Anomaly mediated gaugino mass and path integral measure

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Plan

- Heavy sfermion scenario
- How important anomaly mediation is
- Review derivation by conformal compensator
- “Puzzle” in superspace formulation
- Solution : Path-integral measure

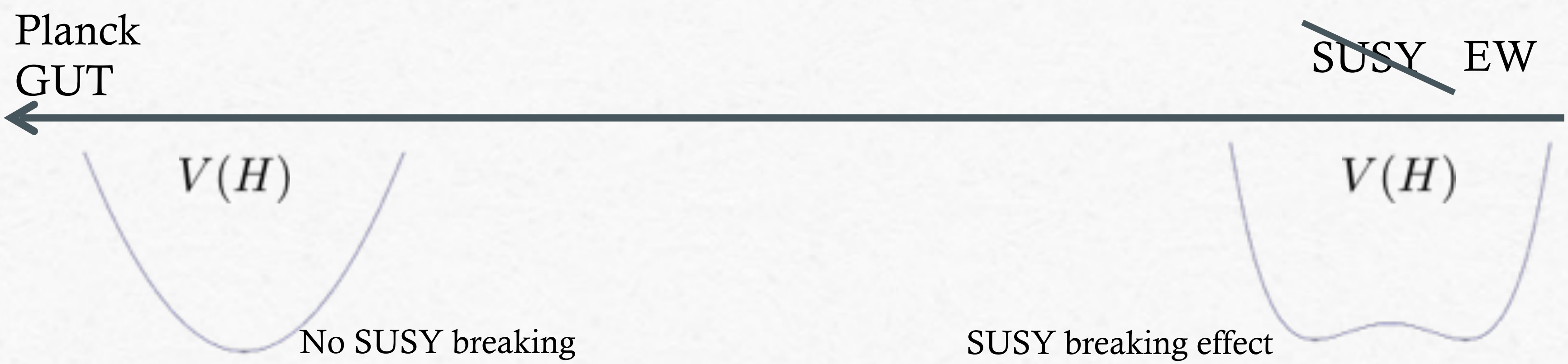
Heavy sfermion scenario

~Theoretical background and motivation~

Why I consider SUSY

- Natural candidate of dark matter : lightest SUSY particle (LSP)
- Electroweak scale may be (nearly) obtained by dimensional transmutation

Witten (1981)



Heavy sfermion scenario

Giudice, Luty, Murayama, Rattazzi (1998)
Wells (2003), Ibe, Moroi, Yanagida (2006)
Hall, Nomura (2012)

SUSY breaking field is charged or composite Z

- Typical for dynamical SUSY breaking models
- No Polonyi problem (moduli problem)

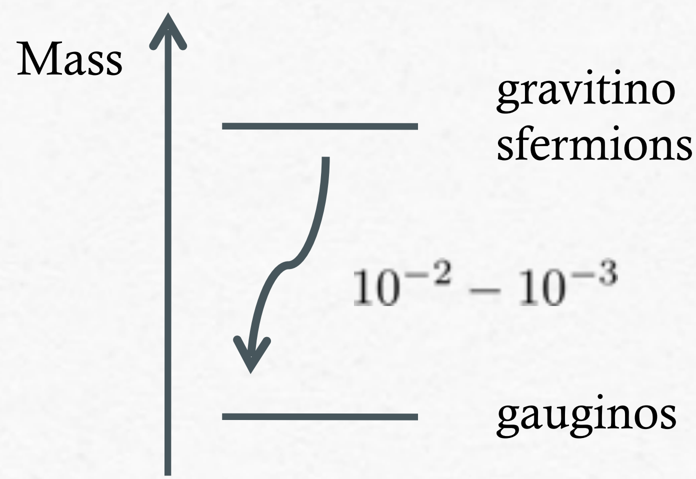
- Soft scalar masses

✔ $$\int d^4\theta \frac{ZZ^\dagger}{M_{\rm PL}^2} QQ^\dagger$$

- Gaugino masses

✗ $$\int d^2\theta \frac{Z}{M_{\rm PL}} W^\alpha W_\alpha$$

only 1-loop effects: anomaly mediation etc.



