Lattice QCD
with Strong External Electric Fields

Arata Yamamoto
(RIKEN)

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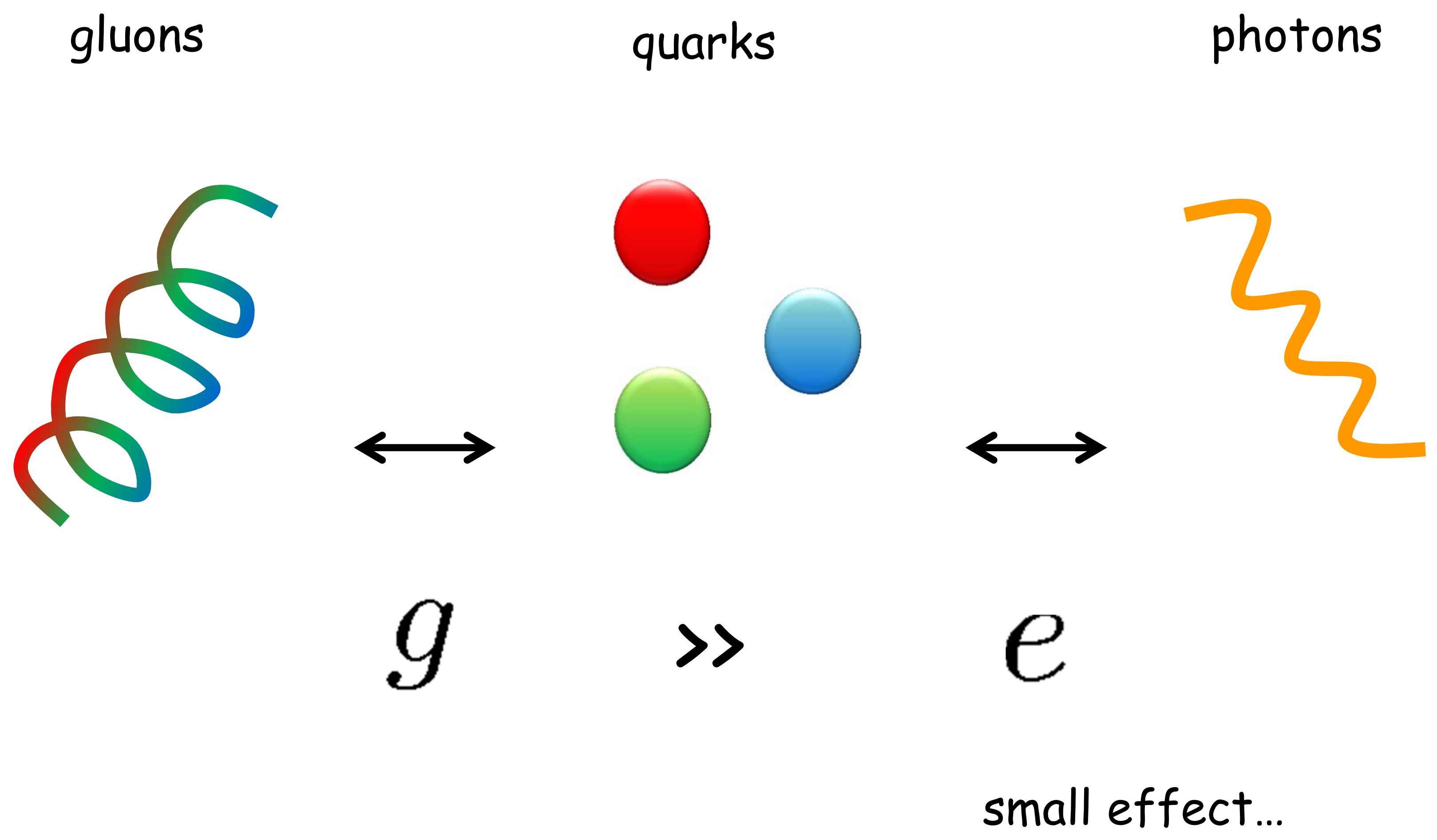
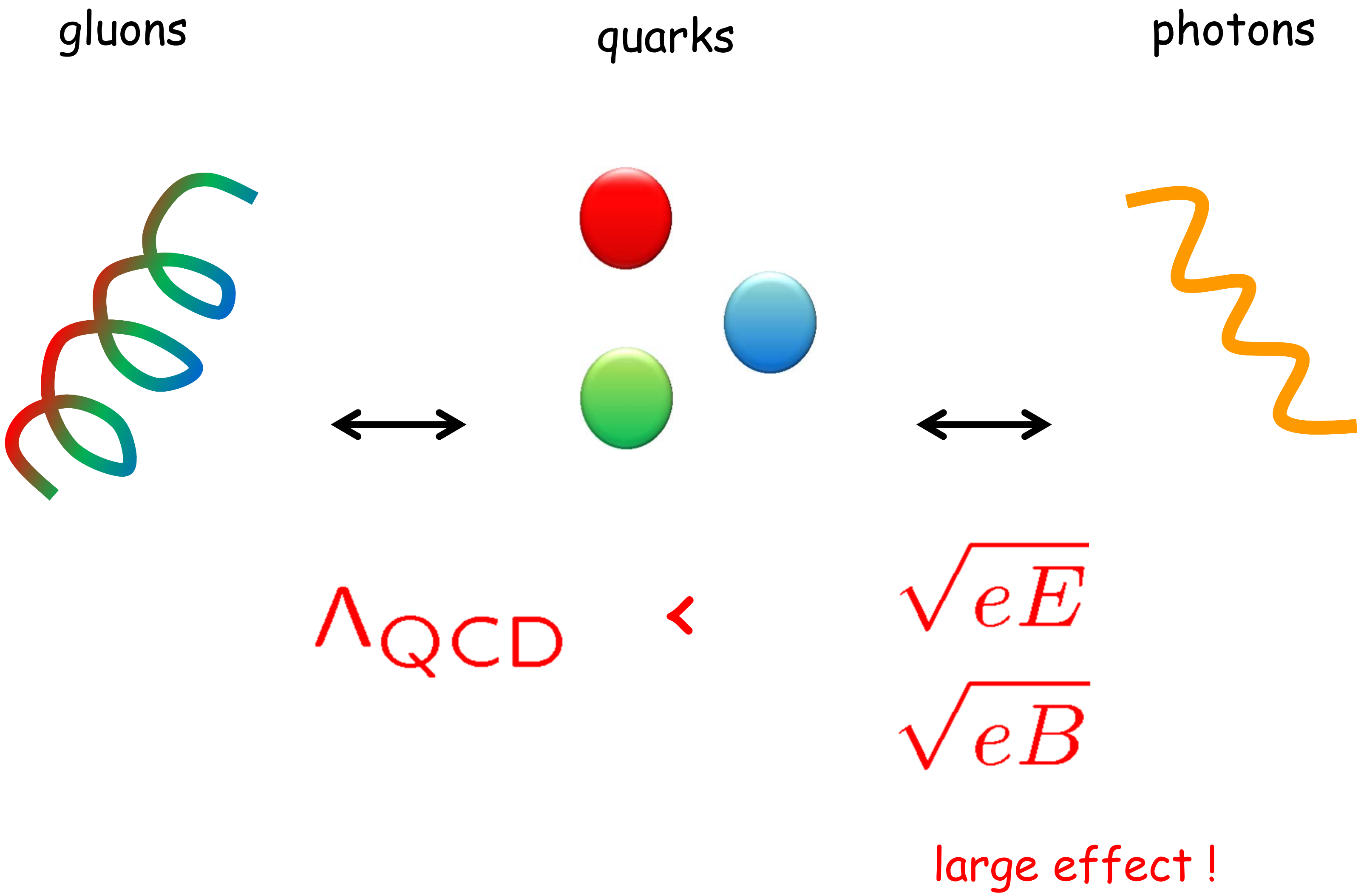

gluons
quarks
photons
$g \gg e$
small effect...

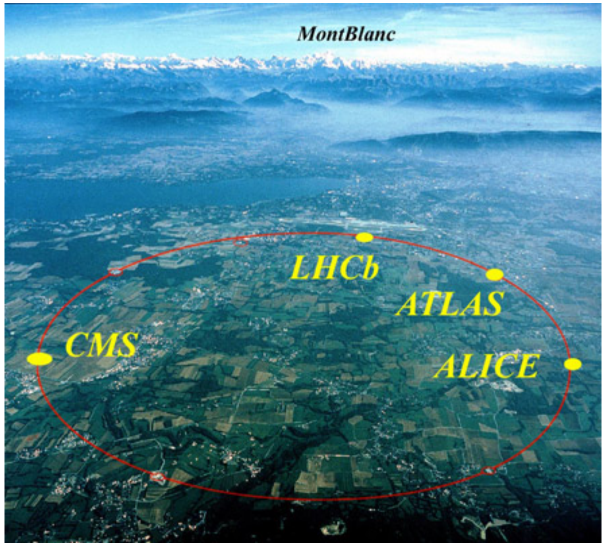

gluons
quarks
photons
Λ_{QCD}
$<$
$\sqrt{eE}$
$\sqrt{eB}$
large effect !

strong magnetic fields:

earth	~ 0.01 eV^2
magnetar	~ 10 MeV^2
heavy ion collision	~ 0.1 GeV^2

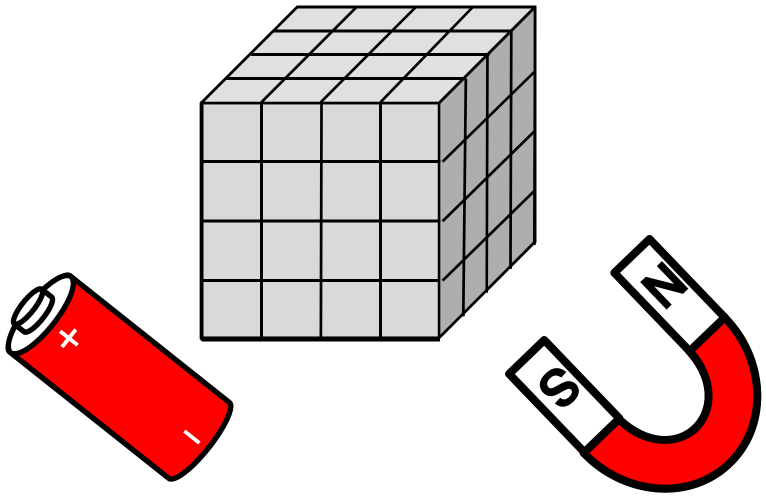
strong electric fields:

thunder cloud	~ 1 eV^2
heavy ion collision (fluctuation)	~ 0.1 GeV^2



Formulation

Lattice QCD + external electromagnetism



$$S = S_{\mathrm{YM}}[U] + S_{\mathrm{quark}}[\bar{\psi}, \psi, U, u] + \cancel{S_{\mathrm{U(1)}}[u]}$$

not dynamical

NOTE : Forget electrons.

