Anomaly or symmetry ?

- fate of axial U(1) at high temperature -

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PRD86 (2012) 114512 [arXiv1209.2061]

and JLQCD collaboration, PRD87(2013)11, 114514[arXiv:1304.6145]; arXiv:1412.5703; PRD93(2016)3,034507

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1. Introduction

Short summary (for M1 students)

- Quantum anomaly (量子異常)
  = symmetry breaking ∝ $\hbar$

- chiral U(1) symmetry has an anomaly.

- We discuss what happens on it at high temperature.

Chiral symmetry breaking

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Our textbook says
"chiral symmetry in QCD is broken
in two different ways" :

$$SU(N_f)_R \times SU(N_f)_L \times U(1)_V \times U(1)_A$$

Spontaneous breaking
[Nambu 1961]

Chap 11 on
Peskin & Schroeder

Anomaly
(explicit breaking)
[Adler 1969, Bell,Jackiw 1

Chap 19 on
Peskin & Schroeder

$$SU(N_f)_V \times U(1)_V$$

SSB and anomaly related ?

However, we'd like to discuss a possibility of

Spontaneous breaking
[Nambu 1961]

relation ?

Anomaly
(explicit breaking)
[Adler 1969, Bell,Jackiw 1969]

in particular, a possibility of BOTH (effective) restoration at the SAME temperature.

Sounds sounds but …

Normal response = “NO KIDDING !”

Anomaly =

explicit breaking at high energy.

There is no reason for its restoration.

But, we would like to show that this issue is not so trivial, and some evidences of (effective or accidental) $U(1)_A$ restoration.

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2. Why $U(1)_A$ restoration possible ?

(in Nf=2 QCD)

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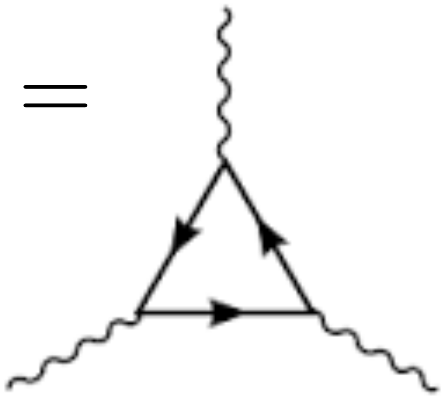
$U(1)_A$ chiral anomaly in textbooks

[Fujikawa 1980]

$$\langle \partial_\mu J_5^\mu \rangle =$$



$$= \frac{g^2}{32\pi^2} \mathrm{Tr} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

gauge field = classical background

Fermion = nearly massless ($m \ll \Lambda_{\mathrm{cut}}$)

Anomaly must disappear (at least) at T=∞

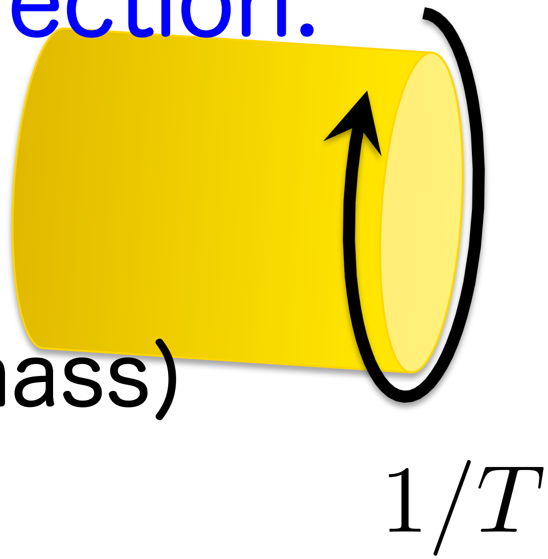
Finite T = compact 4th direction.

Anti-periodic boundary

→ quarks are massive.

First KK-mass (Matsubara mass)

$= \pi T$



T=∞ → 3D theory, quarks decoupled.

There is no source of anomaly.

Gluon integrals

$$\langle \partial_\mu J_5^\mu O \rangle_{q,A_\mu} = \langle q(x) O \rangle_{q,A_\mu} = 0 \ ?$$

$$q(x) = \frac{g^2}{32\pi^2} \mathrm{Tr} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

Cf.) chiral condensate:

$$\langle \bar{q} q \rangle_{q,A_\mu} = \frac{\int \mathrm{Tr} D^{-1}(x,x) \det D^2 e^{-S_G}}{Z}$$

$$\begin{cases} \neq 0 & (T < T_c) \\ = 0 & (T > T_c) \end{cases}$$

Anomaly must disappear (at least) on dim=3 operators

SU(2) and U(1) share the same order parameter(s).

