Dynamics of Revolving D-branes and Stringy Threshold Corrections to the Higgs Mass Term

"Hierarchy problem" Implication from Superstring

Hierarchy problem is a key idea to go beyond SM in particle physics, cosmology, string theory

Satoshi Iso (KEK & Sokendai)

based on collaborations

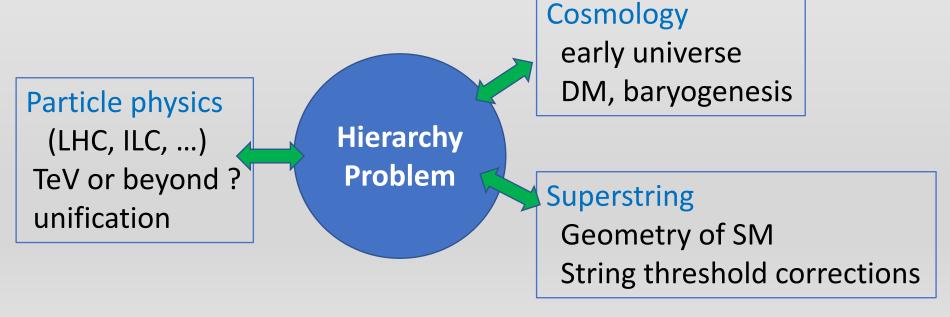
Phenomenology: Yuta Orikasa, Nobu Okada, Michio Hashimoto Cosmology: Kengo Shimada, Pasquale Serpico, Kazu Kohri

String: Nori Kitazawa, Hikaru Ohta, Takao Suyama

Hierarchy problem: Hajime Aoki, Kiyoharu Kawana

Hierarchy problem = Dynamics of EWSB and its Stability

How EWSB occurs dynamically ? Who ordered the Higgs potential ? Why Higgs VEV <H> = 256 GeV , Higgs mass $m_H = 125$ GeV ? Why are they stable against (possibly) higher energy scales? e.g. GUT 10^{16} GeV, Planck 10^{19} GeV, or SUSY? RHv?



Today's talk

[1] Hierarchy problem

Phenomenology: Radiative breaking of EW

Implication to cosmology: Supercooled universe

[2] Implication from Superstring Can the superstring resolve HP? String threshold corrections

[1] What is the Hierarchy problem?

Hierarchy problem = How UV physics affects IR? It is NOT a problem of quadratic divergences.

cf. RG approach to quadratic divergences

H. Aoki, SI PRD (2012)

SM + something

Intermediate scales

UV physics

Before Higgs

SM — MSSM — RHN, SUSY GUT — CY — Superstring

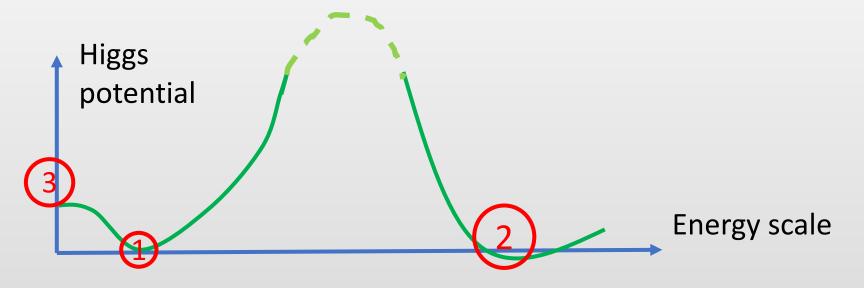
After Higgs

SM + something ----- QG or Superstring

Froggatt Nielsen (96) and Shaposhnikov (07)

Current status of SM and Higgs

LHC → Higgs potential: (at least) 3 important points



- 1 Higgs VEV v= 246 GeV → Particle physics (present universe)
- 2 UV scale $M_{UV} > 10^{10}$ GeV or M_{Planck}
 - → Gravity, String theory & origin of Higgs
- 3 h=0 (origin) → Cosmology (early universe)

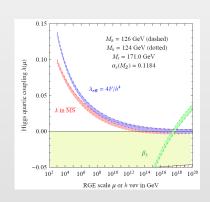
All regions (1, 2, 3) will be related by RG and theories.

