Integrability of cohomogeneity-one strings in AdS_5 spacetime

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Introduction

Physical significance of extended objects

- Topological defects e.g. cosmic strings etc
- Braneworld universe model
- ···
- AdS/CFT correspondence

Extended objects (cosmic strings) are described by PDE

- Nambu-Goto equation, etc.

Cohomogeneity-one（C-1) object

Almost homogeneous,

but one inhomogeneous dimension

C-1 universe

universe models with homogeneous 3-space

e.g. Friedmann universe model,

(Bianchi universe model)

Einstein equations → Friedmann equation

P.D.E. → O.D.E.

$$ds^2 = -dt^2 + a(t)^2\left(dx^2 + dy^2 + dz^2\right)$$

C-1 black hole

black holes with spherical symmetry

e.g. Schwarzschild black hole

$$ds^2 = -\left(1 - \frac{r_g^2}{r^2}\right) dt^2 + \frac{dr^2}{1 - \frac{r_g^2}{r^2}} + r^2 d\Omega_{S^3}^2$$

Σ : r=const surface

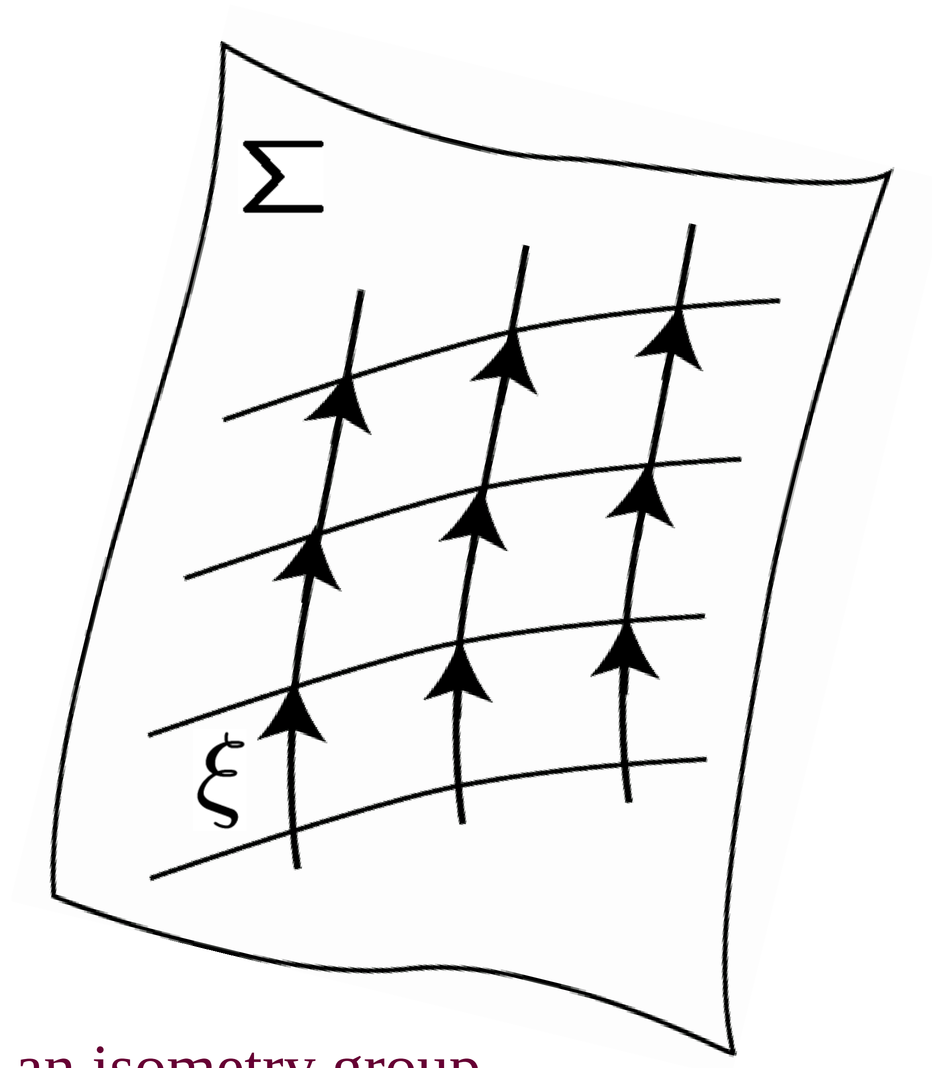
$$ds_\Sigma^2 = -a_0^2 dt^2 + b_0^2 d\Omega_{S^3}^2$$

C-1 String

1-parameter isometry group
of a target space acts
on the string world sheet

⇕

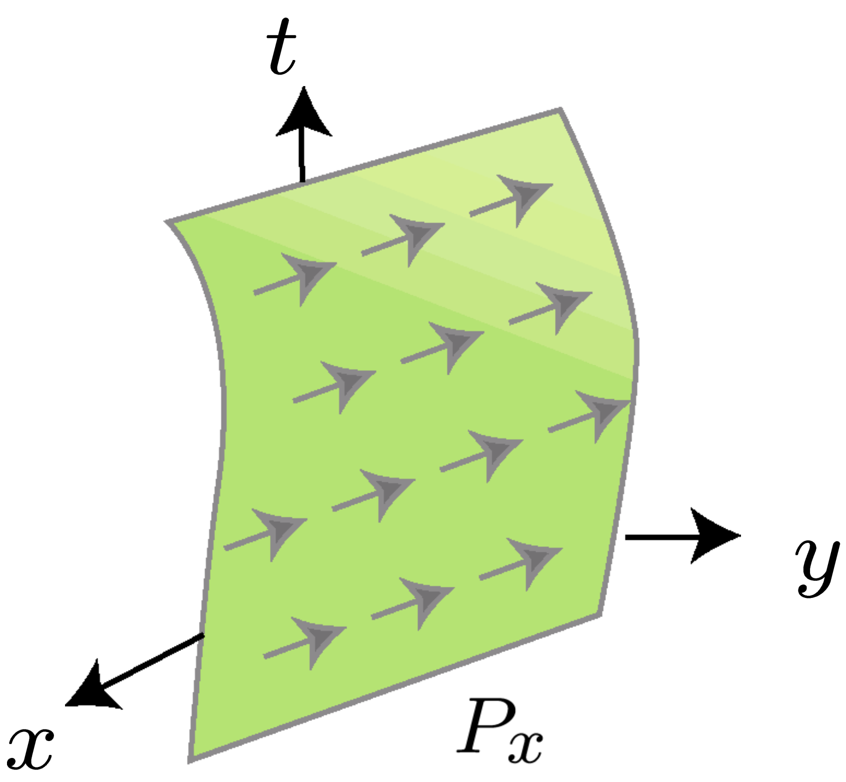
A Killing vector field is
tangent to the world sheet

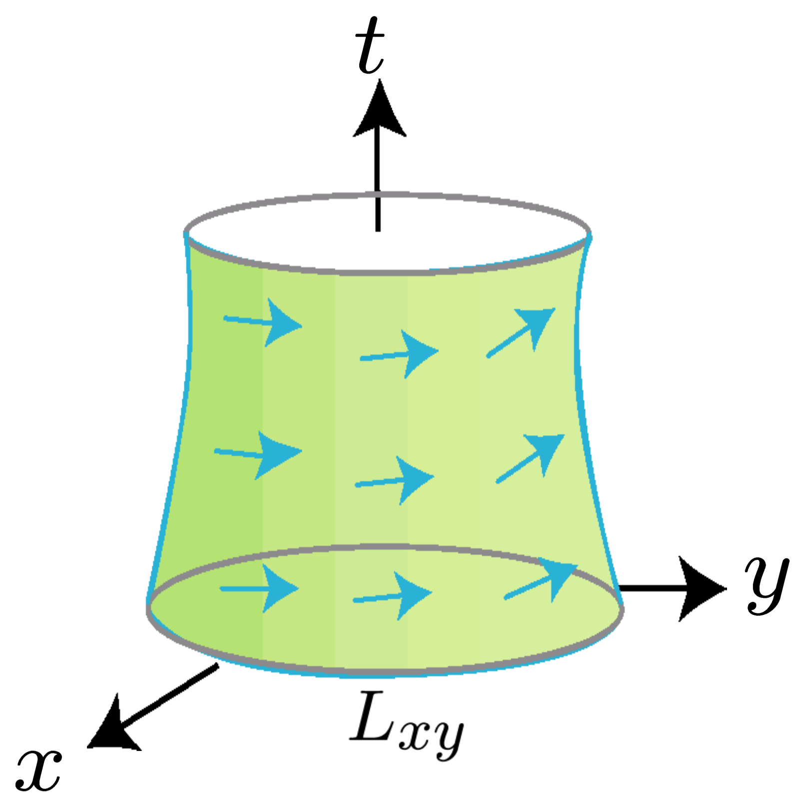


Killing vector : generator of an isometry group
(infinitesimal version of isometry)

We consider cohomogeneity-one strings

Strings with Symmetry





An isometry acts on the string world sheet.