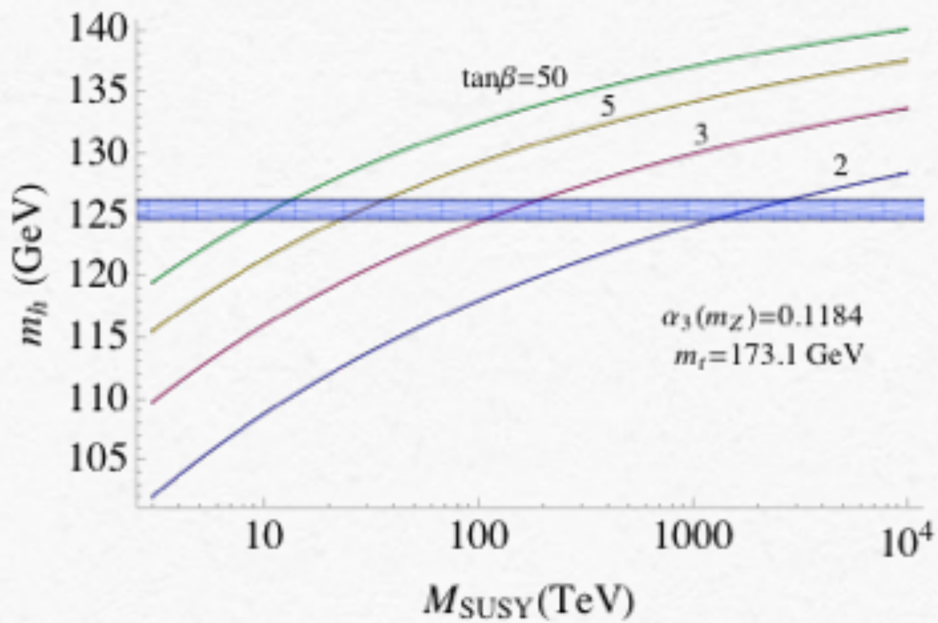
Higgs mass and dark matter

$$(m_h)_{\rm tree} < m_Z$$

$$(m_h)_{\rm obs} \simeq 125 \ {\rm GeV}$$

Suggest large SUSY breaking

Okada, Yamaguchi, Yanagida (1991)
Ellis, Ridolfi, Zwirner (1991) Haber, Hempfling (1991)
Hahn, Heinemeyer, Hollik, Rzehak, Weiglein (2014)



$$m_0 = O(100) \ {\rm TeV}$$

$$m_{1/2} = O(1) \ {\rm TeV}$$

Correct thermal DM abundance

e.g. Anomaly mediation → wino LSP

$$M_2 < 3 \ {\rm TeV}$$

Hisano, Matsumoto, Nagai, Saito, Senami (2007)

Anomaly mediation

Randall, Sundrum (1999)
Giudice, Luty, Murayama, Rattazzi (1998)

Always present in heavy sfermion scenario

$$M_\lambda^{\mathrm{AM}}/g^2 = \beta(g^2)m_{3/2}$$

- Essential ingredient of heavy sfermion scenario
- Quantum effect, theoretically interesting

Derivation of Anomaly mediation

~Anomalous breaking of Super Weyl symmetry~

Note: ・U(1) gauge theory with a massless vector-like matter

$$Q,\ \bar{Q}$$

・ One-loop analysis

A derivation in the literature

- So-called Super conformal formulation

Local Superconformal symmetry

Gauge fixing ↓

SUGRA invariant theory

super conformal compensator

$$\phi = 1 + m_{3/2}\theta^2$$

Possible gaugino mass

$$\propto \int \mathrm{d}^2\theta f(\phi) W^\alpha W_\alpha$$

A derivation in the literature

Randall, Sundrum (1999)
Giudice, Luty, Murayama, Rattazzi (1998)

Ex. Consider a Pauli-Villars regularization

$P, \ \bar{P}$ Pauli-Villars fields

$$K = Q^\dagger Q + P^\dagger P + \bar{Q}^\dagger \bar{Q} + \bar{P}^\dagger \bar{P},$$

