Non-perturbatively renormalizable quantum gravity and hierarchy problem

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Introduction

- Quantum gravity
  - Challenging problem in elementary particle physics
- Einstein gravity describes many phenomena in long distance (low energy).
- Quantization of gravitational field (metric)
  - Perturbatively un-renormalizable
  - Need infinite number of counter term and UV cutoff.
  - Prediction power becomes weak.
- String theory? Loop quantum gravity…?

Introduction

Can we construct quantum gravity by quantum field theory?

- Yes!

Asymptotic safety

Talk plan

1. What is Asymptotic Safety?
2. Functional Renormalization Group
3. Review of pure gravity case
4. Higgs-Yukawa model non-minimally coupled to asymptotically safe gravity

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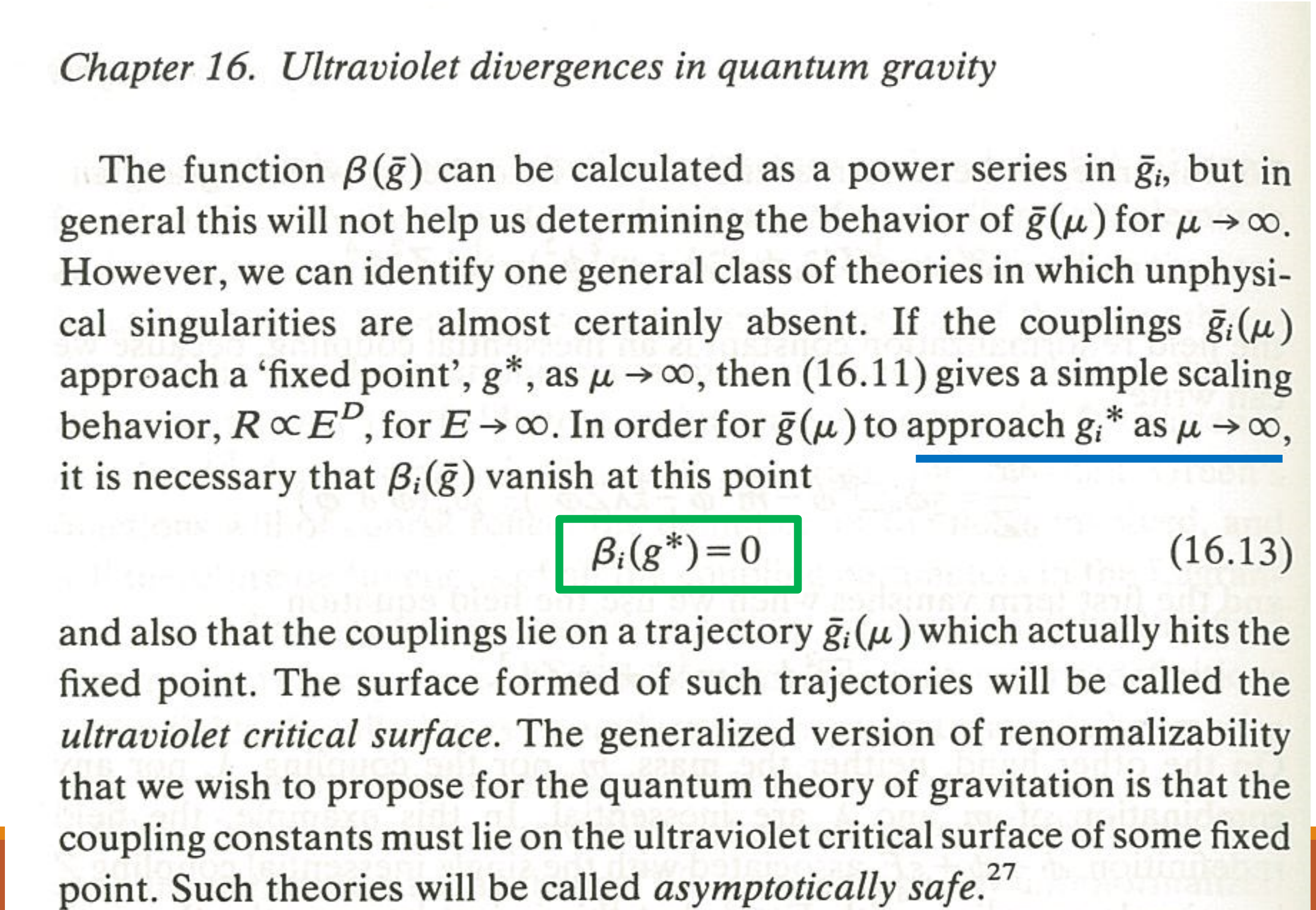
What's Asymptotic Safety?

- Suggested by S. Weinberg

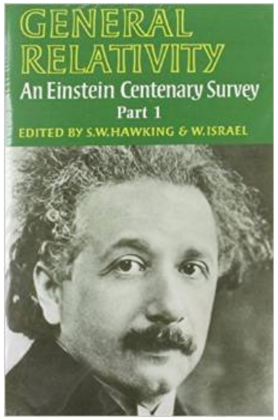


Chapter 16. Ultraviolet divergences in quantum gravity

The function $\beta(\bar{g})$ can be calculated as a power series in $\bar{g}_i$, but in general this will not help us determining the behavior of $\bar{g}(\mu)$ for $\mu \to \infty$. However, we can identify one general class of theories in which unphysical singularities are almost certainly absent. If the couplings $\bar{g}_i(\mu)$ approach a 'fixed point', g^*, as $\mu \to \infty$, then (16.11) gives a simple scaling behavior, $R \propto E^D$, for $E \to \infty$. In order for $\bar{g}(\mu)$ to approach g_i^* as $\mu \to \infty$, it is necessary that $\beta_i(\bar{g})$ vanish at this point

$$\beta_i(g^*) = 0 \tag{16.13}$$

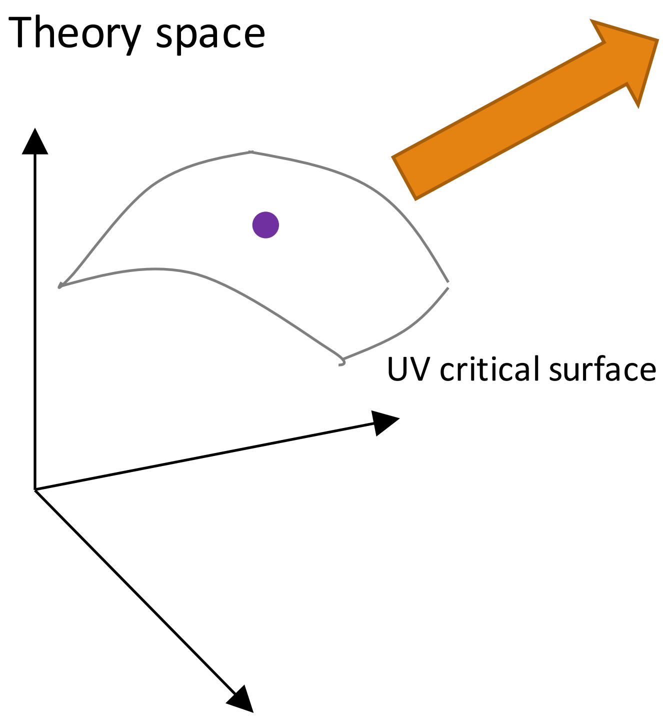
and also that the couplings lie on a trajectory $\bar{g}_i(\mu)$ which actually hits the fixed point. The surface formed of such trajectories will be called the *ultraviolet critical surface*. The generalized version of renormalizability that we wish to propose for the quantum theory of gravitation is that the coupling constants must lie on the ultraviolet critical surface of some fixed point. Such theories will be called *asymptotically safe.*[27]