Advantage of C-1 Objects

Tractable and physically interesting

	Homogeneous	Cohomogeneity-1	No symmetry
To solve	Simplest (algebraic)	Simple (ODE)	Difficult (PDE)
Variety	Poor	Rich	Richest
Physics	Trivial	Non-trivial	General

Examples of C-1 strings

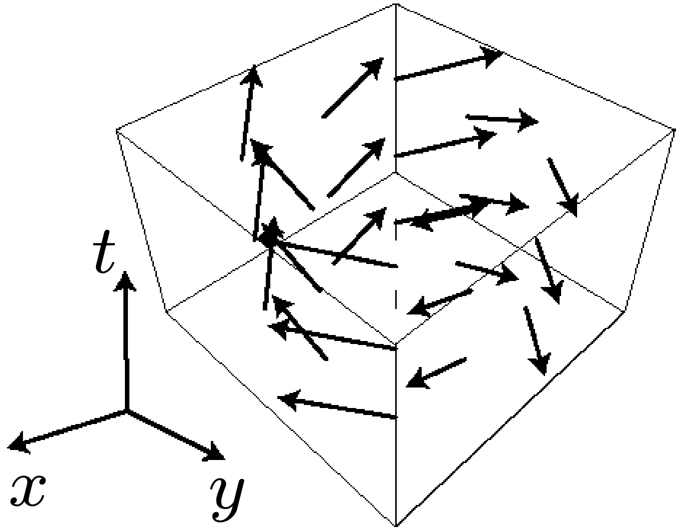
Example:
Stationary Rotating Strings in 4D Minkowski

Target space

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\
&= -dt^2 + d\rho^2 + \rho^2 d\phi^2 + dz^2
\end{aligned}
$$

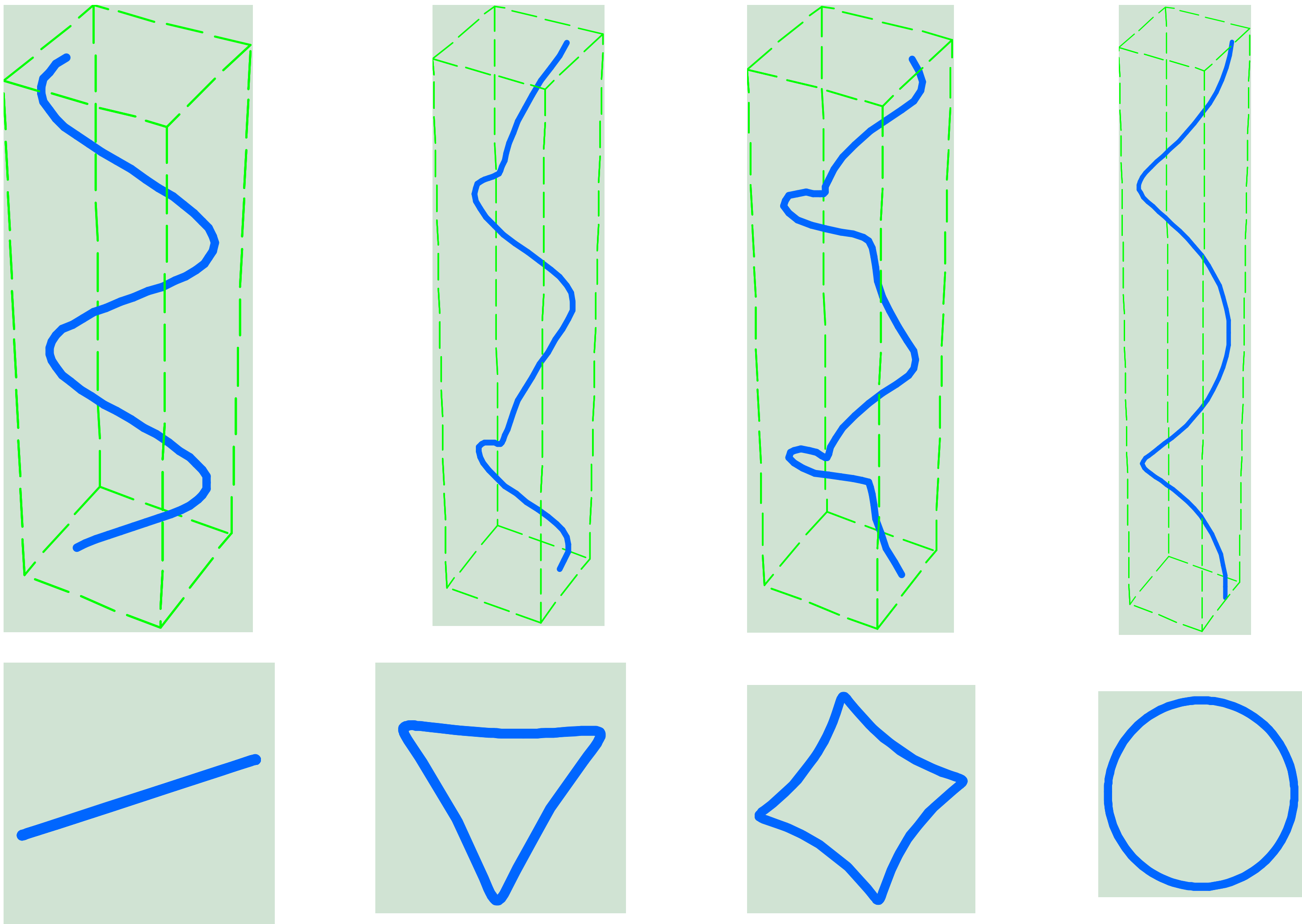
Consider a Killing vector

$$\xi = \partial_t + \Omega \partial_\phi$$

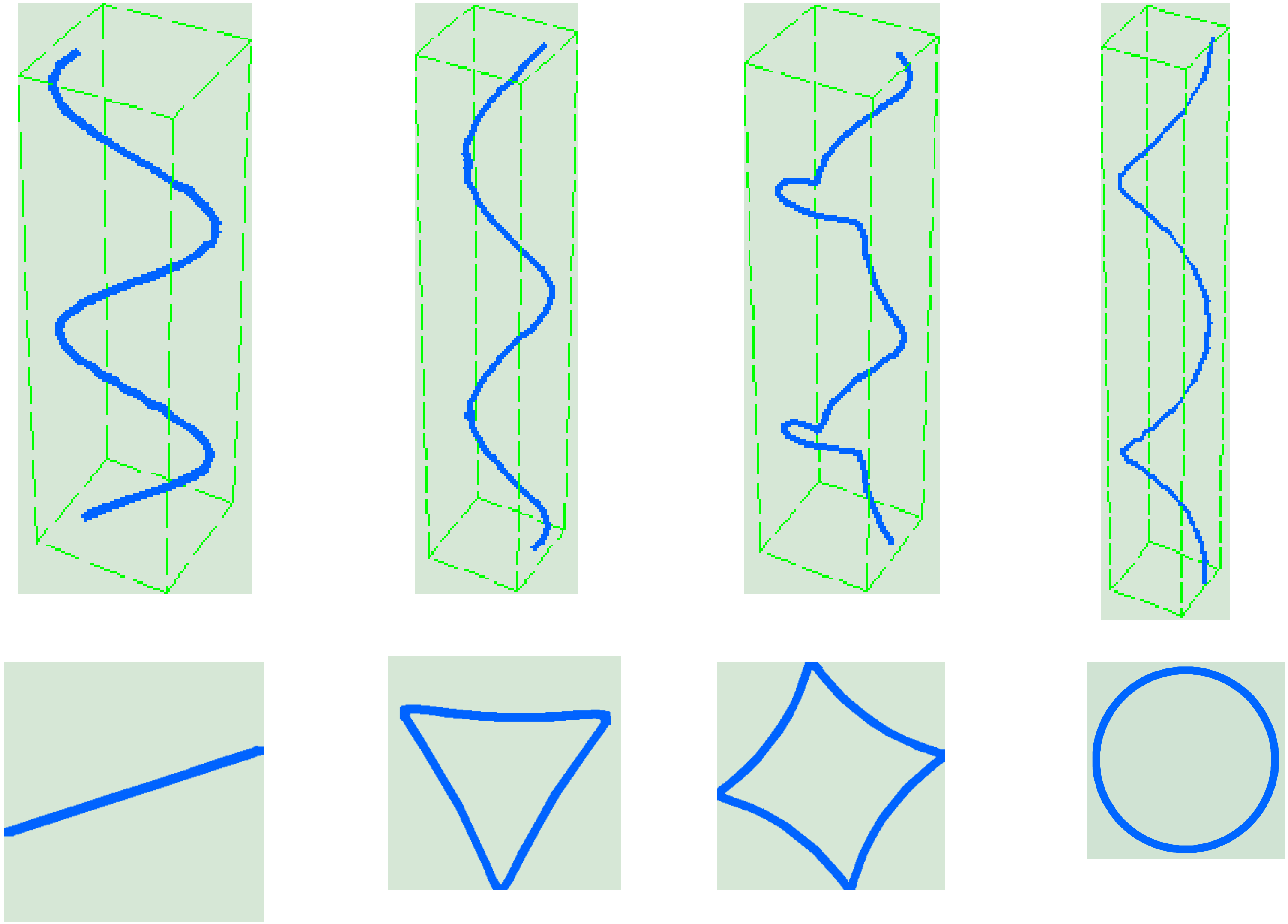


Ogawa, Ishihara,Kozaki,Nakano,Saitoh, PRD78, 023525(2008)

