Predictions of Higgs Mass and Weinberg Angle in 6D Gauge-Higgs Unification

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PLAN

- Introduction
- General argument on Weinberg angle
- One Higgs doublet models
- Two Higgs doublet models
- Summary

Introduction

Higgs boson was discovered and its mass was found to be

125 GeV $\ll M_P$

H^0 MASS

VALUE (GeV)		*DOCUMENT ID*		*TECN*	*COMMENT*
125.09±0.21±0.11	1,2	AAD	15B	LHC	pp, 7, 8 TeV

We must predict this value in new physics beyond the SM

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We must predict this value in new physics beyond the SM

→ Gauge-Higgs Unification

Gauge-Higgs Unification

0 mode of extra component of higher dim. gauge field is identified with the SM Higgs

$A_{\mu,}$ A_i = SM Higgs

(Local) Higgs mass is forbidden by gauge sym.

⇒ Quantum correction to Higgs mass is finite

Checked in various types of models:

1-loop: (D+1)-dim QED on S^1, 5D non-Abelian on S^1/Z_2, 6D non-Abelian on T^2, 6D scalar QED on S^2

2-loop: 5D QED on S^1

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In 5D, No Higgs potential@tree & generated@loops

⇒ Higgs mass is likely to be too small

i.e. $M_H{}^2 \sim M_W{}^2/(16\pi^2)$

In 6D, a Higgs quartic coupling

$g^2[A_5, A_6]^2$ in $Tr(F_{56}F^{56})$

exists at tree level unless $A_5 \propto A_6$

➡ Higgs mass can be made heavy??

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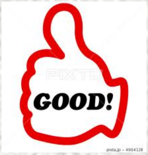
6D SU(3) GHU on T^2/Z_3 (1HD)

$$\lambda_{tree} = \frac{g^2}{2} \quad \text{(like SUSY)}$$

$$M_H^2 = 2\lambda_{tree} v^2, \quad M_W = \frac{1}{2} g v$$

⇒ $M_H = 2M_W$



Scrucca, Serone, Silvestrini, Wulzer (2004)

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Higgs mass can be made heavy??

6D SU(3) GHU on T^2/Z_3 (1HD) $\Rightarrow$ $\sin^2\theta_W = \frac{3}{4}$ ✖

(>> 0.23(exp.))

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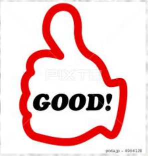
6D G_2 GHU on T^2/Z_4 (1HD) $\Rightarrow$ $\sin^2\theta_W = \frac{1}{4}$



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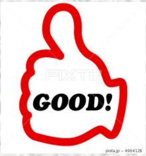

$$\lambda_{tree} = \frac{g^2}{6} \quad \text{(like SUSY)}$$

$$M_H^2 = 2\lambda_{tree} v^2, \quad M_W = \frac{1}{2} gv$$

$M_H = M_Z$ ✖

Csaki, Grojean, Murayama (2003)

In this talk,

we discuss possibilities to predict realistic Higgs mass & Weinberg angle in 6D gauge-Higgs unification models with one or two Higgs doublets

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we discuss possibilities to predict realistic Higgs mass & Weinberg angle in 6D gauge-Higgs unification models with one or two Higgs doublets

1st step toward a realistic model of 6D GHU

General Argument on Weinberg Angle

Weinberg angle & representations under SU(3)

In GHU, the gauge group must be extended to get Higgs doublet from the adjoint rep. ⇒ minimal group = SU(3)

⇒ Knowing of which rep. of SU(3) the Higgs doublet belongs to, Weinberg angle is fixed

Key formula

$$\sin^2\theta_W = \frac{Tr\, I_3^2}{Tr\, Q^2} \left(= \frac{g_1^2}{g_1^2 + g_2^2} \right)$$

Simplest rep. SU(3) triplet

3 = 2 + 1 under SU(2)

$$\begin{pmatrix} q \\ q-1 \\ 1-2q \end{pmatrix}$$

q, $q-1$: SU(2) doublet

$1-2q$ ← TrQ=0

q: electric charge

Weinberg angle under this charge assignment

$$\sin^2\theta_W = \frac{TrI_3^2}{TrQ^2} = \frac{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + 0}{q^2 + (q-1)^2 + (1-2q)^2} = \frac{1}{4\left(3q^2 - 3q + 1\right)}$$

$$\begin{pmatrix} q \\ q-1 \\ 1-2q \end{pmatrix}$$

SU(3) model (Kubo, Lim, Yamashita; Scrucca, Serone, Silvestrini)

Higgs doublet from octet

8 = 3 x 3* = 3 + 2 + 2* + 1

(SU(2) decomp.)

$$\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ H^- & H^{0*} & 0 \end{pmatrix}$$

$Q(H^0) = (q-1) + [-(1-2q)] = 3q-2 = 0 \Rightarrow q = 2/3$

$\therefore \sin^2\theta_W = \frac{3}{4} \gg 0.23$ (exp.)

