Analytic gravitational-wave spectrum from bubble collisions

Masahiro Takimoto (KEK)

Based on arXiv:1605.01403 (by Rysuke Jinno & MT)

Introduction

FIRST DECECTION OF GWS

- LIGO announcement @ 2016/2/11

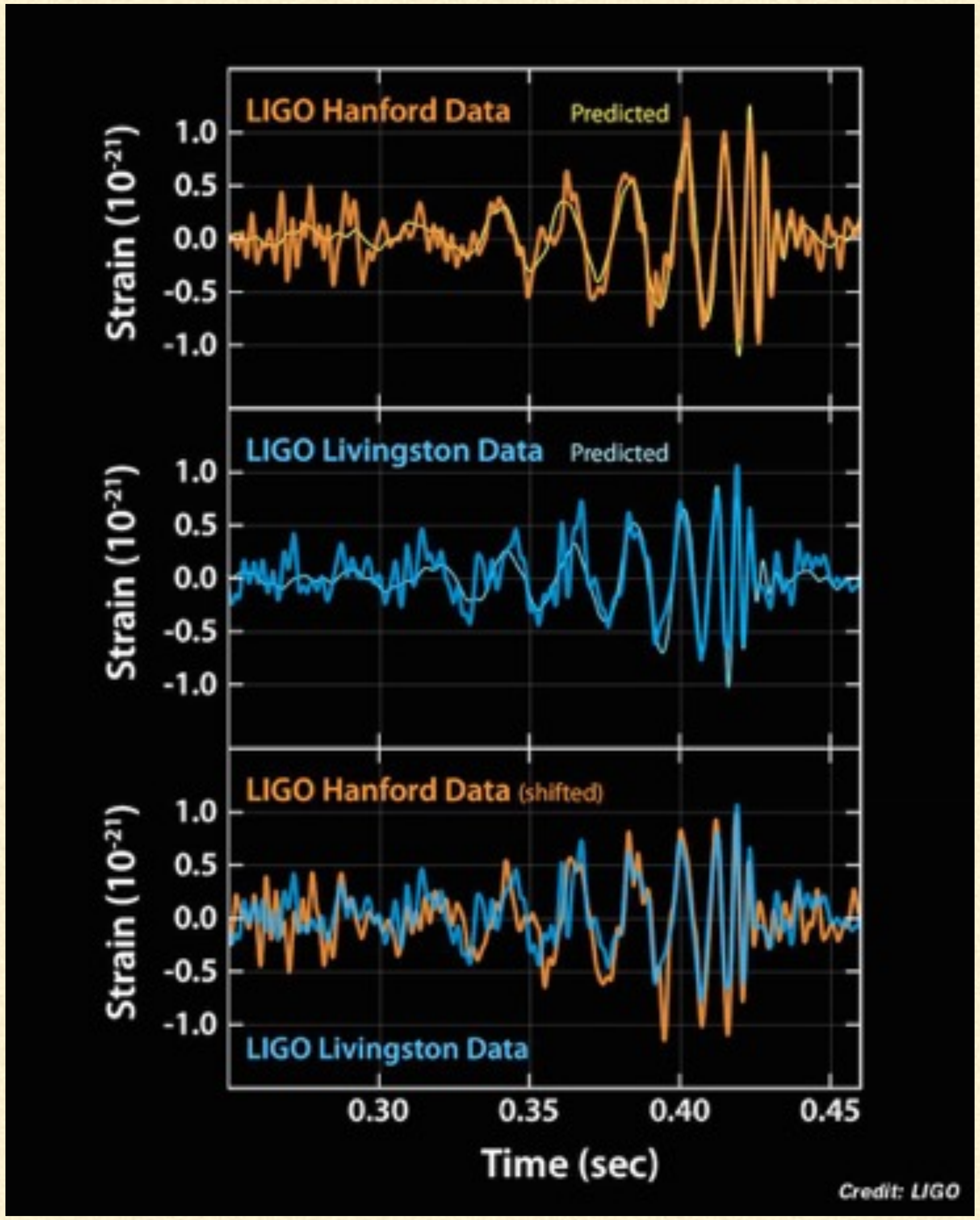
  - Black hole binary

    36M⊙ + 29M⊙ →62M⊙

    with 3.0M⊙ radiated in GWs

  - Frequency ~ 35 to 250 Hz

  - Significance > 5.1σ



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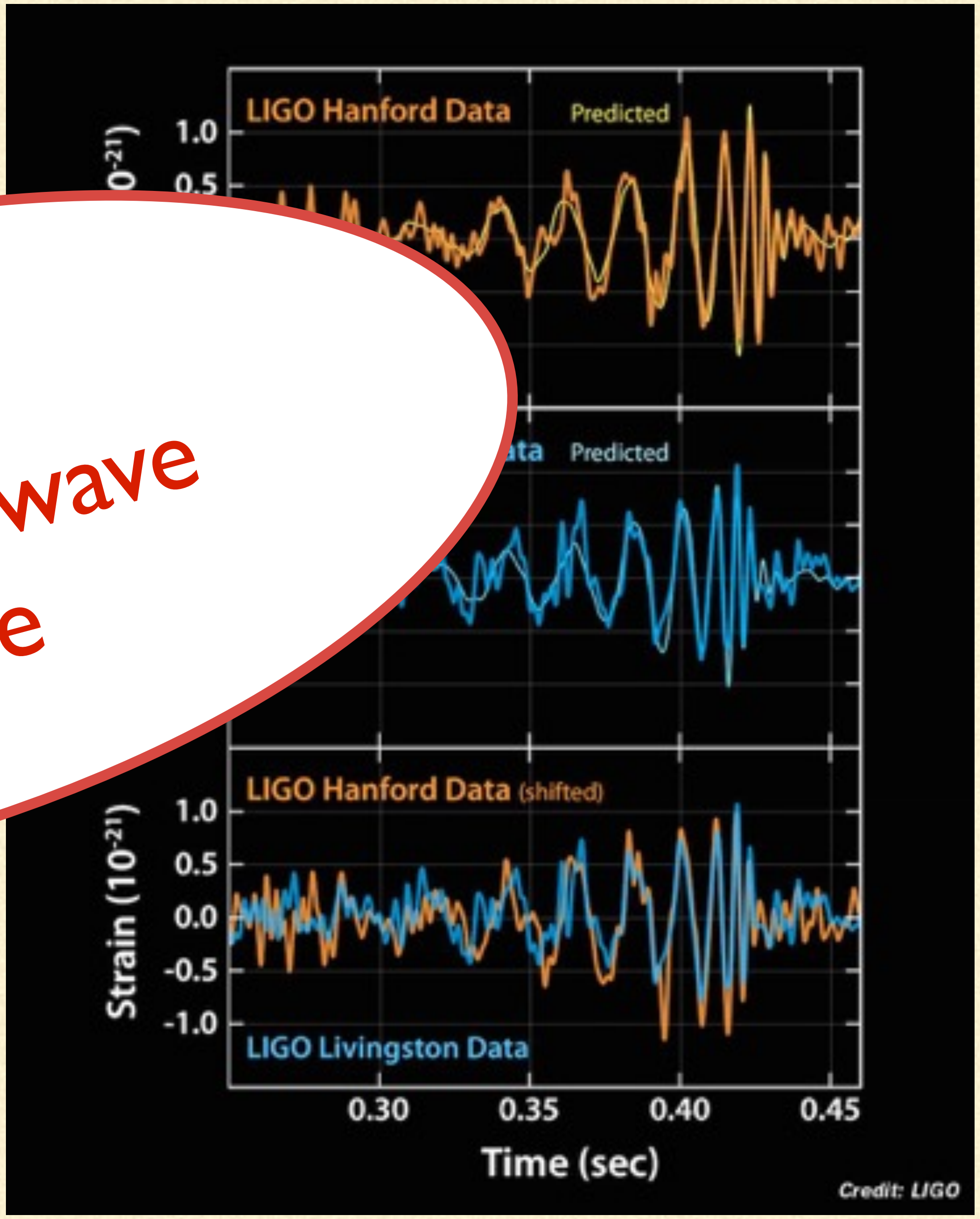

The era of
Gravitational wave
has come

EXPERIMENTAL PROSPECTS

From Ando-san's talk @日本物理学会2014年秋季大会

COSMOLOGICAL SOURCES OF GWS

- Cosmological sources of GWs

  - Inflationary quantum fluctuations (“Primordial GWs”)

  - Preheating

  - Cosmic string

  - First-order phase transition

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TALK PLAN

0. Introduction

1. Phase transition in the early universe

2. GWs produced in phase transition

3. GWs from bubble collision

   - What is usually done - What we did - What we will do

1. Phase transition in the early universe

PHASE TRANSITION IN THE EARLY UNIVERSE

- Particle physics which may accompany phase transition

  - Electroweak

  - Physics related to the naturalness of EW scale

    (e.g. SUSY, classical conformality, ...)