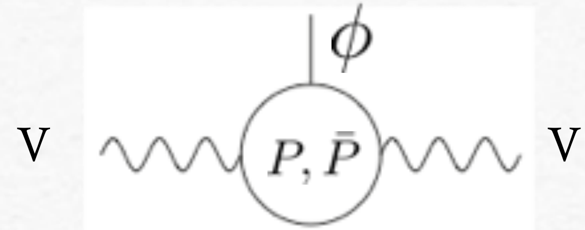
$$W = \phi \Lambda P \bar{P}$$

Λ : Pauli-Villars mass term

(Dependence on the compensator is determined by superconformal symmetry)

A derivation in the literature

Integrate down to μ



$$\mathcal{L}_{\rm eff} \supset \frac{1}{16} \int \mathrm{d}^2\theta \left(\frac{1}{g^2} + \frac{1}{8\pi^2} \ln \frac{\phi^2 \Lambda^2}{\mu^2} \right) W^\alpha W_\alpha$$

$$\phi = 1 + m_{3/2}\theta^2$$

$$M_\lambda / g^2 = \frac{1}{16\pi^2} 2 m_{3/2}$$

Similar for other regularization with cut off

Dimensional reduction : Boyda, Murayama, Pierce (2008)

A derivation in the literature 2

Local Superconformal symmetry

Gauge fixing

SUGRA invariant theory

Superconformal symmetry must be exact

In gauge theory, dilatation is in general anomalous…

Counter term: $$\int d^2\theta \beta(g^2) \ln(\phi) W^\alpha W_\alpha$$

Superspace formulation

- So-called superspace formalism

Wess,Bagger (1992) and refs. therein

Similar thing as the compensator,
i.e. chiral field with non-zero F term:

chiral density $2\mathcal{E} = e(1 - M^*\Theta^2) + \cdots$

EOM : $M^* = -3m_{3/2}$

Naïve candidate:

e: determinant of vielbein

$$\mathcal{L}_{\rm eff} \supset \frac{1}{16}\int \mathrm{d}^2\Theta \left(\frac{1}{g^2} + \frac{1}{4\pi^2}\ln(2\mathcal{E})^{1/3}\right) W^\alpha W_\alpha$$

$\mathcal{E}$ is not a scalar, but a density…

Superspace formulation?

- So-called superspace formalism

Expression consistent with SUGRA invariance was not known

(in local action)

c.f. non-local 1PI action: Bagger, Moroi, Poppitz (2000)

Some people suspected vanishing anomaly mediation de Alwis (2008)

No decisive argument against the suspicion so far.

Considered as “Puzzle”

What I will explain

Harigaya, Ibe (2014)
arXiv: 1409.5029

Let us revisit the anomaly mediation from the very beginning, step by step.

1)How gaugino mass is controlled in SUGRA

- Approximate, classical super Weyl (SW) symmetry

2) How and how much the SW symmetry is broken at quantum level?

- Construct a path-integral measure
- Evaluate the anomaly of the SW symmetry

→ Find the expression of anomaly mediation consistent with SUGRA in superspace

Approximate SW symmetry

Extension of the Weyl & R symmetry to the superspace

Parameterized by a chiral superfield Σ

Ex. Kinetic terms

$$\int \mathrm{d}^2\Theta(2\mathcal{E})W^\alpha W_\alpha, \int \mathrm{d}^2\Theta(2\mathcal{E})(\mathcal{D}^{\dagger 2} - 8R)Q^\dagger Q$$

Invariant under the SW symmetry

SW symmetry is broken only by Planck-suppressed interactions

Absence of gaugino mass

$$2\mathcal{E} = e(1 - M^*\Theta^2) + \cdots$$

EOM : $M^* = -3m_{3/2}$

Consider a SW transformation $\Sigma = F(x)\Theta^2$: F-type SW symmetry (FSW)

$$\delta_{\rm SW} M = 6F^*,$$
$$\delta_{\rm SW}\lambda^\alpha = \delta_{\rm SW} e = 0$$

λ : gaugino

$eM\lambda\lambda$: forbidden by the approximate FSW symmetry Harigaya, Ibe (2014)

(Planck-suppressed FSW violating interaction ➔ $M_\lambda = O(m_{3/2}^2/M_{\rm PL})$

Anomaly of FSW symmetry

Path-integral measure contains quantum anomaly

Fujikawa (1979)

$$\int \underline{[\mathrm{D}f(Q,\cdots)]} \times \underline{\exp[iS]}$$

FSW ?

