Lattice QCD approach to axion dark matter

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Based on

R. Kitano and NY, JHEP 1510, 136 (2015)
+ work in progress

Seminar@Kyoto 2016/1/20

Strong CP problem

Symmetry in the SM does not prohibit the θ term,

$$\mathcal{L}_\theta = \frac{i\theta}{32\pi^2}\epsilon^{\mu\nu\sigma\rho}\mathrm{Tr}(G_{\mu\nu}G_{\sigma\rho}) = \frac{i\theta}{32\pi^2}G\tilde{G}$$

$G_{\mu\nu}$: gluon field strength

✓ Two origins : $\theta = \theta_{\rm QCD} + \theta_{\rm Yukawa}$

✓ $\theta_{\rm QCD}$, $\theta_{\rm Yukawa}$: free parameter

✓ Violate P and CP

✓ NEDM exp: $\theta = \theta_{\rm QCD} + \theta_{\rm Yukawa} \lesssim 10^{-10}$

➡ **Why is θ so small?**

Two possible solutions

✓ $m_{\mathrm{u}} = 0$

Chiral rotation of u_L and/or u_R gets rid of θ.

θ-term $\Rightarrow$ unphysical

✓ Peccei-Quinn mechanism

θ-term dynamically vanishes.

(more explanations below)

$m_{\mathrm{u}} = 0$?

Citation: K.A. Olive *et al.* (Particle Data Group), Chin. Phys. C, **38**, 090001 (2014) and 2015 update

Light Quarks (u, d, s)

OMITTED FROM SUMMARY TABLE

u-QUARK MASS

The u-, d-, and s-quark masses are estimates of so-called "current-quark masses," in a mass- independent subtraction scheme such as $\overline{\mathrm{MS}}$. The ratios m_u/m_d and m_s/m_d are extracted from pion and kaon masses using chiral symmetry. The estimates of d and u masses are not without controversy and remain under active investigation. Within the literature there are even suggestions that the u quark could be essentially massless. The s-quark mass is estimated from SU(3) splittings in hadron masses.

"m_u=0" seems not to be completely excluded.

We have normalized the $\overline{\mathrm{MS}}$ masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
2.3 $^{+0.7}_{-0.5}$ OUR EVALUATION	See the ideogram below.			
2.36±0.24	[1] CARRASCO	14	LATT	$\overline{\mathrm{MS}}$ scheme
2.15±0.03±0.10	[2] DURR	11	LATT	$\overline{\mathrm{MS}}$ scheme
2.24±0.10±0.34	[3] BLUM	10	LATT	$\overline{\mathrm{MS}}$ scheme

Peccei-Quinn mechanism [Peccei and Quinn (77)]

Introduce SM singlet complex scalar $\varphi(x)=|\varphi(x)|e^{i\mathbf{a(x)}/f_a}$
+ some more (model dependent)

$$\Rightarrow \quad \theta \to \theta + \frac{a(x)}{f_a}$$

$$\Rightarrow \quad \mathcal{L}_{\rm eff} = (\partial_\mu a)^2 + \frac{\chi_t}{2}(\theta + \frac{a}{f_a})^2 + \cdots$$

periodic: $V(\theta+a/f_a) = V(\theta+a/f_a+2n\pi)$

$$\theta + \frac{a}{f_a} = 0 \qquad \text{(dynamically selected)}$$

CP conserving vacuum is realized as a potential minimum.
⇒Strong CP problem is gone.

Axion mass

$$\mathcal{L}_{\rm eff} = (\partial_\mu a)^2 + \frac{\chi_t}{2}(\theta + \frac{a}{f_a})^2 + \cdots$$

Axion mass: $m_a^2 = \chi_t / 2 f_a^2$

χ_t : topological susceptibility

$$\chi_t = -\frac{1}{V}\frac{1}{Z}\frac{d^2 Z}{d\theta^2}\bigg|_{\theta=0} = \frac{\langle Q^2 \rangle}{V}$$

Q: topological charge

$$Q = \frac{1}{32\pi^2}\int d^4x \, G\tilde{G}$$

At T=0, χ_t =[70(9) MeV]^4 $\Rightarrow m_a \approx 6 \times 10^{-6}\,\mathrm{eV}\left(\frac{10^{12}\mathrm{GeV}}{f_a/N}\right)$

Constraint on m_a - f_a plane

Axion coupling to SM particles:

$$\frac{a(x)}{f_a} G\tilde{G} \text{ (gluon)}, \qquad \frac{a(x)}{f_a} F\tilde{F} \text{ (photon)}$$

$\sim c \times 1 / f_a$

Search for $a \to \gamma\,\gamma$

$\Downarrow$

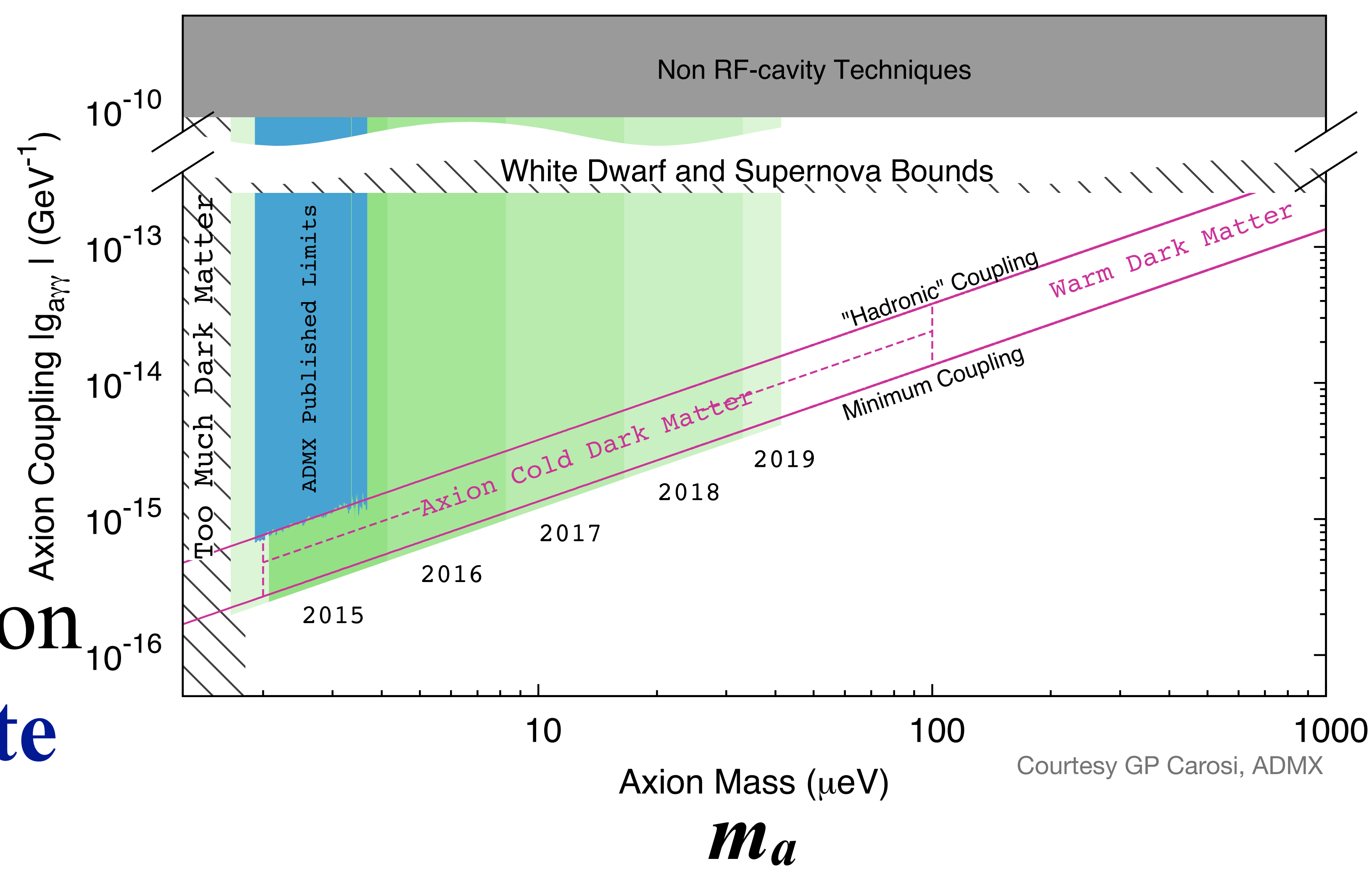
constraint on

m_a-$1/f_a$ plane

Large f_a

$\Rightarrow$ very weak interaction

$\Rightarrow$ **good DM candidate**



axion abundance

Evolution of coherent component:

