# Precise determination of charm and bottom quark masses

# チャーム及びボトムクォーク質量の精密決定

Y. Sumino
(Tohoku Univ.)

# ☆Plan of talk

1. Motivation
2. Current status and History [Particle Data Group]
3. Theoretical analysis methods and Results

   Sum rules, Quarkonium spectrum, Inclusive *B* decays
4. Summary

## Motivation for precise determination of $m_b$ and $m_c$

- Input parameters for precision flavor physics

  LHC$_b$, Belle II e.g. $\Gamma_b \propto m_b^5$, $b, c$-hadron spectroscopy

- Constraints on GUT, BSM for flavor origins

  e.g. $m_b/m_\tau$ ratio in SU(5) GUT

***My personal motivation is more basic...***

# Why and How do we measure quark masses?

$$\mathcal{L}_{\rm QCD}(\alpha_s, m_q) = \sum_{\substack{q=u,d,s,\\ c,b,t}} \overline{\psi_q}\,(i\gamma^\mu D_\mu - m_q)\psi_q \;-\; \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a}$$


"But I didn't know that one could find them inside particles."

M. Gell-Mann

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***A question:*** *Quark mass increases or decreases by strong interaction?*


"But I didn't know that one could find them inside particles."

M. Gell-Mann

# Particle Data Group 2016

## c

$I(J^P) = 0(\frac{1}{2}^+)$

Charge = $\frac{2}{3}$ $e$ Charm = +1

### $c$-QUARK MASS

The $c$-quark mass corresponds to the "running" mass $m_c$ ($\mu = m_c$) in the $\overline{\rm MS}$ scheme. We have converted masses in other schemes to the

| VALUE (GeV) | | | DOCUMENT ID | | TECN |
|---|---|---|---|---|---|
| **1.27** | **±0.03** | **OUR EVALUATION** | See the ideogram below. | | |
| 1.246 | ±0.023 | | [1] KIYO | 16 | THEO |
| 1.2715 | ±0.0095 | | [2] CHAKRABOR... | 15 | LATT |
| 1.288 | ±0.020 | | [3] DEHNADI | 15 | THEO |
| 1.348 | ±0.046 | | [4] CARRASCO | 14 | LATT |
| 1.26 | ±0.05 | ±0.04 | [5] ABRAMOWICZ | 13C | COMB |
| 1.24 | ±0.03 | $^{+0.03}_{-0.07}$ | [6] ALEKHIN | 13 | THEO |
| 1.282 | ±0.011 | ±0.022 | [7] DEHNADI | 13 | THEO |
| 1.286 | ±0.066 | | [8] NARISON | 13 | THEO |
| 1.159 | ±0.075 | | [9] SAMOYLOV | 13 | NOMD |
| 1.36 | ±0.04 | ±0.10 | [10] ALEKHIN | 12 | THEO |
| 1.261 | ±0.016 | | [11] NARISON | 12A | THEO |
| 1.278 | ±0.009 | | [12] BODENSTEIN | 11 | THEO |
| 1.28 | $^{+0.07}_{-0.06}$ | | [13] LASCHKA | 11 | THEO |
| 1.196 | ±0.059 | ±0.050 | [14] AUBERT | 10A | BABR |
| 1.28 | ±0.04 | | [15] BLOSSIER | 10 | LATT |
| 1.279 | ±0.013 | | [16] CHETYRKIN | 09 | THEO |
| 1.25 | ±0.04 | | [17] SIGNER | 09 | THEO |

## b

$I(J^P) = 0(\frac{1}{2}^+)$

Charge = $-\frac{1}{3}$ $e$ Bottom = −1

### $b$-QUARK MASS

The first value is the "running mass" $\overline{m}_b(\mu = \overline{m}_b)$ in the $\overline{\rm MS}$ scheme, and the second value is the $1S$ mass, which is half the mass of the $\Upsilon(1S)$

| $\overline{\rm MS}$ MASS (GeV) | 1S MASS (GeV) | DOCUMENT ID | | TECN |
|---|---|---|---|---|
| **4.18** $^{+0.04}_{-0.03}$ | **OUR EVALUATION** of $\overline{\rm MS}$ Mass. See the ideogram below. | | | |
| **4.66** $^{+0.04}_{-0.03}$ | **OUR EVALUATION** of 1$S$ Mass. See the ideogram below. | | | |
| 4.197±0.022 | 4.671 ± 0.024 | [1] KIYO | 16 | THEO |
| 4.183±0.037 | 4.656 ± 0.041 | [2] ALBERTI | 15 | THEO |
| 4.193 $^{+0.022}_{-0.035}$ | 4.667 $^{+0.024}_{-0.039}$ | [3] BENEKE | 15 | THEO |
| 4.176±0.023 | 4.648 ± 0.026 | [4] DEHNADI | 15 | THEO |
| 4.07 ±0.17 | 4.53 ± 0.19 | [5] ABRAMOWICZ | 14A | HERA |
| 4.201±0.043 | 4.676 ± 0.048 | [6] AYALA | 14A | THEO |
| 4.21 ±0.11 | 4.69 ± 0.12 | [7] BERNARDONI | 14 | LATT |
| 4.169±0.002±0.008 | 4.640 ± 0.002 ± 0.009 | [8] PENIN | 14 | THEO |
| 4.166±0.043 | 4.637 ± 0.048 | [9] LEE | 13O | LATT |
| 4.247±0.034 | 4.727 ± 0.039 | [10] LUCHA | 13 | THEO |
| 4.236±0.069 | 4.715 ± 0.077 | [11] NARISON | 13 | THEO |
| 4.213±0.059 | 4.689 ± 0.066 | [12] NARISON | 13A | THEO |
| 4.171±0.009 | 4.642 ± 0.010 | [13] BODENSTEIN | 12 | THEO |
| 4.29 ±0.14 | 4.77 ± 0.16 | [14] DIMOPOUL... | 12 | LATT |
| 4.235±0.003±0.055 | 4.755 ± 0.003 ± 0.058 | [15] HOANG | 12 | THEO |
| 4.177±0.011 | 4.649 ± 0.012 | [16] NARISON | 12 | THEO |
| 4.18 $^{+0.05}_{-0.04}$ | 4.65 $^{+0.06}_{-0.04}$ | [17] LASCHKA | 11 | THEO |
| 4.186±0.044±0.015 | 4.659 ± 0.050 ± 0.017 | [18] AUBERT | 10A | BABR |
| 4.164±0.023 | 4.635 ± 0.026 | [19] MCNEILE | 10 | LATT |
| 4.163±0.016 | 4.633 ± 0.018 | [20] CHETYRKIN | 09 | THEO |
| 4.243±0.049 | 4.723 ± 0.055 | [21] SCHWANDA | 08 | BELL |

# Particle Data Group 2016

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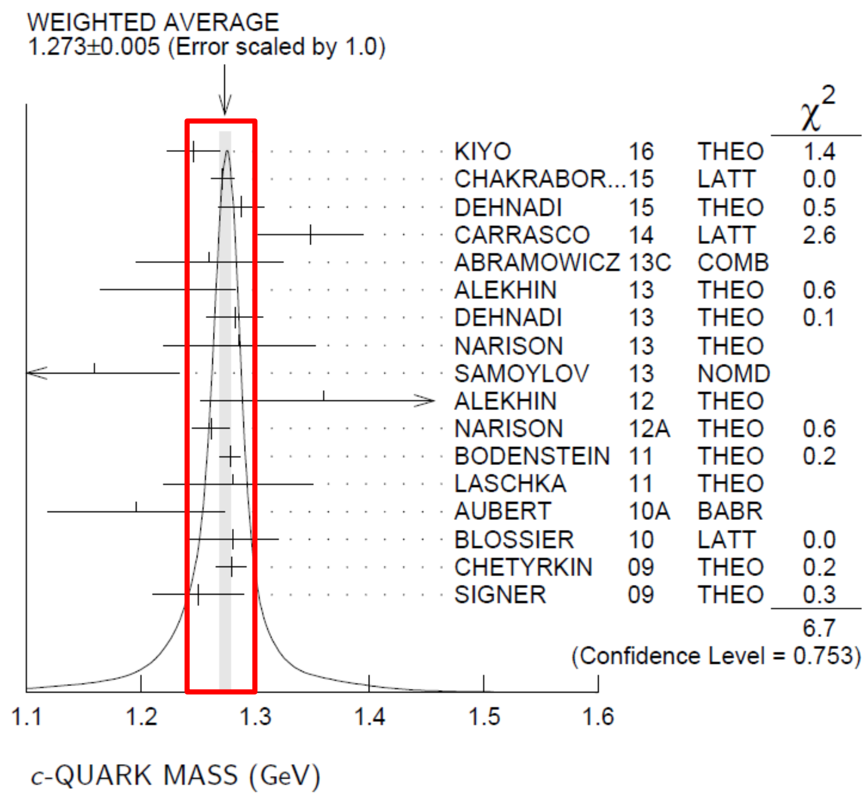


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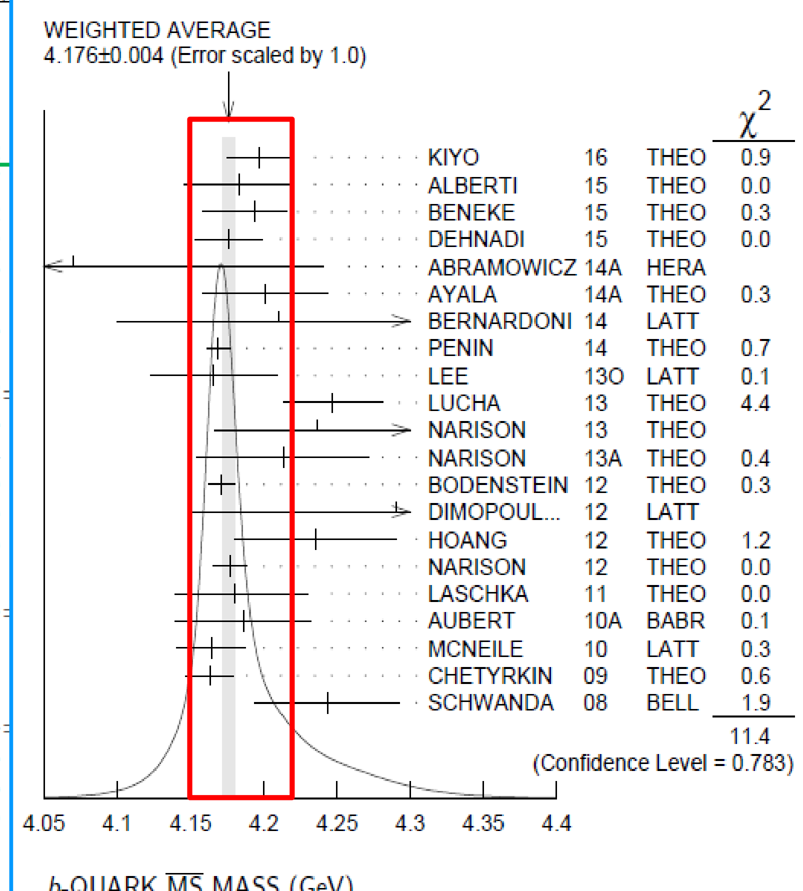

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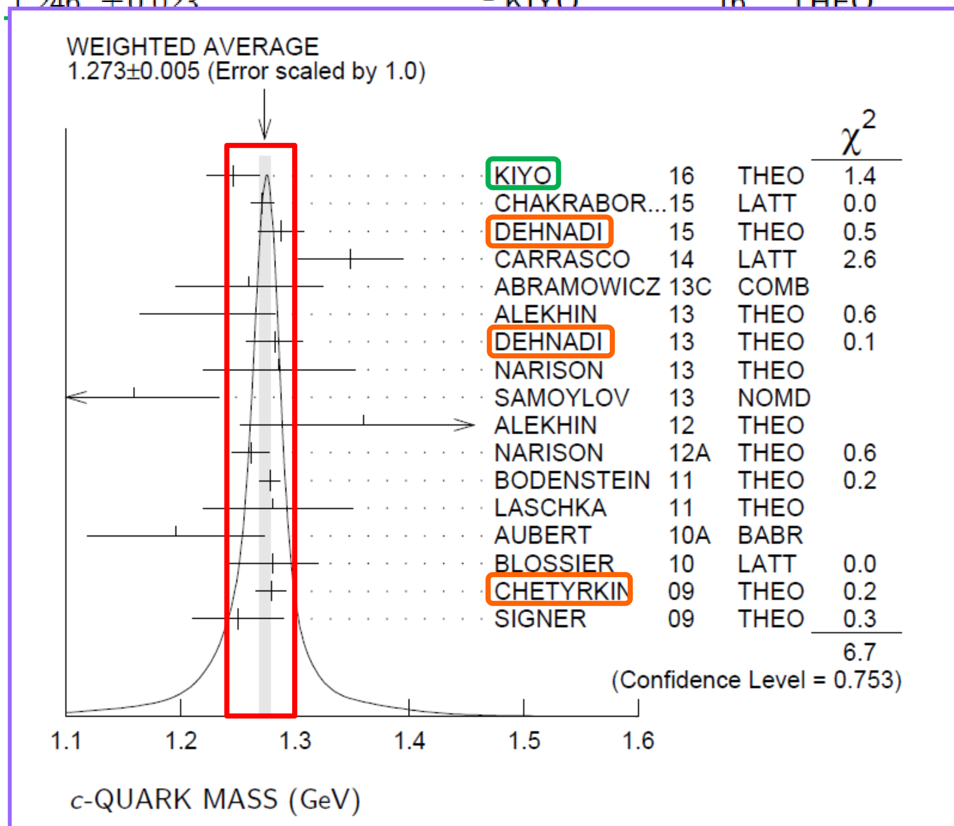


