On the superconformal index of Argyres-Douglas theories

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Introduction

Argyres-Douglas (AD) theories

4d N=2 SCFTs (superconformal field theories)
without useful Lagrangian description

U(1) gauge fields + massless electrons + massless monopoles

Introduction

Argyres-Douglas (AD) theories

N=2 SUSY gauge theory $\xrightarrow{\text{RG-flow}}$ AD theory

$U(1)_R$ is accidental

The physics of AD theories has been unclear for 20 years!

Introduction

Argyres-Douglas (AD) theories

Only little is known about the AD theories...

known	unknown
• flavor symmetry	• partition function
• conformal anomalies	• spectrum of local operators
• physics on the Coulomb branch (Seiberg-Witten curve)	• correlation functions
	etc.

We would like to understand these theories!!

Introduction

superconformal index (Schur limit)

flavor sym.

$$\mathcal{I}(q;\vec{x}) = \mathrm{Tr}_{\mathcal{H}}\,(-1)^F q^{E-R} \prod_{k=1}^{\mathrm{rank}\,G_F} (x_k)^{A_k}$$

$\mathcal{H}$: Hilbert sp. of local operators $\qquad q,\ x_k \in \mathbb{C}$

E : scaling dim. $\qquad R$: $SU(2)_R$ charge $\qquad A_k$: flavor charge

- This index captures the spectrum of BPS local operators.
- Lagrangian $\longrightarrow$ path-integral (localization)

For AD theories, we cannot use path-integral...

Q: What is $\mathcal{I}(q;\vec{x})$ of AD theories ???

Our answer

- We conjecture exact expressions for the superconformal indices (in the Schur limit) of two infinite series of AD theories.

(A_1, A_{2n-3}) theory
$$\mathcal{I}_{(A_1,A_{2n-3})}(q;x) = \sum_R d_R\, \tilde{f}_R^{(n)}(x)$$

(A_1, D_{2n}) theory
$$\mathcal{I}_{(A_1,D_{2n})}(q;x_1,x_2) = \sum_R \tilde{f}_R^{(n)}(x_1) f_R^{\text{reg}}(x_2)$$

(n = 1, 2, 3, 4, 5, ...)

(R : irreducible representations of su(2))

Outline

1. An example of Argyres-Douglas theory
2. Generalized AD theories
3. Superconformal index
4. Our conjecture

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Argyres-Douglas theories

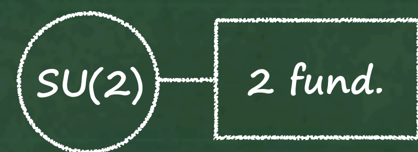
[Argyres-Douglas]
[Argyres-Plesser-Seiberg-Witten]
[Eguchi-Hori-Ito-Yang]

- originally obtained as the IR CFT of an RG-flow.

$$\text{N=2 SUSY gauge theory} \xrightarrow[\text{RG-flow}]{} \text{AD theory}$$

(ex.) 4d N=2 SU(2) gauge theory w/ $N_f = 2$ flavors

$A_\mu,\ \phi$



(1) Turn on quark masses as

$$m_1 = m_2 = m$$

(2) Go to the Coulomb branch

$$u = \langle \mathrm{tr}(\phi^2) \rangle \neq 0$$

Higgs mechanism

U(1) gauge theory w/ $\begin{cases} \text{W-boson} \\ \text{monopole} \end{cases}$

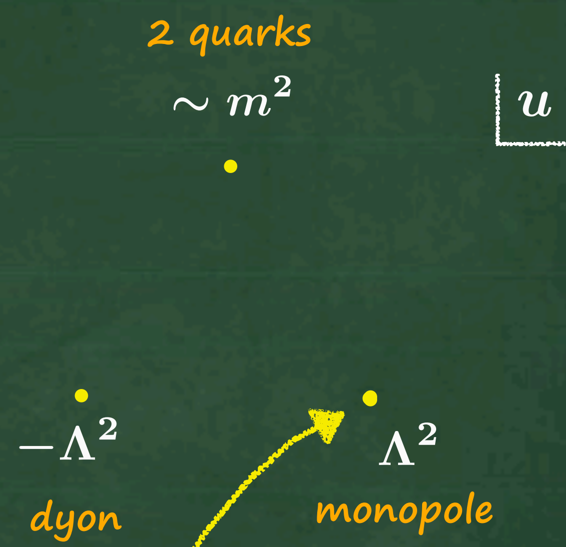
characterized by SW curve

Argyres-Douglas theories

SU(2) — 2 fund.

