Generalization of Higgs Effective Field Theory

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Motivation

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Make universal predictions for the properties of new particles

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Make universal predictions for the properties of *new particles*

new particles = extra scalars with arbitrary charge

$$H^0, \quad H^\pm, \; H^{\pm\pm}, \cdots$$

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Make universal predictions for the properties of

$$H^0,\quad H^\pm,\quad H^{\pm\pm},\ \cdots$$

new particles = extra scalars with arbitrary charge

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Motivation

Make *universal* predictions for the properties of

$$H^0, \quad H^{\pm}, \quad H^{\pm\pm}, \cdots$$

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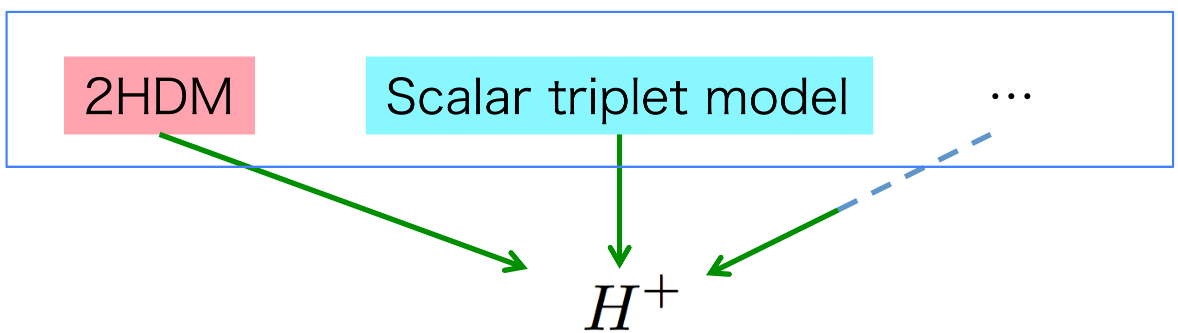
$$H^0, \quad H^{\pm}, \quad H^{\pm\pm}, \cdots$$

universal = don't care about the origin of new scalars

Ex.) H^+ - search



universal = *model-independent*

Motivation

Make universal predictions for the properties of

$$H^0,\quad H^{\pm},\quad H^{\pm\pm}\,,\ \cdots$$

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Motivation

Make universal predictions for the *properties* of

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property = {
- production cross section
- ϕ-V-V couplings

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EFT approach

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Existing EFT : heavy particles are integrated-out

➡ Cannot be used for these purpose

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Goal

Generalization of Higgs effective field theory

Talk Plan

- Introduction
- Extension of HEFT
- 1-loop correction of generalized HEFT
- Phenomenological implication
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Introduction

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Generalization of Higgs effective field theory

Variation of EFT

- SMEFT (Standard Model EFT)
- HEFT (Higgs EFT)

SMEFT

Standard Model Effective Field Theory

$$\mathcal{L}_{\rm SMEFT} = (D_\mu H)^\dagger D^\mu H + \frac{C_1}{f^2}(H^\dagger D_\mu H)^*(H^\dagger D^\mu H) + \cdots$$

- Symmetry : $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Matter contents : H , W_μ^a , B_μ $\cdots$

  SM fields in symmetric phase
- 125 GeV scalar h transforms as part of H

$$H = \begin{pmatrix} \pi^+ \\ \frac{v+h+i\pi^0}{\sqrt{2}} \end{pmatrix}$$

HEFT

Higgs Effective Field Theory

$$\mathcal{L}_{\rm HEFT} = \frac{v^2}{4} F(h) {\rm Tr}[(D_\mu U)^\dagger D^\mu U] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h) + \cdots$$

$$F(h) = 1 + \kappa_1 \frac{h}{v} + \kappa_2 \left(\frac{h}{v}\right)^2 + \cdots \qquad U = \exp\left(\frac{i\pi^a \tau^a}{v}\right)$$

$$V(h) = m_h^2 h^2 + \lambda_3 h^3 + \cdots$$

- Symmetry : $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Matter contents : $W_\mu^\pm,\ Z_\mu,\ h \quad \cdots$ SM fields in broken phase
- 125 GeV scalar h is not necessarily included in H

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- Symmetry : $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$m^2 \pi^+ \pi^- \qquad m'^2 \pi^0 \pi^0$$

In our vacuum, $SU(2)_L \times U(1)_Y$ is realized as the shift symmetry of $\pi^\pm, \pi^0$

and forbid their mass terms

HEFT

Higgs Effective Field Theory

$$\mathcal{L}_{\mathrm{HEFT}} = \frac{v^2}{4} F(h) \mathrm{Tr}[(D_\mu U)^\dagger D^\mu U] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h) + \cdots$$

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- Symmetry : $SU(3)_C \times SU(2)_L \times U(1)_Y$

In the theory with SSB ($\mathcal{G} \to \mathcal{H}$)

$\mathcal{G}$ is not really broken, but remains as non-linearly realized symmetry talk title

(Extension)

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Extension of HEFT

Goal

Generalization of Higgs effective field theory

From now on, we call generalized HEFT as " GHEFT "

~ GHEFT ~

- symmetry : $SU(3)_C \times \quad SU(2)_L \times U(1)_Y$

- matter contents : SM + $H^0, \ H^{\pm}, \ H^{\pm\pm}, \cdots$

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  non-linearly realized

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  Non-trivial

Extension of HEFT

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・symmetry : $SU(3)_C \times SU(2)_L \times U(1)_Y$

non-linearly realized ($SU(2)_L \times U(1)_Y$)

・matter contents : SM + $H^0, H^\pm, H^{\pm\pm}, \cdots$

Non-trivial

Add $H^0, H^\pm, H^{\pm\pm}, \cdots$ respecting non-linearly realized $SU(2)_L \times U(1)_Y$

Extension of HEFT

Mission

Add H^0, $H^\pm$, $H^{\pm\pm}$, $\cdots$ respecting non-linearly realized $SU(2)_L \times U(1)_Y$

→ CCWZ method

Coleman et. al., Phys. Rev. 177. 2239
Callan et. al., Phys. Rev. 177. 2247

We apply CCWZ to " SM + H^0, $H^\pm$, $H^{\pm\pm}$, $\cdots$ " situation

Extension of HEFT

Mission

Add $H^0,\ H^\pm,\ H^{\pm\pm},\ \cdots$ respecting non-linearly realized $SU(2)_L \times U(1)_Y$

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What is the non-linearly realized symmetry ?

Extension of HEFT

Non-linearly realized symmetry

Ex.) $O(N)$ sigma model

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \vec{\phi})^2 + \frac{1}{2}\mu^2(\vec{\phi})^2 - \frac{\lambda}{4}(\vec{\phi})^4$$

$$\vec{\phi} = (\phi_1, \phi_2, \cdots \phi_N)^T$$

SSB $\quad \langle \vec{\phi} \rangle = (0, 0, \cdots, v)^T$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 - \frac{1}{2}(2\mu^2)\sigma^2 + \cdots$$

σ : scalar matter $\qquad$ π^a : NG boson

Q What is the transformation law of σ & π^a under $O(N)$?

Extension of HEFT

Non-linearly realized symmetry

Ex.) $O(N)$ sigma model

Q Transformation law of σ & π^a under $O(N)$?

$$\vec{\phi} = (\pi_1, \pi_2, \cdots, v + \sigma)^T$$

Extension of HEFT

Non-linearly realized symmetry

Ex.) $O(N)$ sigma model

Q Transformation law of σ & π^a under $O(N)$?

$$\vec{\phi} = (v+\sigma)e^{i\frac{\pi^a}{v}X^a}\vec{F} \qquad \vec{F} = (0, 0, \cdots, 1)^T$$