  - Peccei-Quinn PT

  - GUT

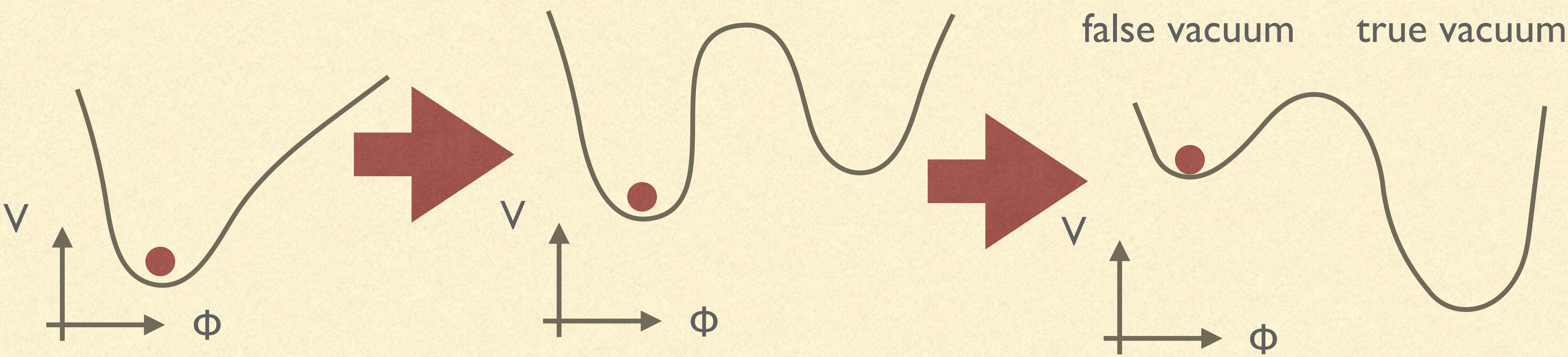
... and so on

PHASE TRANSITION IN THE EARLY UNIVERSE

- How (first order) phase transition occurs

- High temperature

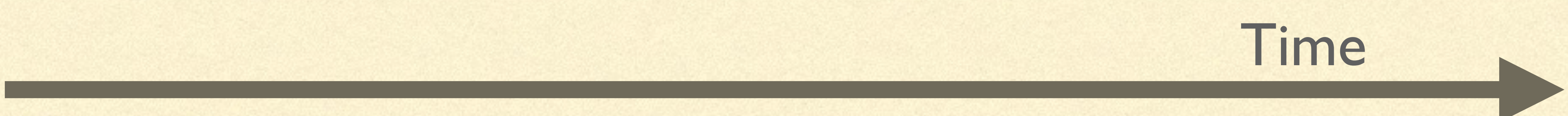
- Low temperature



Trapped at symmetry enhanced point

Another extreme appears

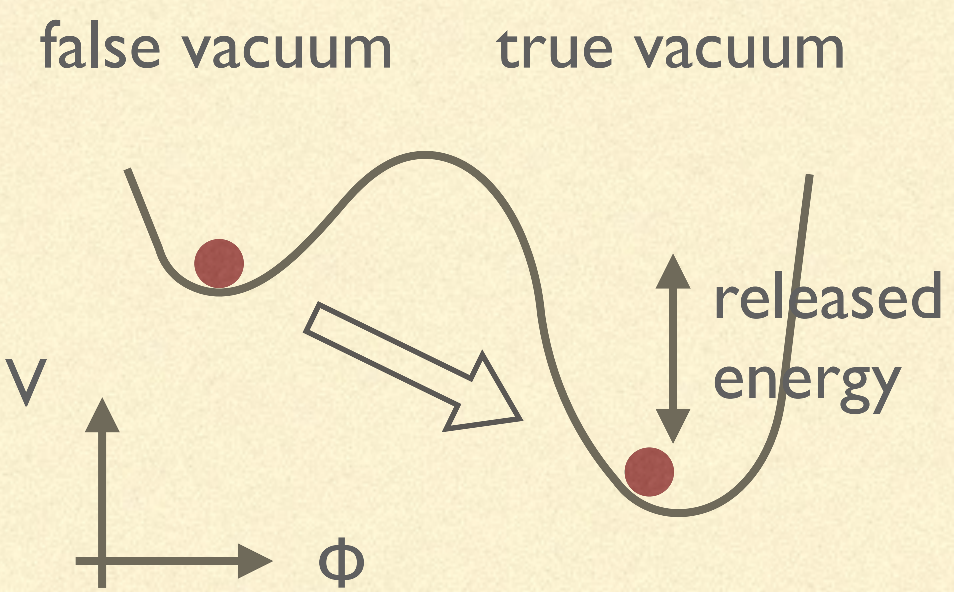
Another extreme becomes stable



PHASE TRANSITION IN THE EARLY UNIVERSE

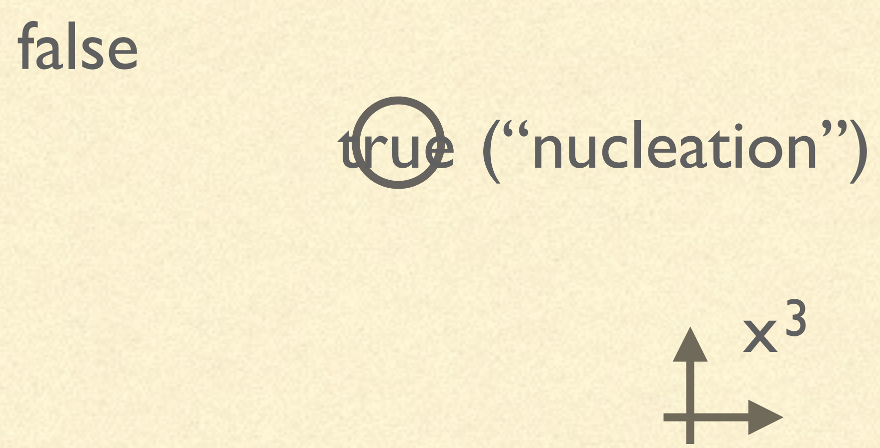
- How first order phase transition occurs

- Field space



Quantum tunneling

- Position space



Bubble formation

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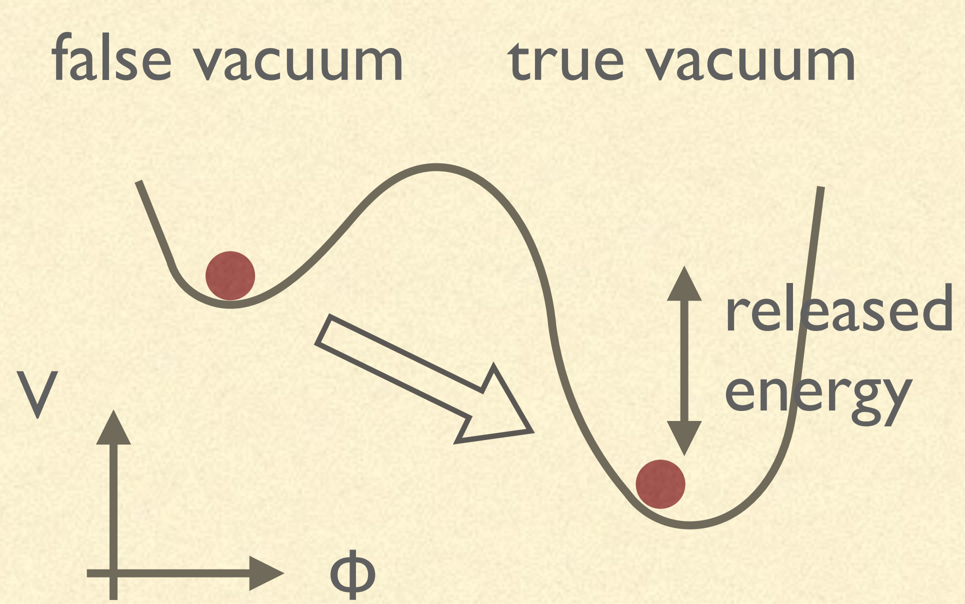

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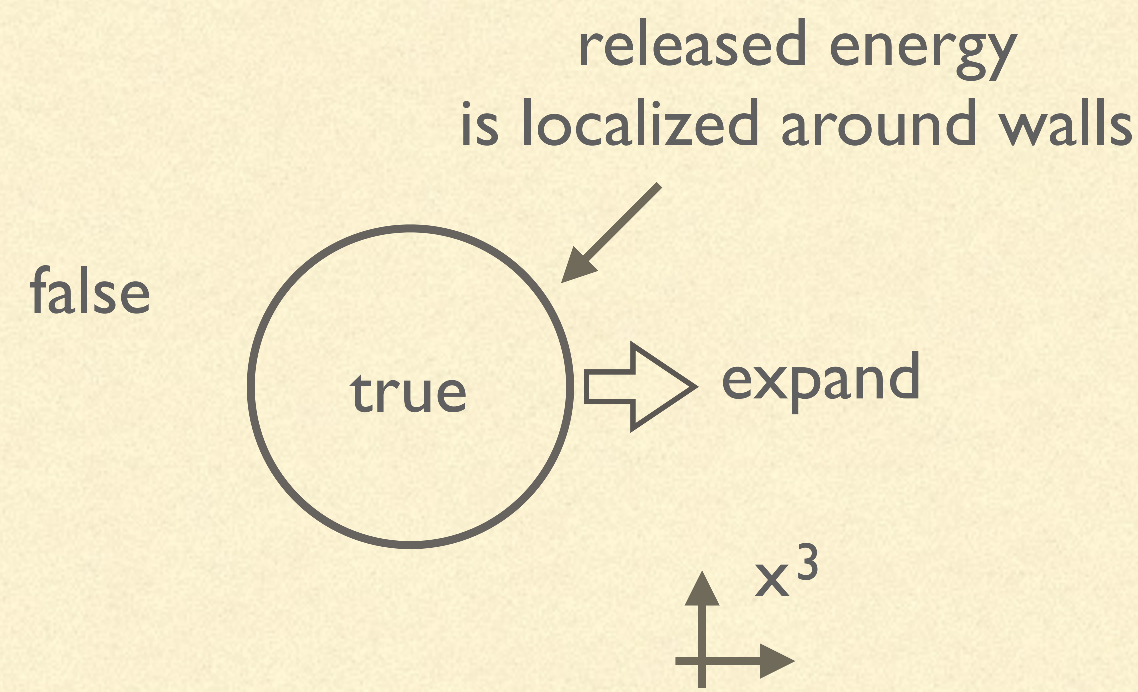

Bubble formation

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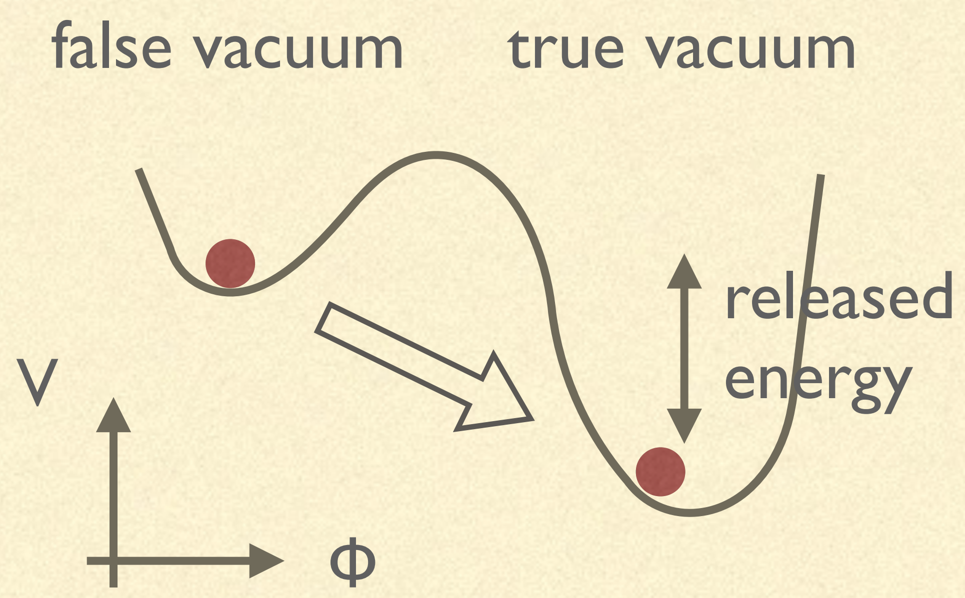

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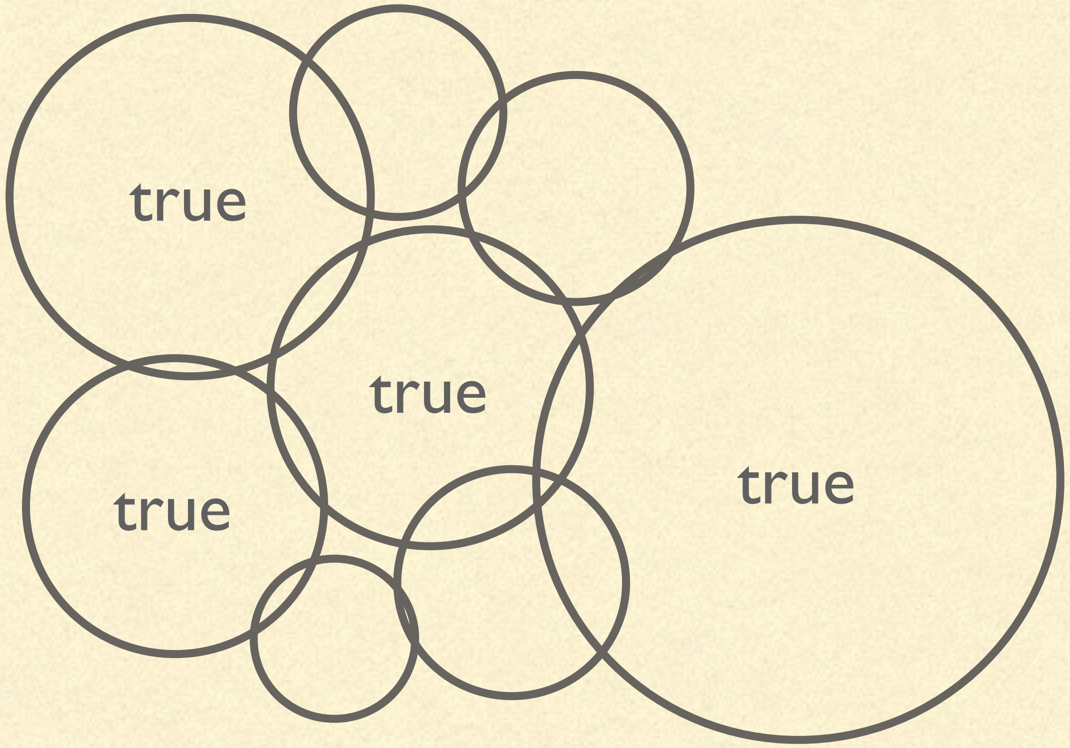