$u = \langle \mathrm{tr}(\phi^2) \rangle \neq 0$ $\qquad m_1 = m_2 = m$

Λ : dynamical scale



- 3 massless singularities on the Coulomb branch.
- What if we tune m so that $m^2 \sim \Lambda^2$?

2 quarks and monopole become massless!!

- At $u \sim \Lambda^2$, the IR CFT contains massless monopole and quarks.

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ||

$\boxed{\text{Argyres-Douglas theory}}$ called the (A_1, A_3) theory

In summary,

IR SCFT

$\mathrm{SU}(2)$ — $\boxed{\text{2 fund.}}$ $\xrightarrow{\text{RG-flow}}$ $\boxed{(A_1, A_3)}$

non-Lagrangian

$\mathrm{tr}(\phi^2)$

Coulomb branch op.

(canonical dim. = 2)

$\mathcal{O}$ w/ $[\mathcal{O}] = \frac{4}{3}$

fractional scaling dim.

accidental $U(1)_R$ charge = 2 x (scaling dim.)

$\longrightarrow$ $\mathcal{O}$ has $U(1)_R$ charge $\frac{8}{3}$

invisible at UV = accidental symmetry at IR

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Generalizations of the (A_1, A_3) theory are given by

class S theories.

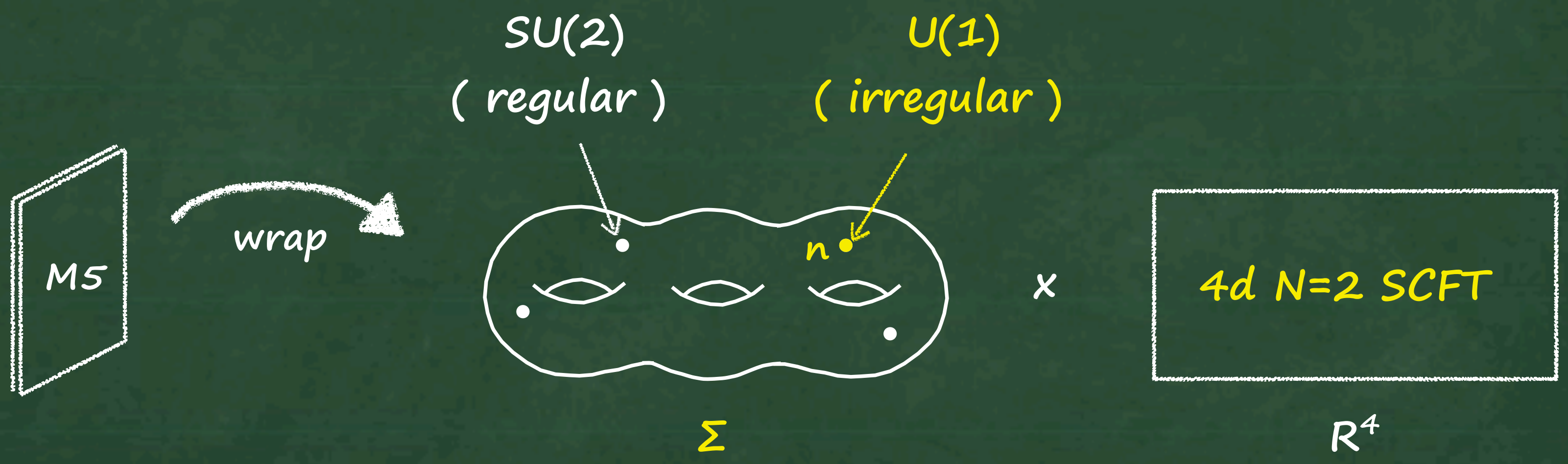
=

4d N=2 theories living on M5-branes wrapping a Riemann surface.