2 important lessons from LHC for the Higgs potential

$$V = -\mu^2 |H|^2 + \lambda (|H|^2)^2$$

"EW" physics may be directly related to Planck scale physics without intermediate scales in between.

Froggatt Nielsen (96) M.Shaposhnikov (07)



- (2) No deviations from SM / no TeV SUSY?
 - → An alternative to the Naturalness (=hierarchy) problem

After Higgs

SM + something ----- QG or Superstring

IR physics

UV physics

Hierarchy problem

(Appelquist Carazzone: Decoupling Theorem) = existence of EFT IR EFT is described by light particles after integrating heavy particles. The effects of heavy particles are renormalization of parameters in EFT.

But Higgs mass in EFT may receive large threshold corrections in UV physics.

$$\delta m_H^2 = \frac{g^2}{16\pi^2} \left(\Lambda^2 + M^2 \log(\Lambda/M) \right)$$

Quadratic divergence is not physical.

The real issue is the logarithmic part with large coefficient M.

From EFT point of view, the following 3 conditions are required:

- (1) EFT contains no bare mass term of Higgs / or small after including (2) scalar field = stable moduli ?
- (2) Threshold corrections of UV physics are suppressed string theory or asymptotic safety
- (3) No intermediate scales non-susy vacua of string theory, ...

3 questions for Higgs and EWSB

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Question 1. Presence of EFT
   How can we get low-energy EFT with a vanishing scalar mass?
          EFT = SM + Higgs + something
                     trial e.g. N. Kitazawa, SI PTEP(2015)
                    → nonsusy vacua of superstring with flat moduli?
Question 2. bottom-up physics
   Is the EWSB radiatively realized in EFT?
          Coleman-Weinberg mechanism → SM must be extended!
                    N.Okada, Y.Orikasa, SI PLB(2009), PRD(2009), PTEP(2012) ...
   What are the phenomenological/cosmological consequences?
           DM, low T B-genesis ... K.Shimada, Y. Orikasa, SI 2010-2018
           supercooled universe P. Serpico, K. Shimada, SI PRL (2017)
Question 3. top-down physics
   Why are threshold corrections by UV physics suppressed?
                    N. Kitazawa, T. Suyama, H. Ohta, SI JHEP (2018) & 2019
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Question 2. Is the EWSB radiatively realized in EFT? "classically conformal (B-L)-extension of SM"

Okada, Orikasa, SI (09) also Lindner et.al. (10)

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IR physics

Standard  
Model  

Hodel  

B-L sector @ TeV

UV physics

Planck scale

V(h)=0

No intermediate scales
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(1) B-L breaking by CW mechanism → triggers EWSB at 100 GeV Everything occurs radiatively.

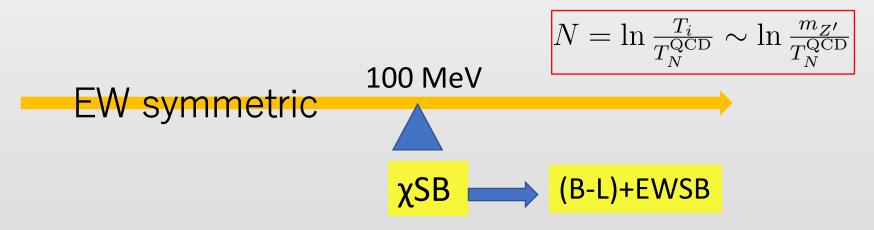
Thus it is free from naturalness problem (no intermediate scales).

(2) Minimal extension of SM & Phenomenologically viable ν oscillation, (resonant) leptogenesis, DM candidate (e.g. right-handed ν with Z_2)

Question 2. Implication to Cosmology

In classically conformal models motivated by naturalness, the early universe is drastically different

extreme SUPERCOOLING and the second inflation with N~10



Interesting cosmological consequences
Stochastic GW, PBH, Cold EWBH, axion abundance, supercool DM

Supercooling is also expected in other models, e.g. Randall-Sundrum models, Harling, Servant (18)

[2] Implication from Superstrings

N. kitazawa, H.Ohta, T.Suyama, SI hep-th/1909.10717

also

N. Kitazawa, SI PTEP (15)
H. Ohta, T. Suyama, SI arXiv:1812.11505
N. Kitazawa, SI arXiv:1812.08912

In the top-down (string/quantum gravity) approach we want to answer the following questions:

Question 1.