Extension of HEFT

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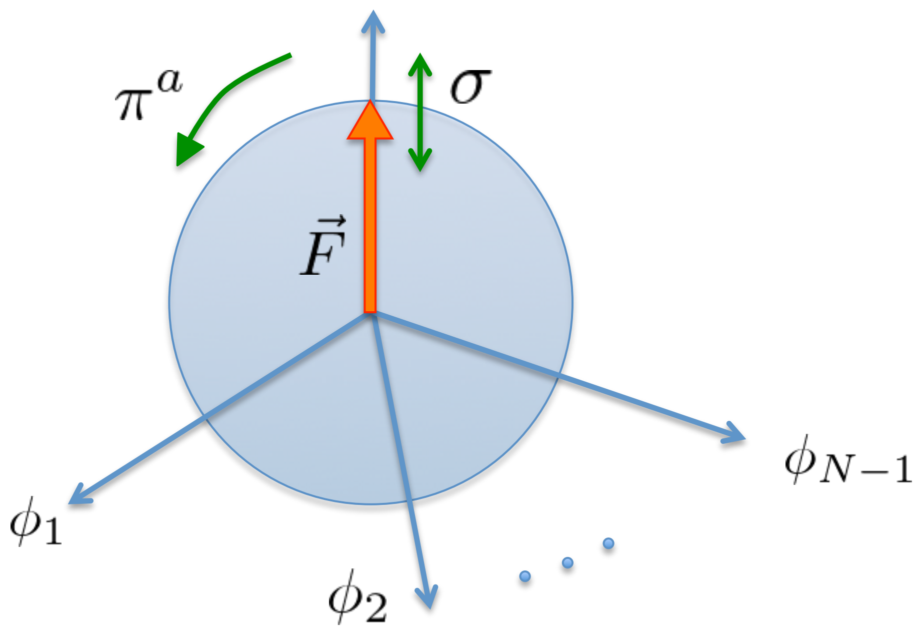

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$$\vec{\phi} = (v+\sigma)e^{i\frac{\pi^a}{v}X^a}\vec{F} \qquad \vec{F} = (0,0,\cdots,1)^T$$

$\vec{\phi}$ transformation under $O(N)$

$$\vec{\phi} \to \mathfrak{g}\cdot\vec{\phi} \qquad \mathfrak{g}\in O(N)$$

$$\Updownarrow$$

$$(v+\sigma)e^{i\frac{\pi^a}{v}X^a}\vec{F} \to (v+\sigma)\,\mathfrak{g}\cdot e^{i\frac{\pi^a}{v}X^a}\vec{F}$$

A $O(N)$: $\sigma \to \sigma$ $\qquad e^{i\frac{\pi^a}{v}X^a} \to \mathfrak{g}\cdot e^{i\frac{\pi^a}{v}X^a}$

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$$(v+\sigma)e^{i\frac{\pi^a}{v}X^a}\vec{F} \rightarrow (v+\sigma)\,\mathfrak{g}\cdot e^{i\frac{\pi^a}{v}X^a}\cdot\mathfrak{h}\cdot\vec{F} \qquad \mathfrak{h}\in O(N-1)$$

A $O(N)$: $\sigma \rightarrow \sigma \qquad e^{i\frac{\pi^a}{v}X^a} \rightarrow \mathfrak{g}\cdot e^{i\frac{\pi^a}{v}X^a}\cdot\mathfrak{h}$

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$\mathfrak{h} \in O(N-1)$

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Extension of HEFT

Non-linearly realized symmetry

Ex.) $O(N)$ sigma model

A $\quad O(N)$: $e^{i\frac{\pi^a}{v}X^a} \to \mathfrak{g}\cdot e^{i\frac{\pi^a}{v}X^a}\cdot\mathfrak{h}$ $\quad \mathfrak{g}\in O(N)$ $\quad \mathfrak{h}\in O(N-1)$

Actually, $\mathfrak{h}$ is necessary …

- w/o $\mathfrak{h}$

$$e^{iX} \to e^{i(S+X)}\cdot e^{iX} = e^{\overbrace{i(S+X)+iX-\frac{1}{2}[S+X,X]+\cdots}^{}}$$

$iX'+iS'$

π'^a cannot be defined

- w/ $\mathfrak{h}$

$$e^{iX} \to e^{i(S+X)}\cdot e^{iX}\cdot e^{-iS''} = e^{iX'}$$

π'^a can be defined

Extension of HEFT

Non-linearly realized symmetry

Ex.) $O(N)$ sigma model

A $O(N)$: $e^{i\frac{\pi^a}{v}X^a} \to \mathfrak{g}\cdot e^{i\frac{\pi^a}{v}X^a}\cdot\mathfrak{h}$ $\mathfrak{g}\in O(N)$ $\mathfrak{h}\in O(N-1)$

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$$e^{iX}\to e^{i(S+X)}\cdot e^{iX}=e^{\overbrace{i(S+X)+iX-\frac{1}{2}[S+X,X]+\cdots}^{}}$$

$\underbrace{\qquad}_{iX'+iS'}$

π'^a cannot be defined

- w/ $\mathfrak{h}$

$$e^{iX}\to e^{i(S+X)}\cdot e^{iX}\cdot e^{-iS''}=e^{iX'}$$

$$=e^{i(\cdots+\frac{1}{v}\mathcal{M}_{ab}\pi^b+\frac{1}{v^2}\mathcal{N}_{abc}\pi^b\pi^c+\cdots)X^a}$$

$=: \pi'^a$

Extension of HEFT

Non-linearly realized symmetry

$O(N)$:

$$e^{i\frac{\pi^a}{v}X^a} \to e^{i(S+X)} \cdot e^{i\frac{\pi^a}{v}X^a} \cdot e^{-iS}$$

$$=: \pi'^a$$

$$= e^{i(\cdots + \frac{1}{v}\mathcal{M}_{ab}\pi^b + \frac{1}{v^2}\mathcal{N}_{abc}\pi^b\pi^c + \cdots)X^a}$$

$\mathfrak{g} \in O(N) \quad \mathfrak{h} \in O(N-1)$

- π'^a is non-linear combination of π^a
- $\mathfrak{h}$ depends on NG boson π^a : $\mathfrak{h} = \mathfrak{h}(\pi)$

In any $\mathcal{G} \to \mathcal{H}$, $\mathcal{G}$: $e^{i\frac{\pi^a}{v}X^a} \to \mathfrak{g} \cdot e^{i\frac{\pi^a}{v}X^a} \cdot \mathfrak{h}$ $\quad \mathfrak{g} \in \mathcal{G} \quad \mathfrak{h} \in \mathcal{H}$

Extension of HEFT

In the case of GHEFT … $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$

NG boson

$$SU(2)_L \times U(1)_Y : \quad e^{i\frac{\pi^a}{v}X^a} \to \mathfrak{g} \cdot e^{i\frac{\pi^a}{v}X^a} \cdot \mathfrak{h}$$

$\mathfrak{g} \in SU(2)_L \times U(1)_Y$

$\mathfrak{h} \in U(1)_{\rm em}$

Scalar matter field

$$SU(2)_L \times U(1)_Y \quad : \quad \phi^I \to [\rho_\phi(\mathfrak{h})]^I{}_J \phi^J \qquad \phi^I = \{h, H^0, H^\pm, H^{\pm\pm}, \cdots\}$$

Ex.) $\quad SU(2)_L \quad : \quad H^{+'} = H^+ + \frac{\theta_L}{2\sqrt{2}}(\pi^- - \pi^+)H^+ + \cdots$

Extension of HEFT

In the case of GHEFT ... $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$

NG boson

$$SU(2)_L \times U(1)_Y : \quad e^{i\frac{\pi^a}{v}X^a} \to \mathfrak{g} \cdot e^{i\frac{\pi^a}{v}X^a} \cdot \mathfrak{h}$$

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Ex.) $SU(2)_L \quad : \quad H^{+\prime} = H^+ + \frac{\theta_{L(x)}}{2\sqrt{2}}(\pi^-_{(x)} - \pi^+_{(x)})H^+ + \cdots$

Even if we take $\theta_L(x) \to \theta_L$

It still depends on x_μ through $\pi^\pm(x), \pi^0(x)$

connection : W_μ

connection : ?

Now we understand that
symmetry is non-linealy realized in broken phase

So, how can we construct GHEFT
respecting non-linearly realized symmetry ?