Analytic Solutions



Strings are Rotating



Example:
Troidal Spirals in 5D Minkowski

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\
&= -dt^2 + d\rho^2 + \rho^2 d\phi^2 + d\zeta^2 + \zeta^2 d\psi^2.
\end{aligned}
$$

$\partial_t, \partial_\phi, \partial_\psi$ are commutable Killing vectors

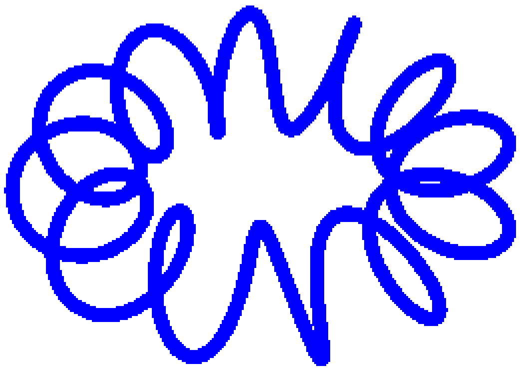
We consider C-1 strings with

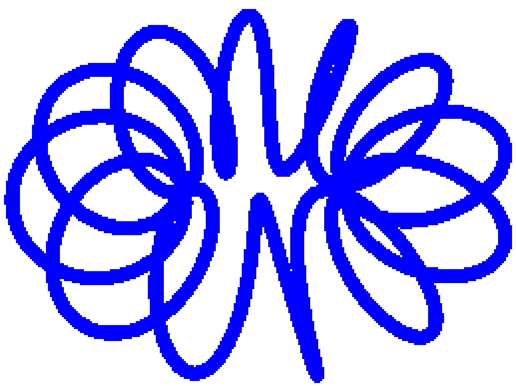
$$\xi = \partial_\phi + \alpha \partial_\psi$$

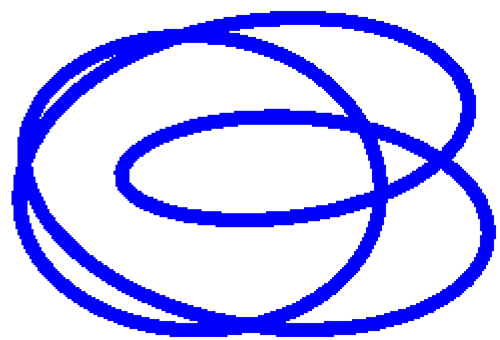
T. Igata, and H. Ishihara (2010)

T. Igata, H. Ishihara and K.Nishiwaki (2012)

Analytic Solutions







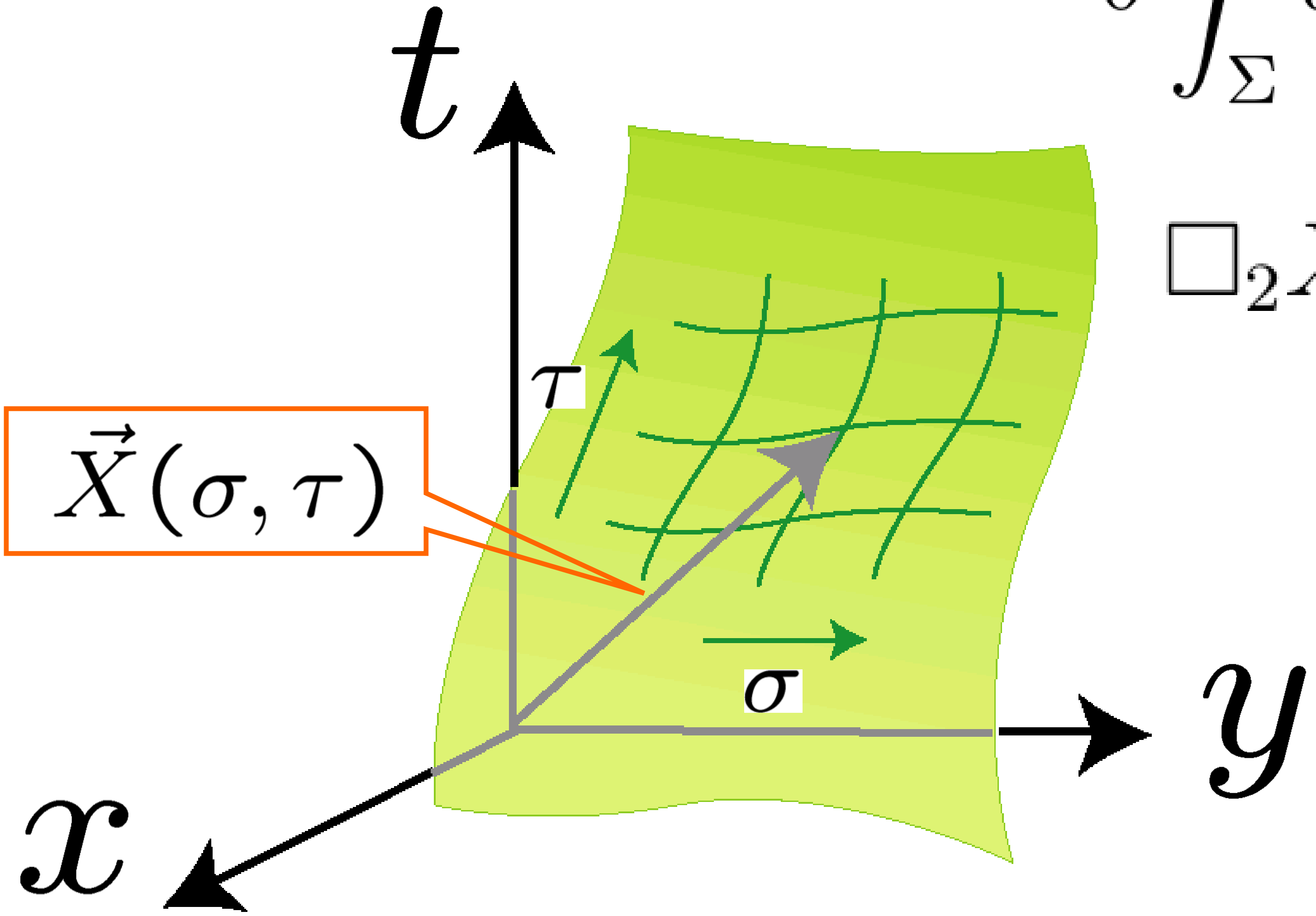
- Cohomogeneity-one strings
- Classification of Killing vectors
  (classification of C-1 strings)
- Orbit space
- Integrability of equations of motion