Note $$\sin^2\theta_W = \frac{1}{4\left(3q^2-3q+1\right)} = \frac{1}{4} \Leftrightarrow q = 0,1$$

q=1 case

$$\begin{pmatrix} q \\ q-1 \\ 1-2q \end{pmatrix} \underbrace{=}_{q=1} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$ $\Rightarrow$ Higgs doublet in SU(3) triplet

Question: SU(3) triplet from adjoint rep.??

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known G_2: 14 = 8 + 3 + 3* SU(3) decomp.

(Manton; Csaki, Grojean, Murayama)

q=0 case

$$\begin{pmatrix} q \\ q-1 \\ 1-2q \end{pmatrix}$$

Note: (q-1) + (1-2q) = −q = 0!!

⇒ Higgs from the 2nd rank sym. tensor (6 rep.) of SU(3)

Question: SU(3) 2nd rank sym. tensor from adjoint rep.??

q=0 case

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Note: (q-1) + (1-2q) = -q = 0!!

⇒ Higgs from the 2nd rank sym. tensor (6 rep.) of SU(3)

Question: SU(3) 2nd rank sym. tensor from adjoint rep.??

Sp(6): 21 = 8 + 6 + 6* + 1

SU(3) decomp.

New!!

3,6,8 reps. = All the 2nd rank tensor of SU(3)

	SU(3) rep.	$\sin^2\theta_W$
SU(3)	8	$\frac{3}{4}$
G_2	3(or 3*)	$\frac{1}{4}$
Sp(6)	6(or 6*)	$\frac{1}{4}$

3,6,8 reps. = All the 2nd rank tensor of SU(3)

	SU(3) rep.	$\sin^2\theta_W$	M_H
SU(3)	8	$\frac{3}{4}$	$2M_W$
G_2	3(or 3*)	$\frac{1}{4}$	M_Z
Sp(6)	6(or 6*)	$\frac{1}{4}$	??

In GHU, Higgs mass can be predicted

↑ Next our interest

One Higgs doublet models

Sp(6) model on T^2/Z_6

$21 = 8 + 6 + 6^* + 1$, Higgs in $6(6^*) \Rightarrow \sin^2\theta_W = \frac{1}{4}$

Z_6 orbifold

$z \to \omega z$ ($\omega^6 = 1$、$z = (x^5 - i\, x^6)/\sqrt{2}$)

$$A_\mu\left(x^\mu, \omega z\right) = P A_\mu\left(x^\mu, z\right) P^\dagger$$

$$A_z\left(x^\mu, \omega z\right) = \omega P A_z\left(x^\mu, z\right) P^\dagger \qquad A_z \equiv \frac{A_5 + iA_6}{\sqrt{2}}$$

$$A_{\bar{z}}\left(x^\mu, \omega z\right) = \bar{\omega} P A_{\bar{z}}\left(x^\mu, z\right) P^\dagger \qquad A_{\bar{z}} \equiv \left(A_z\right)^\dagger$$

$$P = diag\left(\omega, \omega, \omega^4, \bar{\omega}, \bar{\omega}, \bar{\omega}^4\right)$$

In components

$$A_\mu = \begin{pmatrix} 1 & 1 & \bar{\omega}^3 & \omega^2 & \omega^2 & \bar{\omega} \\ 1 & 1 & \bar{\omega}^3 & \omega^2 & \omega^2 & \bar{\omega} \\ \omega^3 & \omega^3 & 1 & \bar{\omega} & \bar{\omega} & \omega^2 \\ \bar{\omega}^2 & \bar{\omega}^2 & \omega & 1 & 1 & \omega^3 \\ \bar{\omega}^2 & \bar{\omega}^2 & \omega & 1 & 1 & \omega^3 \\ \omega & \omega & \bar{\omega}^2 & \bar{\omega}^3 & \bar{\omega}^3 & 1 \end{pmatrix}, \quad A_z = \begin{pmatrix} \omega & \omega & \bar{\omega}^2 & \omega^3 & \omega^3 & 1 \\ \omega & \omega & \bar{\omega}^2 & \omega^3 & \omega^3 & 1 \\ \bar{\omega}^2 & \bar{\omega}^2 & \omega & 1 & 1 & \omega^3 \\ \bar{\omega} & \bar{\omega} & \omega^2 & \omega & \omega & \bar{\omega}^2 \\ \bar{\omega} & \bar{\omega} & \omega^2 & \omega & \omega & \bar{\omega}^2 \\ \omega^2 & \omega^2 & \bar{\omega} & \bar{\omega}^2 & \bar{\omega}^2 & \omega \end{pmatrix}$$