Argyres-Douglas theories

- Class S theories

2 M5-branes compactified on a Riemann surface Σ w/ punctures



regular puncture $\longrightarrow$ SU(2) flavor sym., only 1 type

irregular puncture $\longrightarrow$ U(1) flavor sym., various types labeled by $n = 1,2,3,4,\ldots$

$\parallel$

different B.C. of M5-branes

Argyres-Douglas theories

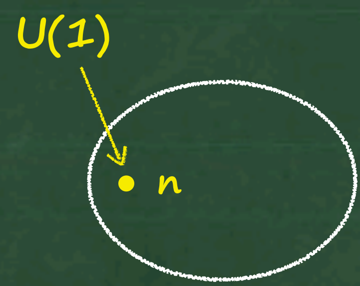
[Xie]

<u>(A_1, A_{2n-3}) theory</u>

genus 0

1 *irregular* puncture

(labeled by n)



×

R^4

(A_1, A_{2n-3}) theory

U(1) flavor sym.

[Xie]

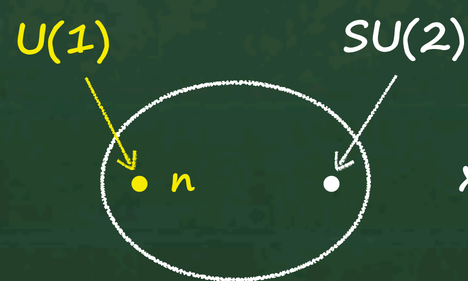
[Bonelli-Maruyoshi-Tanzini]

<u>(A_1, D_{2n}) theory</u>

genus 0

1 *irregular* + 1 regular

(labeled by n)



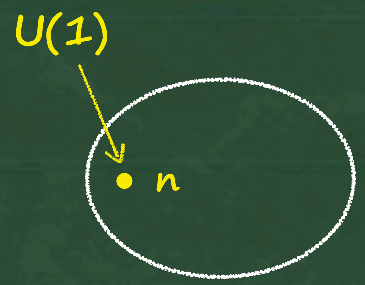
×

R^4

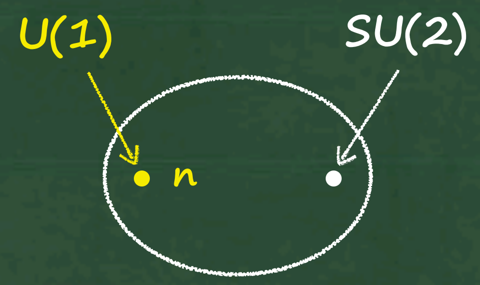
(A_1, D_{2n}) theory

SU(2) × U(1) flavor sym.

Argyres-Douglas theories



(A_1, A_{2n-3}) theory



(A_1, D_{2n}) theory

- Coulomb branch ops. $\mathcal{O}_k$ such that $[\mathcal{O}_k] = 1 + \dfrac{k}{n}$

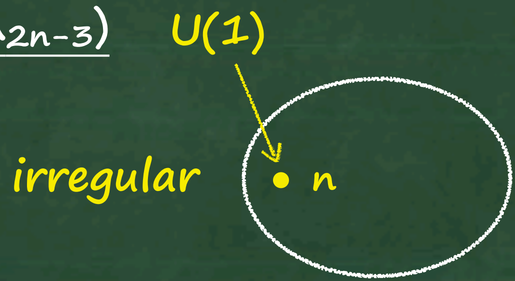
$$\begin{cases} k = 1, 2, 3, \ldots, n-2 \ \text{ for } \ (A_1, A_{2n-3}) \\ k = 1, 2, 3, \ldots, n-1 \ \text{ for } \ (A_1, D_{2n}) \end{cases}$$

(A_1, A_1) = a free hypermultiplet $\qquad$ (A_1, D_2) = 2 free hypermultiplets

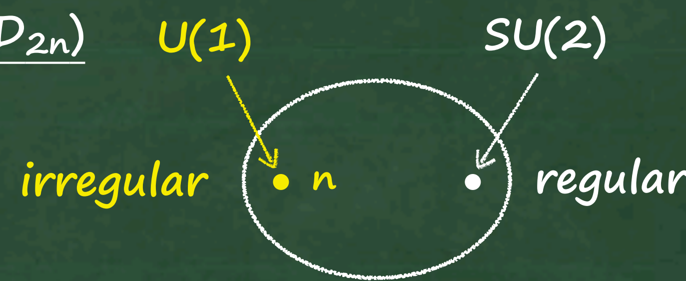
In summary,

- General AD theories are obtained by compactifying M5-branes on S^2 .