Cohomogeneity-one strings

World Sheet

$$\delta \int_{\Sigma} dA = 0$$

$$\Box_2 \vec{X} = 0$$

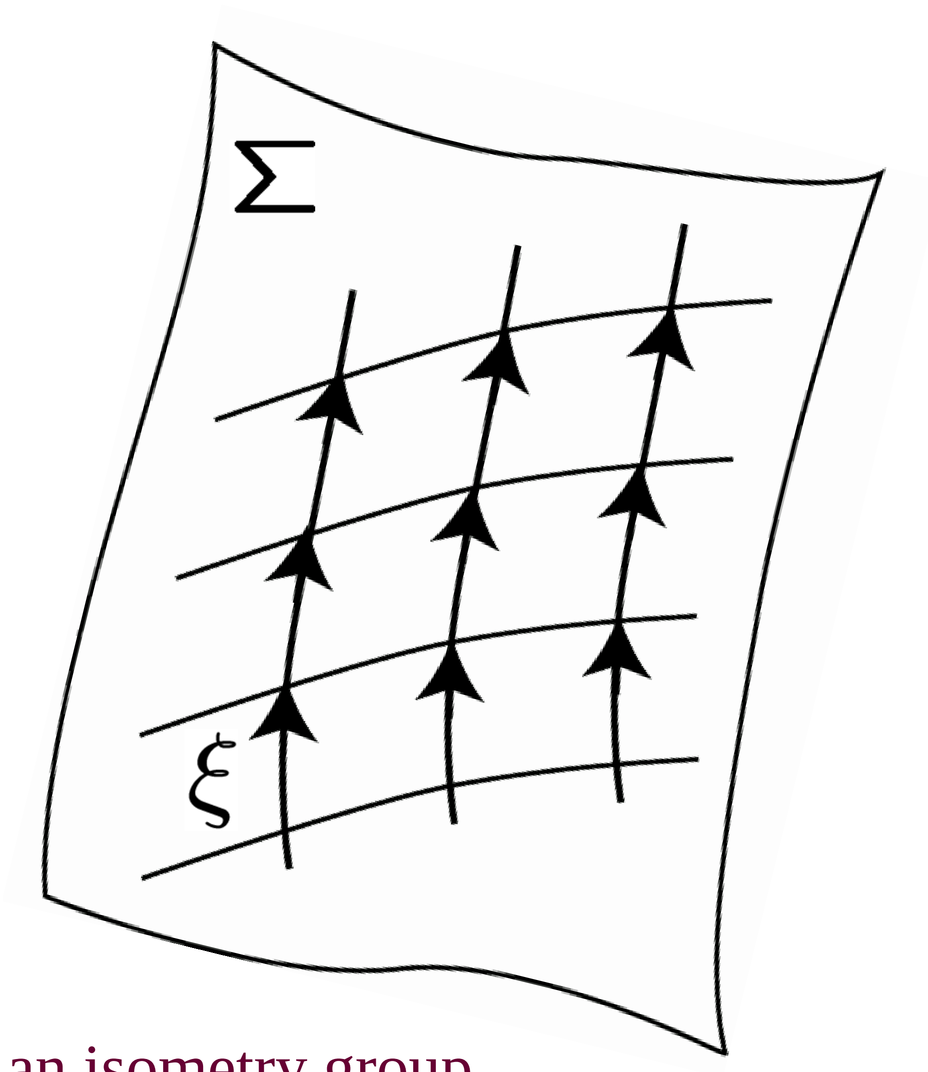


C-1 String

1-parameter isometry group
of a target space acts
on the world sheet

⇕

A Killing vector field is
tangent to the world sheet

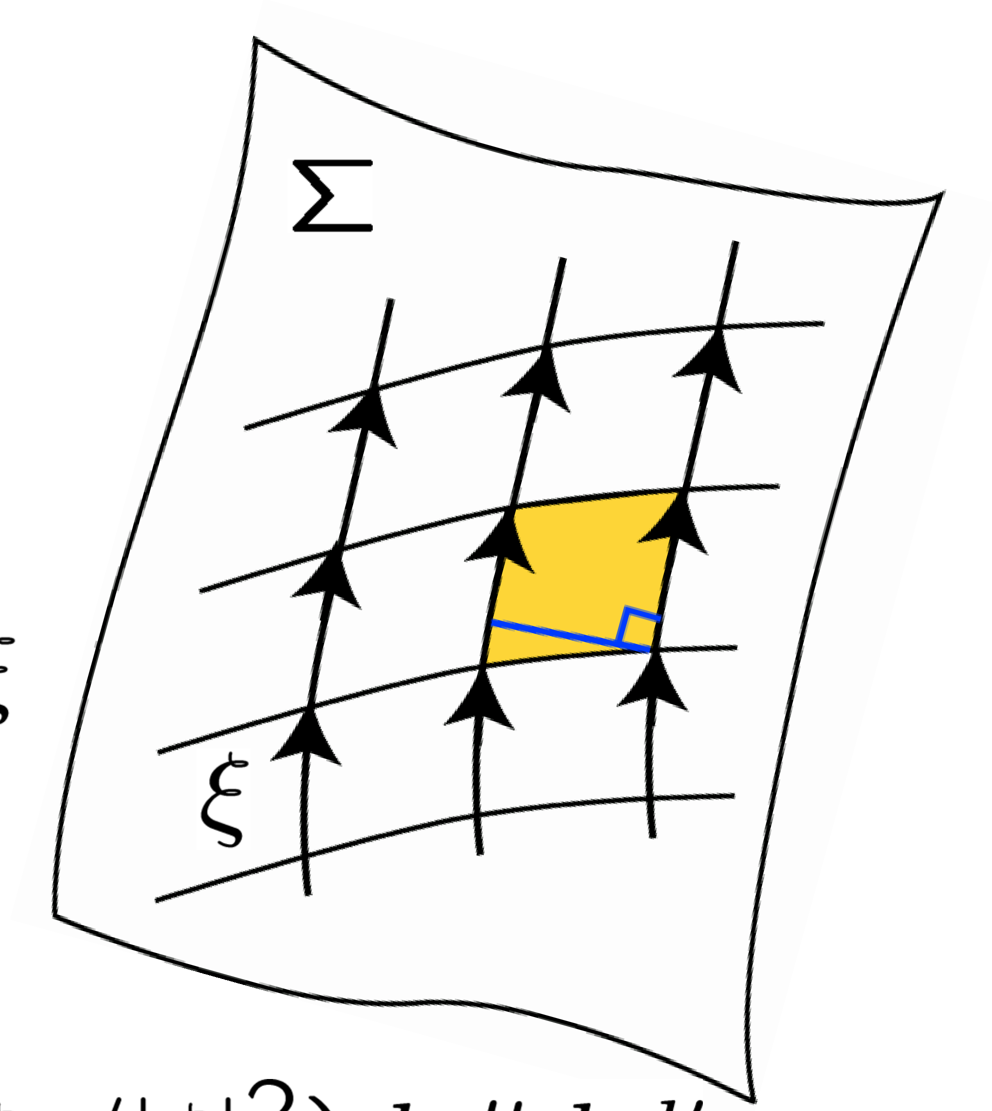


Killing vector : generator of an isometry group
(infinitesimal version of isometry)

Area elements

A Killing vector field is tangent to the world sheet

C-1 string associated with ξ



$$dA = |\xi| dl_\perp$$

$$dl_\perp^2 = (g_{\mu\nu} - \xi_\mu \xi_\nu / |\xi|^2) dx^\mu dx^\nu$$

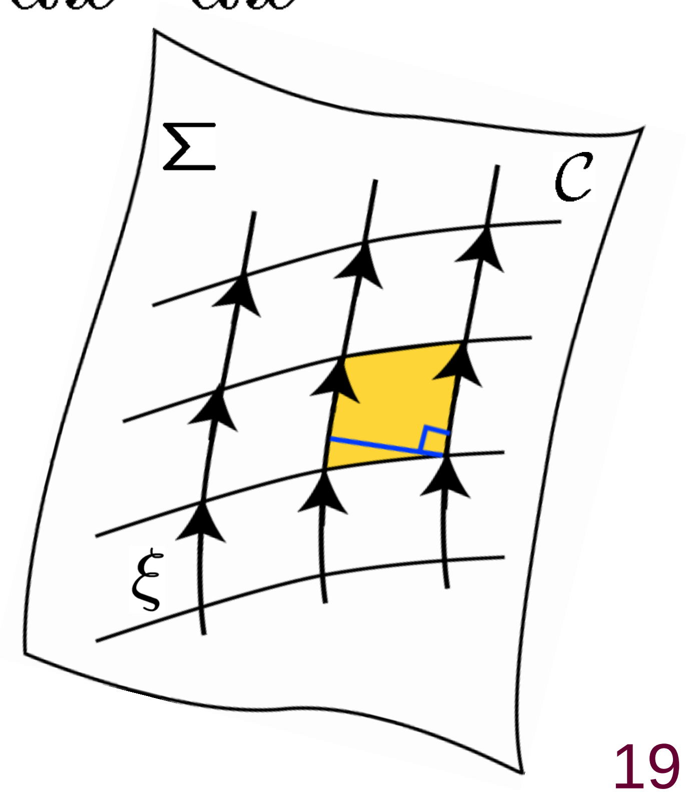
Nambu-Goto action

$$S \propto \int_\Sigma dA = \int_\Sigma |\xi| dl_\perp$$

$$= \int_C \sqrt{(\xi \cdot \xi) h_{\mu\nu} dx^\mu dx^\nu}$$

$$h_{\mu\nu} = g_{\mu\nu} - \frac{\xi_\mu \xi_\nu}{\xi \cdot \xi}$$

We get geodesic action.

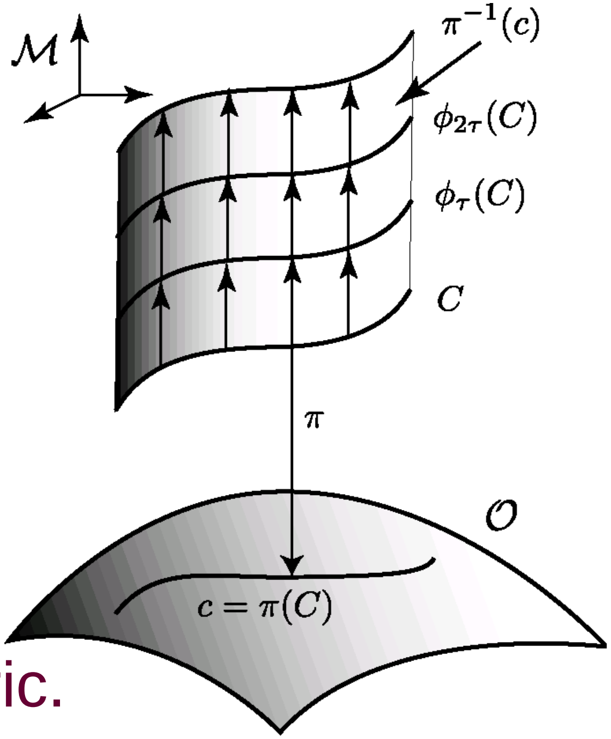


In general

- Let $(\mathcal{M}, g)$ admits isometry group ϕ generated by a Killing vector ξ .
- Consider the orbit space $\mathcal{M}/\phi$.
- We introduce the metric

$$h_{ab} = |\xi \cdot \xi| \left(g_{ab} - \frac{\xi_a \xi_b}{\xi \cdot \xi} \right)$$

on the orbit space $\mathcal{M}/\phi$.