✔ FSW

Gaugino mass May be hidden here

Let us find an appropriate Path-integral measure
and evaluate the anomaly of FSW

SUGRA invariant measure

Einstein scalar field $\psi(x)$ $[\mathrm{D}(e^{1/2}\psi)]$

Fujikawa, Yasuda (1984)

Chiral scalar field $Q(x,\Theta)$ $[D(2\mathcal{E})^{1/2}Q]$

$$\equiv [DQ_{\mathrm{diff}}]$$

Harigaya, Ibe (2014)

Anomaly of FSW symmetry

$$\int \underline{[D(2\mathcal{E})^{1/2}Q]} \times \underline{\exp[iS]}$$

FSW ?

✔ FSW

✔ FSW

FSW ?

FSW invariant measure

$$\delta_{FSW}\mathcal{E} = 6F\Theta^2\mathcal{E}$$

$$\delta_{FSW}Q = -2F\Theta^2 Q$$

$$[D(2\mathcal{E})^{1/3}Q] \equiv [DQ_{\rm SW}]$$

Harigaya, Ibe (2014)

Anomaly mediation

$$\int [DQ_{\rm diff}][DQ^\dagger_{\rm diff}]\exp(iS)$$

$$= \int [DQ_{\rm SW}][DQ^\dagger_{\rm SW}]\exp(iS + i\Delta S)$$

✔ FSW ✗ FSW

$$\Delta S = \frac{1}{16}\frac{1}{2\pi^2} \times \int \mathrm{d}^4x\, \mathrm{d}^2\Theta\, 2\mathcal{E}\ln(2\mathcal{E})^{1/6}\, W^\alpha W_\alpha + \text{h.c.}$$

$$M_\lambda/g^2 = \frac{1}{2}\frac{1}{2\pi^2}\ln(2\mathcal{E})^{1/6}|_{\Theta^2} = +\frac{1}{16\pi^2} \times 2m_{3/2}$$

Harigaya, Ibe (2014)

Anomaly mediation

$$\int [DQ_{\rm diff}][DQ^\dagger_{\rm diff}]\exp(iS)$$

$$= \int [DQ_{\rm SW}][DQ^\dagger_{\rm SW}]\exp(iS + i\Delta S)$$

✔ FSW ✘ FSW

✘ SUGRA ⟷ (counter !) ✘ SUGRA

$$\Delta S = \frac{1}{16}\frac{1}{2\pi^2} \times \int \mathrm{d}^4x\, \mathrm{d}^2\Theta\, 2\mathcal{E} \ln(2\mathcal{E})^{1/6}\, W^\alpha W_\alpha + \mathrm{h.c.}$$

Harigaya, Ibe (2014)

Superconformal, measure

$[DQ]$: Superconformal variant, need counter term

$$\propto \int \mathrm{d}^2\theta \ln\phi W^\alpha W_\alpha$$

Integrate our PV

PV method :

$[DQ][DP]$: Superconformal anomaly cancels,
Only PV mass term matters

Summary

- Heavy sfermion scenario is a natural consequence of SUSY breaking with charged/composite particle
- Anomaly mediation is important there
- We have derived the anomaly mediation in superspace formulation of SUGRA
- Path-integral measure is important to derive the anomaly mediation in a consistent way with SUGRA

Back up

Cosmology

1) $m_{3/2} > 100\ \text{TeV}$: Gravitinos decay before BBN era

2) $\Omega_{\text{LSP}} h^2 = 0.12 \times \frac{m_{\text{LSP}}}{900\text{GeV}} \frac{T_{\text{RH}}}{2 \times 10^9 \text{GeV}}$

