

$N=3$ higher spin holography and Higgs phenomenon

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Based on collaborations with

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1. INTRODUCTION

Higher spin gauge theory

- Higher spin gauge theory

- A totally symmetric rank-s field

$$\varphi_{\mu_1 \dots \mu_s} \sim \varphi_{\mu_1 \dots \mu_s} + \partial_{(\mu_1} \xi_{\mu_2 \dots \mu_s)}$$

- Natural extension of electromagnetism ($s=1$) and gravity ($s=2$)
- **Vasiliev theory** is famous as a non-trivial theory on AdS

- Applications

- Tensionless limit of **superstring theory**
 - Superstring theory includes massive higher spin states, and the tensionless limit should be related to higher spin gauge theory
- Simplified version of **AdS/CFT correspondence**
 - More tractable AdS/CFT correspondence can be constructed than using superstring theory

AdS_4/CFT_3

- Klebanov-Polyakov proposal '02

4d Vasiliev theory \longleftrightarrow 3d $O(N)$ vector model

- Correlation functions [Giombi-Yin '09-'10]
 - CFT correlators are reproduced from Vasiliev theory w/ GKP-W relation
- Role of higher spin symmetry [Maldacena-Zhiboedov '11-'12]
 - Higher spin symmetry is mostly enough to fix the correlators
- **ABJ triality** [Chang-Minwalla-Sharma-Yin '12]
 - 4d extended Vasiliev theory \Leftrightarrow 3d ABJ theory \Leftrightarrow Superstrings on $AdS_4 \times CP^3$
 - Concrete relations between superstrings and higher spin fields via AdS/CFT

AdS_3/CFT_2

- Gaberdiel-Gopakumar proposal '10

3d Vasiliev theory \longleftrightarrow 2d large N minimal model

- More tractable than AdS_4/CFT_3
 - 3d higher spin gauge theory is topological
 - 2d conformal symmetry is enhanced to be infinite dimensional
- Extensions
 - Supersymmetry [CHR,Candu,Gaberdiel,Beccaria,Groher '12-'13]
 - AdS_3 version of ABJ triality [Gaberdiel-Gopakumar,CHR '13-'15]
 - 3d extended Vasiliev theory \Leftrightarrow 2d coset model \Leftrightarrow Superstrings on $AdS_3 \times M^7$
 - Higgs phenomenon of higher spin fields [HR, CH '15, Gaberdiel-Peng-Zadeh '15]

Plan of the talk

1. Introduction
2. Higher spin gauge theory
3. $\text{AdS}_4/\text{CFT}_3$: ABJ triality
4. $\text{AdS}_3/\text{CFT}_2$: Our conjecture
5. Conclusion

2. HIGHER SPIN GAUGE THEORY

Free theory for higher spin fields

- Higher spin gauge symmetry

$$\delta\varphi_{\mu_1\ldots\mu_s} = \partial_{(\mu_1}\xi_{\mu_2\ldots\mu_s)}, \quad \varphi_{\lambda\sigma\mu_3\ldots\mu_s}^{\lambda\sigma} = 0, \quad \xi_{\lambda\mu_3\ldots\mu_s}^{\lambda} = 0$$

- Equations of motion [Fronsdal '78]

$$\mathcal{F}_{\mu_1\ldots\mu_s} \equiv \square\varphi_{\mu_1\ldots\mu_s} - \partial_{(\mu_1}\partial^{\lambda}\varphi_{|\mu_2\ldots\mu_s)\lambda} + \partial_{(\mu_1}\partial_{\mu_2}\varphi_{\mu_3\ldots\mu_s)\lambda}^{\lambda} = 0$$

$$\left(\begin{array}{l} \partial^{\mu}F_{\mu\nu} = \partial^{\mu}\partial_{\mu}A_{\nu} - \partial_{\nu}\partial \cdot A = 0 \text{ (spin 1)} \\ R_{\mu\nu} = \square h_{\mu\nu} - \partial_{\mu}\partial \cdot h_{\nu} - \partial_{\nu}\partial \cdot h_{\mu} + \partial_{\mu}\partial_{\nu}h_{\lambda}^{\lambda} = 0 \text{ (spin 2)} \end{array} \right)$$

