boldsymbol{D}} &= D_M \Gamma^M = (\partial_\mu + A_{n\,\mu})\gamma^\mu + H i \gamma^5, \end{aligned}$$

$$\mathcal{L}_F = \bar{\Psi} i \Gamma^M \boldsymbol{D}_M \Psi = \begin{pmatrix} \bar{\psi}_L & \bar{\psi}_R \end{pmatrix} \begin{pmatrix} i\not{\boldsymbol{D}}_L & \sqrt{\lambda} H \\ \sqrt{\lambda} H^\dagger & i\not{\boldsymbol{D}}_R \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$y = \sqrt{\lambda} \sim g$, (not bad for 3rd generations ?)

Reconstruction of SM

- The fermionic Lagrangian

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$y = \sqrt{\lambda} \sim g$, (not bad for 3rd generations ?)

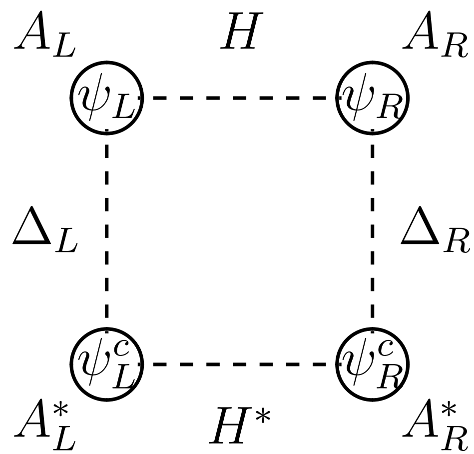
The SM (w/o Higgs vev) is reconstructed.

The Coleman – Weinberg mechanism does not work in the SM.

Proper extension is required for a viable model.

Prospects of extended SM

- $\mathrm{SU}(3)_{\mathrm{c}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}} \times \mathrm{U}(1)_{\mathrm{B\text{-}L}}$ in $M^4 \times Z_2 \times Z_2$ model



$$\Delta_L : (\mathbf{1}, \mathbf{3}+\mathbf{1}, 1, 2), \quad \Delta_R : (\mathbf{1}, \mathbf{1}, 0, 2), \quad H : (\mathbf{1}, \mathbf{2}, \pm 1/2, 0)$$

We impose some symmetry or **ad hoc constraints**…
Ex) $\Delta_L = 0$

$$\boldsymbol{F} = \boldsymbol{dA} + \boldsymbol{A} \wedge \boldsymbol{A}, \quad \mathcal{L} \sim \mathrm{Tr} \boldsymbol{F}^* \boldsymbol{F} \ni V = \lambda_H |H|^4 + \lambda_{\Delta_R} |\Delta_R|^4 + \lambda_{\mathrm{mix}} |H|^2 |\Delta_R|^2,$$

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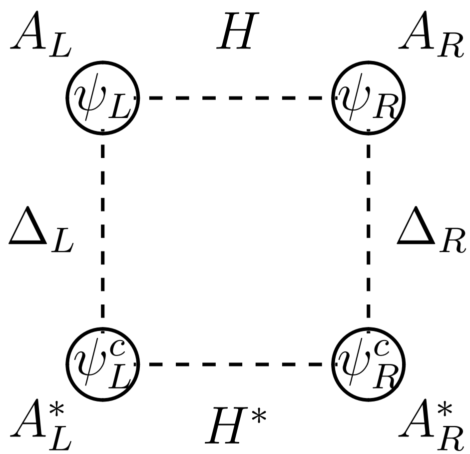

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The extended Higgs potential required successful SSB

$$\boldsymbol{F} = \boldsymbol{dA} + \boldsymbol{A} \wedge \boldsymbol{A}, \quad \mathcal{L} \sim \mathrm{Tr}\boldsymbol{F}^* \boldsymbol{F} \ni V = \underline{\lambda_H |H|^4 + \lambda_{\Delta_R} |\Delta_R|^4 + \lambda_{\mathrm{mix}} |H|^2 |\Delta_R|^2},$$

