Membranes from monopole operators in ABJM theory: Large angular momentum and M-theoretic AdS_4/CFT_3

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- arxiv: 1310.0016

1. Introduction

M-theory, matrix model

AdS_4/CFT_3

large angular momentum.

('94 Hull-Townsend '95 Witten)

- M-theory

membrane (M2)

no coupling const., single scale l_p

D=11 theory

$\downarrow$ on S^1 — $\left(\begin{array}{l} R_{11} = l_p g_s^{\frac{2}{3}} \\ \text{KK mom.} = \text{D0-charge} \end{array} \right.$

D=10 IIA String

- crucial in non-perturbative String theory

- no established formulation. $\left(\begin{array}{l} \text{microscopic DOF?} \\ \text{action ?} \end{array} \right.$

- best candidate.

$\longrightarrow$ Matrix model.

- # Matrix model

'88 de Wit, Hoppe, Nicolai
'96 Banks, Fischler, Shenker, Susskind

- DOF X^α $(\alpha = 1, \cdots, 9)$ $\quad [\quad]$ #D0-branes

- action $S = \mathrm{Tr} \int (D_t X^\alpha)^2 + [X^\alpha, X^\beta]^2 \, dt + \text{Fermions}$

- regularised theory of M2 (D=11 Supermembrane) flat space

- matrix model of M-th. on pp-wave BKG ('02 BMN)

$$S = \mathrm{Tr} \int (D_t X^\alpha)^2 - X^2 + i X[X, X] + [X, X]^2 \, dt$$

- best candidate, though with unsolved issues ((matrix size)$\to\infty$? D=11 Lorentz inv.?)

- <u>questions</u>

  What are good observables of M-theory?

  How to extract observables using M.M?

  How to verify the results of computation?

AdS_4/CFT_3

$R \sim R_{S^7} = 2R_{AdS_4}$

'06 ABJM

- M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ $=$ D=3 ABJM theory

  Φ^A, $A = 1, \cdots, 4$ ← complex scalar

  $U(N) \times U(N)$, A, $\hat{A}$

  CS action $\frac{1}{k}$ ← level

$$\frac{R^6}{l_p^6} = Nk$$

- $S^7/\mathbb{Z}_k \rightarrow$ M-th. circle $R_{11} = \frac{R}{k}$
- IIA regime : $N \gg 1$, $\lambda = \frac{N}{k}$: fixed

$\boxed{\text{M-th. regime : } N \gg 1, \quad k \sim O(1) \text{ (fixed.)}}$ ← this talk

- learn about
  - M-theory
  - AdS/CFT

  e.g. matrix model ⟷ ABJM?

  non-stringy

  all non-planar contributes

- To study M-theory regime,
we consider states with large angular mom. $J \gg 1$
- use $\frac{1}{J} \ll 1$ to introduce
approximation schemes on both sides, then verify AdS/CFT
- WKB approx ~ large quantum number.
- <u>BMN</u> '02 Berenstein, Maldacena, Nastase in AdS_5/CFT_4

String on $AdS_5 \times S^5$	$D=4$, $\mathcal{N}=4$ SYM $g_{YM}^2 N \gg 1$
$\downarrow J \gg 1$	$\downarrow J \gg 1$, near BPS BMN op.
String on pp-wave BKG	eff. coupling $\frac{g_{YM}^2 N}{J^2} \ll 1$

$\longleftrightarrow$
successful check.

Our work

☆ $J \gg 1$ seems to provide good approx. on both sides

AdS side : pp-wave approx & loop exp.

CFT side : Born-Oppenheimer

☆ AdS/CFT Dictionary for $J \gg 1$

analog of BMN op. (based on monopoleoperator)

~ operator in ABJM

corresponding to near BPS fluctuation of membranes.

☆ Successful test at the leading order

of the approx. (near-BPS)

Outline

1. Introduction
2. AdS side
   BPS ground state
   near BPS fluctuation
3. CFT side
   BPS ground state
   near BPS fluctuation
4. Summary & Discussion

☆ $J \gg 1$ → approx. on both sides

☆ AdS/CFT dictionary

☆ test at the leading order for near BPS op.

2. AdS side

pp-wave approx.

membrane Hamiltonian

(matrix size) = J

BPS states ~ Concentric Spherical membranes

fluctuation ; near BPS states

tree vs 1-loop

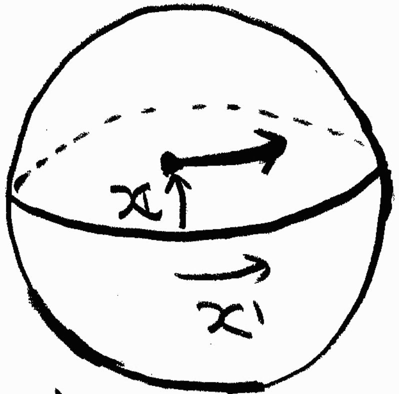
$$N^{\frac{1}{3}} \ll J \ll J^{\frac{1}{2}}$$

- <u>pp-wave approx.</u> (ultrarelativistic (infinite mom.) limit on curved space.)