Euclidean QCD + external electromagnetic field

$$Z = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}U \;\; e^{-S_{\mathrm{YM}}-S_{\mathrm{quark}}}$$

$$= \int \mathcal{D}U \;\; \det D(A_\mu) \;\; e^{-S_{\mathrm{YM}}}$$

Euclidean QCD + external electromagnetic field

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \;\, e^{-S_{\mathrm{YM}} - S_{\mathrm{quark}}}$$

$$= \int \mathcal{D}U \;\; \det D(A_\mu) \;\, e^{-S_{\mathrm{YM}}}$$

$$\boxed{B = \partial_1 A_2 - \partial_2 A_1} \qquad \boxed{E = \partial_0 A_3 - \partial_3 A_0}$$

no problem

sign problem

Monte Carlo simulation

$$Z = \int \mathcal{D}U \, \underbrace{\det D e^{-S_{\mathrm{YM}}}}_{\text{Monte Carlo weight}}$$

$$\langle O[U] \rangle = \sum_{U} O[U]$$

Monte Carlo simulation

$$Z = \int \mathcal{D}U \; \underbrace{\det D e^{-S_{\rm YM}}}_{\text{Monte Carlo weight}}$$

$$\langle O[U] \rangle = \sum_U O[U]$$

if Monte Carlo weight $\ngeq$ 0, "sign problem"

e.g., $\det D(\mu) = |\det D(\mu)| e^{i\phi}$

$$E = \partial_0 A_3 - \partial_3 A_0$$

$$E = \partial_0 A_3 - \partial_3 A_0$$

$$i\partial_0 + qA_0(x) - \mu \xrightarrow{\text{Wick rotation}} -\partial_4 + qA_0(x) - \mu$$

real electric field

sign problem

$$E = \partial_0 A_3 - \partial_3 A_0$$

$$i\partial_0 + qA_0(x) - \mu \xrightarrow{\text{Wick rotation}} -\partial_4 + qA_0(x) - \mu$$

real electric field

sign problem

$$\longrightarrow -\partial_4 + iqA_4(x) - \mu$$

imaginary electric field

no sign problem
but analytic continuation

Lattice QCD + magnetic field

so many papers

Lattice QCD + imaginary electric field

Fiebig Wilcox Woloshyn (1989) Christensen Wilcox Lee Zhou (2005)
Engelhardt (2007) Detmold Tiburzi Walker-Loud (2009) (2010)
D'Elia Mariti Negro (2013)

Lattice QCD + weak (quenched) electric field

Shintani Aoki Ishizuka Kanaya Kikukawa Kuramashi Okawa
Ukawa Yoshie (2007) Shintani Aoki Kuramashi (2008)

Lattice QCD + strong electric field

this study

physical electric charge

$$q = \left(\frac{2e}{3}, -\frac{e}{3}\right)$$

sign problem

physical electric charge

$$q = \left(\frac{2e}{3}, -\frac{e}{3}\right)$$

sign problem

"isospin" electric charge

$$q_3 = \left(\frac{e}{2}, -\frac{e}{2}\right)$$

NO sign problem

cf.) isospin chemical potential

$$\mu_3 = \left(\frac{\mu}{2}, -\frac{\mu}{2}\right)$$

NO sign problem

Lattice QCD + isospin electric field

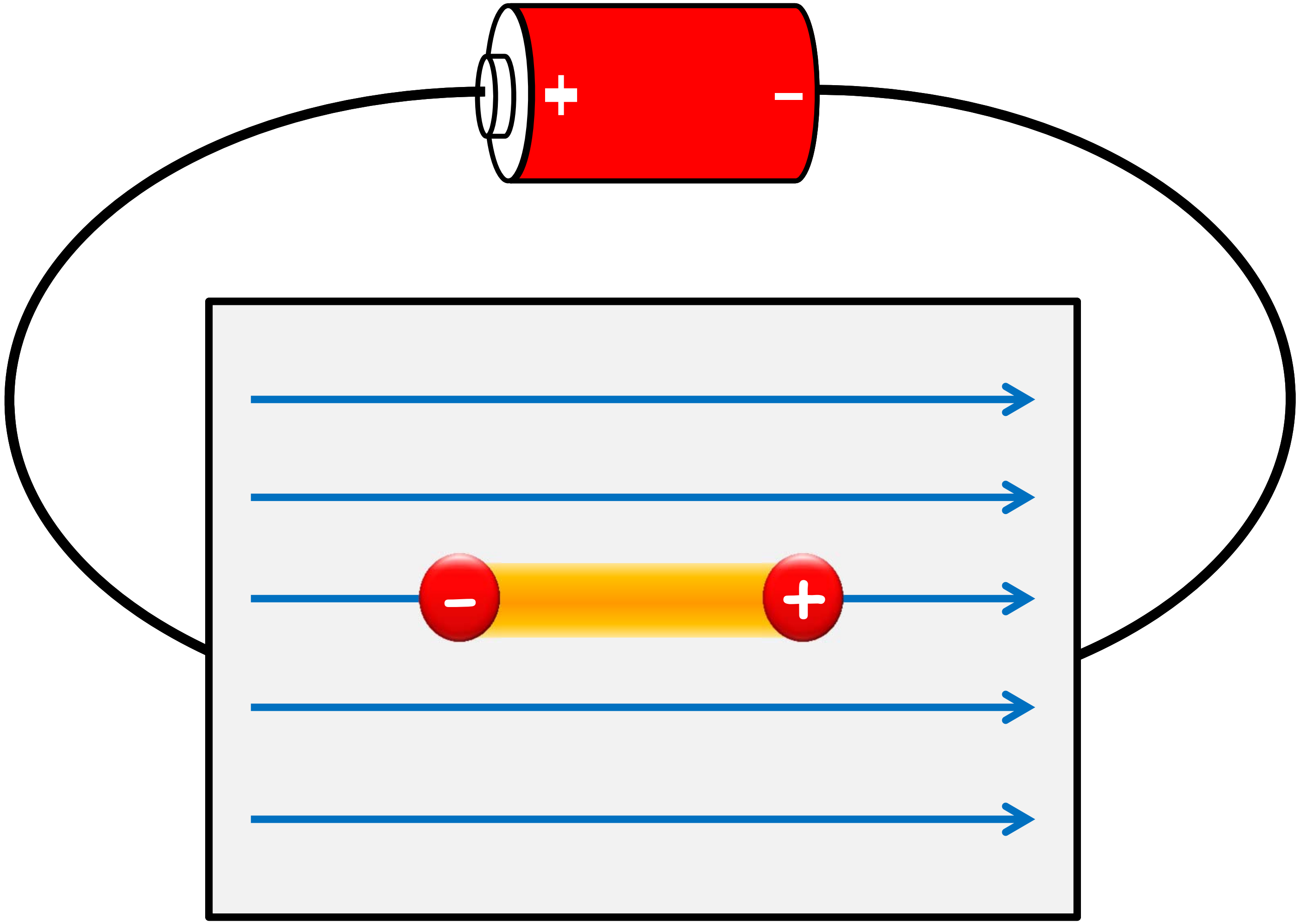
$$Z = \int \mathcal{D}U \ \det D(eA_0) \det D(-eA_0) e^{-S_{\mathrm{YM}}}$$