(A_1, A_{2n-3})



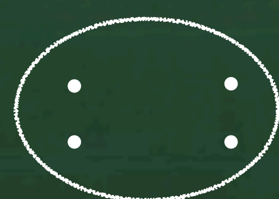
(A_1, D_{2n})



- These are in the same class of theories as $SU(2)$ w/ $N_f = 4$ flavors.

cf.) $SU(2)\ N_f = 4$

4 regulars



flavor $SU(2)^4 \xrightarrow[\text{enhance}]{} SO(8)$

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Superconformal index

superconformal index (Schur limit)

flavor sym.

$$\mathcal{I}(q;\vec{x}) = \mathrm{Tr}_{\mathcal{H}}\,(-1)^F q^{E-R} \prod_{k=1}^{\mathrm{rank}\,G_F} (x_k)^{A_k}$$

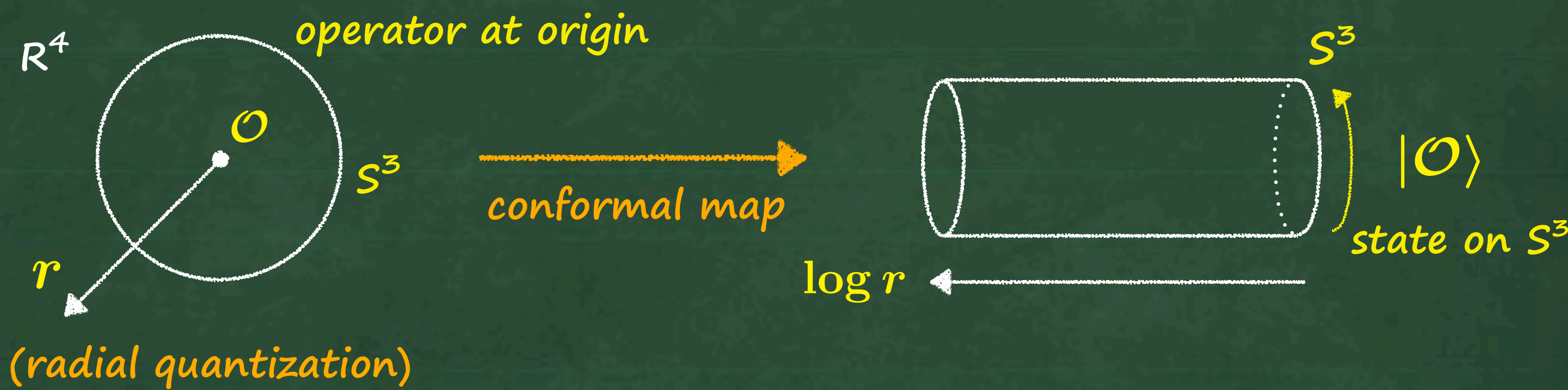
$\mathcal{H}$: Hilbert sp. of local operators $\quad q,\ x_k \in \mathbb{C}$

E : scaling dim. $\quad R$: $SU(2)_R$ charge $\quad A_k$: flavor charge

- E−R and A_k commute with 2 supercharges and their conjugates.
- This index captures the spectrum of BPS local ops.

Superconformal index

- state-operator map



$$\mathcal{I}(q;\vec{x}) = Z_{S^1\times S^3}(\underbrace{\log q; \log x_k}_{\text{background gauge fields}})$$

If there is a Lagrangian description of the theory, the index can be evaluated by path integral on $S^1 \times S^3$.

Superconformal index

A free hypermultiplet: $Q,\ \tilde{Q},\ \psi_Q,\ \psi_{\tilde{Q}}$ SU(2) flavor sym.

$$\mathcal{I}(q;\vec{x}) = \mathrm{Tr}_{\mathcal{H}}\ (-1)^F q^{E-R}\ x^{SU(2)\text{charge}}$$

$$\begin{aligned}
\mathcal{I}(q;x) \ &= \ \overset{1}{1} + q^{\frac{1}{2}}\left(\overset{Q}{x} + \overset{\tilde{Q}}{x^{-1}}\right) + q\left(\overset{Q^2}{x^2} + \overset{Q\tilde{Q}}{1} + \overset{\tilde{Q}^2}{x^{-2}}\right) \\
&\quad + q^{\frac{3}{2}}\left(\chi_4^{su(2)}(x) + \chi_2^{su(2)}(x)\right) \\
&\quad + q^{2}\left(\chi_5^{su(2)}(x) + \chi_3^{su(2)}(x) + \chi_1^{su(2)}(x)\right) + \cdots \\
&= \prod_{k=0}^{\infty} \frac{1}{(1-q^{k+1/2}x)(1-q^{k+1/2}x^{-1})}
\end{aligned}$$

For AD theories, we have no Lagrangian,

and therefore we cannot use path-integral...