- <u>Simple ex.</u>

particle on $R \times S^n$

$|x| \ll R$



radius R.

- $ds^2 \approx -dt^2 + (dx^1)^2 \times \left(1 - \frac{x^2}{R^2}\right) + dx^2$

- $P_0 = -E, \quad P_1 = P = \frac{J}{R} \gg 1$

- $0 = G^{\mu\nu} P_\mu P_\nu$ dynamics of a particle.

$$= -E^2 + P^2 \times \left(1 + \frac{x^2}{R^2}\right) + \mathbb{p}^2$$

$$\rightsquigarrow E - P = \frac{\mathbb{p}^2}{2P} + \frac{P}{2}\frac{x^2}{R^2}$$

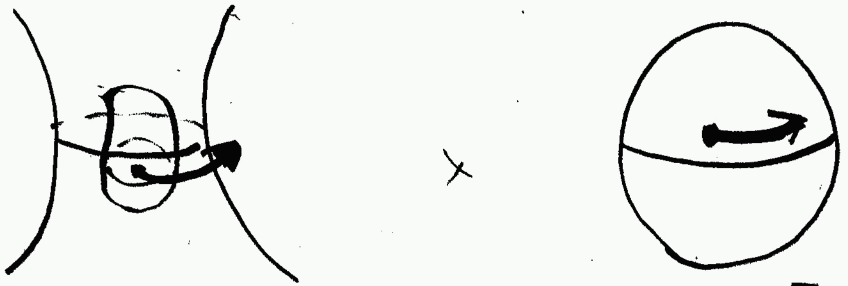
non-rel particle

harm. osc. potential

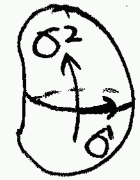
(using $E + P \approx 2P$)

- J : large $\rightarrow$ strong harm. osc. potential.

Hamiltonian of membrane theory $\hbar = 1$. $\ell_p = 1$.

- 

AdS$_4$ (x^1, x^2, x^3) × S^7 $(x^4, \cdots, x^9)$



$x^\alpha(\sigma^1, \sigma^2) \underset{\text{conj.}}{\longleftrightarrow} p^\alpha(\sigma^1, \sigma^2)$

(cf. Dobashi-Shimada-Yoneya)

- $$H = \int \underbrace{\frac{(p^\alpha)^2}{2P}}_{\text{inertia}} + \underbrace{\frac{1}{2P}\{x^\alpha, x^\beta\}^2}_{\text{tension}} + \underbrace{\frac{P}{R^2}x^2}_{\text{pp-wave approx.}} + \underbrace{\frac{1}{R}\epsilon_{\alpha\beta\gamma}x^\alpha\{x^\beta, x^\gamma\}}_{\text{flux in AdS}} d\sigma^1 d\sigma^2$$

($\epsilon_{123} = 1$, perm. otherwise $= 0$)

$$\{f, g\} = \frac{\partial f}{\partial \sigma^1}\frac{\partial g}{\partial \sigma^2} - \frac{\partial f}{\partial \sigma^2}\frac{\partial g}{\partial \sigma^1}$$

- can use old results on membrane, matrix model on pp-wave BKG

('02 Dasgupta, Sheikh-Jabbari, van Raamsdonk
Kim, Plefka
Sugiyama, Yoshida)

$k = 1$

- regularisation.

$$x^\alpha(\sigma^1, \sigma^2), \quad p^\alpha(\sigma^1, \sigma^2).$$

$$\longrightarrow X^\alpha, P^\alpha : \text{matrices}$$

- (matrix size) $= J$

$S^7/\mathbb{Z}_k \rightarrow$ M-th. circle.

$\rightsquigarrow$ $J =$ KK mom. on M-theory circle

$= \# D0$

$=$ matrix size

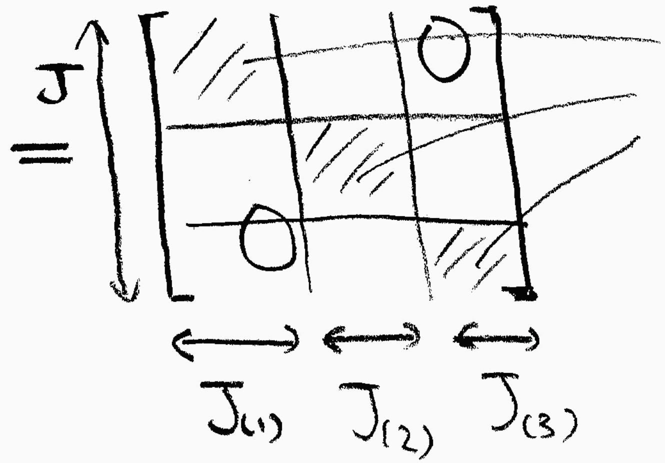
- cf. BMN op. $\sim \mathrm{Tr}\, Z^J \sim$ closed string reg. by lattice with J sites.