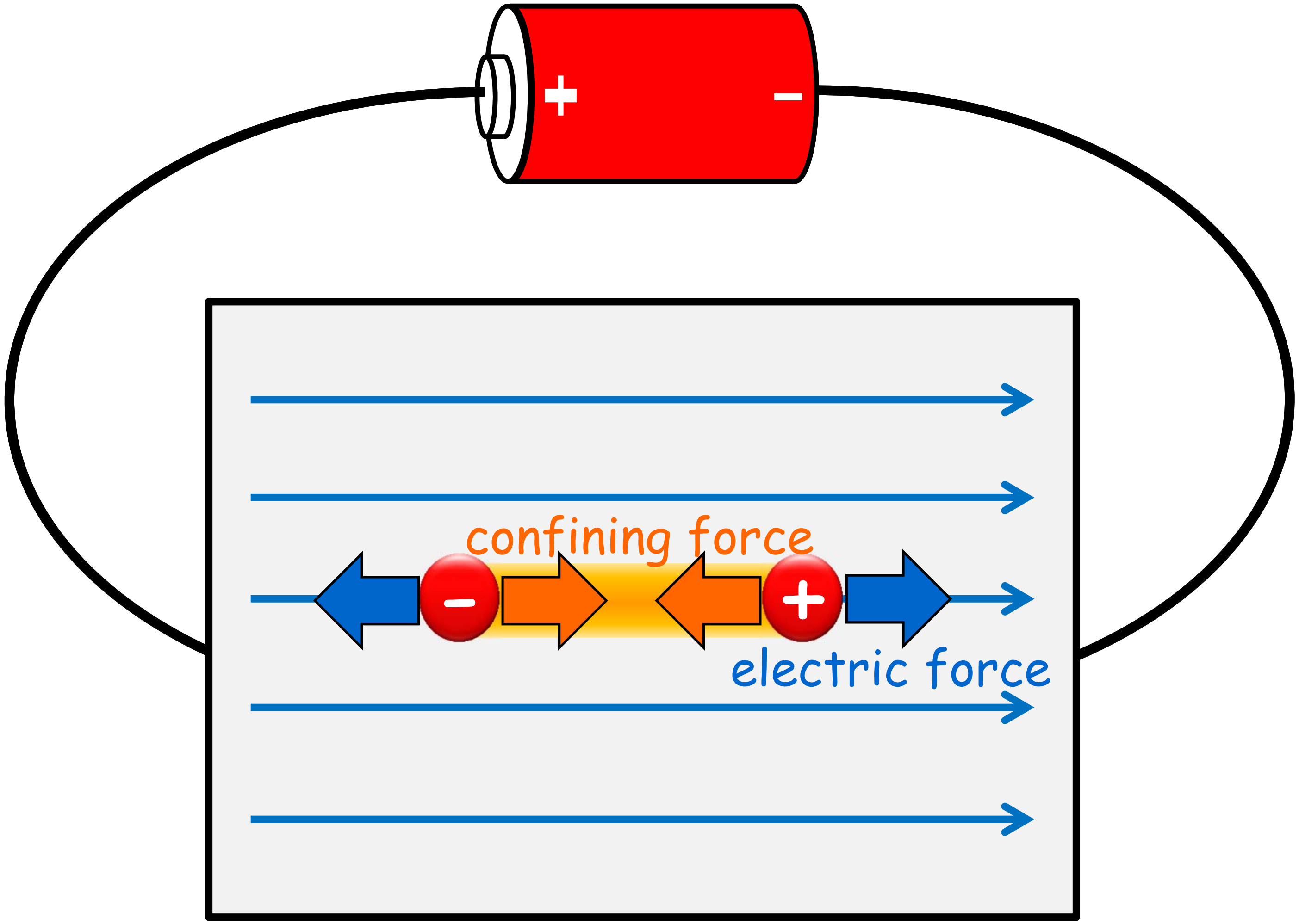
$$= \int \mathcal{D}U \ \underbrace{|\det D(eA_0)|^2 e^{-S_{\mathrm{YM}}}}_{\geqq 0}$$

$$\det D(\pm eA_0) = |\det D(eA_0)| e^{\pm i\phi}$$

Result 1

deconfinement

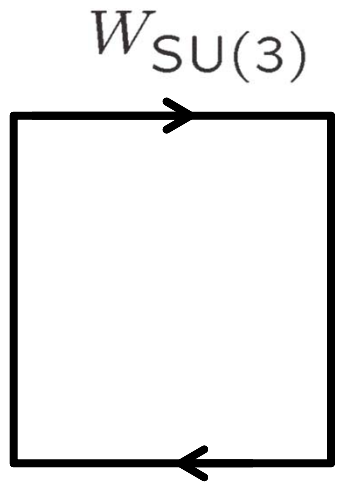




electric force > confining force $\rightarrow$ deconfinement

Wilson loop

$$\langle W_{\mathsf{SU(3)}} \rangle = \langle \mathrm{tr} \prod_{\mathsf{loop}} U_\mu \rangle$$



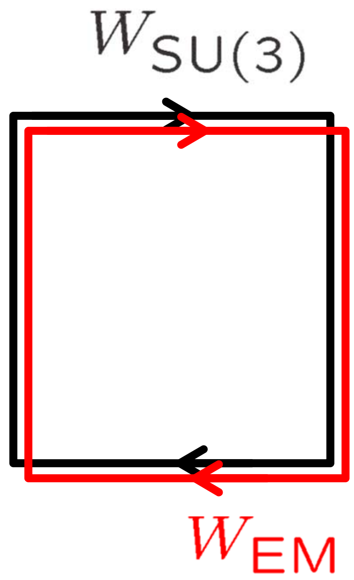
heavy-quark potential

$$V_{\mathsf{SU(3)}}(R) = \sigma R + \frac{A}{R} + \mathrm{const.}$$

"charged" Wilson loop

$$\langle W_C \rangle = \langle \mathrm{tr} \prod_{\text{loop}} U_\mu u_\mu \rangle$$

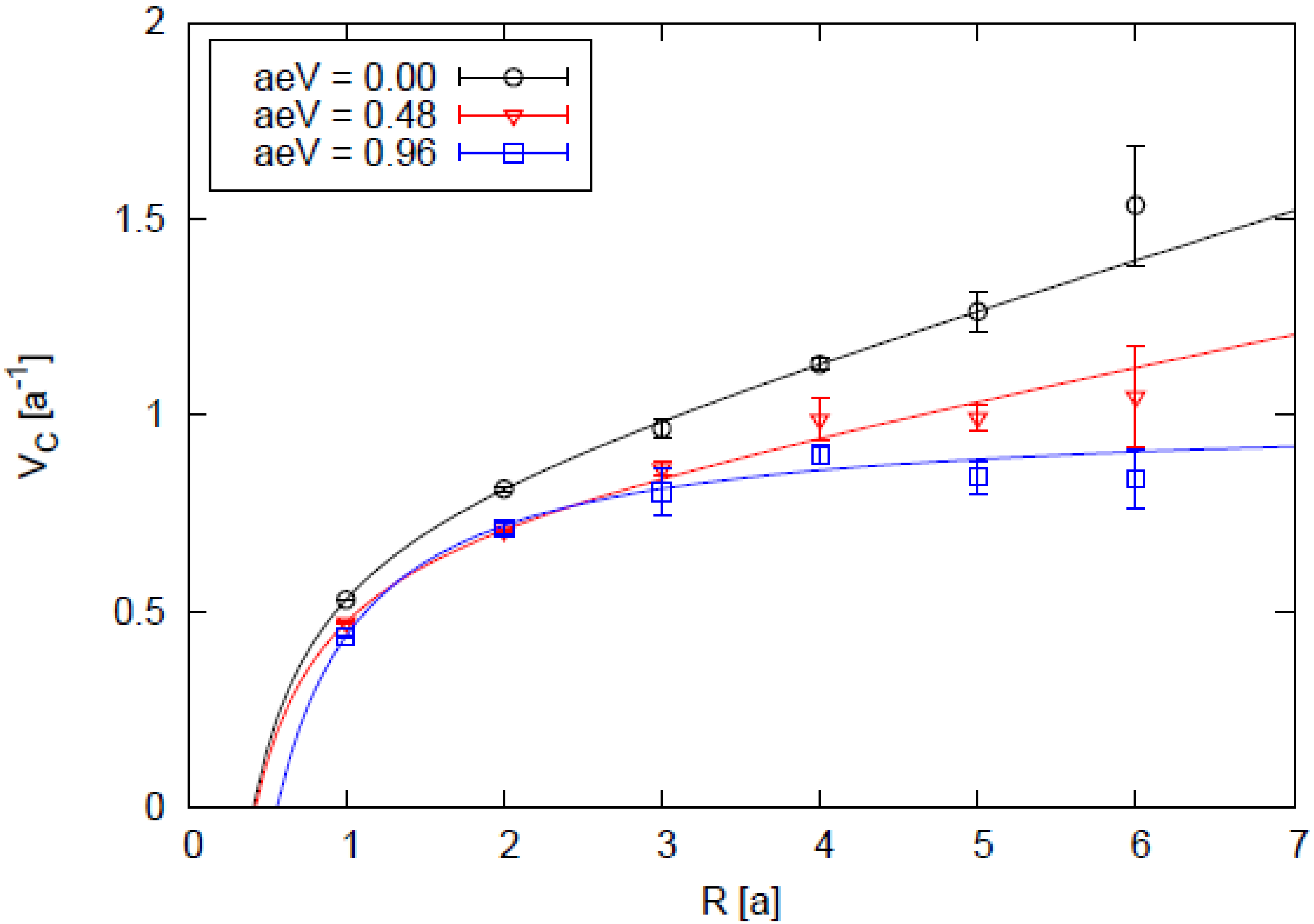
$$\simeq \langle W_{\mathrm{SU(3)}} \rangle W_{\mathrm{EM}}$$