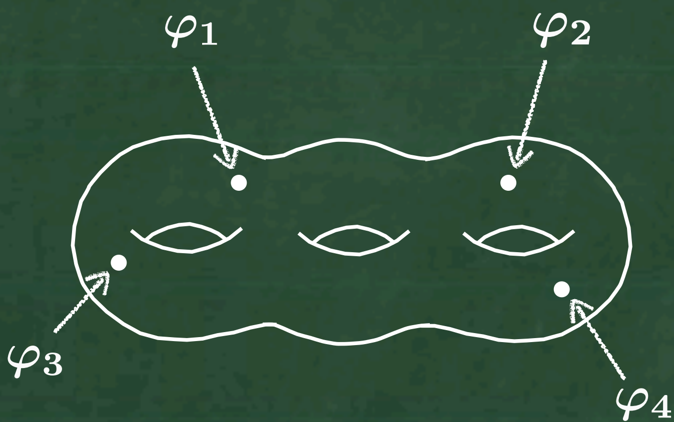
Let's use another expression for the same index.

Superconformal index

- 2 M5-branes on Σ

φ_k : operator insertion at the k-th puncture
(depending on the 4d flavor fugacity x_k)

Σ



m = # of punctures

2d TQFT on Σ
(q-deformed Yang-Mills)

$$\langle \varphi_1 \, \varphi_2 \, \cdots \, \varphi_m \rangle_{q\mathrm{YM}}$$

$$\parallel$$

'11 [Gadde-Rastelli-Razamat-Yan]

x

R^4

$\boxed{\text{4d N=2 SCFT}}$

$$\mathcal{I}(q; \vec{x})$$

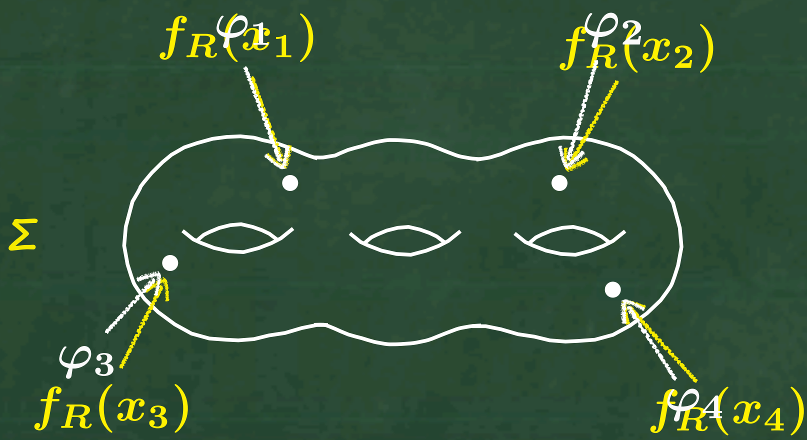
4d Superconformal index

Superconformal index

$$d_R \equiv \frac{[\dim R]_q}{(q^2;q)_\infty} \qquad [k]_q \equiv \frac{q^{k/2}-q^{-k/2}}{q^{1/2}-q^{-1/2}}$$

$$(z;q)_n \equiv \prod_{k=0}^{n-1}(1-q^k z)$$

- 2 M5-branes on Σ



$\longrightarrow$

2d TQFT on Σ

(q-deformed Yang-Mills)

$$\langle \varphi_1\, \varphi_2 \cdots \varphi_m \rangle_{q\mathrm{YM}}$$

$$\parallel$$

$$\sum_{R\,:\,\text{irrep of } su(2)} (d_R)^{2-2g-m} \prod_{k=1}^{m} f_R(x_k)$$

$f_R(x_k)$: wave function for the k-th puncture

regular puncture : $f_R(x) = f_R^{\mathrm{reg}}(x) \equiv \dfrac{\chi_R^{su(2)}(x)}{(q;q)_\infty (qx^2;q)_\infty (qx^{-2};q)_\infty}$ [Gadde-Rastelli-Razamat-Yan]

irregular puncture : $f_R(x) =$??????

