Codimension-2 Solutions in 5D Supergravity

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Minkyu Park & MS, arXiv:1505.05169

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Microstructures of Black Holes

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Yukawa Institute for Theoretical Physics, Kyoto University, Japan

Invited speakers:

Stefano Giusto (Padova)
Masanori Hanada (YITP Kyoto)
Masahiro Hotta (Tohoku)
Hikaru Kawai (Kyoto)
Tetsuji Kimura (Keio)
Daniel Mayerson (Michigan)
Joris Raeymaekers (Prague, Inst. Phys.)
Rodolfo Russo (Queen Mary)
Kostas Skenderis (Southampton)
David Turton (CEA Saclay)
Bert Vercnocke (Amsterdam)
Nicholas Warner (USC)
Ryo Yokokura (Keio)

Introduction

Low codim branes (1)

- Branes: important for nonpertubative physics
- Low codim (≤ 2) branes:
  - Less studied
  - Non-standard features
    - Codim 2 (D7) → destroys asymptotics
    - Codim 1 (D8) → spacetime ends at finite distance

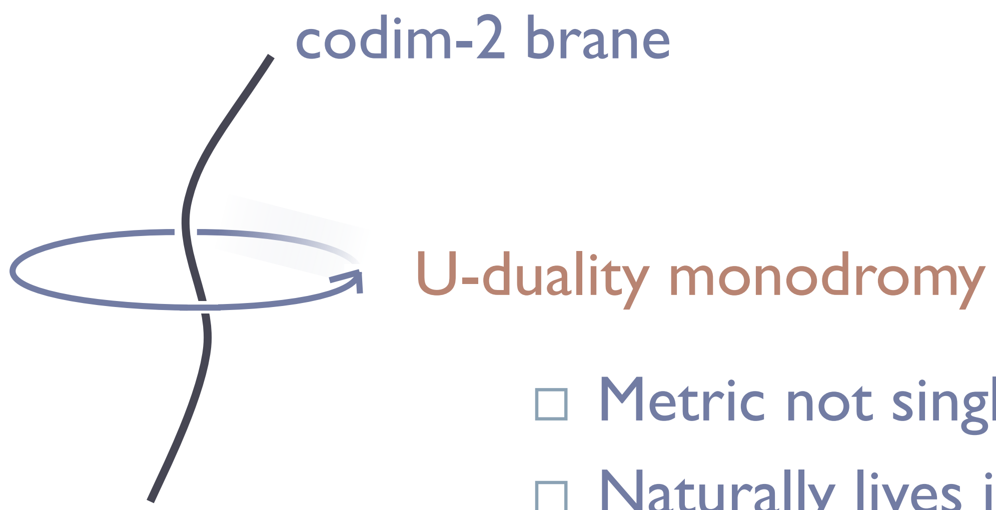
Peculiar, but all the more interesting!

Low codim branes (2)

Why interesting?

- F-theory
- Exotic, non-geometric in general $5^2_2,\ 7_3,\ 1^6_4, \ldots$

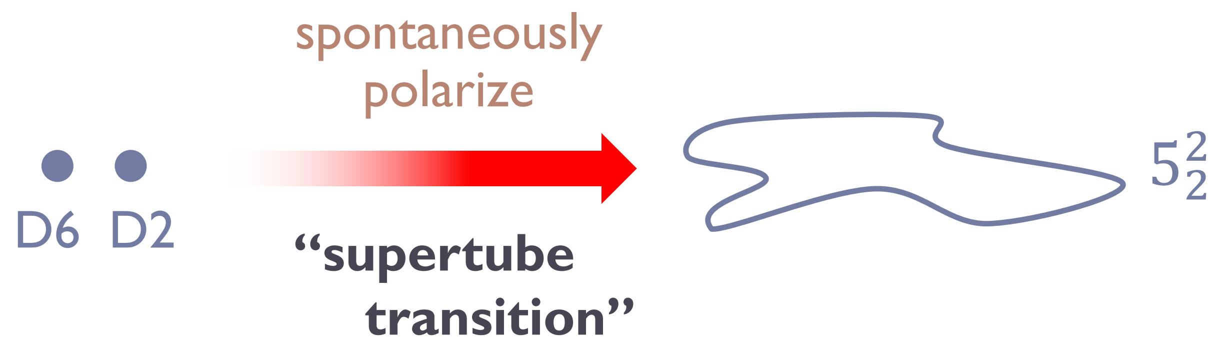
[de Boer+Shigemori '10, '12]



- Metric not single-valued
- Naturally lives in DFT/EFT

Low codim branes (3)

- Can be created out of ordinary branes



  - More common than previously thought
  - Relevance for black hole physics

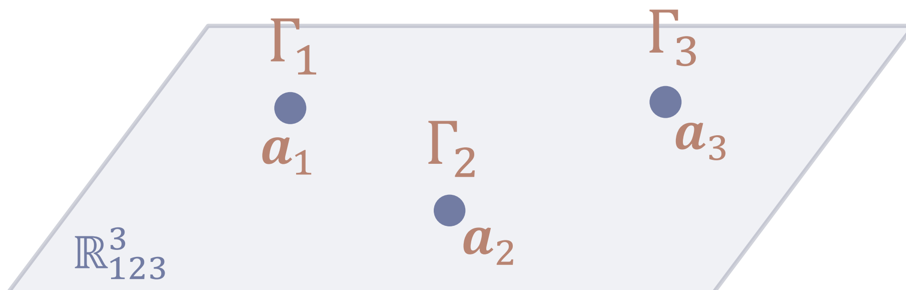
    → Cf. Fuzzball proposal,
    Microstate geometry program

Multi-center solutions

IIA on $T^6 = T^2_{45} \times T^2_{67} \times T^2_{89}$

→ System of $(\mathrm{D6}, \mathrm{D4}, \mathrm{D2}, \mathrm{D0}) \equiv \Gamma$

→ Multi-center solution



moduli, codim-3 source

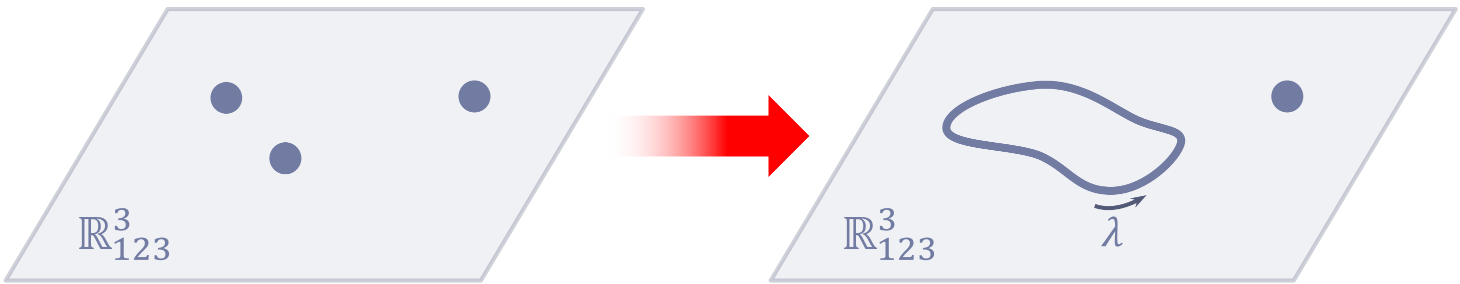
$$H(\boldsymbol{x}) = h + \sum_p \frac{\Gamma_p}{|\boldsymbol{x} - \boldsymbol{a}_p|}$$

⇨ **Paradigm for BH physics**

- Split attractor flow, wall crossing, quiver QM…
- Microstate geometry program

Codim-2 solutions

$$\mathrm{D2}(45) + \mathrm{D2}(67) \to \mathrm{NS5}(\lambda 4567)$$
$$\mathrm{D6}(456789) + \mathrm{D2}(89) \to 5^2_2(\lambda 4567{,}89)$$



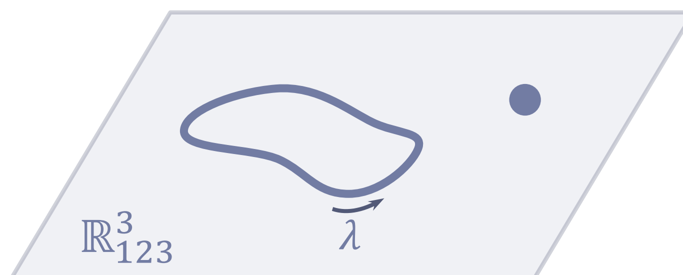
$H(x)$: includes codim-2 source.
multi-valued.