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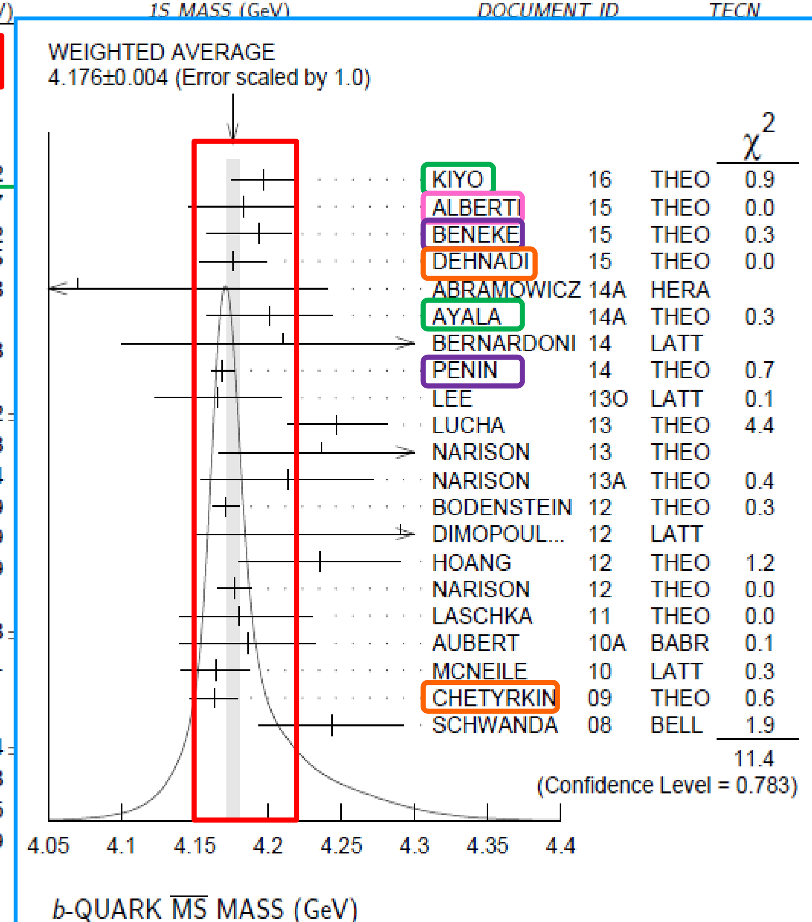
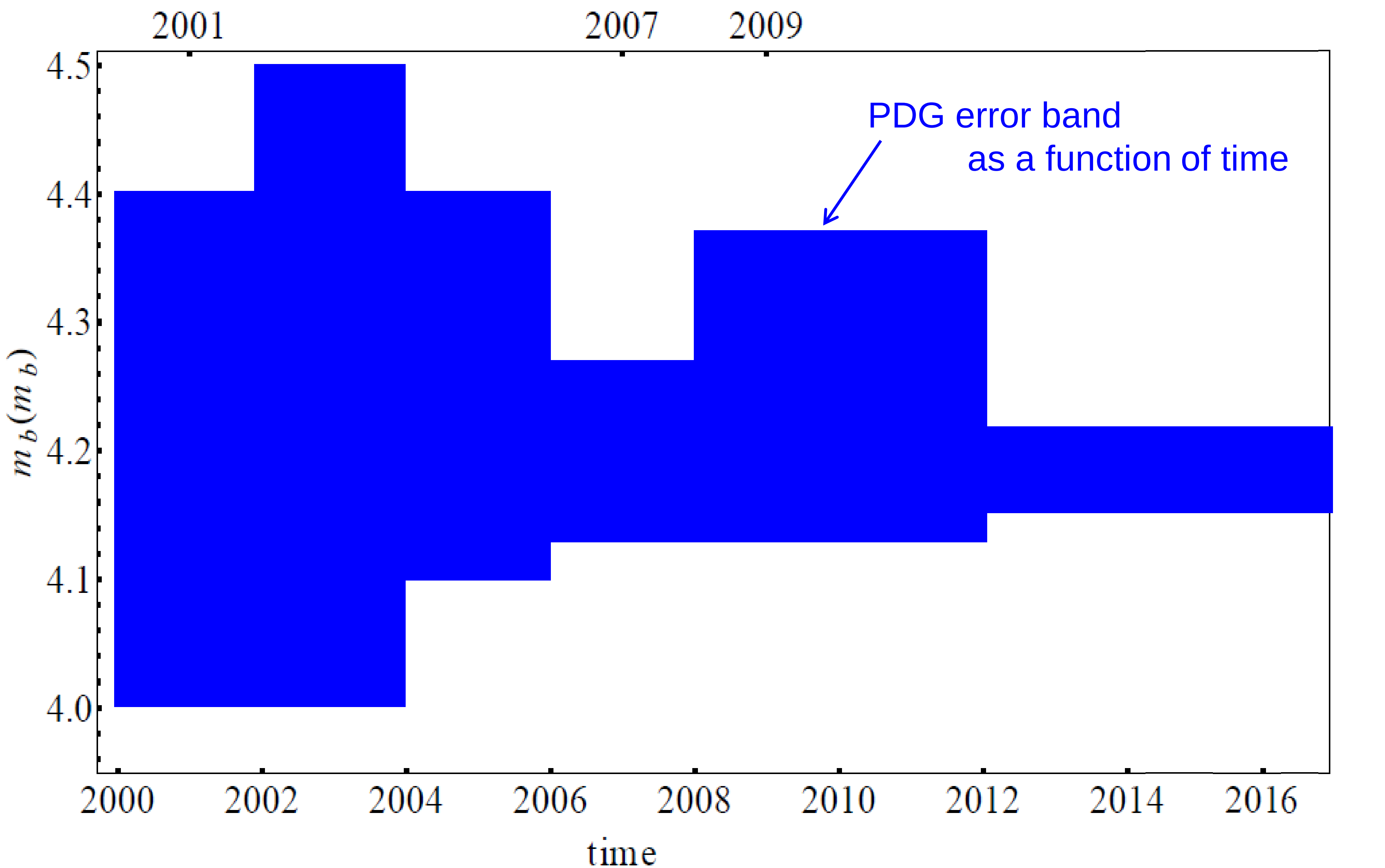
| 4.243±0.049 | | | |

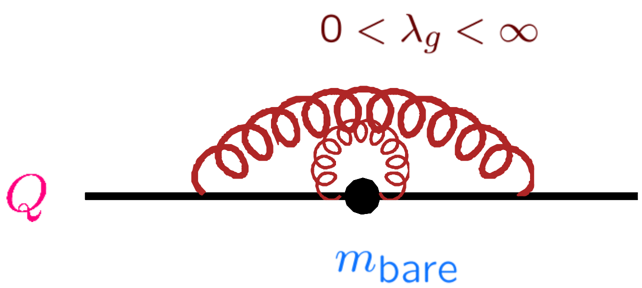




2001
2007
2009
4.5
4.4
4.3
4.2
4.1
4.0
$m_b(m_b)$
PDG error band
as a function of time
2000
2002
2004
2006
2008
2010
2012
2014
2016
time

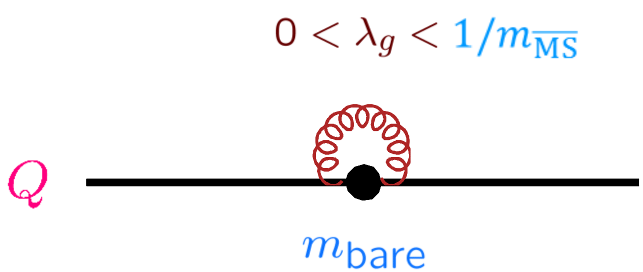
# Representative quark mass definitions in perturbative QCD

Marquard, Smirnov, Smirnov, Steinhauser




Pole mass $m_{\text{pole}}$ = energy of a quark at rest
(= pole position of quark propagator)

$$\frac{1}{p^2 - m_{\text{pole}}^2}$$




$\overline{\text{MS}}$ mass $m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$ = quark mass (a param.)
in $\mathcal{L}_{\text{QCD}}$ renomalized in $\overline{\text{MS}}$ scheme
(only UV div. is subtracted)

---

$$\Sigma_q(m^2) \sim \int d^D k \, \frac{m}{2p.k + k^2} \frac{1}{k^2}\bigg|_{p=(m,\vec{0})}$$

$$\sim \frac{m}{\varepsilon} + \text{finite}$$

# Sum rules

Shifman, Vainshtein, Zakharov

Moments of *R*-ratio

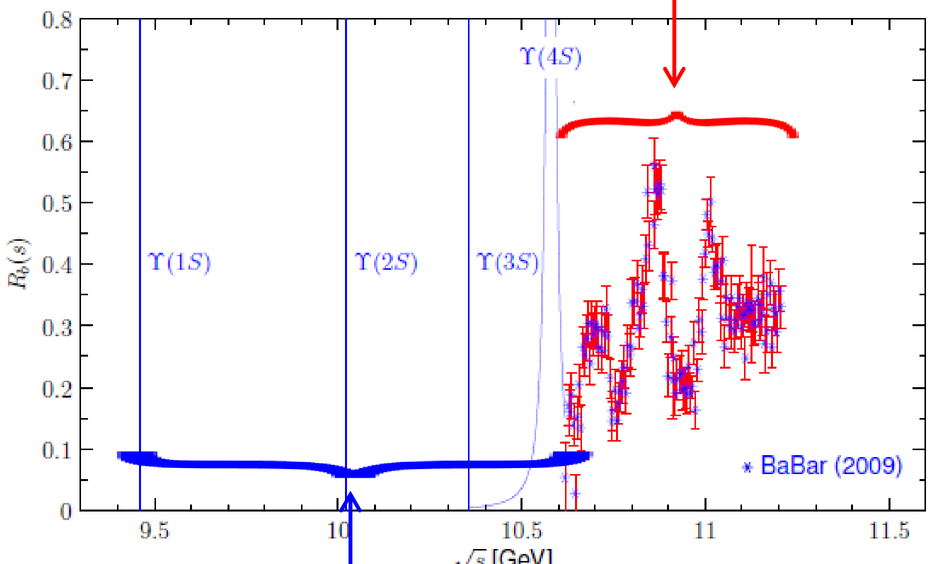
$$\mathcal{M}_n = \int_{s_0}^{\infty} ds \; \frac{R_Q(s)}{s^{n+1}} = \frac{12\pi^2}{n^2}\left(\frac{\partial}{\partial q^2}\right)^n \Pi_Q(q^2)\Big|_{q^2=0}$$