- $$H = \mathrm{Tr}\left(R (P^\alpha)^2 - R\, [X^\alpha, X^\beta]^2 + \frac{1}{R^3} (X^\alpha)^2 + i \frac{1}{R} \epsilon_{\alpha\beta\gamma} X^\alpha [X^\beta, X^\gamma] \right)$$

pp-wave matrix model.

BPS states

- classical ground states $E=0$

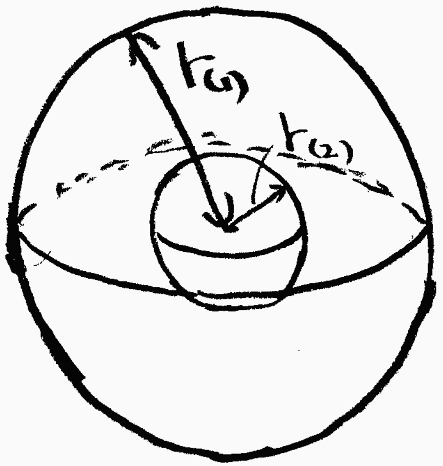
$X^\alpha =$ 

$\alpha = 1,2,3$

$$r_{(i)} \times \frac{1}{J_{(i)}} L_\alpha^{(i)}$$

$$[L_\alpha^{(i)}, L_\beta^{(i)}] = i\epsilon_{\alpha\beta\gamma} L_\gamma^{(i)}, \quad J_{(i)} \times J_{(i)}$$ matrices

- $$\boxed{J = J_{(1)} + J_{(2)} + \cdots}$$

- 

collection of fuzzy spheres

Concentric spherical membranes with radii $r_{(i)}$

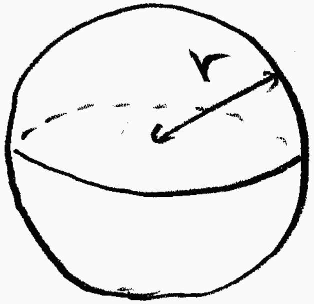
$$r_{(i)} = \frac{J_{(i)}}{R^2}.$$

· pp-wave approx.

$$\frac{r}{R} = \frac{J}{R^3} = \frac{J}{N^{\frac{1}{2}}} \ll 1$$

- **near BPS fluct.**

- single membrane case $J = J_{(1)}$,
- focus on fluct. in S^7-direction ~ 6-scalars ← polorisation



spherical membrane

- oscillation mode corresponds to $Y_{\ell m}(\theta, \varphi)$

$$\omega = \frac{1}{R}\sqrt{\underbrace{\frac{1}{4}}_{\text{harm osc.}} + \underbrace{\ell(\ell+1)}_{\text{tension}}} \quad \leftarrow \text{tree level}$$

- **1-loop correction** ('02 Dasgupta et al.) $H = \underbrace{H_2}_{\text{free}} + H_3 + H_4$

$$\Delta E = \sum_{\psi'} \frac{|\langle\psi|H_3|\psi'\rangle|^2}{E - E'} + \langle\psi|H_4|\psi\rangle$$

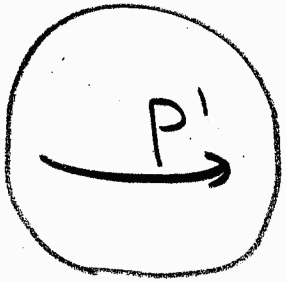
- $J \gg 1 \to H_2$: large
  $\to$ # (intermediate state) : large

SUSY

- $$\frac{\text{1-loop}}{\text{tree}} = \frac{N}{J^3} \leftarrow = \left(\frac{\ell_p}{r}\right)^3$$

- $$N^{\frac{1}{3}} \underbrace{\ll}_{\text{loop exp.}} J \underbrace{\ll}_{\text{pp-wave}} N^{\frac{1}{2}}$$

Summary of 2. AdS side

- 

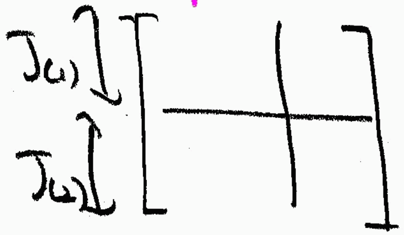
  large (angular) mom. in curved space

  $\longrightarrow$ H contains $\frac{P}{R^2}(x^{\alpha})^2$

- matrix size $= J$

- BPS states   collection of fuzzy spheres

  $J = J_{(1)} + J_{(2)} + \cdots$

- near BPS states

  $Y_{\ell m} \longrightarrow \omega = \sqrt{\frac{1}{4} + \ell(\ell+1)} = \frac{1}{2} + \ell$

- validity of approx.

  $$N^{\frac{1}{3}} \underset{}{\overset{\text{loop exp.}}{\ll}} J \overset{\text{pp-wave approx.}}{\ll} N^{\frac{1}{2}}$$

3. CFT side

monopole op.