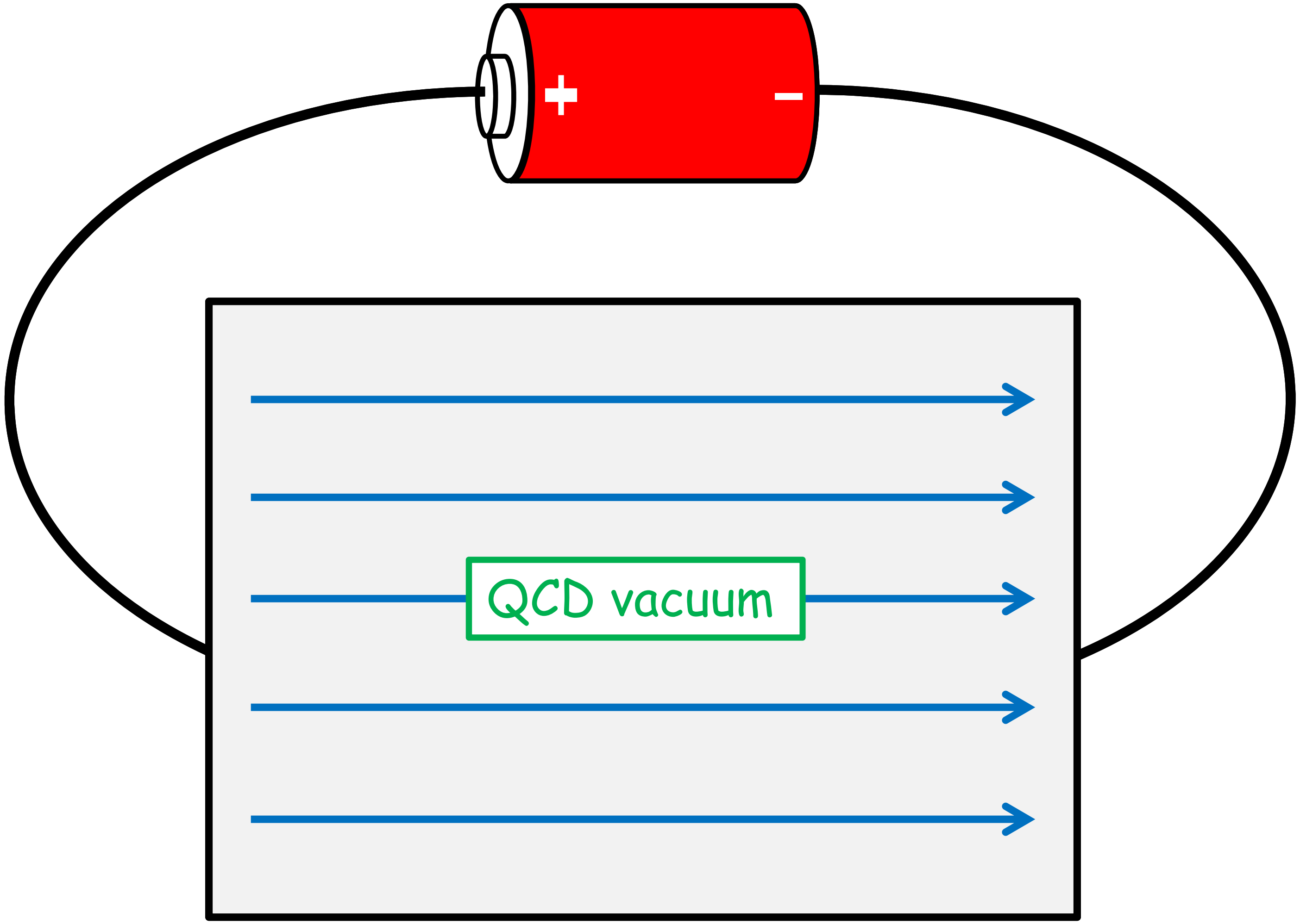
"charged" heavy-quark potential

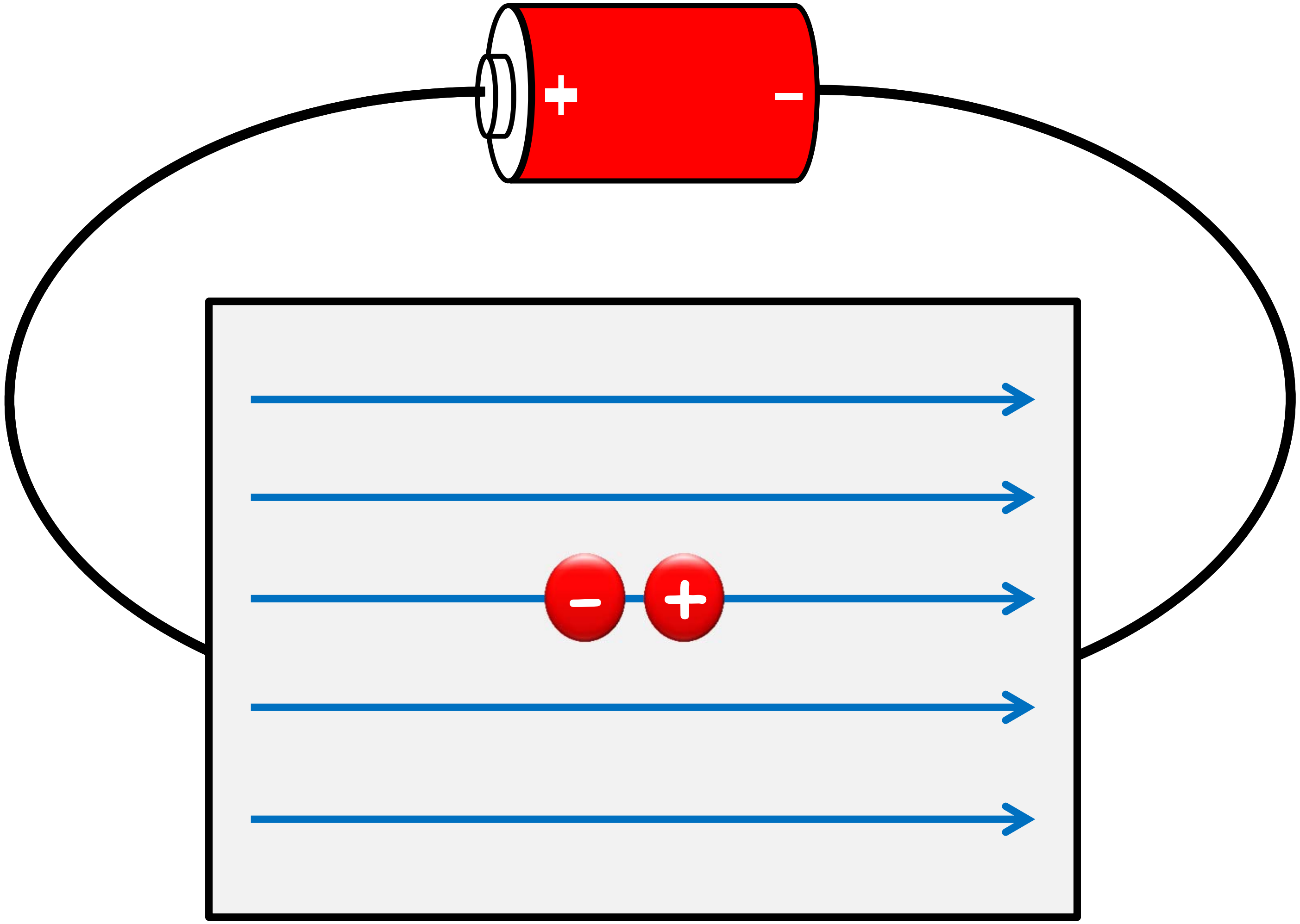
$$V_C(R) \simeq \left(\sigma - \frac{e}{2} E\right) R + \frac{A}{R} + \text{const.}$$