New solutions!

- Various applications
- New microstates (MGP)

Goal:

- Demonstrate that these codim-2 solutions exist by presenting some simple but explicit examples



- We will see monodromy structure characteristic of codim-2 branes

4D/5D Solutions

Setup

- M-theory on $T^6 = T^2_{45} \times T^2_{67} \times T^2_{89}$

⇓

- $D = 5, \mathcal{N} = 1$ sugra with 2 vector multiplets

  gauge fields: A^I_μ , $I = 1,2,3.$ $F^I \equiv dA^I$.

  scalars: X^I, $X^1 X^2 X^3 = 1$

- Action

$$S_{\rm bos} = \int (*_5 R - Q_{IJ} dX^I \wedge *_5 \, dX^I - Q_{IJ} F^I \wedge *_5 \, F^J \\ -\tfrac{1}{6} C_{IJK} F^I \wedge F^J \wedge A^K)$$

$$C_{IJK} = |\epsilon_{IJK}| \, , \qquad Q_{IJ} = \tfrac{1}{2} \mathrm{diag}(1/X^1, 1/X^2, 1/X^3)$$

BPS solutions

[Gutowski-Reall '04] [Bena-Warner '04]
[Gutowski-Gauntlett '04] [Bates-Denef '03]

- Require susy
- Assume $U(1)$ symmetry

⇓

Solution specified by harmonic functions in $\mathbb{R}^3$:

$$H = \left(V, K^I, L_I, M\right), \qquad I = 1,2,3$$

$$\triangle H = 0 \quad \Rightarrow \quad H = h + \sum_p \frac{\Gamma_p}{|\boldsymbol{x} - \boldsymbol{a}_p|}$$

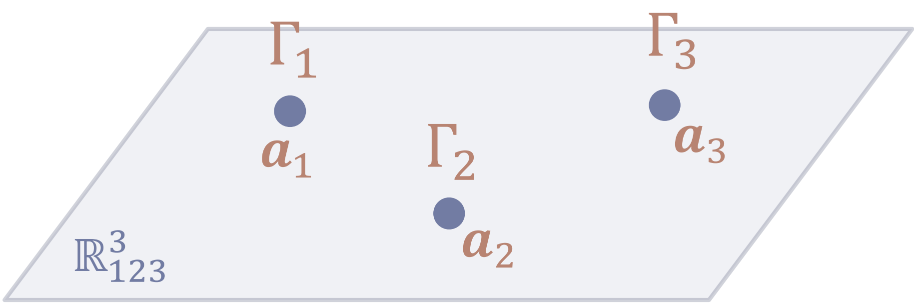
Bates-Denef/Gutowski-Gauntlett/Bena-Warner solution, or

"4D/5D solution"

Multi-center solution

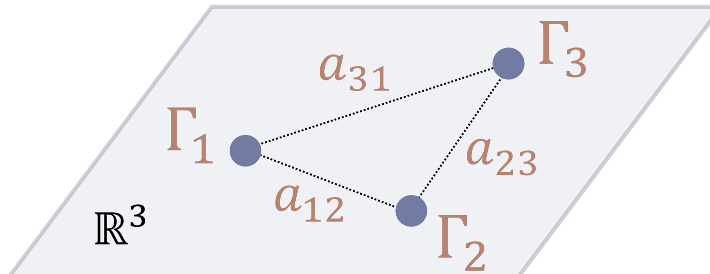
$$H = (V, K^I, L_I, M), \qquad H = h + \sum_p \frac{\Gamma_p}{|\boldsymbol{x} - \boldsymbol{a}_p|}$$

— Describes multi-center config of branes in IIA on T^6_{456789}



$$V \leftrightarrow \mathrm{D6}(456789) \qquad \begin{array}{l} K^1 \leftrightarrow \mathrm{D4}(6789) \\ K^2 \leftrightarrow \mathrm{D4}(4589) \\ K^3 \leftrightarrow \mathrm{D4}(4567) \end{array} \qquad \begin{array}{l} L_1 \leftrightarrow \mathrm{D2}(45) \\ L_2 \leftrightarrow \mathrm{D2}(67) \\ L_3 \leftrightarrow \mathrm{D2}(89) \end{array} \qquad M \leftrightarrow \mathrm{D0}$$

Integrability cond.



- Positions $\boldsymbol{a}_p$ satisfy "integrability cond"

$$\sum_{q(\neq p)} \frac{\langle \Gamma_p, \Gamma_q \rangle}{a_{pq}} = \langle h, \Gamma_p \rangle \qquad a_{pq} \equiv |\mathbf{a}_p - \mathbf{a}_q|$$

- Represents force balance
- Comes from integrability for ω

$$0 = V \triangle M - M \triangle V + \frac{1}{2} \left(K^I \triangle L_I - L_I \triangle K^I \right)$$

10D IIA fields

$$ds^2_{10,\mathrm{str}} = -\frac{1}{\sqrt{V(Z - V\mu^2)}}(dt+\omega)^2 + \sqrt{V(Z - V\mu^2)}\, dx^i dx^i$$
$$+ \sqrt{\frac{Z - V\mu^2}{V}}\left(Z_1^{-1} dx_{45}^2 + Z_2^{-1} dx_{67}^2 + Z_3^{-1} dx_{89}^2\right)$$
$$e^{2\Phi} = \frac{(Z - V\mu^2)^{3/2}}{V^{3/2} Z}\,, \qquad B_2 = \left(V^{-1}K^I - Z_I^{-1}\mu\right) J_I\,, \qquad \cdots$$

$$Z = Z_1 Z_2 Z_3 \qquad J_1 \equiv dx^4 \wedge dx^5, \quad J_2 \equiv dx^6 \wedge dx^7, \quad J_3 \equiv dx^8 \wedge dx^9$$

$$Z_I = L_I + \frac{1}{2} C_{IJK} V^{-1} K^J K^K$$
$$\mu = M + \frac{1}{2} V^{-1} K^I L_I + \frac{1}{6} C_{IJK} V^{-2} K^I K^J K^K$$

$$*_3\, d\omega = V dM - M dV + \frac{1}{2}\left(K^I dL_I - L_I dK^I\right)$$

Torus moduli

Complexified Kähler moduli for T^2_{45}:

$$\tau^1 = B_{45} + i\,\mathrm{vol}(T^2_{45})$$

$$= \left(\frac{K^1}{V} - \frac{\mu}{Z_1}\right) + \frac{i\sqrt{V(Z - V\mu^2)}}{Z_1 V}$$

$R_4 = R_5 = l_s$

We likewise have τ^2, τ^3 for T^2_{67}, T^2_{89}

Related to $SL(2,\mathbb{Z})^3$ duality of STU model

$SL(2,\mathbb{Z}) \times SL(2,\mathbb{Z}) \times SL(2,\mathbb{Z})$

$SL(2,\mathbb{Z})_{45}$: generated by T-duality and shift in B_{45}

$$\tau^1 \to \frac{\alpha_1 \tau^1 + \beta_1}{\gamma_1 \tau^1 + \delta_1} \qquad \alpha_1\delta_1 - \beta_1\gamma_1 = 1$$

$$\begin{pmatrix} K^1 \\ V \end{pmatrix} \to \begin{pmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{pmatrix} \begin{pmatrix} K^1 \\ V \end{pmatrix} \qquad \begin{pmatrix} -2M \\ L_1 \end{pmatrix} \to \begin{pmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{pmatrix} \begin{pmatrix} -2M \\ L_1 \end{pmatrix}$$

$$\begin{pmatrix} -L_2 \\ K^3 \end{pmatrix} \to \begin{pmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{pmatrix} \begin{pmatrix} -L_2 \\ K^3 \end{pmatrix} \qquad \begin{pmatrix} -L_3 \\ K^2 \end{pmatrix} \to \begin{pmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{pmatrix} \begin{pmatrix} -L_3 \\ K^2 \end{pmatrix}$$

We likewise have $SL(2,\mathbb{Z})_{67} \times SL(2,\mathbb{Z})_{89}$ acting on τ^2, τ^3