How can we embed a bottom-up model in string theory? "stringy realization of the SM and the Higgs field"

Question 3.

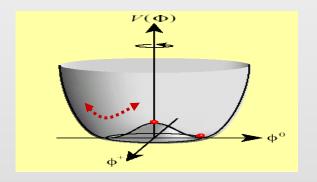
How can we control threshold corrections of Planck scale physics? "string threshold corrections to Higgs potential"

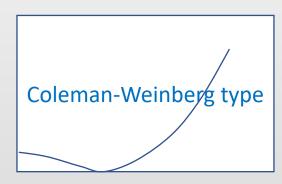
 $(M_{UV})^2 H^2$ M_{UV} : string scale or susy breaking scale = hierarchy problem

What is the Higgs and the Higgs potential?

In a field theoretic approach, we usually construct a Higgs potential first.

"Consider a Higgs potential first!"





Then obtain a solution as a minimum of the potential.

one solution to one Higgs potential

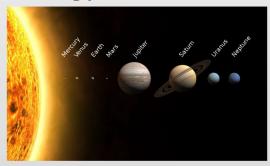
→ We are faced with the naturalness problem

$$=\frac{3y_t^2}{8\pi^2}\Lambda^2$$

Question:

Can we obtain a solution of <H> first and then calculate the corresponding Higgs potential.

Analogy with



There are many solutions.

Higgs potential is calculated for each solution <H>.

Any radius of orbits is a solution to the effective potential.

Initial condition (angular momentum) gives a different orbit.

This may be called "Dynamical-tuning of Higgs potential"

How can we embed Higgs (and SM) in superstring theory?

In string theory, dynamics of field theory is transformed into a problem of geometry.

D-branes, moduli, compactification, ...

Hierarchy problem also becomes geometrical.

Stringy view of our "universe" and "Higgs" sector:

Geometry in string theory = Dynamics in QFT

- [1] (3+1)-dimensional space-time is embedded in d=9+1. Either compactification or brane-world scenario
- [2] Higgs (scalar) field is a geometrical "moduli" field e.g. distance between D-branes volume / shape of extra-dimension etc.
- [3] VEVs of moduli fields are proportional to the geometrical size.

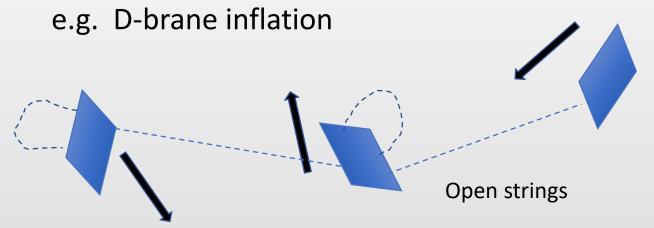
$$N_1$$
 \uparrow d D-branes $U(N_1 + N_2) \rightarrow U(N_1) \times U(N_2)$ $d = \frac{M}{M_s^2}$

How can we obtain a small value of d << I_{string}?

= Hierarchy problem in string theory

D-brane universe

Suppose that many D-branes are moving randomly.



The dynamics is nontrivial.

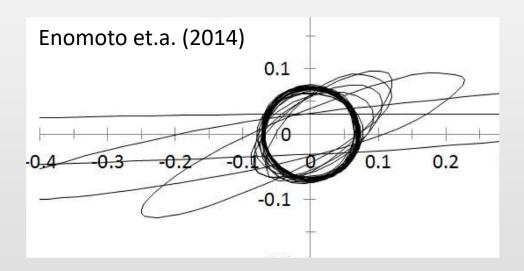
Masses of the open strings stretching between them vary according to their motion.