Generalized HEFT : NGB part

$\xi := e^{i\frac{\pi^a}{v}X^a}$ transforms non-trivially,

but $\frac{1}{i}\xi^\dagger\partial_\mu\xi$ transforms in a simple manner

$$\frac{1}{i}\xi^\dagger\partial_\mu\xi = \alpha^a_{\perp\mu}X^a + \alpha_{\parallel\mu}S$$

X^a : broken generator S : unbroken generator

$$SU(2)_L \times U(1)_Y : \quad \alpha_{\perp\mu} \quad \to \quad \mathfrak{h}\cdot\alpha_{\perp\mu}\cdot\mathfrak{h}^{-1} \qquad \mathfrak{h} = \mathfrak{h}(\pi)$$

NG boson kinetic term : $\mathcal{L}_\pi = v^2\mathrm{Tr}\left[\alpha_{\perp\mu}\alpha_\perp^\mu\right]$

Generalized HEFT : Matter part

$\xi := e^{i\frac{\pi^a}{v}X^a}$ transforms non-trivially,

but $\frac{1}{i}\xi^\dagger\partial_\mu\xi$ transforms in a simple manner

$$\frac{1}{i}\xi^\dagger\partial_\mu\xi = \alpha^a_{\perp\mu}X^a + \alpha_{\parallel\mu}S$$

X^a : broken generator S : unbroken generator

$$SU(2)_L \times U(1)_Y : \quad \alpha_{\parallel\mu} \to \mathfrak{h}\cdot\alpha_{\parallel\mu}\cdot\mathfrak{h}^{-1} - \frac{1}{i}\mathfrak{h}\cdot\partial_\mu\mathfrak{h}^{-1} \qquad \mathfrak{h} = \mathfrak{h}(\pi)$$

$$SU(2)_L \times U(1)_Y : \quad \phi^I \to [\rho_\phi(\mathfrak{h})]^I{}_J\phi^J \qquad \phi^I = \{h, H^0, H^\pm, H^{\pm\pm}, \cdots\}$$

Covariant derivative $(\mathcal{D}_\mu\phi)^I = \partial_\mu\phi^I - i\alpha_{\parallel\mu}[Q_\phi]^I{}_J\phi^J$

Extension of HEFT

Building blocks
$$\begin{cases} \alpha_{\perp\mu} \to \mathfrak{h} \cdot \alpha_{\perp\mu} \cdot \mathfrak{h}^{-1} \\ (\mathcal{D}_\mu \phi)^I \to [\rho_\phi(\mathfrak{h})]^I{}_J (\mathcal{D}_\mu \phi)^J \end{cases}$$

Once we get covariant quantity, we can easily construct GHEFT

Generalized HEFT (GHEFT)

$$\mathcal{L}_{\rm GHEFT} = \frac{1}{2} G_{ab}(\phi) \alpha^a_{\perp\mu} \alpha^{b\mu}_{\perp} + G_{aI}(\phi) \alpha^a_{\perp\mu} (\mathcal{D}^\mu \phi)^I + \frac{1}{2} G_{IJ}(\phi) (\mathcal{D}_\mu \phi)^I (\mathcal{D}^\mu \phi)^J - V$$

$$\phi^I = \{h, H^0, H^\pm, H^{\pm\pm}, \cdots\}$$

$$(\mathcal{D}_\mu \phi)^I = \partial_\mu \phi^I - i \alpha_{\parallel\mu} [Q_\phi]^I{}_J \phi^J$$

$$Q_\phi = \begin{pmatrix} -q_1\sigma_2 & & & & & \\ & \ddots & & & & \\ & & -q_{n_C}\sigma_2 & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{pmatrix}$$

charged

neutral

Extension of HEFT

Generalized HEFT (GHEFT)

$$\mathcal{L}_{\text{GHEFT}} = \frac{1}{2} G_{ab}(\phi) \alpha_{\perp \mu}^{a} \alpha_{\perp}^{b\mu} + G_{aI}(\phi) \alpha_{\perp \mu}^{a} (\mathcal{D}^{\mu}\phi)^{I} + \frac{1}{2} G_{IJ}(\phi) (\mathcal{D}_{\mu}\phi)^{I} (\mathcal{D}^{\mu}\phi)^{J} - V$$

$$\phi^{I} = \{h, H^{0}, H^{\pm}, H^{\pm\pm}, \cdots\}$$

$\frac{1}{2} G_{IJ}(\phi) (\mathcal{D}_{\mu}\phi)^{I} (\mathcal{D}^{\mu}\phi)^{J}$: Non-minimal derivative coupling

Ex.) non-minimal composite Higgs ($SO(6)/SO(5)$)

$$\mathcal{L} = \frac{1}{2} \left[(\partial_{\mu} h)^2 + (\partial_{\mu}\eta)^2 + \frac{(h\partial_{\mu} h + \eta \partial_{\mu}\eta)^2}{f^2 - h^2 - \eta^2} \right] + \cdots$$

η : scalar DM A. Marzocca *et.al.*, JHEP07(2014)107

Extension of HEFT

Generalized HEFT (GHEFT)

$$\mathcal{L}_{\text{GHEFT}} = \frac{1}{2}G_{ab}(\phi)\alpha^a_{\perp\mu}\alpha^{b\mu}_{\perp} + G_{aI}(\phi)\alpha^a_{\perp\mu}(\mathcal{D}^\mu\phi)^I + \frac{1}{2}G_{IJ}(\phi)(\mathcal{D}_\mu\phi)^I(\mathcal{D}^\mu\phi)^J - V$$

$$\to \frac{1}{2}F(h)\delta_{ab}\alpha^a_{\perp\mu}\alpha^{b\mu}_{\perp} + \frac{1}{2}\partial_\mu h\partial^\mu h - V$$: HEFT

In HEFT, we can always eliminate

- off-diagonal interaction : $G_{aI}(\phi)\partial_\mu\pi^a\partial^\mu\phi^I$
- derivative couplings such as $|\vec{\phi}|^2(\partial_\mu\vec{\phi})^2$

by field redefinition

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Generalized HEFT

Motivation

Make universal predictions for the properties of

$$H^0, \quad H^\pm, \quad H^{\pm\pm}, \cdots$$

Let's calculate (precisely-measured) observables in GHEFT

➡ 1-loop correction of GHEFT

Generalized HEFT (GHEFT)

$$\mathcal{L}_{\text{GHEFT}} = \frac{1}{2} G_{ab}(\phi) \alpha^a_{\perp\mu} \alpha^{b\mu}_{\perp} + G_{aI}(\phi) \alpha^a_{\perp\mu} (\mathcal{D}^\mu \phi)^I + \frac{1}{2} G_{IJ}(\phi) (\mathcal{D}_\mu \phi)^I (\mathcal{D}^\mu \phi)^J - V$$

1-loop corrections

Symmetry manifest

$$\mathcal{L}_{\text{GHEFT}} = \frac{1}{2}G_{ab}\alpha^a_{\perp\mu}\alpha^{b\mu}_{\perp} + G_{aI}\alpha^a_{\perp\mu}(\mathcal{D}^\mu\phi)^I + \frac{1}{2}G_{IJ}(\mathcal{D}_\mu\phi)^I(\mathcal{D}^\mu\phi)^J - V$$

$$|||$$

$$\phi^I = \{h, H^0, H^\pm, H^{\pm\pm}, \cdots\}$$

Geometrical description

$$\mathcal{L}_{\text{GHEFT}} = \frac{1}{2}g_{ij}(\Phi)\partial_\mu\Phi^i\partial^\mu\Phi^j - V$$

$$\Phi^i = \{\pi^a, \phi^I\}$$

- easy to calculate physical observables
  - ➡ amplitude must be covariant under $\Phi \to \Phi'$
- easy to calculate 1-loop correction
  - ➡ $\mathcal{L}_{\text{1-loop}} = \frac{1}{(4\pi)^2\epsilon}\left[\frac{1}{12}R^i{}_{jkl}R^j{}_{imn}(\partial\Phi)^4 + \cdots\right]$

Alonso et. al. JHEP08(2016)101

1-loop corrections

Symmetry manifest

$$\mathcal{L}_{\rm GHEFT} = \frac{1}{2}G_{ab}\alpha^a_{\perp\mu}\alpha^{b\mu}_{\perp} + G_{aI}\alpha^a_{\perp\mu}(\mathcal{D}^\mu\phi)^I + \frac{1}{2}G_{IJ}(\mathcal{D}_\mu\phi)^I(\mathcal{D}^\mu\phi)^J - V$$

$$= \frac{1}{2}(\ \cdots\ )\partial_\mu\pi^a\partial^\mu\pi^b + (\ \star\star\star\ )\partial_\mu\pi^a\partial^\mu\phi^I + \frac{1}{2}(\ \circ\circ\circ\ )\partial_\mu\phi^I\partial^\mu\phi^J - V$$

$$= \frac{1}{2}\partial_\mu(\pi^a, \phi^I)\begin{pmatrix} (\ \cdots\ ) & (\ \star\star\star\ ) \\ (\ \star\star\star\ ) & (\ \circ\circ\circ\ ) \end{pmatrix}\partial^\mu\begin{pmatrix} \pi^b \\ \phi^J \end{pmatrix} - V$$

$$= \frac{1}{2}g_{ij}(\Phi)\partial_\mu\Phi^i\partial^\mu\Phi^j - V \qquad \Phi^i = \{\pi^a, \phi^I\}$$