Example: 4-charge BH

- Susy BH in 4D (4 supercharges)

N^0 D6(456789)
N_1 D2(45)
N_2 D2(67)
N_3 D2(89)

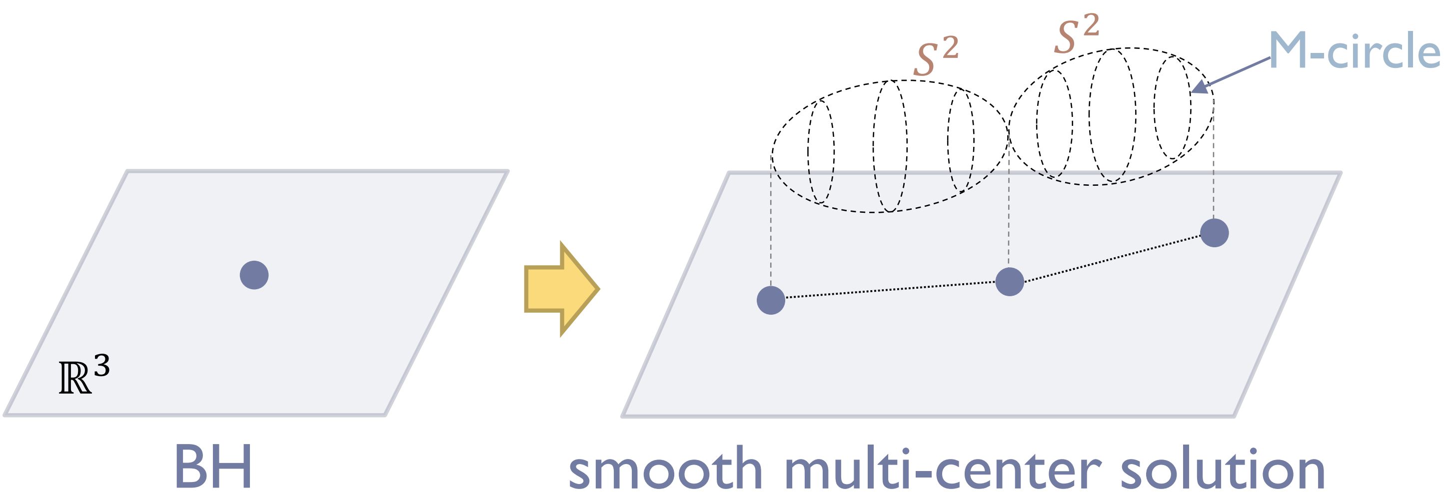
$$V = \frac{N^0}{r} \qquad K^I = 0$$

$$L_I = 1 + \frac{N_I}{r} \qquad M = 0$$

- Single-center
- Macroscopic entropy: $S \sim \sqrt{N^0 N_1 N_2 N_3}$

Microstate geometry program

What portion of the BH entropy
of (supersymmetric) BHs is accounted for
by smooth, horizonless solutions of classical sugra?



- 4D/5D solution: paradigm for MGP

[Bena, Warner '06] [Berglund, Gimon, Levi '06]

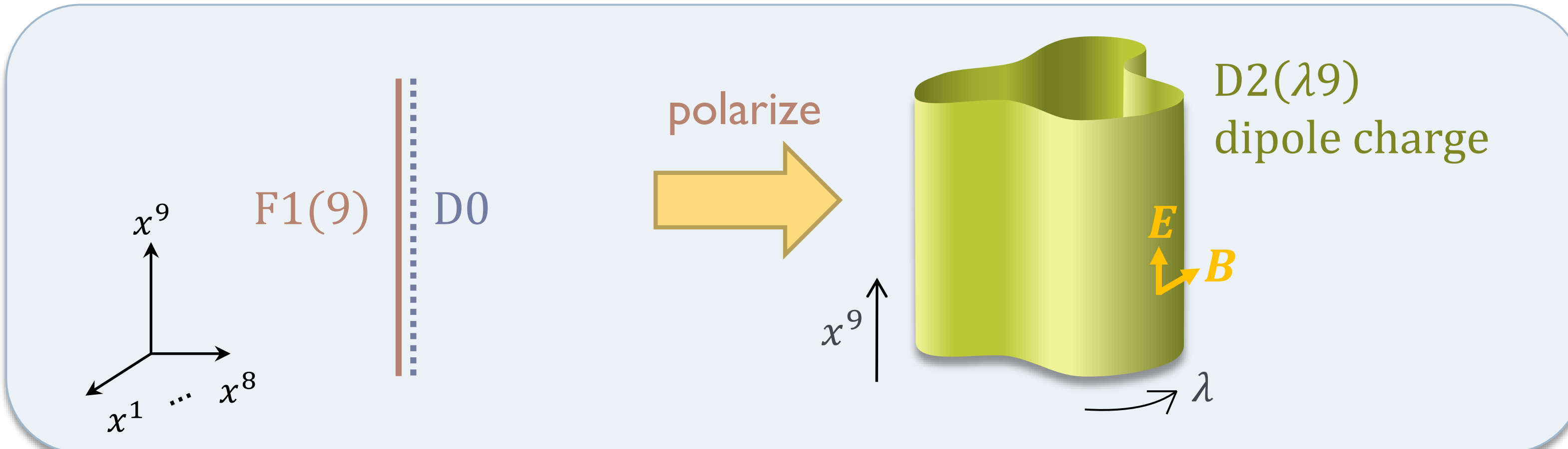
- Not enough microstates found so far

[Bena, Bobev, Giusto, Ruef, Warner '11] [de Boer, El-Showk, Messamah, Van den Bleeken '09]

Supertube transition

Supertube transition [Mateos+Townsend 2001]

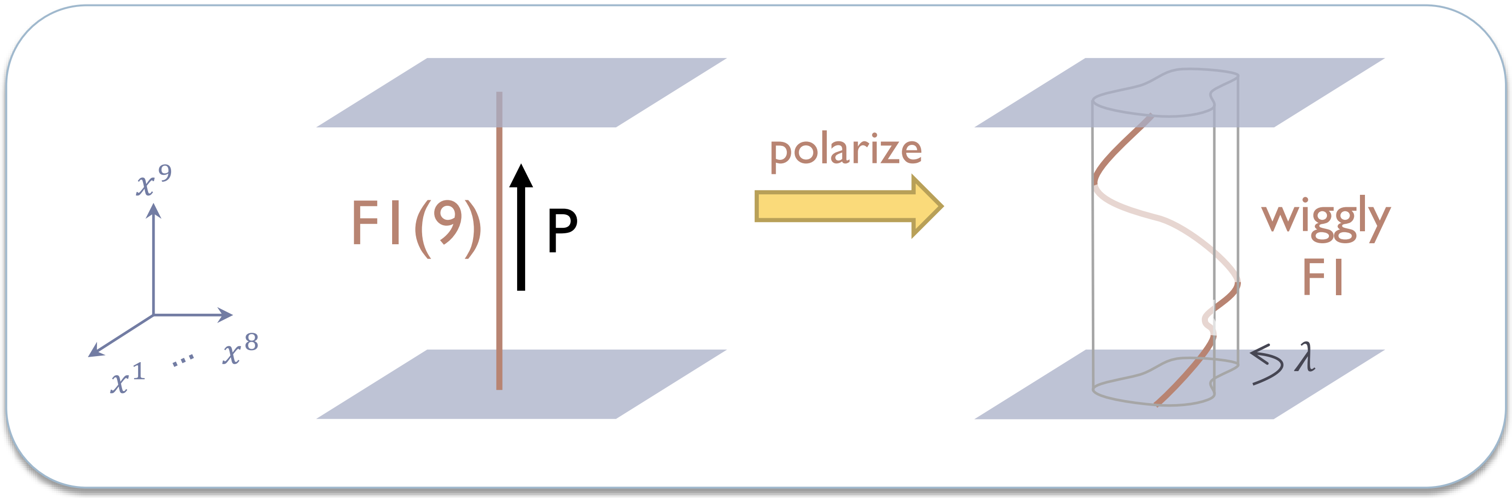
$$\mathrm{D0} + \mathrm{F1}(9) \longrightarrow \mathrm{D2}(\lambda 9)$$



- Spontaneous polarization phenomenon
  (cf. Myers effect)
- Produces new dipole charge
- Cross section = *arbitrary* curve

F1-P frame

$\mathrm{F1}(9) + \mathrm{P}(9) \Rightarrow \mathrm{F1}(\lambda)$



- To carry momentum, F1 must wiggle in transverse $\mathbb{R}^8$
- Projection onto transverse $\mathbb{R}^8$ is an arbitrary curve

Dualizing supertubes

Original supertube effect:

$$\mathrm{D0} + \mathrm{F1}(9) \to \mathrm{D2}(\lambda 9)$$

dualize!