- Action

$$S = \frac{1}{2} \int d^D x \varphi^{\mu_1\ldots\mu_s} \left(\mathcal{F}_{\mu_1\ldots\mu_s} - \frac{1}{2} \eta_{(\mu_1\mu_2} \mathcal{F}_{\mu_3\ldots\mu_s)\lambda}^{\lambda} \right)$$

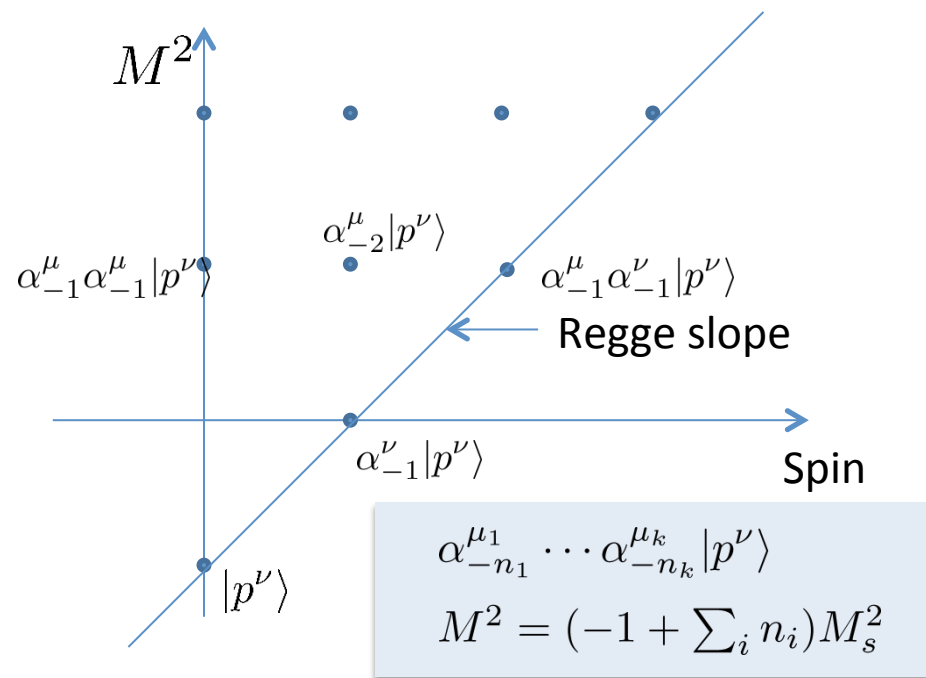
– Uniquely fixed by the gauge transformation

Interacting theory

- Difficulty
 - Free theory of higher spin fields is not so difficult
 - No-go theorems [e.g., Weinberg '64] forbid **non-trivially interacting** higher spin gauge theory (with some assumptions)
- Non-trivial theories
 - Vasiliev theory
 - Defined on AdS space with all higher spins ($s = 2, 3, \dots, \infty$)
 - Only equations of motion are known
 - Higher spin AdS_3 gravity
 - Topological theory (Chern-Simons descriptions)