“charged” heavy-quark potential in full QCD

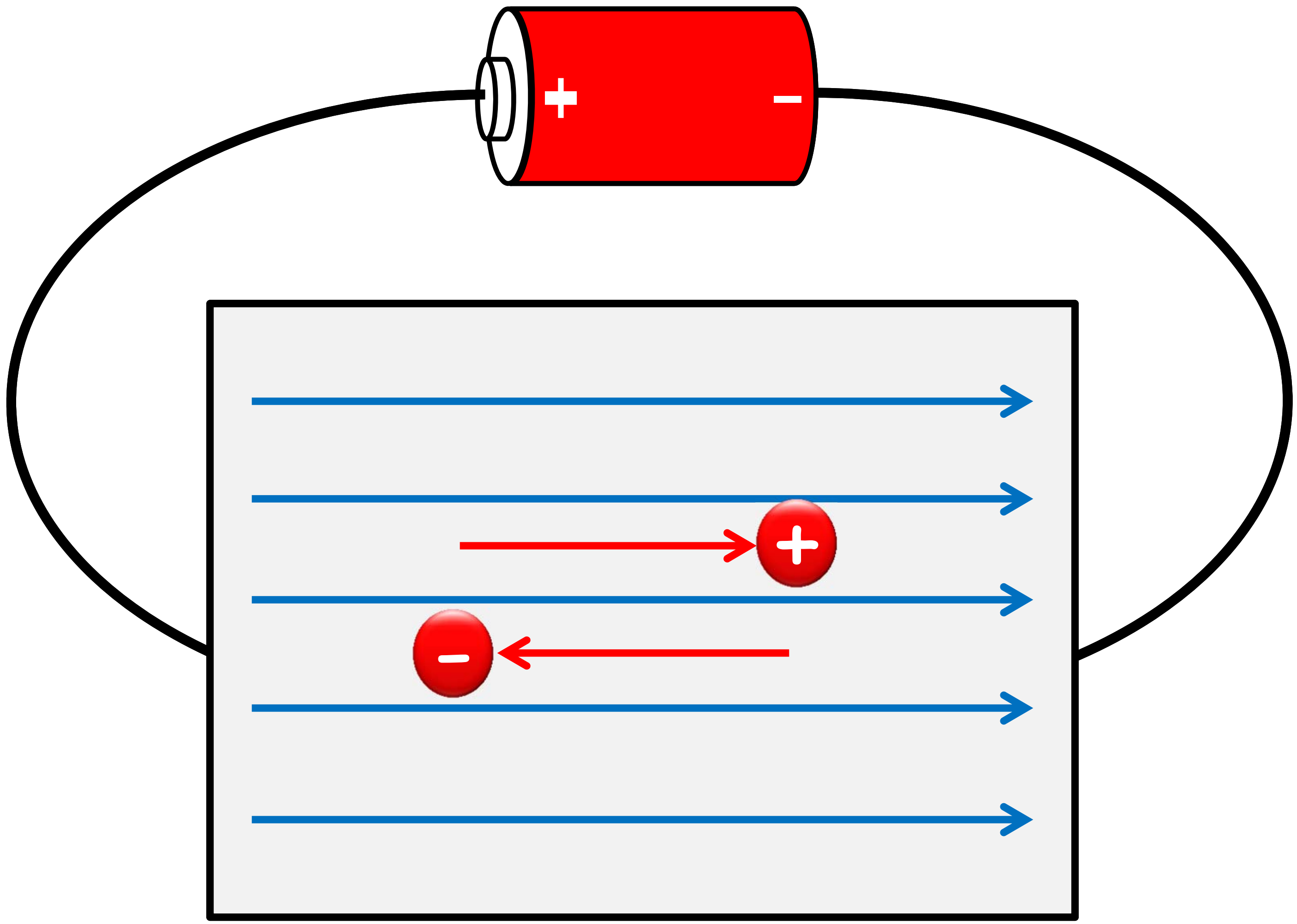


Result 2
charge distribution


+
−
QCD vacuum

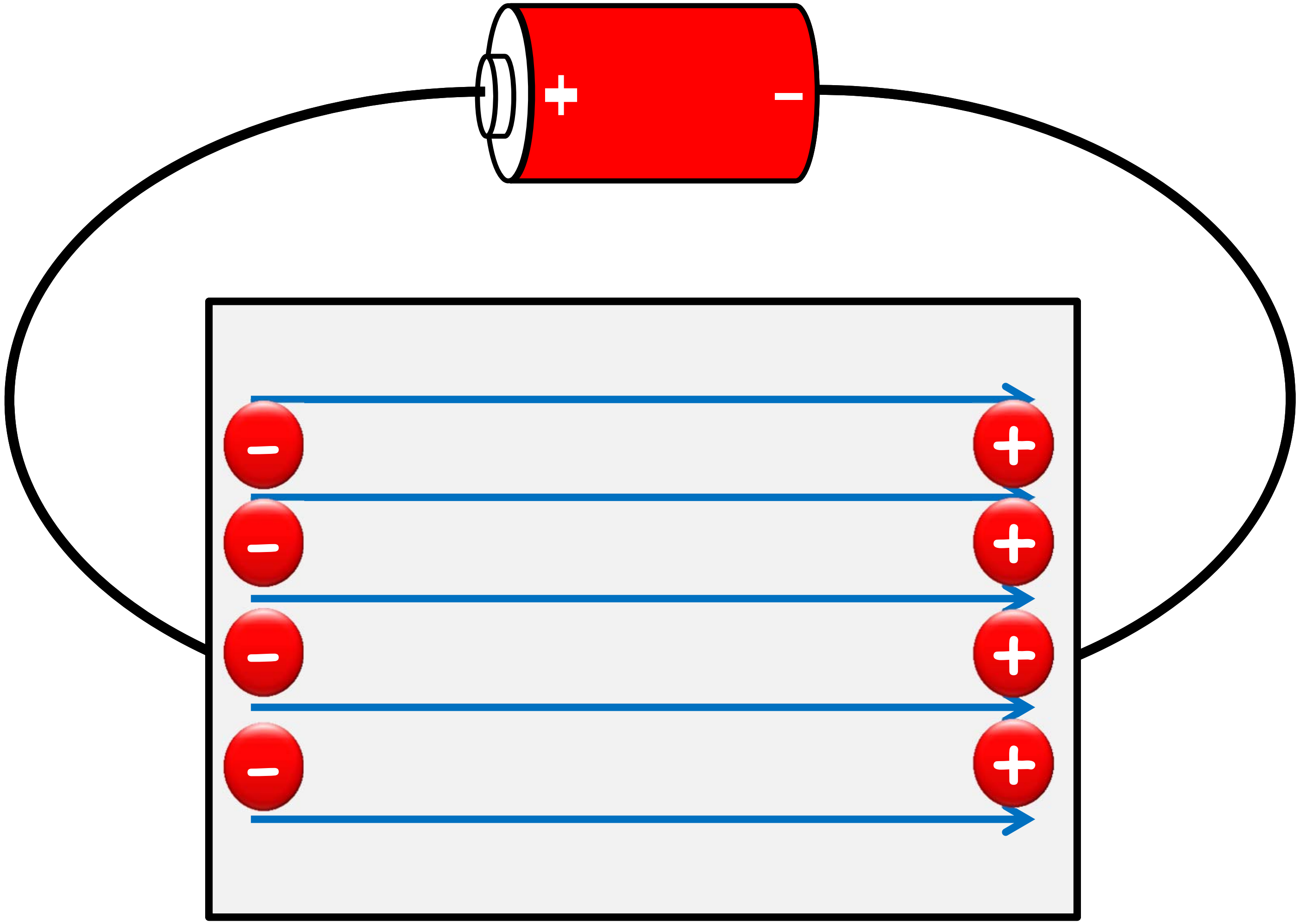


Schwinger mechanism



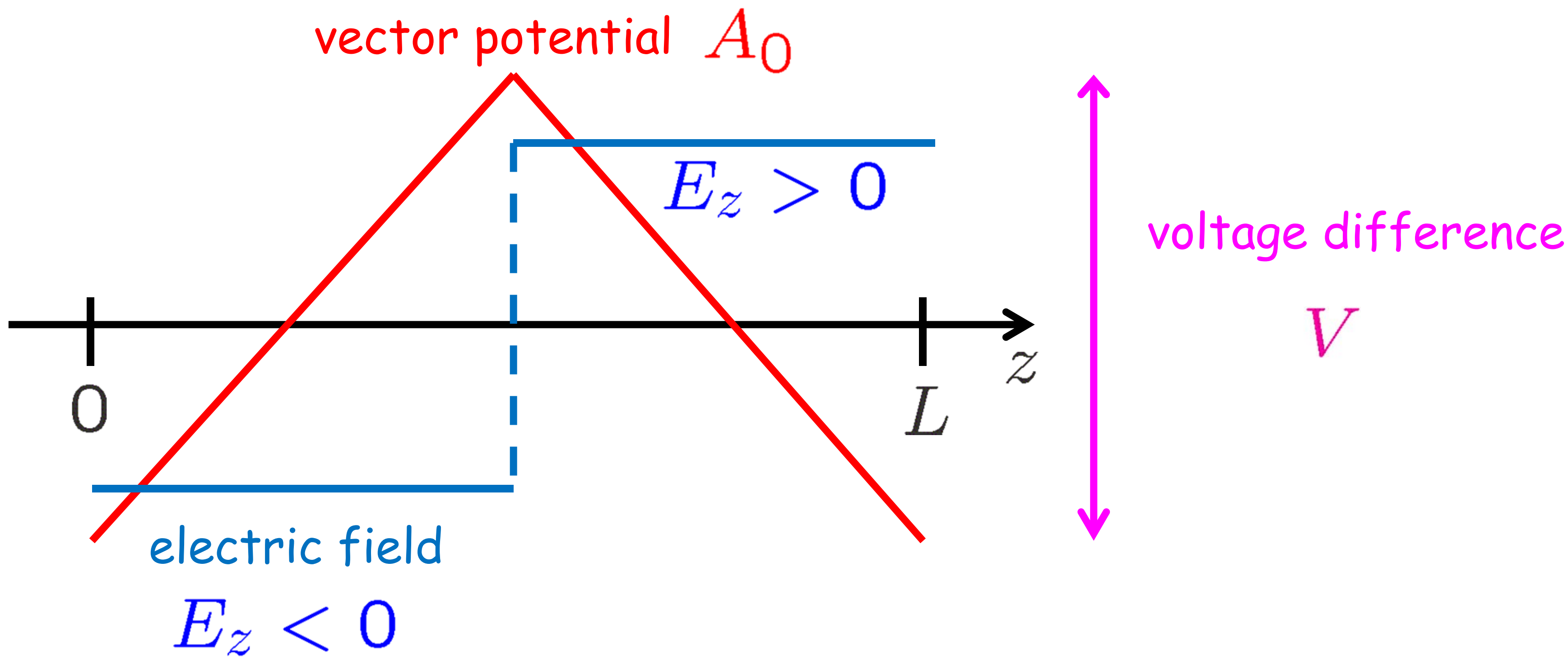
Schwinger mechanism

electric current