$$\ddot{a} + 3H(T)\dot{a} + m_a(T)^2 a = 0$$

$$3H(T)^2 M_{\rm pl}^2 = \rho(T) \quad \propto T^4$$

$$m_a^2(T) = \chi_t(T) \,/\, 2 f_a^2$$

Initially, $3H(T) > m_a(T) \Rightarrow \dot{a} = 0$

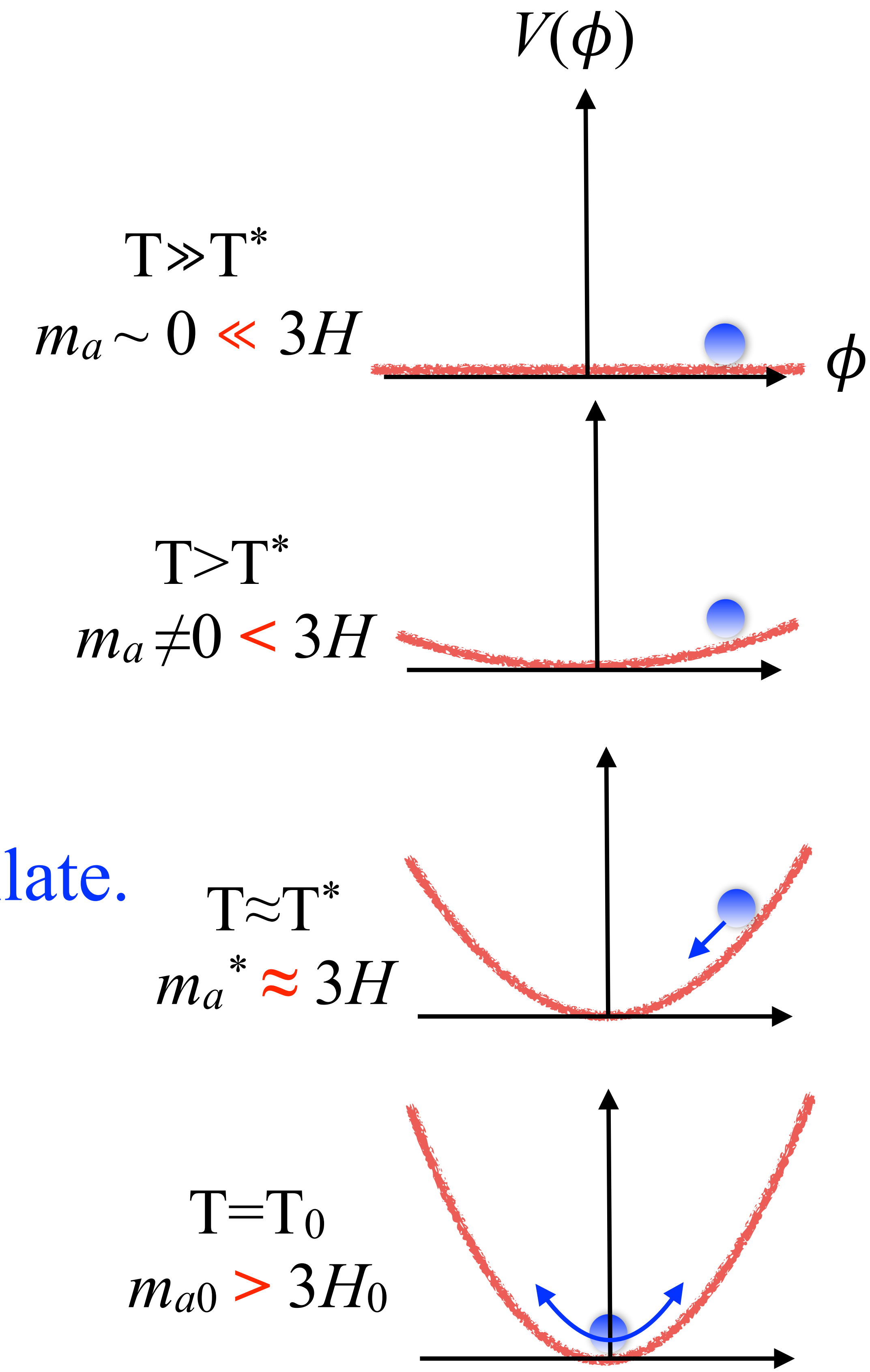
When $3H \sim m_a$ $(T = T^*)$ $\Rightarrow$ a starts to oscillate.

$$n_a(T_0) \approx n_a(T^*)(T_0/T^*)^3$$

$$\sim m_a(T^*)\, f_a^2\, \theta_{a,\mathrm{i}}^2\, (T_0/T^*)^3$$

$$(\theta_{a,\mathrm{i}} = \theta + a_{\rm ini} \,/\, f_a)$$

Need to know T dependence of $\chi_t(T)$



Current estimate of $\chi_t(T)$

Dilute Instanton Gas Approximation (DIGA)

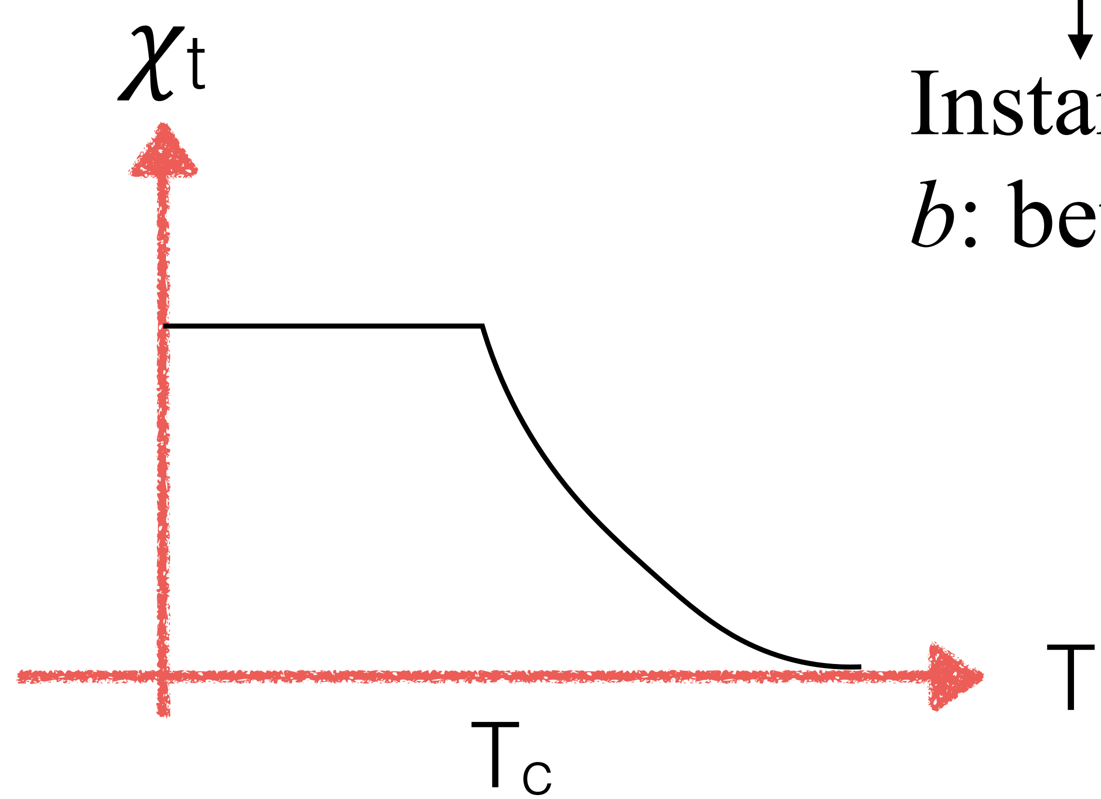
[Gross, Pisarski, Yaffe (1981)]

Assuming non-interacting isolated instantons

$$\chi_t(T) = m_a^2(T) f_a^2 \propto m_u m_d m_s \Lambda_{\mathrm{QCD}}^b T^{-8}$$

Instanton action = $e^{8\pi^2/g^2}$

b: beta function



Seems valid (only) at very high temperature

Dilute Instanton Gas Approximation (DIGA)

$$\chi_t(T) = m_a^2(T) f_a^2 \propto m_u m_d m_s \Lambda_{\rm QCD}^b T^{-8}$$

➡ $T^* \sim O(1)$ GeV

$$\Omega_a \simeq 0.2 \cdot \theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ {\rm eV}} \right)^{-7/6}$$