→ If the change is nonadiabatic,

D-branes lose their energy by emitting light open string modes. (similar to preheating mechanism) "Beauty is attractive"

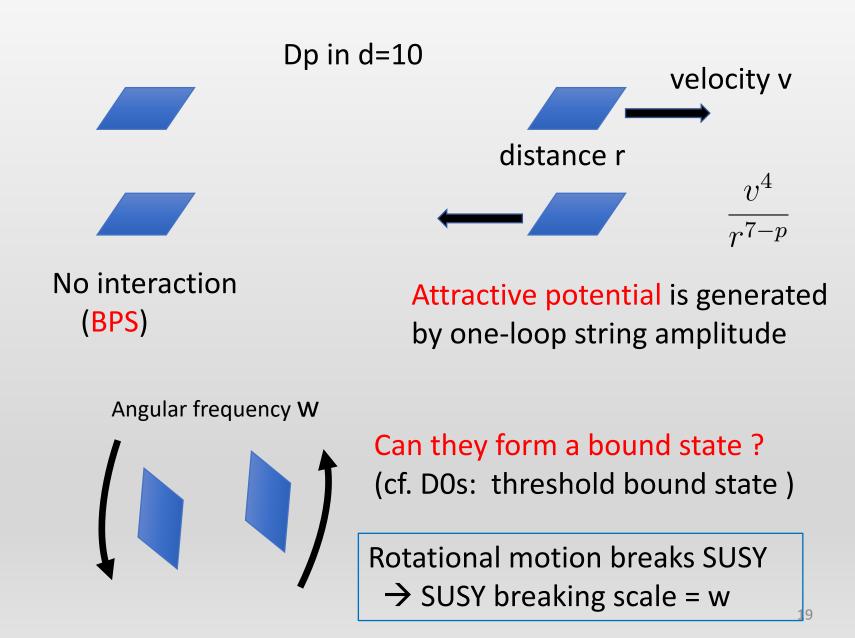
Kofman et.a. (04)

What is the fate of D-branes?

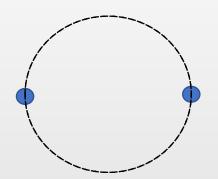


In the bosonic string, they form a bound state (if closed string emission is neglected.)

In superstring theory



Interaction potential between revolving D3-branes



Suppose that two parallel D3s are rotating in a transverse plane

Calculation of the potential in string theory

= one-loop open string amplitude with rotating boundary condition

$$V(r) = -\int_0^\infty \frac{dt}{t} e^{-\frac{r^2}{2\pi\alpha'}t} Z(t)$$

Z(t): partition function of open string $(\eta(t), \theta_{ab}(t))$ in a simple case but generally difficult to obtain because the quantization of open strings is not performed.

In field theory, the stringy calculation is interpreted as a sum of one-loop amplitudes of infinitely many fields massless (SYM) + massive fields

$$V \sim \frac{1}{2} \sum_{N=0}^{\infty} (-1)^{F} d_{N} \operatorname{tr} \log(p^{2} + m_{N}^{2})$$

$$= \sum_{N=0}^{\infty} d_{N} \frac{(-1)^{F}}{64\pi^{2}} \left(\Lambda^{2} m_{N}^{2} + m_{N}^{4} \log m_{N}^{2} / \Lambda^{2}\right)$$

$$m_{N}^{2} = f(m_{\text{str}}, r^{2}, \omega^{2}) \sim (N m_{str})^{2} + \mathcal{O}(r^{2}, \omega^{2})$$

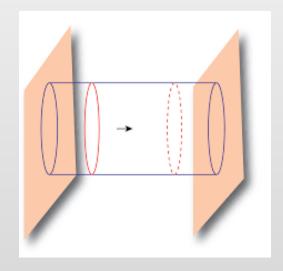
Massless states (SYM) may dominantly contribute to V(r). supersymmetry at w=0, V(r) $\sim \omega^2 r^2$

Infinitely many massive states can also contribute to $V(r) = \sum \omega^2 r^2$ How to calculate the stringy threshold corrections? = hierarchy problem

A new method to calculate the stringy threshold corrections in D-brane models

Basic idea = Partial Modular Transformation

(open-closed string duality)



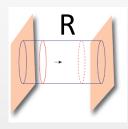
one-loop open string amplitude = closed string exchange

Summing both open and closed is a double counting.