BPS states

near BPS fluctuation

Born - Oppenheimer approx.

large J in ABJM

- J : R-charge in ABJM.

  4 complex scalars

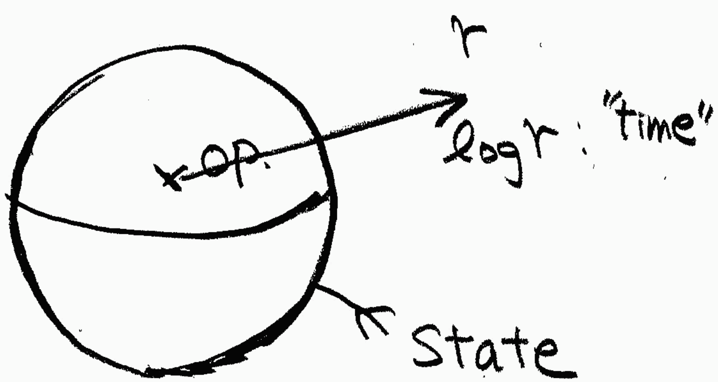
  $\phi^1, \phi^2, \phi^3, \boxed{\phi^4}$: bi-fundamental of $U(N) \times U(N)$

- ϕ^4 carries unit charge

- $\cancel{\mathrm{Tr}(\phi^4)^J}$ ← wrong, not gauge inv.

  needs monopole op. ('06 ABJM '02 Kapustin et al.)

- State op. mapping



ABJM on $S^2 \times \mathbb{R}^1$

ϕ^A : mass $= \frac{1}{2}$

Energy = conformal dimension

ABJM on $S^2 \times \mathbb{R}^1$

$$\begin{cases} A = 1,2,3,4 & SU(4) \text{ flavour} \\ i = 1,\cdots,N & U(N) \text{ colour} \\ \hat{i} = 1,\cdots,N & \text{the other } U(N) \text{ colour} \end{cases}$$

- $$\begin{pmatrix} \phi^{A\,i}{}_{\hat{j}} & \overset{\text{conj.}}{\longleftrightarrow} & \Pi_A{}^{\hat{i}}{}_{j} \\ \phi^{\dagger}{}_A{}^{\hat{i}}{}_{j} & \longleftrightarrow & \Pi^{\dagger A\,i}{}_{\hat{j}} \end{pmatrix} \qquad \begin{pmatrix} A_r{}^{i}{}_{j} \\ \hat{A}_r{}^{\hat{i}}{}_{\hat{j}} \end{pmatrix} \qquad r = 1,2 \text{ sphere.}$$

- $$H = \mathrm{tr}\int \frac{1}{\sin\theta}\Pi^A\Pi_A + g^{rs} D_r\phi^A D_s\phi_A \sin\theta + \frac{1}{4}\phi^A\phi_A \sin\theta + \left(\frac{2\pi}{k}\right)^2 V_6(\phi) + \cdots \, d\theta\, d\varphi$$

  $\frac{1}{4}\phi^A\phi_A$: mass from radial qtz. $\qquad V_6(\phi)$: ϕ^6-potential

- Gauß' law constr.

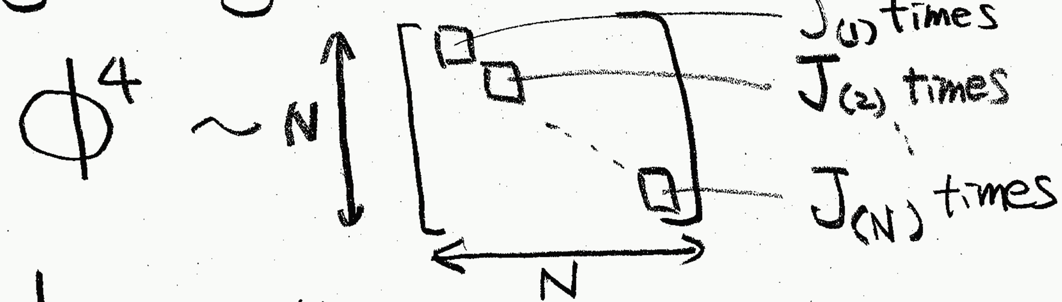
$$\frac{k}{2\pi} F_{\theta\varphi} = \rho \,, \quad -\frac{k}{2\pi}\hat{F}_{\theta\varphi} = \hat{\rho}$$