$Sp(6) \rightarrow SU(2) \times U(1) \times U(1)$

One Higgs doublet

Zero modes

$$A_\mu = \begin{pmatrix} a_\mu & 0 \\ 0 & -a_\mu^* \end{pmatrix} + A_\mu^9 T^9, \quad A_z = \begin{pmatrix} 0 & a_z \\ 0 & 0 \end{pmatrix}$$

$$a_\mu = \begin{pmatrix} \frac{\sqrt{6}}{6} Z_\mu & \frac{1}{2} W_\mu^+ & 0 \\ \frac{1}{2} W_\mu^- & -\frac{\sqrt{2}}{4}\gamma_\mu - \frac{\sqrt{6}}{12} Z_\mu & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4}\gamma_\mu - \frac{\sqrt{6}}{12} Z_\mu \end{pmatrix}, \quad a_z = \frac{1}{2}\begin{pmatrix} 0 & 0 & \phi^+ \\ 0 & 0 & \phi^0 \\ \phi^+ & \phi^0 & 0 \end{pmatrix}$$

Mass ratios of W & Higgs

$$\mathcal{L} = -\frac{1}{2}Tr\left(F^{MN}F_{MN}\right) = -\frac{1}{2}Tr\left(F^{\mu\nu}F_{\mu\nu}\right) + 2Tr\left(F_z^{\mu}F_{\mu\bar{z}}\right) + Tr\left(F_{z\bar{z}}\right)^2$$

$$2Tr\left(F_z^{\mu}F_{\mu\bar{z}}\right) \to -2g^2Tr\left(\left[A^{\mu},A_z\right]\left[A_{\mu},A_{\bar{z}}\right]\right) = g^2\left|\phi^0\right|^2\left(\frac{1}{4}W^{+\mu}W_{\mu}^{-} + \frac{1}{6}Z^{\mu}Z_{\mu}\right)$$

$$Tr\left(F_{z\bar{z}}\right)^2 \to -g^2Tr\left(\left[A_z,A_{\bar{z}}\right]^2\right) = -\frac{g^2}{4}\left|\phi^0\right|^4$$

$$\underbrace{\Rightarrow}_{\langle\phi^0\rangle = v/\sqrt{2}} \quad M_W^2 = \frac{g^2}{8}v^2,\ M_Z^2 = \frac{g^2}{6}v^2,\ M_H^2 = \frac{g^2}{2}v^2 \Rightarrow M_H = 2M_W$$

Same prediction as SU(3) model on T^2/Z_3

SU(4) model on T^2/Z_6

$15 = 8 + 3 + 3^* + 1 \Rightarrow$ Higgs in $3(3^*)$ as G_2 case

Parity matrix: $P = \mathrm{diag}(1, 1, \omega^3, \omega)$ $(\omega^6 = 1)$

$$A_\mu = \begin{pmatrix} \frac{1}{2}\gamma_\mu - \frac{\sqrt{3}}{6}Z_\mu & \frac{1}{\sqrt{2}}W_\mu^+ & 0 & 0 \\ \frac{1}{\sqrt{2}}W_\mu^- & \frac{\sqrt{3}}{3}Z_\mu & 0 & 0 \\ 0 & 0 & -\frac{1}{2}\gamma_\mu - \frac{\sqrt{3}}{6}Z_\mu & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \text{Extra U(1)}$$

$$A_z = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & \phi^+ \\ 0 & 0 & 0 & \phi^0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

SU(4)→SU(2)xU(1)xU(1)

One Higgs doublet

$$2Tr\left(F_z^{\mu} F_{\mu\bar{z}}\right) \rightarrow 2g^2\left|\phi^0\right|^2\left(\frac{1}{4}W^{+\mu}W_{\mu}^{-} + \frac{1}{6}Z^{\mu}Z_{\mu}\right)$$

$$Tr\left(F_{z\bar{z}}\right)^2 \rightarrow -\frac{g^2}{2}\left|\phi^0\right|^4$$

$$\underbrace{\Rightarrow}_{\langle\phi^0\rangle = v/\sqrt{2}} \quad M_W^2 = \frac{g^2}{4}v^2,\ M_Z^2 = \frac{g^2}{3}v^2,\ M_H^2 = \frac{g^2}{2}v^2$$

$$\Rightarrow M_H = 2M_W$$

Same prediction as SU(3) & Sp(6) models

Two Higgs doublet models

SU(3) model with two Higgs doublets on T^2/Z_2

Assumption: $\left|\vec{l}_1\right| = \left|\vec{l}_2\right| = 2\pi R,\ \vec{l}_1 \perp \vec{l}_2$

($\vec{l}_{1,2}$: lattice vectors along 2 cycles of the torus)

Orbifold:
$$A_\mu(-x^5,-x^6) = P A_\mu(x^5,x^6) P^{-1}$$
$$A_{5,6}(-x^5,-x^6) = - P A_{5,6}(x^5,x^6) P^{-1}$$

P=diag(1,1,−1)

$$\Rightarrow \quad A_{5,6}^{(0,0)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & \phi_{1,2}^{+} \\ 0 & 0 & \phi_{1,2}^{0} \\ \phi_{1,2}^{-} & \phi_{1,2}^{0*} & 0 \end{pmatrix}, \quad H_{1,2} = \begin{pmatrix} \phi_{1,2}^{+} \\ \phi_{1,2}^{0} \end{pmatrix}$$