- → partial sum of open and closed strings
- → approximate each by lowest modes
- = SYM +SUGRA with an appropriate cutoff

less than 3% accuracy of calculation can be achieved

Ex. Potential between D-branes: Bosonic p-brane



$$V(R) = -\int_0^\infty \frac{dt}{t} e^{-\frac{R^2}{2\pi\alpha'}t} (8\pi^2\alpha't)^{-\frac{1}{2}(p+1)} \eta(it)^{-24}$$

$$\eta(it)^{-24} = \sum_{n=-1}^{\infty} d_n e^{-2\pi nt}$$

$$d_{-1} = 1, d_0 = 24, d_1 = 324...$$

open tachyon, massless, massive

(1) Large R behavior (UV region at t=0)

 \rightarrow modular tr. (s = 1/t = ∞ IR region)

$$V(R) = -(8\pi^2\alpha')^{-\frac{1}{2}(p+1)} \int_0^\infty ds \, e^{-\frac{R^2}{2\pi\alpha'}s^{-1}} s^{\frac{1}{2}(p-25)} \eta(is)^{-24}$$

$$d_{-1} = 1, d_0 = 24, d_1 = 324...$$

closed tachyon, massless, massive



n=0 mode
$$V(R) \ \sim \ -(4\pi)^{-\frac{1}{2}(p+1)}(2\pi\alpha')^{11-p}\Gamma(\frac{23-p}{2})R^{p-23},$$

Massless closed string modes dominate. (if tachyons are neglected)

Newton potential

(2) Small R behavior ($t = \infty$ region)

Not only the massless modes dominate, since the contribution by n-th modes is given by

$$\int_{1/\Lambda^2}^{\infty} \frac{dt}{t} e^{-\frac{R^2}{2\pi\alpha'}t} t^{-\frac{1}{2}(p+1)} e^{-2\pi nt}$$

Coleman-Weinberg potential

D3-brane (p=3)

$$\int_{1/\Lambda^2}^{\infty} \frac{dt}{t} e^{-\frac{R^2}{2\pi\alpha'}t} t^{-\frac{1}{2}(p+1)} e^{-2\pi nt} = \frac{\Lambda^4}{2} - \Lambda^2 x + \frac{3 - 2\gamma}{4} x^2 - \frac{1}{2} x^2 \log(x/\Lambda^2)$$

$$x = 2n\pi + \frac{R^2}{2\pi\alpha'}$$

mass of n-th closed mode

Large-n modes equally contribute to V(R).

→ Need to sum up all n to obtain string threshold correction to V(R). In order to obtain the potential V(r) at small r region, do we need to calculate the one-loop open string amplitude exactly?

In many cases, it is not possible.



"Partial Modular Transformation"

Nori Kitazawa, Hikaru Ohta Takao Suyama and SI, 2019

Schwinger parameter of open string amplitude

$$t = [0, \infty] \implies t = [0,1] + t = [1,\infty] \implies s = [1,\infty] + t = [1,\infty]$$

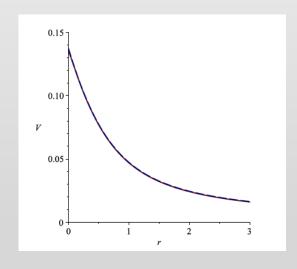
open UV open IR closed IR open IR
approximated by SUGRA + SYM

Potential V(R) \equiv sum of SYM and SUGRA with UV cutoff but free from double counting due to the appropriate UV cut-off at t=s=1.

How good is the approximation by SYM + SUGRA?

Example: D3-branes at angle. For simplicity, we set $\phi_1=\phi_2=\phi_3=\phi, \ \phi_4=0$

$$V(R) = -\int_0^\infty \frac{dt}{t} (8\pi^2 \alpha' t)^{-\frac{1}{2}} e^{-\frac{R^2}{2\pi\alpha'}t} \frac{i \prod_{a=1}^4 \vartheta_{11}(\frac{i}{\pi} \phi'_a t, it)}{\eta(it)^3 \prod_{a=1}^3 \vartheta_{11}(\frac{i}{\pi} \phi_a t, it)},$$



$$e^{-2\pi} = 0.001867$$

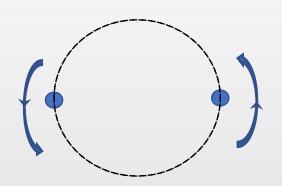
red line = exact 1-loop potential

blue broken line = SYM + SUGRA (only the lowest modes are taken into account)

for the simple bosonic case

$$\sum_{n=0}^{\infty} d_n e^{-2\pi n} = \eta(i)^{-24} - e^{2\pi} = 1.026 d_0.$$

Interaction potential between revolving D3-branes



Two parallel D3s are revolving around each other in a transverse plane

Since the boundary conditions are complicated, only a perturbative calculation of 1-loop open string amplitude is possible.