Among quark bi-linears $\langle \bar{q}\Gamma q(x) \rangle$,

Only $\langle \bar{q}q(x) \rangle$ can have a VEV.

SU(2)xSU(2) restoration

⇔ no U(1)anomaly at d<=4 operators.

* Higher dim. Op. ? -> we discuss later.

Both are related to Dirac spectrum (& instanton physics)

SU(2) SSB ⇆ (near) zero-modes of D

Banks-Casher relation [1980]

$$\pi\rho(\lambda=0) = \langle\bar{q}q\rangle \equiv \Sigma.$$

$$\left(\rho(\lambda) \equiv \lim_{V\to\infty} \sum_{\lambda_i \geq 0} \left\langle \frac{\delta(\lambda_i - \lambda)}{V} \right\rangle .\right)$$

λ : Dirac eigenvalue

U(1) anomaly ⇆ zero-modes of D

Atiyah-Singer index theorem [1963]

$$n_+ - n_- = \frac{1}{32\pi^2}\int d^4x \mathrm{Tr} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Anomaly often disappears

Non-CFT at UV

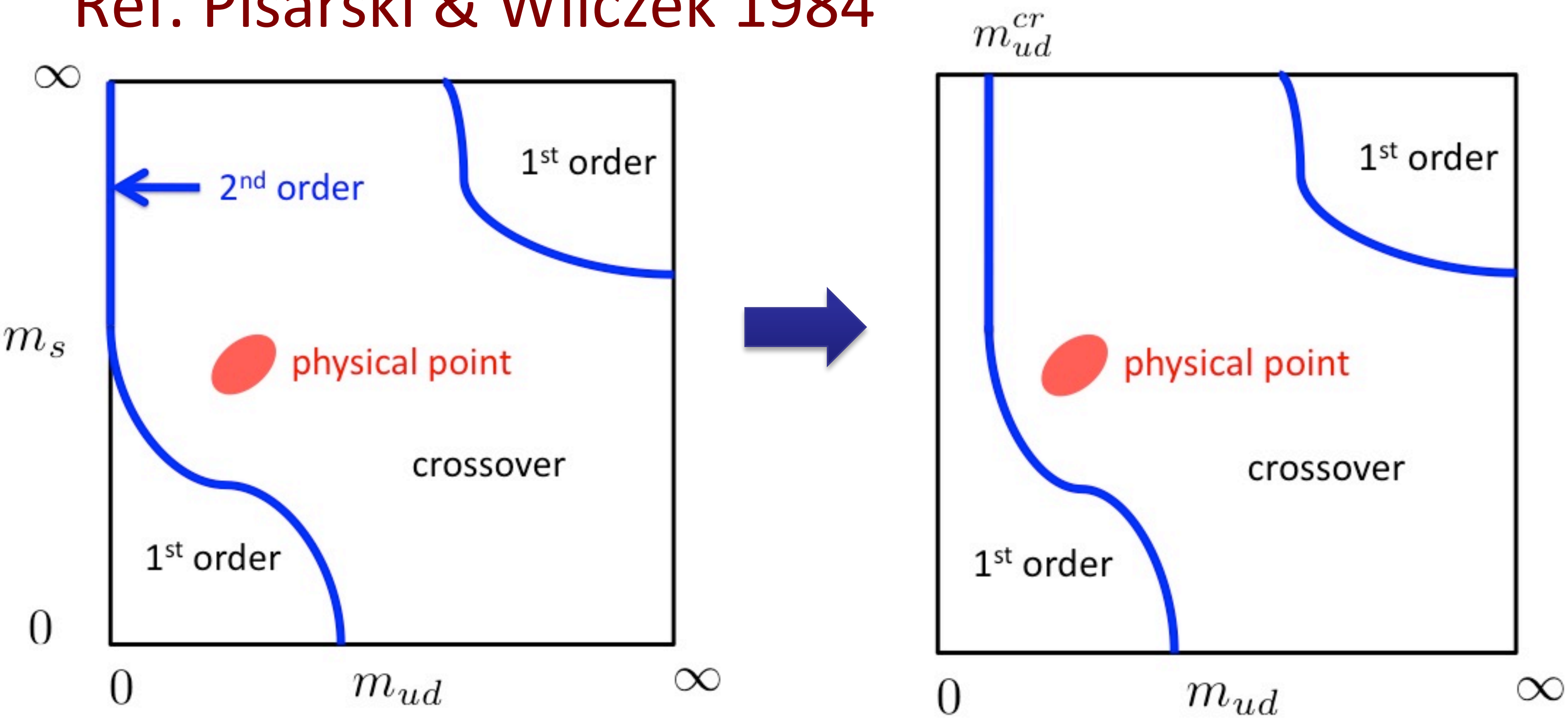
↓ ← some non-trivial dynamics

CFT at IR

= scale anomaly disappears !