Kawasaki, Moroi (1995)
Kawasaki, Kohri, Moroi (2005)

Thermal leptogenesis possible for $T_R > 10^9\ \text{GeV}$

Fukugita, Yanagida (1986)

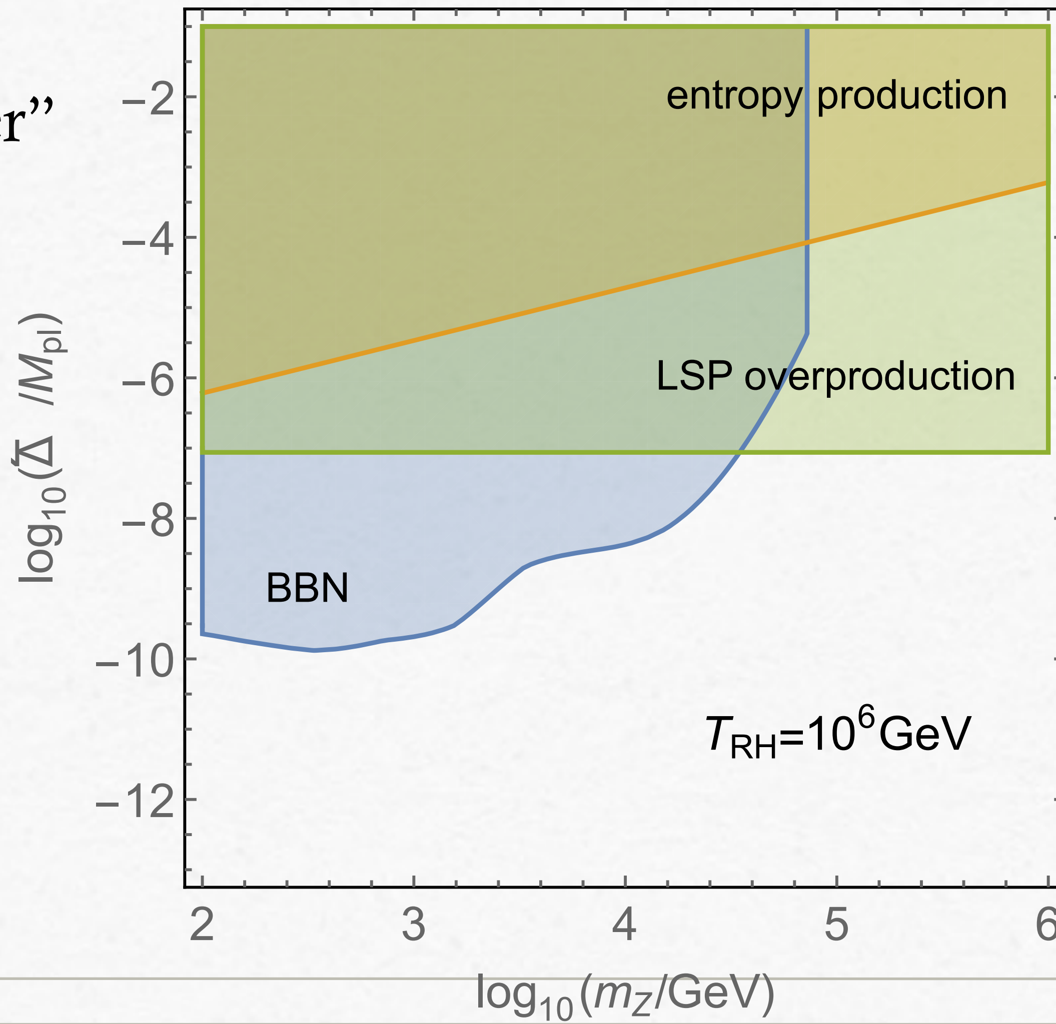
Giudice, Notari, Raidal, Riotto, Strumia (2004)
Buchmuller, Bari, Plumacher (2005)