SYM with cutoff at t=1 + SUGRA with cutoff at s=1

$$\tilde{V}_{o}(R) = -\int_{1}^{\infty} \frac{dt}{t} \int \frac{d^{D}k}{(2\pi)^{D}} \sum_{\substack{\text{light open} \\ \text{open}}} e^{-2\pi t E_{o}(k) - \frac{R^{2}}{2\pi\alpha'}t},$$

$$\tilde{V}_{c}(R) = -\int_{1}^{\infty} ds \sum_{\substack{\text{massless} \\ \text{closed}}} \int \frac{d^{D'}k}{(2\pi)^{D'}} \langle B|c \rangle \langle c|B' \rangle e^{-2\pi s E_{c}(k) - \frac{R^{2}}{2\pi\alpha'}s^{-1}}.$$

(a) SYM calculations in background field gauge

$$S = \frac{1}{g^2} \int d^{p+1}x \operatorname{Tr} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D_{\mu} \Phi_I D^{\mu} \Phi^I + \frac{1}{4} ([\Phi_I, \Phi_J])^2 + \frac{i}{2} \bar{\Psi} \Gamma^{\mu} D_{\mu} \Psi + \frac{1}{2} \bar{\Psi} \Gamma^I [\Phi_I, \Psi] \right]$$

$$\Phi_I = B_I + \phi_I,$$

$$B_I = b_I(t)\sigma_3, \qquad b_8 = r\cos\omega\tau, \qquad b_9 = r\sin\omega\tau,$$

$$b_8 = r\cos\omega\tau,$$

$$b_9 = r \sin \omega \tau$$

$$t = -i\tau$$

t=-i au, (Euclidean time au)



$$\log \left[\det(-\partial^2 + r^2)^{-6} \det(E_{B+}(-i\partial))^{-1} \det(E_{B-}(-i\partial))^{-1} \right]$$

$$\times \det(E_{F+}(-i\partial))^4 \det(E_{F-}(-i\partial))^4$$

(b) SUGRA calculations

$$X^{\mu} = X^{\mu}(\zeta)$$

Interactions between Dp-brane and SUGRA fields are obtained from DBI +CS

$$S_{\text{DBI+CS}} = T_p \int d^{p+1} \zeta \left[e^{\frac{1}{4}(p-3)\Phi} \sqrt{-\hat{g}} + \hat{C}_{p+1} \right] \frac{\hat{g}_{\alpha\beta} = \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\nu} g_{\mu\nu},}{\hat{C}_{\alpha_1 \cdots \alpha_{p+1}}^{(p+1)} = \partial_{\alpha_1} X^{\mu_1} \cdots \partial_{\alpha_{p+1}} X^{\mu_{p+1}} C_{\mu_1 \cdots \mu_{p+1}}^{(p+1)}}$$

Propagators

dilaton:
$$\Delta(x) := 2\kappa_{10}^2 \int \frac{d^{10}k}{(2\pi)^{10}} \frac{e^{ik \cdot x}}{k^2},$$

graviton:
$$\Delta_{\mu\nu;\rho\sigma}(x) := \left(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{1}{4}\eta_{\mu\nu}\eta_{\rho\sigma}\right)\Delta(x),$$

R-R field:
$$\Delta_{\mu_0 \cdots \mu_p; \nu_0 \cdots \nu_p}(x) := \sum_{\sigma \in \mathcal{S}_{p+1}} \operatorname{sgn}(\sigma) \, \eta_{\mu_0 \nu_{\sigma(0)}} \cdots \eta_{\mu_p \nu_{\sigma(p)}} \Delta(x),$$