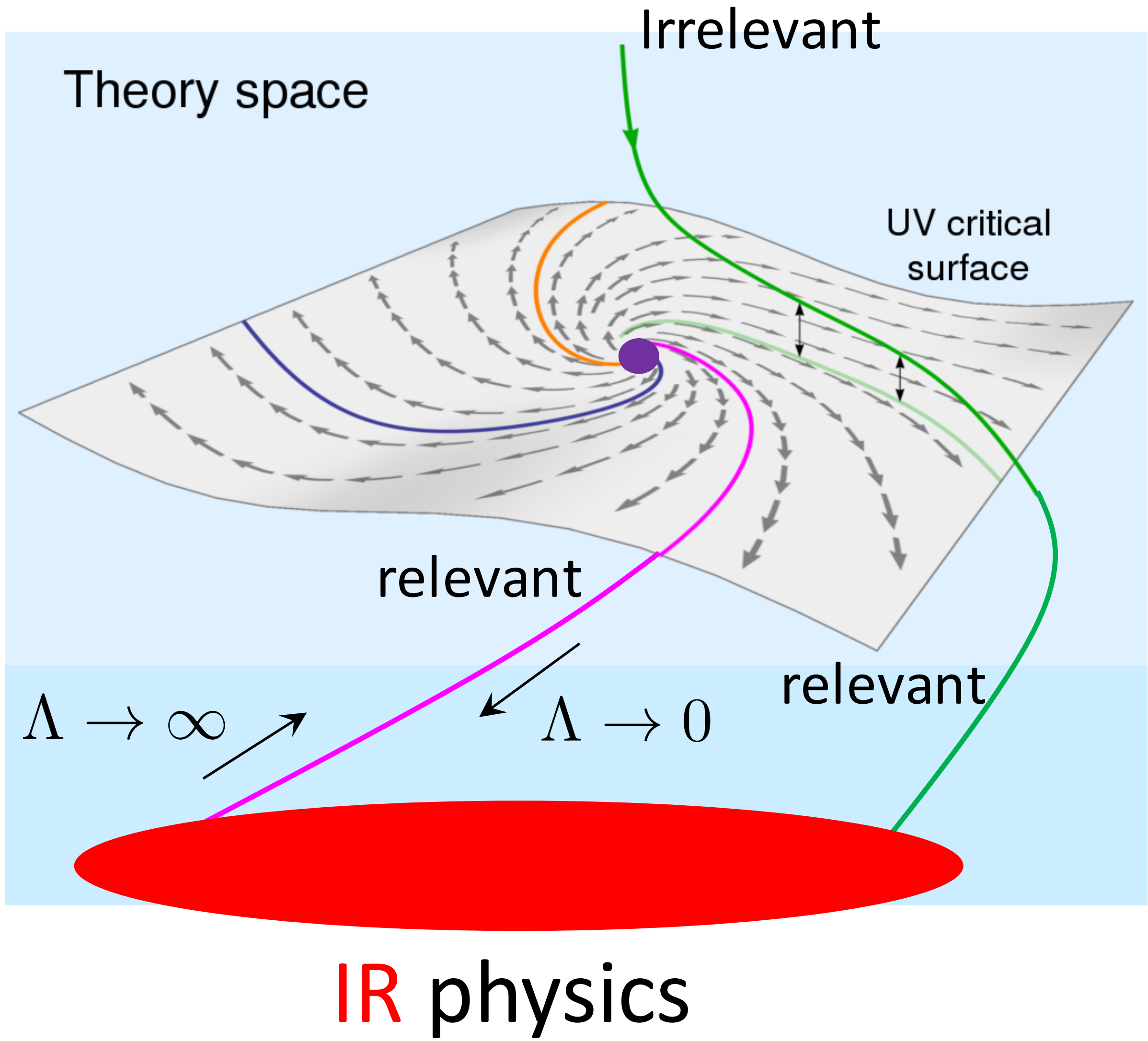


g^*

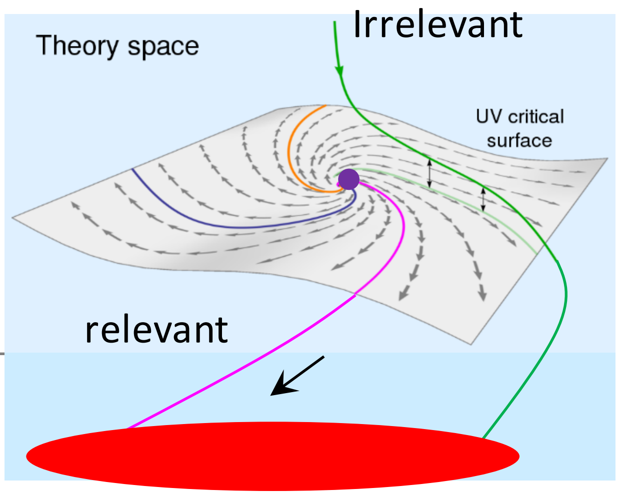
UV Fixed point

If UV fixed point exists,





Asymptotic safety



- UVFP must exist.
- UV critical surface is defined.
  - Spanned by relevant couplings
- Irrelevant flow is controlled by relevant flow in IR limit $\Lambda \to 0$.
- In UV limit $\Lambda \to \infty$ with fixed IR physics,
  - RG flow on the UV critical surface have no divergence.
  - It takes infinite RG transformation steps near the UVFP.
  - UV critical surface is UV complete theory.
- Finite number of relevant couplings = renormalizable
  - **Their coupling constants are free parameters.**
- Generalization of asymptotic free

What Weinberg showed

- $d = 2 + \epsilon$ gravity with ϵ expansion
  - Found a UVFP of Newton constant:

$$g^* = \frac{3}{38}\epsilon$$

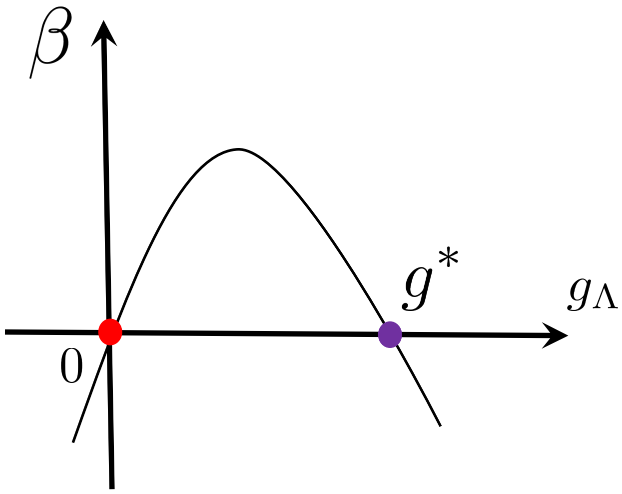
  - When $\epsilon > 0$, for $\Lambda \to \infty$

$$G_\Lambda \simeq \frac{g^*}{\Lambda^\epsilon} \to 0$$

Asymptotic free

- How is $d = 4$ gravity?