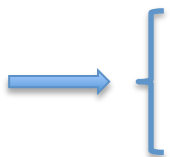
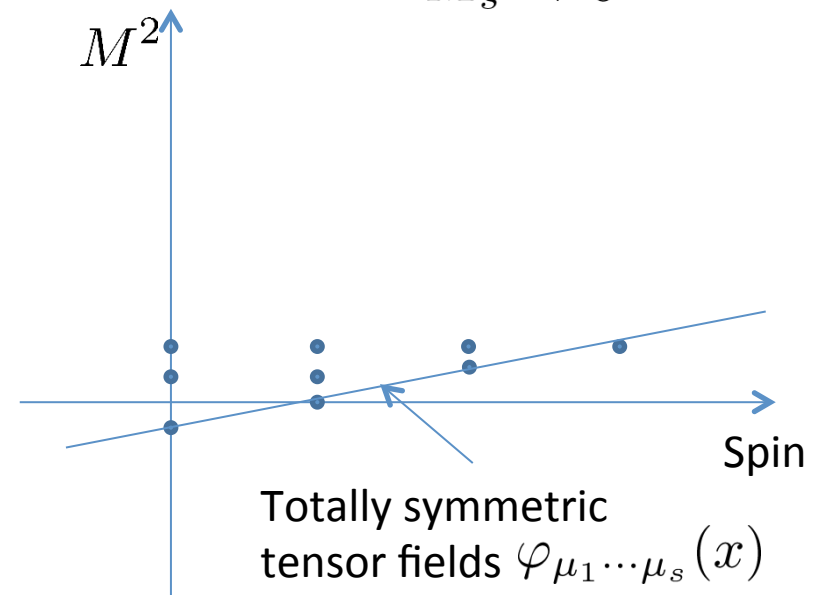
Strings \Leftrightarrow Higher spin fields

- String spectrum



- Tensionless limit

$$M_s \rightarrow 0$$



- Higher spin gauge symmetry may appear at the tensionless limit
- The theory can be examined using the (broken) large symmetry

Towards the Gross's speculation

- Backgrounds
 - Superstring theory may be given by a broken phase of higher spin gauge theory [Gross '88]
 - Recent developments are made by working on **AdS space**
 - **Vasiliev theory** as a non-trivial higher spin gauge theory on AdS space
 - **AdS/CFT** correspondence utilizing the Vasiliev theory
- Concrete proposals
 - Relations between superstrings and higher spin fields via AdS/CFT
 - $\text{AdS}_4/\text{CFT}_3 \rightarrow$ ABJ triality [Chang-Minwalla-Sharma-Yin '12]
 - $\text{AdS}_3/\text{CFT}_2 \rightarrow$ AdS_3 versions [Gaberdiel-Gopakumar, CHR '13-'15], Higgsing [HR, CH '15, Gaberdiel-Peng-Zadeh '15]

3. ADS_4/CFT_3 : ABJ TRIALITY

Klebanov-Polyakov

- Klebanov-Polyakov conjecture '02

4d Vasiliev theory \longleftrightarrow 3d $O(N)$ **vector** model

- $O(N)$ vector model

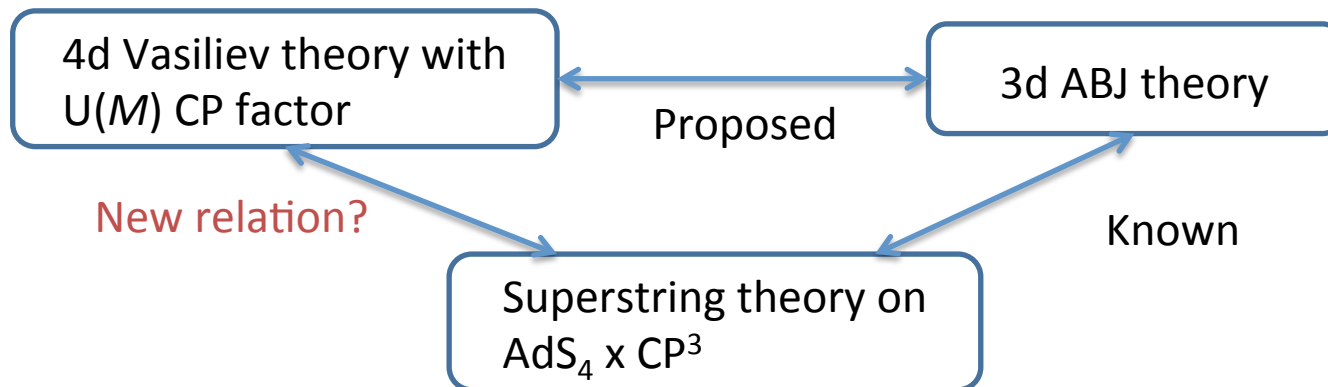
$$S = \sum_{i=1}^N \int d^3x \partial^\mu h^i \partial_\mu h^i \quad + \text{ } O(N) \text{ invariant constraint}$$