Prospects of extended SM

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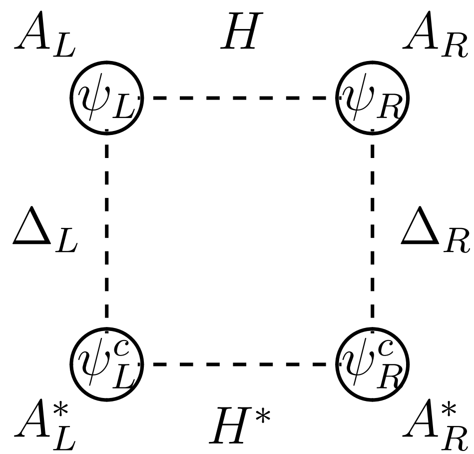

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We impose some symmetry or **ad hoc constraints**…

(recently Δ_L model with CCI is constructed)

Higgs Triplet Model with CCI, Okada, Orikasa, Yagyu, 1510.00799

$$\boldsymbol{F} = \boldsymbol{dA} + \boldsymbol{A} \wedge \boldsymbol{A}, \quad \mathcal{L} \sim \mathrm{Tr}\boldsymbol{F}^*\boldsymbol{F} \ni V = \underline{\lambda_H |H|^4 + \lambda_{\Delta_R} |\Delta_R|^4 + \lambda_{\mathrm{mix}} |H|^2 |\Delta_R|^2,}$$

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- Ex) **The (partial) flat potential** $\lambda_H = \lambda_{\mathrm{mix}} = 0$ Iso, Orikasa, '13,

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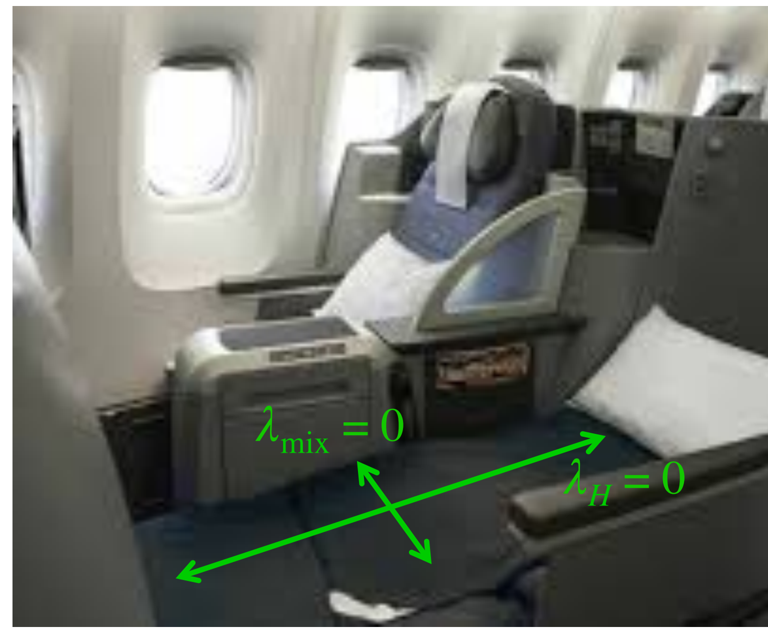
$$\boldsymbol{A} = A + \Phi dy + \Delta dz,$$

$$\ni \begin{pmatrix} 0 & \Phi & 0 & 0 \\ \Phi^\dagger & 0 & 0 & 0 \\ 0 & 0 & 0 & \Phi^* \\ 0 & 0 & \Phi^T & 0 \end{pmatrix} dy + \begin{pmatrix} 0 & 0 & \Delta_L & 0 \\ 0 & 0 & 0 & \Delta_R \\ \Delta_L^\dagger & 0 & 0 & 0 \\ 0 & \Delta_R^\dagger & 0 & 0 \end{pmatrix} dz,$$