$$g_\Lambda = \Lambda^\epsilon G_\Lambda$$



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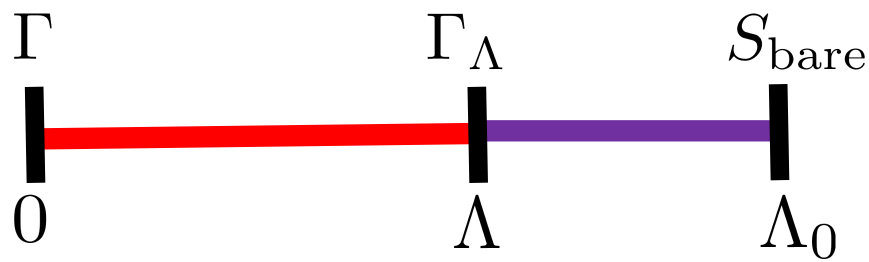
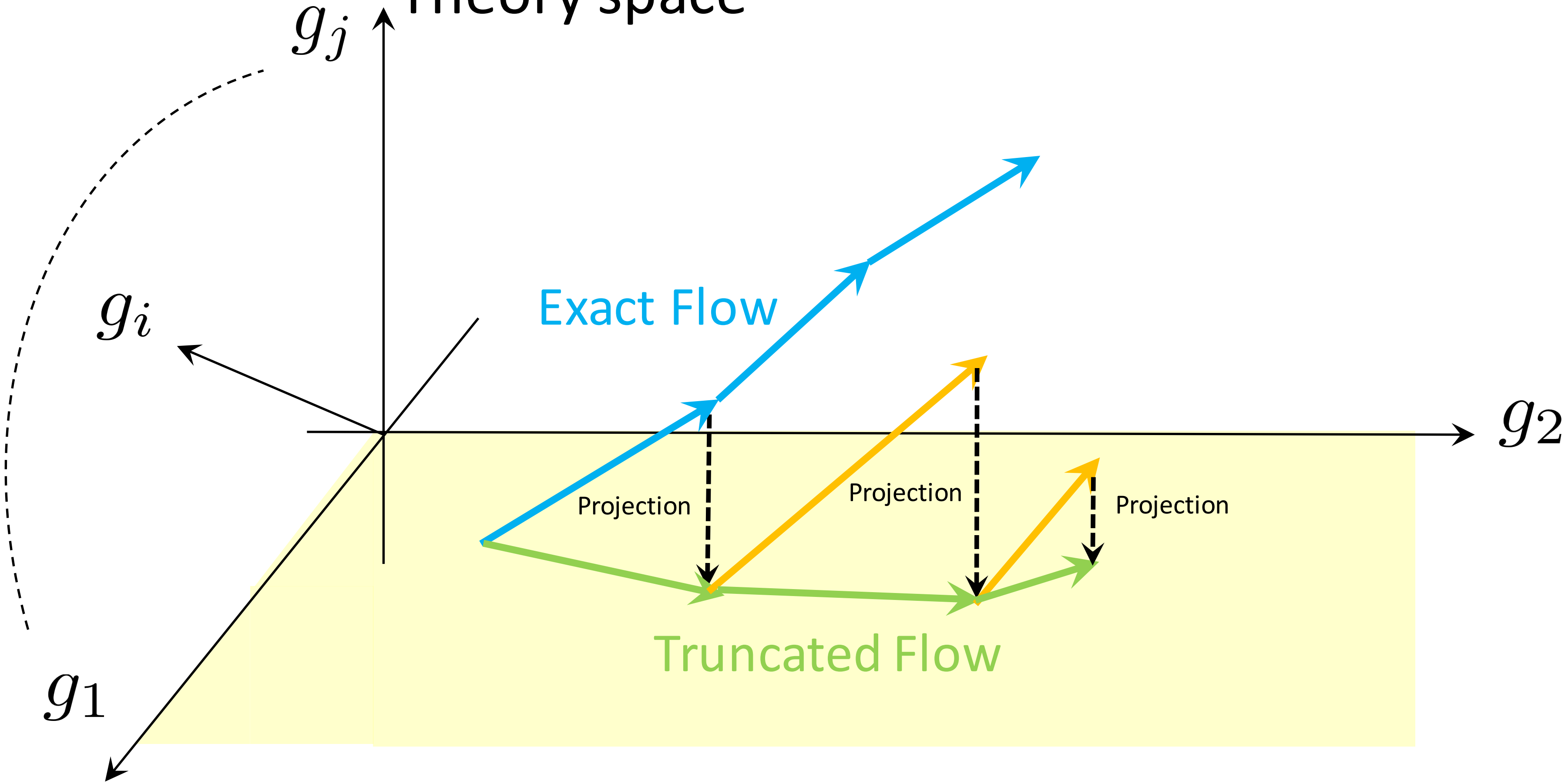
Functional Renormalization Group

- Also called Wilsonian RG, Exact RG, Non-perturbative RG...etc.
- Integrating higher momentum fluctuation, the low energy effective theory is defined.



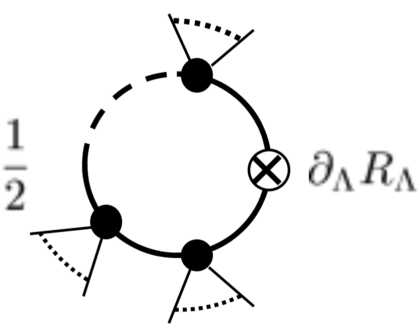
- Advantage:
  - Because of not asymptotic expansion, it is can be applied to strong dynamics and arbitrary dimension.
  - We can easily extend a theory space.
- Disadvantage:
  - The cutoff breaks gauge symmetry.
  - Dependence of gauge and cutoff scheme (due to the truncation).

Theory space



Wetterich equation

$$\partial_\Lambda \Gamma_\Lambda[\Phi] = \frac{1}{2}\mathrm{STr}\left\{\left[\frac{\overrightarrow{\delta}}{\delta\Phi}\Gamma_\Lambda[\Phi]\frac{\overleftarrow{\delta}}{\delta\Phi} + R_\Lambda\right]^{-1}\cdot(\partial_\Lambda R_\Lambda)\right\} = \frac{1}{2}$$



Approximation

- Expand effective action into operators
  - e.g. scalar theory:

$$\Gamma_\Lambda = \int d^d x \left[\frac{Z_\Lambda}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m_\Lambda^2 \phi^2 + \frac{\lambda_\Lambda}{4} \phi^4 + \cdots \right]$$

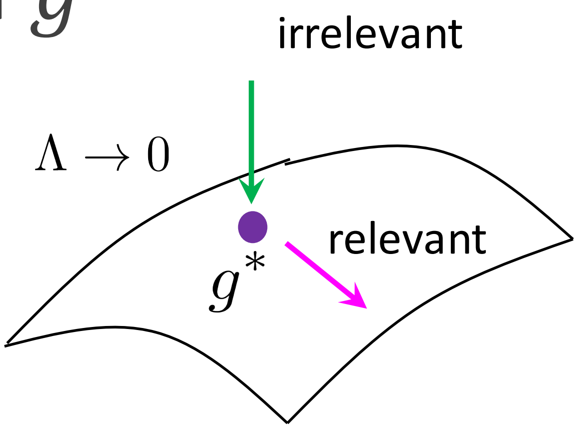
$$\Rightarrow \quad \begin{aligned} \partial_\Lambda Z_\Lambda &= \beta_Z(Z_\Lambda, m_\Lambda, \lambda_\Lambda, \cdots) \\ \partial_\Lambda m_\Lambda &= \beta_m(Z_\Lambda, m_\Lambda, \lambda_\Lambda, \cdots) \\ \partial_\Lambda \lambda_\Lambda &= \beta_\lambda(Z_\Lambda, m_\Lambda, \lambda_\Lambda, \cdots) \quad \ldots \end{aligned}$$

  - Including operators, approximation is improved.