Bubble formation

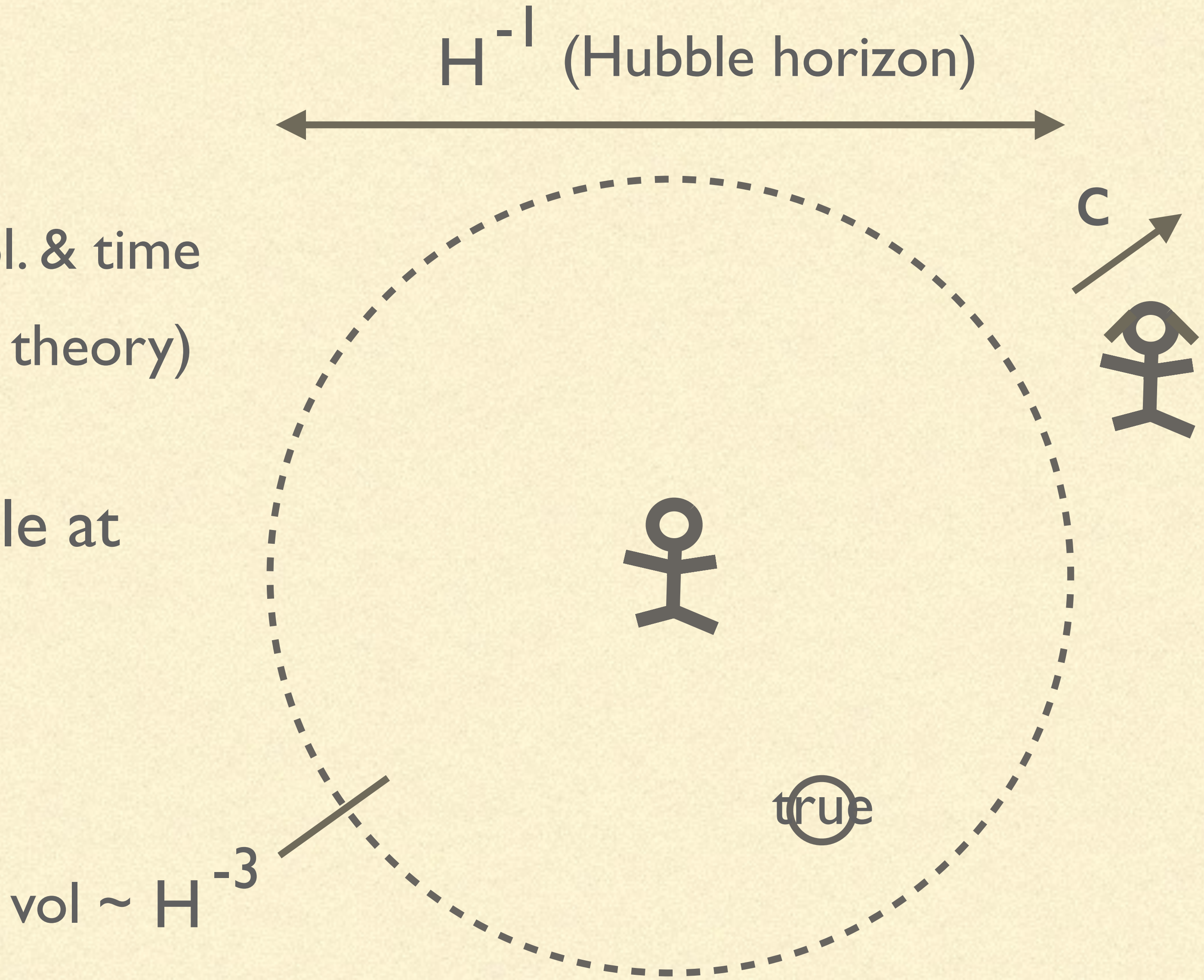
PHASE TRANSITION IN THE EARLY UNIVERSE

- When first order phase transition occurs
  - $\boxed{\text{Transition occurs at } \Gamma \sim H^4}$

Γ : nucleation rate per unit vol. & time (determined by underlying theory)

because  finds one bubble at

$$\Gamma \times \underbrace{\text{vol}}_{H^{-3}} \times \underbrace{t}_{H^{-1}} \sim 1$$



PHASE TRANSITION IN THE EARLY UNIVERSE

- How long first order phase transition lasts

  - $\boxed{\text{Duration of PT}}$ is determined by the changing rate of Γ (= β)

$$\Gamma = \Gamma_* e^{\beta(t - t_*)} \quad : \quad \text{Taylor exp. around transition time } t_*$$

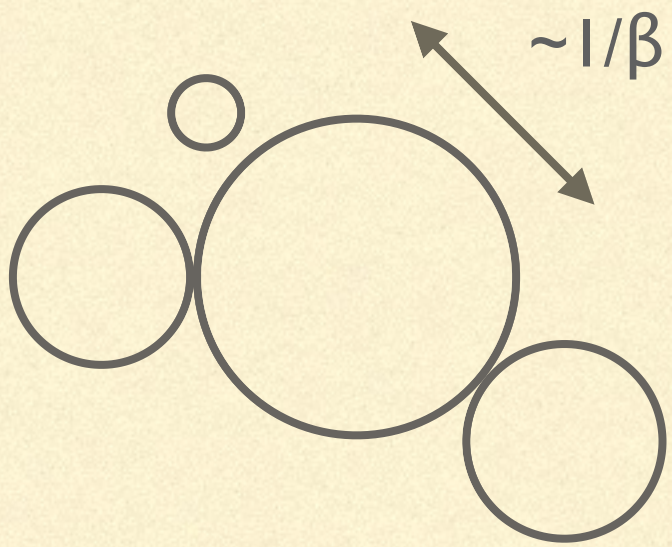
because

1. Γ significantly changes

   with time interval $\delta t \sim 1/\beta$

2. So, the first bubble typically collides

   with others δt after nucleation



PHASE TRANSITION IN THE EARLY UNIVERSE

- How long first order phase transition lasts

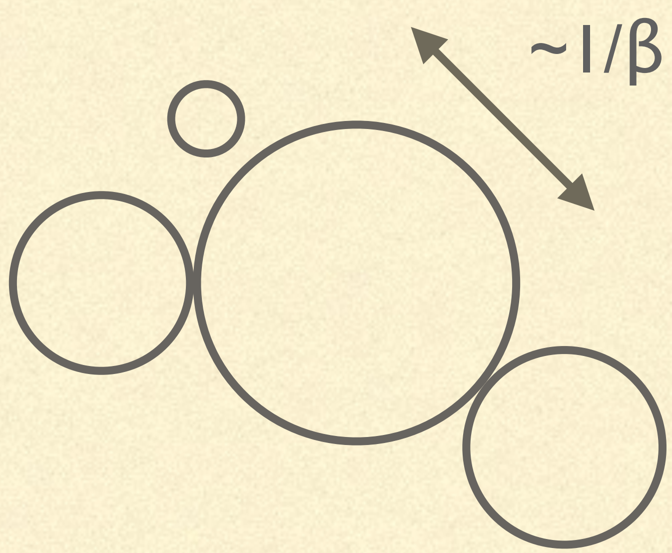
  - $\boxed{\text{Duration of PT}}$ is determined by the changing rate of Γ ($= \beta$)

$$\Gamma = \Gamma_* e^{\beta(t - t_*)} \quad : \quad \text{Taylor exp. around transition time } t_*$$

To summarize, duration of PT is $\boxed{\delta t \sim 1/\beta}$

large β (fast-changing Γ) : short duration

small β (slowly-changing Γ) : long duration



PHASE TRANSITION IN THE EARLY UNIVERSE

Duration of phase transition $\boxed{\delta t \sim 1/\beta}$

- Estimation of the transition rate

  - Then we can calculate β

  - $\boxed{\text{Transition occurs at } \Gamma \sim H^4}$

  (= how fast transition rate changes, $\Gamma \sim \Gamma_* e^{\beta(t - t_*)}$) $\Gamma \sim O(T^4)\, e^{-S_3/T}$

  since it is just Taylor expansion coefficient

  bounce action

$$\boxed{\beta \simeq \frac{d(S_3/T)}{dt} \simeq H \frac{d(S_3/T)}{d\ln T}}$$

普通は

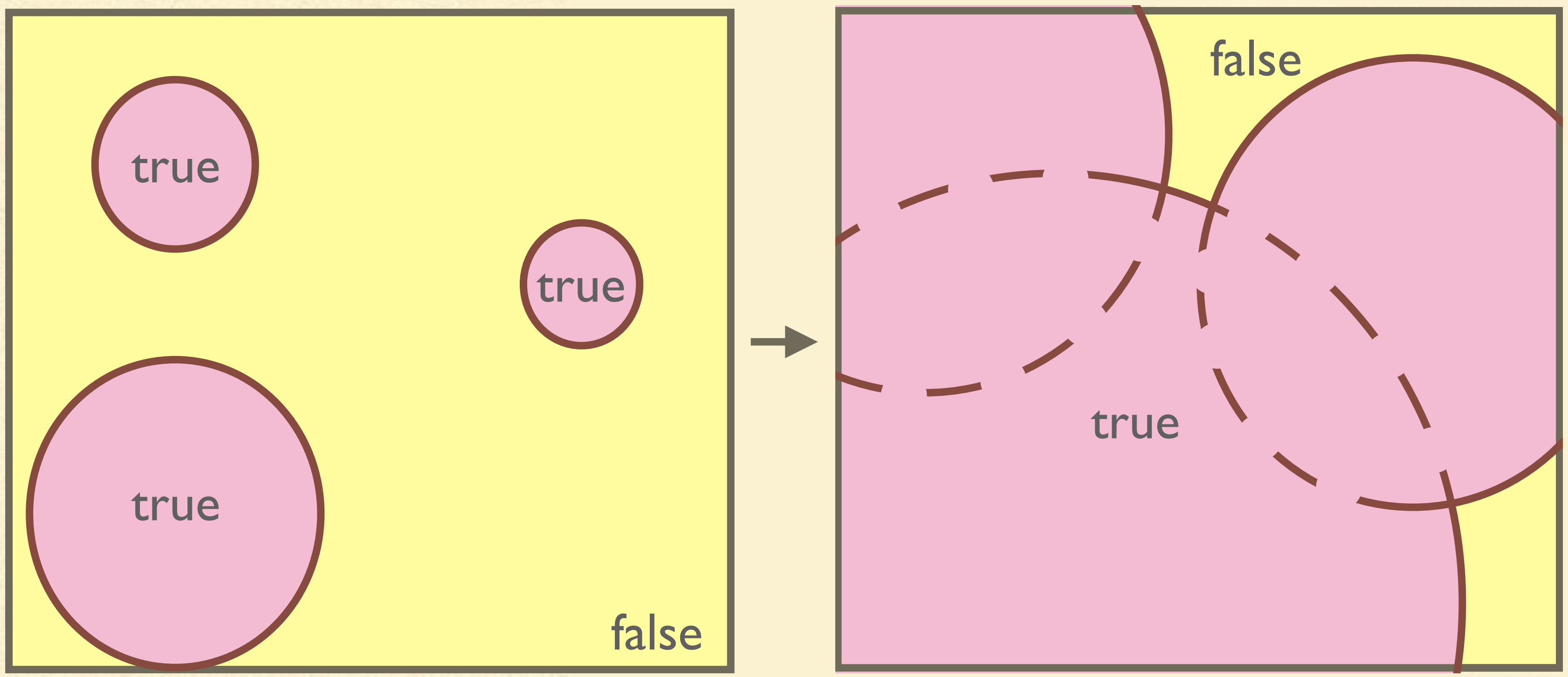
$\beta/H \gtrsim 1$

Note again: small beta→ Large gravitational waves

2. GWs produced in phase transition

2. GWS PRODUCED IN PHASE TRANSITION

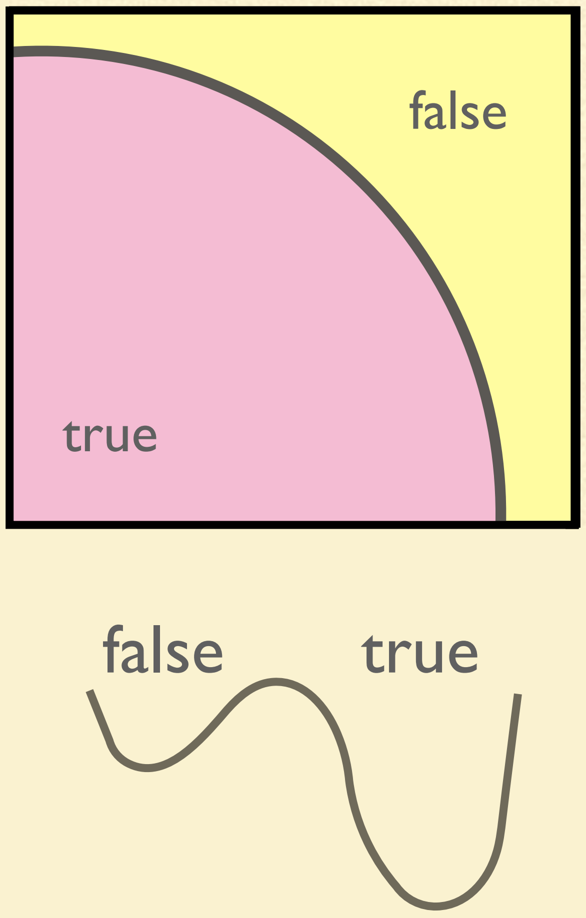
- What do we need to calculate GW production ?