The 4d Index from 2d TQFT

$$\mathcal{I}(q;\vec{x}) = \sum_{R\,:\,\text{irrep of } su(2)} (d_R)^{2-2g-m} \prod_{k=1}^{m} f_R(x_k)$$

(2-2g-m = 1)

(A_1, A_{2n-3}) $$\mathcal{I}_{(A_1,A_{2n-3})}(q;x) = \sum_R d_R\, \tilde{f}_R^{(n)}(x)$$

wave function for an irregular puncture $\tilde{f}_R^{(n)}(x)$ $f_R^{\text{reg}}(y)$

(A_1, D_{2n}) $$\mathcal{I}_{(A_1,D_{2n})}(q;x,y) = \sum_R \tilde{f}_R^{(n)}(x) f_R^{\text{reg}}(y)$$

(2-2g-m = 0)

Q: What is $\tilde{f}_R^{(n)}(x)$????

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Our conjecture

$\underline{(A_1, A_{2n-3})}$

$$\mathcal{I}_{(A_1,A_{2n-3})}(q;x) = \sum_R d_R\, \tilde{f}_R^{(n)}(x)$$

$\underline{(A_1, D_{2n})}$

$$\mathcal{I}_{(A_1,D_{2n})}(q;x,y) = \sum_R \tilde{f}_R^{(n)}(x) f_R^{\text{reg}}(y)$$

w/

$$\tilde{f}_R^{(n)}(x) = \frac{q^{nC_2(R)}}{(q;q)_\infty} \text{Tr}_R \left[x^{2J_3} q^{-n(J_3)^2} \right]$$

$C_2(R)$: quadratic Casimir $\qquad$ J_3 : Cartan of su(2)

This is our conjecture! Very simple!!!!

Consistency checks

- (A_1, A_1) = 1 free hypermultiplet

$$\mathcal{I}_{(A_1,A_1)}(q;x) = \sum_R d_R\, \tilde{f}_R^{(2)}(x)$$ our conjecture

$$= \prod_{k=0}^{\infty} \frac{1}{(1-q^{k+1/2}x)(1-q^{k+1/2}x^{-1})}$$ path-integral

- (A_1, D_2) = 2 free hypermultiplets

$$\mathcal{I}_{(A_1,D_2)}(q;x,y) = \sum_R \tilde{f}_R^{(1)}(x)\, f_R(y)$$ our conjecture

$$= \prod_{s_1,s_2=\pm1} \prod_{k=0}^{\infty} \frac{1}{(1-q^{k+1/2}x^{s_1}y^{s_2})}$$ path-integral

The next simplest examples are (A_1, A_3) and (A_1, D_4).

For these theories,

there is another conjecture for the index.

Consistency checks

- 2d chiral algebra '13 [Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees]

  For any 4d N=2 SCFT $\mathcal{T}$ with flavor symmetry G_F, there exists a 2d chiral algebra (VOA) such that

  1. VOA $\supset$ Virasoro algebra, affine G_F algebra

  2. $\mathcal{I}_{\mathcal{T}}(q;\vec{x}) = \mathrm{Tr}_{\mathrm{VOA}}(-1)^F q^{L_0} \prod_{k=1}^{\mathrm{rank}\, G_F} (x_k)^{A_k}$

     (character of VOA)

<u>conjectures</u>

$\mathcal{T}$ = (A_1, A_3) $\longrightarrow$ VOA = $\widehat{su}(2)_{-4/3}$

$\mathcal{T}$ = (A_1, D_4) $\longrightarrow$ VOA = $\widehat{su}(3)_{-3/2}$

(Virasoro is given by Sugawara)

($\mathcal{T}$ = SU(2) w/ 4 flavors $\longrightarrow$ VOA = $\widehat{so}(8)_{-2}$)

($\mathcal{T}$ = Minahan-Nemeshanski E_6 $\longrightarrow$ VOA = $(\widehat{E_6})_{-3}$)

Consistency checks

- (A_1, A_3)

$$\mathcal{I}_{(A_1,A_3)}(q;x) = \sum_R d_R\, \tilde{f}_R^{(3)}(x) \quad \text{our conjecture}$$
$$= 1 + q\chi_3^{su(2)}(x) + q^2\left[1 + \chi_3^{su(2)}(x) + \chi_5^{su(2)}(x)\right] + \cdots$$
$$= \mathrm{Tr}_{\widehat{su}(2)_{-4/3}}\, q^{L_0} x^J \quad \text{chiral algebra}$$

- (A_1, D_4)

$$\mathcal{I}_{(A_1,D_4)}(q;x,y) = \sum_R \tilde{f}_R^{(2)}(x)\, f_R(y) \quad \text{our conjecture}$$
$$= 1 + q\chi_8^{su(3)}(x) + q^2\left[1 + \chi_8^{su(3)}(x) + \chi_{27}^{su(3)}(x)\right] + \cdots$$
$$= \mathrm{Tr}_{\widehat{su}(3)_{-3/2}}\, q^{L_0} (yx^{1/3})^{J_1} (x^{2/3})^{J_2} \quad \text{chiral algebra}$$

Perfectly consistent !!!!!