- State counting

	Bulk fields	Higher spin currents
Vector -like model	One higher spin field $\varphi_{\mu_1 \dots \mu_s}$	$\sum_{i=1}^N h^i \partial_{(\mu_1} \dots \partial_{\mu_s)} h^i$
Matrix-like model	Many string states with fixed total spin $\alpha_{-n_1}^{\mu_1} \dots \alpha_{-n_k}^{\mu_k} p\rangle$	$\text{tr}[\nabla^{m_1} X \nabla^{m_2} X \dots \nabla^{m_j} X]$

ABJ triality

- Klebanov-Polyakov proposal '02
 - 4d Vasiliev theory \Leftrightarrow 3d $O(N)$ vector model
- ABJ triality [Chang-Minwalla-Sharma-Yin '12]
 - HS side: 4d Vasiliev theory with $U(M)$ Chan-Paton factor
 - CFT side: 3d $U(N)_k \times U(M)_{-k}$ Chern-Simons-Matter theory (ABJ theory)
 - String side: Superstring theory on $AdS_4 \times CP^3$



Adding CP factor

- 3d ABJ theory
 - Bi-fundamentals under $U(N) \times U(M)$ gauge symmetry

$$A_i^\alpha, B_\beta^j \quad (i, j = 1, 2, \dots, N, \quad \alpha, \beta = 1, 2, \dots, M)$$

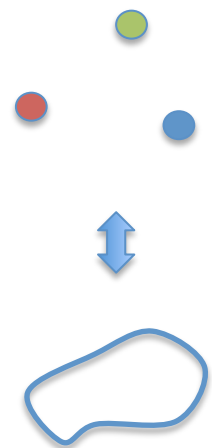
- Higher spin region: $M \ll N$
 - 't Hooft parameter is stronger for $U(N)$ than $U(M)$

$U(N)$ invariant currents

Higher spin fields

$$[J_{\mu_1 \dots \mu_s}]_\beta^\alpha = A_i^\alpha \partial_{(\mu_1} \dots \partial_{\mu_s)} B_\beta^i \quad \longleftrightarrow \quad [\varphi_{\mu_1 \dots \mu_s}]_\beta^\alpha$$

- String region: $M \approx N \gg 1$
 - $\text{tr}[ABAB \dots AB] \Leftrightarrow$ strings
 - Single-string state \Leftrightarrow Multi-particle state of higher spin fields



4. $\text{ADS}_3/\text{CFT}_2$: OUR CONJECTURE

Gaberdiel-Gopakumar

- Gaberdiel-Gopakumar conjecture '10

3d Vasiliev theory \longleftrightarrow 2d W_N minimal model

- 3d Vasiliev theory [Prokushkin-Vasiliev '98]
 - Massless sector: Gauge fields with spin $s = 2, 3, \dots, \infty$
 - Massive sector: Complex scalar fields with mass $M^2 = -1 + \lambda^2$
- Minimal model w.r.t. W_N algebra
 - Coset description: $(\mathfrak{su}(N)_k \oplus \mathfrak{su}(N)_1) / \mathfrak{su}(N)_{k+1}$
 - 't Hooft limit: $k, N \rightarrow \infty$, $\lambda = N/(N+k)$: finite
- Evidence
 - Symmetry, partition function, correlation functions,...

Lower dimensional triality

- Gaberdiel-Gopakumar proposal '10
 - 3d Vasiliev theory \Leftrightarrow 2d W_N minimal model
- Extension [CHR '13] (c.f. [Gaberdiel-Gopakumar '13] for $M=2$)
 - HS side: 3d Vasiliev theory with $U(M)$ CP factor
 - CFT side: 2d coset-type model at 't Hooft limit
$$\frac{\mathfrak{su}(N+M)_k \oplus \mathfrak{so}(2NM)_1}{\mathfrak{su}(N)_{k+M} \oplus \mathfrak{u}(1)}$$
- Related superstring theory
 - $N=4$ holography [Gaberdiel-Gopakumar '13-'15]
 - $N=4$ SUSY \rightarrow Superstrings on $AdS_3 \times M^7$ ($M^7 = S^3 \times S^3 \times S^1$ or $S^3 \times T^4$)
 - $U(2)$ CP factor \rightarrow String bit picture is obscure
 - Holography with $U(M)$ CP factor [CHR '14, HR '15]
 - $N=3$ SUSY at $k=N+M \rightarrow M^7 = SO(5)/SO(3)$ (or $SU(3)/U(1)$)??
 - BPS spectrum is shown to agree (cf. [Argurio-Giveon-Shomer '00])