$\theta_{\rm ini}$: initial value of $\theta_{\rm ini} = a_{\rm ini}/f_a$

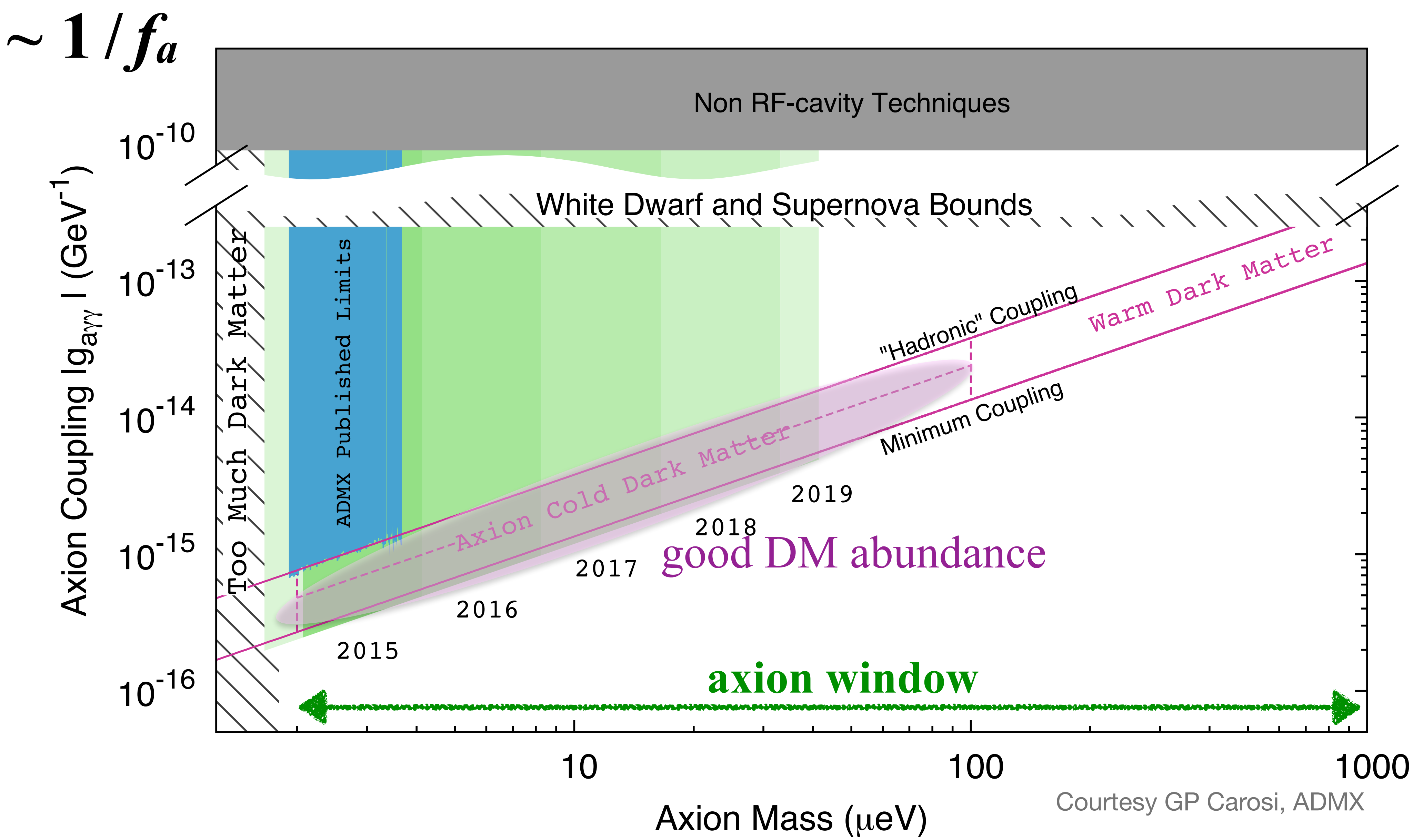
$$\theta_{\rm ini}{}^2 = \begin{cases} \frac{\pi^2}{3} & \text{SSB after Inflation} \\ \text{random in } [0, \pi^2] & \text{SSB before Inflation} \end{cases}$$

cf. IILM (interacting instanton liquid model) predicts $T^{-6.7}$.

[Wantz, Shellard (2010)]

Axion = good candidate of DM

$$\Omega_a \simeq 0.2 \cdot \theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ {\rm eV}}\right)^{-7/6}$$



Other possibility?

Low T ($\sim\Lambda_{QCD}$) $\Rightarrow$ no validity in instanton picture
The following extreme behaviors are suggested or yet-allowed.

✓ Step function like behavior [Aoki, Fukaya, Taniguchi (2012)]

$$\chi_t(T) \sim \chi_t(T=0)\ \theta(T_c - T) \quad \text{if } m_{u,d} < m_q^{crit} \ (\leftarrow \text{unknown})$$