We solve geodesic equations in the metric.

H.Ishihara and H.Kozaki, Phys.Rev. D72 (2005) 061701.

T. Koike, H. Kozaki, and H. Ishihara, Phys.Rev. D77 (2008) 125003

H. Kozaki, T. Koike, and H. Ishihara, Class.Quant.Grav. 27 (2010) 10500

Classification of Killing vectors

Equivalence of Killing vectors

Equivalence class of isometry

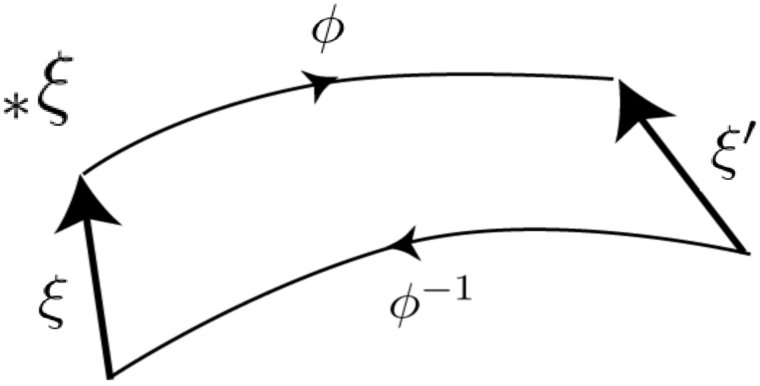
$$g,\ g' \in \mathrm{Isom}\mathcal{M} \quad g \sim g'$$

$$\Leftrightarrow \exists\phi \in \mathrm{Isom}\mathcal{M} \text{ s.t. } g' = \phi g \phi^{-1}$$

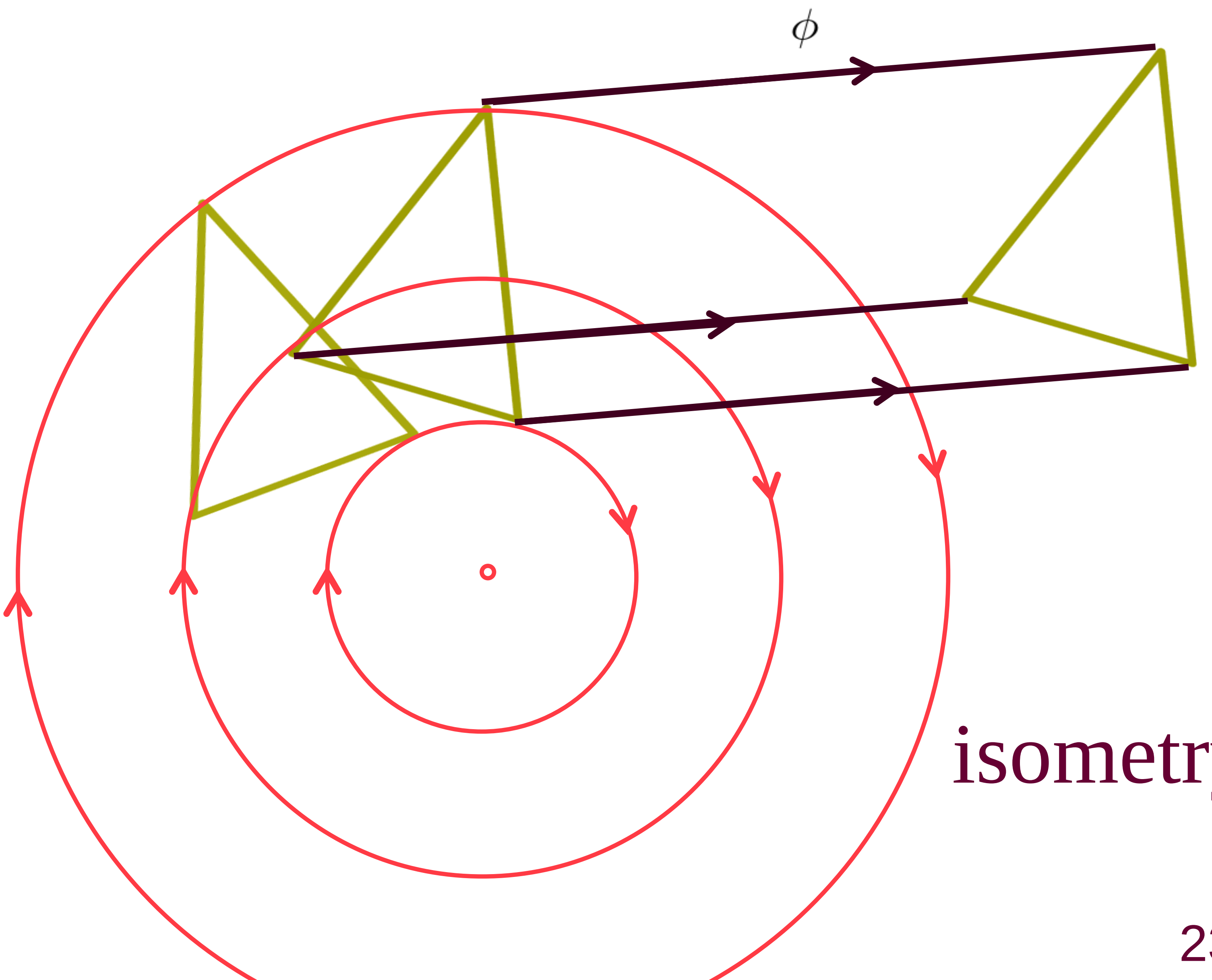
Conjugacy class

Equivalence of Killing vector

$$\xi \sim \xi' \quad \Leftrightarrow \quad \xi' = \phi_* \xi$$

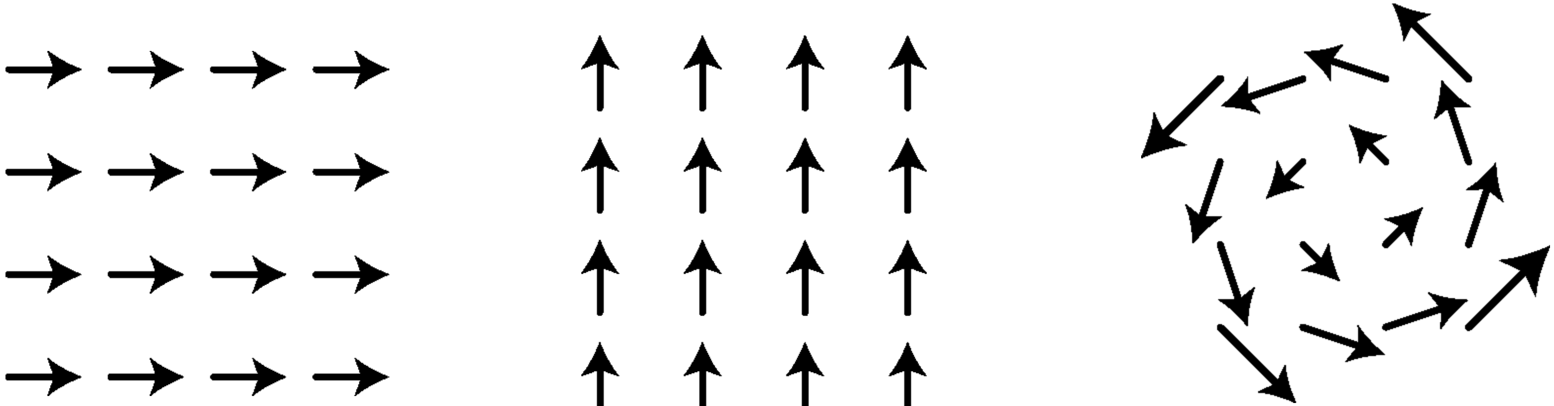


Equivalent triangles



isometry

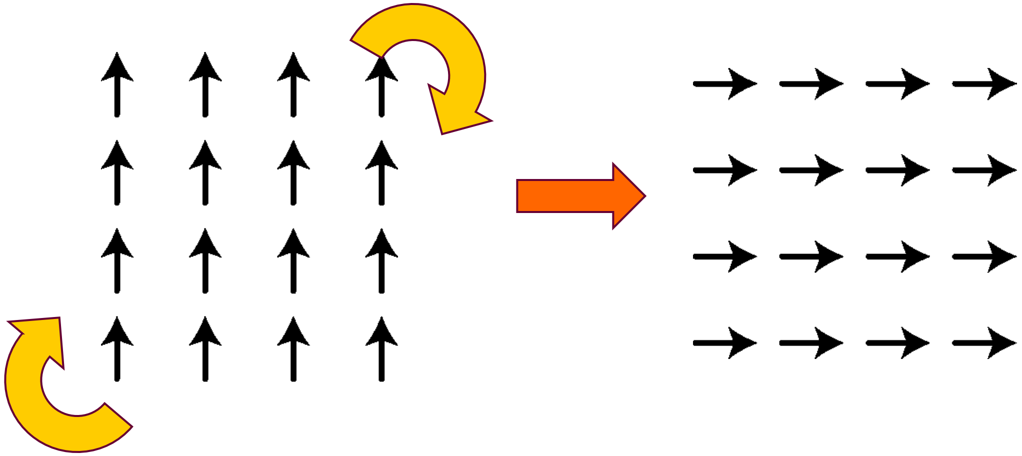
Isometries in x-y Plane



$$P_x = \partial_x \qquad P_y = \partial_y \qquad L_{xy} = x\partial_y - y\partial_x$$