What is interesting ?

Ref. Pisarski & Wilczek 1984



What is interesting ?

Axion dark matter

$$\frac{\langle Q^2 \rangle}{V} \propto m_{axion}^2 .$$

low $<Q^2>$ ⇆ low m_{axion}

References

Berkowits et al.15 Kitano&Yamada 15, Mages et al. 15, Trunin et al. 15, Botani et al. 15

Short summary of sec. 2

1. Anomaly often disappears (cf. CFT at IR)
2. $U(1)_A$ anomaly at finite T is non-trivial.
3. Instantons & Dirac zero-modes
   affect both of $SU(2)_L xSU(2)_R$ and $U(1)_A$.
4. Then $U(1)_A$ may be restored as an (effective or accidental) symmetry at T>Tc.
5. Phenomenogically interesting : order of chiral phase transition & dark matter.

Current status = still controversial

Cohen (1996) [analytic, continuum] : YES.
Lee & Hatsuda (1996)) [analytic, continuum] : NO.
Aoki-F-Taniguchi (2012)) [analytic, lattice] : Yes.
HotQCD,LLNL/RBC (2011-15)[domain-wall(DW)] : NO.
JLQCD(2013-16)[overlap, Mobius DW]: Yes.
TWQCD(2013)[Mobius DW]: Yes.
Dick et al. (2015)[ov on HISQ sea]: No.
Ejiri et al.(2015) [Wilson w/ large Nf]: No.
If (partially) YES,
Pelissetto & Vicari (2013) : 2nd order still possible.
Sato & Yamada (2014) : not simple O(4) scaling.
Nakayama & Ohtuski(2015): IR fixed point exists.
(2016): partial restoration can't be extended to full U(1)
If NO,
Kanazawa & Yamamoto (2015) : finite V may cause a fake restoration.

3. Analytic evidences

[Aoki, F & Taniguchi 2012]

Our goal

＝generalization of Banks-Casher relation

→may link SU(2) SSB and U(1) anomaly.

$SU(2)_L x SU(2)_R$ breaking/restoration

⇕

(near) zero mode spectrum of Dirac operator

⇕

$U(1)_A$ breaking/restoration

What's new in this work ?

Note : T. Cohen 1996 obtained (almost) the same conclusion as ours [in continuum, assuming no chiral zero-mode's effect].

In our work,

1. Analytic study on a lattice :

   No UV divergence, $V\to\infty$ limit can be taken.

2. Exact chiral symmetry

   SU(2) fully recovered.

3. Well-defined zero-modes (= instantons)

Exact chiral symmetry on the lattice

[Neuberger 1998]

Overlap lattice fermion action (Nf=2):

$$S = \bar{\psi}[D + mF(D)]\psi, \quad F(D) = 1 - \frac{Ra}{2}D$$

Ginsparg-Wilson relation [1982]:

$$D\gamma_5 + \gamma_5 D = aRD\gamma_5 D$$

If m=0, action is invariant under *chiral rotation* :

$$\psi \to e^{i\gamma_5\alpha(1-aD)}\psi, \quad \bar{\psi} \to \bar{\psi}e^{i\gamma_5\alpha}$$

[Luscher 1998]

Eigenvalue decomposition

Quark propagator in Dirac eigen-mode decomposition

$$S(x,y) = \sum_n \left[\frac{\phi_n(x)\phi_n^\dagger(y)}{f_m\lambda_n + m} + \frac{\gamma_5\phi_n(x)\phi_n^\dagger(y)\gamma_5}{f_m\bar{\lambda}_n + m} \right] + \sum_{k=1}^{N_{R+L}} \frac{1}{m}\phi_k(x)\phi_k^\dagger(y) + \sum_{K=1}^{N_D} \frac{Ra}{2}\phi_K(x)\phi_K^\dagger(y)$$

bulk modes(non-chiral) zero modes(chiral) real doubler modes (chiral)