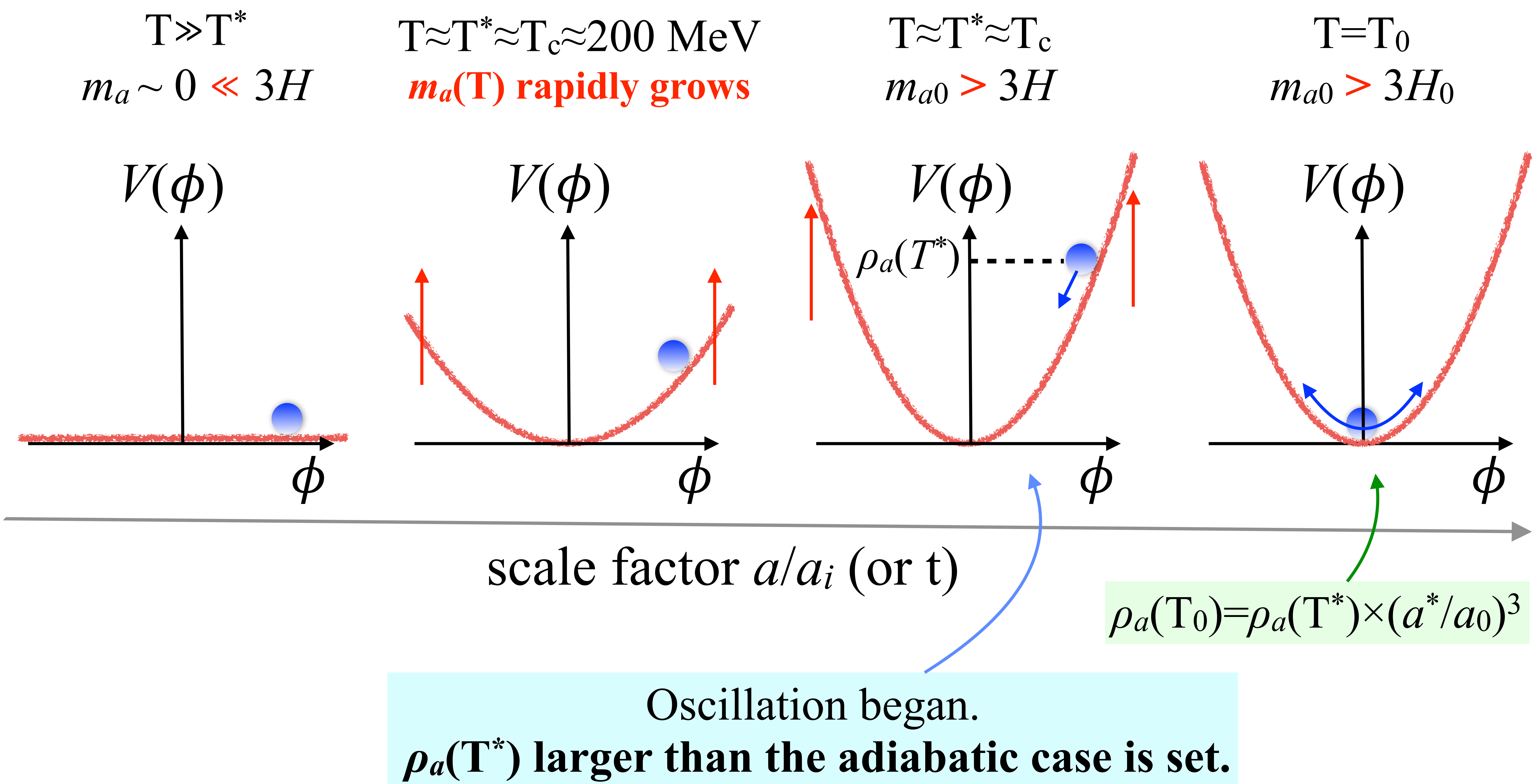
✓ A bit milder case

$$\chi_t(T) \sim \chi_t(T=0) \qquad \text{for } T \lesssim T_c$$

$$\sim \chi_t(T=0) \exp[\, -2c(m_q)\, T^2/T_c^2 \,] \quad \text{for } T > T_c$$

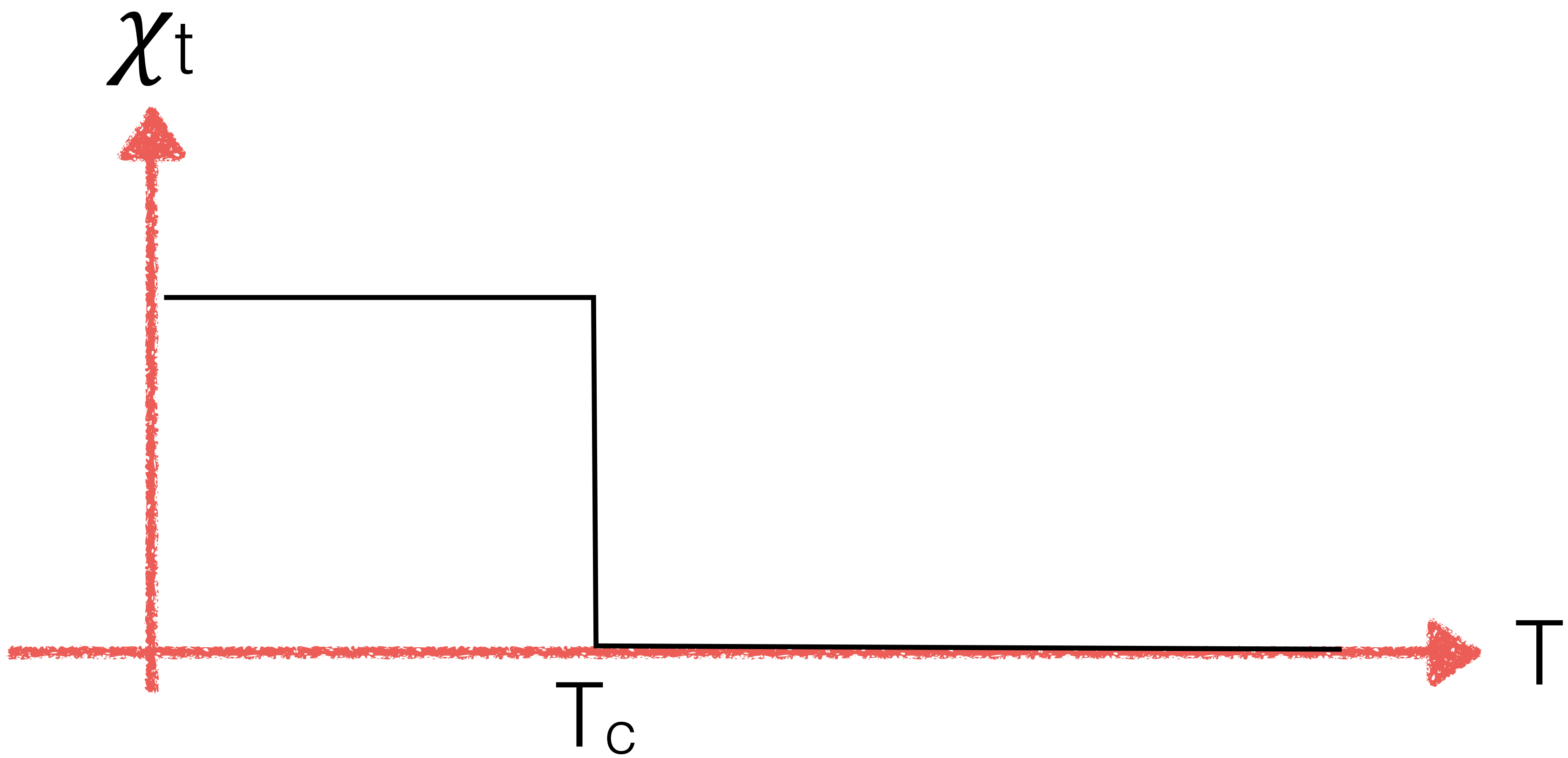
In extreme cases

[Kitano, Yamada (2015)]

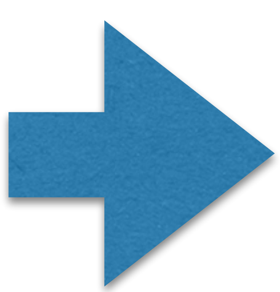


if χ_t=0 above T_c~150MeV,

the axion suddenly starts to oscillate at $T=T_c$

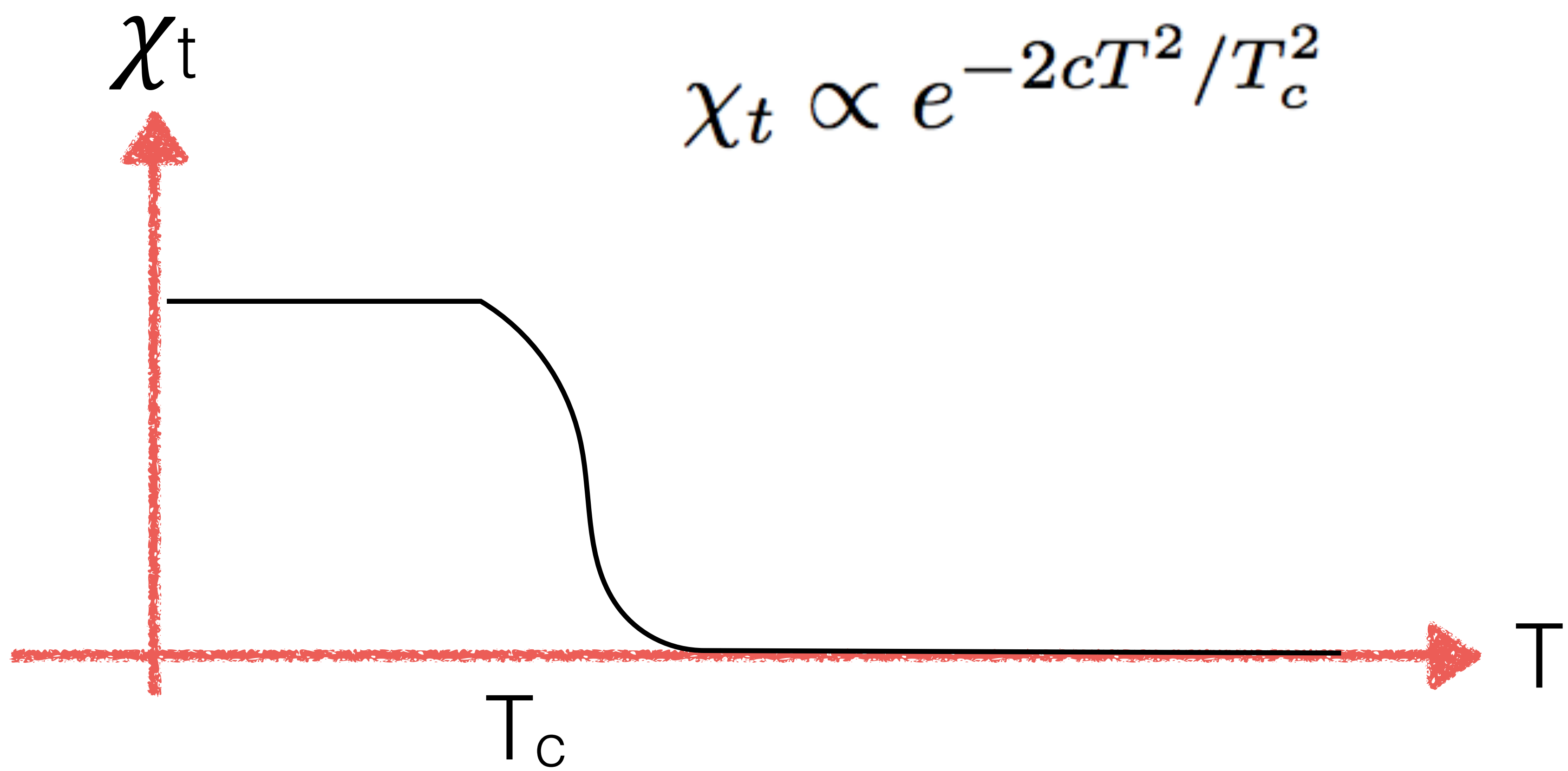


$$\Omega_a \sim 2 \times 10^5 \theta_{\rm ini}^2$$ independent of m_a



axion window is gone.

a bit milder case



$$\Omega_a \sim 0.2\theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ {\rm eV}}\right)^{-1} \times 2.5c \qquad (c \gg 1)$$

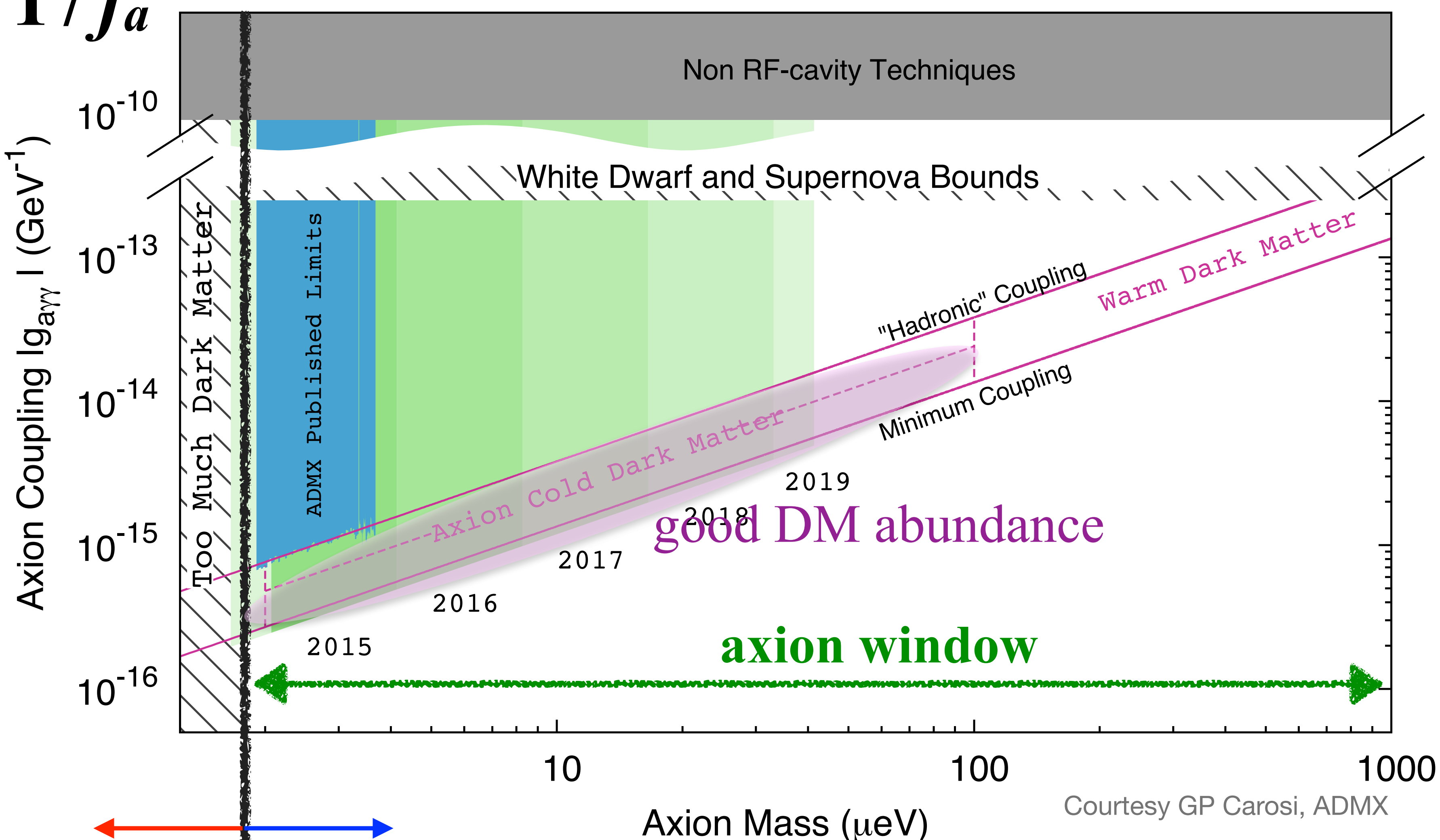
enhancement due to the non-adiabatic evolution of the potential.