3 linearly independent Klling vector fields

Equivalence Class



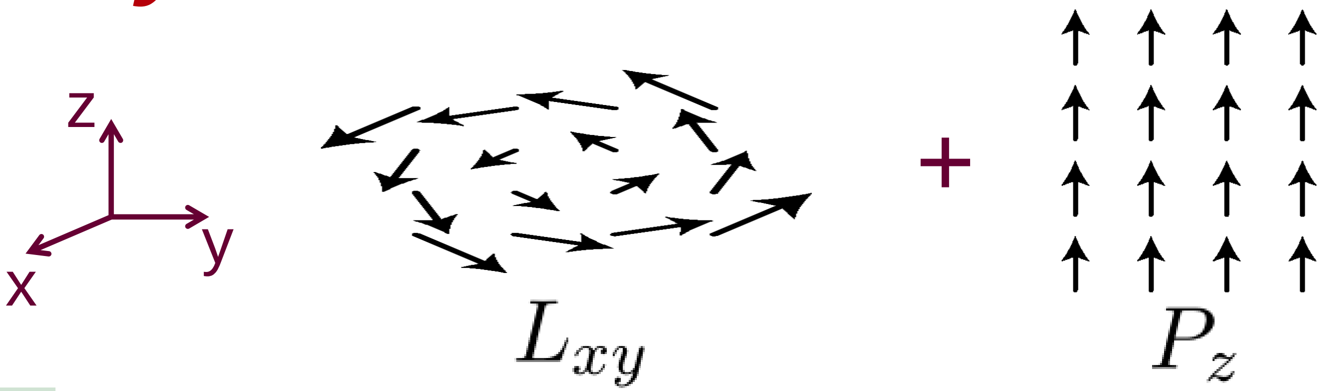
$$P_y \sim P_x$$

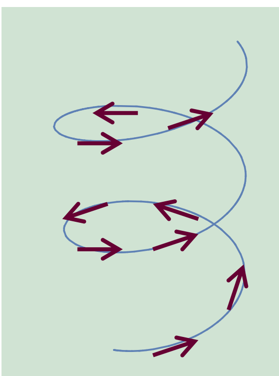
$$\alpha P_x + \beta P_y \sim P_x$$

$$\alpha P_x + \beta P_y + L_{xy} \sim L_{xy}$$

Equivalence classes $\{P_x, L_{xy}\}$

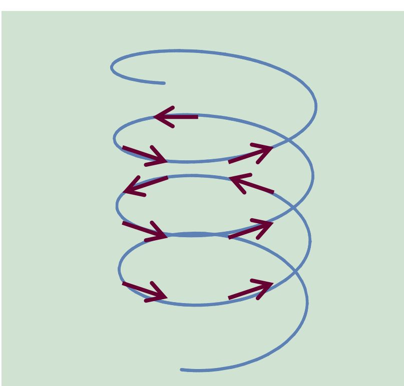
Isometry in R^3





$$\xi = L_{xy} + aP_z$$

$$a \neq a' \Rightarrow \xi \neq \xi'$$



$$\xi' = L_{xy} + a'P_z$$

Classification of Killing vectors

4-dim. Euclid

Type	Canonical form
I	$aP_z + bL_{xy}$
II	$aL_{zw} + bL_{xy}$

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Classification of Killing vectors

4-dim. Euclid

Type	Canonical form
I	$aP_z + bL_{xy}$
II	$aL_{zw} + bL_{xy}$

P : translation

L : rotation

K : Lorentz boost

4-dim. Minkowski

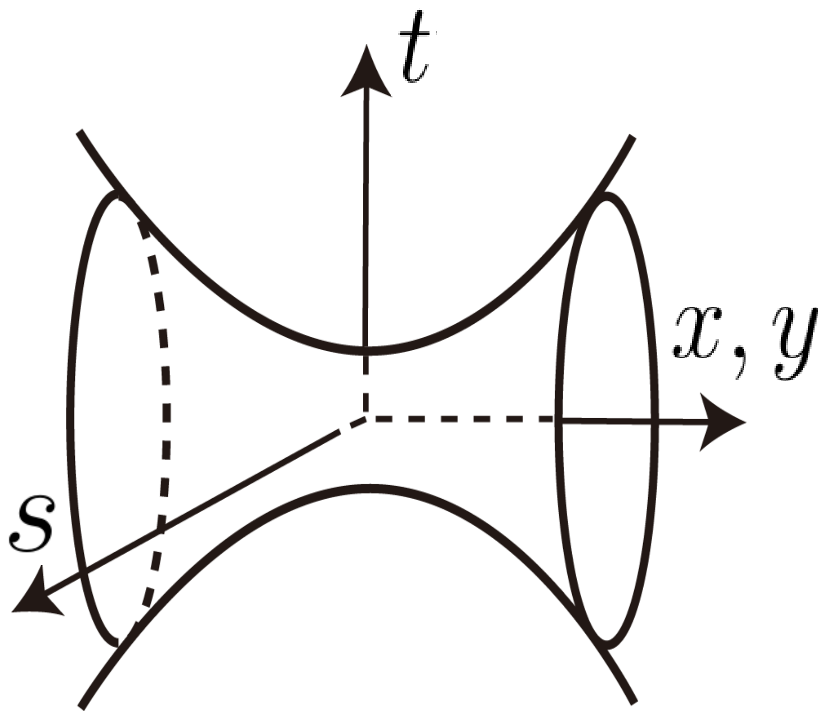
Type	Canonical form
I	$aP_t + bL_{xy}$
II	$a(P_t + P_z) + bL_{xy}$
III	$aP_z + bL_{xy}$
IV	$aP_z + b(K_{ty} + L_{xy})$
V	$aP_z + bK_{ty}$
VI	$aP_x + b(K_{ty} + L_{xy})$
VII	$aK_{tz} + bL_{xy}$

Ishihara and Kozaki PRD(2005)

5-dim. AdS spacetimev

$$dS^2 = -dt^2 - ds^2 + dx^2 + dy^2 + dz^2 + dw^2$$

$$-t^2 - s^2 + x^2 + y^2 + z^2 + w^2 = -1$$



Killing Vector Fields in AdS^5

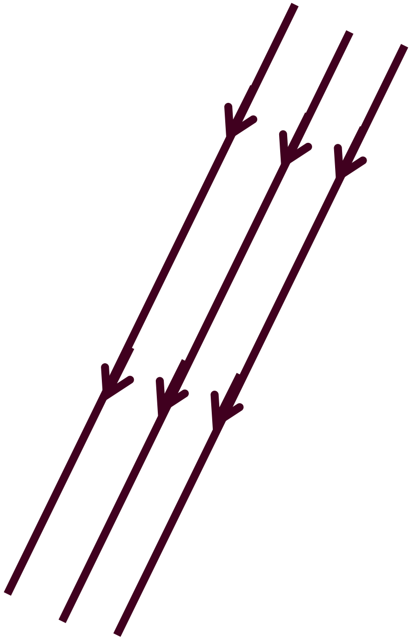
Type	Killing vector field
I	$K_{tx} + K_{sy} + L_{xy} + \widetilde{L}_{st} + 2(L_{yz} + K_{tz})$
II	$K_{tx} + \widetilde{L}_{st} + aL_{yz}$
III	$K_{tx} + L_{xy} + aL_{zw}$
IV	$K_{tx} + L_{xy} + aK_{sz}$
V	$K_{tx} + K_{xy} + L_{sw} + L_{wz} + a(L_{xw} - L_{ts} - L_{zy})$
VI	$K_{tx} + K_{sy} + aL_{zw} + b(\widetilde{L}_{st} - L_{xy})$
VII	$K_{tx} + L_{xy} + K_{sy} + \widetilde{L}_{st} + aL_{zw} + b(L_{xy} - \widetilde{L}_{st})$
VIII	$K_{tx} + L_{xy} + K_{sy} + \widetilde{L}_{st} + aL_{zw} + b(K_{ty} + K_{sx})$
IX	$a\widetilde{L}_{st} + bL_{xy} + cL_{zw} \qquad (a^2 + b^2 + c^2 = 1)$
X	$aK_{tx} + bK_{sy} + cL_{zw} \qquad (a^2 + b^2 + c^2 = 1, a \neq \pm b)$

C-1 strings in AdS_5 are classified in 10 families.