How to find relevant direction?

- linearizing beta function around g^*

$$\partial_\Lambda g_i = \beta_i(g) \simeq \underbrace{\beta(g^*)}_{=0} + \sum_{j=1}^{N} \left.\frac{\partial \beta_i}{\partial g_j}\right|_{g=g^*} (g_j - g_j^*)$$



$$g_i(\Lambda) = g_i^* + \sum_j^N \zeta_j^i \left(\frac{\Lambda_0}{\Lambda}\right)^{\theta_j}$$

θ_j Critical exponent

$\theta_j > 0$
Go away from g^*

Count number of positive θ.

$\theta_j < 0$ close to g^*

Summary of this part

- FRG is useful method to study asymptotic safe gravity.

- Counting positive critical exponent, we can find dimension of UV critical surface.

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Einstein-Hilbert truncation

- Einstein-Hilbert truncation (Euclidean)

$$\Gamma_\Lambda = \frac{1}{16\pi G_\Lambda}\int d^4x\sqrt{g}\,[\lambda_\Lambda - R + \alpha_\Lambda R^2 + \cdots] + S_{\rm gh} + S_{\rm gf}$$

2 dimensional theory space

- Expand metric around back-ground field:

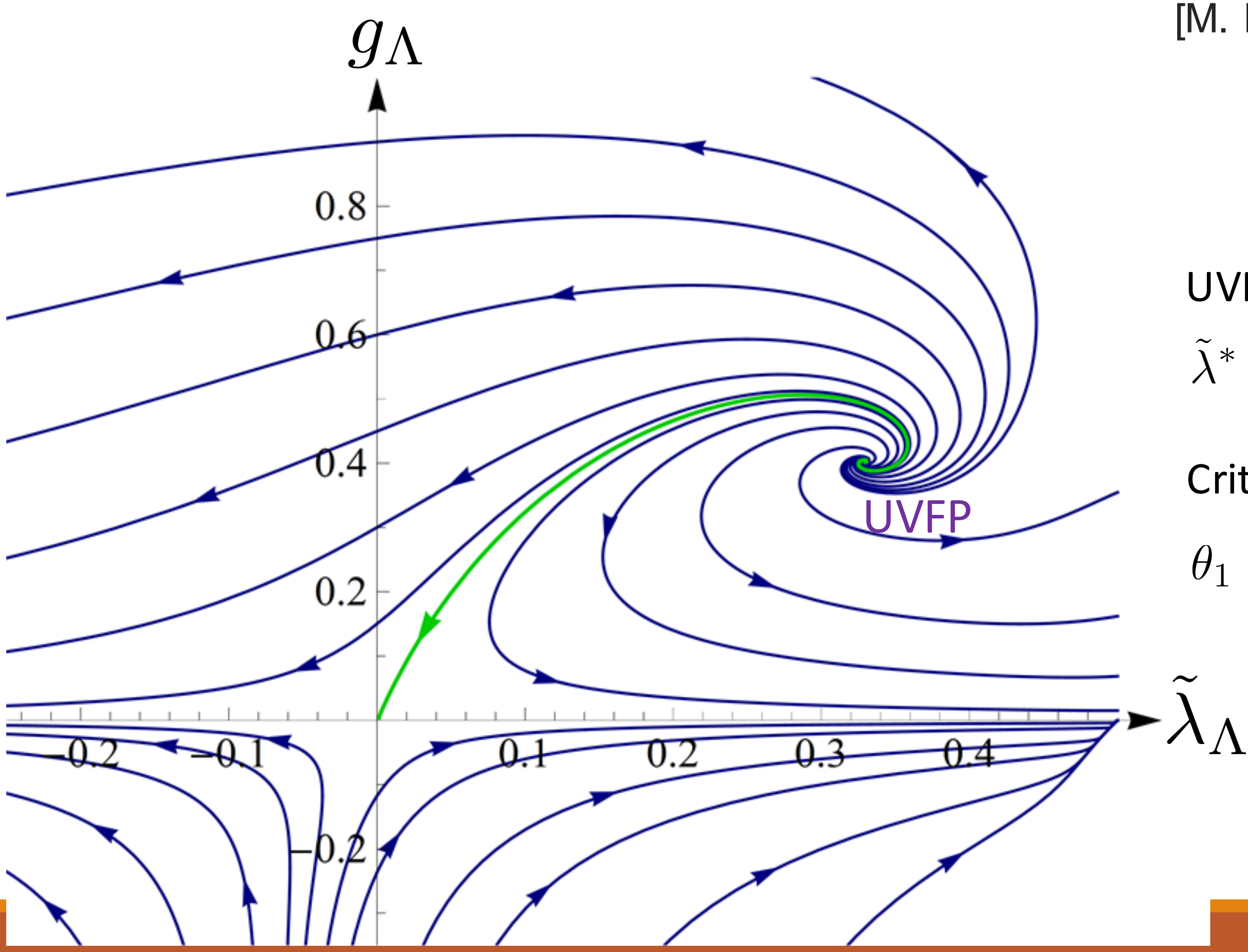
$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

- Take de-Donder gauge α=0, β=1

$$S_{\rm gf} = \frac{1}{2\alpha}\int d^4x\sqrt{g}\,F(\phi^2)\bar{g}^{\mu\nu}\Sigma_\mu\Sigma_\nu \qquad \Sigma_\mu = \partial^\nu h_{\nu\mu} - \frac{\beta+1}{4}\partial_\mu\bar{g}^{\rho\sigma}h_{\rho\sigma}$$

Einstein-Hilbert truncation

[M. Reuter, F. Saueressig, '02]



UVFP:

$$\tilde{\lambda}^* = 0.33, \quad g^* = 0.403$$

Critical exponents:

$$\theta_1 = 1.941, \quad \theta_2 = 3.147$$

relevant

Higher operators

[A. Codello, R. Percacci, C. Rahmede, '07]
[P. Machado, F. Saueressig, '07]
[A. Codello, R. Percacci, C. Rahmede, '09]
[A. Bonanno, A. Contillo, R. Percacci, '11]
[K. Falls, D.F. Litim, K. Nikolakopoulos, C. Rahmede, '13]

- $f(R)$ truncation:

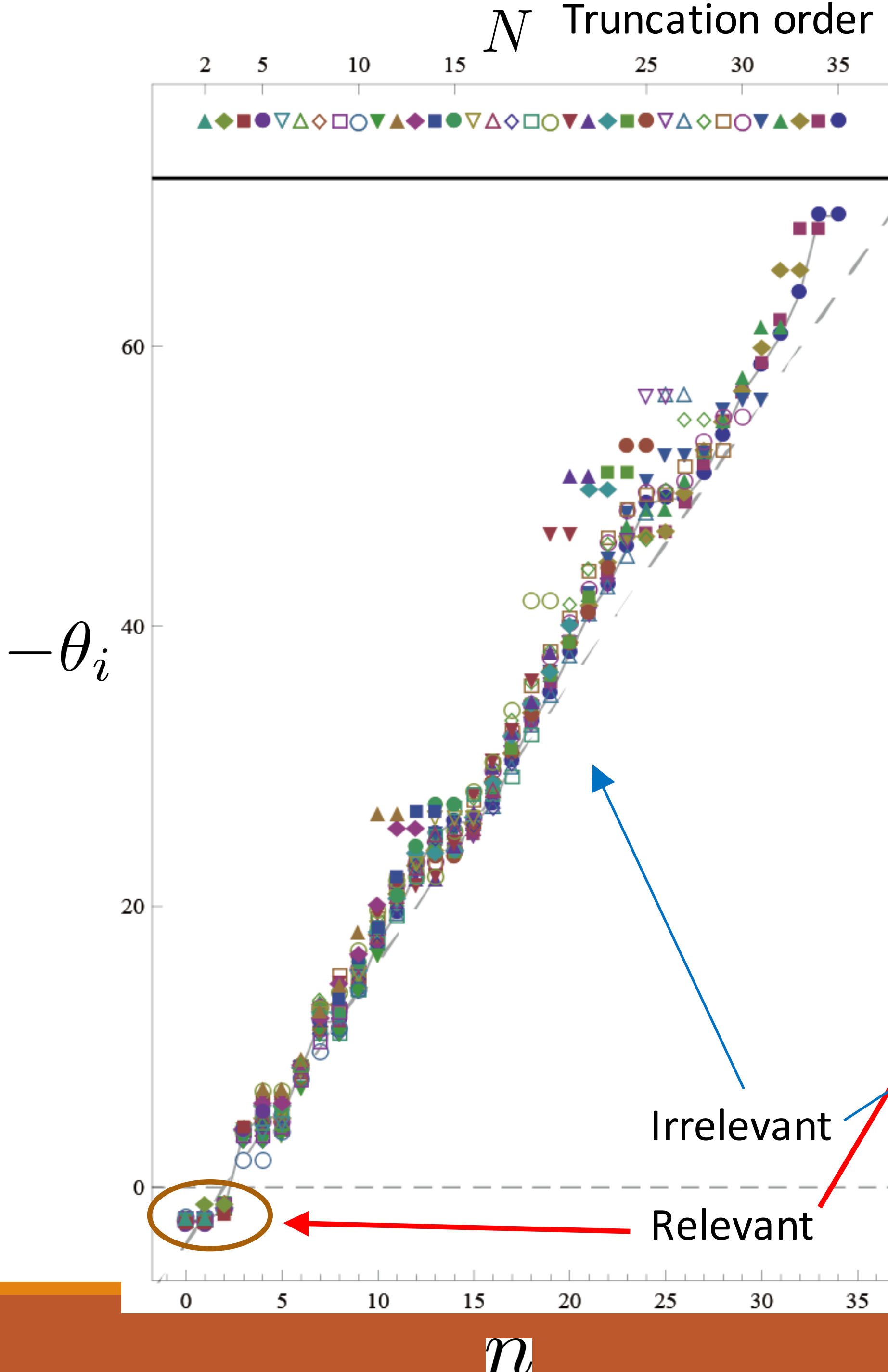
$$\Gamma_\Lambda[g] = \int d^4x \sqrt{g} f_\Lambda(R)$$

$$f_\Lambda(R) = \sum_{n=0}^{N} u_{n,\Lambda} R^n = u_{0,\Lambda} + u_{1,\Lambda} R + u_{2,\Lambda} R^2 + \cdots$$

$$= \sum_{n=0}^{N} g_n \left(\frac{R}{\Lambda^2} \right)^n \Lambda^4$$

$$-\Lambda \frac{\partial}{\partial \Lambda} g_n = \beta_n(\{g_n\})$$

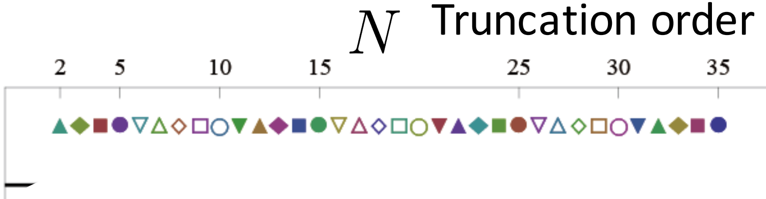
[K. Falls, D.F. Litim, K. Nikolakopoulos, C. Rahmede, '13]



$$f_\Lambda(R) = \sum_{n=0}^{N} g_n \left(\frac{R}{\Lambda^2}\right)^n \Lambda^4$$

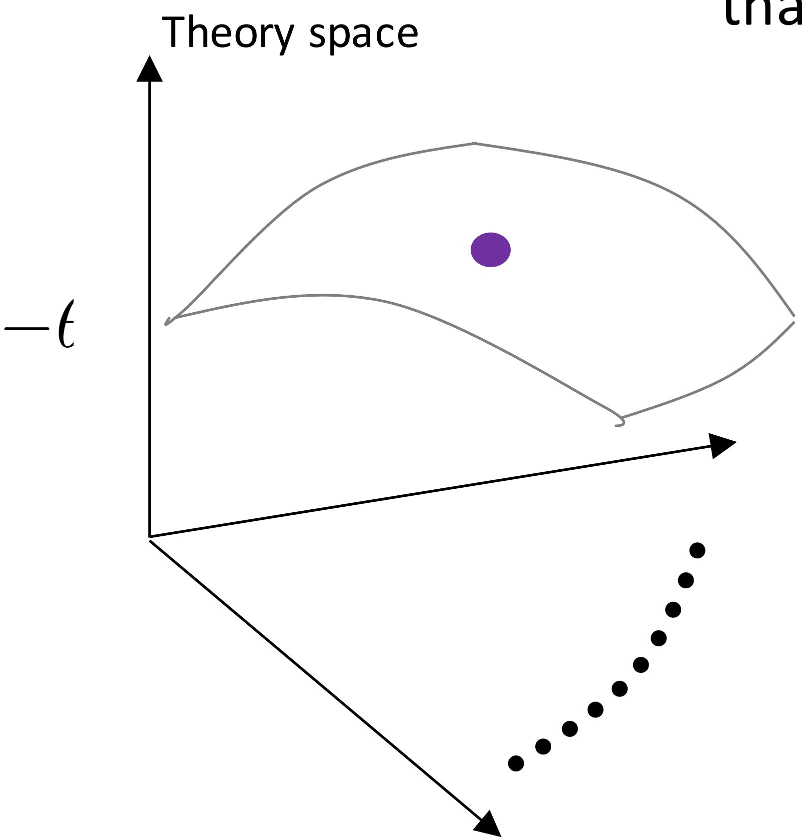
$$= g_0\Lambda^4 + g_1\left(\frac{R}{\Lambda^2}\right)\Lambda^4 + g_2\left(\frac{R}{\Lambda^2}\right)^2\Lambda^4 + \cdots$$

$$g_n(\Lambda) = g_n^* + \sum_i \zeta_n^i \left(\frac{\Lambda_0}{\Lambda}\right)^{\theta_i}$$



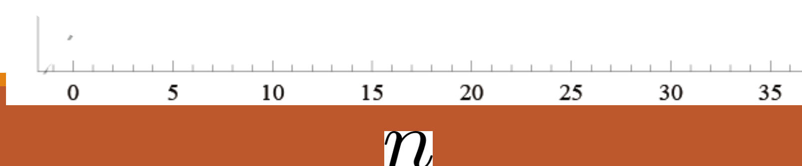
[K. Falls, D.F. Litim, K. Nikolakopoulos, C. Rahmede, '13]

UV critical surface is spanned by <u>3 operators</u>, that is, **<u>3 dimensional space</u>**.



- <u>**3 free parameters**</u>
- Renormalizable!

At low energy, gravity should be Einstein gravity.