$\boldsymbol{Q} = \boldsymbol{b}$ or $\boldsymbol{c}$


$$\mathcal{M}_n^{\mathrm{exp}} = \mathcal{M}_n^{\mathrm{th}}(\mathrm{NNNLO})$$

$$R_Q(s) = R_{tot}^{\mathrm{exp}}(s) - R_{bkg}^{\mathrm{th}}(s)$$

Sensitive energy regions




$1 \leq n \leq 4$ Relativistic sum rule

relativistic pert. quarks

$n \gg 1$ Non-relativistic sum rule

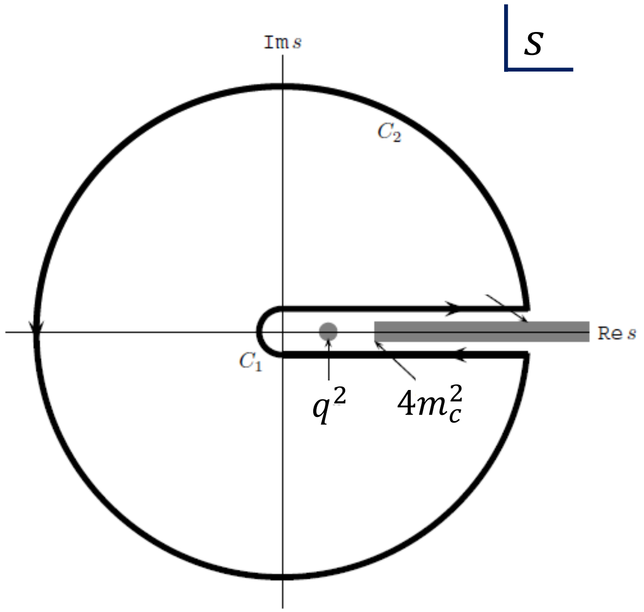
non-rel. bound-state theory

$$R_Q(s) \propto \mathrm{Im}\sum_n \frac{|\psi_n(0)|^2}{\sqrt{s} - M_n + i\Gamma_n/2}$$

Dispersion integral relating $\Pi_c$ and $R_c$:

$$\Pi_c(q^2) = \frac{q^2}{12\pi^2}\int \mathrm{d}s \frac{R_c(s)}{s(s-q^2)} + Q_c^2 \frac{3}{16\pi^2}\bar{C}_0,$$

$$R_c(s) \propto \mathrm{Im}\,\Pi_c(s)$$




Integral path of $\int_C ds \frac{\Pi_c(s)}{s(s-q^2)}$

$$\Pi_c(q^2) = Q_c^2 \frac{3}{16\pi^2}\sum_{n\geqslant 0} \bar{C}_n z^n, \qquad (17)$$

with $Q_c = 2/3$ and $z = q^2/(4m_c^2)$ where $m_c = m_c(\mu)$ is the $\overline{\mathrm{MS}}$ charm quark mass at the scale $\mu$.

# Sum rules

Shifman, Vainshtein, Zakharov

Moments of *R*-ratio

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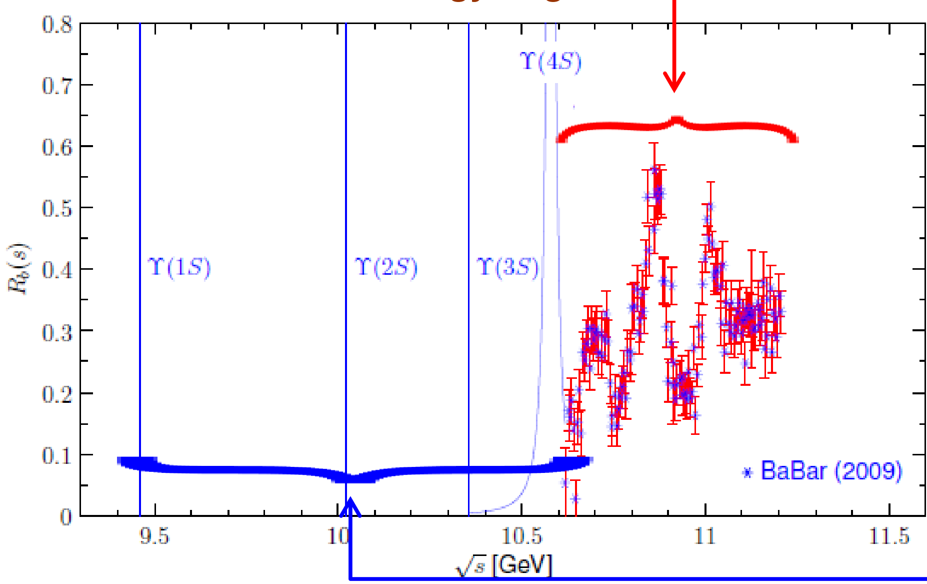
$Q = b$ or $c$


$$\mathcal{M}_n^{\text{exp}} = \mathcal{M}_n^{\text{th}}(\text{NNNLO})$$

$$R_Q(s) = R_{tot}^{\text{exp}}(s) - R_{bkg}^{\text{th}}(s)$$

Sensitive energy regions




$1 \le n \le 4$ Relativistic sum rule

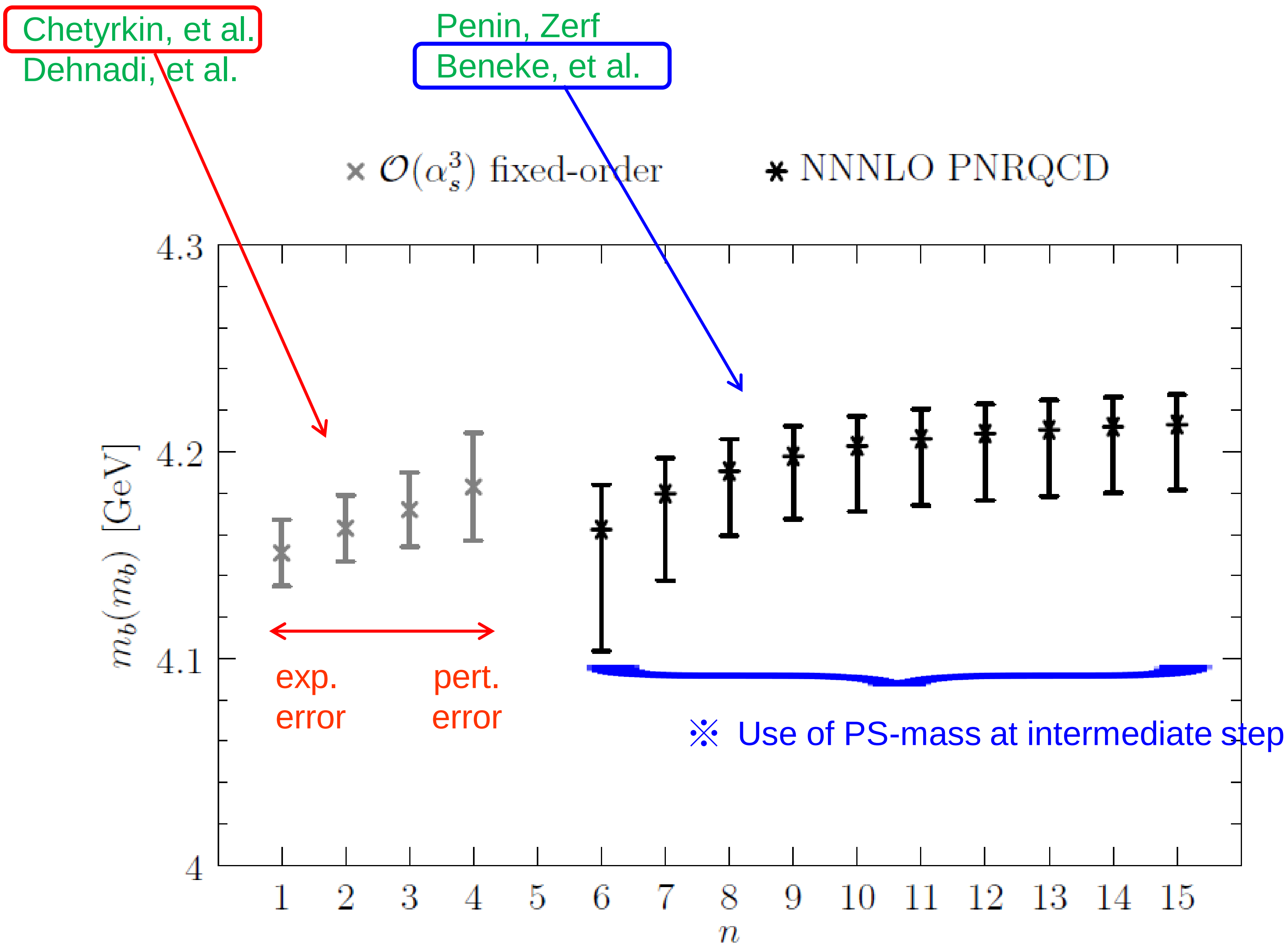
relativistic pert. quarks

$n \gg 1$ Non-relativistic sum rule

non-rel. bound-state theory

$$R_Q(s) \propto \operatorname{Im}\sum_n \frac{|\psi_n(0)|^2}{\sqrt{s} - M_n + i\Gamma_n/2}$$

# Relativistic vs. non-relativistic sum rules



## Heavy quarkonium states ($t\bar{t}, b\bar{b}, c\bar{c}, b\bar{c}$)

Unique system: Properties of individual hadrons predictable in pert. QCD

Two theoretical foundations for computing higher-order corr. systematically

- EFT (pNRQCD, vNRQCD) — Pineda, Soto, Brambilla, Vairo; Luke, Manohar, Rothstein
- Threshold expansion — Beneke, Smirnov

## Computation of full spectrum up to NNNLO

Kiyo, YS: 1408.5590

$$\mathcal{L}_{\rm pNRQCD} = S^\dagger\left(i\partial_t - \hat{H}_S\right)S + O^{a\dagger}\left(iD_t - \hat{H}_O\right)^{ab}O^b + g\, S^\dagger \vec{r}\cdot\vec{E}^a O^a + \cdots$$

$S, O^a$: color singlet and octet composite-state fields

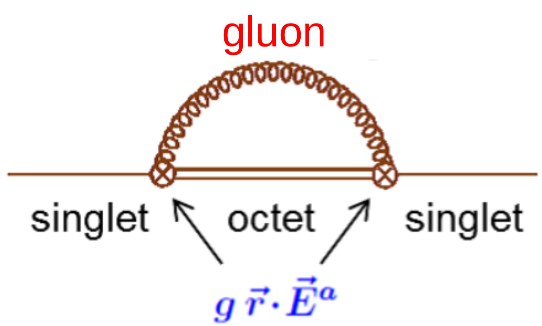
Energy levels given by poles of the full propagator of $S$ in pNRQCD.

singlet

$$\frac{1}{E - \hat{H}_S + i\varepsilon}$$

↓

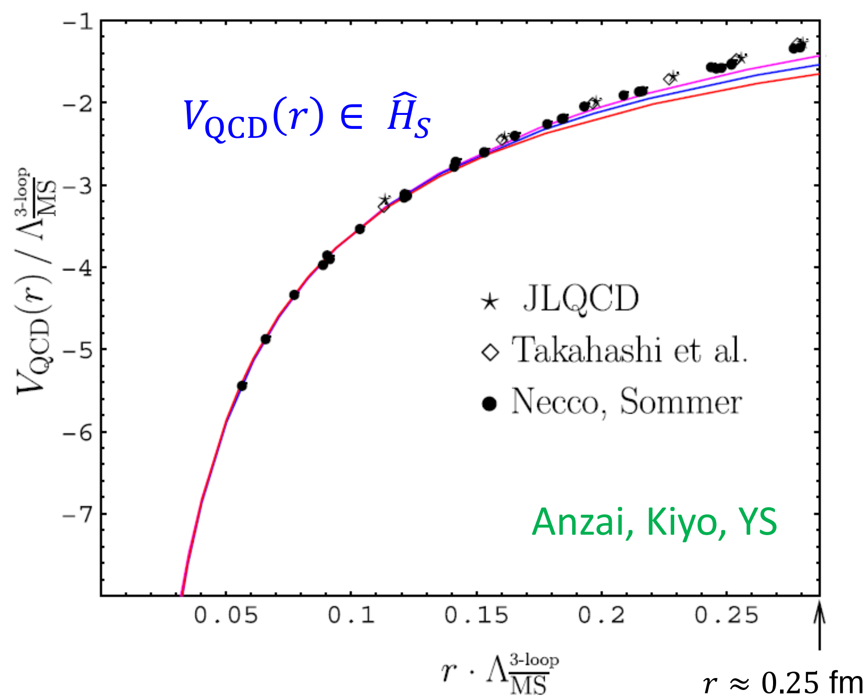
Pert. theory in Quantum Mech.