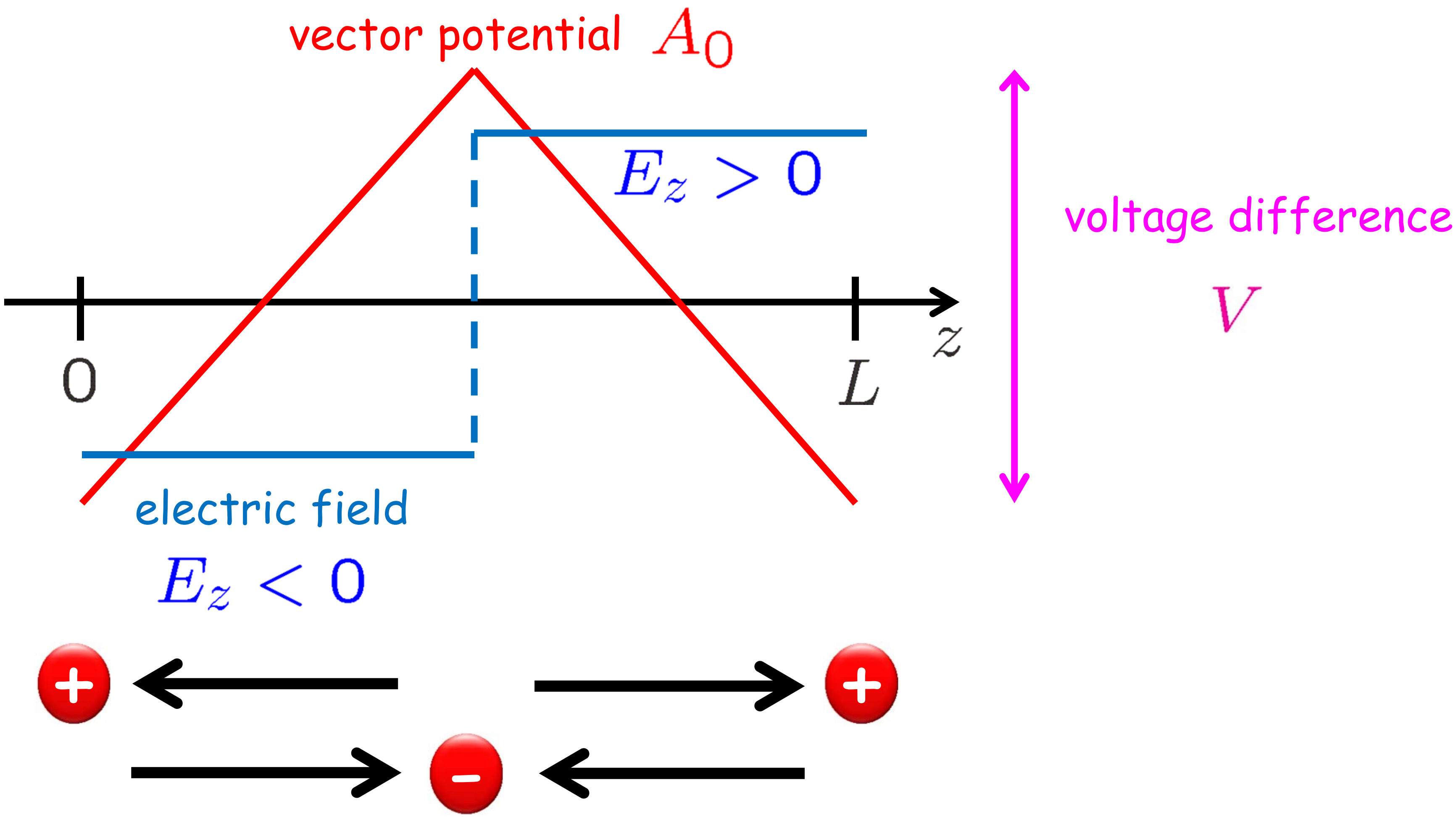


Schwinger mechanism

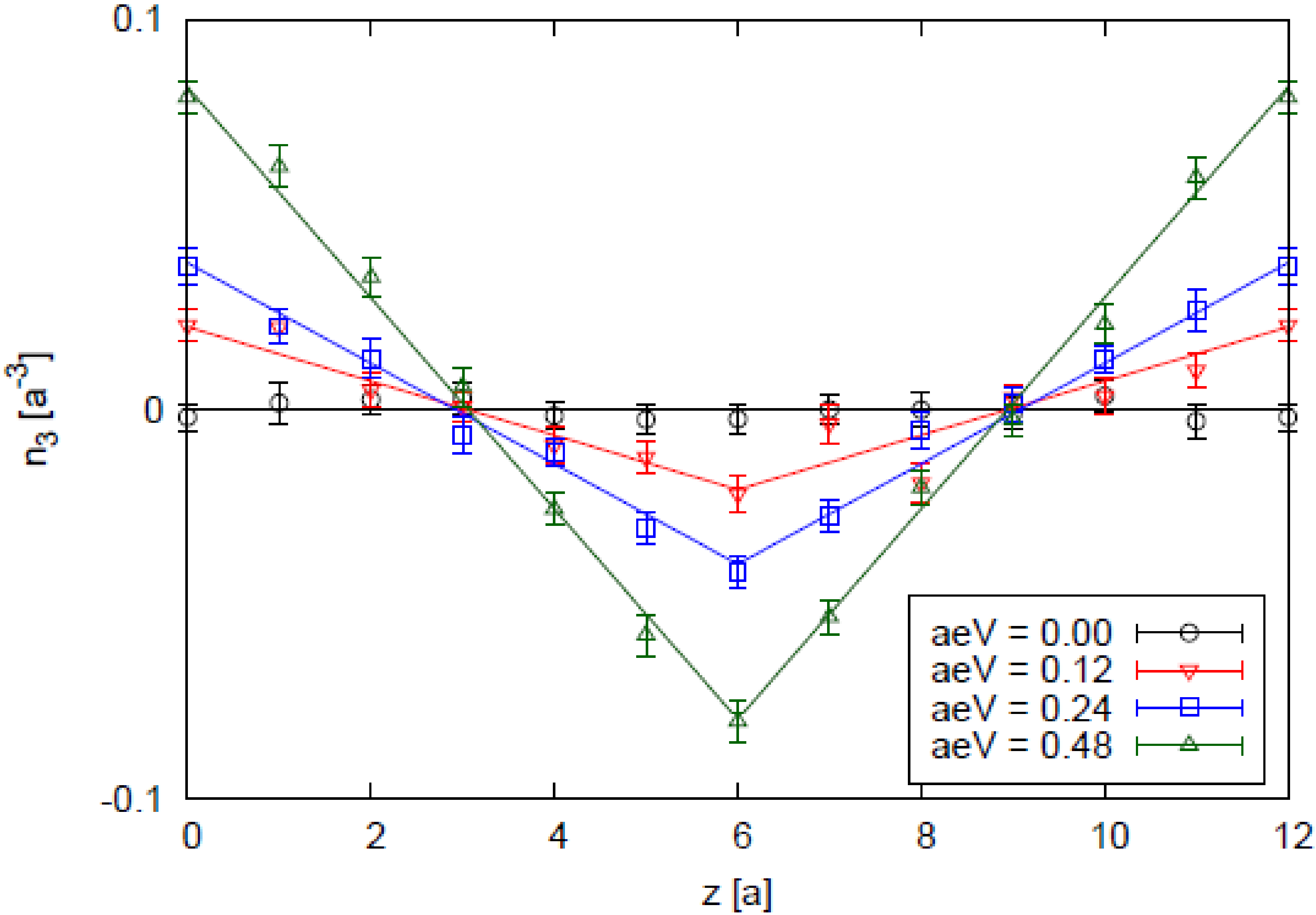
→ electric current → charge distribution (nearly) constant electric field in a periodic box



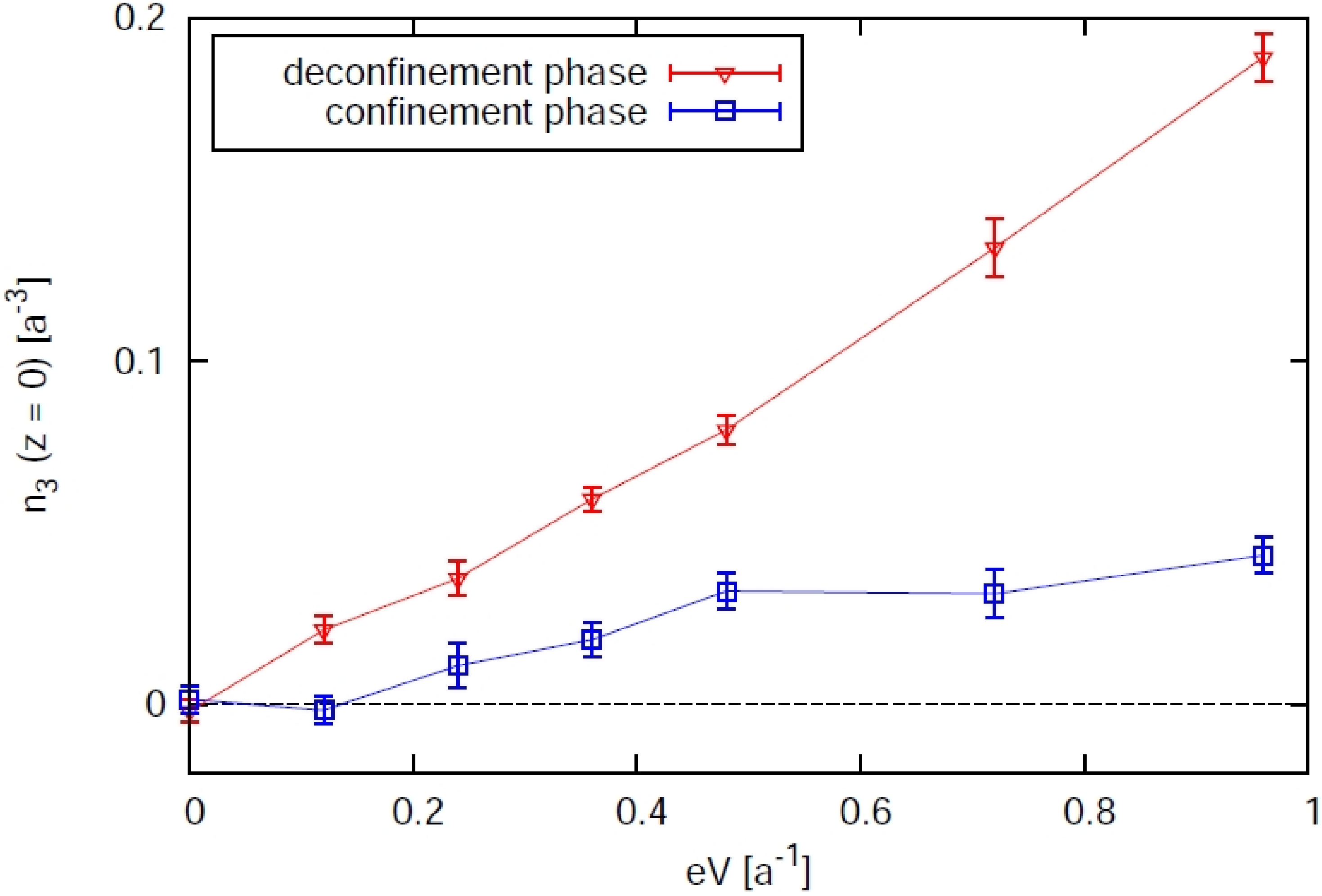
(nearly) constant electric field in a periodic box