If we impose

$$dy \wedge dy = dy \wedge dz = 0, ??$$



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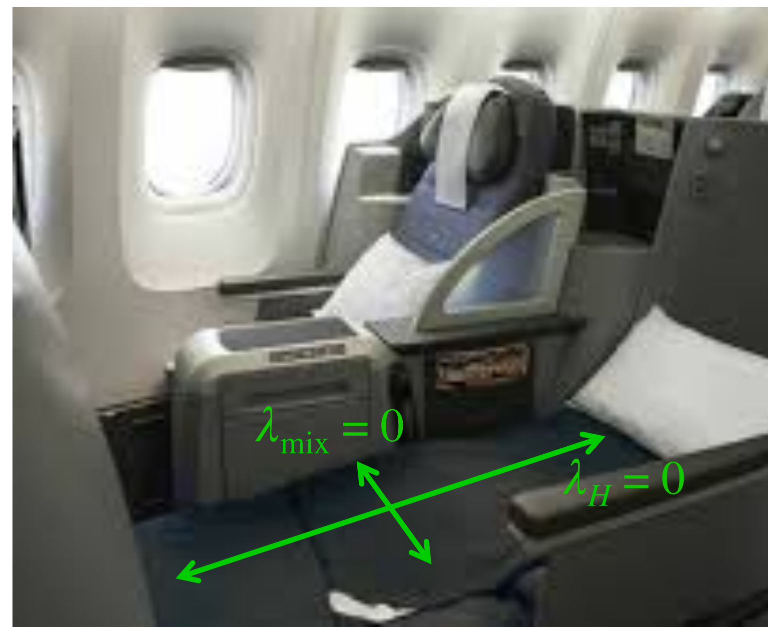

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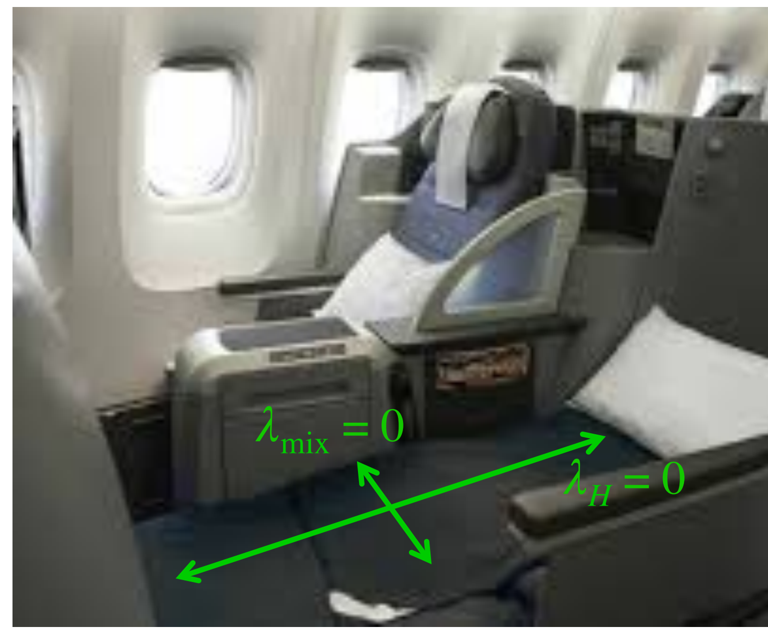