Consistency checks

- (A_1, A_3)

$$\mathcal{I}_{(A_1,A_3)}(q;x) = \sum_R d_R\, \tilde{f}_R^{(3)}(x) \quad \underline{\text{our conjecture}}$$

$$= 1 + q\chi_3^{su(2)}(x) + q^2\left[1 + \chi_3^{su(2)}(x) + \chi_5^{su(2)}(x)\right] + \cdots$$

$$= \mathrm{Tr}_{\widehat{su}(2)_{-4/3}} q^{L_0} x^J \quad \underline{\text{chiral algebra}}$$

- (A_1, D_4)

$$\mathcal{I}_{(A_1,D_4)}(q;x,y) = \sum_R \tilde{f}_R^{(2)}(x)\, f_R(y) \quad \underline{\text{our conjecture}}$$

$$= 1 + q\chi_8^{su(3)}(x,y) + q^2\left[1 + \chi_8^{su(3)}(x,y) + \chi_{27}^{su(3)}(x,y)\right] + \cdots$$

$$= \mathrm{Tr}_{\widehat{su}(3)_{-3/2}} q^{L_0} (yx^{1/3})^{J_1} (x^{2/3})^{J_2} \quad \underline{\text{chiral algebra}}$$

Perfectly consistent !!!!!

Summary

- We conjectured exact expressions for the superconformal indices of (A_1, A_{2n-3}) and (A_1, D_{2n}) theories (in the Schur limit) in terms of 2d qYM on sphere.

$$\mathcal{I}_{(A_1,A_{2n-3})}(q;x) = \sum_R d_R\, \tilde{f}_R^{(n)}(x)$$

$$\mathcal{I}_{(A_1,D_{2n})}(q;x,y) = \sum_R \tilde{f}_R^{(n)}(x) f_R^{\mathrm{reg}}(y)$$

- Our formula passes a lot of non-trivial consistency checks!

  - reproduces free hypermultiplet indices
  - agrees with the 2d chiral algebra conjecture
  - consistent with S-dualities of AD theories
  - consistent with RG-flows of AD theories
  - consistent with the $q \to 1$ limit

Talk to me if you're interested in these!

- The Macdonald limit of the index was also conjectured in our latest paper.

(1509.05402)

$q \to 1$ limit

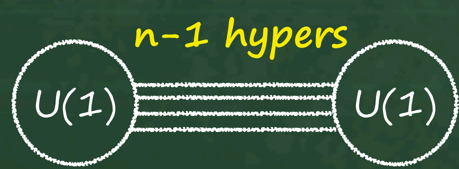
- In the limit $q \to 1$, we checked that our Schur index reduces to the S^3 partition function of the 3d reduction of the 4d theory.

[Dolan-Spiridonov-Vartanov]
[Gadde-Yan]
[Imamura]

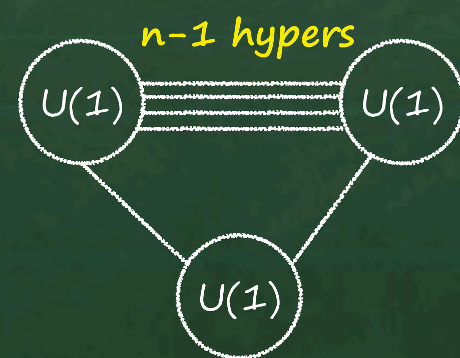
$$\mathcal{I}(q;\vec{x}) \xrightarrow[q \to 1]{} \mathcal{Z}_{S^3}(\vec{x})$$

The 3d mirror is a 3d N=4 gauge theory. [Boalch] [Xie]

(A_1, A_{2n-3})



(A_1, D_{2n})



4d $SU(2)_R \times U(1)_R$ $\longrightarrow$ a subgroup of 3d $SU(2)_R \times SU(2)_L \times$ topological $U(1)$