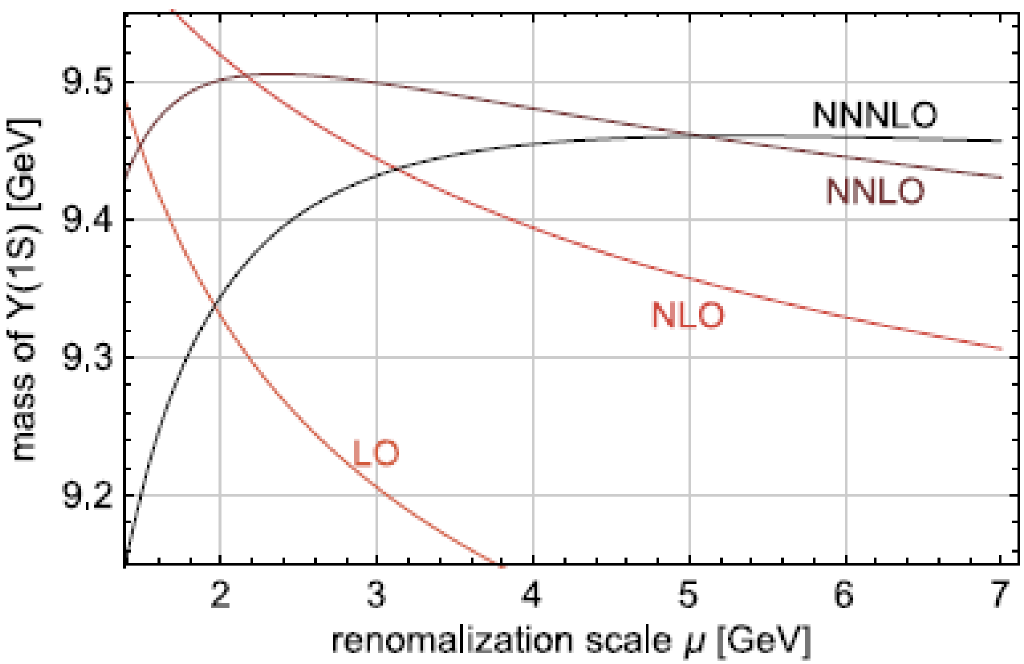

NNNLO

same as Lamb shift

3-loop pert. QCD vs. lattice comp.



# Scale dependence

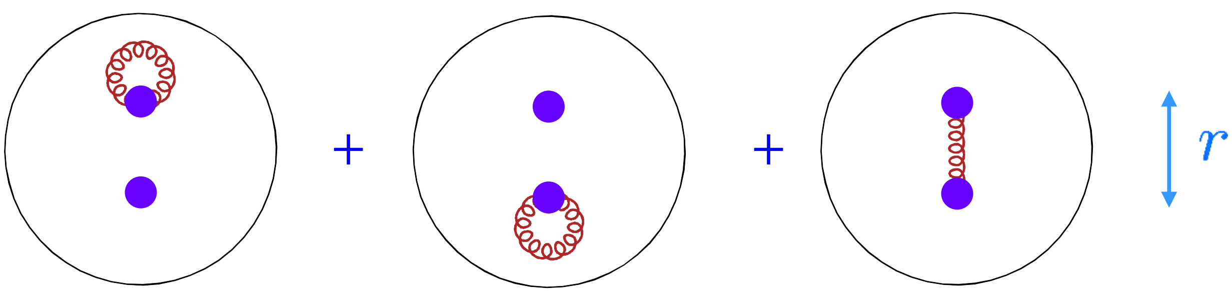



Kiyo, Mishima, YS

Use $\overline{\mathrm{MS}}$ mass $\bar{m}_b \equiv m_b^{\overline{\mathrm{MS}}}(m_b^{\overline{\mathrm{MS}}})$

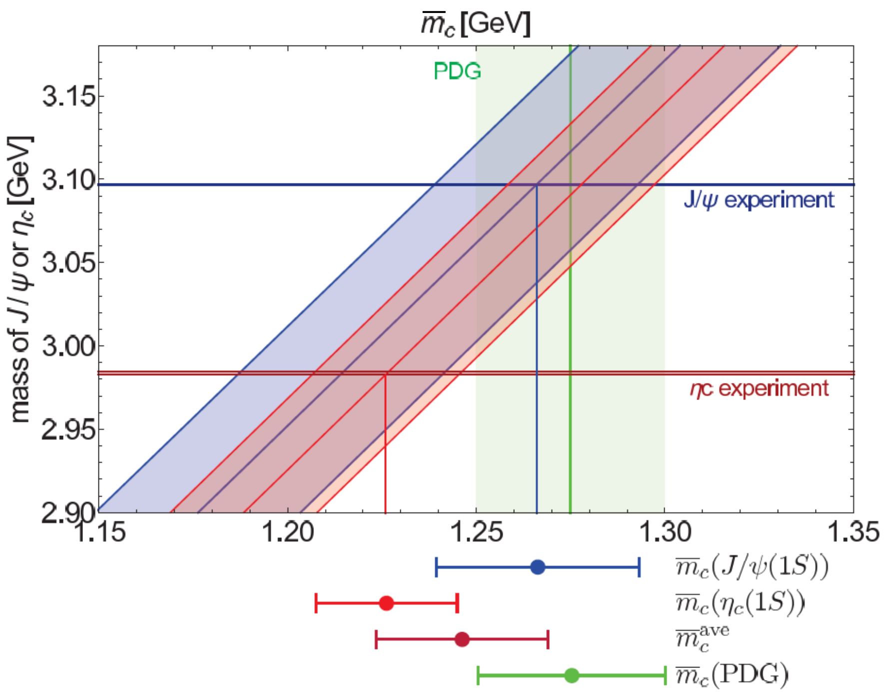
$$M_n = 2\,\bar{m}_b\,(1 + c_{n,1}\alpha_s + c_{n,2}\alpha_s^2 + \cdots)$$

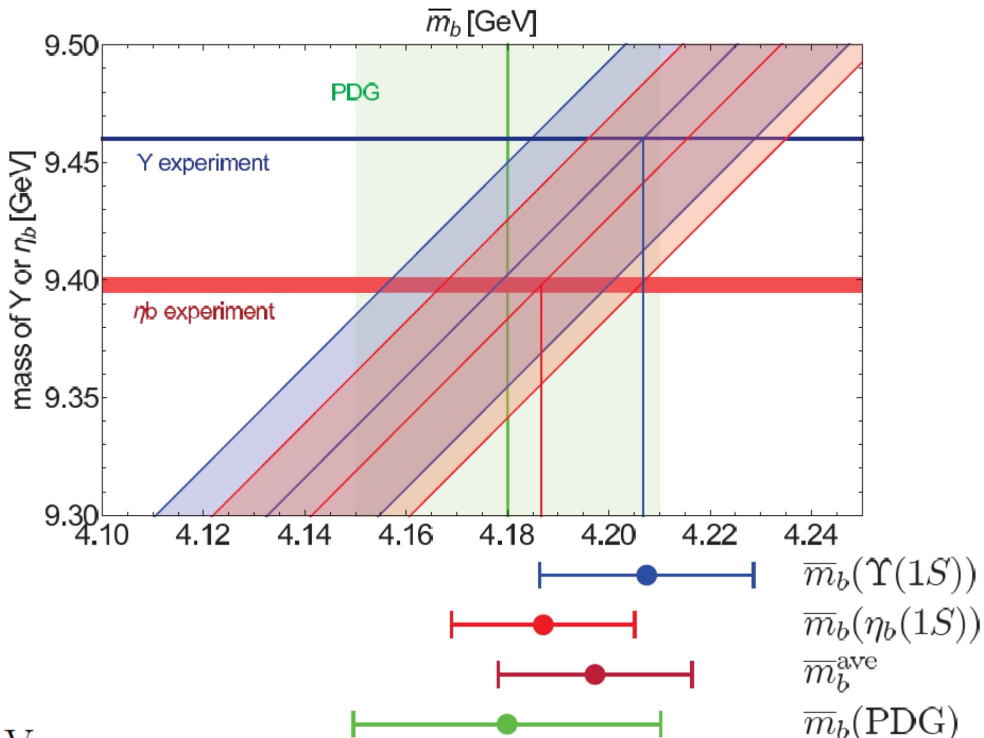
$M_{\Upsilon(1S)} =$



# $\overline{m}_b, \overline{m}_c$ determination

Kiyo, Mishima, YS



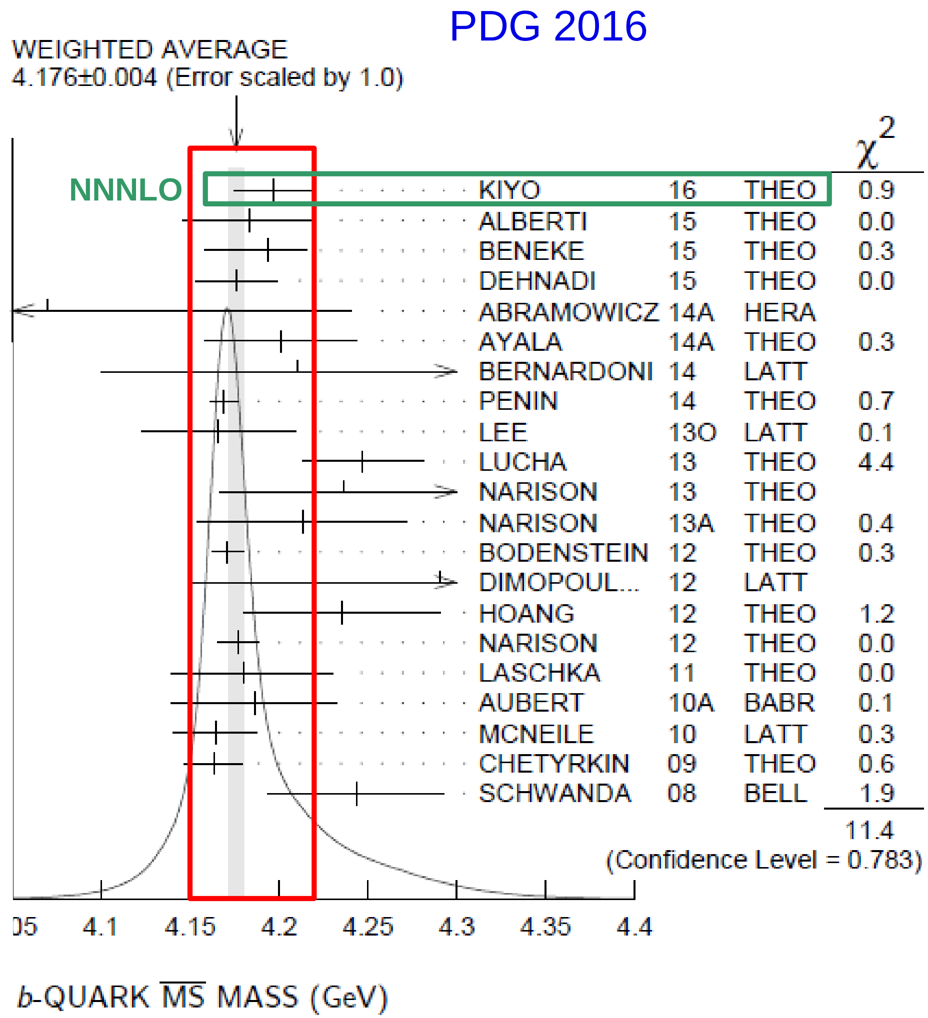



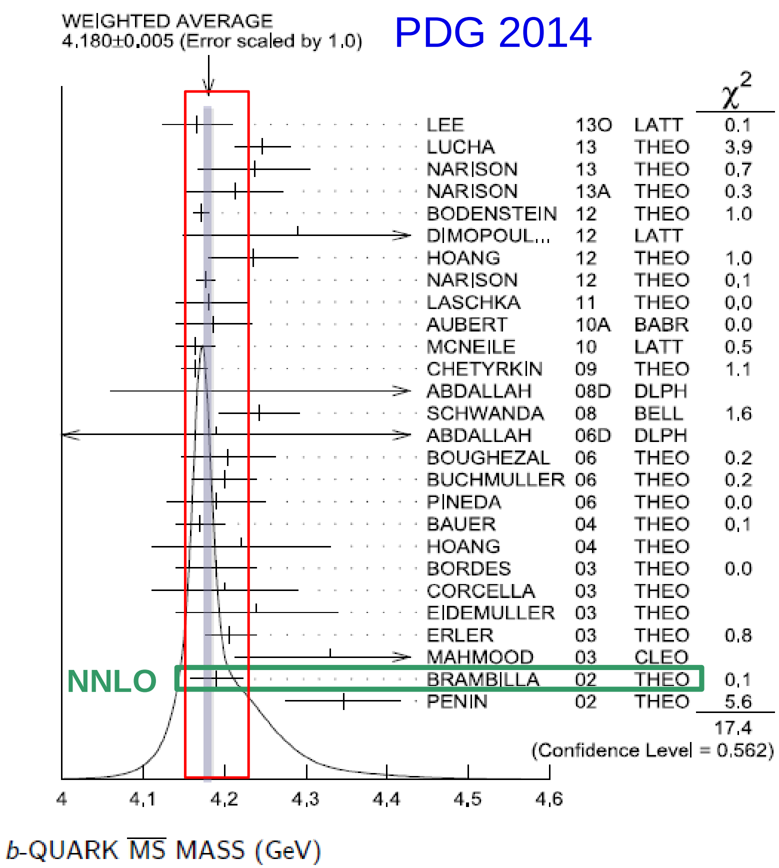

$\overline{m}_c^{\rm ave} = 1246 \pm 2\ (d_3)\ \pm 4\ (\alpha_s)\ \pm 23\ (\rm h.o.)\quad \rm MeV$