3) No Polonyi problem

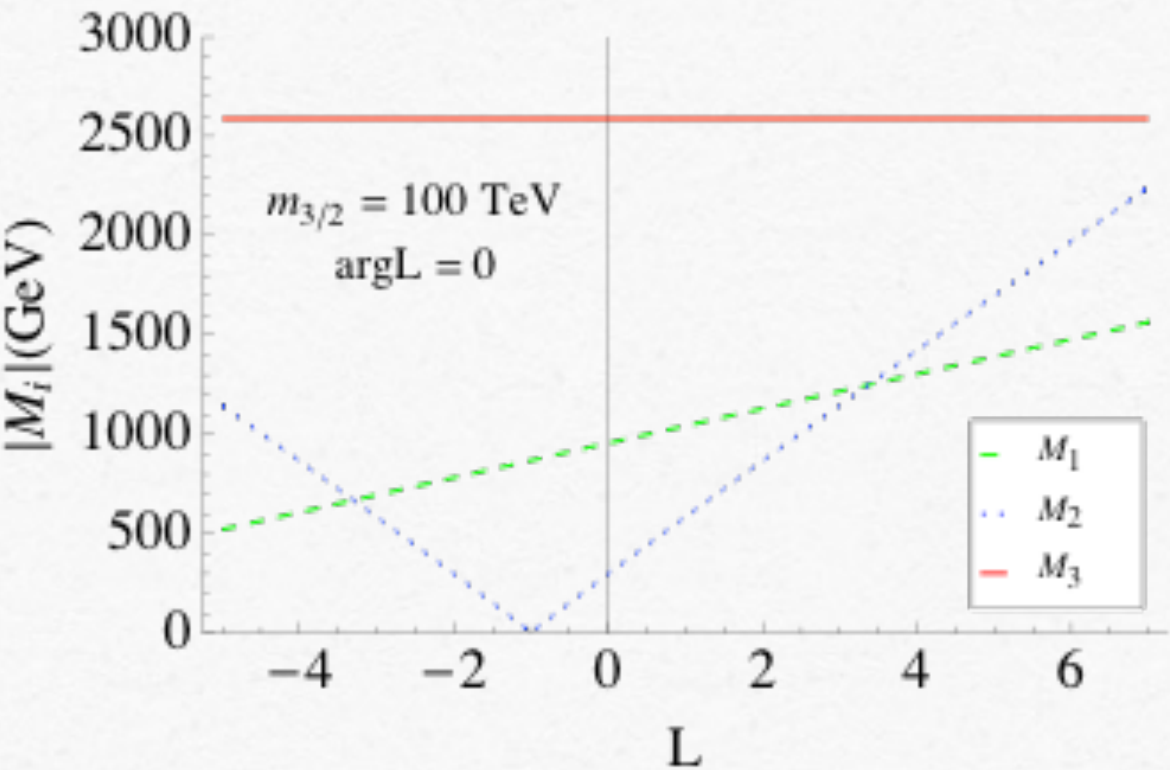
Polonyi problem

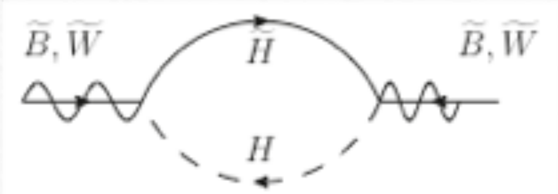
Singlet field : No "center"