- Transition rate (considered before)
- Energy momentum tensor of the system

2. GWS PRODUCED IN PHASE TRANSITION

- Single bubble profile



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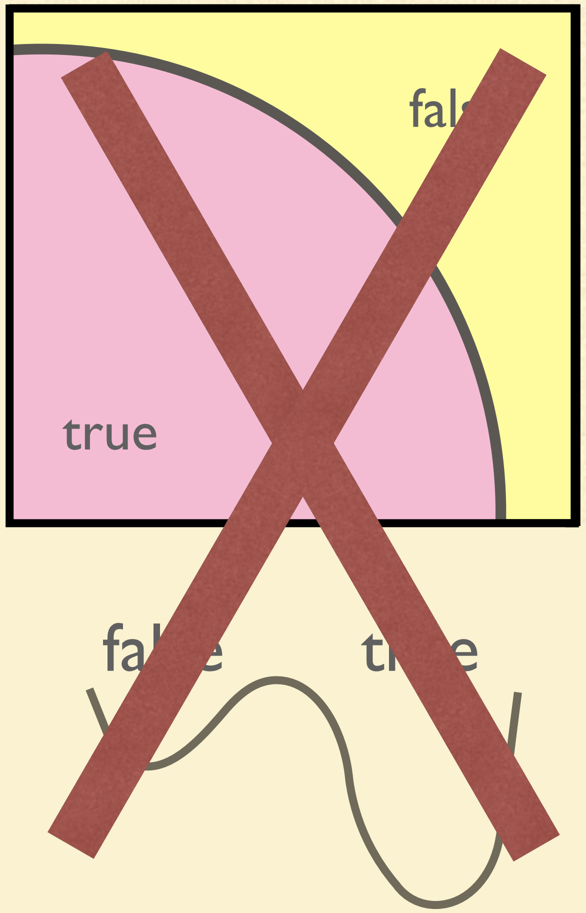


Not this simple

2. GWS PRODUCED IN PHASE TRANSITION

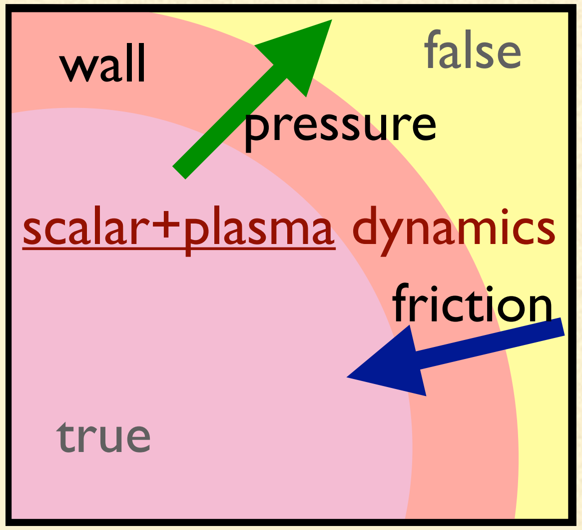
- Single bubble profile

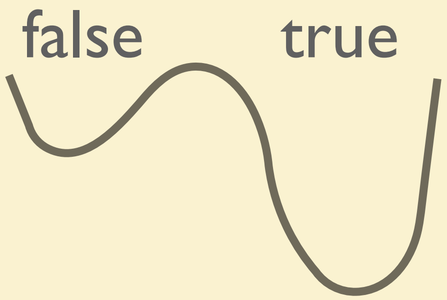
  - Main players : scalar field & plasma

  - Wall (where the scalar field value changes) wants to expand ("pressure")

  - Wall is pushed back by plasma ("friction")

  - These dynamics are generally complicated & hard to solve, so, let's try some qualitative classification





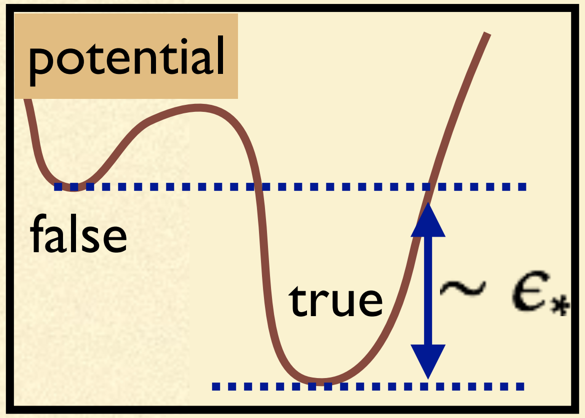
2. GWS PRODUCED IN PHASE TRANSITION

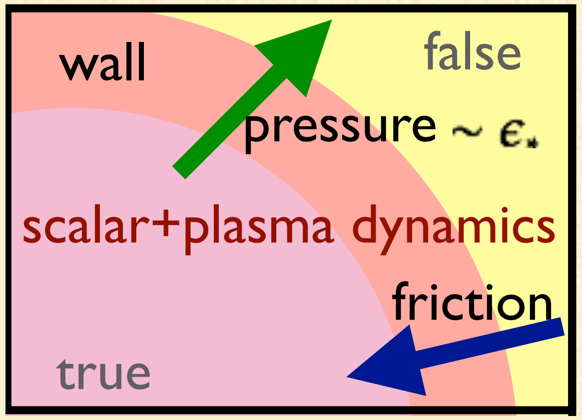
- Single bubble profile : Qualitative classification

  - Roughly speaking,

$$\alpha \equiv \epsilon_* / \rho_{\mathrm{radiation}}$$

determines the late-time behavior of the bubble wall





(R) Runaway case : Pressure dominates friction

→ Wall velocity approaches speed of light

$\alpha \gtrsim O(1)$

Energy is dominated by <u>scalar</u> motion (wall itself)

(T) Terminal velocity case : Pressure & friction are in balance

→ Bubble walls reach a terminal velocity (<c)

$\alpha \lesssim O(1)$

Energy is dominated by <u>plasma</u> around walls

2. GWS PRODUCED IN PHASE TRANSITION

- Gravitational-wave sources

  - Usually categorized into 3 classes

    (1) Bubble wall collision : Scalar field dynamics
    (& also plasma)

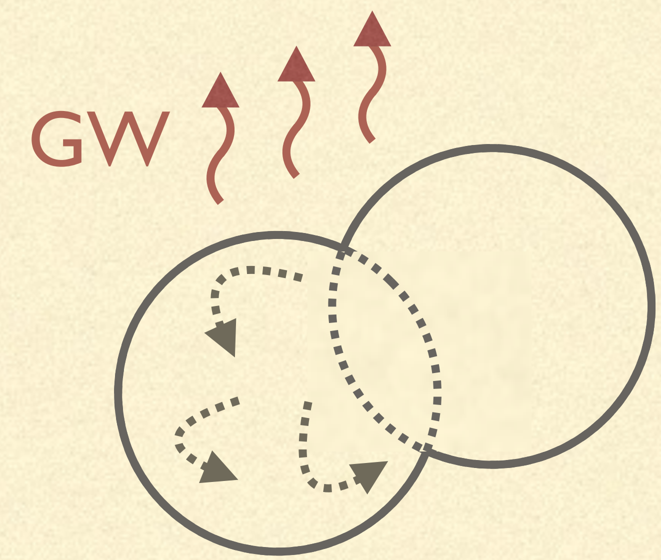
    (2) Sound wave : Plasma dynamics after collision

    (3) Turbulence : Plasma dynamics after collision

  - Which is important in (R)Runaway & (T)Terminal vel. cases?

    (R) → (1) Bubble wall collision

    (T) → (2,3) Sound wave (& Turbulence)



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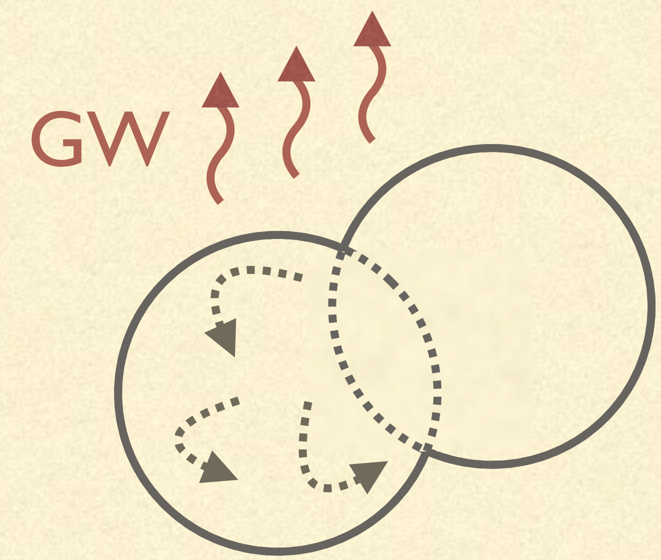
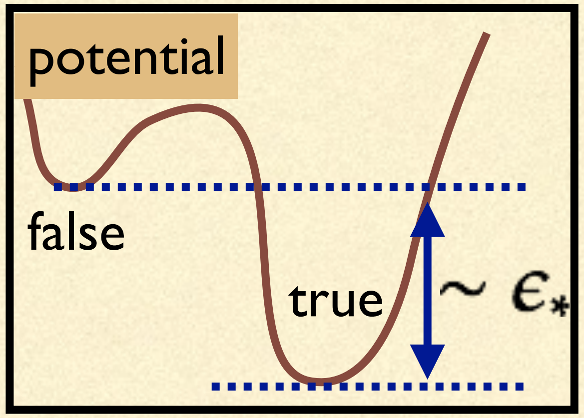


2. GWS PRODUCED IN PHASE TRANSITION

- Why do we focus on bubble wall collision?

  - For any GW sources, following factor exists

$$\Omega_{\mathrm{GW}} \propto \left(\frac{\alpha}{1+\alpha}\right)^2 \qquad (\alpha \equiv \epsilon_* / \rho_{\mathrm{radiation}})$$



  - Taking future observations into account,

    it would be reasonable to focus on large α case

    (= (R)Runaway case for the bubble wall behavior)

2. GWS PRODUCED IN PHASE TRANSITION

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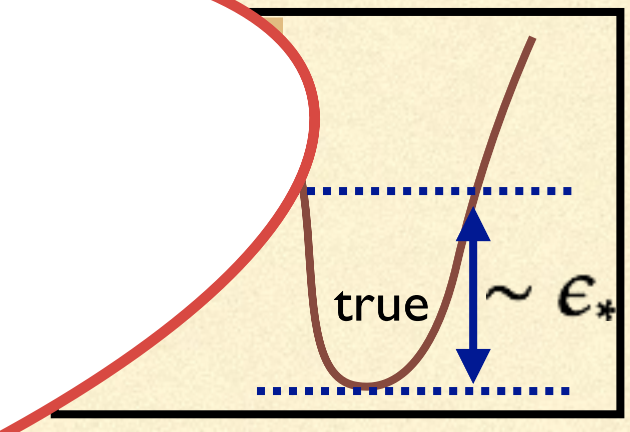


Fixing theoretical prediction
in this extreme case
is important

  - Taking

  it wou … on large α case

  (= (R)Runaway case for the bubble wall behavior)

2. GWS PRODUCED IN PHASE TRANSITION

参考までに、、

★Rough estimation of GW amplitude

Present GW amplitude & frequency

are obtained just by redshifting

~quadrupole factor ~radiation fraction today

$$h^2\Omega_{\rm GW,peak} \sim \mathcal{O}(10^{-2})\mathcal{O}(10^{-5})\left(\frac{\beta}{H_*}\right)^{-2}\left(\frac{\alpha}{1+\alpha}\right)^2$$