Marginal deformation & Higgsing

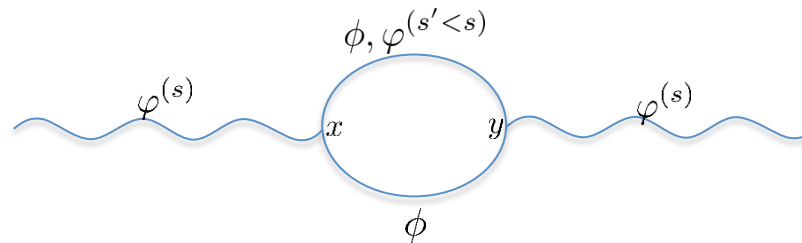
[HR '15]

- Turn on **string tension**

Superstring theory	CFT	Higher spin gauge theory
Tensionless limit	2d $N=3$ coset model	3d $N=3$ Vasiliev theory
Turning on string tension	Double-trace type deformations	Change of boundary conditions for bulk fields

← [Witten '01] →

- Higgs mass of **spin s fields** from one loop effects



– **Non-standard boundary conditions** for ϕ induces non-trivial mass term

- For a massive graviton [Porrati '01, Duff-Liu-Sati '02, Kiritsis '06, Aharony-Clark-Karch '06]
- For higher spin fields on AdS_4 [Girardello-Porrati-Zaffaroni '02]

CFT methods

- Higgs phenomenon from CFT

- Conserved current \Leftrightarrow Higher spin fields are massless

$$\partial \cdot J^{(s)} = 0$$

- Non-conserved current \Leftrightarrow Higher spin fields are massive

$$\partial \cdot J^{(s)} = \alpha \mathcal{O}^{(s-1)}$$

- Higgs mass from scaling dimension

- Scaling dimension can be computed by

$$\begin{aligned} |\partial \cdot J^{(s)}|^2 &\propto (\Delta - s - d + 2) \langle J^{(s)} | J^{(s)} \rangle \\ &= |\alpha \mathcal{O}^{(s-1)}|^2 = \alpha^2 \langle \mathcal{O}^{(s-1)} | \mathcal{O}^{(s-1)} \rangle \end{aligned}$$

- Dictionary for $\text{AdS}_{d+1}/\text{CFT}_d$

$$M_{(s)}^2 = \Delta(\Delta - d) - (d + s - 2)(s - 2)$$

Our results

- The masses of **spin s fields**
 - Leading in $1/N$ (or $1/c$) but all order in f^2 [HR, CH '15]

$$\begin{cases} M_{(s)}^2 = 0 & (\text{so}(3)_R \text{ singlet}) \\ M_{(s)}^2 = \frac{12(s-1)}{c} \frac{f^2}{(1+f^2)^2} & (\text{so}(3)_R \text{ triplet}) \end{cases}$$

- Comments $\left(c = \frac{3}{2}MN = \frac{3}{2G_N}\right)$
 - $M^2 = 0$
 - Similar results were obtained at the leading order of f in [Gaberdiel-jin-Li '13]
 - Probably masses are generated at the higher order of M/N except for $s=2$
 - $M^2 \propto s-1$
 - Superstrings with pure NSNS-flux?? (M/N -corrections should be checked)
 - $M^2 \approx s \log(s) \Leftrightarrow$ superstrings with pure RR-flux [Gaberdiel-Peng-Zadeh '15]