General analysis of Higgs potential & Higgs mass

General form of the Higgs effective potential up to quartic

$$V\left(H_1,H_2\right)=-\lambda Tr\left(\left[A_5^{(0,0)},A_6^{(0,0)}\right]^2\right)$$

$$+aTr\left[\left(A_5^{(0,0)}\right)^2+\left(A_6^{(0,0)}\right)^2\right]+ibTr\left(Y\left[A_5^{(0,0)},A_6^{(0,0)}\right]\right)$$

$$=\frac{\lambda}{2}\left[\left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right)+\left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right)-\left(H_2^\dagger H_1\right)^2-\left(H_1^\dagger H_2\right)^2\right]$$

$$+a\left(H_1^\dagger H_1+H_2^\dagger H_2\right)-\frac{i}{2}b\left(H_1^\dagger H_2-H_2^\dagger H_1\right)$$

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$$= \frac{\lambda}{2}\left[\left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + \left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right) - \left(H_2^\dagger H_1\right)^2 - \left(H_1^\dagger H_2\right)^2\right]$$

$$+ a\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) - \frac{i}{2} b\left(H_1^\dagger H_2 - H_2^\dagger H_1\right)$$

Quartic coupling (1st term)

from $Tr\,(F_{56})^2$ at classical level

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$$+a\left(H_1^\dagger H_1+H_2^\dagger H_2\right)-\frac{i}{2}b\left(H_1^\dagger H_2-H_2^\dagger H_1\right)$$

2nd terms are generated at 1-loop, nonlocal and finite due to the Wilson loop $P\left(\exp\left(ig\oint A_{5,6}\,dx_{5,6}\right)\right)$

$\vec{l}_1 \perp \vec{l}_2 \Rightarrow$ No mixing A_5A_6 $\quad$ $\left|\vec{l}_1\right|=\left|\vec{l}_2\right| \Rightarrow$ Same coefficient of $(A_5)^2$ & $(A_6)^2$

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$$=\frac{\lambda}{2}\left[\left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right)+\left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right)-\left(H_2^\dagger H_1\right)^2-\left(H_1^\dagger H_2\right)^2\right]$$

$$+a\left(H_1^\dagger H_1+H_2^\dagger H_2\right)-\frac{i}{2}b\left(H_1^\dagger H_2-H_2^\dagger H_1\right)$$

The last term from
the brane localized tadpole term Tr (YF_{56})
for $U(1)_Y$ unbroken on the fixed points

$$V(H_1,H_2)=\frac{\lambda}{2}\left[\left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right)+\left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right)-\left(H_2^\dagger H_1\right)^2-\left(H_1^\dagger H_2\right)^2\right]$$
$$+a\left(H_1^\dagger H_1+H_2^\dagger H_2\right)-\frac{i}{2}b\left(H_1^\dagger H_2-H_2^\dagger H_1\right)$$

$$\langle H_{1,2}\rangle=\left(\phi_{1,2}^+,\phi_{1,2}^0\right)^T$$

$$V\left(\phi_1^0,\phi_2^0\right)=2\lambda\left[\mathrm{Im}\left(\phi_1^{0*}\phi_2^0\right)\right]^2+a\left(\left|\phi_1^0\right|^2+\left|\phi_2^0\right|^2\right)+b\,\mathrm{Im}\left(\phi_1^{0*}\phi_2^0\right)$$

$$\left|\phi_{1,2}^0\right|=v_{1,2}/\sqrt{2},\ \ \phi_1^{0*}\phi_2=v_1v_2e^{-i\theta}/2\ \ \left(v_{1,2}\geq 0\right)$$

$$V\left(v_1,v_2,\theta\right)=\frac{\lambda}{2}\left(v_1v_2\sin\theta\right)^2+\frac{a}{2}\left(v_1^2+v_2^2\right)-\frac{b}{2}v_1v_2\sin\theta$$
$$\Rightarrow v_1=v_2=\sqrt{\frac{|b|-2a}{2\lambda}}=\frac{v}{\sqrt{2}},\ \ \theta=sign(b)\frac{\pi}{2}$$

Not flat direction θ=0

Mass spectrum

Useful basis

$$H \equiv \frac{1}{\sqrt{2}}\left[H_1 + i\, sign(b) H_2\right]$$

$$\tilde{H} \equiv \frac{1}{\sqrt{2}}\left[H_1 - i\, sign(b) H_2\right]$$

$$\Rightarrow \quad \langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \quad \langle \tilde{H} \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

NG bosons $G^{\pm}$, G^0 & physical Higgs in this basis

$$H = \begin{pmatrix} G^+ \\ \frac{v + h + iG^0}{\sqrt{2}} \end{pmatrix}, \quad \tilde{H} = \begin{pmatrix} h^+ \\ \frac{\tilde{h} + iP}{\sqrt{2}} \end{pmatrix}$$