$$\Delta Z \sim M_{\rm Pl}$$



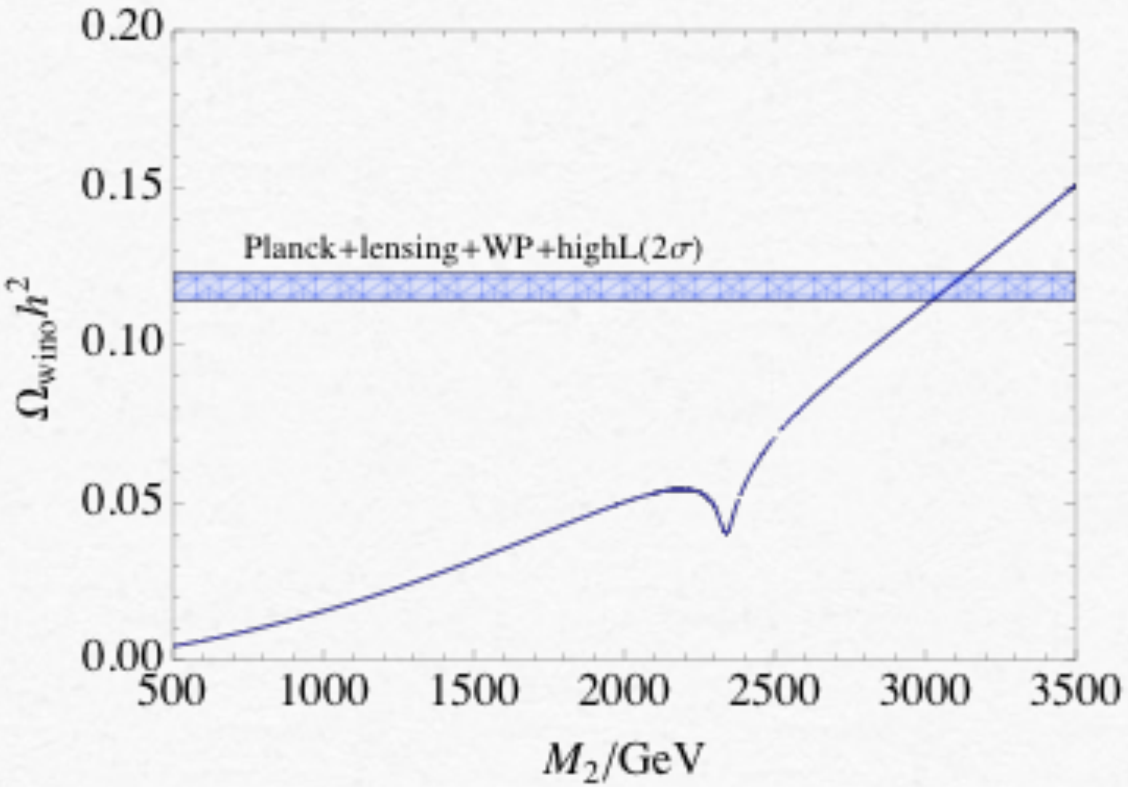
Gaugino masses





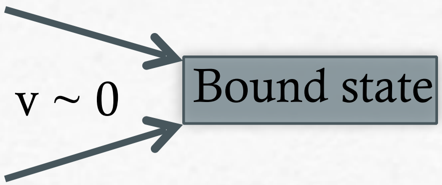
$$L \equiv \frac{\mu\, m_A^2 \sin 2\beta}{|\mu|^2 - m_A^2} \ln \frac{|\mu|^2}{m_A^2}$$

Wino LSP



Dip:

Resonance



Hiano, Matsumoto, Nojiri, Saito (2005)

Supergravity, 1PI

Bagger, Moroi, Poppitz (2000)

They found the 1PI term reproducing the anomaly of SW

$$\Delta S \simeq -\frac{1}{16}\frac{1}{8\pi^2}\int \mathrm{d}^4x\,\mathrm{d}^2\Theta\,2\mathcal{E}\,\frac{1}{\Box_+}\left(\mathcal{D}^{\dagger 2}-8R\right)R^\dagger\,W^\alpha W_\alpha + \mathrm{h.c.}$$