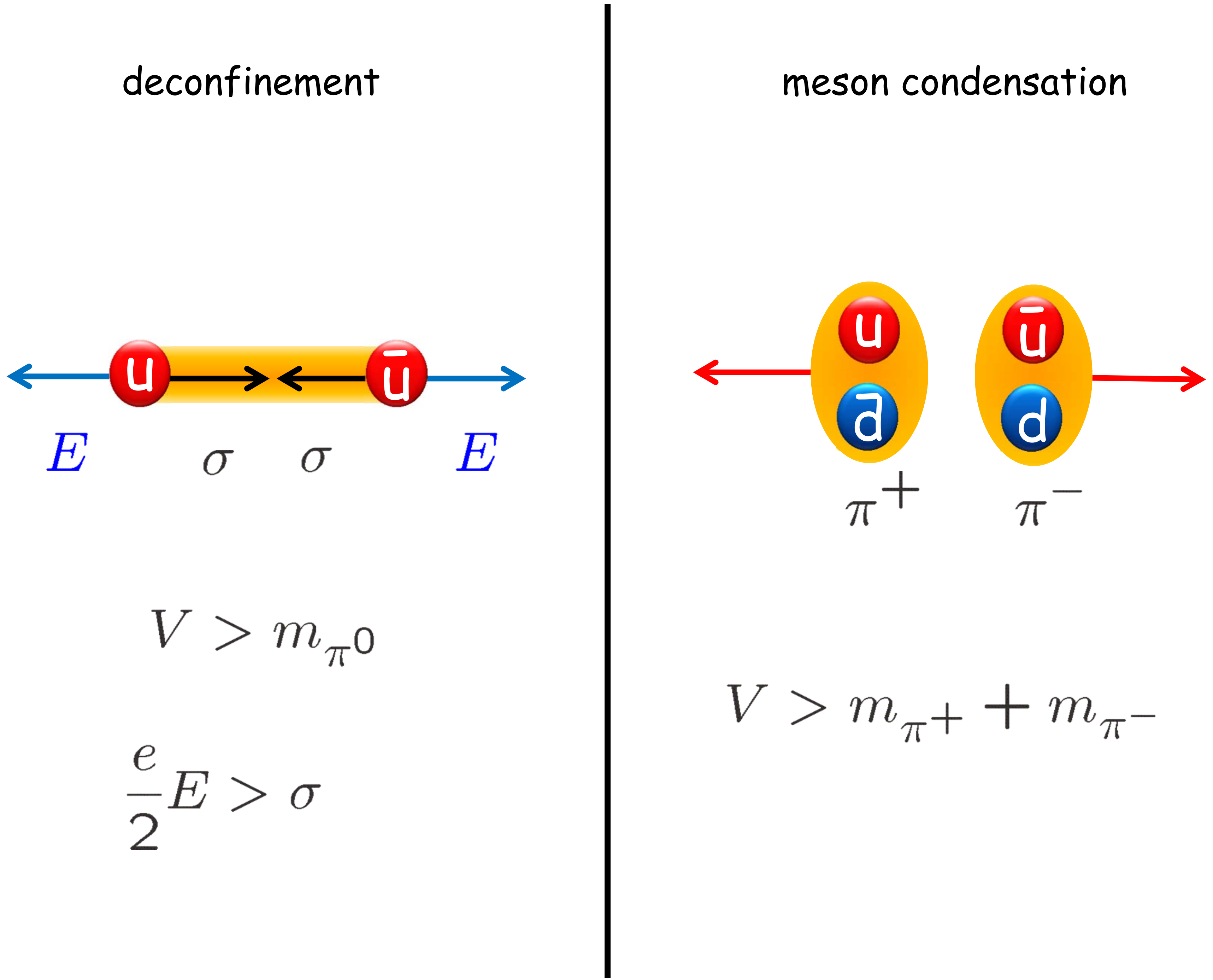


charge density distribution (deconfinement phase)

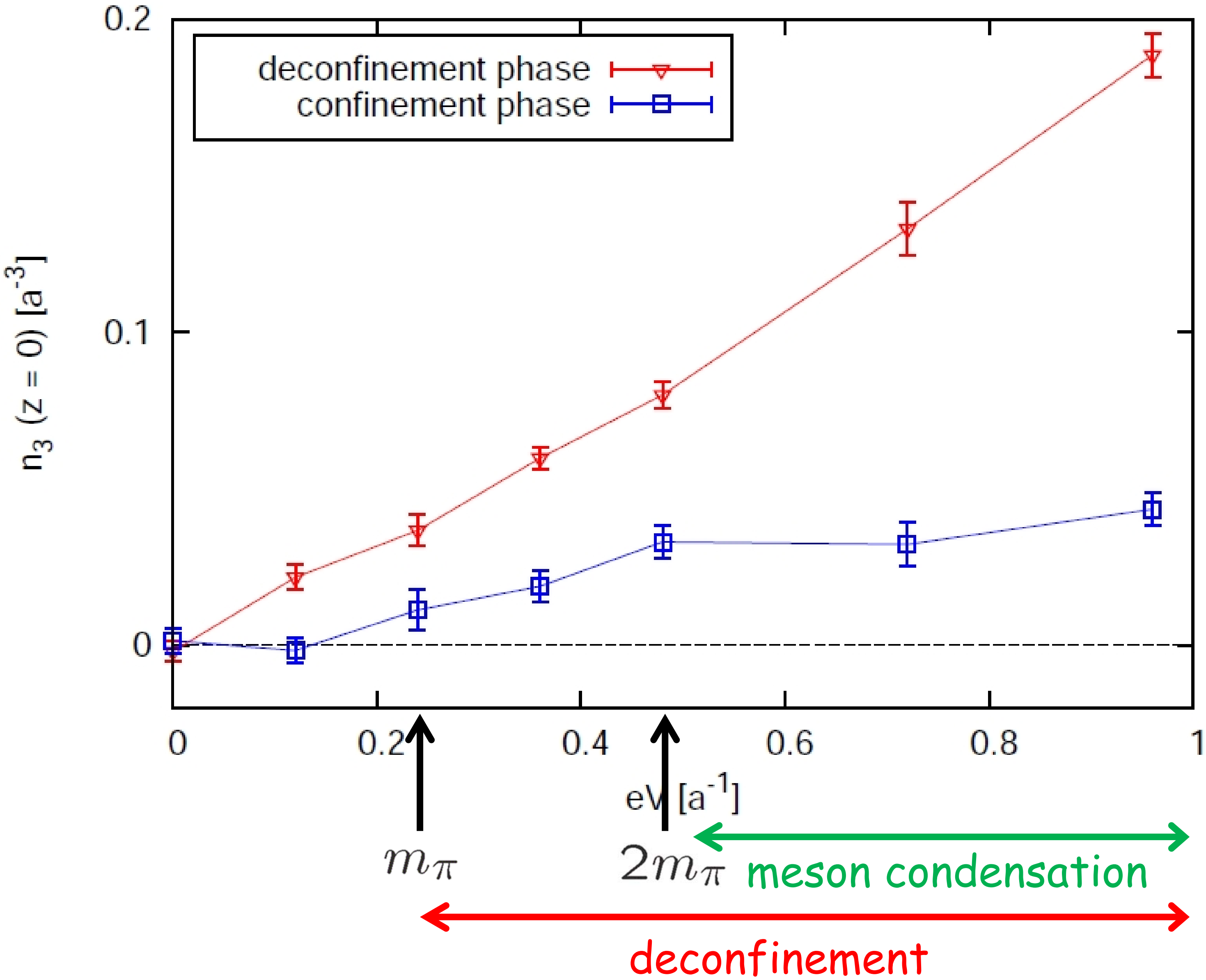


voltage dependence




deconfinement
u
ū
E
σ
σ
E
$V > m_{\pi^0}$
$\frac{e}{2}E > \sigma$
meson condensation
u
d̄
ū
d
π^+
π^-
$V > m_{\pi^+} + m_{\pi^-}$

voltage dependence



Summary

- ✓ I studied lattice QCD with a strong electric field.
- ✓ I studied deconfinement by electric force.
- ✓ I studied charged particle generation.

おまけ

T. Hayata, Y. Hidaka, AY, arXiv:1309.0012

Basar, Dunne, Kharzeev (2010)

(1+3)-dim. QCD with a strong external magnetic field

$\downarrow$ lowest Landau level approximation

(1+1)-dim. QCD

$\downarrow$ chemical potential

chiral (magnetic) spiral

$$\langle \bar{c}c \rangle = \langle \bar{c}c \rangle_0 \cos(2\mu x)$$

$$\langle \bar{c}i\gamma^5 c \rangle = \langle \bar{c}c \rangle_0 \sin(2\mu x)$$

Basar, Dunne, Kharzeev (2010)

(1+3)-dim. QCD with a strong external magnetic field

lowest Landau level approximation

(1+1)-dim. QCD

chemical potential

chiral (magnetic) spiral

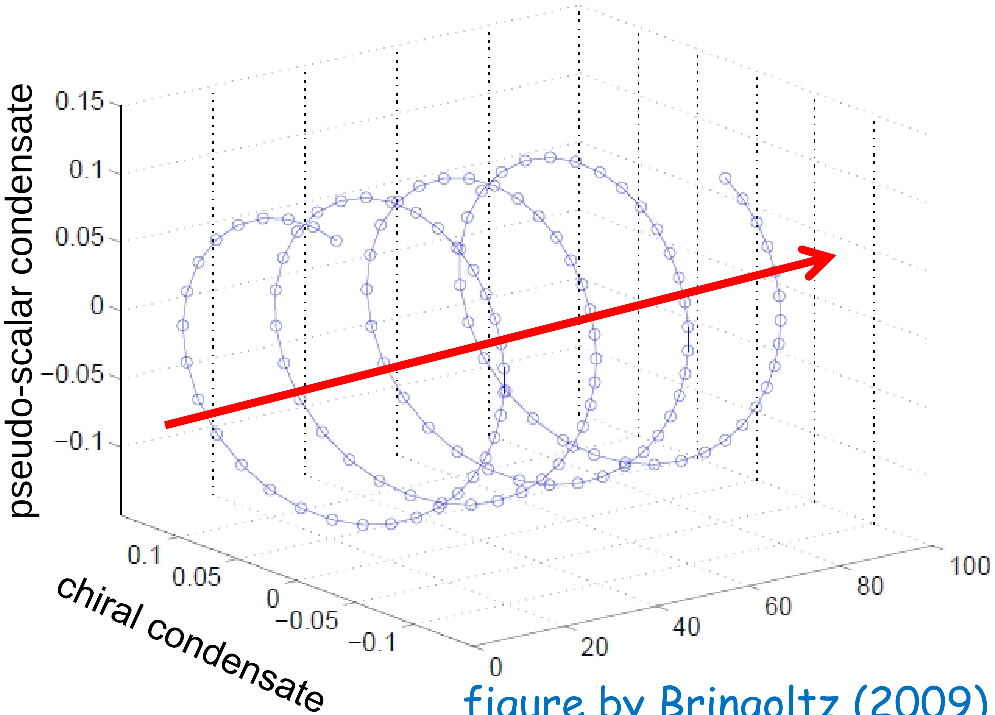


figure by Bringoltz (2009)