Eigenvalue density for a given configuration A:

$$\rho^A(\lambda) = \lim_{V\to\infty} \frac{1}{V} \sum_n \delta(\lambda - \sqrt{\bar{\lambda}_n^A \lambda_n^A}),$$

Eigenvalue decomposition

Example : scalar density

$$\langle -\bar{q}q\rangle_m = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\right\rangle$$

$$= \lim_{V\to\infty}\frac{1}{V}\left\langle \frac{N^A_{R+L}}{m} + \sum_{n\ (\mathrm{Im}\lambda^A_n>0)} \frac{2m}{\bar{\lambda}^A_n\lambda^A_n + m^2}\right\rangle_m + \mathcal{O}(a)$$

$$= \lim_{V\to\infty}\left[\frac{1}{mV}\langle N^A_{R+L}\rangle_m + \int_0^{2/Ra} d\lambda \langle\rho^A(\lambda)\rangle_m \frac{2m}{\lambda^2+m^2}\right] + \mathcal{O}(a)$$

Note : In the m→0 limit, BC rel. (re)appears.

$$\lim_{m\to 0}\frac{2m}{\lambda^2+m^2} = \pi\delta(\lambda) \quad \Rightarrow \quad \lim_{m\to 0}\langle -\bar{q}q\rangle_m = \pi\langle\rho^A(0)\rangle$$

Our assumptions

1. SU(2) x SU(2) fully recovered at Tc.
2. if $\mathcal{O}(A)$ is m-independent ,

$$\langle \mathcal{O}(A) \rangle_m = f(m^2) \qquad f(x) \text{ is analytic at } x=0$$

3. if $\mathcal{O}(A)$ is m-independent and positive, and satisfies

$$\lim_{m\to 0} \frac{1}{m^{2k}} \langle \mathcal{O}(A) \rangle_m = 0$$

$$\Longrightarrow \langle \mathcal{O}(A) \rangle_m = m^{2(k+1)} \underbrace{\int \mathcal{D}A\, \hat{P}(m^2, A) \mathcal{O}(A)}_{\text{finite}}$$

$$\Longrightarrow \langle \mathcal{O}(A)^l \rangle_m = m^{2(k+1)} \int \mathcal{D}A\, \hat{P}(m^2, A) \mathcal{O}(A)^l = O(m^{2(k+1)})$$

4. $$\rho^A(\lambda) \equiv \lim_{V\to\infty} \frac{1}{V} \sum_n \delta\left(\lambda - \sqrt{\bar{\lambda}_n^A \lambda_n^A}\right) = \sum_{n=0}^{\infty} \rho_n^A \frac{\lambda^n}{n!} \quad \text{at } \lambda = 0 \ (\lambda < \epsilon \ )$$

(4 can be removed.)

(pseudo) scalar operators and our recipe

$$\mathcal{O}_{n_1,n_2,n_3,n_4} = (P^a)^{n_1}(S^a)^{n_2}(P^0)^{n_3}(S^0)^{n_4}$$

$a = 1,2,3 : \mathrm{SU}(2)$ triplet $\quad 0 :$ singlet

$$S^{(a/0)} = \int d^4x\ \bar{q}\tau^{(a/0)}q(x), \quad P^{a/0} = \int d^4x\ \bar{q}\gamma_5\tau^{(a/0)}q(x),$$

1. at T > Tc, $SU(2)_A$ rotation is zero (in m→0 limit):

$$\langle \delta^a O_{n_1,n_2,n_3,n_4} \rangle = 0.$$

2. → constraints on Dirac eigenvalue density.

3. → examine if any effects to $U(1)_A$ rotation :

$$\langle \delta^0 O_{n_1,n_2,n_3,n_4} \rangle = 0\ ???$$

Axial SU(2) and U(1) rotations

$$\mathcal{O}_{n_1,n_2,n_3,n_4} = (P^a)^{n_1}(S^a)^{n_2}(P^0)^{n_3}(S^0)^{n_4}$$

$$SU(2)\ :\ \delta_a q = i\gamma_5\tau_a q$$

$$\delta_a\mathcal{O}_{n_1,n_2,n_3,n_4} = -2n_1\mathcal{O}_{n_1-1,n_2,n_3,n_4+1} + 2n_2\mathcal{O}_{n_1,n_2-1,n_3+1,n_4} - 2n_3\mathcal{O}_{n_1,n_2+1,n_3-1,n_4} + 2n_4\mathcal{O}_{n_1+1,n_2,n_3,n_4-1}$$