Potential is given by

$$\tilde{V}_c = -2\kappa_{10}^2 \int d^{p+1}\zeta \int d^{p+1}\tilde{\zeta} \,\Delta(X-\tilde{X}) \left(F_{\Phi}(X,\tilde{X}) + F_g(X,\tilde{X}) + F_C(X,\tilde{X})\right)$$

$$F_{\Phi}(X,\tilde{X}) = \left(\frac{p-3}{4}\right)^2 T_p^2 \sqrt{-\det\hat{\eta}_{\alpha\beta}(X)} \sqrt{-\det\hat{\eta}_{\gamma\delta}(\tilde{X})}, \qquad F_C(X,\tilde{X}) = T_p^2 \det(\partial_{\alpha}X \cdot \partial_{\beta}\tilde{X}).$$

$$F_g(X,\tilde{X}) = T_p^2 \sqrt{-\det \hat{\eta}_{\alpha\beta}(X)} \sqrt{-\det \hat{\eta}_{\gamma\delta}(\tilde{X})} \left(-\frac{(p+1)^2}{16} + \frac{1}{2} \hat{\eta}^{\alpha\beta}(X) (\partial_{\beta}X \cdot \partial_{\delta}\tilde{X}) \hat{\eta}^{\delta\gamma}(\tilde{X}) (\partial_{\gamma}\tilde{X} \cdot \partial_{\alpha}X) \right)$$

Potential between Revolving D-branes: result

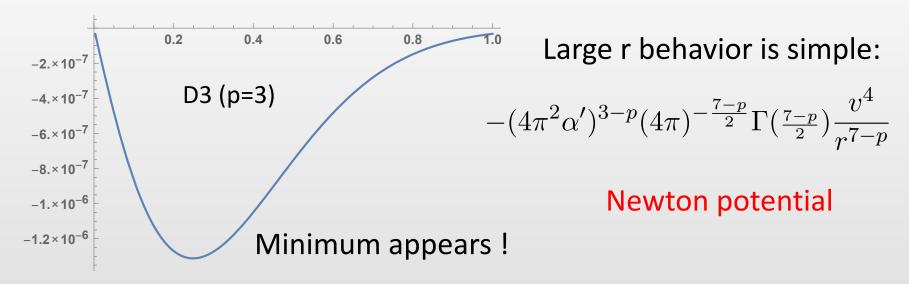
SYM part = effective potential from massless modes

$$\tilde{V}_{o,B} = -\int_{\Lambda^{-2}}^{\infty} \frac{dt}{t} \int \frac{d^{p+1}k}{(2\pi)^{p+1}} e^{-t(k^2+4r^2)} \\
\times \left[6 + 2e^{-t\omega^2 + t\frac{8(r\omega)^2}{k^2 + 4r^2}} \cosh\left(t\sqrt{4\omega^2 k_0^2 + \left(\frac{8(r\omega)^2}{k^2 + 4r^2}\right)^2}\right) \right] \\
\tilde{V}_{o,F} = 4 \int_{\Lambda^{-2}}^{\infty} \frac{dt}{t} \int \frac{d^{p+1}k}{(2\pi)^{p+1}} e^{-t(k^2+4r^2)} e^{-t\cdot\frac{\omega^2}{4}} \cdot 2 \cosh\left(t\sqrt{\omega^2 k_0^2 + 4(r\omega)^2}\right) \right]$$

SUGRA part = threshold corrections from stringy massive modes

$$\tilde{V}_{c}(r) = -\kappa_{10}^{2} T_{p}^{2} V_{p+1} (4\pi)^{-\frac{10-p}{2}} \frac{v^{4}}{1-v^{2}} \int_{\tilde{\Lambda}^{-2}}^{\infty} ds \, s^{-\frac{10-p}{2}} \times \int d\zeta \exp\left[-\frac{1}{4s} \left(\zeta^{2} + 2r^{2} (1 + \cos \omega \zeta)\right)\right] (1 + \cos \omega \zeta)^{2}$$

Effective potential V(r) at fixed w: r = moduli field = Higgs?



Small r behavior is more involved:

$$\tilde{V}_{o} = \frac{\beta^{2} \omega^{4}}{\pi^{2}} \left(-\frac{m_{s}^{2}}{\omega^{2}} \left(1 - E_{2}(\omega^{2}/m_{s}^{2}) \right) + \int_{\omega^{2}/m_{s}^{2}}^{\infty} \frac{dt}{t} e^{-t/4} F(\frac{1}{2}, \frac{3}{2}; \frac{t}{4}) \right) + \mathcal{O}(\beta^{4})$$

As expected,

$$\tilde{V}_o(2r) \sim -\frac{\omega^2 r^2}{\pi^2}$$

The dominant contribution to V(r) at $r \sim 0$ comes from SYM.