Higgs potential

$$V\left(H,\tilde{H}\right)=\frac{\lambda}{2}\left[\left(H^{\dagger}H\right)^{2}+\left(\tilde{H}^{\dagger}\tilde{H}\right)^{2}-\left(H^{\dagger}H\right)\left(\tilde{H}^{\dagger}\tilde{H}\right)-\left(H^{\dagger}\tilde{H}\right)\left(\tilde{H}^{\dagger}H\right)\right]$$
$$+a\left(H^{\dagger}H+\tilde{H}^{\dagger}\tilde{H}\right)-\frac{|b|}{2}\left(H^{\dagger}H-\tilde{H}^{\dagger}\tilde{H}\right)$$

$$\Rightarrow\quad V\left(H,\tilde{H}\right)_{quadratic}=0\times\left|G^{+}\right|^{2}+\left(\frac{3}{2}a+\frac{|b|}{4}\right)\left|h^{+}\right|^{2}+0\times\left(G^{0}\right)^{2}$$
$$+\frac{1}{2}(2a)P^{2}+\frac{1}{2}\left(|b|-2a\right)h^{2}+\frac{1}{2}(2a)\tilde{h}^{2}$$

Charged Higgs: $M_{h^{+}}^{2}=2a+M_{W}^{2}$

CP-odd Higgs: $M_{P}^{2}=2a$

CP-even Higgs: $M_{h}^{2}=\left(2M_{W}\right)^{2},\ M_{\tilde{h}}^{2}=2a$

If $a < 2M_W^2$, Higgs mass is predicted to be

$$M_H = M_{\tilde{h}} = \sqrt{2a} < 2M_W$$

In this case, the light Higgs is...

$\tilde{h}$: doublet without VEV ⇒ NOT SM Higgs

$$H = \begin{pmatrix} G^+ \\ \dfrac{v + h + iG^0}{\sqrt{2}} \end{pmatrix}, \quad \tilde{H} = \begin{pmatrix} h^+ \\ \dfrac{\tilde{h} + iP}{\sqrt{2}} \end{pmatrix}$$

MSSM is instructive

$$\begin{pmatrix} H_{SM} \\ H' \end{pmatrix} = \begin{pmatrix} -\cos\beta & \sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \tilde{H}_d \\ H_u \end{pmatrix} \Rightarrow \langle H_{SM} \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \langle H' \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\tan\beta = \frac{v_u}{v_d}, \langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \tilde{H}_d \equiv i\sigma_2 H_d^*$$

In general, light(h^0) & heavy(H^0) Higgses are mixed

$$H_{SM} = \begin{pmatrix} G^+ \\ \left(v + \left\{-\sin(\alpha-\beta)h^0 + \cos(\alpha-\beta)H^0\right\} + iG^0\right)/\sqrt{2} \end{pmatrix} \underbrace{=}_{\alpha=\beta} \begin{pmatrix} G^+ \\ \left(v + H^0 + iG^0\right)/\sqrt{2} \end{pmatrix}$$

$$H' = \begin{pmatrix} H^+ \\ \left(\left\{\cos(\alpha-\beta)h^0 + \sin(\alpha-\beta)H^0\right\} + iA^0\right)/\sqrt{2} \end{pmatrix} \underbrace{=}_{\alpha=\beta} \begin{pmatrix} H^+ \\ \left(h^0 + iA^0\right)/\sqrt{2} \end{pmatrix}$$

Our case corresponds to α=β in MSSM

due to simple compactification $\left|\vec{l}_1\right| = \left|\vec{l}_2\right|,\ \vec{l}_1 \perp \vec{l}_2$

⇒ Asymmetric torus can change this situation

Check if the form of quadratic terms

$$V\left(H_1, H_2\right)_{quadratic} = a\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) - \frac{i}{2} b\left(H_1^\dagger H_2 - H_2^\dagger H_1\right)$$

are correct by calculating

Scalar & Fermion loop corrections

Scalar case

SU(3) triplet is expanded as

$$\Phi\left(x^{\mu},x^{5},x^{6}\right)=\begin{pmatrix}\varphi_1\\ \varphi_2\\ \varphi_3\end{pmatrix}=\sum_{n_1,n_2=-\infty}^{\infty}\frac{1}{2\pi R}\begin{pmatrix}\cos\left(\left(n_1x^5+n_2x^6\right)/R\right)\varphi_1^{(n_1,n_2)}\left(x^{\mu}\right)\\ \cos\left(\left(n_1x^5+n_2x^6\right)/R\right)\varphi_2^{(n_1,n_2)}\left(x^{\mu}\right)\\ i\sin\left(\left(n_1x^5+n_2x^6\right)/R\right)\varphi_3^{(n_1,n_2)}\left(x^{\mu}\right)\end{pmatrix}$$

under the parity $\Phi(-x^5,-x^6) = P\Phi(x^5,x^6)$, $P = \mathrm{diag}(1,1,-1)$

Calculate effective potential of $H_{1,2}$ by use of background field method

$$A_{5,6}^{(0,0)}=\frac{1}{\sqrt{2}}\begin{pmatrix}0_{2\times 2} & H_{1,2}\\ H_{1,2}^{\dagger} & 0\end{pmatrix}\Rightarrow D_{5,6}=i\begin{pmatrix}\frac{n_{1,2}}{R}I_2 & \frac{g}{\sqrt{2}}H_{1,2}\\ \frac{g}{\sqrt{2}}H_{1,2}^{\dagger} & \frac{n_{1,2}}{R}\end{pmatrix}$$