Over-closure bound sensitive to T-dependence

Instanton: $\chi_t \sim T^{-8}$ $\Rightarrow$ $\Omega_a \simeq 0.2 \cdot \theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ {\rm eV}}\right)^{-7/6}$

Extreme case: $\chi_t \sim \theta(T_c - T)$ $\Rightarrow$ $\Omega_a \sim 2 \times 10^5 \cdot \theta_{\rm ini}^2$

$\sim 1/f_a$



Mild T-dependence

Large T-dependence

Over-closure bound

Lattice determination of $\chi_t(T)$

χ_t on the lattice

$$\chi_t = \frac{\langle Q^2 \rangle}{V}$$

we just need to measure Q in each configuration.

$$Q = \int d^4x \frac{1}{32\pi^2} F\tilde{F}$$ (Bosonic definition)

$$= n_L - n_R$$ (index theorem)

χ_t in pure Yang-Milles

in 2015, three independent calculations appeared.

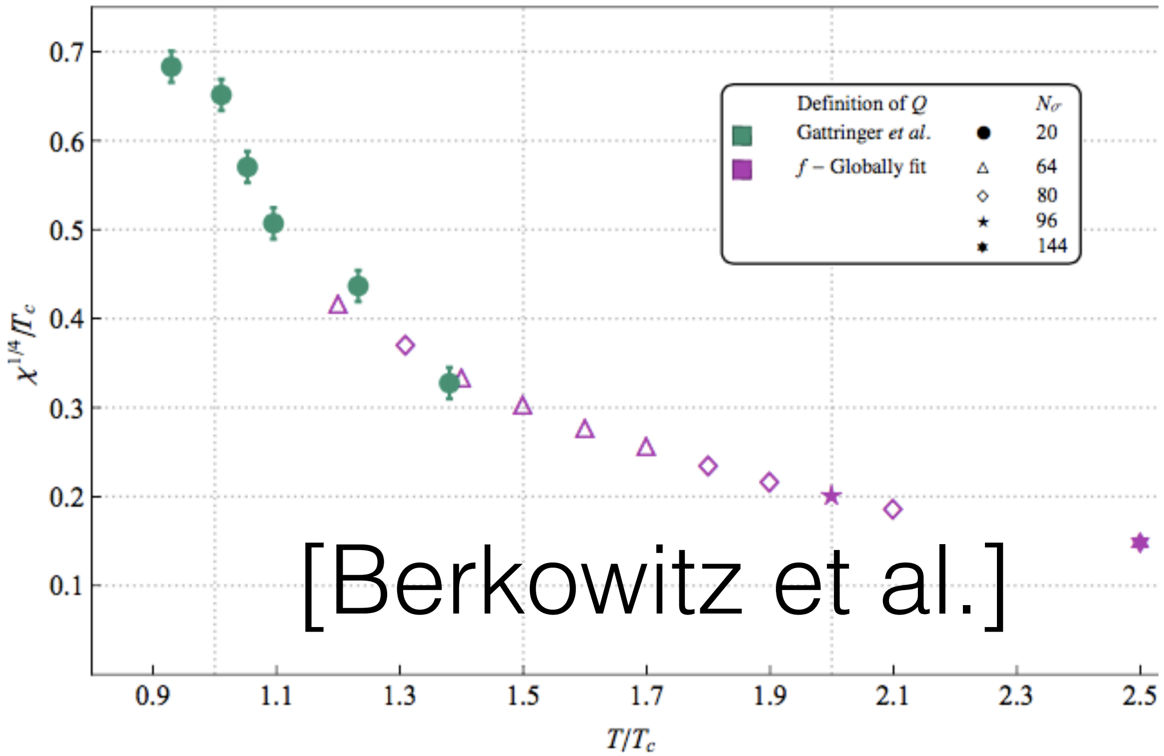
(in the SU(3) Yang-Milles theory, **no quarks yet**)

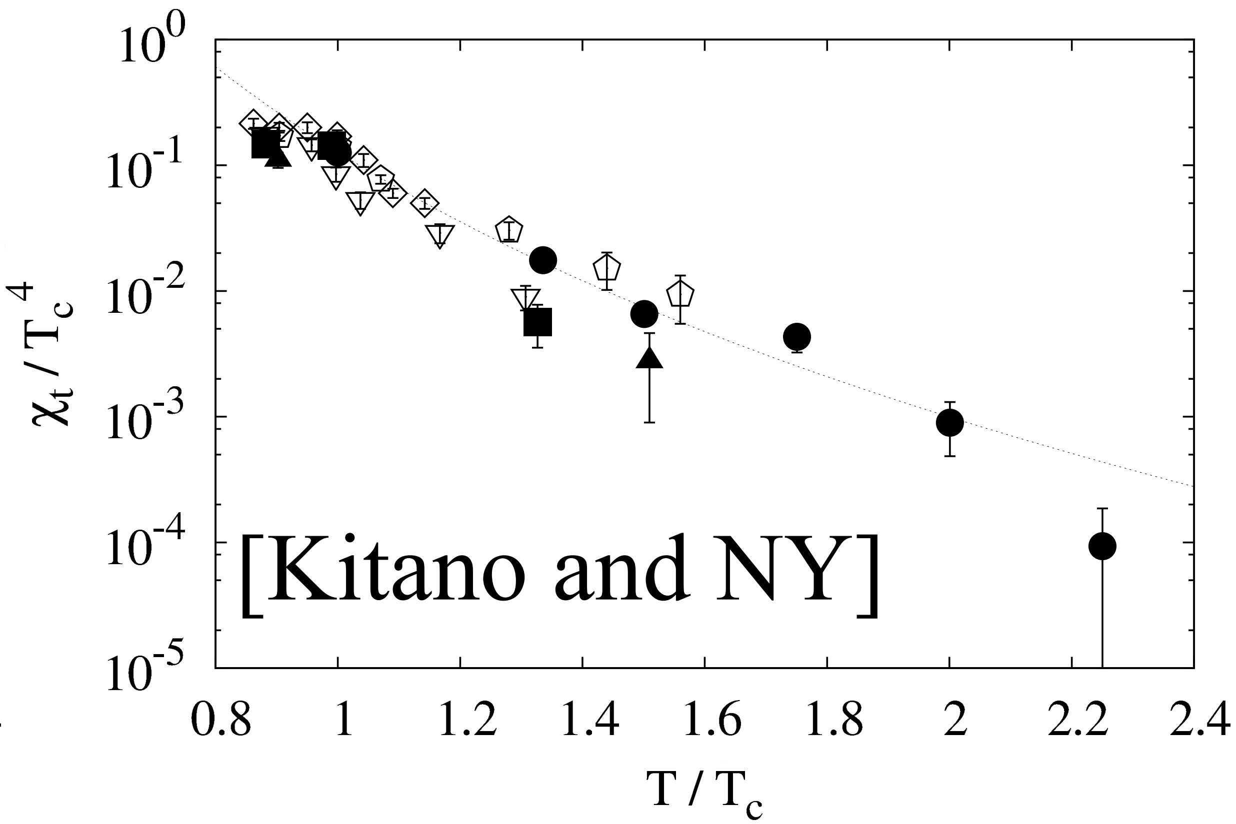
E. Berkowiz, M. Buchoff, E. Rinaldi (LLNL)
Bosonic (cooling)