Geometrical description

1-loop corrections

Symmetry manifest

$$\mathcal{L}_{\rm GHEFT} = \frac{1}{2}G_{ab}\alpha^a_{\perp\mu}\alpha^{b\mu}_{\perp} + G_{aI}\alpha^a_{\perp\mu}(\mathcal{D}^\mu\phi)^I + \frac{1}{2}G_{IJ}(\mathcal{D}_\mu\phi)^I(\mathcal{D}^\mu\phi)^J - V$$

$G_{ij}(\phi) \leftrightarrow g_{ij}(\phi)$ dictionary

$$g_{1I} = G_{1I} + \frac{1}{2}G_{3I}\pi^2 - \frac{1}{6}G_{1I}\pi^2\pi^2 + \frac{1}{6}G_{2I}\pi^1\pi^2 + \mathcal{O}((\pi)^3)\,,$$

$$g_{2I} = G_{2I} + \frac{1}{2}G_{3I}\pi^1 + \frac{1}{6}G_{1I}\pi^1\pi^2 - \frac{1}{6}G_{2I}\pi^1\pi^1 + \mathcal{O}((\pi)^3)\,,$$

$$\vdots$$

R. Nagai, M. Tanabashi, K. Tsumura, Y.U.
Phys. Rev. D **100**, 075020

$$= \frac{1}{2}g_{ij}(\Phi)\partial_\mu\Phi^i\partial^\mu\Phi^j - V \qquad \Phi^i = \{\pi^a, \phi^I\}$$

Geometrical description

1-loop corrections

Motivation

Make universal predictions for the properties of

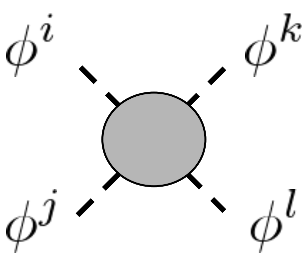
$$H^0, \quad H^{\pm}, \quad H^{\pm\pm}, \cdots$$

Ex.) **aQGC**

$$\mathcal{L}_{\text{1-loop}} = \frac{1}{(4\pi)^2\epsilon}\left[\frac{1}{12}R^i{}_{jkl}R^j{}_{imn}(\partial\Phi)^4 + \cdots\right]$$

Alonso et. al. JHEP08(2016)101

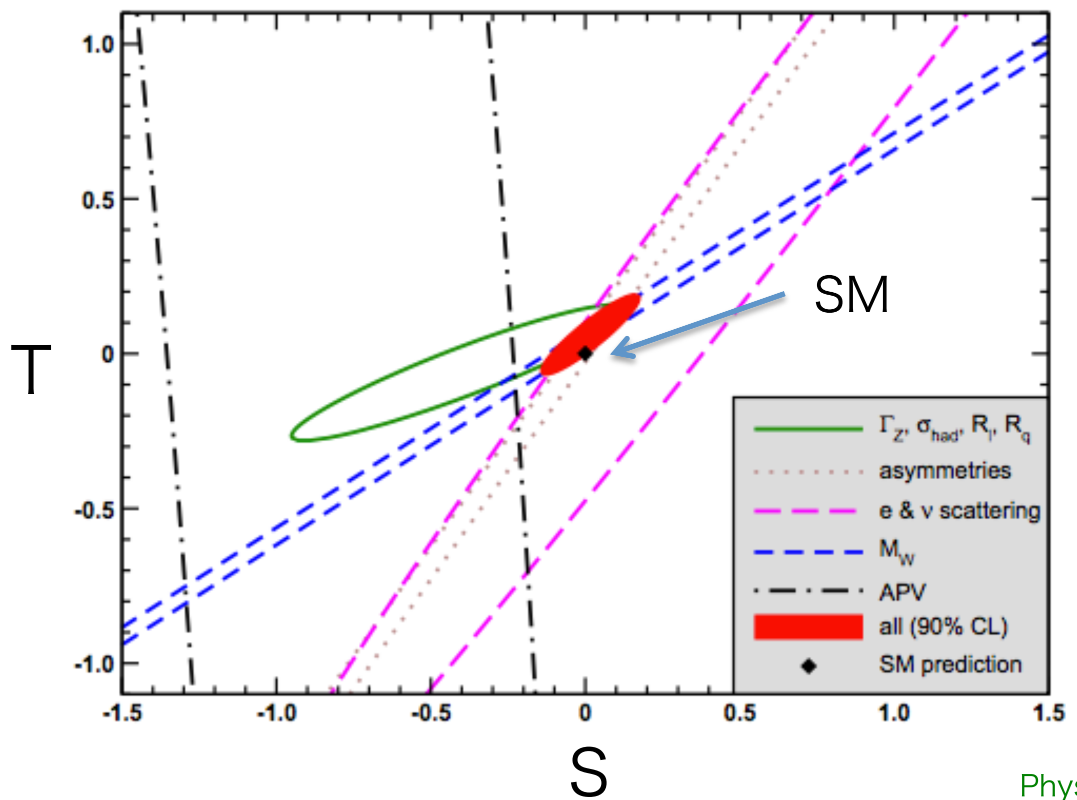
R_{ijkl} : 4-scalar amplitude



$$\underset{E \gg m_\phi}{\sim} \frac{s}{3}(\bar{R}_{iklj} + \bar{R}_{ilkj}) + \frac{t}{3}(\bar{R}_{ijlk} + \bar{R}_{iljk}) + \frac{u}{3}(\bar{R}_{ijkl} + \bar{R}_{ikjl})$$

1-loop corrections

Peskin-Takeuchi's S, T, U parameter



Tanabashi et al, (PDG), Phys. Rev. D 98, 030001(2018)

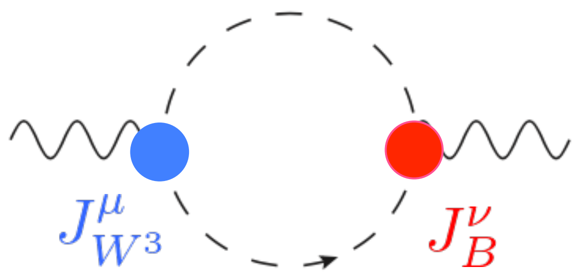
$$S = 0.02 \pm 0.07 \qquad T = 0.06 \pm 0.06$$

1-loop corrections

Peskin-Takeuchi's S, T, U parameter

Killing vector $w^i_a(\phi)$, $y^i(\phi)$ $\sim$ conserved current

$S = W^3_\mu$



$B_\mu \propto (\bar{w}^i_a)_{;j}(\bar{y}^j)_{;i}$

GHEFT is non-renormalizable $\rightarrow$ S parameter diverges !!

$$S_{\mathrm{div}} = -\frac{1}{12\pi}(\bar{w}^i_3)_{;j}(\bar{y}^j)_{;i} \ln\frac{\Lambda^2}{\mu^2}$$

Talk Plan

- Introduction
- Extension of HEFT
- 1-loop correction of generalized HEFT
- Phenomenological implication
- Summary

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Correlation among parameters

Scattering amplitude

$$\mathcal{M} \sim V_{;(ijkl)} + R_{ijkl}E^2 + \frac{1}{(4\pi)^2}\ln\frac{\Lambda^2}{\mu^2}\;(R\cdot R)_{ijkl}E^4 + \cdots$$

Peskin-Takeuchi's S, T, U parameter

$$S_{\rm div} = -\frac{1}{12\pi}(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i}\ln\frac{\Lambda^2}{\mu^2}$$

$w_a^i(\phi)$, $y^i(\phi)$: Killing vector

Correlation among parameters

Scattering amplitude

$$\mathcal{M} \sim V_{;(ijkl)} + R_{ijkl}E^2 + \frac{1}{(4\pi)^2} \ln \frac{\Lambda^2}{\mu^2} \; (R \cdot R)_{ijkl} E^4 + \cdots$$