Effective potential

$$V_{eff}^{(s)} = \frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4} \sum_{n_1,n_2=-\infty}^{\infty} Tr\log\left(p_E^2 I_3 + \mathcal{M}_{n_1,n_2}^2\right)$$

$$\mathcal{M}_{n_1,n_2}^2 = -D_5^2 - D_6^2 = \frac{n_1^2+n_2^2}{R^2} I_3 + \begin{pmatrix} \frac{g^2}{2}\left(H_1 H_1^\dagger + H_2 H_2^\dagger\right) & \sqrt{2} g \frac{n_1 H_1 + n_2 H_2}{R} \\ \sqrt{2} g \frac{n_1 H_1^\dagger + n_2 H_2^\dagger}{R} & \frac{g^2}{2}\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) \end{pmatrix}$$

Quadratic terms of $H_{1,2}$

2nd order

1st order

$$V_2^{(s)} = \frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4} \sum_{n_1,n_2=-\infty}^{\infty} \left[\frac{g^2\left(H_1^\dagger H_1 + H_2^\dagger H_2\right)}{p_E^2 + \frac{n_1^2+n_2^2}{R^2}} - 2g^2 \frac{\frac{\left(n_1 H_1^\dagger + n_2 H_2^\dagger\right)\left(n_1 H_1 + n_2 H_2\right)}{R^2}}{\left(p_E^2 + \frac{n_1^2+n_2^2}{R^2}\right)^2} \right]$$

$$= \underbrace{\frac{g^2}{2}\int \frac{d^4 p_E}{(2\pi)^4} \sum_{n_1,n_2=-\infty}^{\infty} \left[\frac{1}{p_E^2 + \frac{n_1^2+n_2^2}{R^2}} - \frac{\frac{n_1^2+n_2^2}{R^2}}{\left(p_E^2 + \frac{n_1^2+n_2^2}{R^2}\right)^2} \right]}_{a^{(s)}} \left(H_1^\dagger H_1 + H_2^\dagger H_2\right), \quad b^{(s)} = 0$$

Effective potential

Cross terms $H_1^{\dagger}H_2$, $H_2^{\dagger}H_1$ vanish

$$\because \quad \sum_{n_1,n_2=-\infty}^{\infty} \frac{n_1 n_2}{\left(p_E^2+\left(n_1^2+n_2^2\right)/R^2\right)^2}=0$$

$n_1^2 H_1^{\dagger}H_1 \rightarrow \frac{1}{2}(n_1^2+n_2^2)H_1^{\dagger}H_1$ etc.

$$+\mathcal{M}_{n_1,n_2}^2 \Big)$$

$$\sqrt{2}g\frac{n_1H_1+n_2H_2}{R}$$

$$\left(H_1^{\dagger}H_1+H_2^{\dagger}H_2\right)$$

Quadratic terms of $H_{1,2}$

$$V_2^{(s)}=\frac{1}{2}\int\frac{d^4p_E}{(2\pi)^4}\sum_{n_1,n_2=-\infty}^{\infty}\left[\frac{g^2\left(H_1^{\dagger}H_1+H_2^{\dagger}H_2\right)}{p_E^2+\frac{n_1^2+n_2^2}{R^2}}-2g^2\frac{\frac{\left(n_1H_1^{\dagger}+n_2H_2^{\dagger}\right)\left(n_1H_1+n_2H_2\right)}{R^2}}{\left(p_E^2+\frac{n_1^2+n_2^2}{R^2}\right)^2}\right]$$

$$=\underbrace{\frac{g^2}{2}\int\frac{d^4p_E}{(2\pi)^4}\sum_{n_1,n_2=-\infty}^{\infty}\left[\frac{1}{p_E^2+\frac{n_1^2+n_2^2}{R^2}}-\frac{\frac{n_1^2+n_2^2}{R^2}}{\left(p_E^2+\frac{n_1^2+n_2^2}{R^2}\right)^2}\right]}_{a^{(s)}}\left(H_1^{\dagger}H_1+H_2^{\dagger}H_2\right),\ b^{(s)}=0$$

Skipping details of calculations, we get final results

$$a^{(s)} = \frac{g^2}{16\pi^5 R^2} \sum_{(k_1, k_2) \neq (0,0)} \frac{1}{\left(k_1^2 + k_2^2\right)^2}$$

$$b^{(s)} = 0$$

$$V\left(H_1, H_2\right)_{quadratic} = a\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) - \frac{i}{2} b\left(H_1^\dagger H_2 - H_2^\dagger H_1\right)$$