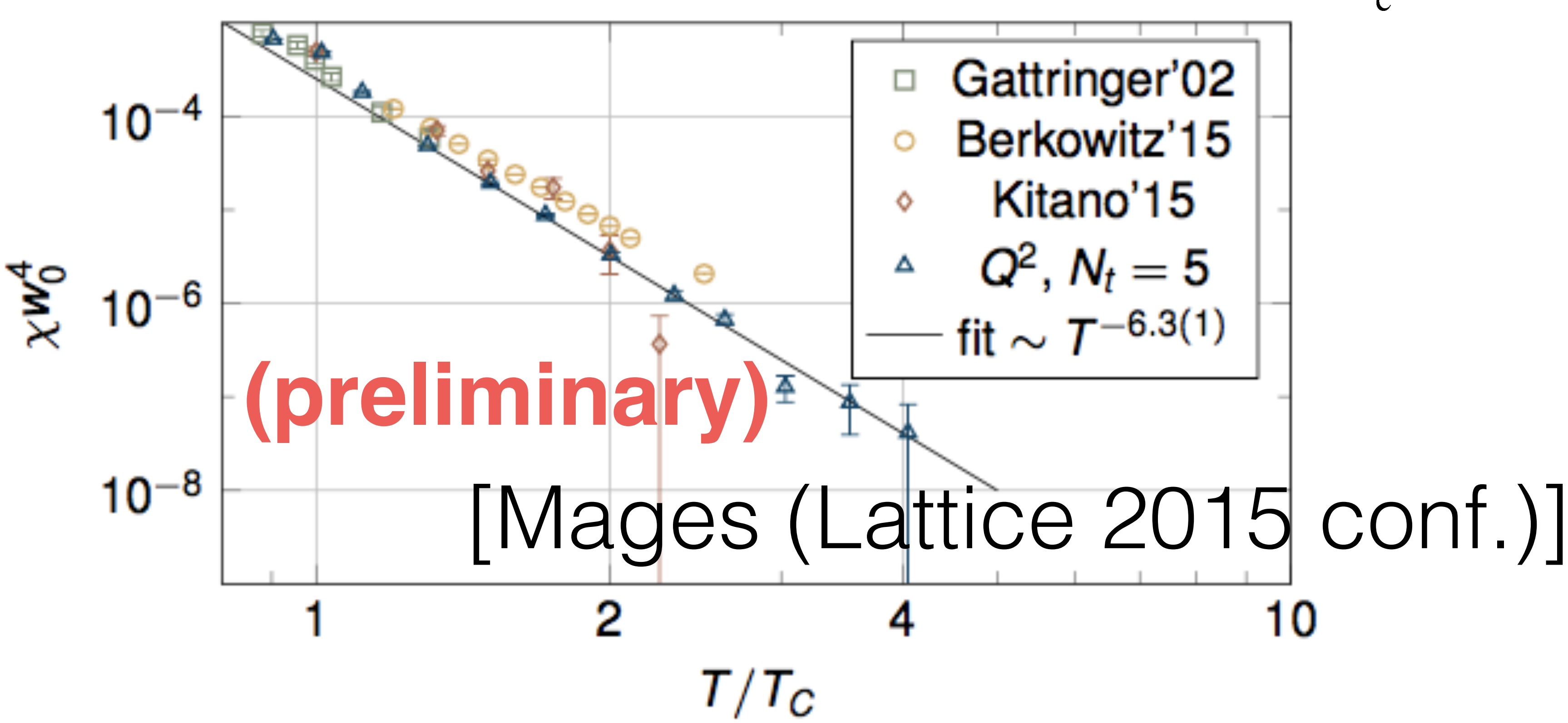
R. Kitano and NY (KEK) Index theorem

S. Mages et al (BMW) Bosonic (Wilson Flow)

lattice results

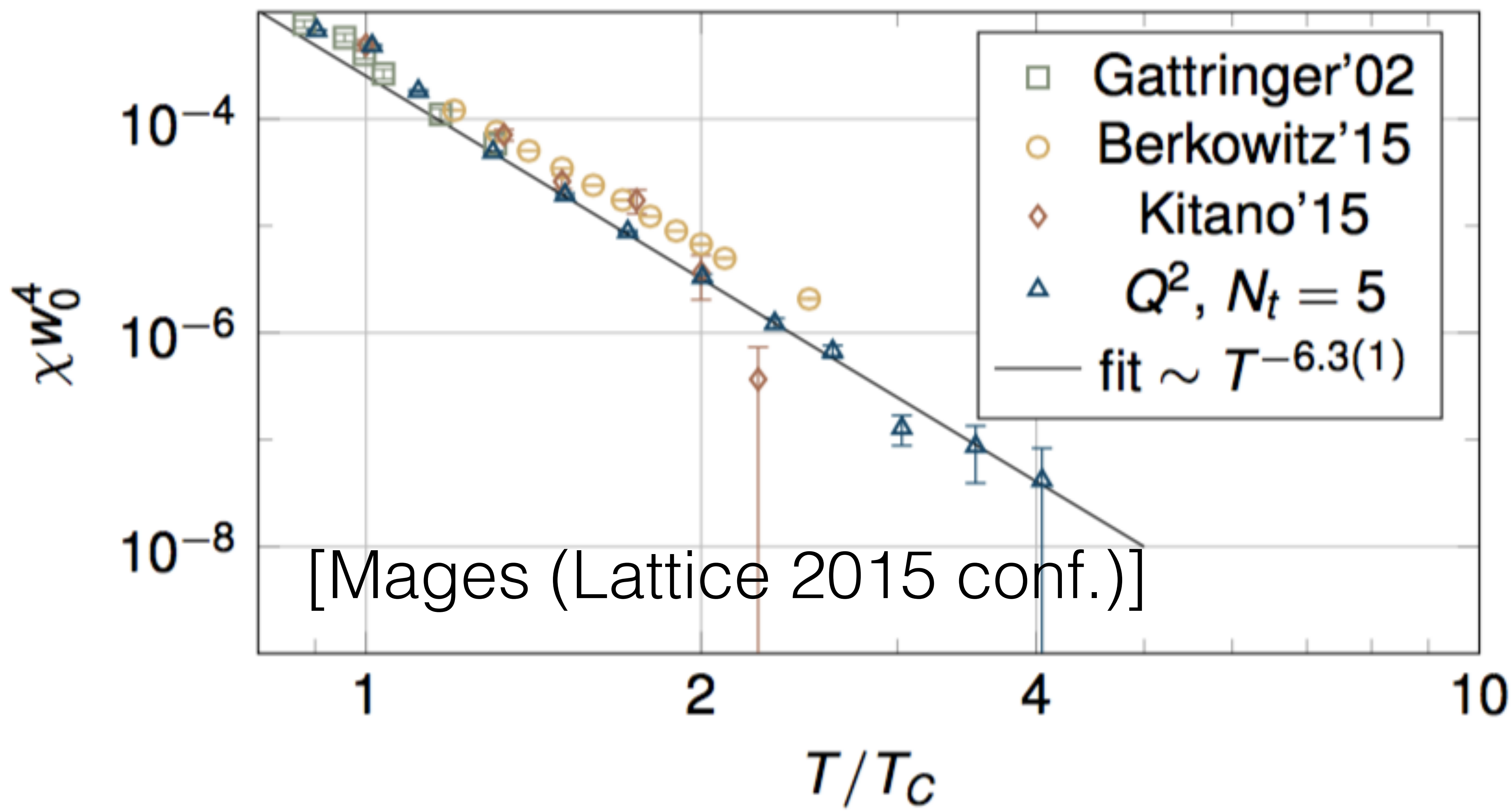






All look consistent

(at least qualitatively)



We see a clear power law even at a very low temperature.

T^{-n}: n= 5.64 - 7.14 ⇔ Consistent with DIGA (n=7) in pure YM!

Problems in measuring χ_t at high T

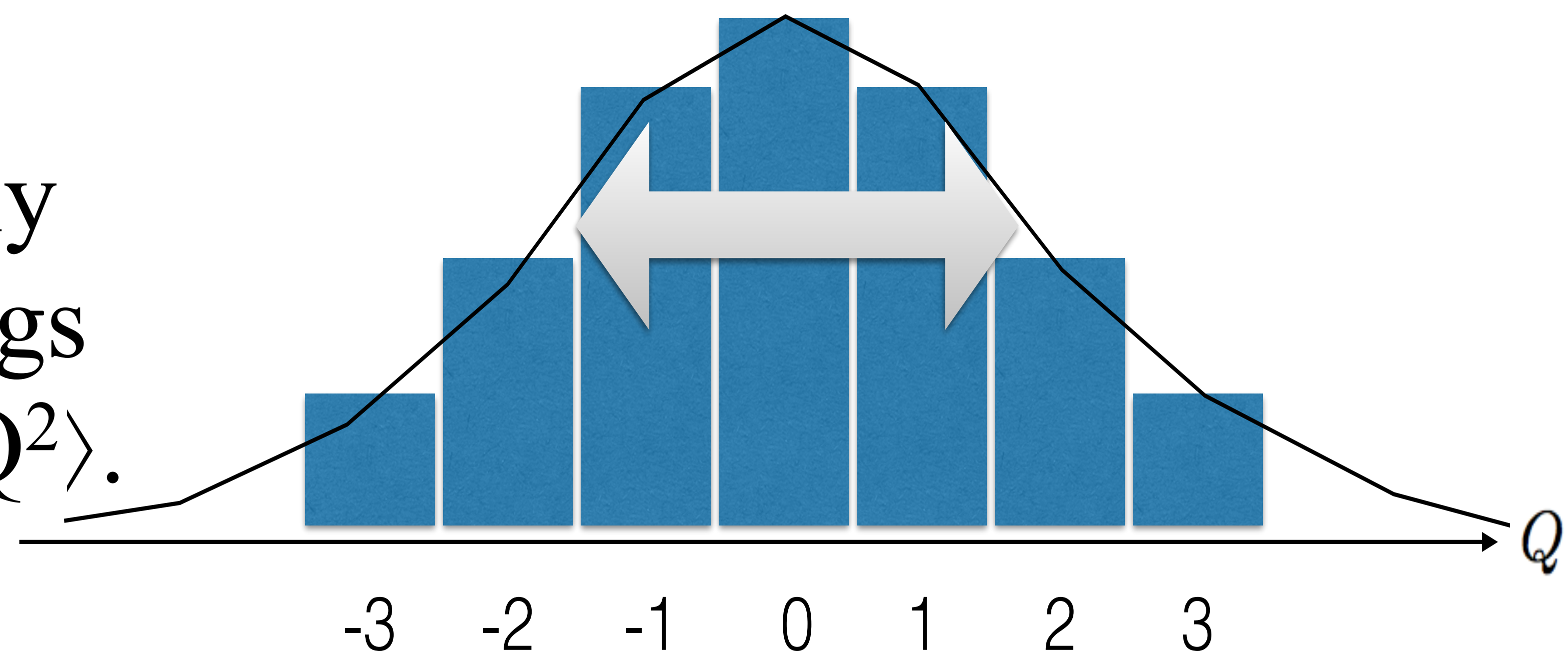
Besides the inclusion of dynamical quarks