Peskin-Takeuchi's S, T, U parameter

Thanks to the Killing eq. we can rewrite $S_{\rm div}$

$$S_{\rm div} \sim \frac{1}{(4\pi)^2} R^i{}_{jkl} \cdot \vec{w}\vec{w}(\partial \vec{w}) \; \ln \frac{\Lambda^2}{\mu^2}$$

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Phenomenological implication

$$S_{\rm div} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}_c^k)(\bar{w}_3^l)\bar{R}^i{}_{jkl}(\bar{w}_b^j)_{;i} + \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

$\bar{w}_a^i$, $\bar{y}^i$: Killing vector for $SU(2)_L$, $U(1)_Y$

$$\sum_{k,l}\bar{w}^k\bar{w}^l\bar{R}^i{}_{jkl} = \bar{w}^{\pi}\bar{w}^{\pi}\bar{R}^i{}_{j\pi\pi} + \bar{w}^{\phi^I}\bar{w}^{\phi^J}\bar{R}^i{}_{j\phi^I\phi^J}$$

Phenomenological implication

$$S_{\rm div} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}^k_c)(\bar{w}^l_3)\bar{R}^i{}_{jkl}(\bar{w}^j_b)_{;i} + \epsilon_{3bc}(\bar{w}^k_b)(\bar{w}^l_c)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

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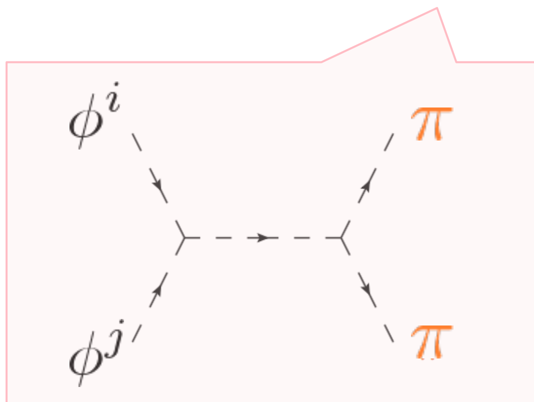

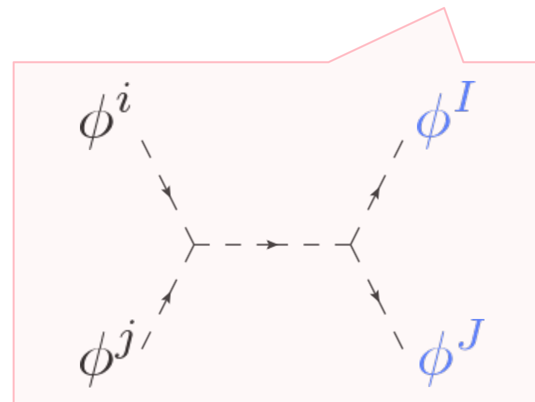

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$$S_{\rm div} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}^k_c)(\bar{w}^l_3)\bar{R}^i{}_{jkl}(\bar{w}^j_b)_{;i} + \epsilon_{3bc}(\bar{w}^k_b)(\bar{w}^l_c)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

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$$S_{\rm div} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}_c^k)(\bar{w}_3^l)\bar{R}^i{}_{jkl}(\bar{w}_b^j)_{;i} + \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

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$$\pi \to \pi + \langle h\rangle \qquad \phi^I \to \phi^I + i[Q_\phi]^I{}_J\phi^J$$

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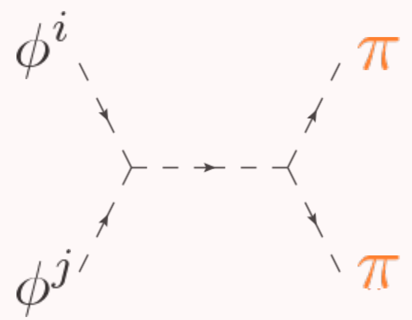

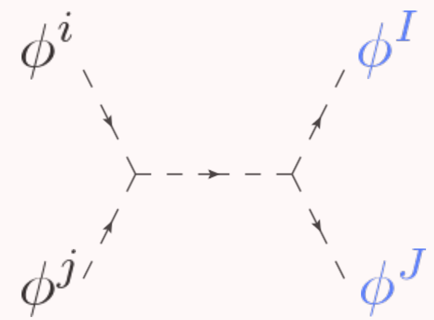

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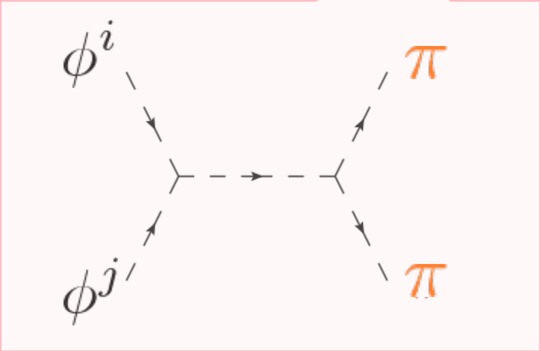

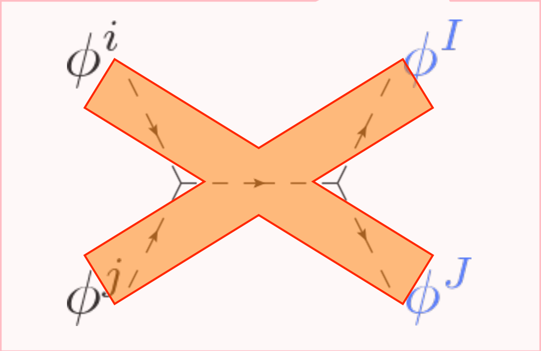

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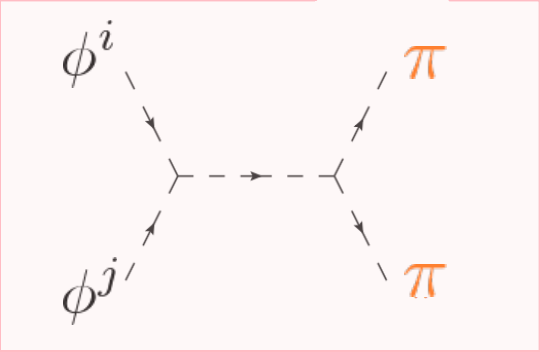

Phenomenological implication

$$S_{\text{div}} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}^k_c)(\bar{w}^l_3)\bar{R}^i{}_{jkl}(\bar{w}^j_b)_{;i} + \epsilon_{3bc}(\bar{w}^k_b)(\bar{w}^l_c)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

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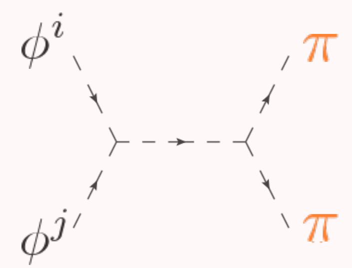

When we require $S_{\text{div}} = 0$ $\cdots$

imposing $\bar{R}_{ijkl} = 0$ are too strict
(perturbative unitary)

only $\bar{R}^i{}_{j\pi\pi} = 0$ is enough

Phenomenological implication

$$S_{\rm div} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}_c^k)(\bar{w}_3^l)\bar{R}^i{}_{jkl}(\bar{w}_b^j)_{;i} + \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