Skipping details of calculations, we get final results

$$a^{(s)} = \frac{g^2}{16\pi^5 R^2} \sum_{(k_1,k_2)\neq(0,0)} \frac{1}{\left(k_1^2 + k_2^2\right)^2}$$

$$b^{(s)} = 0$$

$$V\left(H_1,H_2\right)_{quadratic} = a\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) - \frac{i}{2} b\left(H_1^\dagger H_2 - H_2^\dagger H_1\right)$$

$$v_1 = v_2 = \sqrt{\frac{|b| - 2a}{2\lambda}} \Rightarrow |b| > 2a$$

Not satisfied in the scalar case

Fermion case

SU(3) triplet, 6D Weyl fermion

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}, \ \Gamma_7 \Psi = -\Psi$$

$$\Gamma^\mu = \gamma^\mu \otimes I_2, \ \Gamma^5 = \gamma^5 \otimes i\sigma_1, \ \Gamma^6 = \gamma^5 \otimes i\sigma_2$$

$$\Gamma_7 = \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^5 \Gamma^6 = -\gamma^5 \otimes \sigma_3$$

4D fermions: ψ_R with σ_3=+1 or ψ_L with σ_3=-1 (γ^5=+1(R), -1(L))

Mode expansion

$$\Psi\left(x^\mu, x^5, x^6\right) = \sum_{n_1, n_2 = -\infty}^{\infty} \frac{1}{2\pi R} \begin{pmatrix} \cos\left(\frac{n_1 x^5 + n_2 x^6}{R}\right) \psi_{1L}^{(n_1,n_2)}\left(x^\mu\right) + i \sin\left(\frac{n_1 x^5 + n_2 x^6}{R}\right) \psi_{1R}^{(n_1,n_2)}\left(x^\mu\right) \\ \cos\left(\frac{n_1 x^5 + n_2 x^6}{R}\right) \psi_{2L}^{(n_1,n_2)}\left(x^\mu\right) + i \sin\left(\frac{n_1 x^5 + n_2 x^6}{R}\right) \psi_{2R}^{(n_1,n_2)}\left(x^\mu\right) \\ i \sin\left(\frac{n_1 x^5 + n_2 x^6}{R}\right) \psi_{3L}^{(n_1,n_2)}\left(x^\mu\right) + \cos\left(\frac{n_1 x^5 + n_2 x^6}{R}\right) \psi_{3R}^{(n_1,n_2)}\left(x^\mu\right) \end{pmatrix}$$

under Z_2 parity

$$\Psi\left(-x^5, -x^6\right) = P\left(-i\Gamma^5\Gamma^6\right)\Psi\left(x^5, x^6\right) = -P\left(I_4 \otimes \sigma_3\right)\Psi\left(x^5, x^6\right)$$

Effective potential

$$V_{eff}^{(f)} = -\frac{1}{2} \times 2 \int \frac{d^4 p_E}{(2\pi)^4} \sum_{n_1, n_2 = -\infty}^{\infty} Tr \log\left(p_E^2 I_3 + \tilde{\mathcal{M}}^2_{n_1, n_2} \right)$$

$$\tilde{\mathcal{M}}^2_{n_1, n_2} = \left(D_5 \Gamma^5 + D_6 \Gamma^6 \right)^2 = \mathcal{M}^2_{n_1, n_2} - g^2 \Gamma^5 \Gamma^6 \left[A_5, A_6 \right]$$

$$= \mathcal{M}^2_{n_1, n_2} + \frac{i}{2} g^2 \left(I_4 \otimes \sigma_3 \right) \begin{pmatrix} H_1 H_2^\dagger - H_2 H_1^\dagger & 0 \\ 0 & H_1^\dagger H_2 - H_2^\dagger H_1 \end{pmatrix}$$

Same as the scalar field case

Effective potential

$$V_{eff}^{(f)} = -\frac{1}{2} \times 2 \int \frac{d^4 p_E}{(2\pi)^4} \sum_{n_1,n_2=-\infty}^{\infty} Tr \log\left(p_E^2 I_3 + \tilde{\mathcal{M}}^2_{n_1,n_2} \right)$$

$$\tilde{\mathcal{M}}^2_{n_1,n_2} = \left(D_5 \Gamma^5 + D_6 \Gamma^6 \right)^2 = \mathcal{M}^2_{n_1,n_2} - g^2 \Gamma^5 \Gamma^6 \left[A_5, A_6 \right]$$

$$= \mathcal{M}^2_{n_1,n_2} + \frac{i}{2} g^2 \left(I_4 \otimes \sigma_3 \right) \begin{pmatrix} H_1 H_2^\dagger - H_2 H_1^\dagger & 0 \\ 0 & H_1^\dagger H_2 - H_2^\dagger H_1 \end{pmatrix}$$

Tr of the 2nd term has only nonvanishing 0 mode contribution

$\Psi_{1L,2L}^{(0,0)}$: $\sigma_3 = -1$, $\Psi_{3R}^{(0,0)}$: $\sigma_3 = +1$

$$Tr\left[\begin{pmatrix} -I_2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} H_1 H_2^\dagger - H_2 H_1^\dagger & 0 \\ 0 & H_1^\dagger H_2 - H_2^\dagger H_1 \end{pmatrix} \right] = 2\left(H_1^\dagger H_2 - H_2^\dagger H_1 \right)$$