- $$\rho = i\phi^A\Pi_A - i\Pi^{\dagger A}\phi^{\dagger}{}_A + \cdots, \quad \hat{\rho} = i\phi^{\dagger}{}_A\Pi^{\dagger A} - i\Pi_A\phi^A + \cdots$$

monopole op. & BPS states ('06 ABJM, '09 Seok Kim)

- BPS state ~ lowest energy state with given charge J
- excite 0-modes of ϕ^4 J times

only diagonal elements should be excited ↗ (cf. '09 Simon-Sheikh-Jabbari) matches with AdS side



$$\boxed{J = J_{(1)} + \cdots + J_{(N)}}$$

(GNO charges)

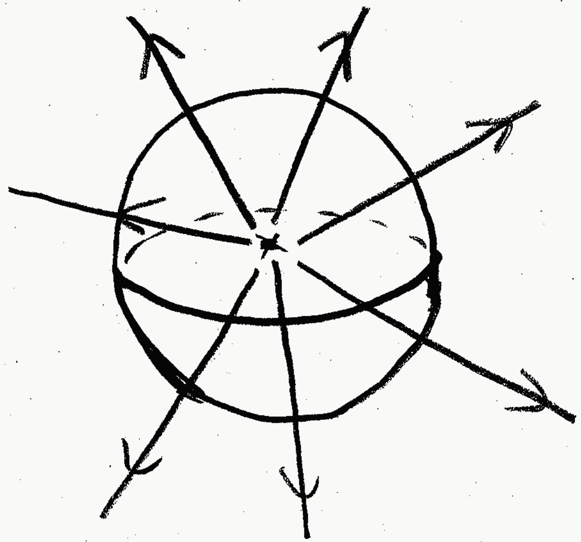
- Gauss law constr.

$$\frac{k}{2\pi} F_{\theta\varphi} = \rho = \begin{bmatrix} J_{(1)} & & & \\ & J_{(2)} & & \\ & & \ddots & \\ & & & J_{(N)} \end{bmatrix}$$

- needs to introduce BKG flux. $g_{(i)}$

GNO charge

~ monopole op.



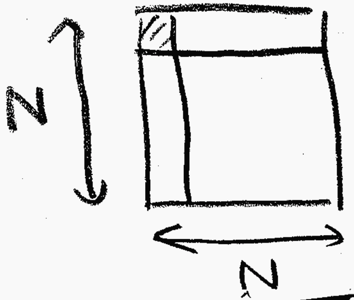
- <u>near BPS fluctuation</u>
- Consider single to GNO charge
  ~ Single membrane
- to study fluctuatiation
  needs gauge fixing (BKG gauge + unitary gauge + Coulomb gauge)

$$\frac{k}{2\pi} F_{12} = \rho = i\phi\,\Pi - i\Pi\,\phi$$

canonical conj.

$$(\langle\phi\phi^*\rangle \sim J,\ \langle\Pi\Pi^*\rangle \sim J,\ \langle\phi\rangle = 0,\ \langle\Pi\rangle = 0)$$

- <u>near BPS fluctuation</u> S^7- direction ~ 6 polorisation



excite (1,1) component of ϕ^1, ϕ^2, ϕ^3
corresponding to $Y_{\ell m}(\theta,\varphi)$

$$\boxed{\Delta \sim \sqrt{\left(\tfrac{1}{2}\right)^2 + \ell(\ell+1)}}$$

mass term

spatial kinetic term

→ matches with AdS side

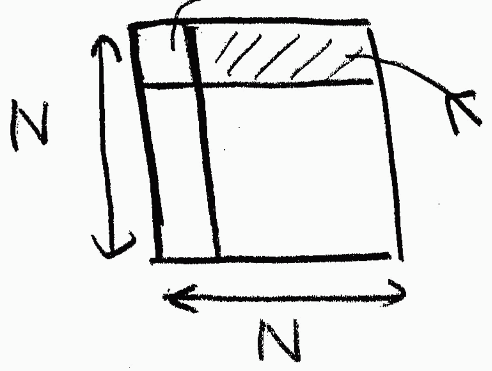
S^2 in radial qtz of ABJM

~ S^2 of M2 in AdS

Born-Oppenheimer approx.