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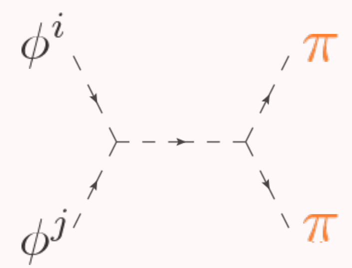
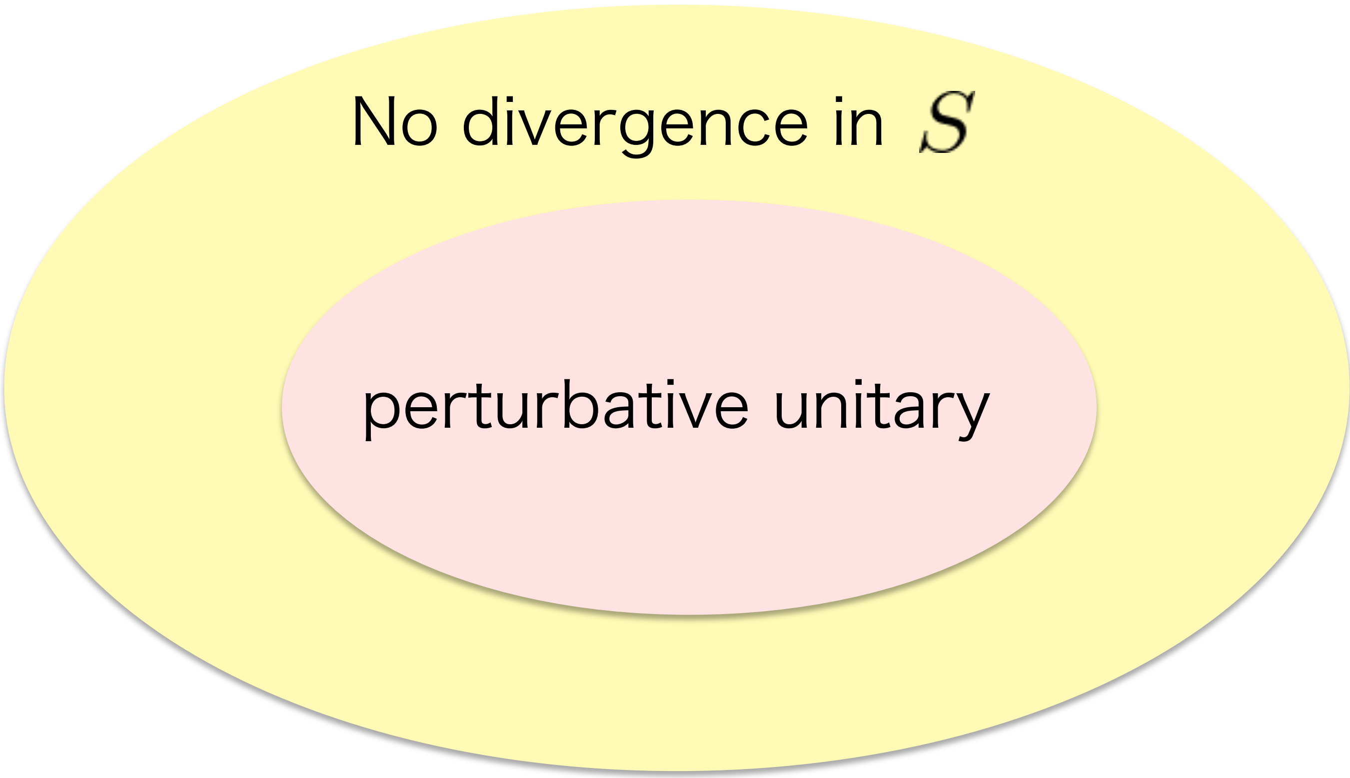


Finiteness of S
(*i.e.* $S_{\rm div} = 0$)

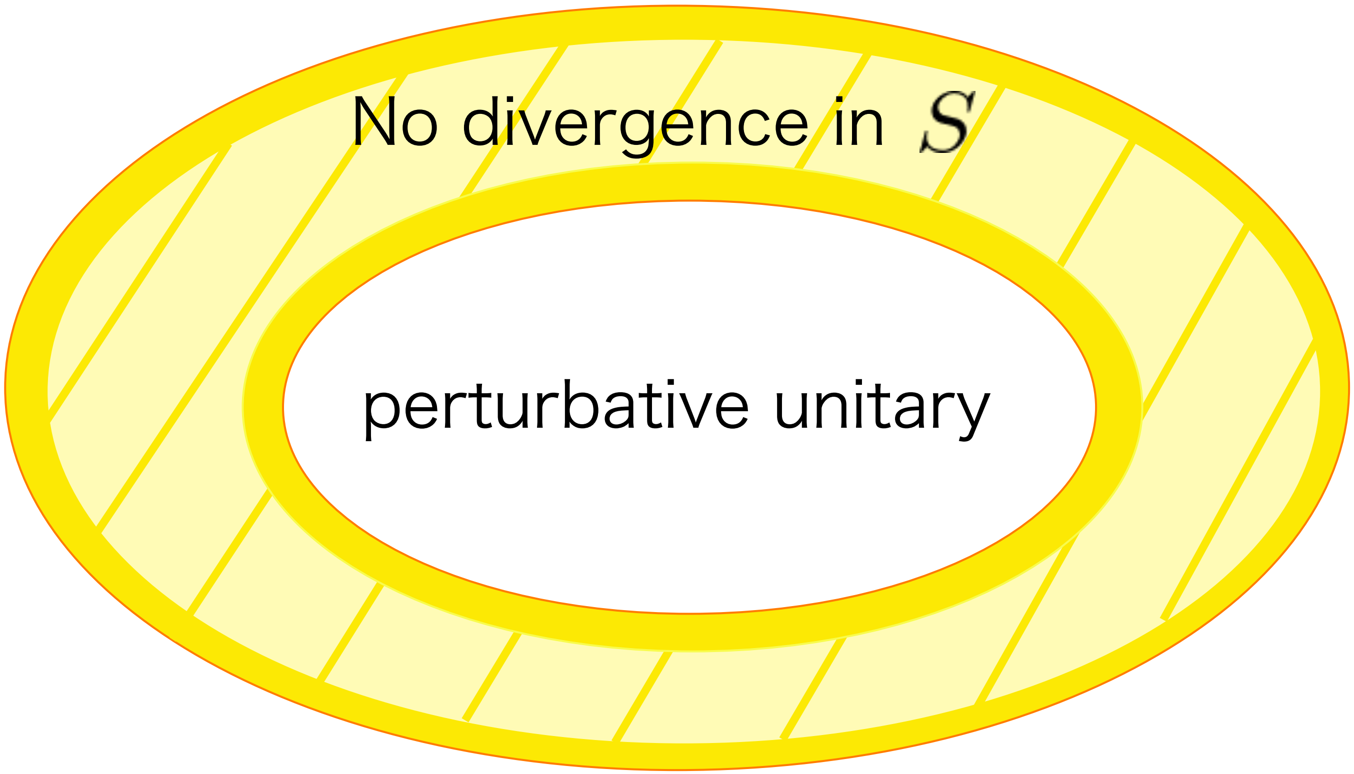
$<$

perturbative unitarity
(*i.e.* $\bar{R}_{ijkl} = 0$)