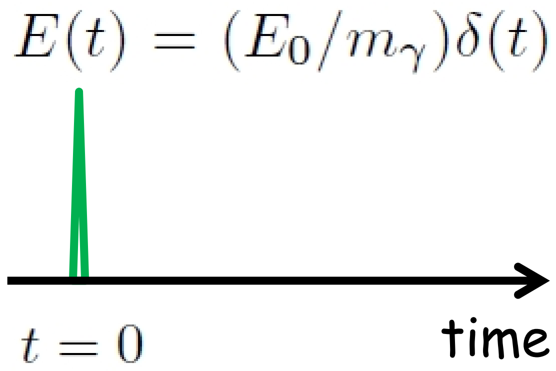
(1+3)-dim. QCD with a strong external magnetic field

$\downarrow$ lowest Landau level approximation

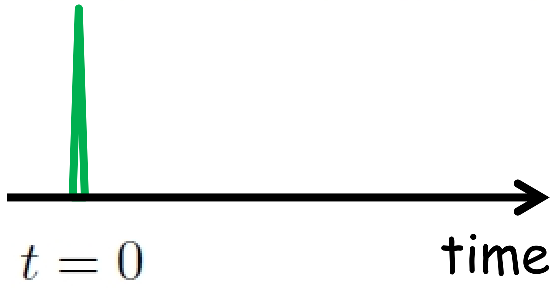
(1+1)-dim. QCD

$\downarrow$ external electric field + QED backreaction

temporal chiral spiral

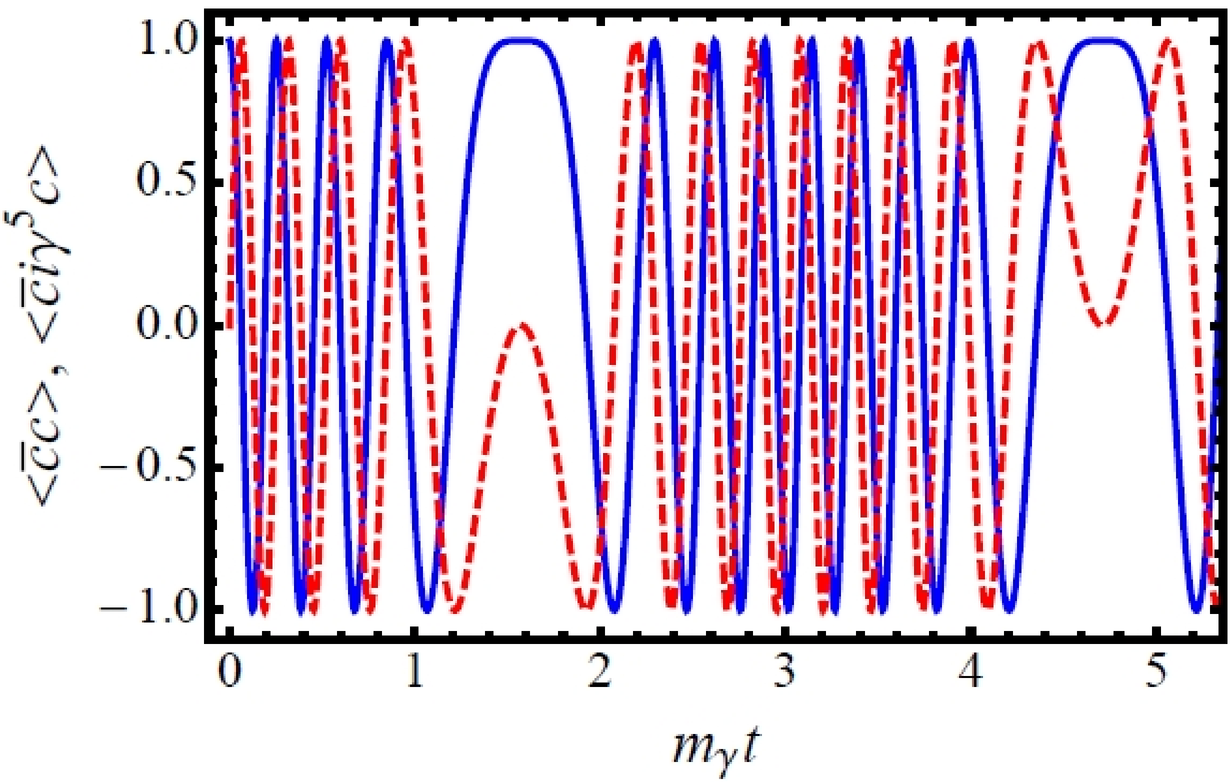

$E(t) = (E_0/m_\gamma)\delta(t)$
$t = 0$
time

$$E(t) = (E_0/m_\gamma)\delta(t)$$

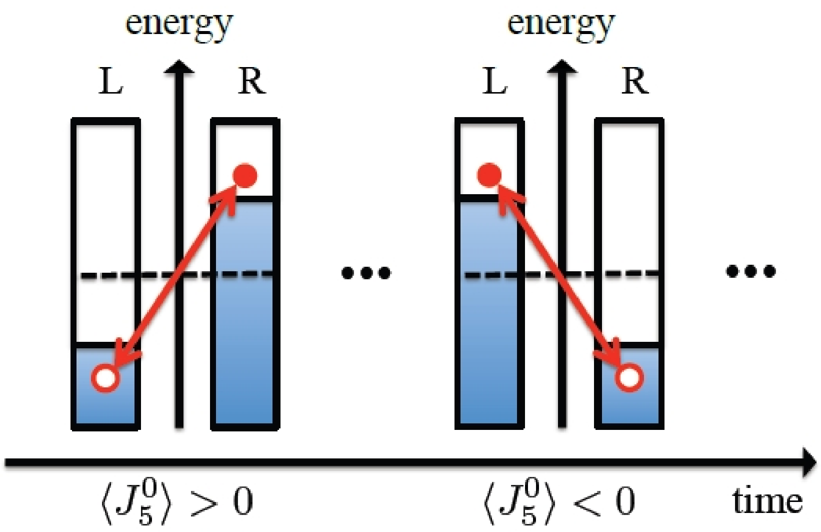


$$\langle \bar{c}c \rangle = \langle \bar{c}c \rangle_0 \cos\left(\frac{2e}{m_\gamma^2} E_0 \sin m_\gamma t \right)$$

$$\langle \bar{c}i\gamma^5 c \rangle = \langle \bar{c}c \rangle_0 \sin\left(\frac{2e}{m_\gamma^2} E_0 \sin m_\gamma t \right)$$



temporal chiral spiral



spatial chiral spiral (~ Fulde-Ferrell-Larkin-Ovchinnikov)

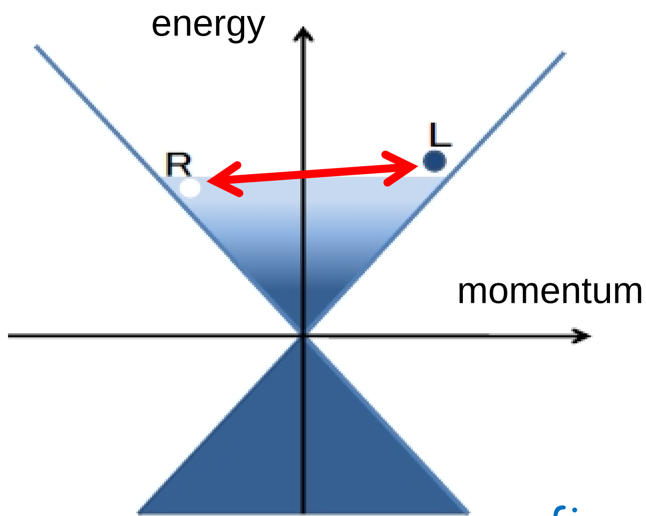
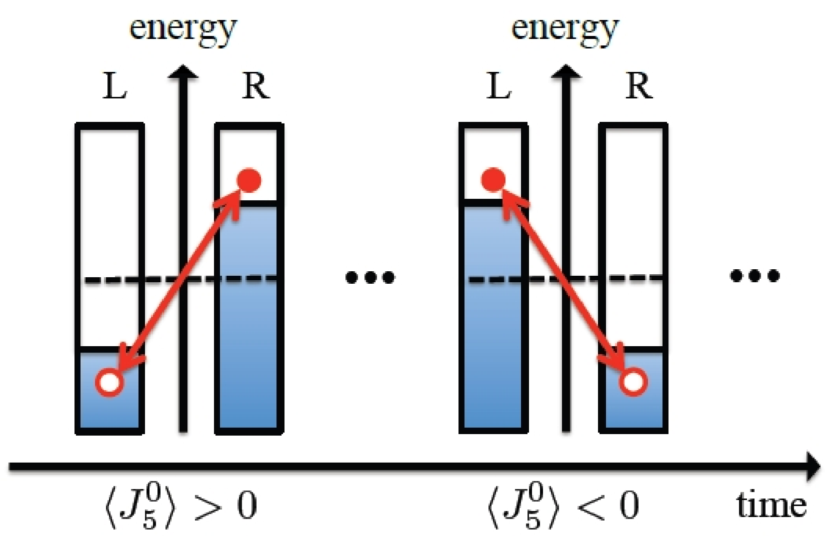


figure by Kojo et al (2010)

temporal chiral spiral



- 1+1 dimensions (= LLL approximation)

  solvable, non-dissipative

- 1+3 dimensions (= beyond LLL)

  dissipative?

おわり