"slow" or "light" modes

- fields in ABJM



do not feel flux, "free" $\rightarrow$ Y_{lm} $\omega=\sqrt{\left(\frac{1}{2}\right)^2+l(l+1)}$ $l=0,1,\cdots$

feel flux $\rightarrow$ Y_{glm} $l=g,g+1,\cdots$ $g\sim J$

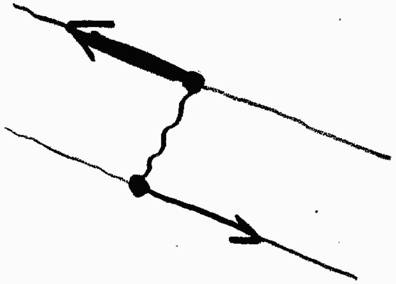
$\omega=\sqrt{\left(\frac{1}{2}\right)^2+l(l+1)}$ "fast" or "heavy" modes

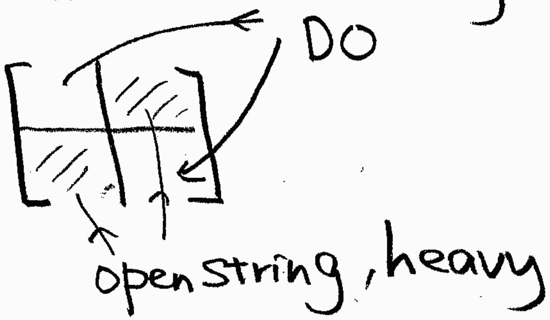
- order J gap in the frequency or energy

$\rightarrow$ integrate out fast modes ~ B-O approx.

should introduce exp. param $\left(\frac{1}{J}\right)^{\ominus}$

- SUSY important ~ cancellation of large contribution to the potential from zero pt. osc. of fast modes
- analogous to D-brane scattering





SUSY

$\rightarrow$ $\frac{v^4}{r^7}\ll 1$

Some feature of gauge fixing

- $$\phi^{4\prime} = \underbrace{\overset{\text{const.}}{f} + u}_{\text{Re}} + i \underbrace{v}_{\text{Im}}, \quad v = 0 \leftarrow \text{unitary gauge}$$

- solve P_v using Gauß' law constr. $\frac{k}{2\pi} F_{\theta\varphi} = \rho$

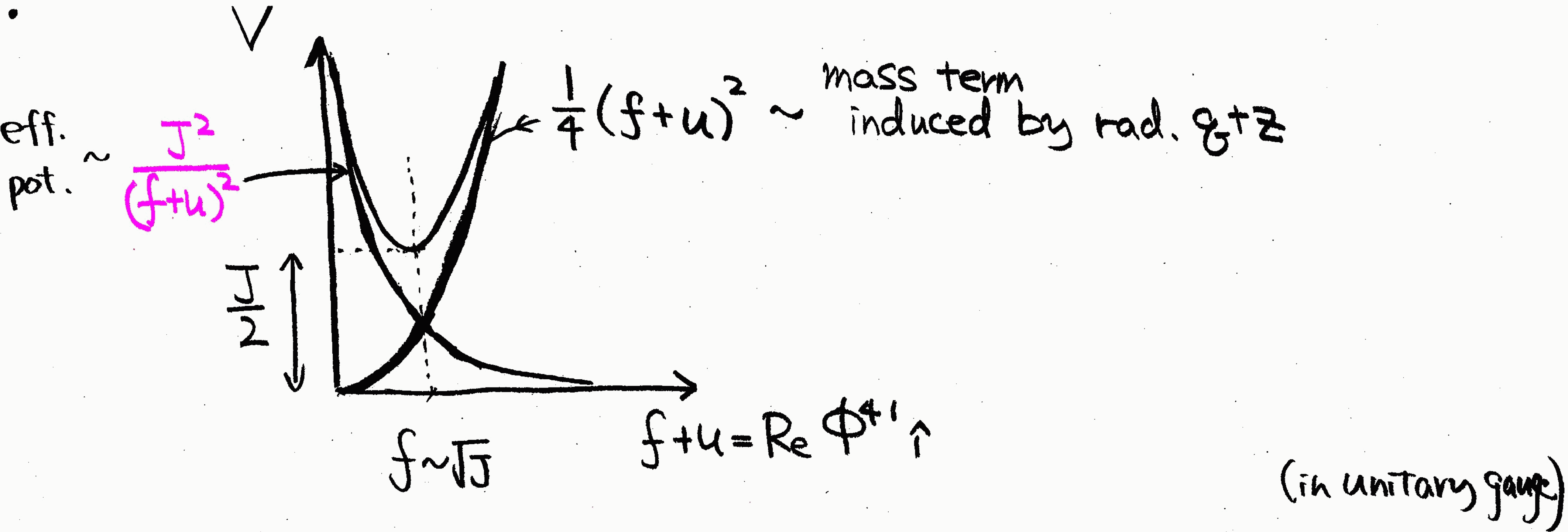
$$\leadsto \frac{k}{2\pi}(\xi + \cdots) = (f+u)P_v - vP_u + \cdots$$

$$\leadsto P_v \sim \frac{J}{f+u}$$

- Substitute into original H, containing P_v^2.

$$H_{\text{gauge-fixed}} \sim \frac{J^2}{(f+u)^2} + \cdots$$

effective potential.