Prediction of Higgs mass

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Before discovery of Higgs

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Asymptotic safety of gravity and the Higgs boson mass

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A B S T R A C T

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well. For $A_\lambda < 0$ one finds m_H in the interval $m_{\min} < m_H < m_{\max} \simeq 174$ GeV, now sensitive to A_λ and other properties of the short distance running. The case $A_\lambda > 0$ is favored by explicit computations existing in the literature.

Summary of this part

- Numerous studies show that
  - UVFP exists
  - UV critical surface is 3 dimensional.
- Succeed prediction of the Higgs mass.

Asymptotic safe gravity is one of good candidates of quantum gravity!

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Higgs-Yukawa model

- Non-minimally coupled to asymptotically safe gravity

$$\Gamma_\Lambda = \int d^4x\sqrt{g}\Big[V_\Lambda(\phi^2) - F_\Lambda(\phi^2)R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \bar{\psi}\nabla\!\!\!/\,\psi + y_\Lambda\phi\bar{\psi}\psi\Big] + S_{\text{gh}} + S_{\text{gf}}$$

- Potentials

$$V(\phi^2) = \Lambda_{\text{cc}} + m^2\phi^2 + \lambda\phi^4 + \cdots$$

Cosmological Const.

$$F(\phi^2) = M_{\text{pl}}^2 + \xi\phi^2 + \cdots$$

Planck mass (Newton const.)

Non-minimal coupling

Why consider this model?

- Toy model of Higgs inflation
- Higgs inflation

[F. Bezrukov, M. Shaposhnikov, '08]

$$S_J = \int d^4x\sqrt{-g}\left\{\left(1+\xi\frac{h^2}{M_{\rm pl}^2}+\cdots\right)\frac{M_{\rm pl}^2}{2}R+\frac{1}{2}\partial_\mu h\partial^\mu h - V(h^2)\right\}$$

$$\left(1+\xi\frac{h^2}{M_{\rm pl}^2}+\cdots\right)g_{\mu\nu}\to g^E_{\mu\nu} \quad \Longrightarrow \quad V(h^2)\to\frac{V(h^2)}{\left(1+\xi\frac{h^2}{M_{\rm pl}^2}+\cdots\right)^2}$$

- To realize Higgs inflation, needs large ξ
  - At least $\xi\sim 10$ … Is it possible?

Without fermion

[R. Percacci, D. Perini' 03]
[G. Narain, R. Percacci '09]

- Scalar-gravity system
  - 5 dimensional theory space: $\{M_{\mathrm{P}}^2,\ \Lambda_{\mathrm{cc}},\ m^2,\ \xi,\ \lambda\}$
  - UVFP exists.
  - Critical exponents:

	$M_{\mathrm{P}}^2,\ \Lambda_{\mathrm{cc}}$	$m^2,\ \xi$	λ
$\theta_i =$	$2.143 \pm 2.879i$	$0.143 \pm 2.879i$	-2.627

  - Non-minimal coupling $\xi\phi^2 R$ found to be **relevant**.
  - ξ is a free parameter.
  - Then, in principle, ξ can have large value.

With fermion

[K-y Oda, **M. Y.**, '15]

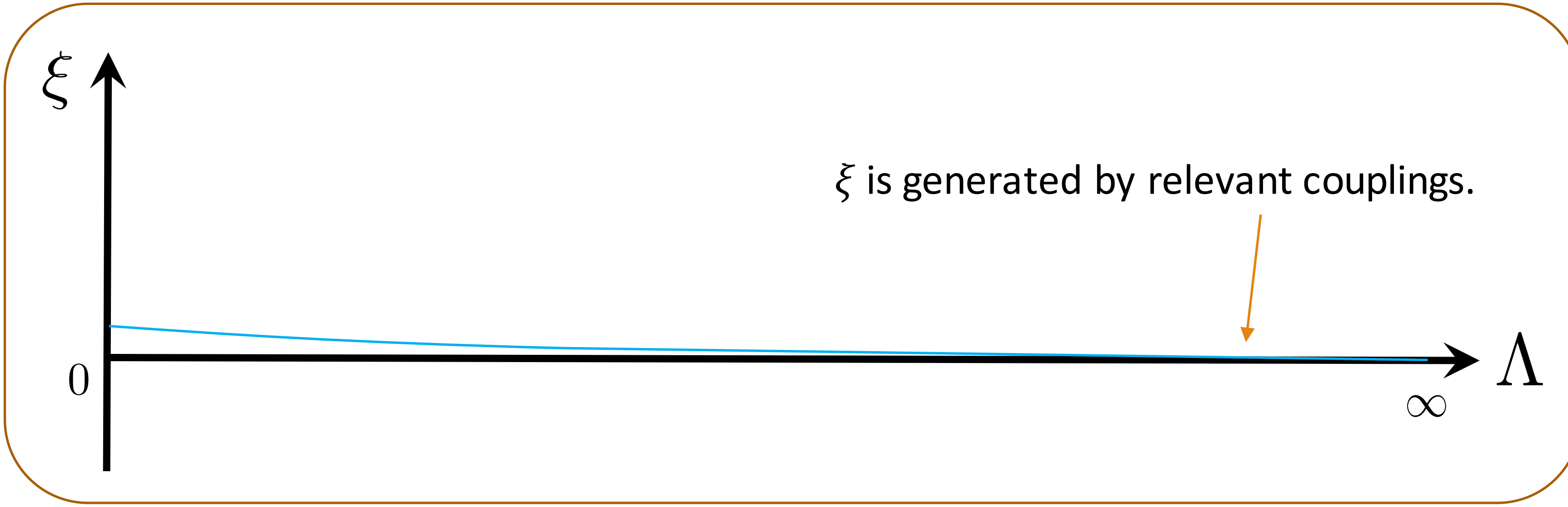
- 6 dimensional theory space: $\{M_{\rm P}^2,\ \Lambda_{\rm cc},\ m^2,\ \xi,\ \lambda,\ y\}$
- Critical exponents:

	$M_{\rm P}^2,\ \Lambda_{\rm cc}$	$m^2,\ \xi$	λ	y
$\theta_i =$	$1.509 \pm 2.4615i$	$-0.4909 \pm 2.461i$	-2.6069	-1.464

- Fermion fluctuation makes non-minimal coupling $\xi\phi^2 R$ **irrelevant**.
- $m^2,\ \xi$ cannot be a free parameter!

Questions

- Can the non-minimal coupling become large?
  - In the Higgs inflation scenario, the non-minimal coupling should be large.
- How is the hierarchy problem?
  - At least we do not have to do fine-tuning of Higgs mass.