$T^* \sim O(10) \times T_c \Rightarrow$ need to explore higher T

$\langle Q^2 \rangle = \chi_t V$: Width of histogram of Q

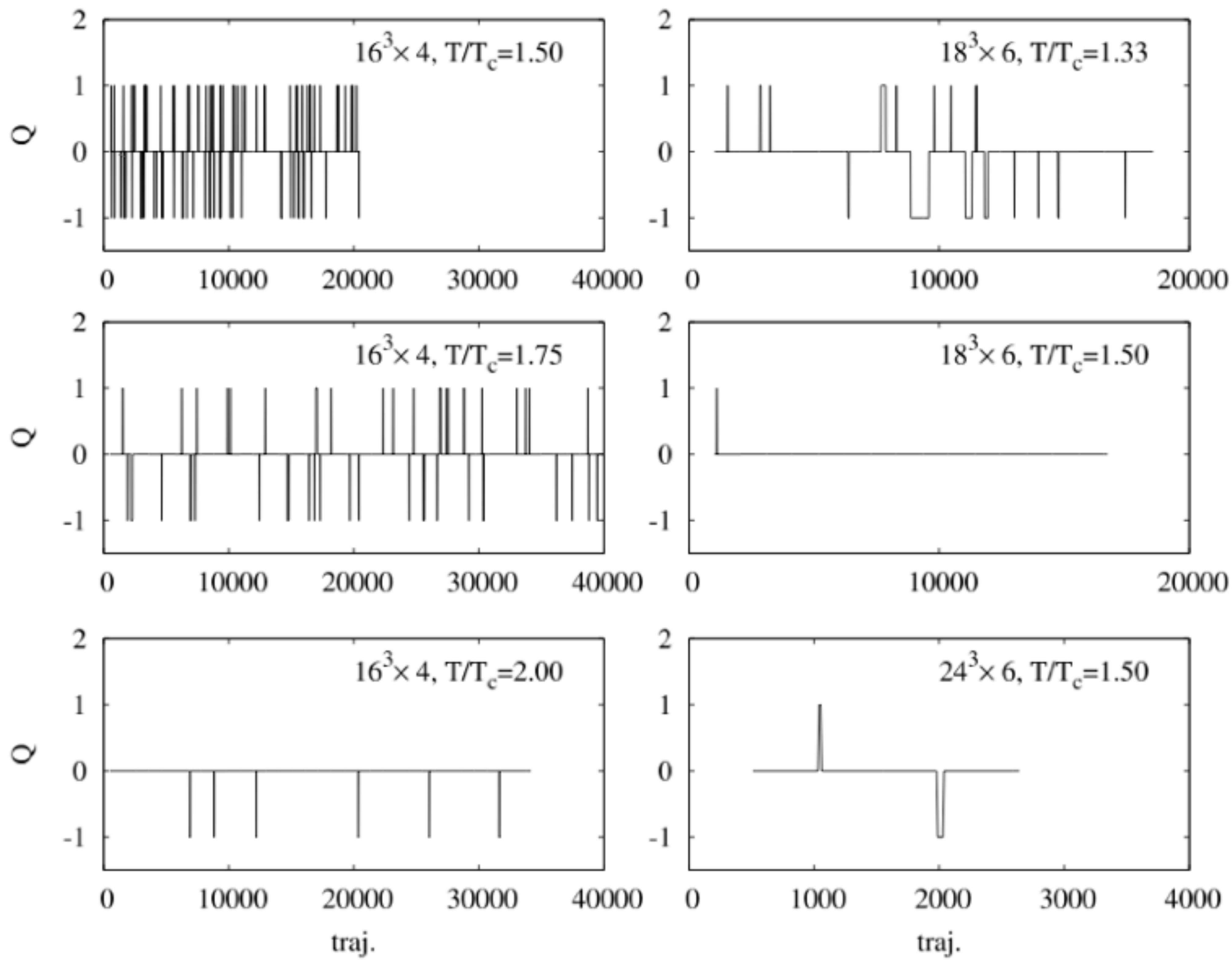
At high T, $\chi_t V$ ↘

At some T, we can only accumulate Q=0 configs and fail to calculate $\langle Q^2 \rangle$.



$$\langle Q^2 \rangle = \chi_t V$$

Frozen Q



[R. Kitano, NY (2015)]

New Method

Kitano, Frison, Matsufuru, Mori, NY, in progress

New Method [Kitano, Frison, Matsufuru, Mori, NY, in progress]

⇒ **able to explore *T* dependence of $\chi_t(T)$ at arbitrary high temperature**

Consider quenched SU(3) (pure YM)

$$Z_Q(\beta) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\,\delta(Q-\hat{Q})e^{-S_g(\beta)}$$

$$\langle\hat{O}\rangle_\beta^{(Q)} = \frac{1}{Z_Q(\beta)}\int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\,\delta(Q-\hat{Q})\,e^{-S_g(\beta)}\,\hat{O}$$

$$Z(\beta) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\,e^{-S_g(\beta)} = \sum_{Q=-\infty}^{+\infty} Z_Q(\beta)$$

$$\langle\hat{O}\rangle_\beta = \frac{1}{Z(\beta)}\sum_{Q=-\infty}^{+\infty} Z_Q(\beta)\langle\hat{O}\rangle_\beta^{(Q)}$$

$$\chi_t V = \langle\hat{Q}^2\rangle_\beta = \frac{1}{Z(\beta)}\sum_{Q=-\infty}^{+\infty} Z_Q(\beta)\langle\hat{Q}^2\rangle_\beta^{(Q)} = \frac{1}{Z(\beta)}\sum_{Q=-\infty}^{+\infty} Z_Q(\beta)Q^2$$

$$\text{At } T/T_c > 1,\quad \chi_t V \ll 1 \Rightarrow \chi_t V \approx \frac{2Z_1(\beta)}{Z_0(\beta)}$$

New Method [Kitano, Frison, Matsufuru, Mori, NY, in progress]

$$\frac{\partial \ln Z_Q(\beta)}{\partial \beta} = \frac{1}{Z_Q(\beta)} \int \mathcal{D}U\, \delta(Q-\hat{Q}) e^{-S_g(\beta)} \left[-\frac{\partial S_g(\beta)}{\partial \beta} \right] \Rightarrow \frac{Z_Q(\beta_2)}{Z_Q(\beta_1)} \quad (\beta=6/g^2)$$

$$\left(S_g(\beta) = 6\, N_{\rm site}\, \beta \left\{ (c_0 + 2c_1) - \hat{P} \right\} \right)$$

$$\chi_t(\beta) V_4(\beta) \approx \frac{2\, Z_1(\beta)}{Z_0(\beta)} = \frac{\frac{Z_1(\beta)}{Z_1(\beta_{\rm ref})}}{\frac{Z_0(\beta)}{Z_0(\beta_{\rm ref})}} \times \frac{2 Z_1(\beta_{\rm ref})}{Z_0(\beta_{\rm ref})}$$

$$= \exp\left[6 N_{\rm site} \int_{\beta_{\rm ref}}^{\beta} d\beta' \left(\langle \hat{P} \rangle_{\beta'}^{(1)} - \langle \hat{P} \rangle_{\beta'}^{(0)} \right) \right] \times \chi_t(\beta_{\rm ref}) V_4(\beta_{\rm ref})$$

$$\frac{d}{d \ln T} \left(\ln \frac{\chi_t(\beta)}{\chi_t(\beta_{\rm ref})} \right) \approx N_{\rm site} \beta_g \beta^2 \left(\langle \hat{P} \rangle_{\beta}^{(1)} - \langle \hat{P} \rangle_{\beta}^{(0)} \right) + 4$$

QCD β function

Lattice coupling $\beta=6/g^2$

Difference of the Wilson loop between the Q=±1 and 0 sectors

High T Limit ($g^2 \to 0$ limit)

$$\frac{d}{d\ln T}\left(\ln\frac{\chi_t(\beta)}{\chi_t(\beta_{\rm ref})}\right) \approx N_{\rm site}\beta_g\beta^2\left(\langle\hat{P}\rangle_\beta^{(1)} - \langle\hat{P}\rangle_\beta^{(0)}\right) + 4$$

$$\approx -\beta_g/g^2\left(\langle S_g{}^{(1)}\rangle - \langle S_g{}^{(0)}\rangle\right) + 4$$

$$\beta_g = \frac{dg^2}{d\ln a} = 2g\frac{dg}{d\ln a} = 2b_0\,g^4 + 2b_1\,g^6 + O(g^8),\ b_0 = \frac{11}{(4\pi)^2},\ b_1 = \frac{102}{(4\pi)^4}$$