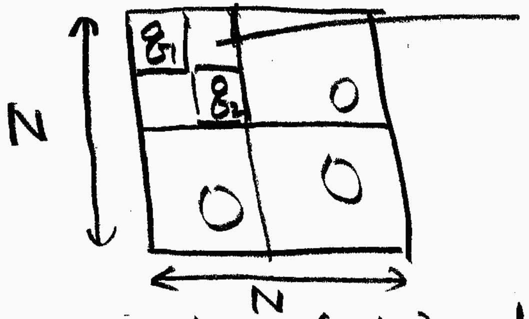


- $f \sim \sqrt{J}$: vac. expectation value of $\mathrm{Re}\,\Phi^{4'}{}_{\uparrow}$

→ Contributes to spectrum & vertices
via Φ^6 -potential, ...
resembles standard Higgs mechanism

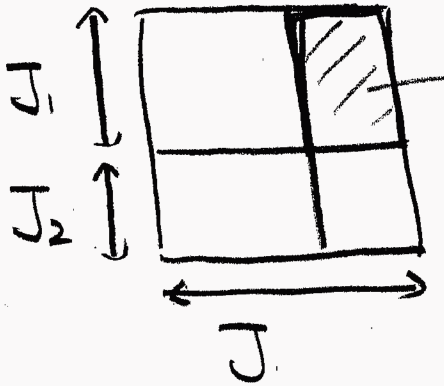
Case with two membranes

- ABJM (CFT) side $\quad q_1 - q_2 \sim O(1)$



$Y_{q\ell m}$ with $q = q_1 - q_2$ not heavy

- matrixmodel (AdS) side $\quad J_1 \sim q_1,\ J_2 \sim q_2 \quad J_1 - J_2 \sim O(1)$



rectangular matrices can be decomposed by matrices corresponding to $Y_{q\ell m}$ $\quad q \sim J_1 - J_2$

(Ishiki, Shimasaki, Takayama, Tsuchiya '06)

- off-diag. DOF in ABJM

$\longleftrightarrow$ block off-diag. in matrix model

4. Summary

- $J \gg 1 \longrightarrow$ approx. on both sides $\quad N^{\frac{1}{3}} \ll J \ll N^{\frac{1}{2}}$

-

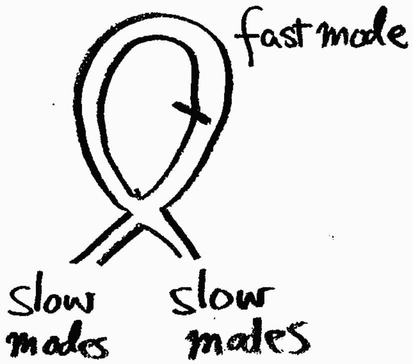
	AdS	CFT
framework	pp-wave matrix model	radial g^{tz} with large flux
approx.	$J \gg 1 \longrightarrow$ large H_2	$J \gg 1 \longrightarrow$ off diag heavy $\longrightarrow$ Born-Oppenheimer approx.
exp. param.	$\frac{Nk}{J^3} \ll 1$, $\frac{J^2}{Nk} \ll 1$	?
BPS	Fuzzy spheres $J = J_{(1)} + J_{(2)} + \cdots$	const. flux $g = g_{(1)} + g_{(2)} + \cdots$
near BPS fluctuation	modes labelled by Tem $\quad l = 0, 1, \cdots$ $\omega = \sqrt{\left(\frac{1}{2}\right)^2 + l(l+1)}$ $S^7: X^4, \cdots, X^9$ $(AdS_4: X^1, X^2, X^3)$	ϕ^1, ϕ^2, ϕ^3 $(\phi^4, A, \hat{A})$

· Important next step.