PDG value $\overline{m}_c = 1275 \pm 25$ MeV

$\overline{m}_b^{\rm ave} = 4197 \pm 2\ (d_3)\ \pm 6\ (\alpha_s)\ \pm 20\ (\rm h.o.) \pm 5\ (m_c)\quad \rm MeV$

PDG value $\overline{m}_b = 4.18 \pm 0.03$ GeV



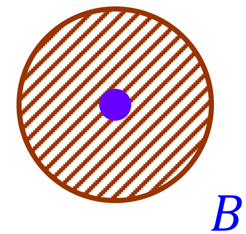

Uncertainty is nearly saturated by renormalon ~ $\Lambda_{\text{QCD}} \cdot (\Lambda_{\text{QCD}}\, r_{1S})^2$

# Simultaneous determination of $|V_{cb}|$ and $m_b^{\overline{\rm MS}}(m_b^{\overline{\rm MS}})$ from inclusive semileptonic $B$ decays

Alberti, et al

Observables in inclusive $B \to X_c \ell \nu$ decays

$$\langle E_\ell^n \rangle = \frac{1}{\Gamma_{E_\ell > E_{\rm cut}}} \int_{E_\ell > E_{\rm cut}} E_\ell^n \frac{d\Gamma}{dE_\ell} dE_\ell ,$$
$$\langle m_X^{2n} \rangle = \frac{1}{\Gamma_{E_\ell > E_{\rm cut}}} \int_{E_\ell > E_{\rm cut}} m_X^{2n} \frac{d\Gamma}{dm_X^2} dm_X^2 ,$$




$m_X$: invariant hadronic mass

OPE in $1/m_b$ expansion

$$\Gamma_{\rm sl} = \Gamma_0 \left[ 1 + a^{(1)} \frac{\alpha_s(m_b)}{\pi} + a^{(2,\beta_0)} \beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 + a^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 \right.$$
$$+ \left(-\frac{1}{2} + p^{(1)} \frac{\alpha_s}{\pi}\right) \frac{\mu_\pi^2}{m_b^2} + \left(g^{(0)} + g^{(1)} \frac{\alpha_s}{\pi}\right) \frac{\mu_G^2(m_b)}{m_b^2}$$
$$\left. + d^{(0)} \frac{\rho_D^3}{m_b^3} - g^{(0)} \frac{\rho_{LS}^3}{m_b^3} + \text{higher orders} \right],$$

$$\mu_\pi^2 = \frac{1}{2M_B} \langle \bar{B} | \bar{b}_v (i\vec{D})^2 b_v | \bar{B} \rangle$$
$$\mu_G^2 = -\frac{1}{2M_B} \langle \bar{B} | \bar{b}_v \frac{g_s}{2} G_{\mu\nu} \sigma^{\mu\nu} b_v | \bar{B} \rangle$$

Observables are mostly sensitive to $\approx m_b - 0.8\, m_c$

⇨ Input $\overline{m}_c(3\ \text{GeV}) = 0.986(13)\ \text{GeV}$

Fit experimental data of the observables and determine $\mu_\pi^2, \rho_D^3, \mu_G^2, \rho_{\rm LS}^3, |V_{cb}|, \overline{m}_b$.

Results:

- $\chi^2$/d.o.f.$\approx 0.4$
- $|V_{\rm cb}| = (42.21 \pm 0.78) \times 10^{-3}$,

  c.f. Determination from exclusive $B \to D^* \ell \nu$ decays

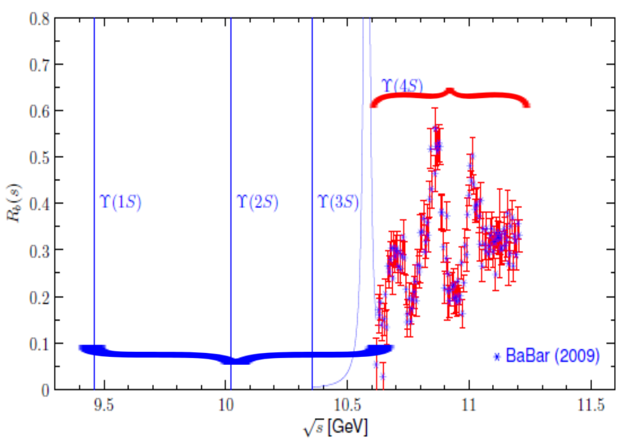
  $$|V_{\rm cb}| = (39.04 \pm 0.49_{\rm exp} \pm 0.53_{\rm lat} \pm 0.19_{\rm QED}) \times 10^{-3}$$

- $\overline{m}_b \equiv m_b^{\overline{\rm MS}}(m_b^{\overline{\rm MS}}) = 4183 \pm 37$ MeV

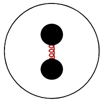
# Summary

- Current theoretical analyses can extract the $\overline{\mathrm{MS}}$ masses $\bar{m}_b$ and $\bar{m}_c$ consistently with ~30 MeV accuracy from different observables.

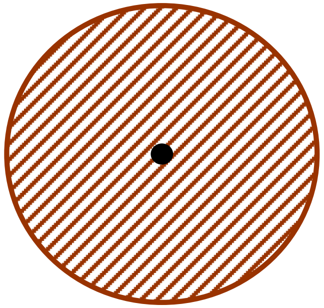
  Relativistic/Non-relativistic sum rules, Quarkonium 1S energy levels, Inclusive observables in semileptonic *B* decays, $\cdots$







$\Upsilon(1S)$, $\eta_b(1S)$



*B*

- Theoretical tools: pert. QCD, EFT, OPE, lattice QCD, $\cdots$

  Higher-order computations, renormalons vs. non-pert. matrix element

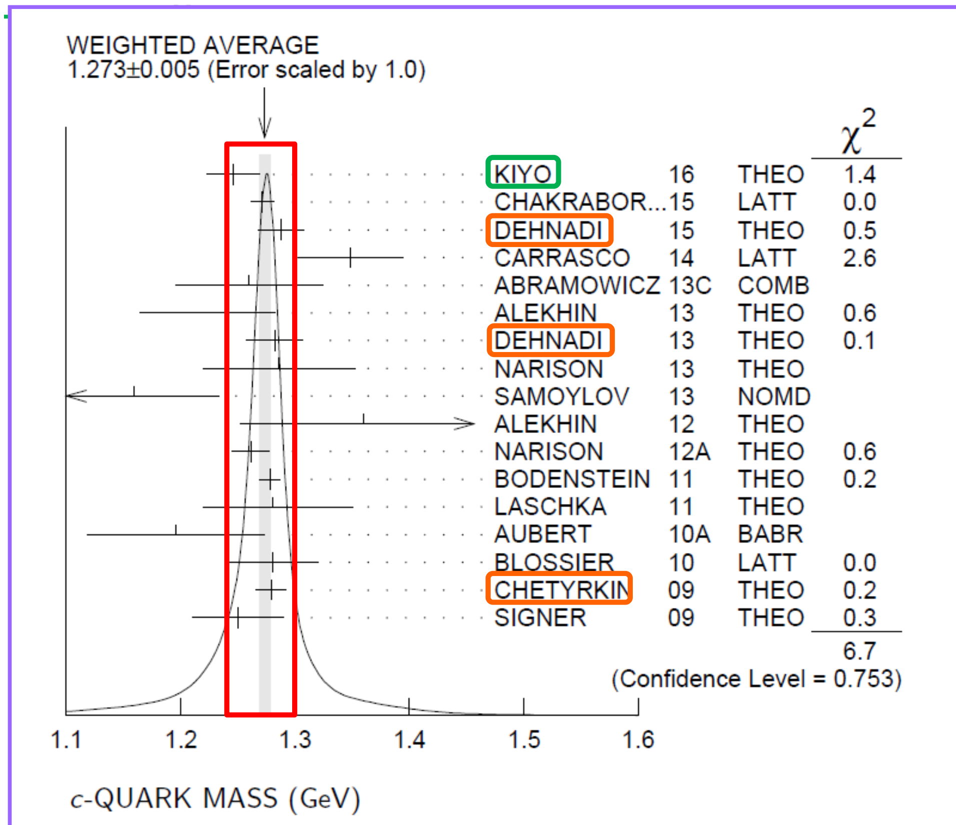
# Summary

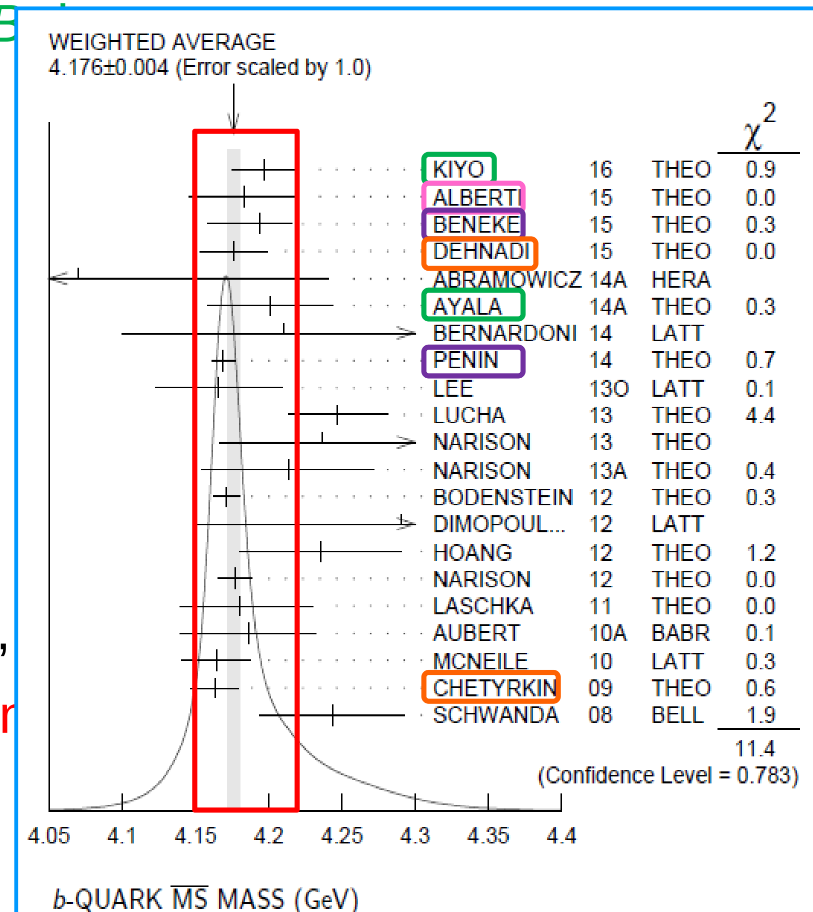
- Current theoretical analyses can extract the $\overline{\mathrm{MS}}$ masses $\bar{m}_b$ and $\bar{m}_c$ consistently with ~30 MeV accuracy from different observables.