- Quantum correction is not large.
- ξ does not become large at low energy scale.

$$\partial_t \xi_2 = -\frac{1}{576\pi^2}\Bigg[\cdots$$

$$+\frac{27\left(1+2\xi_2\right)\left(1-10\xi_2-16\xi_2^2\right)}{\left(1+2\lambda_2\right)^2\left(\xi_0-\lambda_0\right)}+\frac{108\xi_0\xi_2\left(1+2\xi_2\right)^2}{\left(1+2\lambda_2\right)^2\left(\xi_0-\lambda_0\right)^2}+\frac{72\lambda_4}{\left(1+2\lambda_2\right)^2}\frac{1+12\xi_2+2\lambda_2}{1+2\lambda_2}\Bigg]$$

$$+\frac{\partial_t\xi_0-2\xi_0}{1152\pi^2\xi_0}\Bigg[\frac{1+2\lambda_2}{\xi_0-\lambda_0}\left(3+\frac{18\xi_0}{\xi_0-\lambda_0}+\frac{20\xi_0^2}{\left(\xi_0-\lambda_0\right)^2}\right)+\frac{15\xi_2}{\xi_0}-\frac{6\left(1+\xi_2\right)}{\xi_0-\lambda_0}-\frac{10\xi_0\left(3+4\xi_2\right)}{\left(\xi_0-\lambda_0\right)^2}$$

$$-\frac{20\xi_0^2\left(1+2\xi_2\right)}{\left(\xi_0-\lambda_0\right)^3}-\frac{3\left[\lambda_0-\xi_0\left(5-4\xi_2\right)\right]\left(1+2\xi_2\right)}{\left(1+2\lambda_2\right)\left(\xi_0-\lambda_0\right)^2}+\frac{36\xi_0\xi_2\left(1+2\xi_2\right)^2}{\left(1+2\lambda_2\right)^2\left(\xi_0-\lambda_0\right)^2}\Bigg]$$

$$+\frac{\partial_t\xi_2}{1152\pi^2\xi_0}\Bigg[-15+\frac{54\xi_0}{\xi_0-\lambda_0}+\frac{20\xi_0^2}{\left(\xi_0-\lambda_0\right)^2}-\frac{6\xi_0\left(7+2\xi_2\right)}{\left(1+2\lambda_2\right)\left(\xi_0-\lambda_0\right)}-\frac{144\xi_0\xi_2\left(1+2\xi_2\right)}{\left(1+2\lambda_2\right)\left(\xi_0-\lambda_0\right)}\Bigg]$$

$$-\frac{N_f y^2}{48\pi^2}, \qquad (55)$$

How is Hierarchy problem?

- Scalar mass is **irrelevant**.
  - No bare mass term in bare theory.
- Our result indicates that the theory is asymptotically conformal in matter sector.
- Is it a solution for the hierarchy problem?
- No…

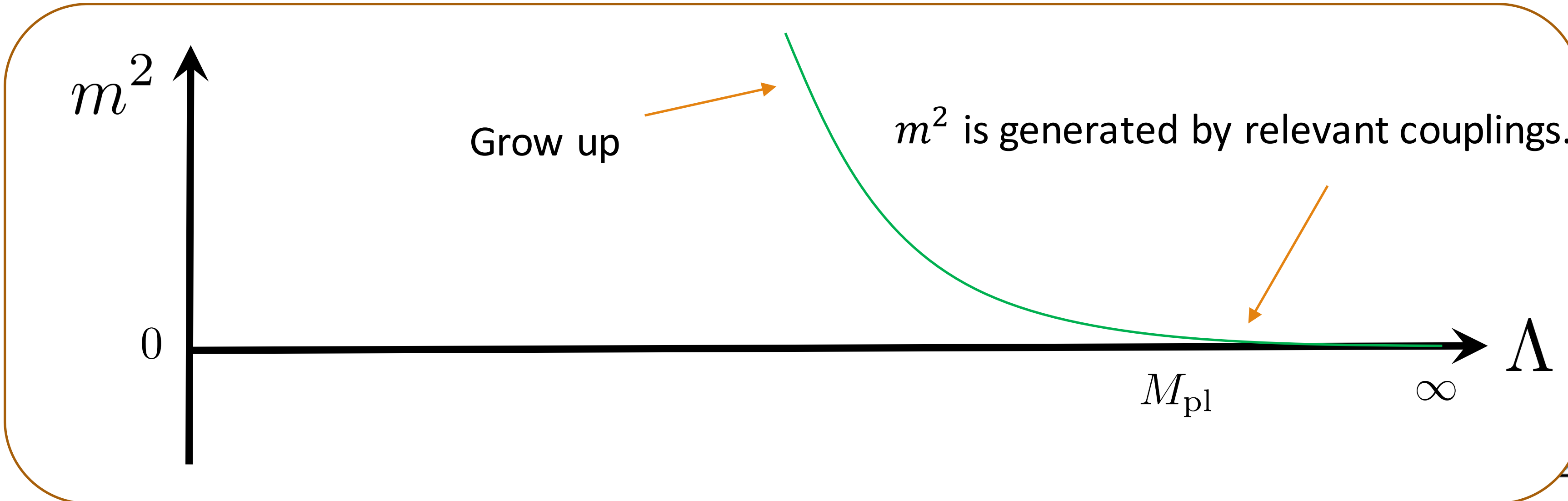
How is Hierarchy problem?

- Hierarchy problem in FRG.
  - E.g. simplified Yukawa model:

$$-\Lambda \frac{\partial}{\partial \Lambda} m^2 = -\frac{y^2}{4\pi^2}\Lambda^2$$

$$m^2(\Lambda^2)\Big|_{\Lambda \to 0} = m_0^2 - \frac{y^2}{8\pi^2}\Lambda_0^2$$

Fine-tuning between bare mass and loop correction



$$+\frac{\partial_t\xi_0-2\xi_0}{96\pi^2\xi_0}\left[-\frac{2\xi_2}{\xi_0}+\frac{3\xi_0\left(1+2\xi_2\right)}{2\left(\xi_0-\lambda_0\right)^2}-\frac{3\xi_0\left(1+2\xi_2\right)^2}{2\left(1+2\lambda_2\right)\left(\xi_0-\lambda_0\right)^2}\right]$$
$$+\frac{1}{96\pi^2}\frac{\partial_t\xi_2}{\xi_0}\left[2-\frac{3\xi_0}{\xi_0-\lambda_0}+\frac{6\xi_0\left(1+2\xi_2\right)}{\left(1+2\lambda_2\right)\left(\xi_0-\lambda_0\right)}\right]-\frac{N_{\rm f}y^2}{8\pi^2}, \qquad (56)$$

- Canonical scaling appears.
- Due to this term, scalar mass becomes large.
- Fine-tuning is still required.

Summary

- Asymptotically safe gravity
  - One of candidates of quantum gravity
- Higgs-Yukawa model
  - Toy model of Higgs inflation
- We showed that fermionic fluctuation makes ξ irrelevant.
- ξ cannot become large.
- Hierarchy problem still remains.
- Extension of theory space.
  - Gauge fields

Appendix

Details of calculation

- Expand fields into background and fluctuation

$$\widehat{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu},$$
$$\widehat{\phi} = \phi + \varphi,$$
$$\widehat{\psi} = \psi + \chi,$$

$$\Gamma_\Lambda[g_{\mu\nu}, \phi, \psi; h_{\mu\nu}, \varphi, \chi] = \int d^4x \sqrt{\widehat{g}} \left\{ V_\Lambda\left(\widehat{\phi}^2\right) - F_\Lambda\left(\widehat{\phi}^2\right)\widehat{R} + \frac{1}{2}\widehat{g}^{\mu\nu}\,\partial_\mu\widehat{\phi}\,\partial_\nu\widehat{\phi} + \bar{\widehat{\psi}}\widehat{\not{D}}\widehat{\psi} + y_\Lambda\widehat{\phi}\bar{\widehat{\psi}}\widehat{\psi} \right\} + S_{\mathrm{GF}} + S_{\mathrm{gh}}, \tag{10}$$