$\Rightarrow$ contribution to tadpole term

Calculations can be done similarly as the scalar case

$$a^{(f)} = -\frac{g^2}{8\pi^5 R^2} \sum_{(k_1,k_2)\neq(0,0)} \frac{1}{\left(k_1^2 + k_2^2\right)^2}$$

$$b^{(f)} = 2g^2 \int \frac{d^4 p_E}{(2\pi)^4} \frac{1}{p_E^2}$$

$$V\left(H_1, H_2\right)_{quadratic} = a\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) - \frac{i}{2} b\left(H_1^\dagger H_2 - H_2^\dagger H_1\right)$$

Tadpole term is generated by only zero mode loop

Quadratically divergent ⇒ Renormalization or

Parity $A_5(x^5, x^6) \rightarrow -A_5(-x^5, x^6)$

$A_6(x^5, x^6) \rightarrow A_6(-x^5, x^6)$

⇒ $F_{56} \rightarrow -F_{56}$

SU(4) model on T^2/Z_2 with two Higgs doublets

SU(3) model: $\sin^2\theta_W = \frac{3}{4}$

$\Rightarrow$ possible model with observed m_H & $\sin^2\theta_W = \frac{1}{4}$??

$$A_\mu(-x^5,-x^6) = PA_\mu(x^5,x^6)P^{-1},\ A_{5,6}(-x^5,-x^6) = -PA_{5,6}(x^5,x^6)P^{-1}$$

P = diag(1,1,1,-1)@(0,0) SU(4)→SU(3)xU(1)

diag(1,1,-1,-1)@(πR, πR) SU(4)→SU(2)xSU(2)xU(1)

$$A_\mu = \begin{pmatrix} (+,+) & (+,+) & (+,-) & (-,-) \\ (+,+) & (+,+) & (+,-) & (-,-) \\ (+,-) & (+,-) & (+,+) & (-,+) \\ (-,-) & (-,-) & (-,+) & (+,+) \end{pmatrix},\quad A_{5,6} = \begin{pmatrix} (-,-) & (-,-) & (-,+) & (+,+) \\ (-,-) & (-,-) & (-,+) & (+,+) \\ (-,+) & (-,+) & (-,-) & (+,-) \\ (+,+) & (+,+) & (+,-) & (-,-) \end{pmatrix}$$

$SU(4)\rightarrow SU(2)_L \times U(1)_Y \times U(1)_X$

Two Higgs doublets

SU(4) model on T^2/Z_2 with two Higgs doublets

SU(3) model: $\sin^2\theta_W = \frac{3}{4}$

⇒ possible model with observed m_H & $\sin^2\theta_W = \frac{1}{4}$??

$$A_\mu(-x^5,-x^6) = PA_\mu(x^5,x^6)P^{-1},\ A_{5,6}(-x^5,-x^6) = -PA_{5,6}(x^5,x^6)P^{-1}$$

P = diag(1,1,1,-1)@(0,0) SU(4)→SU(3)xU(1)

diag(1,1,-1,-1)@(πR, πR) SU(4)→SU(2)xSU(2)xU(1)

Zero modes

$$A_\mu = \begin{pmatrix} \frac{1}{2}\gamma_\mu - \frac{\sqrt{3}}{6}Z_\mu & \frac{1}{\sqrt{2}}W_\mu^+ & 0 & 0 \\ \frac{1}{\sqrt{2}}W_\mu^- & \frac{\sqrt{3}}{3}Z_\mu & 0 & 0 \\ 0 & 0 & -\frac{1}{2}\gamma_\mu - \frac{\sqrt{3}}{6}Z_\mu & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + X_\mu \frac{\sqrt{6}}{12}\mathrm{diag}(1,1,1,-3),\quad A_{5,6} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & \phi_{1,2}^+ \\ 0 & 0 & 0 & \phi_{1,2}^0 \\ 0 & 0 & 0 & 0 \\ \phi_{1,2}^- & \phi_{1,2}^{0*} & 0 & 0 \end{pmatrix}$$

SU(4) model on T^2/Z_2 with two Higgs doublets

Calculation of effective potential $V(H_1, H_2)$ goes in the same way as SU(3) model

$M_H < 2M_W$ is realized by introducing fermions in the fundamental rep. of SU(4)

Summary

- A question if Higgs mass & Weinberg angle can be successfully predicted in 6D GHU with one or two Higgs doublets addressed
- Higgs doublets are embedded in the 2nd rank tensor reps. of SU(3)
  - One Higgs doublet (Sp(6), SU(4)) ⇒ $M_H = 2M_W$
  - Two Higgs doublet ⇒ $M_H < 2M_W$ possible
- Getting M_H=125GeV in more general Z_2 orbifold is left for future work
- Hope this work provides a guideline for constructing realistic models