Ex. $U(1)_R$ anomaly $\supset \frac{\partial_a b^a}{\Box} F\tilde{F}$

Super diffeomorphism invariant measure

$$Q' = Q - \eta^M(x,\Theta)\partial_M Q \ , \qquad Z^M = (x^m, \Theta^\alpha)$$

$$\mathcal{E}' = \mathcal{E} - \eta^M(x,\Theta)\partial_M \mathcal{E} - (-)^M \left(\partial_M \eta^M(x,\Theta)\right)\mathcal{E}$$

$$Q_{\text{diff}} = (2\mathcal{E})^{1/2} Q$$

$$Q'_{\text{diff}} = Q_{\text{diff}} - \eta^M(x,\Theta)\partial_M Q_{\text{diff}} - \frac{1}{2}(-)^M \left(\partial_M \eta^M(x,\Theta)\right) Q_{\text{diff}}$$

$$[DQ'_{\text{diff}}] = [DQ_{\text{diff}}] \times \exp\left[\text{sTr}_{z',z}\,\mathcal{O}(z',z)\right] \ ,$$

$$\mathcal{O}(z',z) \equiv -\left[\eta^M \partial_M + \frac{1}{2}(-1)^M \left(\partial_M \eta^M\right)\right]\delta^6(z'-z) \ ,$$

$$\text{sTr}\,\mathcal{O}(z',z) = \int \mathrm{d}^6 z \mathrm{d}^6 z' \delta^6(z'-z)\mathcal{O}(z',z)$$

Super diffeomorphism invariant measure

$$\delta^6(z'-z) = \int \frac{\mathrm{d}^4k}{(2\pi)^4} d^2\tau \Psi_{-k,-\tau}(z')\Psi_{k,\tau}(z) \ ,$$

$$\Psi_{k,\tau}(z) \equiv \exp(ikx + 2i\tau\Theta)$$

$$\mathrm{sTr}\, \mathcal{O}(z', z) = -\int \mathrm{d}^6 z \int \frac{\mathrm{d}^4k}{(2\pi)^4} d^2\tau \Psi_{-k,-\tau}(z) \left[\eta^M \partial_M + \frac{1}{2}(-)^M \left(\partial_M \eta^M\right)\right] \Psi_{k,\tau}(z)$$

$$\int \mathrm{d}^6 z \Psi_{k,\eta}(z) \left[\eta^M \partial_M + \frac{1}{2}(-)^M \left(\partial_M \eta^M\right)\right] \Psi_{k,\eta}(z) = \frac{1}{2}(-)^M \int \mathrm{d}^6 z \partial_M \left[\Psi_{k,\eta}(z)\, \eta^M \Psi_{k,\eta}(z)\right]$$
$$= 0$$

$$\begin{aligned}\mathrm{sTr}\, \mathcal{O}(z', z) = &-\frac{1}{2} \int \mathrm{d}^6 z \int \frac{\mathrm{d}^4k}{(2\pi)^4} d^2\tau \left(\Psi_{k,\tau}(z) + \Psi_{-k,-\tau}(z)\right) \\ &\left[\eta^M \partial_M + \frac{1}{2}(-)^M \left(\partial_M \eta^M\right)\right] \left(\Psi_{k,\tau}(z) + \Psi_{-k,-\tau}(z)\right) \\ &= -\frac{1}{4}(-)^M \int \mathrm{d}^6 z \int \frac{\mathrm{d}^4k}{(2\pi)^4} d^2\tau \partial_M \left[\left(\Psi_{k,\tau}(z) + \Psi_{-k,-\tau}(z)\right) \eta^M \left(\Psi_{k,\tau}(z) + \Psi_{-k,-\tau}(z)\right)\right] \\ &= 0 \ .\end{aligned}$$

Pauli-Villars regularization

PV mass term

$$\int \mathrm{d}^2\Theta 2\mathcal{E}\Lambda P\bar{P}$$

More relevant than kinetic term

$$\delta_{FSW}\mathcal{E} = 6F\Theta^2\mathcal{E}$$
$$\delta_{FSW}P = -3F\Theta^2 P$$

$$[\mathrm{D}P_{\mathrm{diff}}] = [\mathrm{D}P_{\mathrm{SW}}]$$

Only the measure of massless matter is relevant

Superconformal, measure

$[DQ]$: Superconformal variant, need counter term

$$\propto \int \mathrm{d}^2\theta \ln\phi W^\alpha W_\alpha$$

PV method :

Integrate our PV

$[DQ][DP]$: Superconformal anomaly cancels,
Only PV mass term matters