Qustion 3. String threshold corrections to Higgs potential

$$V \sim \frac{1}{2} \sum_{N=0}^{\infty} (-1)^F d_N \operatorname{tr} \log(p^2 + m_N^2)$$
$$= \sum_{N=0}^{\infty} d_N \frac{(-1)^F}{64\pi^2} \left(\Lambda^2 m_N^2 + m_N^4 \log m_N^2 / \Lambda^2\right)$$

Infinitely many massive open-string modes equally contribute to the CW potential V(r) at $r \sim 0$. Due to bulk SUSY at $\omega = 0$, we naively expect

V(r) ~ (large coefficient)
$$\times \omega^2 r^2$$
 ω : SUSY breaking scale in SUGRA

hierarchy problem in string theory

The infinite sum of massive open string modes, approximated by SUGRA, becomes

$$\tilde{V}_c(2r) \quad = \quad -\frac{\omega^4}{16\pi^2} \left[1 - \left(1 + 4r^2/m_s^2 \right) e^{-4r^2/m_s^2} \right] + \mathcal{O}(\omega^6) \sim -\frac{\omega^4 r^4}{2\pi^2 m_s^4} \quad \text{at small r}$$

$$\sum_{infinite\ open\ massive\ modes} \omega^2 r^2 = 0$$
!!

Naive hierarchy problem does not arise in this context. Probably due to large SUSY in the bulk.

Comment 1. A possibility of a bound state

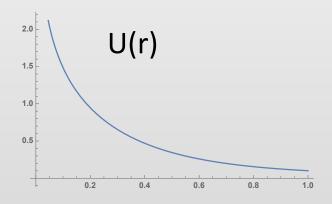
$$U(r) := \frac{L^2}{4T_3r^2} + \tilde{V}(r)$$

Centrifugal potential

L: angular momentum for unit volume of D3-brane

$$\omega = \frac{L}{T_3 r^2}$$

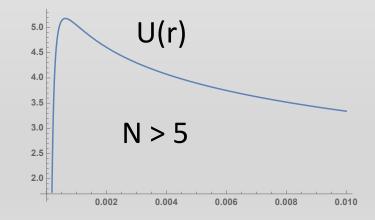
Potential = induced potential + centrifugal potential



no bound states exist.

If N-stack of D3-branes are revolving together,

$$U_N(r) := \frac{NL^2}{4T_3r^2} + N^2\tilde{V}(r)$$

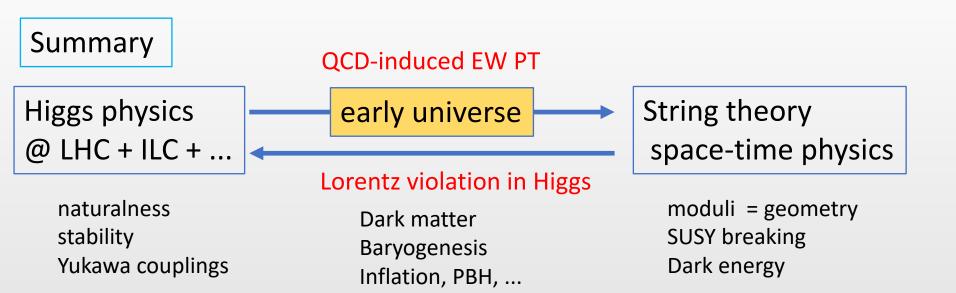


Bound states may exist.

Comment 2. Lorentz violation in the Higgs sector Experimental test of the geometric scenario

Lorentz violation occurs only in the Higgs sector (Coriolis force for Higgs field since it is geometrical.)

$$\omega^2 = M^2 + (1 + \frac{4\omega_0^2}{M^2})p^2 + 16\frac{\omega_0^4}{M^6}p^4 + \cdots$$
 N. Kitazawa, SI ('18) ω_0 < 0.1 GeV



Hierarchy problem is a key idea to go beyond SM in particle physics, cosmology, string theory

Implication to Cosmology:

Supercooling of EWPT, Different scenario of DM, B, L genesis Implication from Superstring: stringy threshold corrections

Mass terms of Higgs may not be generated

Thank you for your attention