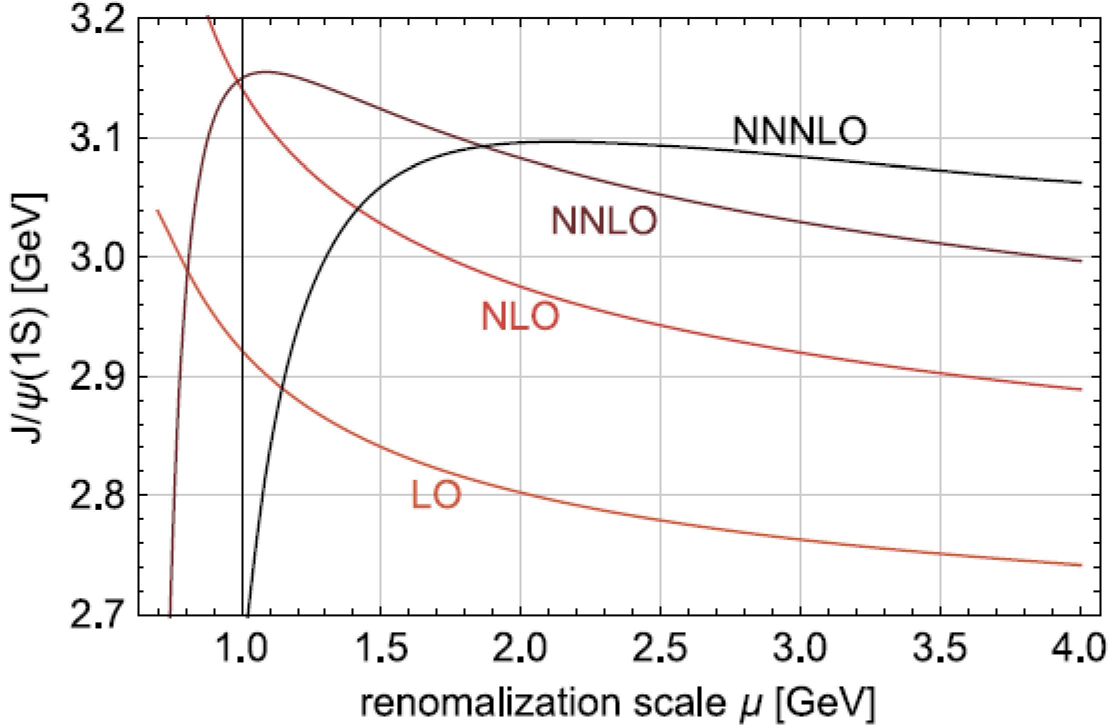
Relativistic/Non-relativistic sum rules, Quarkonium 1S energy levels, Inclusive observables in semileptonic B

OPE,

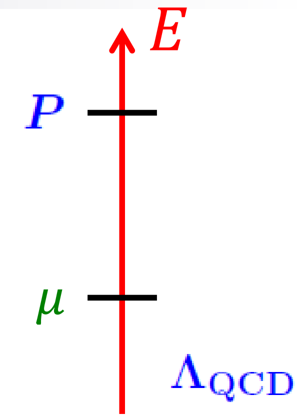
rmalon



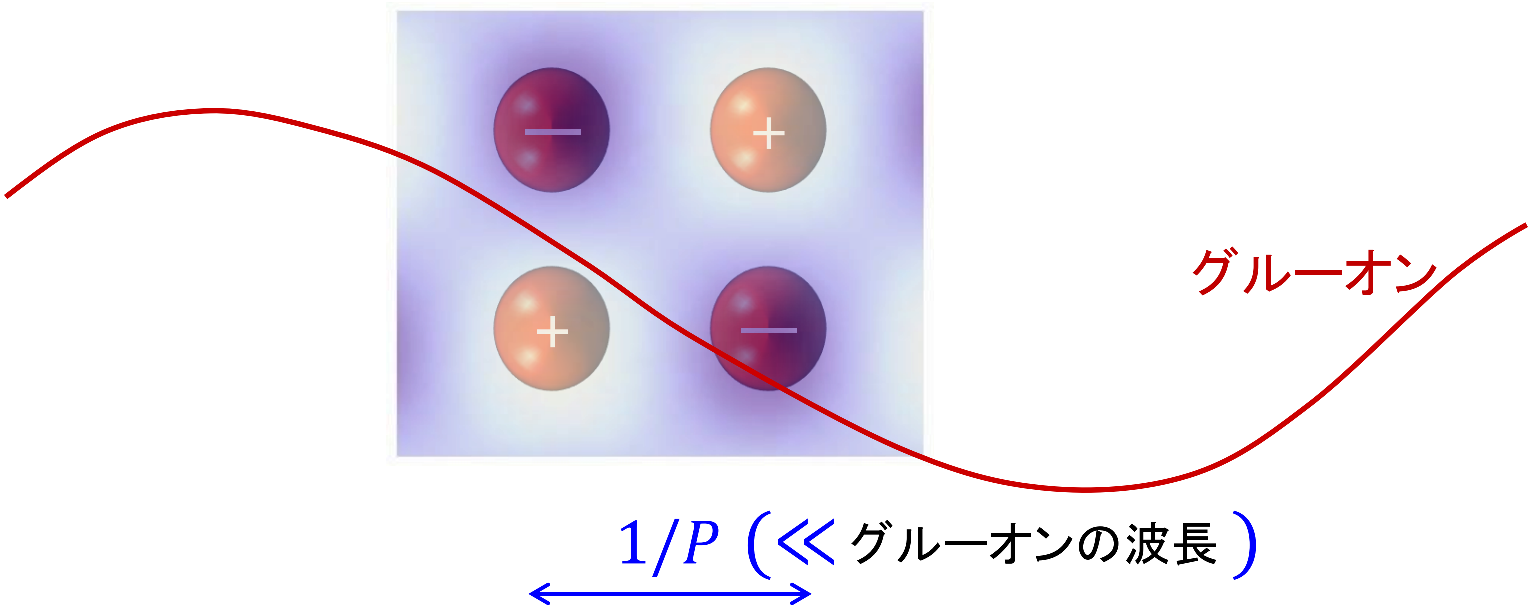

3.2
3.1
3.0
2.9
2.8
2.7
J/ψ(1S) [GeV]
NNNLO
NNLO
NLO
LO
1.0
1.5
2.0
2.5
3.0
3.5
4.0
renomalization scale μ [GeV]

# 有効理論におけるOPE



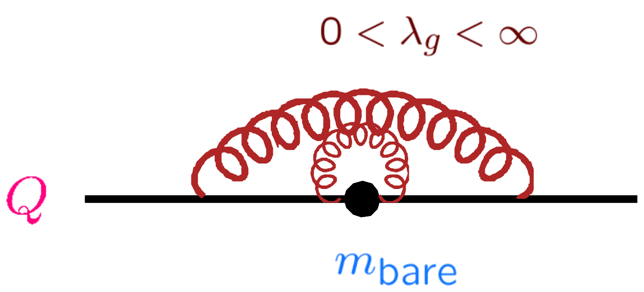

高いエネルギースケール $P \gg \Lambda_{\mathrm{QCD}}$ を含む物理量

$$A(P) = g_1(\mu/P)\,\langle n\,|\,\mathcal{O}(x)\,|\,n\,\rangle + \frac{g_2(\mu/P)}{P^2}\,\langle n\,|\,\partial_\alpha \mathcal{O}(x)\partial^\alpha \mathcal{O}(x)\,|\,n\,\rangle + \frac{g_3(\mu/P)}{P^4}\,\langle n\,|\,\partial_\alpha\partial_\beta \mathcal{O}(x)\partial^\alpha\partial^\beta \mathcal{O}(x)\,|\,n\,\rangle + \cdots$$



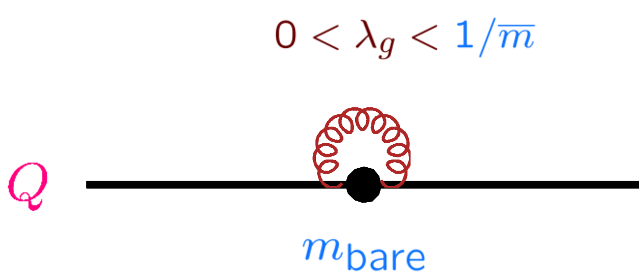
# Representative quark mass definitions in perturbative QCD

Marquard, Smirnov, Smirnov, Steinhauser




Pole mass $m_{\text{pole}}$ = energy of a quark at rest
(= pole position of quark propagator)

$$\frac{1}{p^2 - m_{\text{pole}}^2}$$




$\overline{\text{MS}}$ mass $m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$ = quark mass (a param.)
in $\mathcal{L}_{\text{QCD}}$ renomalized in $\overline{\text{MS}}$ scheme
(only UV div. is subtracted)

---

$$\Sigma_q(m^2) \sim \int d^D k \, \frac{m}{2p.k + k^2} \frac{1}{k^2}\bigg|_{p=(m,\vec{0})}$$

$$\sim \frac{m}{\varepsilon} + \text{finite}$$

# Representative quark mass definitions in perturbative QCD

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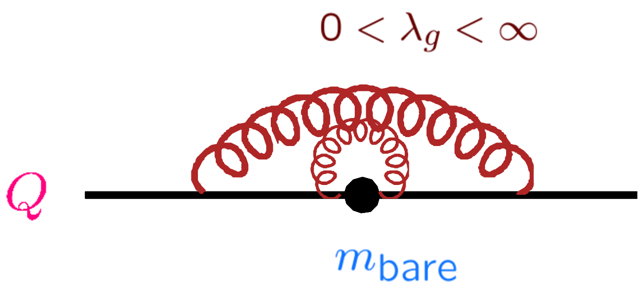


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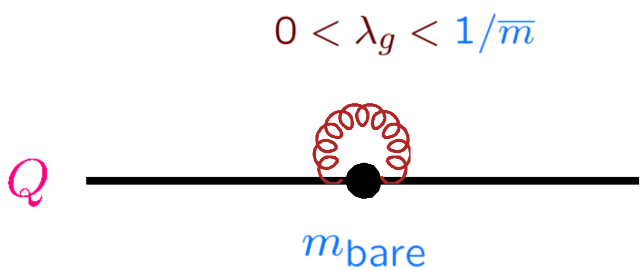


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in $\mathcal{L}_{\text{QCD}}$ renomalized in $\overline{\text{MS}}$ scheme
(only UV div. is subtracted)

---

$$\Sigma_q(m^2) \sim \int d^D k \, \frac{m}{2p.k + k^2} \frac{1}{k^2}\bigg|_{p=(m,\vec{0})} - \int d^D k \frac{\partial}{\partial k_\mu}\left\{k_\mu \frac{m}{(D-4)(k^2-m^2)^2}\right\}$$

$$= \frac{4m^3}{D-4}\int d^D k \frac{1}{(k^2-m^2)^3} + \int d^D k \left\{\frac{m}{2p.k + k^2}\frac{1}{k^2}\bigg|_{p=(m,\vec{0})} - \frac{m}{(k^2-m^2)^2}\right\}$$

$$\sim \frac{m}{\varepsilon} + \text{finite}$$

Only UV structure is required for subtraction
(insensitive to IR structure).

# Sum rules

[Shifman, Vainshtein, Zakharov '78]

Consider moments:

$$\mathcal{M}_n = \int_{s_0}^{\infty} ds \, \frac{R_b(s)}{s^{n+1}} = \frac{12\pi^2}{n!} \left( \frac{\partial}{\partial q^2} \right)^n \Pi_b(q^2) \Big|_{q^2=0} ,$$

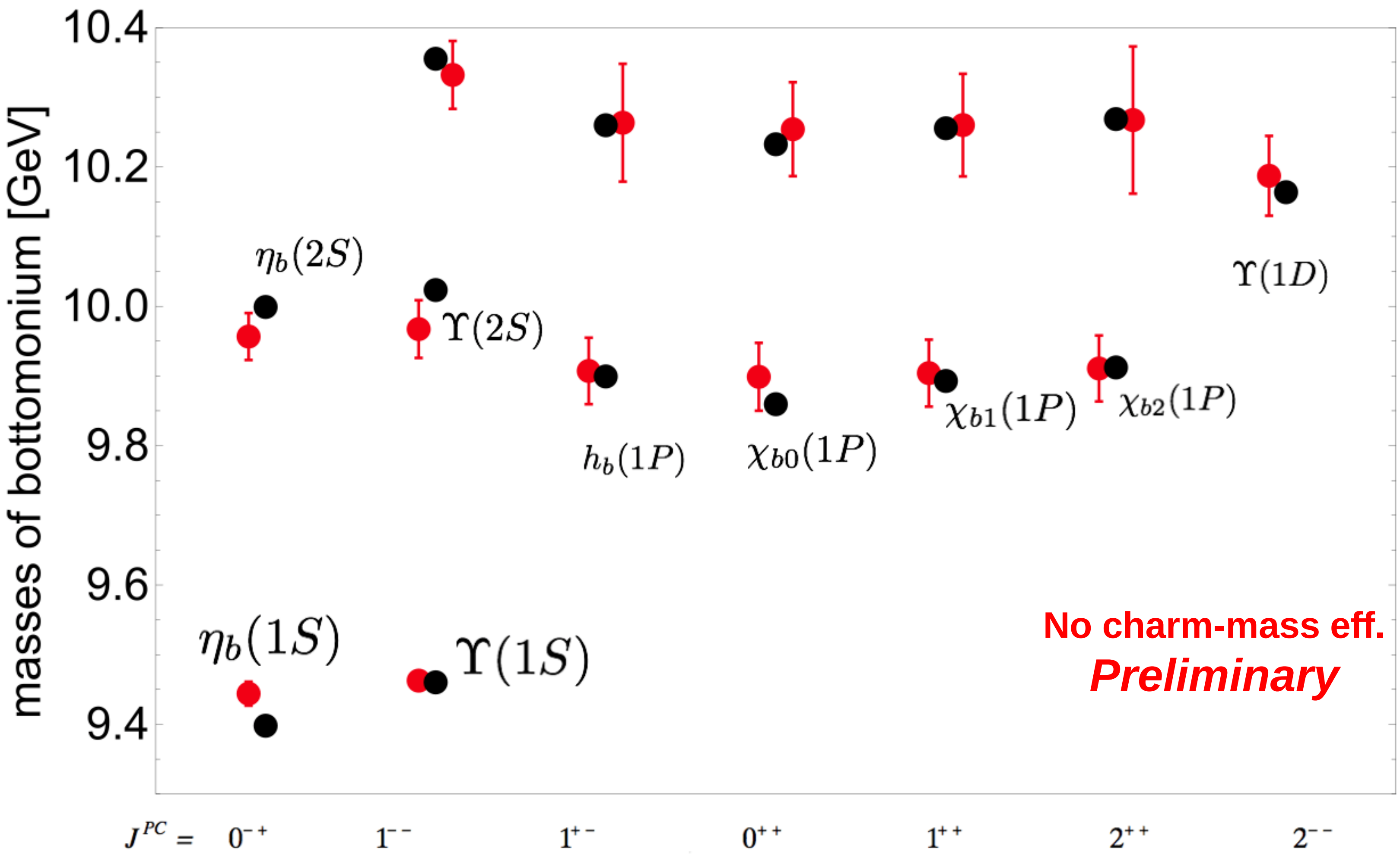
$$\mathcal{M}_n^{\text{exp}} = \mathcal{M}_n^{\text{th}}$$