FRG

$$\partial_t \Gamma_\Lambda = \frac{1}{2}\mathrm{Tr}\,\frac{\partial_t \mathcal{R}_\Lambda}{\Gamma_\Lambda^{(1,1)} + \mathcal{R}_\Lambda}\bigg|_{h^\perp h^\perp} + \frac{1}{2}\mathrm{Tr}'\,\frac{\partial_t \mathcal{R}_\Lambda}{\Gamma_\Lambda^{(1,1)} + \mathcal{R}_\Lambda}\bigg|_{\xi\xi} + \frac{1}{2}\mathrm{Tr}''\,\frac{\partial_t \mathcal{R}_\Lambda}{\Gamma_\Lambda^{(1,1)} + \mathcal{R}_\Lambda}\bigg|_{\mathrm{SS}}$$
$$- \mathrm{Tr}\,\frac{\partial_t \mathcal{R}_\Lambda}{\Gamma_\Lambda^{(1,1)} + \mathcal{R}_\Lambda}\bigg|_{\overline{\chi}\chi} - \mathrm{Tr}\,\frac{\partial_t \mathcal{R}_\Lambda}{\Gamma_\Lambda^{(1,1)} + \mathcal{R}_\Lambda}\bigg|_{\bar{C}^\perp C} - \mathrm{Tr}\,\frac{\partial_t \mathcal{R}_\Lambda}{\Gamma_\Lambda^{(1,1)} + \mathcal{R}_\Lambda}\bigg|_{\bar{C}C},$$

Need two point function with respect to fluctuations

Two point function

$$
\begin{aligned}
\Gamma^{(2)}_{\Lambda}[\Phi;\Upsilon] = \frac{1}{2}\int \mathrm{d}^4x\sqrt{g}\Big[& -\frac{1}{2}F(\phi^2)\,h^{\mu\nu}\partial^2 h_{\mu\nu} + \frac{1}{2}F(\phi^2)\,h\partial^2 h - F(\phi^2)\,h\partial_\mu\partial_\nu h^{\mu\nu} + F(\phi^2)\,h^{\mu\nu}\partial_\mu\partial_\rho h^{\rho}{}_{\nu} \\
& + \left(\frac{1}{4}h^2 - \frac{1}{2}h_{\mu\nu}h^{\mu\nu}\right)\left(V(\phi^2) + y\phi\overline{\psi}\psi - F(\phi^2)\,R\right) \\
& + F(\phi^2)\,hh^{\mu\nu}R_{\mu\nu} - F(\phi^2)\,h_\rho{}^\nu h^{\mu\rho}R_{\mu\nu} - F(\phi^2)\,h^{\mu\nu}R_{\rho\mu\sigma\nu}h^{\rho\sigma} \\
& - \frac{1}{16}h_\rho{}^\mu\partial_\nu h_{\sigma\mu}\overline{\psi}\gamma^\nu[\gamma^\rho,\gamma^\sigma]\psi\Big] \\
+ \int \mathrm{d}^4x\sqrt{g}\,\varphi\Big[& -2\phi F'(\phi^2)\left\{\partial_\mu\partial_\nu - \partial^2 g_{\mu\nu}\right\}h^{\mu\nu} \\
& + h\left\{\phi V'(\phi^2) + \frac{1}{2}y\overline{\psi}\psi - \phi F'(\phi^2)\,R\right\} + h^{\mu\nu}\left\{2\phi F'(\phi^2) + R_{\mu\nu}\right\}\Big] \\
+ \int \mathrm{d}^4x\sqrt{g}\,h & \left[\frac{1}{2}y\phi\left(\overline{\psi}\chi + \overline{\chi}\psi\right)\right] \\
+ \frac{1}{2}\int \mathrm{d}^4x\sqrt{g}\,\varphi & \Big[\left\{-\partial^2 + 2V'(\phi^2) + 4\phi^2V''(\phi^2)\right\} - R\left\{2F'(\phi^2) + 4\phi^2F''(\phi^2)\right\}\Big]\varphi \\
+ \int \mathrm{d}^4x\sqrt{g} & \left[\frac{1}{4}\left(-\partial_\mu h + \partial_\nu h^\nu{}_\mu\right)\left(\overline{\psi}\gamma^\mu\chi - \overline{\chi}\gamma^\mu\psi\right)\right] \\
+ \int \mathrm{d}^4x\sqrt{g}\,\varphi & \left[y\left(\overline{\psi}\chi + \overline{\chi}\psi\right)\right] + \int \mathrm{d}^4x\sqrt{g}\,\overline{\chi}\left[\partial\!\!\!/ + y\phi\right]\chi + S_{\mathrm{GF}} + S_{\mathrm{gh}}, \qquad (17)
\end{aligned}
$$

3.3 York decomposition

We decompose the graviton fluctuation as [131]

$$h_{\mu\nu} = h^{\perp}_{\mu\nu} + \partial_\mu \tilde{\xi}_\nu + \partial_\nu \tilde{\xi}_\mu + \left(\partial_\mu \partial_\nu - \frac{1}{D} g_{\mu\nu} \partial^2\right) \tilde{\sigma} + \frac{1}{D} g_{\mu\nu} h, \tag{19}$$

where $\partial^2 := g^{\mu\nu}\partial_\mu\partial_\nu$; $h^{\perp}_{\mu\nu}$ is the transverse and traceless tensor field with spin 2; $\tilde{\xi}_\mu$ is the transverse vector field with spin 1; and $\tilde{\sigma}$ and $h := g^{\mu\nu}h_{\mu\nu}$ are the scalar fields with spin 0. These fields satisfy the following conditions: $g^{\mu\nu}h^{\perp}_{\mu\nu} = 0$, $\partial^\nu h^{\perp}_{\mu\nu} = 0$, and $\partial^\mu \tilde{\xi}_\mu = 0$.

We decompose the ghosts into the transverse and scalar components:

$$\begin{aligned} C_\mu &= C^{\perp}_\mu + \partial_\mu \tilde{C}, \\ \bar{C}_\mu &= \bar{C}^{\perp}_\mu + \partial_\mu \bar{C}, \end{aligned} \tag{20}$$

where $\tilde{C}$, $\bar{C}$ are spin-0 scalar fields and $C^{\perp}_\mu$, $\bar{C}^{\perp}_\mu$ are spin-1 transverse vector fields that satisfy $\partial^\mu C^{\perp}_\mu = \partial^\mu \bar{C}^{\perp}_\mu = 0$.

Cut off scheme

$$R_\Lambda(p) \sim \begin{cases} 0 & (\Lambda \to 0) \\ \infty & (\Lambda \to \Lambda_0) \end{cases}$$

- Exponential cut-off

$$R_\Lambda(z; s) = \frac{sz}{\exp(sz) - 1} \qquad \text{for} \qquad s > 0$$

- Sharp cut-off

$$R_\Lambda(z) = \hat{R}\theta(\Lambda^2 - z)$$

- Optimized cut-off

$$R_\Lambda(z) = (\Lambda^2 - z)\theta(\Lambda^2 - z)$$

Gauge fixing and ghost action

- Gauge fixing action

$$S_{\rm gf} = \frac{1}{2\alpha}\int d^4x\sqrt{\bar{g}}F(\phi^2)\bar{g}^{\mu\nu}\Sigma_\mu\Sigma_\nu$$

$$\Sigma^\mu = \bar{\partial}_\nu h^{\nu\mu} - \frac{\beta+1}{d}\bar{\partial}^\mu h$$

- Ghost action

$$S_{\rm gh} = \int d^4x\sqrt{\bar{g}}\bar{C}_\mu\left[-\delta^\rho_\mu\bar{\partial}^2 - \left(1-\frac{1+\beta}{2}\right)\bar{\partial}_\mu\bar{\partial}_\rho + \bar{R}^\rho_\mu\right]C_\rho$$