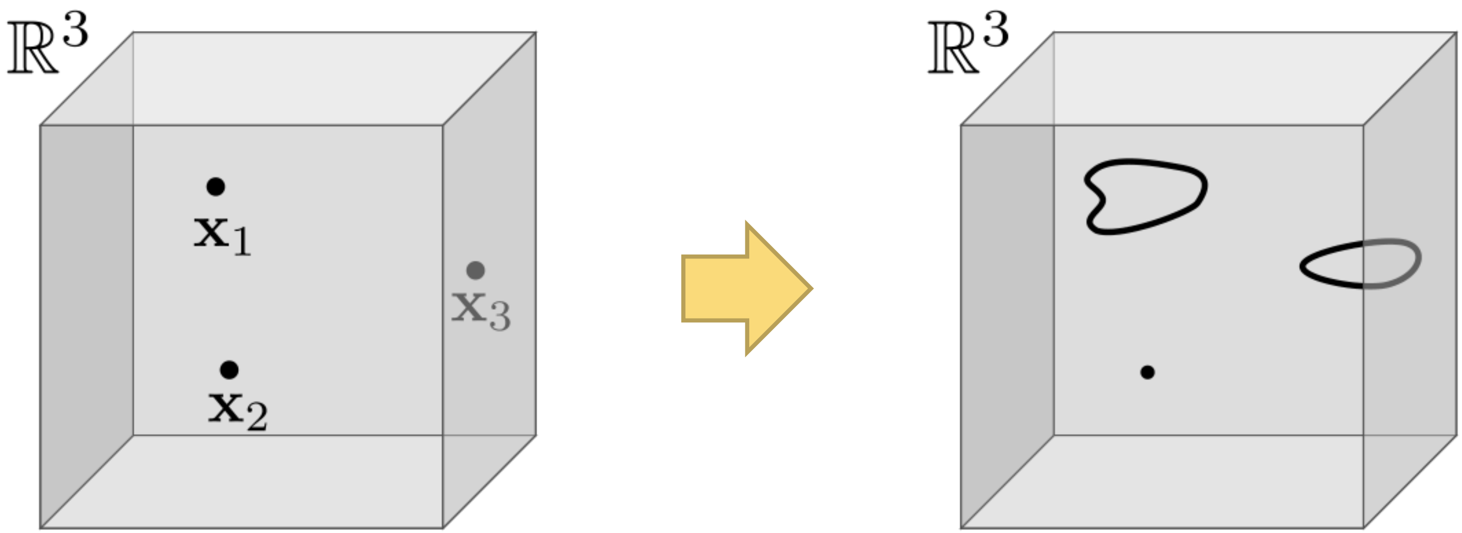
$$\mathrm{F1}(9) + \mathrm{P}(9) \to \mathrm{F1}(\lambda 9)$$

$$\mathrm{D2}(45) + \mathrm{D2}(67) \to \mathrm{NS5}(\lambda 4567)$$

$$\mathrm{D6}(456789) + \mathrm{D2}(45) \to 5^2_2(\lambda 6789,45)$$

- Constituents of 4-chg BH!
- Need to incorporate codim-2 objects in 4D/5D sol'n

Sol'n with codim-2 centers



5_2^2-brane

	1	2	3	4	5	6	7	8	9
NS5	·	·	○	○	○	○	○	~	~

$$\tau \to \tau + 1$$

$$\tau \equiv B_{89} + i \, \mathrm{vol}(T^2_{89})$$

T-duality along x^8

	1	2	3	4	5	6	7	8	9
KKM	·	·	○	○	○	○	○	⊙	~

T-duality along x^9

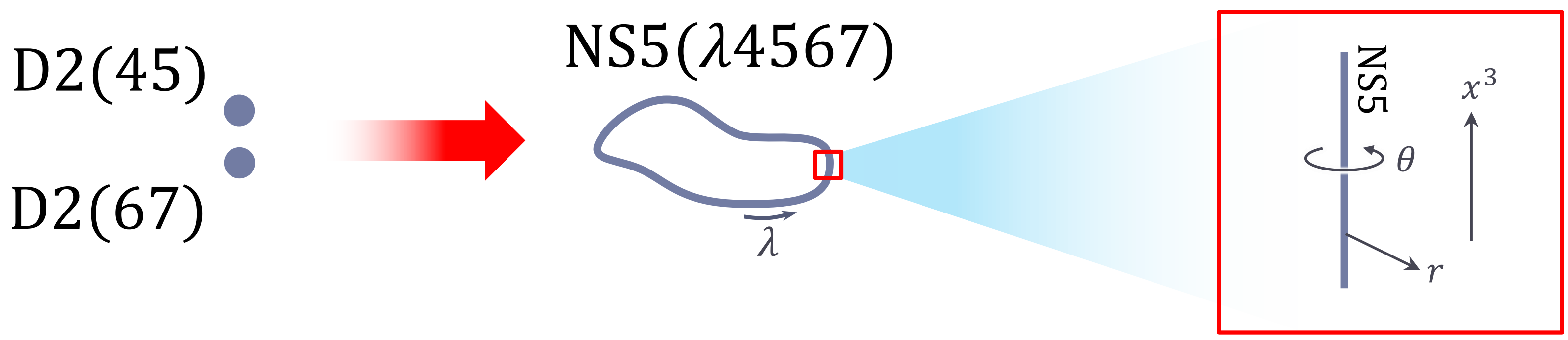
	1	2	3	4	5	6	7	8	9
5_2^2	·	·	○	○	○	○	○	⊙	⊙

$$\tau \to \frac{\tau}{-\tau + 1}$$

$$\tau' \to \tau' + 1, \qquad \tau' \equiv -\frac{1}{\tau}$$

Codim-2 solutions (1)

Straight config (1)



$$V = 1, \qquad K^1 = K^2 = 0, \qquad K^3 = \frac{\theta}{2\pi}$$

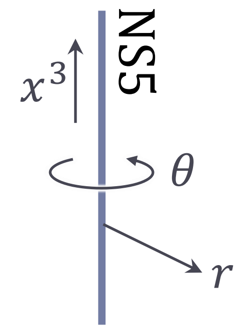
$$L_1 = 1 + Q_1 \log\frac{\Lambda}{r}, \quad L_2 = 1 + Q_2 \log\frac{\Lambda}{r}, \quad L_3 = 1; \quad M = -\frac{\theta}{4\pi}$$

$R_8 = R_9 = l_s$

Straight config (2)

$L_{1,2} = 1 + Q_{1,2} \log \frac{\Lambda}{r}$: D2 source along x^3 axis

$K^3 = \frac{\theta}{2\pi}$: monodromy around x^3 axis



➡ $B_2 = \frac{\theta}{2\pi} dx^8 \wedge dx^9$ ➡ NS5(34567) charge

- Metric is single-valued (no exotic brane, NS5 only)

4D modulus:

$$\tau^3 = B_{89} + i\,\mathrm{vol}\left(T^2_{89}\right) = \frac{\theta}{2\pi}$$

$$\tau^3 \to \tau^3 + 1$$ $SL(2,\mathbb{Z})$ monodromy

Straight config (3)

Monodromy in terms of harmonic functions:

$$\tau^3 \to \tau^3 + 1 \qquad \begin{pmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} K^3 \\ V \end{pmatrix} \to \begin{pmatrix} K^3 + V \\ V \end{pmatrix} \qquad \begin{pmatrix} -2M \\ L_3 \end{pmatrix} \to \begin{pmatrix} -2M + L_3 \\ L_3 \end{pmatrix}$$