Larger $n$:

+ Lower sensitivity to unknown high-energy continuum
+ Higher sensitivity to $m_b$
- Larger non-perturbative effects

- Bottomonium spectroscopy

*Based on* Kiyo, YS
(Plot made by G. Mishima)



## Postdictions or predictions?

- Fine and hyperfine splittings of charmonium/bottomonium reproduced.

  Two exceptions around 2003:

  Recksiegel, Y.S.; Kniehl, Penin, Pineda, Smirnov, Steinhauser

  charmonium hyperfine splitting $\Psi(2S)-\eta_c(2S)$

  bottomonium hyperfine splitting $\Upsilon(1S)-\eta_b(1S)$

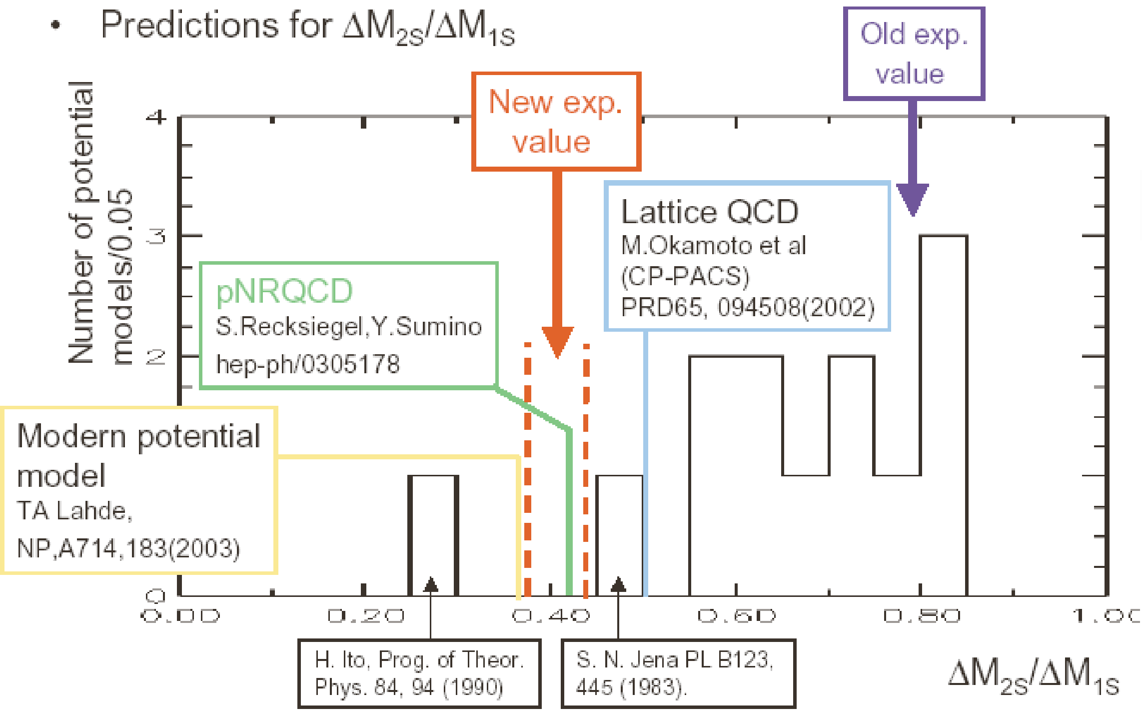
  *Both are solved in favor of pert. QCD predictions.*

Slides from Skwarnicki's plenary talk at Lepton-Photon 2003

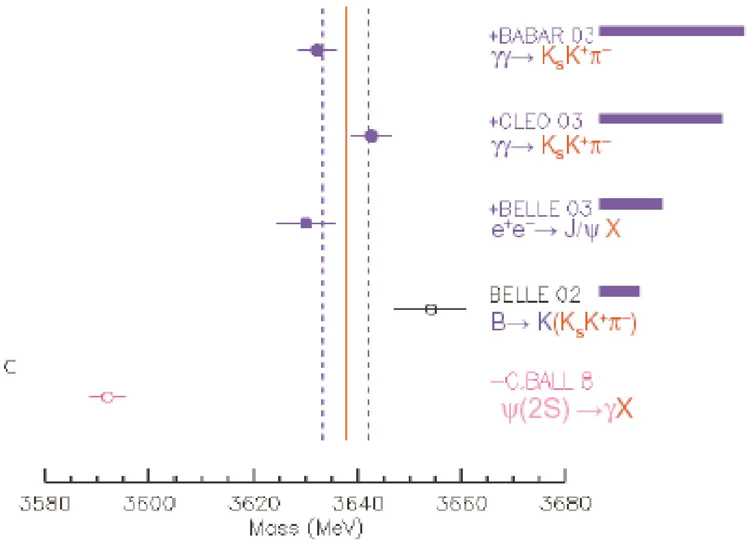


## Predictions for hyperfine splitting ratio

- For 20 years theorists were exposed to the experimental value of $\Delta M_{2S}$=M($\psi$(2S))-M($\eta_c$(2S)) which was wrong by a factor of 2
- Predictions for $\Delta M_{2S}/\Delta M_{1S}$




3637.7±4.4 MeV




*CL=14% scale factor=1.3*

*New measurements of mass are consistent*

$$H_S^{(d)} = \frac{\hat{\vec{p}}^2}{m} + V_S^{(d)}(r), \qquad H_O^{(d)} = \frac{\hat{\vec{p}}^2}{m} + V_O^{(d)}(r),$$

$$V_S^{(d)}(r) = -C_F \frac{\alpha_s}{r} (\bar{\mu} r)^{2\epsilon} A(\epsilon), \qquad V_O^{(d)}(r) = \left(\frac{C_A}{2} - C_F\right) \frac{\alpha_s}{r} (\bar{\mu} r)^{2\epsilon} A(\epsilon),$$

$$A(\epsilon) = \frac{\Gamma(\frac{1}{2} - \epsilon)}{\pi^{\frac{1}{2} - \epsilon}}.$$

Regularization needed to deal with $\delta(\vec{r})$ in commutation relations.

$$V_S^{(d)}(r) \to -C_F \frac{\alpha_s}{r} (\bar{\mu} r)^{2(\epsilon+u)} A(\epsilon),$$

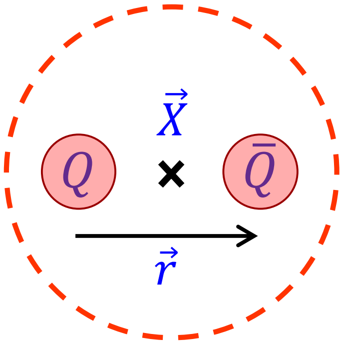
$$V_O^{(d)}(r) \to \left(\frac{C_A}{2} - C_F\right) \frac{\alpha_s}{r} (\bar{\mu} r)^{2(\epsilon+u)} A(\epsilon).$$

## pNRQCD Lagrangian

$Q\bar{Q}$ composite fields

$$S(\vec{X},\vec{r};t)_{\sigma\bar{\sigma}}\,, \qquad O^a(\vec{X},\vec{r};t)_{\sigma\bar{\sigma}}$$

singlet    octet




$$\mathcal{L}_{\mathrm{pNRQCD}} = S^\dagger\left(i\partial_t - \hat{H}_S\right)S + O^{a\dagger}\left(iD_t - \hat{H}_O\right)^{ab}O^b$$
$$+g\, S^\dagger \vec{r}\cdot\vec{E}^a O^a + g\, O^{a\dagger}\vec{r}\cdot\vec{E}^a S + \cdots$$


Color electric field $\vec{E}^a = -\vec{\nabla}A_0^a - \partial_t\vec{A}^a - gf^{abc}A_0^b\vec{A}^c$ at position $\vec{X}$, originating from multipole exp. of $A_\mu\left(\vec{X}\pm\vec{r}/2\right)$ in $\vec{r}$.

$\hat{H}_S, \hat{H}_O$: Quantum mechanical Hamiltonian for singlet and octet states

$$\left(\hat{H}_S\right)_{LO} = \frac{\vec{p}^2}{m} - C_F\frac{\alpha_s}{r}, \qquad \left(\hat{H}_O\right)_{LO} = \frac{\vec{p}^2}{m} + \left(\frac{C_A}{2} - C_F\right)\frac{\alpha_s}{r}$$

$\Rightarrow$ $\beta \sim \frac{p}{m} \sim \frac{1}{mr} \sim \alpha_s$

(in the c.m. frame)

$$\left(\hat{H}_S\right)_{LO} = \frac{\vec{p}^2}{m} - C_F \frac{\alpha_s}{r},$$

$$\left(\hat{H}_S\right)_{NLO} = -C_F \frac{\alpha_s}{r} \cdot \left(\frac{\alpha_s}{4\pi}\right) \cdot \{\beta_0 \log(\mu'^2 r^2) + a_1\},$$

$$\left(\hat{H}_S\right)_{NNLO} = -\frac{\vec{p}^4}{4m^3} - C_F \frac{\alpha_s}{r} \cdot \left(\frac{\alpha_s}{4\pi}\right)^2 \cdot \left\{\beta_0^2 \left[\log^2(\mu'^2 r^2) + \frac{\pi^2}{3}\right] + (\beta_1 + 2\beta_0 a_1) \log(\mu'^2 r^2) + a_2\right\}$$
$$+ \frac{\pi C_F \alpha_s}{m^2} \delta^3(\vec{r}) + \frac{3 C_F \alpha_s}{2m^2 r^3} \vec{L} \cdot \vec{S} - \frac{C_F \alpha_s}{2m^2 r}\left(\vec{p}^2 + \frac{1}{r^2} r_i r_j p_j p_i\right) - \frac{C_A C_F \alpha_s^2}{2mr^2}$$
$$- \frac{C_F \alpha_s}{2m^2}\left\{\frac{\vec{S}^2}{r^3} - 3\frac{\left(\vec{S}\cdot\vec{r}\right)^2}{r^5} - \frac{4\pi}{3}\left(2\vec{S}^2 - 3\right)\delta^3(\vec{r})\right\}$$

$\left(\hat{H}_S\right)_{NNNLO} =$ known (Wilson coeffs. include IR div.)

Kniehl, Penin, Smirnov, Steinhauser

($a_3$: Anzai, Kiyo, YS; Smirnov, Steinhauser)

# Physics Predictions

At NNLO (~2000)

- Global level structure of bottomonium is reproduced. Brambilla, Y.S., Vairo
- Fine and hyperfine splittings of charmonium/bottomonium reproduced.