☆ quantum correction on CFT side

leading order term on AdS side $\sim \frac{Nk}{J^3}$

· 1-loop in fast modes ~ $(1, i')$ modes $(i' = 2, \cdots, N)$

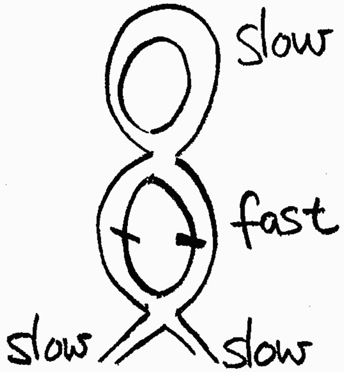


$\sim N k^{n-3} J^{2-n}$ $\left(\begin{array}{l} n: \text{number of} \\ \text{cancellations} \end{array} \right)$

$\leftarrow$ cannot be $\sim \frac{Nk}{J^3}$

should be negligible (or $=0$)

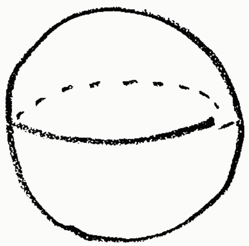
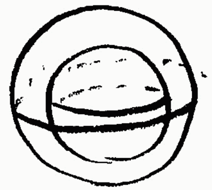
· 1-loop in fast & 1-loop in slow



$\sim N k^{n-3} J^{n-1} \leftarrow \sim \frac{Nk}{J^3}$ for $n=4$

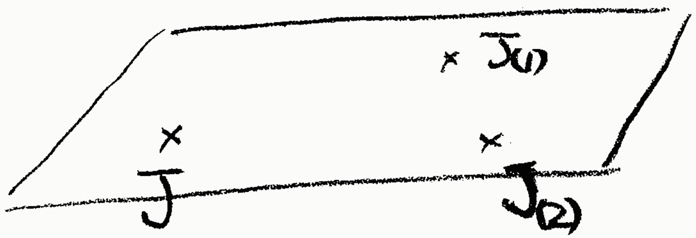
· constraints by SUSY?

☆ Interaction of membranes

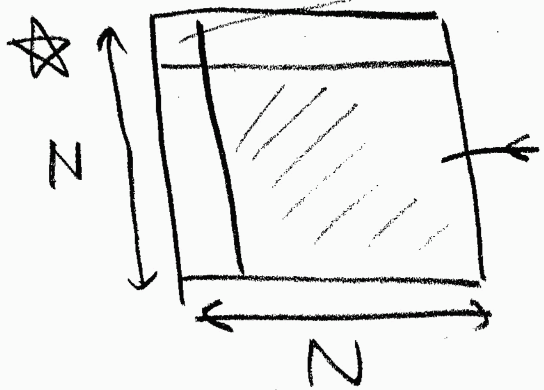
- $J = J$  $\Longrightarrow$ $J = J_{(1)} + J_{(2)}$ 

- matrix model may describe correctly splitting/joining of membranes (longi. mom. transfer)

- Compare to 3-pt. func. of ABJM th.



· Observation



excite → (create) phonon on spherical membrane
1st q+z.

excite → Create small membrane
2nd q+z

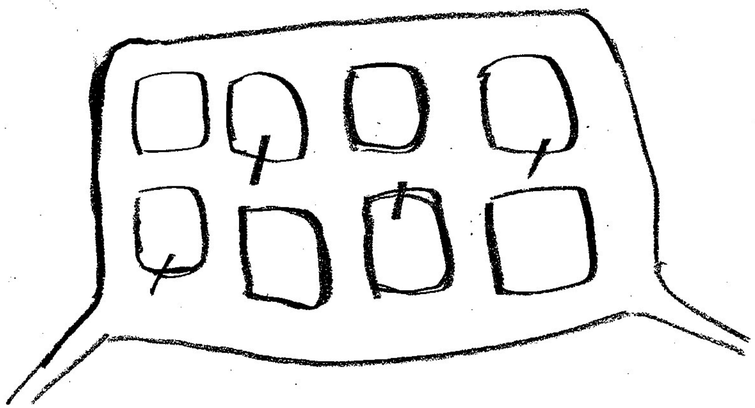
AdS	CFT
bulk /	boundary

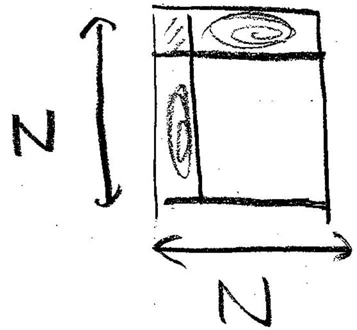
☆ rather direct correspondence

spherical membrane in bulk ⟷ sphere used in radial q+z.

· We hope that our work leads to
good approximation
and then to deeper understanding of
M-theory, matrix model, AdS/CFT

New large N expansion?





$i = 1, \underbrace{2, \cdots, N}_{i'}$

- L : number of loop, p : number of primed loop, n : number of cancellation

$$k^{-L}(N-1)^{p}\left(\frac{J}{k}\right)^{2-n}$$

- for $n = L + p + 2$

$$\rightarrow \left(\frac{Nk}{J^3}\right)^{L-p}\left(\frac{J^2}{Nk}\right)^{L-2p}$$

- for $p = \frac{L}{2}$

$$\left(\frac{Nk}{J^3}\right)^{\frac{L}{2}}$$