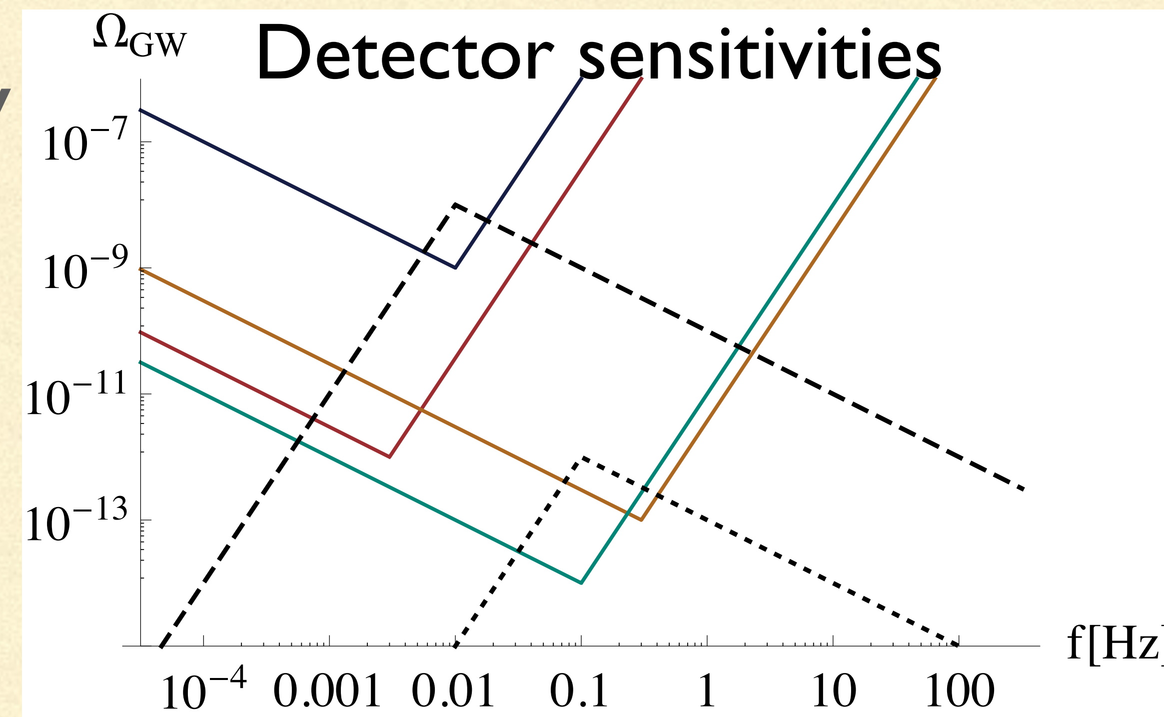
duration time

$$f_{\rm peak} \sim \frac{\beta}{H_*}\frac{T_*}{10^8 {\rm GeV}}[{\rm Hz}]$$

$\Omega_{\rm GW} = \rho_{\rm GW}/\rho_{\rm tot}$

T_* : temp. just after transition

H_* : H just after transition



To have large GW,
small β/H_* and large α are preferred!!

cf. SM with $m_H \sim 10$ GeV → $\beta/H \sim \mathcal{O}(10^5),\ \alpha \sim \mathcal{O}(0.001)$

3. GWs from bubble collisions

- What is usually done - What we did - What we will do

3. GWS FROM BUBBLE COLLISION

- What is usually done

  - Following system is solved

Def. of GWs $$ds^2 = -dt^2 + (\delta_{ij} + 2\underline{h_{ij}})dx^i dx^j$$

Evolution eq. of GWs $$\ddot{h}_{ij}(t,k) + k^2 h_{ij}(t,k) = 8\pi G \underbrace{K_{ij,kl}(\hat{k}) T_{kl}(t,k)}_{=\,\Pi_{ij}(t,k)}$$ source

K : projection to tensor mode $\quad$ T : energy-momentum tensor

Energy density of GWs $$\rho_{\rm GW} = \frac{\langle \dot{h}_{ij} \dot{h}_{ij} \rangle}{8\pi G}$$ (or GW spectrum $\Omega_{\rm GW} \equiv \frac{1}{\rho_{\rm tot}} \frac{d\rho_{\rm GW}}{d\ln k}$)

$\langle \cdots \rangle$: oscillation & ensemble average

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$$\Pi_{ij}(t,k)$$

$$||$$

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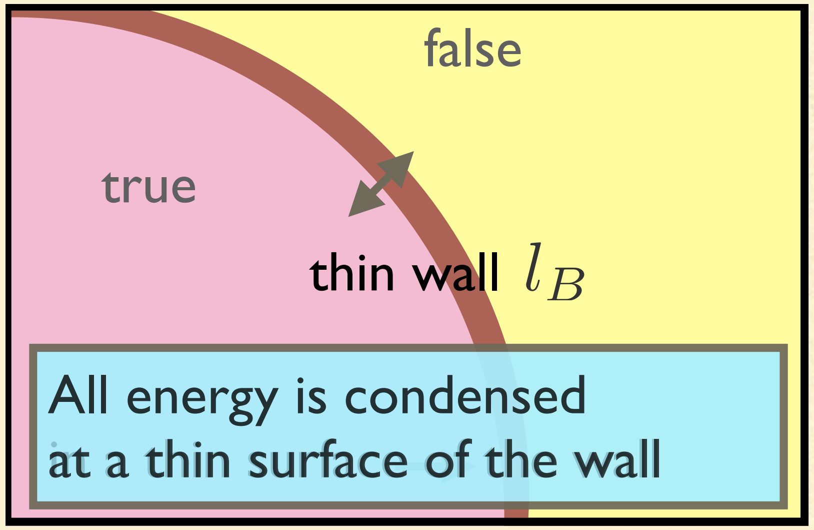
$\langle \cdots \rangle$: oscillation & ensemble average

3. GWS FROM BUBBLE COLLISION

- What is usually done

  - With thin-wall & envelope approximations for bubble walls, ...

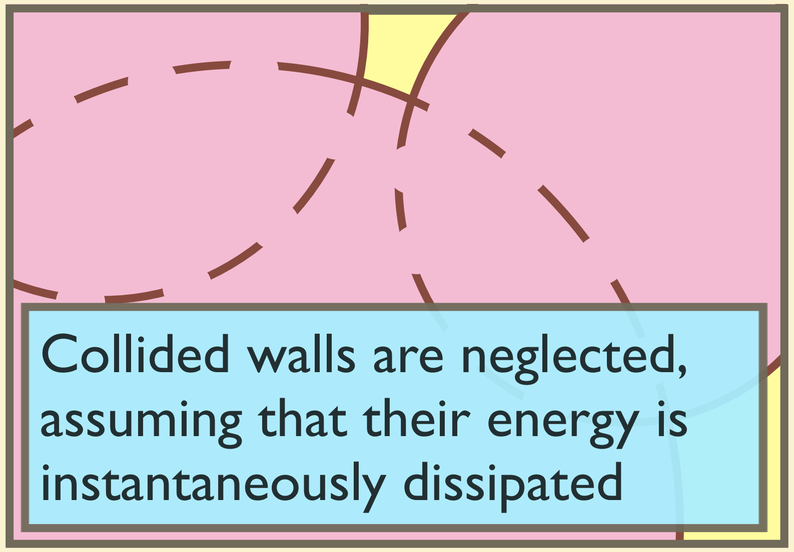
Thin-wall



$$T_{ij}(t,x) = \frac{4\pi}{3} r_B(t)^3 \frac{\kappa \epsilon_*}{4\pi r_B(t)^2 l_B} \times \hat{x}_i \hat{x}_j$$

※Note :
Valid for (R)Runaway case

Envelope



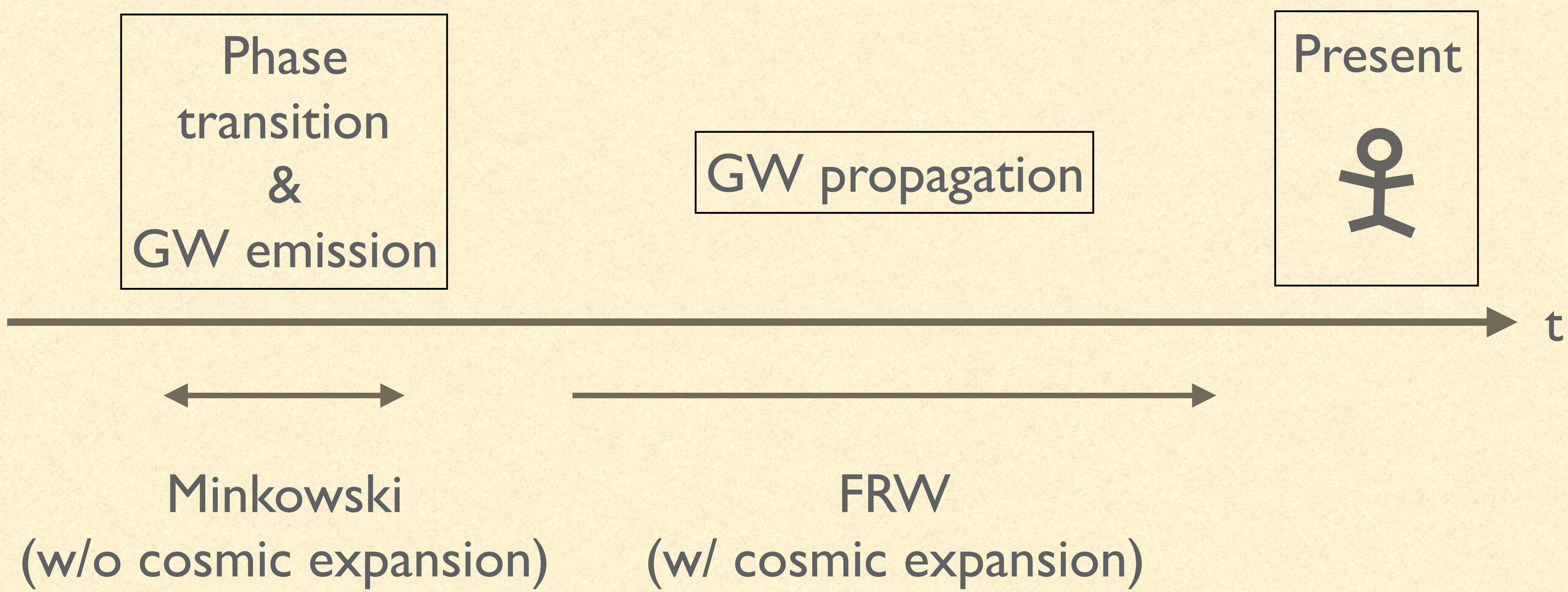
[Kosowsky, Turner, Watkins, PRD45 ('92)]

※Note :
Valid for (R)Runaway case
with some assumptions

3. GWS FROM BUBBLE COLLISION

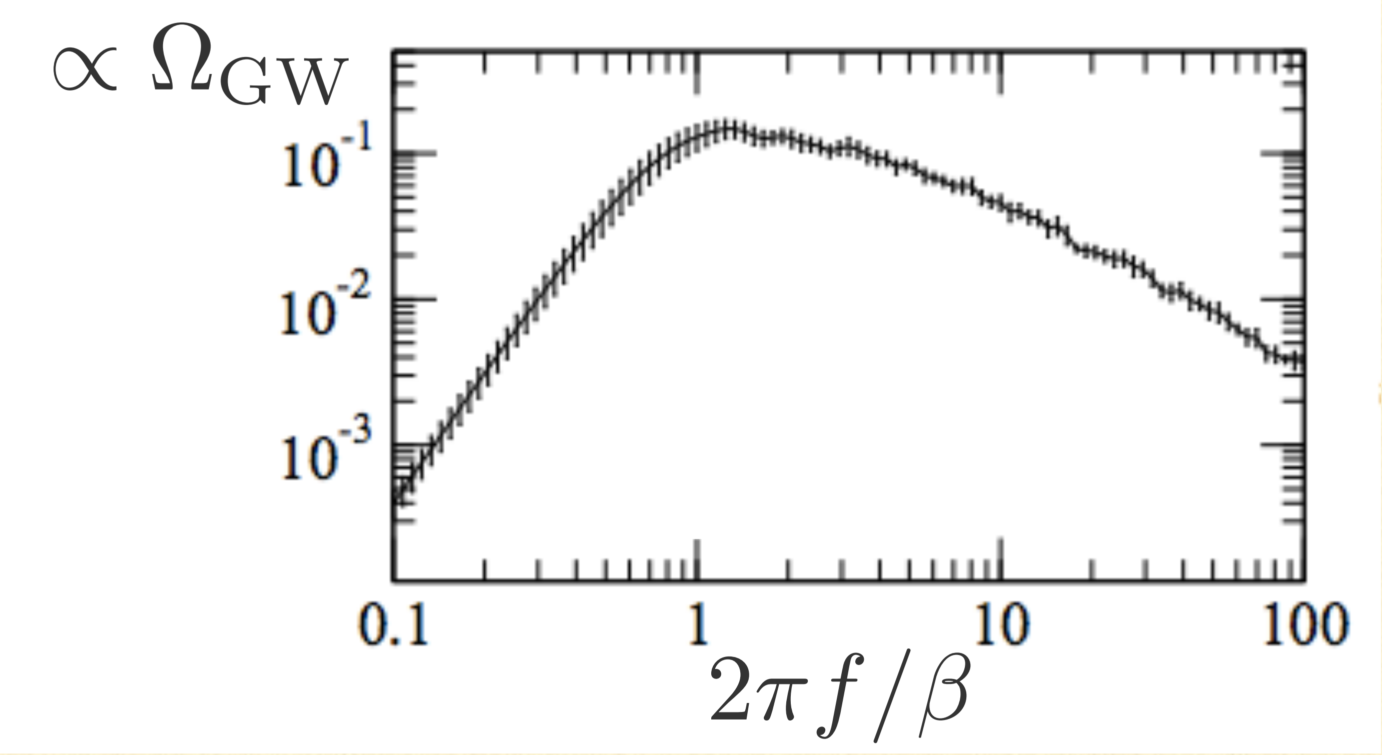
- What is usually done

  - ... and assuming flat background during PT, ...