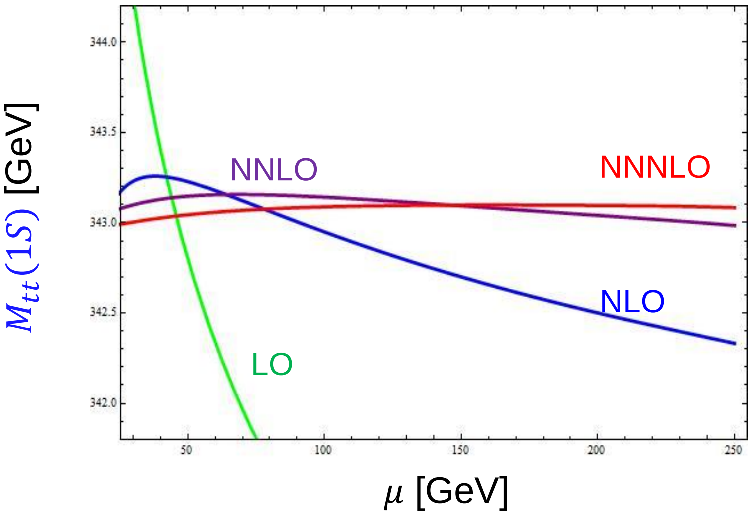
  Two exceptions in ~2003: Recksiegel, Y.S.; Kniehl, Penin, Pineda, Smirnov, Steinhauser

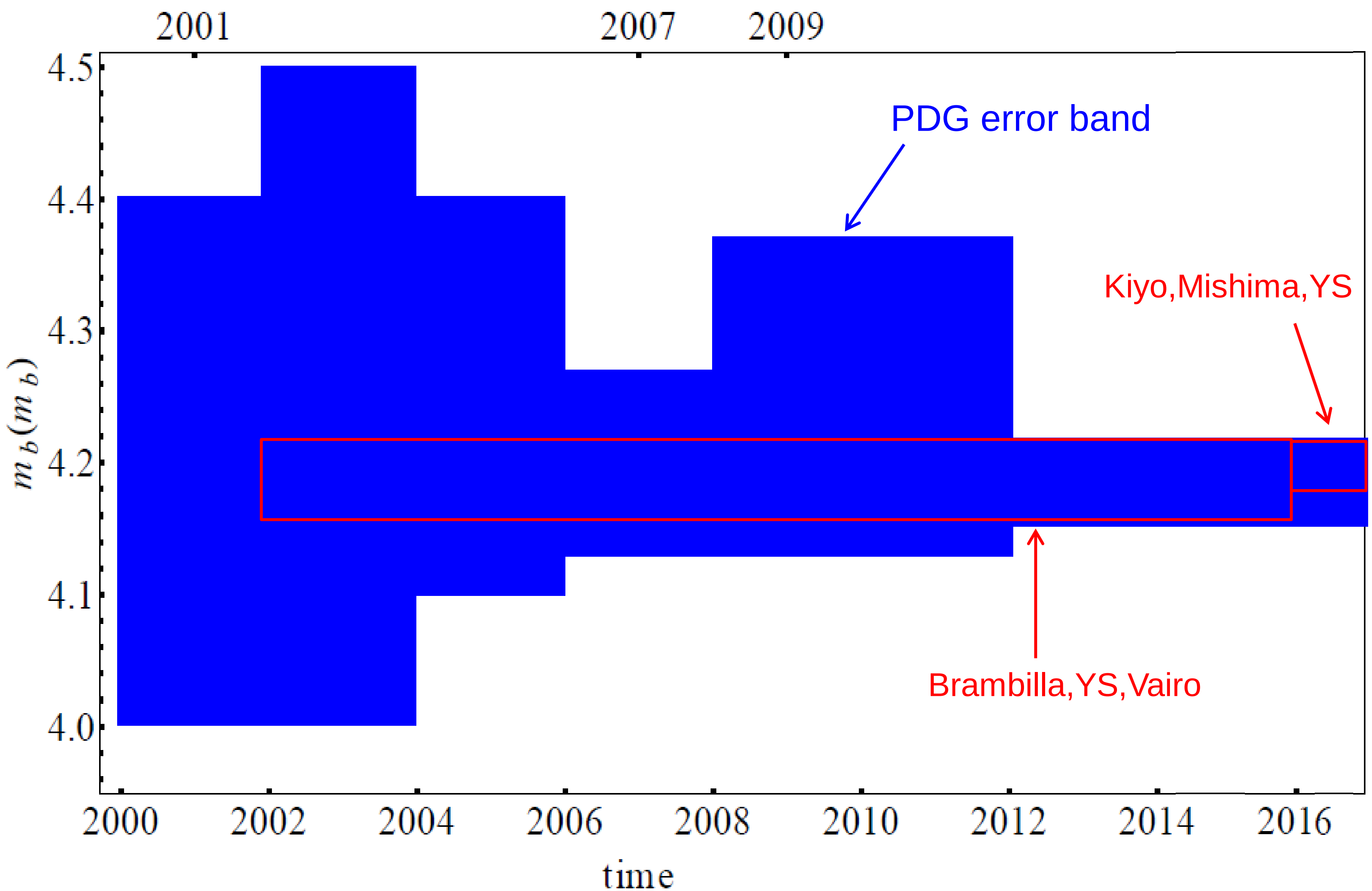
  charmonium hyperfine splitting $\Psi(2S) - \eta_c(2S)$

  bottomonium hyperfine splitting $\Upsilon(1S) - \eta_b(1S)$

  *Solved in favor of pert. QCD predictions.*

At NNNLO




2001
2007
2009
PDG error band
Kiyo,Mishima,YS
Brambilla,YS,Vairo
4.5
4.4
4.3
4.2
4.1
4.0
$m_b(m_b)$
2000
2002
2004
2006
2008
2010
2012
2014
2016
time

# Summary

- Current theoretical analyses can extract the $\overline{\mathrm{MS}}$ masses $\bar{m}_b$ and $\bar{m}_c$ consistently at ~30 MeV accuracy from different observables.

Relativistic/Non-rel. sum rules, Quarkonium 1S energy levels,
Inclusive observables in semileptonic *B* dec

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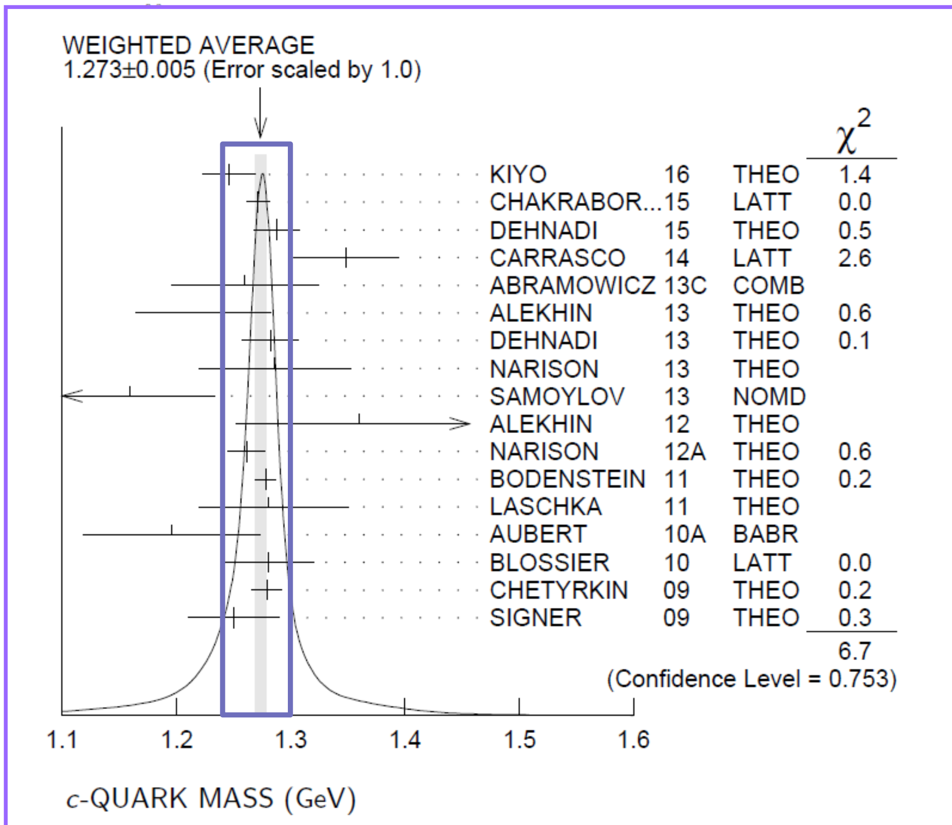


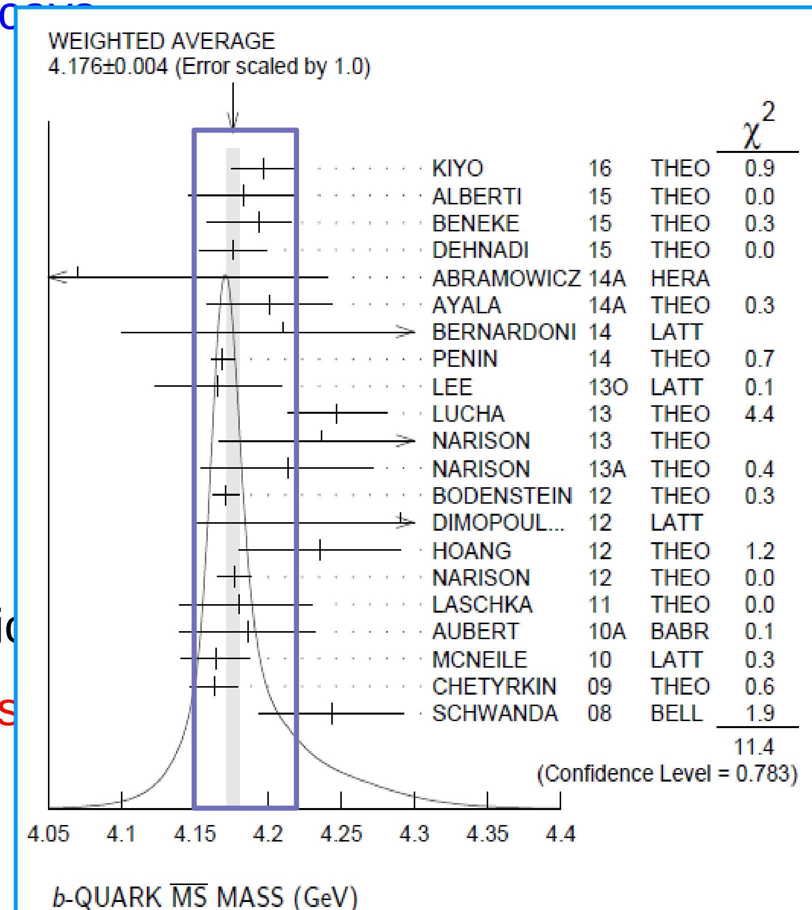

# Predictable observables in pert. QCD

$$\sum_{q,g} |q,g\rangle\langle q,g| = \sum_{hadr.} |hadr.\rangle\langle hadr.| = \mathbf{1}$$

*testable hypothesis* ⇩

(a) Inclusive observables (hadronic inclusive) $\cdots$ insensitive to hadronization

- Inclusive cross sections/decay widths

e.g. $$R(E) \equiv \frac{\sigma(e^+e^- \to hadrons; E)}{\sigma(e^+e^- \to \mu^+\mu^-; E)}$$

- Distributions of non-colored particles, $\ell, \gamma, W, H, \cdots$

(b) Observables of heavy quarkonium states (only individual hadronic states)

- spectrum, leptonic decay width, transition rates

## What’s quark mass?

クォークはハドロン中に confine されているため単独で取り出すことは出来ない。したがって単独のクォークの質量というものを直接測定することもできない筈である。ではそのような現実世界を、クォークとグルーオンの言葉で書かれた QCD 理論はどのように説明し、クォークの質量とはどのように定義され、また決定されているのだろうか。—— 実際のところ、現在のチャームクォークとボトムクォークの質量の決定精度は、Particle Data Group (PDG) によると共に 30–40 MeV 程度である。これは典型的なハドロン化スケール $\Lambda_{QCD} \approx 300$ MeV と比べて 10 倍ほど小さい。それはつまり、様々なハドロンによって異なる「ハドロン化の影響」を剥ぎ取った上で、各クォーク固有の質量が上記精度で consistent に決定できているということを意味する。この講演では、QCD による現在の理解に基づく精密なクォーク質量の定義と描像を解説し、最先端の研究におけるチャームクォークとボトムクォーク質量の幾つかの決定方法について、各々の決定精度の限界を定めている要因なども含めてレヴューする。