If we impose

$$dy \wedge dy = \underbrace{dy \wedge dz}_{(???)} = 0, ??$$

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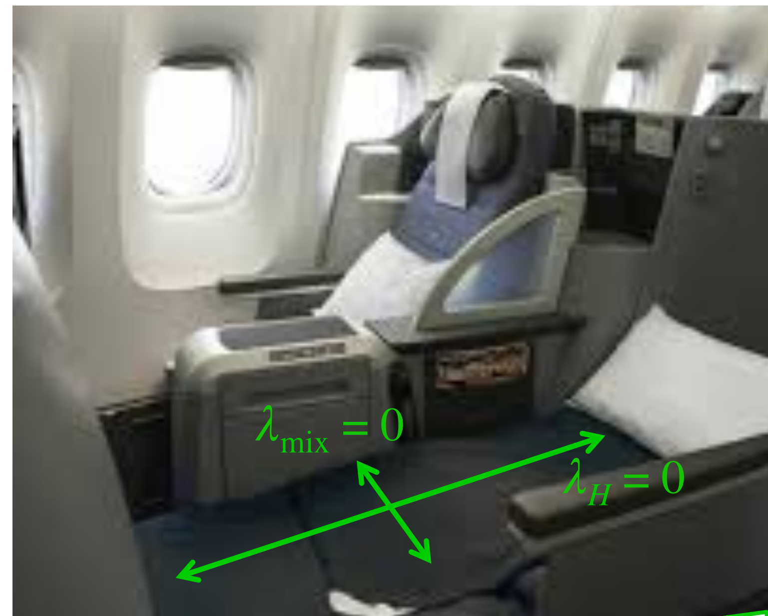


If we impose

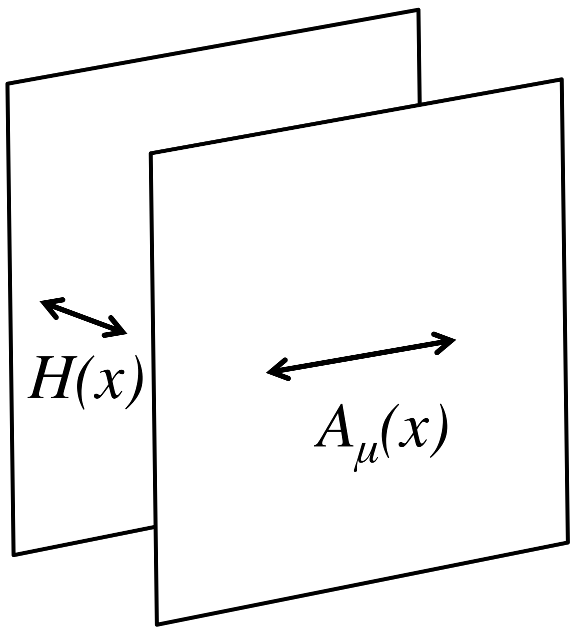
$$dy \wedge dy = \underbrace{dy \wedge dz}_{\text{(???)}} = 0, ??$$

$$\boldsymbol{F} = \boldsymbol{dA} + \boldsymbol{A} \wedge \boldsymbol{A}, \quad \mathcal{L} \sim \mathrm{Tr}\boldsymbol{F}^*\boldsymbol{F} \ni V = \lambda_H \ldots |\Delta_R|^2,$$
$$= 0 \qquad = 0$$

Under construction!!

Conclusion

- The paper today introduced shows



The Higgs boson

=

A Gauge boson

along the NCG discrete extra dimension

  – Yang-Mills-Higgs model in NCG ⇒ SM w/ CCI
  – The possibility the hierarchy problem is solved in the context of NCG.

That’s all. Thank you!

Back up

Differential algebra

Leibniz rule $$d(\xi \wedge \omega) = d\xi \wedge \omega + (-1)^r \xi \wedge d\omega$$

Commutative

- $x^\mu\, y = y\, x^\mu,$
- $dx^\mu\, y = y\, dx^\mu,\ dy\, x^\mu = x^\mu\, dy,$
- $dx^\mu \wedge dy = -\, dy \wedge dx^\mu,$
- Anti-sym. wedge product

Noncommutative

- $y^2 = 1,$
- $y\, dy = -\, dy\, y,$
- $dy \wedge dy = dy \wedge dy,$
- $f(y)\, dy = dy\, f(\text{-}y),$
- Sym. wedge product

“Geometry”? (they are just matrices?)