3. GWS FROM BUBBLE COLLISION

- What is usually done
  - ... they do numerical simulation



[Huber, Konstandin, JCAP0809 ('08)]

3. GWS FROM BUBBLE COLLISION

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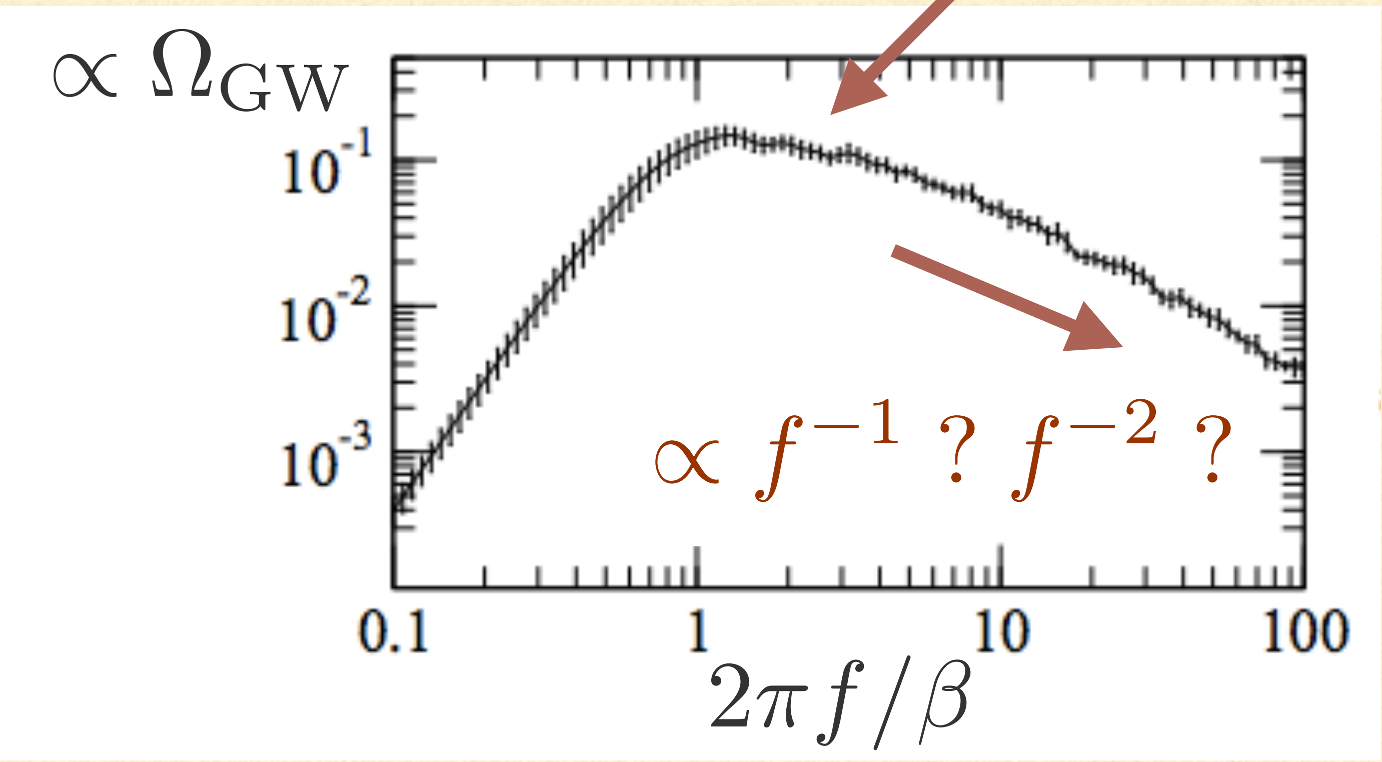
Peak amplitude？



[Huber, Konstandin, JCAP0809 ('08)]

ちょっと

待って。

(Wait a bit)

手で

解けません？

(Can’t we solve it analytically?)

3. GWS FROM BUBBLE COLLISION

- What we did - why analytically calculable ?

Step 1 : GW spectrum is essentially "expectation value" of EM tensor

$\boxed{\text{Green function}}$ $\boxed{\text{Source}}$

$$\Omega_{\rm GW} \equiv \frac{1}{\rho_{\rm tot}} \frac{d\rho_{\rm GW}}{d\ln k} \propto \int dt_x \int dt_y \cos(k(t_x - t_y)) \underline{\Pi(t_x, t_y, k)}$$

with $\langle \Pi_{ij}(t_x, \vec{k}) \Pi^*_{ij}(t_y, \vec{q}) \rangle = (2\pi)^3 \delta^3(\vec{k} - \vec{q}) \underline{\Pi(t_x, t_y, k)}$ & $\Pi_{ij} = K_{ij,kl} T_{kl}$

$\wr$

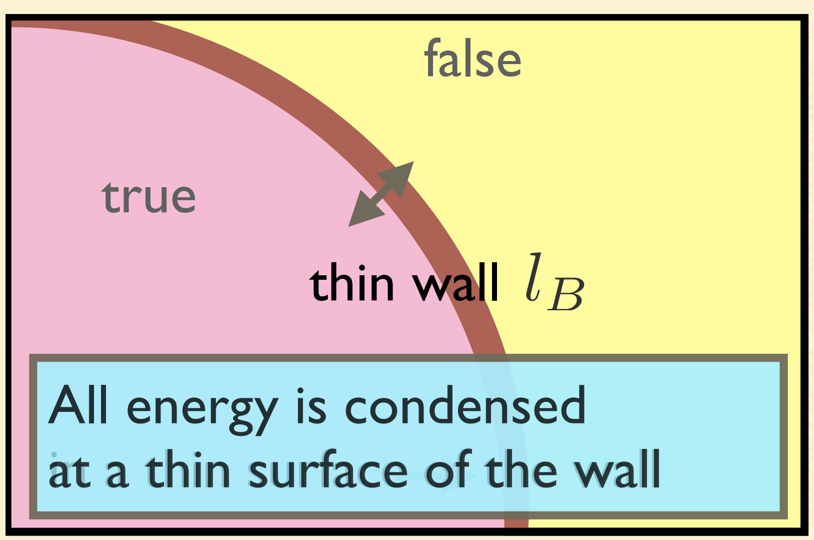
$\underline{\langle T(x) T(y) \rangle}$ x, y : spacetime points

3. GWS FROM BUBBLE COLLISION

- Assumption $\Gamma = \Gamma_* e^{\beta(t - t_*)}$

  - With thin-wall & envelope approximations for bubble walls, ...

Thin-wall



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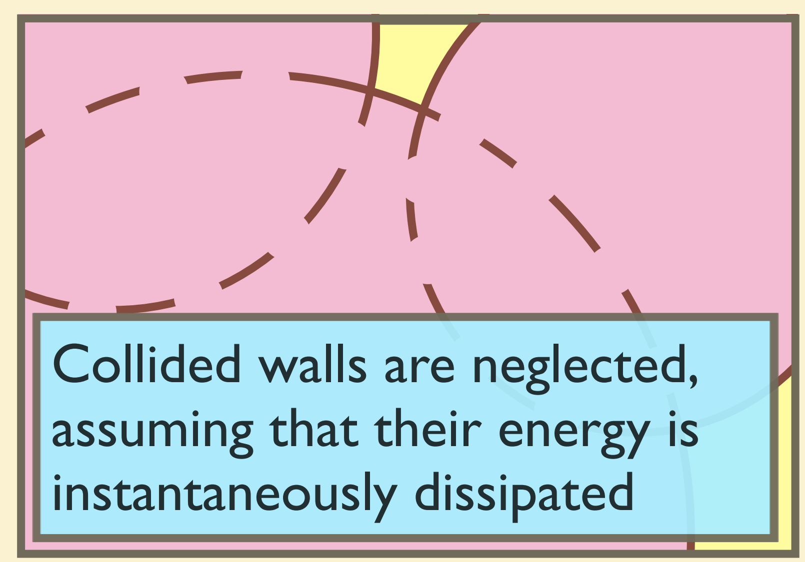

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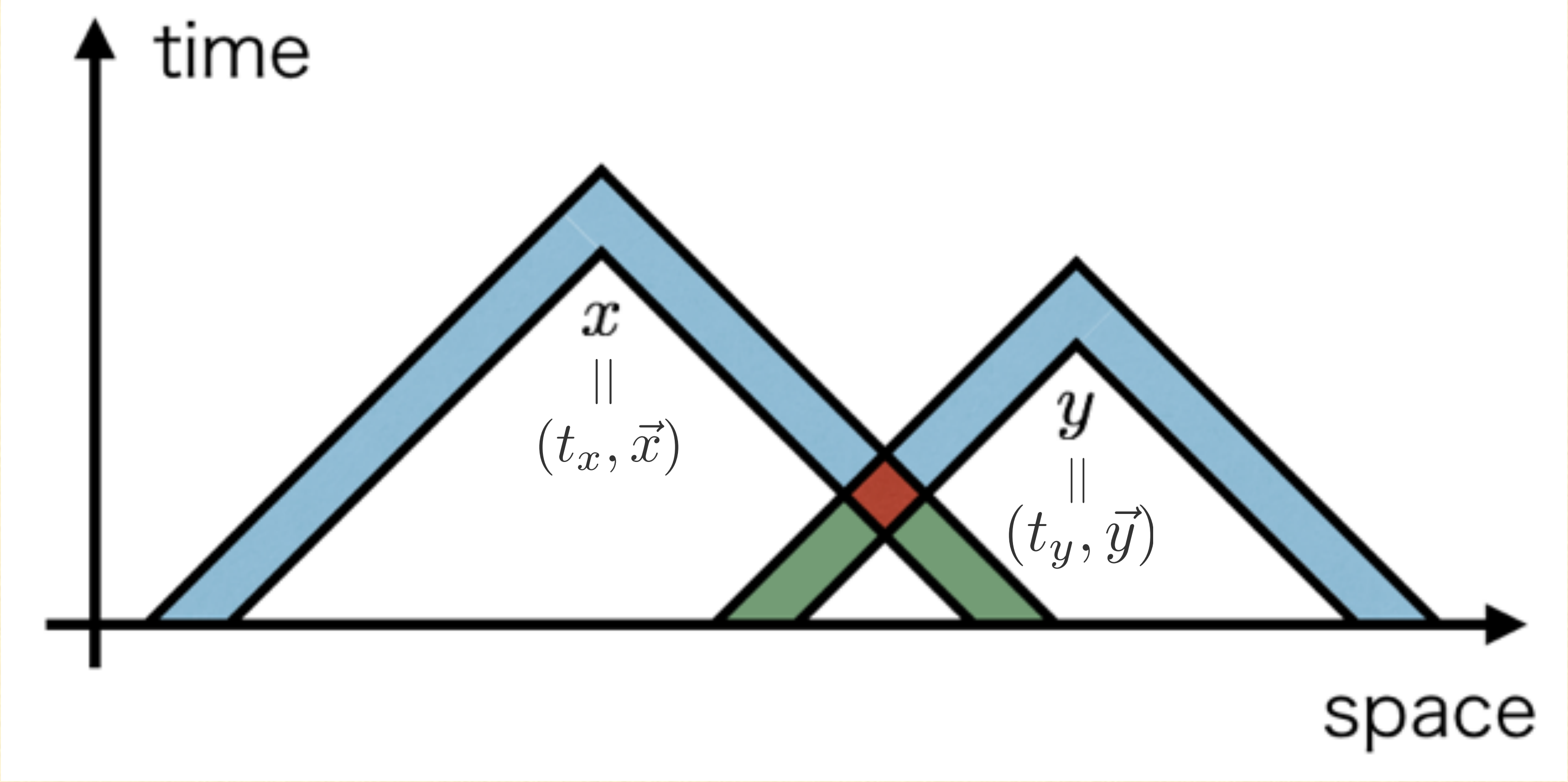
- What we did - why analytically calculable ?