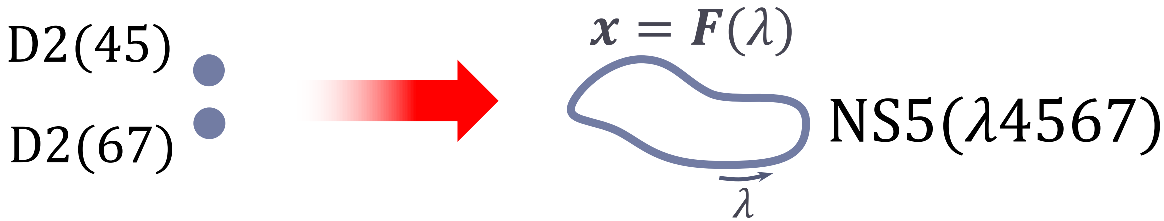
$$\begin{pmatrix} -L_1 \\ K^2 \end{pmatrix} \to \begin{pmatrix} -L_1 + K^2 \\ K^2 \end{pmatrix} \qquad \begin{pmatrix} -L_2 \\ K^1 \end{pmatrix} \to \begin{pmatrix} -L_2 + K^1 \\ K^1 \end{pmatrix}$$

$$V = 1, \qquad K^1 = K^2 = 0, \quad K^3 = \frac{\theta}{2\pi}$$

$$L_1 = 1 + Q_1 \log\frac{\Lambda}{r}, \quad L_2 = 1 + Q_2 \log\frac{\Lambda}{r}, \quad L_3 = 1; \qquad -2M = \frac{\theta}{2\pi}$$

D2+D2→NS5 (1)

NS5 along general curve:



$$V = 1; \quad K^{1,2,3} = 0; \quad L_1 = 1 + \frac{Q_1}{r}, \quad L_2 = 1 + \frac{Q_2}{r}, \quad L_3 = 1; \quad M = 0$$

$$V = 1, \quad K^1 = 0, \quad K^2 = 0, \quad K^3 = \gamma$$
$$L_1 = f_2, \quad L_2 = f_1, \quad L_3 = 1, \quad M = -\frac{\gamma}{2}$$

$$f_1 = 1 + \frac{Q_1}{L}\int_0^L \frac{d\lambda}{|\boldsymbol{x} - \boldsymbol{F}(\lambda)|}, \qquad f_2 = 1 + \frac{Q_1}{L}\int_0^L \frac{|\dot{\boldsymbol{F}}(\lambda)|^2 d\lambda}{|\boldsymbol{x} - \boldsymbol{F}(\lambda)|}$$

$$d\gamma = *_3\, d\alpha, \qquad \alpha_i = \frac{Q_1}{L}\int_0^L \frac{\dot{F}_i(\lambda) d\lambda}{|\boldsymbol{x} - \boldsymbol{F}(\lambda)|}$$

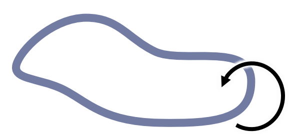
D2+D2→NS5 (2)

$$V = 1, \quad K^1 = 0, \quad K^2 = 0, \quad K^3 = \gamma$$
$$L_1 = f_2, \quad L_2 = f_1, \quad L_3 = 1, \qquad M = -\frac{\gamma}{2}$$

- Obtained by dualizing known supertube solution

  E.g. [Emparan+Mateos+Townsend '01]

- γ multi-valued → multi-valued harmonic functions



$\gamma \to \gamma + \text{const.}$

- Integrability condition satisfied

$$V \triangle M - M \triangle V + \frac{1}{2}\left(K^I \triangle L_I - L_I \triangle K^I\right) = -\triangle\gamma \equiv 0$$

no δ func source

D2+D2→NS5 (3)

- 10D metric is single-valued

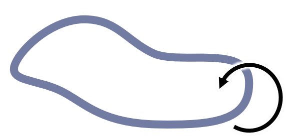
$$Z_1 = f_2,\ \ Z_2 = f_1,\ \ Z_3 = 1,\ \ \mu = 0$$

$$ds_{10}^2 = -(f_1 f_2)^{-1/2}(dt-\alpha)^2 + (f_1 f_2)^{1/2} dx^i dx^i$$
$$+ (f_1/f_2)^{1/2} dx_{45}^2 + (f_2/f_1)^{1/2} dx_{67}^2 + (f_1 f_2)^{1/2} dx_{89}^2$$

Asymptotically $\mathbb{R}^{1,3} \times T^6$

- Torus moduli

$$\tau^1 = i\sqrt{\frac{f_1}{f_2}},\quad \tau^2 = i\sqrt{\frac{f_2}{f_1}},\quad \tau^3 = \gamma + i\sqrt{f_1 f_2}$$

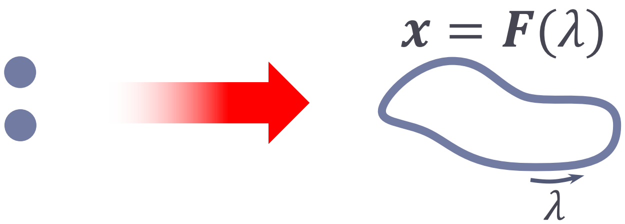


$\tau^3 \to \tau^3 + 1$ → NS5 charge

D6+D2→ 5^2_2 (1)

D6(456789)

D2(89)



5^2_2 (λ4567,89)

$$V = f_2, \quad K^1 = \gamma, \quad K^2 = \gamma, \quad K^3 = 0$$

$$L_1 = 1, \quad L_2 = 1, \quad L_3 = f_1, \quad M = 0$$

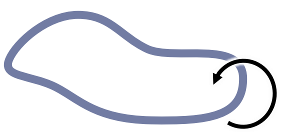
D6+D2→ 5_2^2 (2)

- 10D metric: multi-valued

$$Z_1 = Z_2 = 1, \;\; Z_3 = f_1 F, \quad \mu = f_2^{-1}\gamma \qquad F \equiv 1 + \frac{\gamma^2}{f_1 f_2}$$

$$ds_{10}^2 = -(f_1 f_2)^{-1/2}(dt-\alpha)^2 + (f_1 f_2)^{1/2} dx^i dx^i + (f_1/f_2)^{1/2}\left(dx_{4567}^2 + f_1^{-1}F^{-1}dx_{89}^2\right)$$
$$e^{2\Phi} = f_1^{1/2} f_2^{-3/2} F^{-1}, \qquad B_2 = -\frac{\gamma}{f_1 f_2 F}\, dx^8 \wedge dx^9,$$

  - Asymptotically $\mathbb{R}^{1,3} \times T^6$
  - Non-geometric



T-duality twist

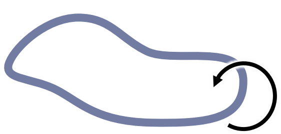
D6+D2→ 5^2_2 (3)

- Torus moduli

$$\tau^3 = B_{89} + i\,\mathrm{vol}(T^2_{89}) = -\frac{\gamma}{f_1 f_2 F} + \frac{i}{\sqrt{f_1 f_2 F}}$$

T-dual modulus:

$$\tau'^3 \equiv -\frac{1}{\tau^3} = \gamma + i\sqrt{f_1 f_2}$$



$\tau'^3 \to \tau'^3 + 1$

→ 5^2_2 brane

Summary so far

- Codim-2 solutions exist
- Constituents of BH systems

$$\left.\begin{array}{l}\mathrm{D6}(456789)\\ \mathrm{D2}(45)\end{array}\right\}\ \text{can polarize into NS5}$$

$$\left.\begin{array}{l}\mathrm{D2}(67)\\ \mathrm{D2}(89)\end{array}\right\}\ \text{can polarize into } 5_2^2$$

→ must be important for BH physics

Codim-2 solutions (2)

More general solutions?