Classification using SO(4,2) ~ SU(2,2)

T.Koike, H.Kozaki, H.Ishihara, Phys.Rev. D77 (2008) 125003

Orbit space

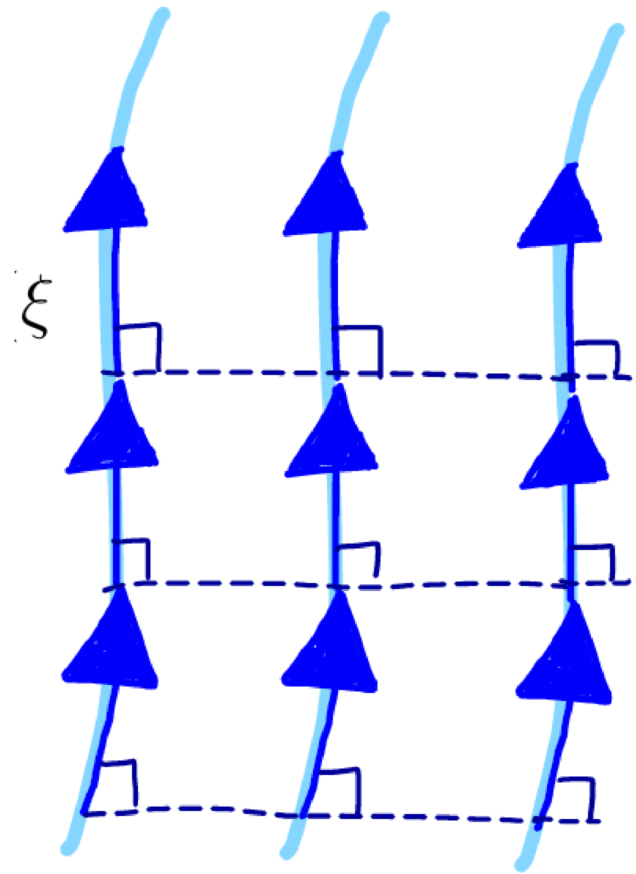


Isometry group acts

“断面”を調べる: orbit space !

Metric on an orbit space

$$\tilde{h}_{ab} = (\xi \cdot \xi)\left(g_{ab} - \frac{\xi_a \xi_b}{\xi \cdot \xi}\right)$$



この計量のもとで測地線を解く

Orbit space の対称性は ξ の選び方に依存する.

Trivial example :4-dim. Euclid space

R^4 $$ds^2 = dx^2 + dy^2 + dz^2 + dw^2$$

Killing vector for reduction

$$\xi = \partial_w \qquad \xi \cdot \xi = 1$$

Reduced metric

$$d\tilde{s}^2 = (\xi \cdot \xi) \left(g_{\mu\nu} - \frac{\xi_\mu \xi_\nu}{\xi \cdot \xi} \right) dx^\mu dx^\nu$$

$$= dx^2 + dy^2 + dz^2$$

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Non trivial example :4-dim. Euclid space

R^4 $$ds^2 = dr_1^2 + r_1^2 d\phi_1^2 + dr_2^2 + r_2^2 d\phi_2^2$$

Killing vector for reduction

$$\xi = a_1 \partial_{\phi_1} + a_2 \partial_{\phi_2} \qquad \xi \cdot \xi = a_1^2 r_1^2 + a_2^2 r_2^2$$

Reduced metric

$$d\tilde{s}^2 = (\xi \cdot \xi) \left(g_{\mu\nu} - \frac{\xi_\mu \xi_\nu}{\xi \cdot \xi} \right) dx^\mu dx^\nu$$

$$= (a_1^2 r_1^2 + a_2^2 r_2^2)(dr_1^2 + dr_2^2) - r_1^2 r_2^2 \, d\psi^2$$

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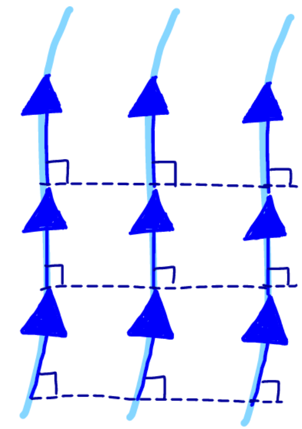
Geodesics on an orbit space

$$S = \int \sqrt{-\tilde{h}_{ab} dx^a dx^b} \ , \quad \tilde{h}_{ab} = (\xi \cdot \xi)\left(g_{ab} - \frac{\xi_a \xi_b}{\xi \cdot \xi}\right)$$



ξ

Classically equivalent

$$S = \frac{1}{2}\int \left(N^{-1}\tilde{h}_{ab}\frac{dx^a}{d\lambda}\frac{dx^b}{d\lambda} - N\right) d\lambda$$

$$= \int (p_a \dot{x}^a - NH)\, d\lambda \ , \qquad H = \frac{1}{2}(\tilde{h}^{ab} p_a p_b + 1)$$

Hamilton equations

$$\dot{x}^a = N\{H, x^a\}$$

$$\dot{p}^a = N\{H, p^a\}$$

$$H \approx 0$$

Geodesic equations

Can we integrate a C-1 string
in anti-de Sitter spacetime ?

Can we integrate geodesics
in an orbit space of a Killing vector?

The system with the degree of freedom N
is integrable in Liouville's sence,
if the number of constants of motion is N.

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Killing vector and constant of motion

Orbit space の計量 $\tilde{h}_{ab}$ が Killing vector η^c を許せば

$$\mathcal{L}_\eta \tilde{h}_{ab} = \nabla_a \eta_b + \nabla_b \eta_a = 0$$

$\eta^c p_c$ が測地線に沿って保存量となる

$$\begin{aligned}
\{H, \eta^c p_c\} &= \left\{ \frac{1}{2}(\tilde{h}^{ab} p_a p_b + 1), \eta^c p_c \right\} \\
&= \left(\tilde{h}^{ab} \partial_b \eta^c - \partial_c \tilde{h}^{ab} \eta^c \right) p_a p_c \\
&= (\nabla^a \eta^c + \nabla^c \eta^a) p_a p_c \\
&= 0
\end{aligned}$$

$\tilde{h}_{ab}$が十分な数のLie可換なKilling vector を許せば
十分な数のPoisson可換な保存量が存在する. ⇨ 積分可能

対称性の高いg_{ab}を target space とすればg_{ab} は
多くのKilling vector を許す.

$$\tilde{h}_{ab} = (\xi \cdot \xi)\left(g_{ab} - \frac{\xi_a \xi_b}{\xi \cdot \xi}\right)$$

ξと可換なg_{ab} のKilling vector は $\tilde{h}_{ab}$ のKilling vector になる.

e^{ξ}を中心とするg_{ab}の isometry の中心化群は $\tilde{h}_{ab}$ のisometry.
その代数の可換な最大部分代数の次元を調べる.

Symmetry of orbit space

4-dim. Minkowski spacetime

Type	Killing vector field	# of commutable KV
I	$aP_t + bL_{xy}$	2
II	$a(P_t + P_z) + bL_{xy}$	2
III	$aP_z + bL_{xy}$	2
IV	$aP_z + b(K_{ty} + L_{xy})$	2
V	$aP_z + bK_{ty}$	2
VI	$aP_x + b(K_{ty} + L_{xy})$	2
VII	$aK_{tz} + bL_{xy}$	1

5-dim. Anti-de Sitter spacetime

Type	Killing vector field	# of commutable KV
I	$K_{tx} + K_{sy} + L_{xy} + \widetilde{L}_{st} + 2(L_{yz} + K_{tz})$	3
II	$K_{tx} + \widetilde{L}_{st} + aL_{yz}$	2
III	$K_{tx} + L_{xy} + aL_{zw}$	2
IV	$K_{tx} + L_{xy} + aK_{sz}$	2
V	$K_{tx} + K_{xy} + L_{sw} + L_{wz} + a(L_{xw} - L_{ts} - L_{zy})$	2
VI	$K_{tx} + K_{sy} + aL_{zw} + b(\widetilde{L}_{st} - L_{xy})$	2
VII	$K_{tx} + L_{xy} + K_{sy} + \widetilde{L}_{st} + aL_{zw} + b(L_{xy} - \widetilde{L}_{st})$	2
VIII	$K_{tx} + L_{xy} + K_{sy} + \widetilde{L}_{st} + aL_{zw} + b(K_{ty} + K_{sx})$	2
IX	$a\widetilde{L}_{st} + bL_{xy} + cL_{zw} \quad (a^2 + b^2 + c^2 = 1)$	2
X	$aK_{tx} + bK_{sy} + cL_{zw} \quad (a^2 + b^2 + c^2 = 1, a \neq \pm b)$	2

足りない！

$a = b = c$ in type X symmetry is enhanced ⟶ ‘ Fubini-Study’

Discusion using root diagram by Morisawa

Killing tensor and constant of motion

If the metric admits a Killing vector

$$\mathcal{L}_\xi g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)} = 0$$

The quantity $Q = \xi^\mu p_\mu$ is conserved.

$$\dot{Q} = \{Q, H\}$$

$$= \{\xi^\mu p_\mu, H\} = (\nabla^\mu \xi^\nu) p_\mu p_\nu = 0$$

Killing tensor and constant of motion

If the metric admits a Killing vector

$$\mathcal{L}_\xi g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)} = 0$$

The quantity $Q = \xi^\mu p_\mu$ is conserved.

$$\begin{aligned}\dot{Q} &= \{Q, H\} \\ &= \{\xi^\mu p_\mu, H\} = (\nabla^\mu \xi^\nu) p_\mu p_\nu = 0\end{aligned}$$

If the metric admits a Killing tensor

$$\nabla_{(\lambda} K_{\mu\nu)} = 0$$

The quantity $Q_{(2)} = K^{\mu\nu} p_\mu p_\nu$ is conserved.

$$\begin{aligned}\dot{Q}_{(2)} &= \{Q_{(2)}, H\} \\ &= \{K^{\mu\nu} p_\mu p_\nu, H\} = (\nabla^\lambda K^{\mu\nu}) p_\lambda p_\mu p_\nu = 0\end{aligned}$$

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$\eta^a, \; \zeta^b$ がKilling vector のとき,

$$K^{ab} = \eta^a \zeta^b$$ は, Killing tensor

Reducible Killing tensor

Spherically symmetric case

L_{xy}, L_{yz}, L_{zx} Killing vectors

$$L^2 = L_{xy}L_{xy} + L_{yz}L_{yz} + L_{zx}L_{zx}$$ Reducible Killing tensor

Kerr black hole は irreducible Killing tensor をもつ

Results

- In the 5-dimensional anti-de Sitter spacetime,
  All possible orbit spaces with the metric

$$h_{ab} = |\xi \cdot \xi| \left(g_{ab} - \frac{\xi_a \xi_b}{\xi \cdot \xi} \right)$$

  are geodesically integrable thanks to a Killing tensor in addition to Killing vectors.