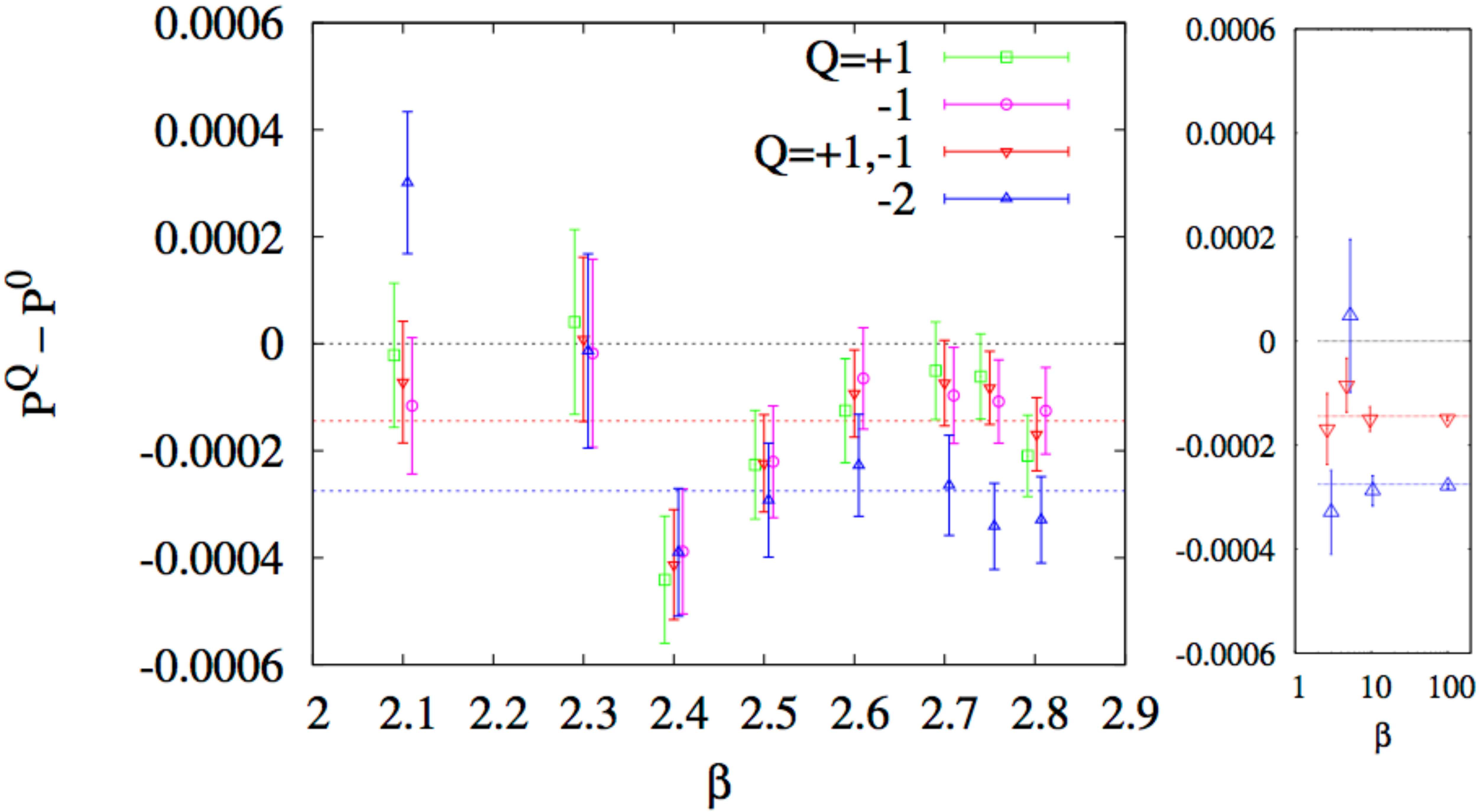
$$\text{Hight } T \text{ Limit} \Rightarrow S_g^{(Q)}|_{\rm BPST} = \frac{8\pi^2}{g^2}|Q|$$

$$\frac{d}{d\ln T}\left(\ln\frac{\chi_t(\beta)}{\chi_t(\beta_{\rm ref})}\right)\Bigg|_{\rm BPST} \approx -16\pi^2(b_0 + b_1\,g^2)|Q| + 4 \approx -11\,|Q| + 4$$

With $|Q|=1$, DIGA results $\chi_t(T) \sim T^{-7}$ is reproduced.

Preliminary Results [Kitano, Frison, Matsufuru, Mori, NY, in progress]

Test in the quenched approximation



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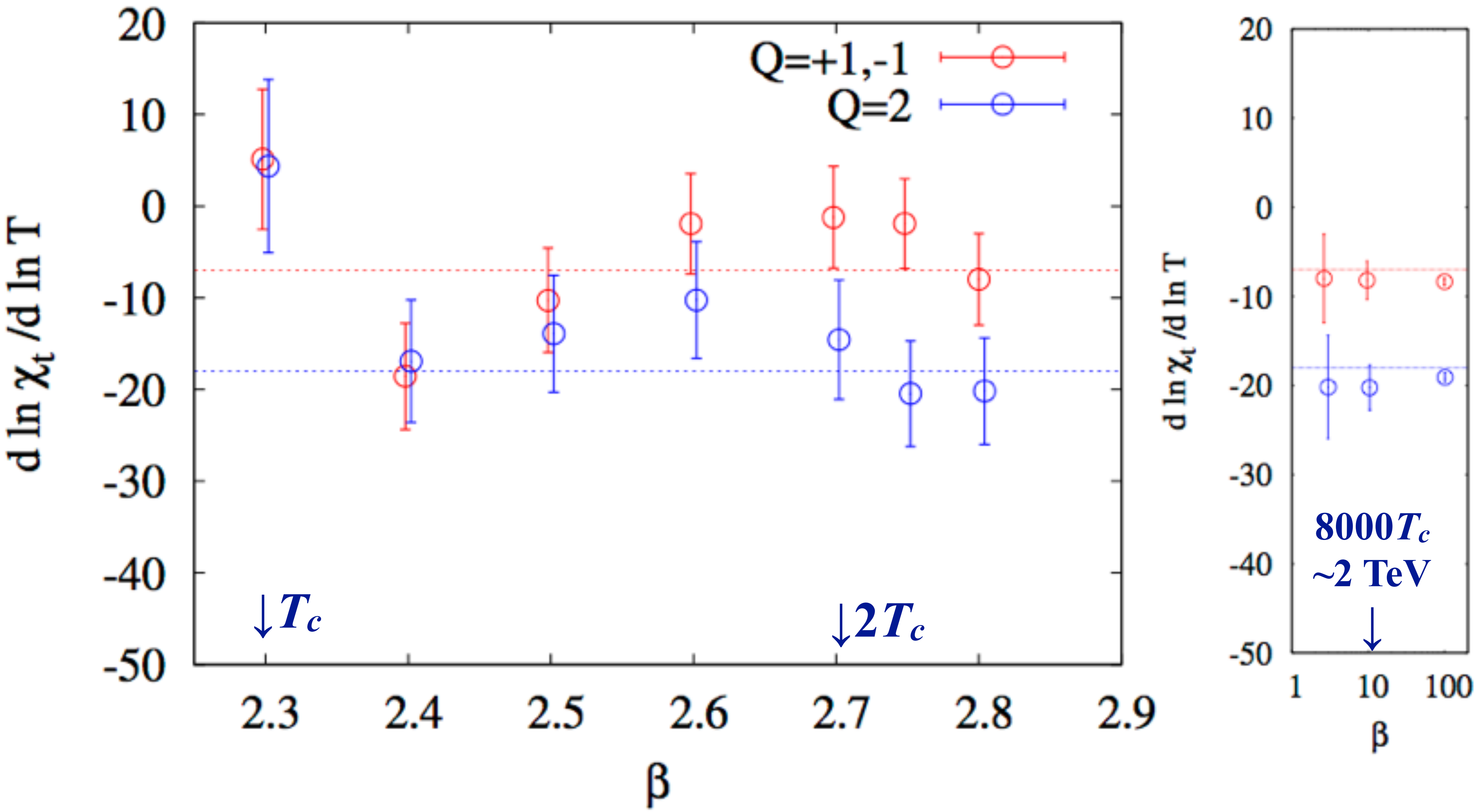

Consistent with DIGA value $\chi_t \propto T^{-7}$ down to 2 T_c.

Unquenched simulation is the next to do.

Summary

- ✓ Lattice QCD can constrain axion physics through the determination of $\chi_t(T)$!
- ✓ χ_t(O(1) GeV) is important to axion DM, but difficulty arises at $T >$ a few $\times T_c$ since Q tends to freeze.
- ✓ We proposed a method to directly calculate the T-dependence of $\chi_t(T)$ at arbitrary high T, which looks promising.
- ✓ Extension to full QCD is on going.