- Need to find harmonic functions such that
  - Have desired monodromy

$$SL(2,\mathbb{Z})_{45}: \quad \begin{pmatrix} K^1 \\ V \end{pmatrix} \to \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} K^1 \\ V \end{pmatrix} \qquad \begin{pmatrix} -2M \\ L_1 \end{pmatrix} \to \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} -2M \\ L_1 \end{pmatrix}$$

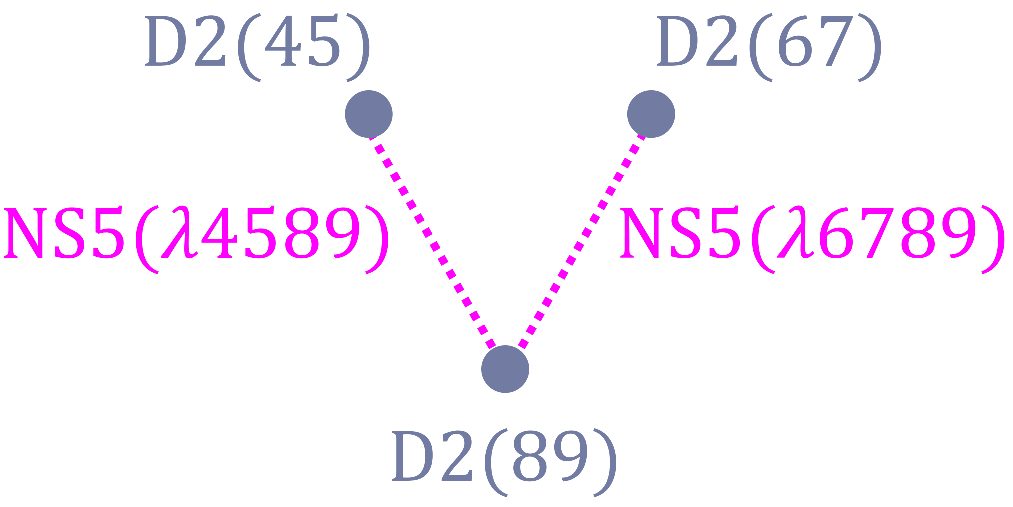
$$\begin{pmatrix} -L_2 \\ K^3 \end{pmatrix} \to \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} -L_2 \\ K^3 \end{pmatrix} \qquad \begin{pmatrix} -L_3 \\ K^2 \end{pmatrix} \to \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} -L_3 \\ K^2 \end{pmatrix}$$

  - Satisfies integrability condition

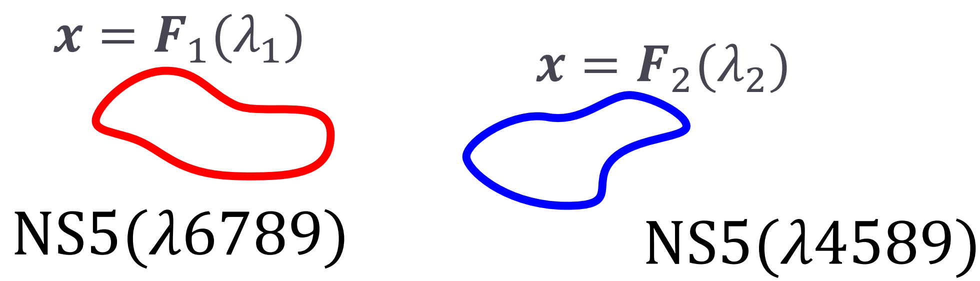
$$0 = V \triangle M - M \triangle V + \frac{1}{2} \left(K^I \triangle L_I - L_I \triangle K^I \right)$$

- Difficult in general

2-dipole solution



D2+D2+D2→NS5+NS5 (1)



$$V = 1, \qquad K^1 = \gamma_1, \quad K^2 = \gamma_2, \quad K^3 = 0,$$

$$L_1 = 1 + Q_2 \int \frac{d\lambda_2}{R_2}, \qquad L_2 = 1 + Q_1 \int \frac{d\lambda_1}{R_1}$$

$$L_3 = 1 + Q_1 \int \frac{|\dot{\boldsymbol{F}}_1|^2 d\lambda_1}{R_1} + Q_2 \int \frac{|\dot{\boldsymbol{F}}_2|^2 d\lambda_2}{R_2}$$

$$+ Q_1 Q_2 \iint d\lambda_1 d\lambda_2 \left(\frac{\dot{\boldsymbol{F}}_1 \cdot \dot{\boldsymbol{F}}_2}{2 R_1 R_2} - \frac{\dot{F}_{1i} \dot{F}_{2j} (R_{1i} R_{2j} - R_{1j} R_{2i})}{F_{12} R_1 R_2 (F_{12} + R_1 + R_2)} \right) - K^1 K^2$$

$$M = \frac{1}{2} Q_1 Q_2 \iint d\lambda_1 d\lambda_2 \frac{\epsilon_{ijk} \dot{F}_{12i} R_{1j} R_{2k}}{F_{12} R_1 R_2 (F_{12} + R_1 + R_2)} - \frac{1}{2} (K^1 L_1 + K^2 L_2)$$

$\boldsymbol{R}_p(\lambda_p) \equiv \boldsymbol{x} - \boldsymbol{F}_p(\lambda_p)$, $\boldsymbol{F}_{12} \equiv \boldsymbol{F}_1(\lambda_1) - \boldsymbol{F}_2(\lambda_2)$, $R_p \equiv |\boldsymbol{R}_p|$, $F_{12} \equiv |\boldsymbol{F}_{12}|$

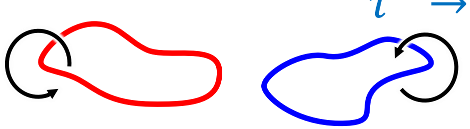
D2+D2+D2→NS5+NS5 (2)

$$V = 1, \quad K^1 = \gamma_1, \quad K^2 = \gamma_2, \quad K^3 = 0, \quad L_1 = 1 + Q_2 \int \frac{d\lambda_2}{R_2}, \quad L_2 = 1 + Q_1 \int \frac{d\lambda_1}{R_1}$$

$$L_3 = 1 + Q_1 \int \frac{|\dot{F}_1|^2 d\lambda_1}{R_1} + Q_2 \int \frac{|\dot{F}_2|^2 d\lambda_2}{R_2} + Q_1 Q_2 \iint d\lambda_1 d\lambda_2 \left(\frac{\dot{F}_1 \cdot \dot{F}_2}{2R_1 R_2} - \frac{\dot{F}_{1i} \dot{F}_{2j} (R_{1i} R_{2j} - R_{1j} R_{2i})}{F_{12} R_1 R_2 (F_{12} + R_1 + R_2)} \right) - K^1 K^2$$

$$M = \frac{1}{2} Q_1 Q_2 \iint d\lambda_1 d\lambda_2 \frac{\epsilon_{ijk} \dot{F}_{12i} R_{1j} R_{2k}}{F_{12} R_1 R_2 (F_{12} + R_1 + R_2)} - \frac{1}{2} (K^1 L_1 + K^2 L_2)$$

- Obtained by smearing and dualizing "superthread" solution
  [Niehoff+Vasilakis+Warner '12]
- L_3, M are multi-valued but harmonic by non-trivial cancellations
- Z_3, μ are single-valued → metric is single-valued
- Correct monodromy for NS5



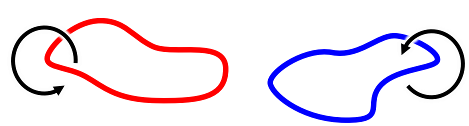
D2+D2+D2→NS5+NS5 (3)

$$V=1,\qquad K^1=\gamma_1,\quad K^2=\gamma_2,\quad K^3=0,\quad L_1=1+Q_2\int\frac{d\lambda_2}{R_2},\qquad L_2=1+Q_1\int\frac{d\lambda_1}{R_1}$$

$$L_3=1+Q_1\int\frac{|\dot F_1|^2 d\lambda_1}{R_1}+Q_2\int\frac{|\dot F_2|^2 d\lambda_2}{R_2}+Q_1Q_2\iint d\lambda_1 d\lambda_2\left(\frac{\dot F_1\cdot\dot F_2}{2R_1R_2}-\frac{\dot F_{1i}\dot F_{2j}\left(R_{1i}R_{2j}-R_{1j}R_{2i}\right)}{F_{12}R_1R_2(F_{12}+R_1+R_2)}\right)-K^1K^2$$

$$-2M=-Q_1Q_2\iint d\lambda_1 d\lambda_2\frac{\epsilon_{ijk}\dot F_{12i}R_{1j}R_{2k}}{F_{12}R_1R_2(F_{12}+R_1+R_2)}+K^1L_1+K^2L_2$$

$$\begin{pmatrix}\alpha_1 & \beta_1\\ \gamma_1 & \delta_1\end{pmatrix}=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$$