Integrability of geodesics on an orbit space

Integrability of Hamiltonian system

The Hamiltonian system with the degree of freedom *N*
is integrable in Liouville's sence,
if the number of independent Poisson commuting invariants
(including the Hamiltonian itself) is *N*.

$$\{H, Q_i\} = \{Q_i, Q_j\} = 0, (i = 1, 2, \cdots N - 1)$$

Restriction of Hamiltonian

$$g^{\mu\nu}p_\mu p_\nu = \left(g^{\mu\nu} - \frac{\xi^\mu \xi^\nu}{\xi \cdot \xi}\right) p_\mu p_\nu + \frac{\xi^\mu \xi^\nu}{\xi \cdot \xi} p_\mu p_\nu$$

$$= h^{\mu\nu} p_\mu p_\nu + \frac{(\xi^\mu p_\mu)(\xi^\nu p_\nu)}{\xi \cdot \xi}$$

$$H_h = \frac{1}{2} h^{\mu\nu} p_\mu p_\nu$$

$$= \frac{1}{2} g^{\mu\nu} p_\mu p_\nu |_{\xi p = 0} = H_g |_{\xi p = 0}$$

If

$$H_g = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu$$

is integrable

$$\xi^\mu p_\mu = 0$$

then

$$H_h = \frac{1}{2} h^{\mu\nu} p_\mu p_\nu$$

is integrable

$$H_g = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu \quad \longrightarrow \quad \tilde{H}_g = \frac{1}{2} (\xi \cdot \xi)^{-1} g^{\mu\nu} p_\mu p_\nu$$

$$\xi^\mu p_\mu = 0 \ \Big\downarrow \qquad\qquad \xi^\mu p_\mu = 0 \ \Big\downarrow$$

$$H_h = \frac{1}{2} h^{\mu\nu} p_\mu p_\nu \quad \longrightarrow \quad \tilde{H}_h = \frac{1}{2} (\xi \cdot \xi)^{-1} h^{\mu\nu} p_\mu p_\nu$$

If

$$H_g = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu \quad \longrightarrow \quad \tilde{H}_g = \frac{1}{2} (\xi \cdot \xi)^{-1} g^{\mu\nu} p_\mu p_\nu$$

is integrable

$$\xi^\mu p_\mu = 0 \quad \Big\downarrow \qquad\qquad \xi^\mu p_\mu = 0 \quad \Big\downarrow$$

then

$$H_h = \frac{1}{2} h^{\mu\nu} p_\mu p_\nu \quad \longrightarrow \quad \tilde{H}_h = \frac{1}{2} (\xi \cdot \xi)^{-1} h^{\mu\nu} p_\mu p_\nu$$

is integrable

Our aim

We show the system with the Hamiltonian

$$H_g = \frac{1}{2}(\xi \cdot \xi)^{-1} g^{\mu\nu} p_\mu p_\nu$$

is integrable, in the case of the metric $g_{\mu\nu}$ is AdS_5 and ξ is any Killing vector on AdS_5 .

If it is true, the system with the Hamiltonian

$$H_h = \frac{1}{2}(\xi \cdot \xi)^{-1} h^{\mu\nu} p_\mu p_\nu$$

is integrable.

The Hamiltonian system

$$\tilde{H}_g = \frac{1}{2}(\xi \cdot \xi)^{-1} g^{\mu\nu} p_\mu p_\nu = E$$

is equivalent to the system

$$H'_g = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu - E(\xi \cdot \xi) \approx 0$$

We find constants of motion of the system H'_g .

"Jaccobi's Hamiltonian"

We assume the constant of motion in the form

$$Q = \frac{1}{2} K_{(2)}^{\mu\nu} p_\mu p_\nu + E K_{(0)}$$

$$\{Q, H'_g\} = \{\frac{1}{2} K_{(2)}^{\mu\nu} p_\mu p_\nu + E K_{(0)}, \frac{1}{2} g^{\mu\nu} p_\mu p_\nu - E(\xi \cdot \xi)\}$$

$$\cdots$$

$$= \nabla^\lambda K_{(2)}^{\mu\nu}\, p_\lambda p_\mu p_\nu + E\left(K_{(2)}^{\mu\nu} \partial_\mu (\xi \cdot \xi) - \partial_\mu K_{(0)} g^{\mu\nu}\right) p_\nu$$

$$= 0$$

We have

$$\nabla^{(\lambda} K_{(2)}^{\mu\nu)} = 0$$

$$\partial^\mu K_{(0)} - K_{(2)}^{\mu\nu} \partial_\nu (\xi \cdot \xi) = 0$$

T.Igata,T.Koike, H.Ishihara,Phys.Rev. D83 (2011) 065027

$$\nabla^{(\lambda} K_{(2)}^{\mu\nu)} = 0$$

Killing tensor eqs. for AdS5
reducible Killing tensor

$$\partial^\mu K_{(0)} - K_{(2)}^{\mu\nu} \partial_\nu (\xi \cdot \xi) = 0$$

$$\partial^{[\lambda} \partial^{\mu]} K_{(0)} = \partial^{[\lambda} K_{(2)}^{\mu]\nu} \partial_\nu (\xi \cdot \xi) = 0$$

Integrability condition for $K_{(0)}$

commutability

$$\{\xi p, \eta_i p\} = \{\eta_j p, \eta_i p\} = 0$$

$$\{\xi p, K_{(2)} pp\} = \{\eta_j p, K_{(2)} pp\} = 0$$

Integrate

$$\partial^\mu K_{(0)} - K^{\mu\nu}_{(2)} \partial_\nu (\xi \cdot \xi) = 0$$

We have

$$Q = \frac{1}{2}\left(K^{\mu\nu}_{(2)} + K_{(0)} g^{\mu\nu} \right) p_\mu p_\nu$$

for all Killing vectors ξ

List of Killing tensors

II: $\xi = L_{xt} - L_{st} - aL_{yz}$

$$K_2 = (L_{xy} + L_{ys})^2 + (L_{xz} + L_{zs})^2 + a^2(L_{xw}^2 - L_{xs}^2 - L_{xt}^2 - L_{ws}^2 - L_{wt}^2 + L_{st}^2)$$

$$K_0 = -a^2(x-s)^2$$

V: $\xi = L_{xt} + L_{ys} - L_{yz} \mp L_{xw} + a(L_{xy} + L_{st} \mp L_{zw})$

$$K_2 = (L_{xz} - L_{xs} \pm L_{yw} - L_{yt})^2 + (\pm L_{xw} - L_{xt} - L_{yz} + L_{ys})^2 + 4a[(L_{xz} - L_{xs} \pm L_{yw} - L_{yt})(L_{zs} \mp L_{wt}) + (\pm L_{xw} - L_{xt} - L_{yz} + L_{ys})(L_{zt} \pm L_{ws})]$$

$$K_0 = 4a^2[(z-s)^2 + (w \mp t)^2]$$

X: $\xi = aL_{xt} + bL_{ys} + cL_{zw}$

$$K_2^{(1)} = (b^2 + c^2)\left(L_{xy}^2 - L_{xs}^2 - L_{yt}^2 + L_{st}^2\right) + (a^2 - b^2)\left(-L_{yz}^2 - L_{yw}^2 + L_{zs}^2 + L_{ws}^2\right)$$

$$K_0^{(1)} = (a^2 - b^2)(b^2 + c^2)(y^2 - s^2)$$

Results

- We consider the 5-dimensional AdS, for example.
- We show all possible orbit spaces with the metric

$$\tilde{h}_{\mu\nu} = (\xi \cdot \xi)\left(g_{\mu\nu} - \frac{\xi_\mu \xi_\nu}{\xi \cdot \xi}\right)$$

are geodesically integrable.

Cohomogeneity-one strings in AdS_5 are integrable.

Future works

- Arbitrary dimensions
- AdS x M
- Extream black holes
- Black holes in AdS