NCG review, F. Lizzi, 0811.0268

- Gelfand – Naimark theorem (1943)

Set of C functions on Haussdorff space	$\Longleftrightarrow$ 1 to 1	commutative C* algebra

orthonormal set
of functions on $\mathbb{R}$

Infinite dimension
commutative algebra

$$\delta(x-a) \Leftrightarrow e^{ipx}$$

on $\mathbb{R}$

$$e^{ipx}e^{ip'x} = e^{i(p+p)'x}$$

$$[e^{ipx}, e^{ip'x}] = 0$$

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Roughly, Algebra w/ complex conjugate

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1 to 1

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Set of C functions on Haussdorff space	⇔ non 1 to 1	noncommutative C* algebra

Infinite dimension
noncommutative algebra

c.f., algebra of functions
on $GL(n,\mathbb{C})$

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Infinite dimension
noncommutative algebra

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on $\cancel{\mathbb{R}}$?

c.f., algebra of functions
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Sometimes, the underlying manifold does not exist.

c.f. The algebra of position and momentum of ordinary QM

離散空間上のゲージ理論

曲率

$$F = dA + A \wedge A$$

$$= \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu + F_{\mu\bullet}\, dx^\mu \wedge dy + F_{\bullet\bullet}\, dy \wedge dy$$

$$\mathscr{L} = \sum_{y=\pm} \frac{1}{g^2} F \wedge *F, \quad A \to gA$$

$$\cdots {}^{\mu}\Phi)^\dagger D_\mu \Phi + \frac{g^2}{2}\left(\left(\frac{\sqrt{2}M}{g}\right)^2 - \Phi^\dagger\Phi\right)^2,$$

余剰次元としてはZ_μはA_μの
1st KK 励起と解釈できる

ヒッグス質量 $\cdots = \frac{\cdots}{g}, \quad \lambda = \frac{g^2}{2}, \quad m_h = \sqrt{2\lambda}\, v = gv.$

破れたゲージボソン $Z_\mu = \frac{A_\mu^+ - A_\mu^-}{\sqrt{2}}$

$$\boxed{m_Z = \sqrt{2} g v = \sqrt{2} m_h.}$$

質量関係式 (@ tree level)！

離散空間上のゲージ理論

曲率 $$F = dA + A \wedge A$$

$$= \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu + F_{\mu\bullet}\, dx^\mu$$

$$\mathscr{L} = \sum_{y=\pm} \frac{1}{g^2} F \wedge *F, \quad A$$

余剰次元としてはZ_μはA_μの
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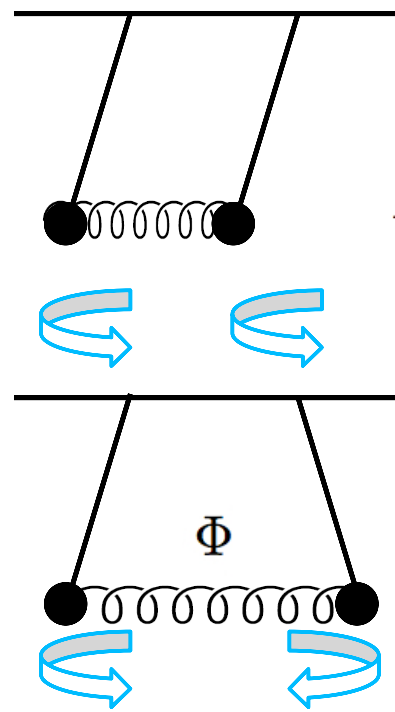
ヒッグス質量 $= \frac{\quad}{g}, \quad \lambda =$

破れたゲージボソン $$Z_\mu = \frac{A_\mu^+ - A_\mu^-}{\sqrt{2}}$$

SSBの物理的起源

Ex) 連成振動系



0 mode

$$A_\mu = \frac{A_\mu^+ + A_\mu^-}{\sqrt{2}}$$

1 mode

$$Z_\mu = \frac{A_\mu^+ - A_\mu^-}{\sqrt{2}}$$

質量関係式 (@ tree level)！