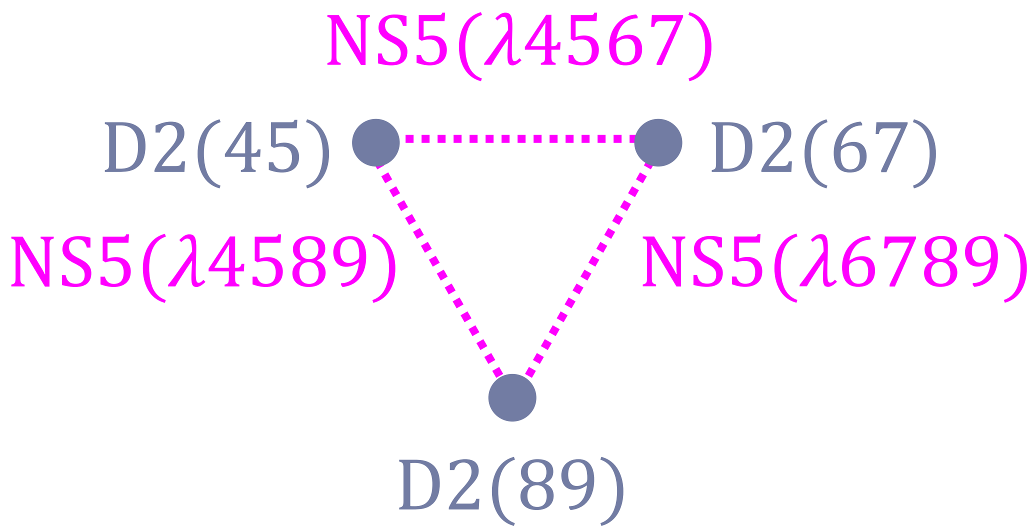


$$\begin{pmatrix}\alpha_2 & \beta_2\\ \gamma_2 & \delta_2\end{pmatrix}=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$$

$$\begin{pmatrix}K^1\\ V\end{pmatrix}\to\begin{pmatrix}K^1+V\\ V\end{pmatrix}\qquad\begin{pmatrix}-2M\\ L_1\end{pmatrix}\to\begin{pmatrix}-2M+L_1\\ L_1\end{pmatrix}\qquad\begin{pmatrix}K^2\\ V\end{pmatrix}\to\begin{pmatrix}K^2+V\\ V\end{pmatrix}\qquad\begin{pmatrix}-2M\\ L_2\end{pmatrix}\to\begin{pmatrix}-2M+L_2\\ L_2\end{pmatrix}$$

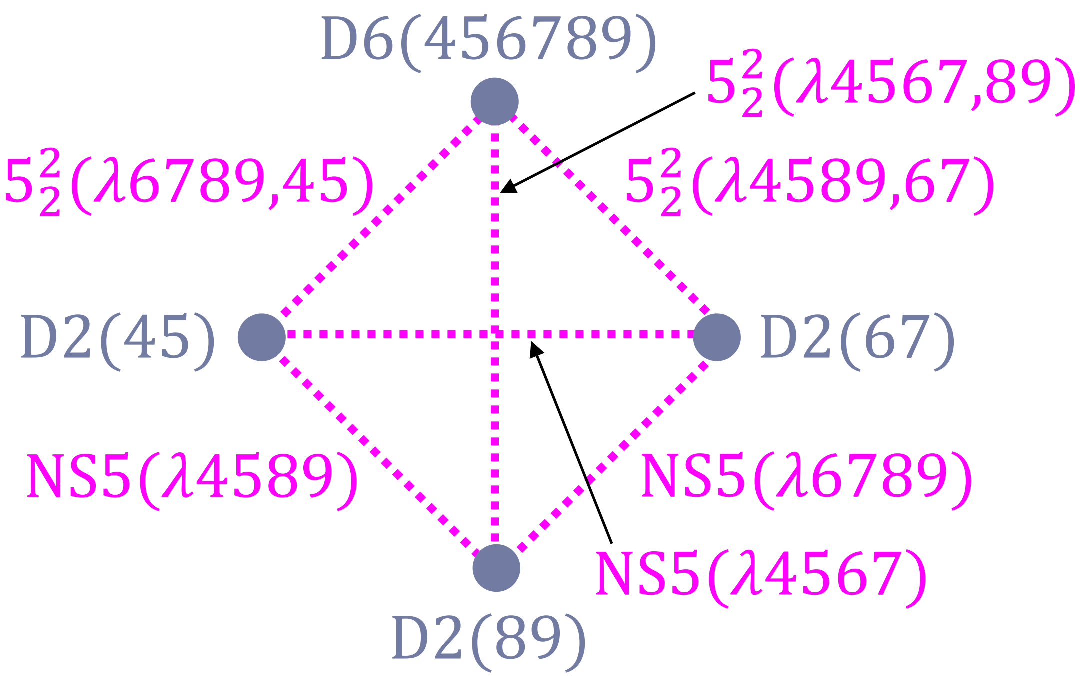
$$\begin{pmatrix}-L_2\\ K^3\end{pmatrix}\to\begin{pmatrix}-L_2+K^3\\ K^3\end{pmatrix}\qquad\begin{pmatrix}-L_3\\ K^2\end{pmatrix}\to\begin{pmatrix}-L_3+K^2\\ K^2\end{pmatrix}\qquad\begin{pmatrix}-L_3\\ K^1\end{pmatrix}\to\begin{pmatrix}-L_3+K^1\\ K^1\end{pmatrix}\qquad\begin{pmatrix}-L_1\\ K^3\end{pmatrix}\to\begin{pmatrix}-L_1+K^3\\ K^3\end{pmatrix}$$

3-dipole solution



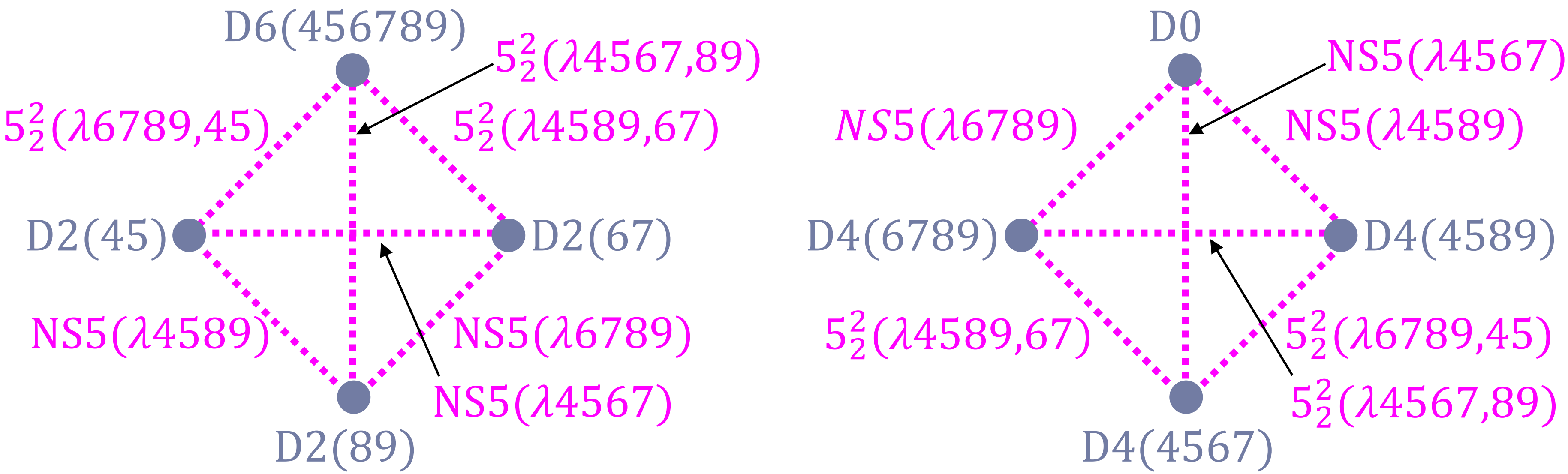
- Can be found, but more complicated and implicit