Step 2 : When does $T(x)T(y)$ has nonzero value ?



3. GWS FROM BUBBLE COLLISION

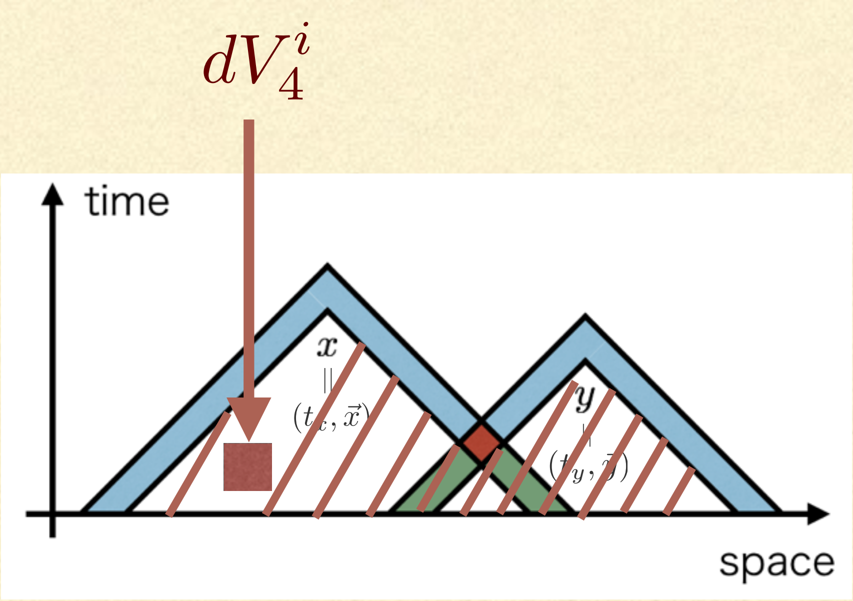
- What we did - why analytically calculable ?

Step 2 : When does $T(x)T(y)$ has nonzero value ?

- First, x & y must be in false vac. before t_x, t_y , respectively

Probability for x & y to be in false vacuum

$$P(x,y) = \prod_i (1 - \Gamma dV_4^i) = e^{-\int d^4z\, \Gamma(z)}$$



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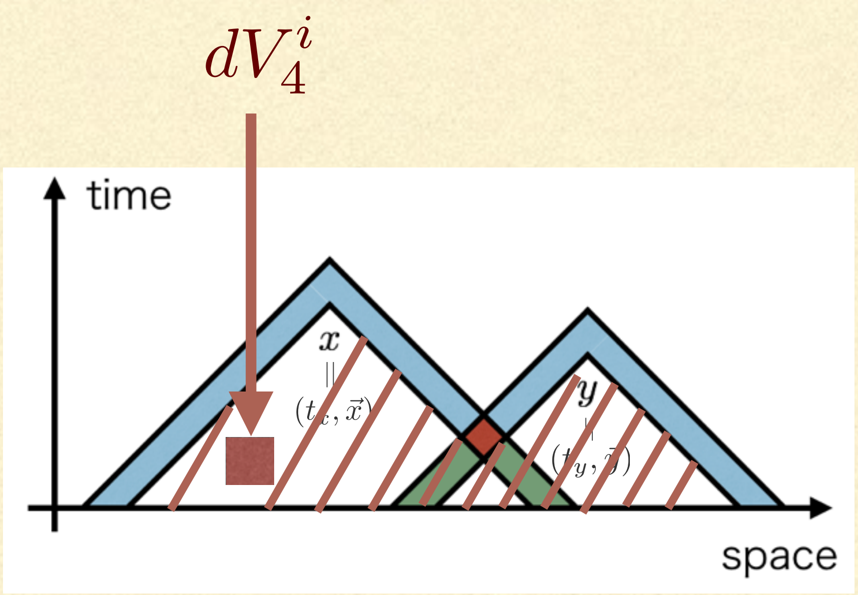
Probability for x & y to be in false vacuum

$$P(x,y) = \prod_i (1-\Gamma dV_4^i) = e^{-\int d^4z\, \Gamma(z)}$$

calculable



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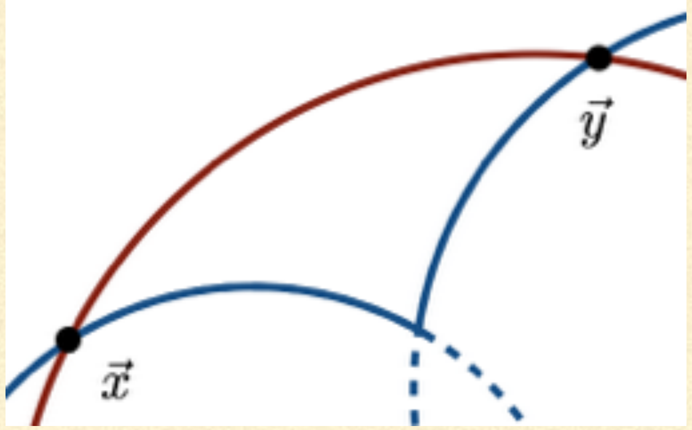
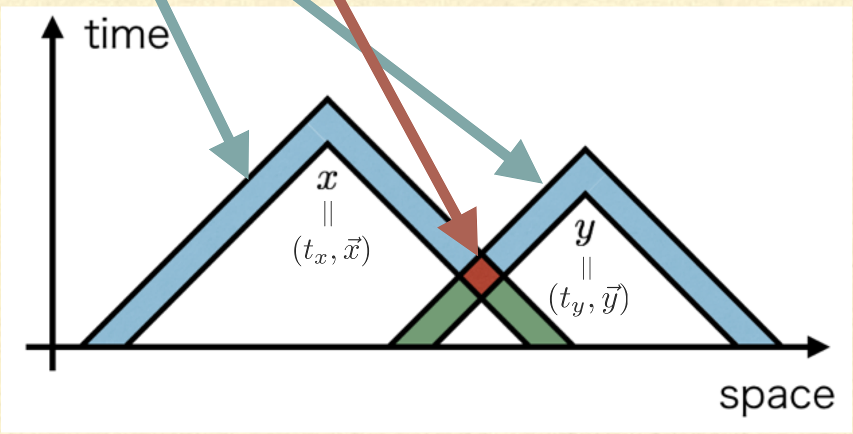
- Second, one single bubble must be nucleated in region,

or two different bubbles must be nucleated in region





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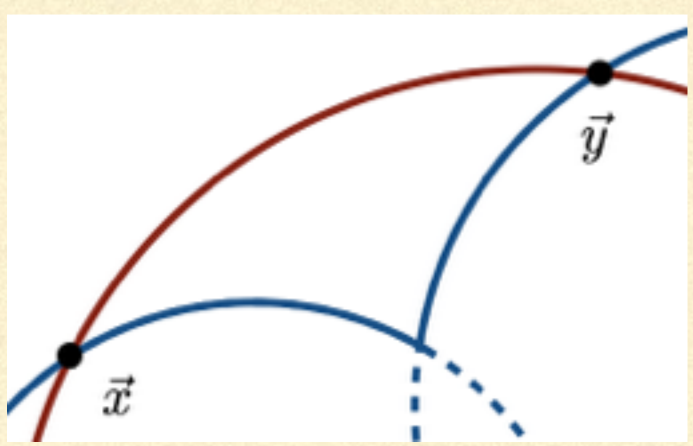
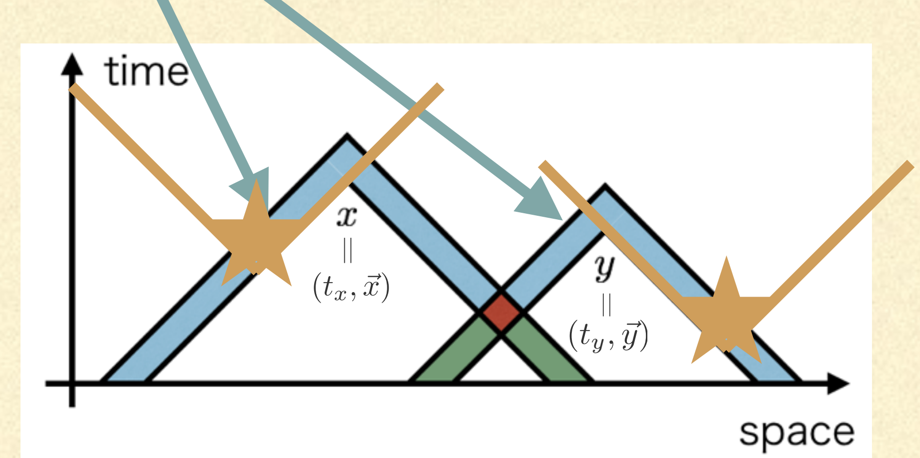




centers of bubbles

3. GWS FROM BUBBLE COLLISION

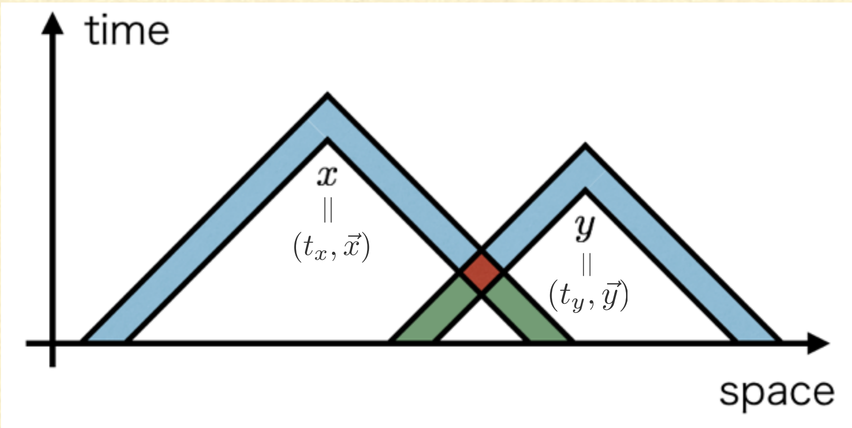
- What we did - why analytically calculable ?

Step 2 : When does $T(x)T(y)$ has nonzero value ?

- Second, one single bubble must be nucleated in region,

or two different bubbles must be nucleated in region

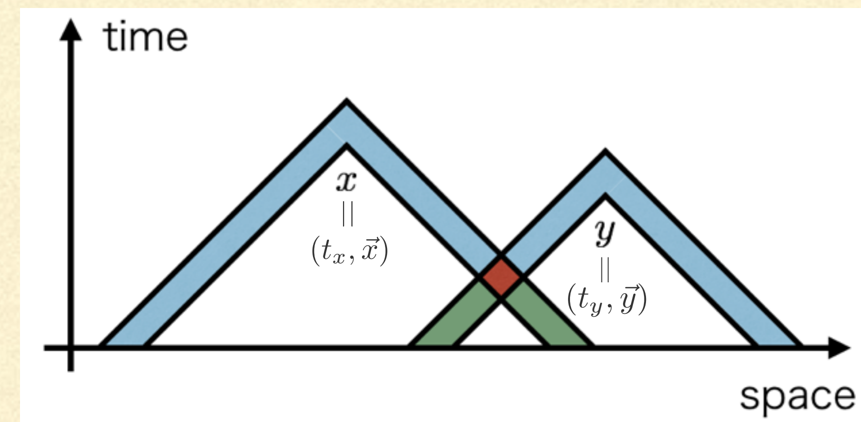
calculable



3. GWS FROM BUBBLE COLLISION

- What we did - why analytically calculable ?

  - Explicit expression (single-bubble case)



$$\langle T_{ij} T_{kl} \rangle^{(s)}(t_x, t_y, \vec{r}) = P(t_x, t_y, r) \int_{-\infty}^{t_{xy}} dt \, \mathcal{T}_{ij,kl}^{(s)}(t, t_x, t_y, \vec{r})$$

Expectation val. of energy-momentum tensor	False vac. probability for spacetime points x & y	Value of T(x)T(y) if single bubble is nucleated and passed through x & y

3. GWS FROM BUBBLE COLLISION

- What we did - why analytically calculable ?