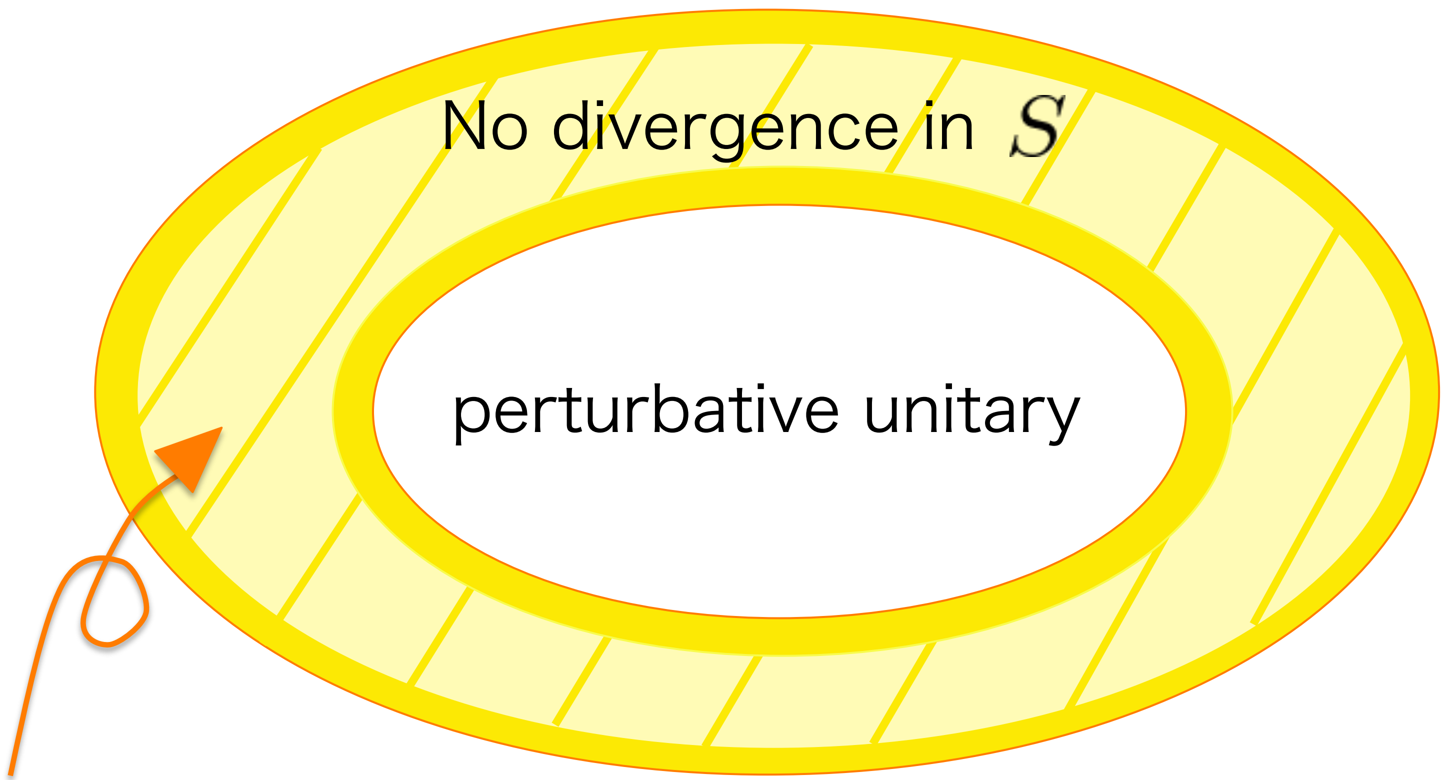
Phenomenological implication



Phenomenological implication



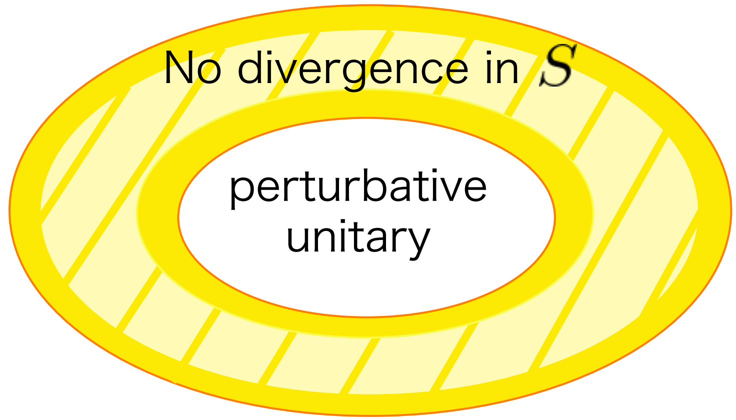
Phenomenological implication



Tree level unitary ×

S 1-loop finite ○

Phenomenological implication

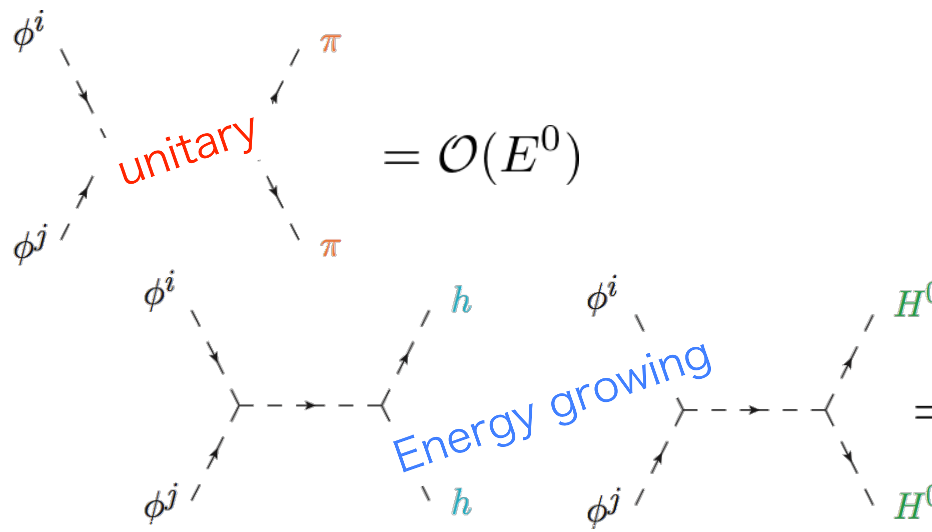


$S_{\rm div} = 0$ if we require $\bar{R}^i{}_{j\pi\pi} = 0$

$\bar{w}^\pi \bar{w}^\pi \bar{R}^i{}_{j\pi\pi}$

$\bar{w}^h \bar{w}^h \bar{R}^i{}_{jhh}$

$\bar{w}^{H^0} \bar{w}^{H^0} \bar{R}^i{}_{jH^0H^0}$



$= \mathcal{O}(E^0)$

$= \mathcal{O}(E^2)$

Summary

- We formulate a generalization of HEFT (GHEFT) to including arbitrary number of extra neutral and charged scalars.
- Thanks to CCWZ method, we construct GHEFT where nonlinear SU(2)xU(1) invariance is manifest.
- In order to calculate 1-loop correction to GHEFT, we rewrite GHEFT so that its geometry is explicit.
- Focusing the geometry of the GHEFT, we find that oblique parameters are deeply related to the perturbative unitarity of the GHEFT.

Prospects

- So far, we focus on the scalar extension of HEFT. Now, we try to add fermions to GHEFT.

Step1: GHEFT + Majorana fermion

Step2: GHEFT + Dirac fermion

$\mathcal{M}(\chi\chi \to \phi^i) \;\propto$ Covariant ϕ^i-derivative of M_χ ?

χ : Majorana fermion

- We also plan to add vector bosons to GHEFT

- We calculate Riemann tensors in each models and made lists.

Back Up

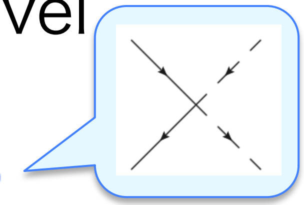
Loop expansion : tree level

tree level $\mathcal{L} = \Delta\mathcal{L}_{\rm LO}$

$$\Delta\mathcal{L}_{\rm LO} = \frac{v^2}{4}F(h)\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h) + \cdots$$

Loop expansion : 1-loop level

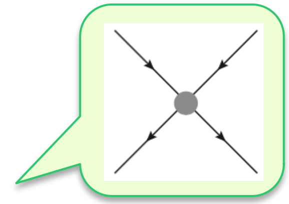
tree level $\mathcal{L} = \Delta\mathcal{L}_{\rm LO}$



$\bar{\psi}\psi \to \phi\phi$

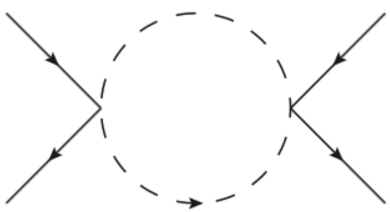
1-loop level $\mathcal{L} = \Delta\mathcal{L}_{\rm LO} + \Delta\mathcal{L}_{\rm NLO}$

(counterterm)



4-fermi

Renormalization condition



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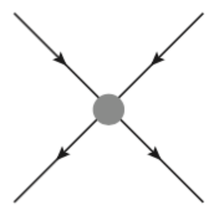