More dipoles



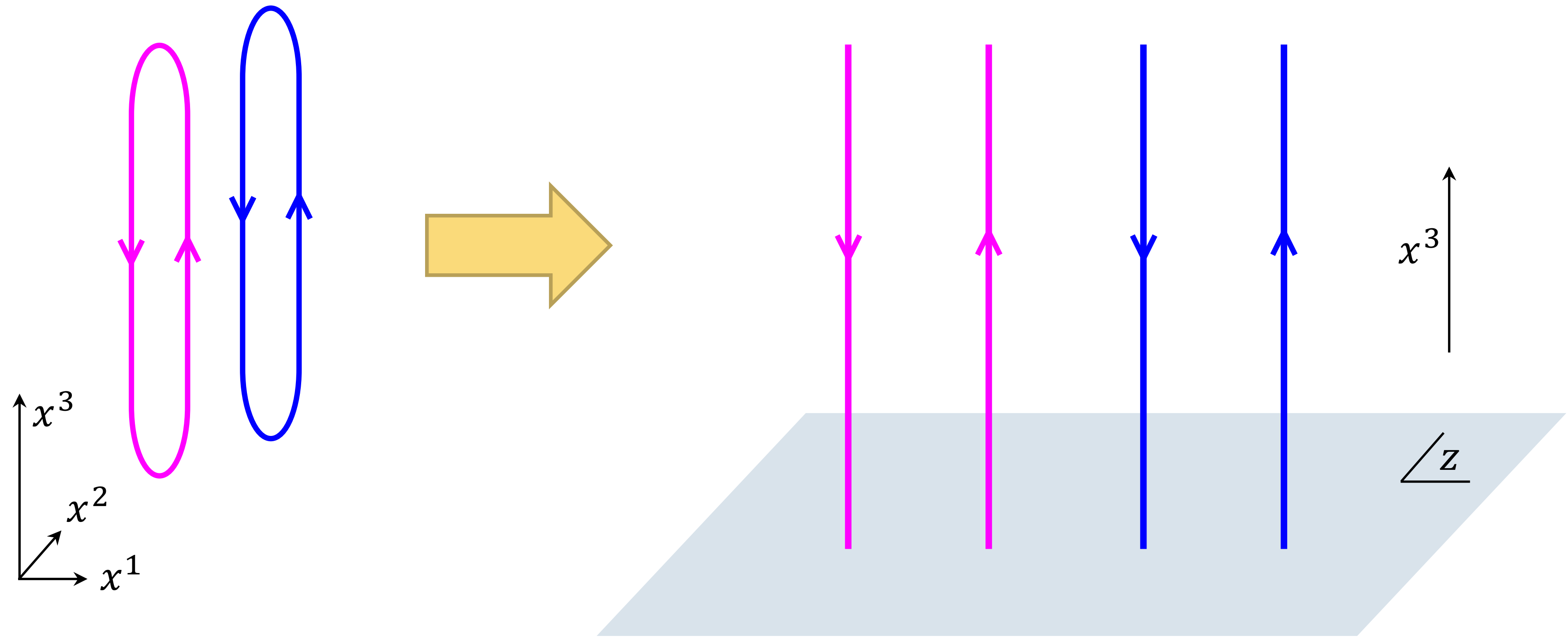
- Represents microstates of 4-charge BH
- Difficult to find…not successful so far

Most general



2D configurations

2D configs



Config with $\tau^3(z)$

General config with $\tau^1 = \tau^2 = i,\ \tau^3 = \tau^3(z)$:

$$V = \frac{1}{2}(g + \bar{g}) \qquad K^1 = K^2 = \frac{i}{2}(g - \bar{g}) \qquad K^3 = \frac{1}{2}(f + \bar{f})$$

$$L_1 = L_2 = -\frac{i}{2}(f - \bar{f}) \qquad L_3 = \frac{1}{2}(g + \bar{g}) \qquad M = -\frac{1}{4}(f + \bar{f})$$

$$\tau^3(z) = \frac{f(z)}{g(z)} = \frac{K^3 + iL_1}{V - iK^1}$$

Can describe:

$$\begin{array}{llll} D2(45) & +D2(67) & \to & ns5(\lambda 4567) \\ D2(89) & +D6(456789) & \to & 5^2_2(\lambda 4567, 89) \\ D4(4589) & +D4(6789) & \to & 5^2_2(\lambda 4567, 89) \\ D0 & +D4(4567) & \to & ns5(\lambda 4567) \end{array}$$

Finding $f(z), g(z)$

How to find holomorphic $f(z), g(z)$ with $\mathrm{Im}\,\tau^3 > 0$?

Seiberg-Witten:

Consider torus fibration over z-plane

$$\begin{pmatrix} f(z) \\ g(z) \end{pmatrix} = h(z) \begin{pmatrix} \partial_z a_D \\ \partial_z a \end{pmatrix}$$

$\partial_z a_D = \int_1 \omega \qquad \partial_z a = \int_2 \omega \qquad \omega$: holomorphic 1-form

Behavior near singularity

$$\partial_z a_D \sim \frac{1}{2\pi} \log z \qquad \partial_z a \sim i \qquad h(z) \sim A + iB$$

$$V \sim -B \qquad K^1 = K^2 \sim -A \qquad K^3 \sim \frac{1}{4\pi}\left(\underbrace{iB \log\left(\frac{z}{\bar{z}}\right)}_{\text{NS5}} + \underbrace{A \log(z\bar{z})}_{\text{D4(4567)}} \right)$$

$$L_1 = L_2 \sim -\overbrace{\frac{iA}{2} \log\left(\frac{z}{\bar{z}}\right)}^{\text{NS5}} + \overbrace{\frac{B}{2} \log(z\bar{z})}^{\text{D2(45),D2(67)}} \qquad L_3 \sim -B$$

$$M \sim -\frac{1}{8\pi}\left(iB \log\left(\frac{z}{\bar{z}}\right) + \underbrace{A \log(z\bar{z})}_{\text{D0}} \right) \qquad \tau^3(z) = \frac{f(z)}{g(z)} = \frac{K^3 + iL_1}{V - iK^1}$$

Does describe:

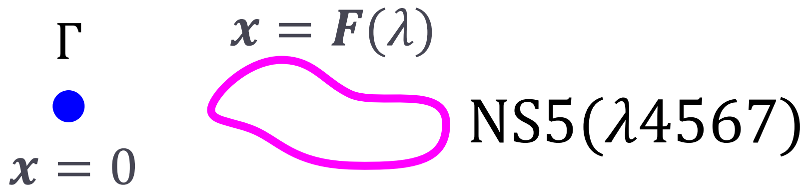
D0 + D4(4567) → ns5(λ4567)

D2(45) + D2(67) → ns5(λ4567)

Can describe antibranes too

Mixed configurations

Codim-2 & codim-3



$$V = n_0 + \frac{n}{r}$$

$$K^1 = k_0^1 + \frac{k^1}{r}, \qquad K^2 = k_0^2 + \frac{k^2}{r}, \qquad K^3 = k_0^3 + \gamma + \frac{k^3}{r}$$

$$L_1 = l_1^0 + f_2 + \frac{l_1}{r}, \qquad L_2 = l_2^0 + f_1 + \frac{l_2}{r}, \qquad L_3 = l_3^0 + \frac{l_3}{r}$$

$$M = m_0 - \frac{\gamma}{2} + \frac{m}{r}$$

Integrability condition

(a) $$0 = n_0 m - m_0 n + \frac{1}{2}(k_0^I l_I - l_I^0 k^I) - \frac{1}{2}\frac{Q}{L}\int_0^L d\lambda \frac{k^1|\dot{\mathbf{F}}(\lambda)|^2 + k^2}{|\mathbf{F}(\lambda)|},$$

(b) $$0 = n + l_3,$$

(c) $$0 = k_0^2 + \frac{k^2}{|\mathbf{F}(\lambda)|} + |\dot{\mathbf{F}}(\lambda)|^2 \left(k_0^1 + \frac{k^1}{|\mathbf{F}(\lambda)|}\right) \qquad \text{for each value of } \lambda.$$

(a): total force from tube to the $r = 0$ brane is 0

→ Easy to satisfy

(c): force by the $r = 0$ brane on each point along tube is 0

→ Constrain curve $\boldsymbol{F}(\lambda)$

→ Only 2 components in $\boldsymbol{F}(\lambda)$ are free

Conclusions

Conclusions

- 4D/5D solution: a paradigm for BH research
- Only codim-3 solutions studied so far
- Codim-2 solutions also important
  - Only scratched the surface
  - Interesting dynamics
  - General solutions?
  - Applications

Future directions

- Applications
  - Split attractor flow, wall crossing, quiver QM, …
  - Microstate Geometry Program
- Lower codimension
  - Superstratum
- Connect to DFT/EFT

Thanks!