  - The rest is trivial

$$\langle T(x)T(y)\rangle$$

$$\Big\downarrow \text{Fourier transform}$$

$$\langle TT\rangle(t_x, t_y, \vec{k}) \sim \Pi(t_x, t_y, k)$$

$$\Big\downarrow \text{substitution}$$

$$\Omega_{\rm GW} \propto \frac{d\rho_{\rm GW}}{d\ln k} \propto \int dt_x \int dt_y \cos(k(t_x - t_y))\Pi(t_x, t_y, k)$$

3. GWS FROM BUBBLE COLLISION

- What we did - why analytically calculable ?

  - Final expression $\Omega_{\mathrm{GW}} \propto \Delta$

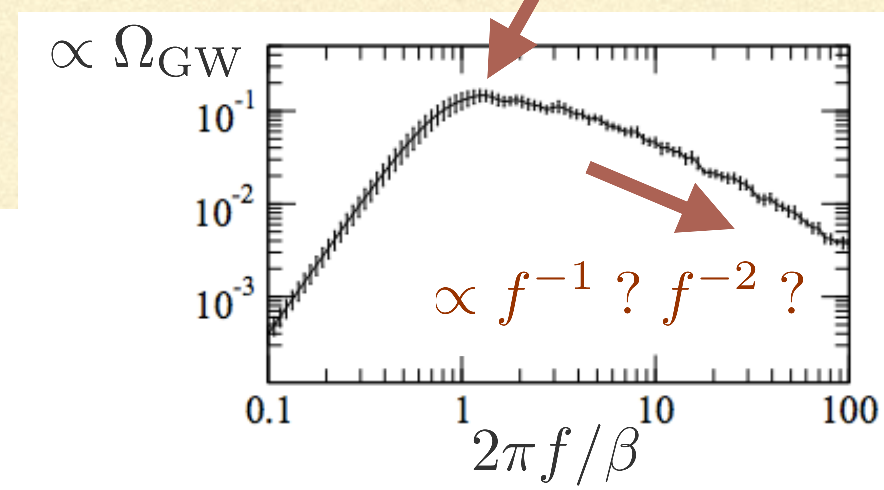
single	$$\Delta^{(s)} = \frac{k^3}{12\pi} \int_0^\infty dt_d \int_{t_d}^\infty dr \, \frac{e^{-r/2} \cos(kt_d)}{r^3 \mathcal{I}(t_d, r)} \times \left[j_0(kr) F_0 + \frac{j_1(kr)}{kr} F_1 + \frac{j_2(kr)}{k^2 r^2} F_2 \right]$$	$$\begin{aligned} F_0 &= 2(r^2 - t_d^2)^2 (r^2 + 6r + 12), \\ F_1 &= 2(r^2 - t_d^2) \left[-r^2 (r^3 + 4r^2 + 12r + 24) + t_d^2 (r^3 + 12r^2 + 60r + 120) \right], \\ F_2 &= \frac{1}{2} \left[r^4 (r^4 + 4r^3 + 20r^2 + 72r + 144) - 2t_d^2 r^2 (r^4 + 12r^3 + 84r^2 + 360r + 720) + t_d^4 (r^4 + 20r^3 + 180r^2 + 840r + 1680) \right] \end{aligned}$$
double	$$\Delta^{(d)} = \frac{k^3}{96\pi} \int_0^\infty dt_d \int_{t_d}^\infty dr \, \frac{e^{-r} \cos(kt_d)}{r^4 \mathcal{I}(t_d, r)^2} \times \frac{j_2(kr)}{k^2 r^2} G(t_d, r) G(-t_d, r)$$	$$G(t_d, r) = (r^2 - t_d^2) \left[(r^3 + 2r^2) + t_d (r^2 + 6r + 12) \right]$$

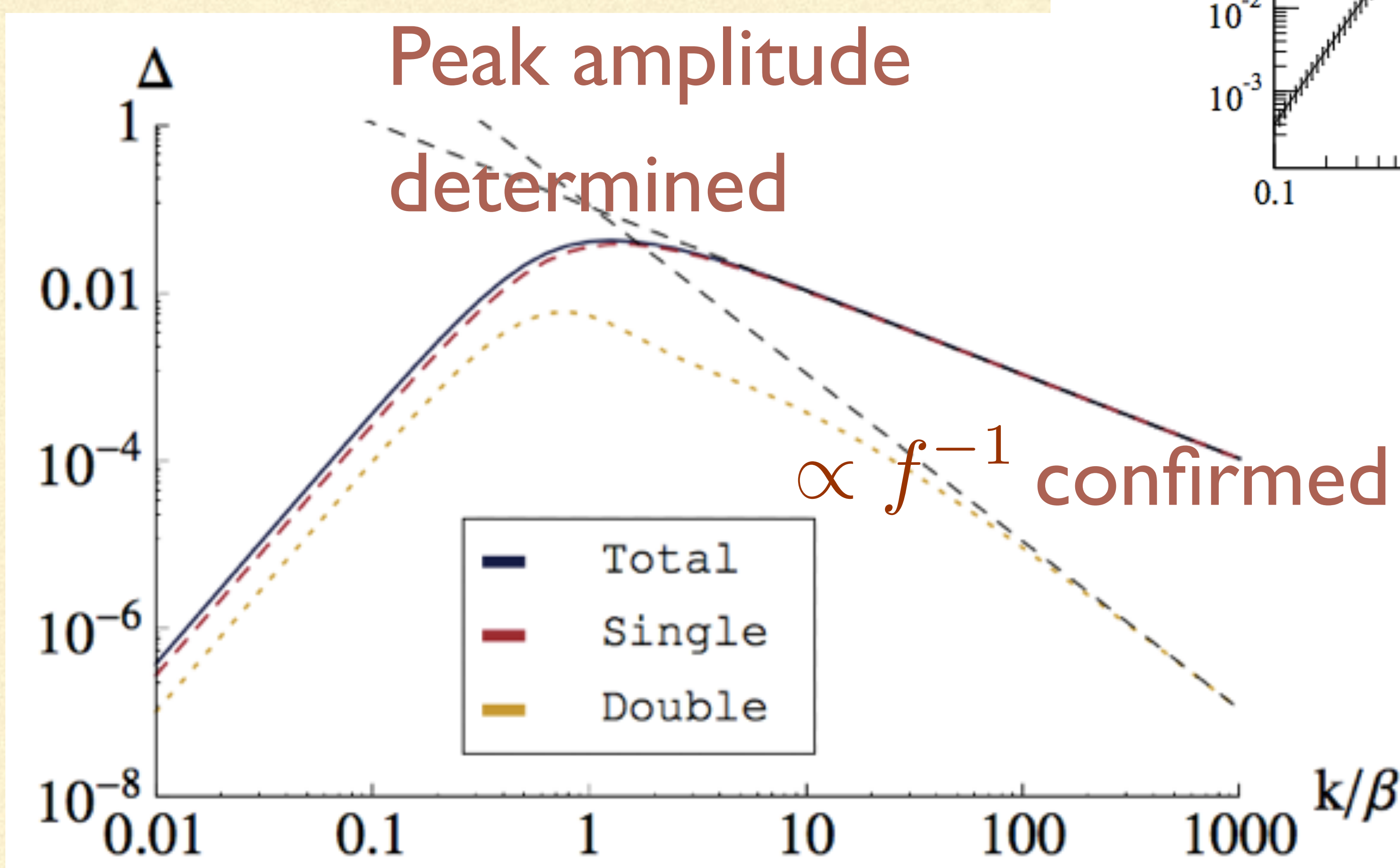
3. GWS FROM BUBBLE COLLISION

- What we did

  - Numerical plot

Peak amplitude ?

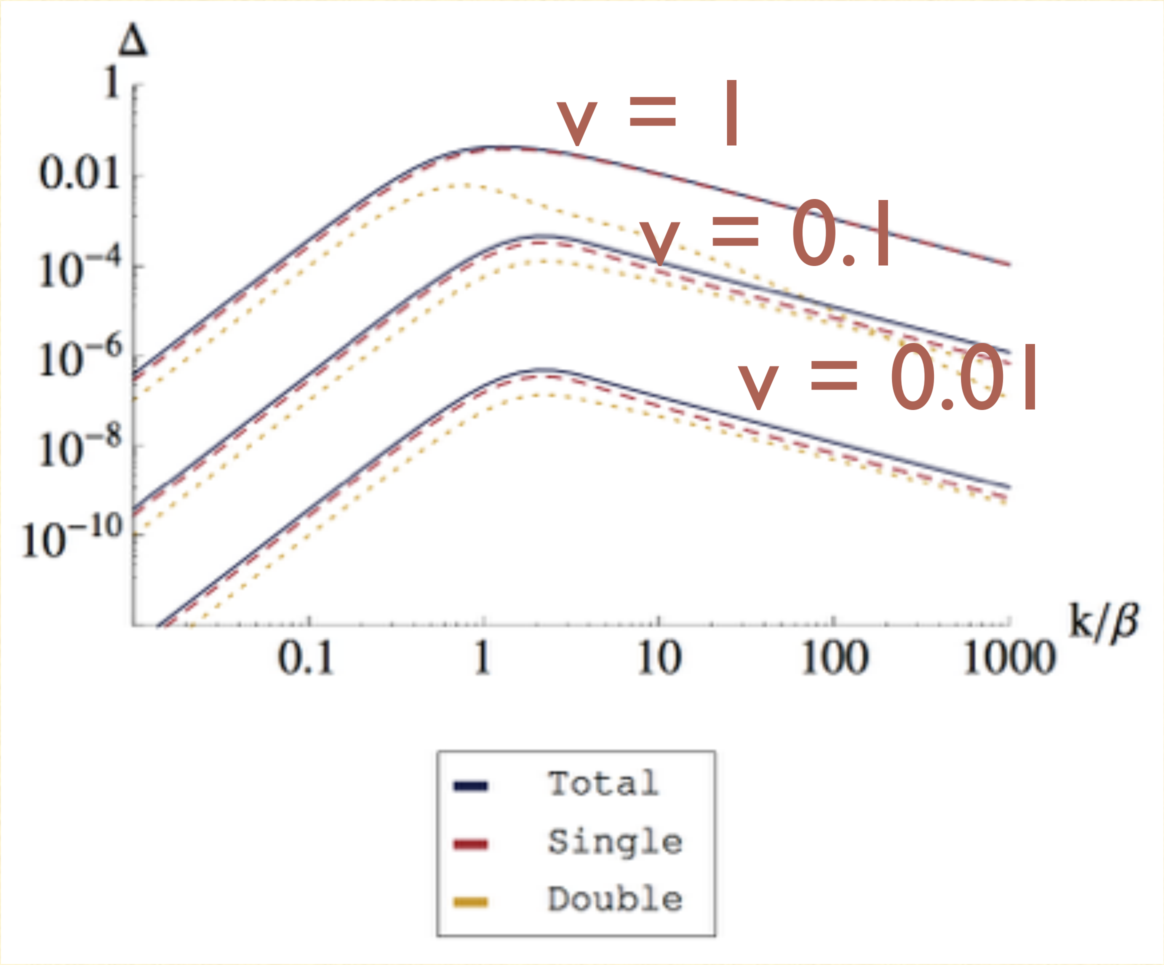




3. GWS FROM BUBBLE COLLISION

- What we did

  - Wall velocity dependence



Wall velocity dependence determined

3. GWS FROM BUBBLE COLLISION

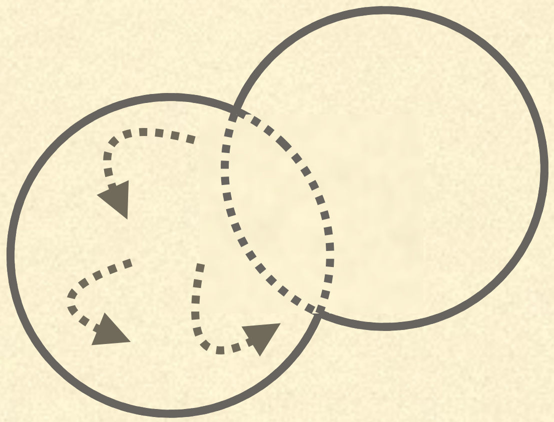
- What we will do (we can extend our method to more general setups)

  - How is envelope approximation good ?

  - Effect of cosmic expansion

  - Validity of “long-lasting”

※some people say that sound-wave can be a long-lasting source by a factor of β/H



3. GWS FROM BUBBLE COLLISION

- What we will do

  - How is envelope approximati

  - Effect of

  - Val

Let's prepare for
GW observation
with precise theoretical prediction

e people say that
sound-wave can be
a long-lasting source
by a factor of β/H