$= \mathcal{O}_{\rm exp}$

1-loop diagram w/ $\Delta\mathcal{L}_{\rm LO}$ vertex

counterterm in $\Delta\mathcal{L}_{\rm NLO}$

Observables not used for R.C. can be predicted

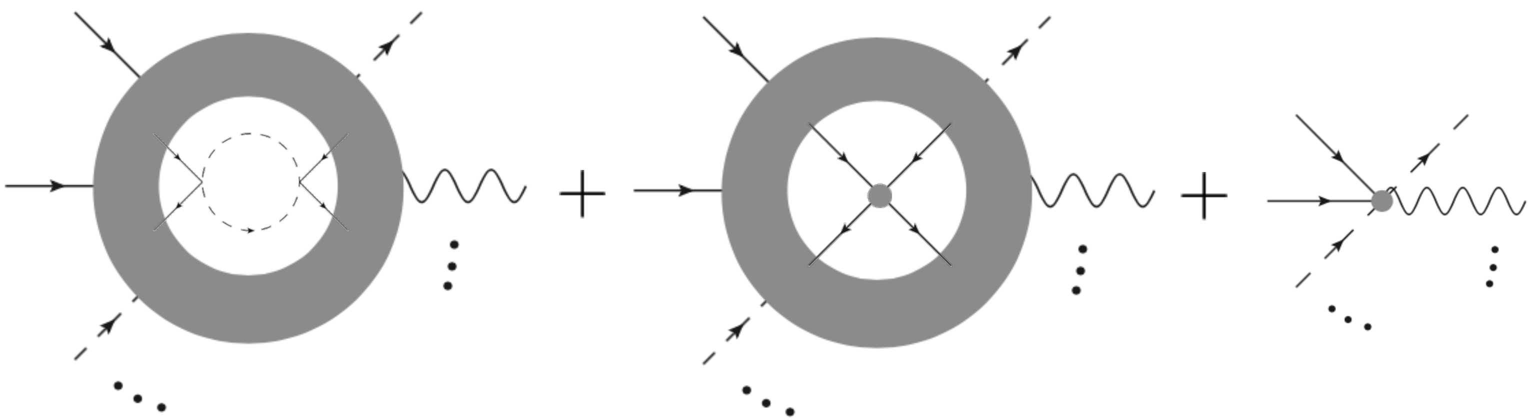
Loop expansion : 2-loop level

tree level $\mathcal{L} = \Delta\mathcal{L}_{\rm LO}$

1-loop level $\mathcal{L} = \Delta\mathcal{L}_{\rm LO} + \Delta\mathcal{L}_{\rm NLO}$

2-loop level $\mathcal{L} = \Delta\mathcal{L}_{\rm LO} + \Delta\mathcal{L}_{\rm NLO} + \Delta\mathcal{L}_{\rm NNLO}$

Renormalization condition



$$= \mathcal{O}'_{\rm exp}$$

2-loop diagram w/ $\Delta\mathcal{L}_{\rm LO}$ vertex

1-loop diagram w/ $\Delta\mathcal{L}_{\rm NLO}$ vertex

counterterm in $\Delta\mathcal{L}_{\rm NNLO}$

Observables not used for R.C. can be predicted

Power counting

表 7.1: 相互作用の分類と、NLO に含まれる個数

LO の vertex	φ^{2i}	$\overline{\psi}_{L(R)}\psi_{R(L)}\varphi^k$	$X_\mu\varphi^l$	$X_\mu^2\varphi^s$	X_μ^4	X_μ^3	$\overline{\psi}_{L(R)}\psi_{L(R)}X_\mu$
因子	p^2/v^{2i-2}	y/v^{k-1}	gp/v^{l-2}	g^2/v^{s-2}	g^2	gp	g
$\mathcal{D}_L$ に含まれる個数	n_i	ν_k	m_l	r_s	x	u	$z_L(z_R)$

L-loop diagram : $\mathcal{D}_L$

$$\mathcal{D}_L \sim \frac{(yv)^\nu (gv)^{m+2r+2x+u+z}}{v^{F_L+F_R-2}} \frac{p^d}{\Lambda^{2L}} \bar{\psi}_L^{F_L^1} \psi_L^{F_L^2} \bar{\psi}_R^{F_R^1} \psi_R^{F_R^2} \left(\frac{X_{\mu\nu}}{v}\right)^V \left(\frac{\varphi}{v}\right)^B$$

$$d \equiv 2L + 2 - \frac{F_L + F_R}{2} - V - \nu - m - 2r - 2x - u - z$$

1-loop corrections

Peskin-Takeuchi's S, T, U parameter

$$S_{\rm div} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}^k_c)(\bar{w}^l_3)\bar{R}^i{}_{jkl}(\bar{w}^j_b)_{;i} + \epsilon_{3bc}(\bar{w}^k_b)(\bar{w}^l_c)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

$$U_{\rm div} = \frac{1}{12\pi}\left(\epsilon_{1bc}(\bar{w}^k_b)(\bar{w}^l_c)\bar{R}^i{}_{jkl}(\bar{w}^j_1)_{;i} - \epsilon_{3bc}(\bar{w}^k_b)(\bar{w}^l_c)\bar{R}^i{}_{jkl}(\bar{w}^j_3)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

$$\begin{aligned} T_{\rm div} \sim & \left[(\bar{w}^i_1)(\bar{w}^j_1) - (\bar{w}^i_3)(\bar{w}^j_3)\right] \\ & \times \Big\{\bar{R}_{ikjl}[\bar{g}^{km}\bar{g}^{ln}\bar{V}_{;mn} + \bar{w}^k_a\bar{w}^l_a + \bar{y}^k\bar{y}^l] \\ & \qquad - 4g_W^2(\bar{w}^k_a)_{;i}(\bar{w}^l_a)_{;j}\bar{g}_{kl} - 4g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl}\Big\}\ln\frac{\Lambda^2}{\mu^2} \end{aligned}$$

T parameter

$$T_{\rm div} \sim \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\right]$$
$$\times \Big\{ \bar{R}_{ikjl}[\bar{g}^{km}\bar{g}^{ln}\bar{V}_{;mn} + \bar{w}_a^k \bar{w}_a^l + \bar{y}^k \bar{y}^l]$$
$$- 4g_W^2(\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl}\Big\} \ln\frac{\Lambda^2}{\mu^2}$$

T parameter

$$
\begin{aligned}
T_{\rm div} \sim & \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\right] \\
& \times \Big\{ \bar{R}_{ikjl}[\bar{g}^{km}\bar{g}^{ln}\bar{V}_{;mn} + \bar{w}_a^k \bar{w}_a^l + \bar{y}^k \bar{y}^l] \\
& \qquad - 4g_W^2(\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl} \Big\} \ln\frac{\Lambda^2}{\mu^2}
\end{aligned}
$$

T parameter

$$T_{\text{div}} \sim \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\right]$$

$$\times \left\{ \bar{R}_{ikjl}[\bar{g}^{km}\bar{g}^{ln}\bar{V}_{;mn} + \bar{w}_a^k\bar{w}_a^l + \bar{y}^k\bar{y}^l] - 4g_W^2(\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl} \right\} \ln\frac{\Lambda^2}{\mu^2}$$

if $\bar{R}_{ikjl} = 0$

$$T_{\text{div}} \sim \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\right]$$

$$\times \left\{ - 4g_W^2(\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl} \right\} \ln\frac{\Lambda^2}{\mu^2}$$

= 0 (SM)

= 0 (2HDM)

$\neq$ 0 (Georgi Machacek Model)

SMEFT

Standard Model Effective Field Theory

$$\mathcal{L}_{\rm SMEFT} = (D_\mu H)^\dagger D^\mu H + \underbrace{\frac{C_1}{f^2}(H^\dagger D_\mu H)^*(H^\dagger D^\mu H) + \cdots}$$

v/f expansion

- $v \ll f$: Good approximation

$$\mathcal{L}_{\rm MCHM} = (D_\mu H)^\dagger (D_\mu H) - \frac{2}{3f^2}|H|^2 (D_\mu H)^\dagger (D^\mu H) + \cdots$$

- $v \lesssim f$: Not valid

$$\mathcal{L}_{\rm MCHM} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{g^2}{v^2}\left(|W|^2 + \frac{1}{2c_W^2}Z^2\right)\left[2\sqrt{1-\xi}\frac{h}{v} + (1-2\xi)\frac{h^2}{v^2} + \cdots\right]$$

Extension of HEFT

Generalized HEFT (GHEFT)

$$\mathcal{L}_{\text{GHEFT}} = \frac{1}{2} G_{ab}(\phi) \alpha_{\perp\mu}^{a} \alpha_{\perp}^{b\mu} + G_{aI}(\phi) \alpha_{\perp\mu}^{a} (\mathcal{D}^{\mu}\phi)^{I} + \frac{1}{2} G_{IJ}(\phi) (\mathcal{D}_{\mu}\phi)^{I} (\mathcal{D}^{\mu}\phi)^{J} - V$$

$$\phi^{I} = \{H^{0}, H^{\pm}, H^{\pm\pm}, \cdots\} \qquad (\mathcal{D}_{\mu}\phi)^{I} = \partial_{\mu}\phi^{I} - i\alpha_{\parallel\mu}[Q_{\phi}]^{I}{}_{J}\phi^{J}$$

HEFT limit : $\phi^{I} = h$

$$G_{ab}(\phi) \to F(h)\delta_{ab}$$

$$G_{aI}(\phi) \to 0$$

$$G_{IJ}(\phi) \to 1$$