5. CONCLUSION

Summary

- Three trialities among higher spin fields, strings and CFT

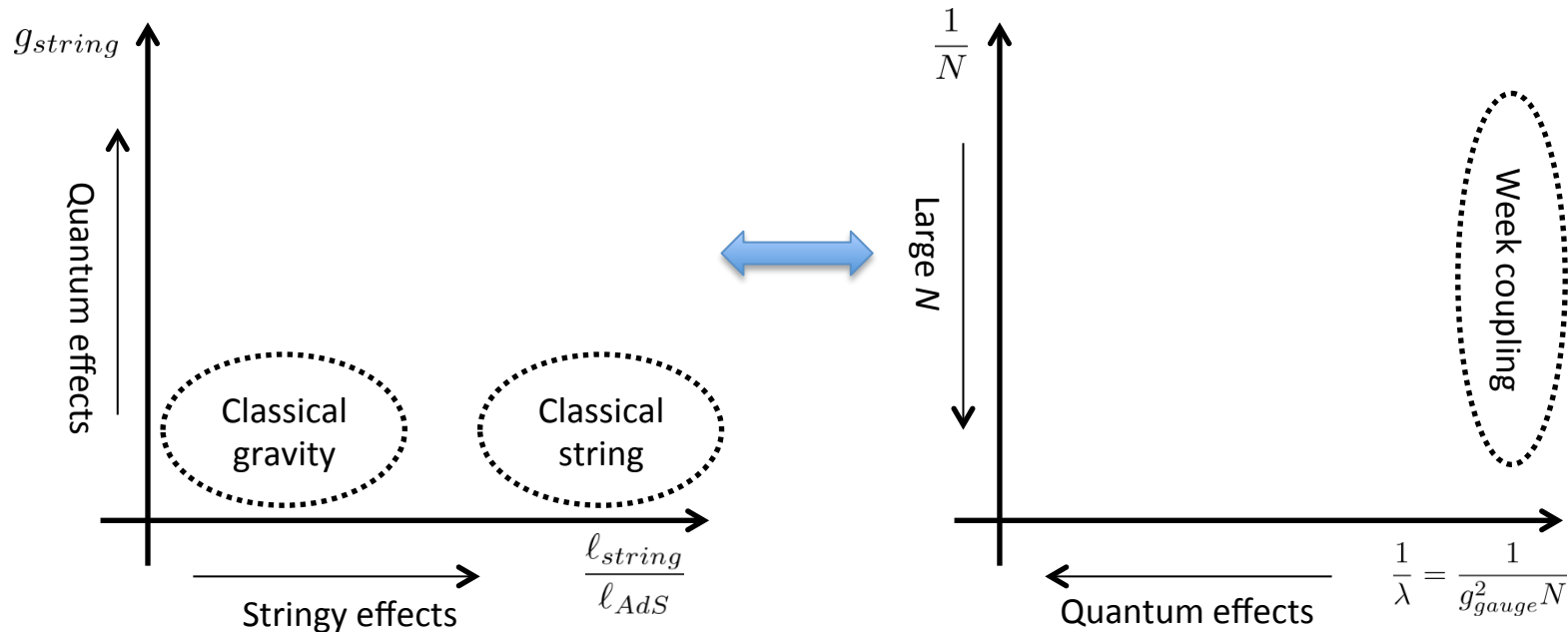
	CFT \Leftrightarrow Strings	HS \Leftrightarrow Strings	Tractability
ABJ triality (AdS_4)	○	○	△
$N=4$ triality (AdS_3)	○	△	○
$N=3$ triality (AdS_3)	△	○	○

- Higgs masses from the symmetry breaking
 - Compare to string spectrum
 - M/N -corrections should be computed
 - Understand $\text{AdS}_3/\text{CFT}_2$ with $N=3$ SUSY
 - Generalize to the ABJ triality
 - The methods for 3d CFTs have been developed

The map of AdS/CFT

- Superstrings on $AdS_5 \times S^5$

- 4d $U(N)$ gauge theory



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- Tensionless limit of string theory (**higher spin gauge theory**) can be dual to a perturbative region of gauge theory
 - Higher spin gauge theory is easier to solve than string theory