$$U(1)\ :\ \delta_0 q = i\gamma_5 q$$

$$\delta_0\mathcal{O}_{n_1,n_2,n_3,n_4} = -2n_1\mathcal{O}_{n_1-1,n_2+1,n_3,n_4} + 2n_2\mathcal{O}_{n_1+1,n_2-1,n_3,n_4} - 2n_3\mathcal{O}_{n_1,n_2,n_3-1,n_4+1} + 2n_4\mathcal{O}_{n_1,n_2,n_3+1,n_4-1}.$$

$N = n_1 + n_2 + n_3 + n_4 = 1$

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SU(2) symmetry: $$\frac{1}{V}\langle \delta^a P^b \rangle = -\frac{2}{V}\delta^{ab}\langle S^0 \rangle = 0.$$

$\rightarrow$ eigenvalue decomposition

$$\frac{1}{V}\langle S^0 \rangle = \left\langle \frac{N^A_{R+L}}{mV} + \pi\rho^A_0 \right\rangle = 0.$$

Banks-Casher relation

$\rightarrow$ $U(1)_A$ anomary is invisible !

$$\frac{1}{V}\langle \delta^0 P^0 \rangle = -\frac{1}{V}\langle S^0 \rangle = 0.$$

note:

$$\langle \delta^0 S^0 \rangle = \langle P^0 \rangle = 0,$$
$$\langle \delta^0 S^a \rangle = \langle P^a \rangle = 0,$$
$$\langle \delta^0 P^a \rangle = -\langle S^a \rangle = 0.$$

$N = n_1 + n_2 + n_3 + n_4 = 2$

SU(2) symmetry:

$$\chi^{\sigma-\pi} \equiv \frac{1}{V^2}\langle \delta^a (P^a S^0)\rangle = \frac{1}{V^2}\langle (P^a)^2 - (S^0)^2\rangle = 0,$$

$$\chi^{\eta-\delta} \equiv \frac{1}{V}\langle \delta^a (S^a P^0)\rangle = \frac{1}{V}\langle (P^0)^2 - (S^a)^2\rangle = 0.$$

→ eigenvalue decomposition

$$\chi^{\sigma-\pi} = -4\left\langle\left(\frac{N^A_{R+L}}{mV} + \pi\rho^A_0\right)^2\right\rangle + O(1/V), \quad \chi^{\eta-\delta} = 2\left\langle\frac{-N_f Q(A)^2}{m^2} + 2\rho^A_1\right\rangle + O(1/V).$$

$$(N^A_{R+L} = n_+ + n_-, \quad Q(A) = n_+ - n_-)$$

⬇

$$\langle \rho^A_0\rangle_m = O(m^2) \qquad \left\langle\frac{N^A_{R+L}}{V}\right\rangle = O(m^4) \quad \langle \rho^A_1\rangle = \frac{N_f^2\langle Q(A)^2\rangle}{m^2 V}$$

$N = n_1 + n_2 + n_3 + n_4 = 3, 4$

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SU(2) symmetry: WT identities

$$\langle \mathcal{O}_{2001} \rangle_m \to 0, \quad \langle -\mathcal{O}_{0201} + 2\mathcal{O}_{1110} \rangle_m \to 0, \quad \langle \mathcal{O}_{0021} + 2\mathcal{O}_{1110} \rangle_m = 0$$
$$\langle -\mathcal{O}_{0003} + 2\mathcal{O}_{2001} \rangle_m \to 0, \quad \langle \mathcal{O}_{0021} - \mathcal{O}_{0201} + \mathcal{O}_{1110} \rangle_m \to 0,$$

$$\langle \mathcal{O}_{4000} - \mathcal{O}_{0004} \rangle_m \to 0, \quad \langle \mathcal{O}_{4000} - 3\mathcal{O}_{2002} \rangle_m \to 0,$$
$$\langle \mathcal{O}_{0400} - \mathcal{O}_{0040} \rangle_m \to 0, \quad \langle \mathcal{O}_{0400} - 3\mathcal{O}_{0220} \rangle_m \to 0,$$
$$\langle \mathcal{O}_{2020} - \mathcal{O}_{0202} \rangle_m \to 0, \quad \langle \mathcal{O}_{2200} - \mathcal{O}_{0022} \rangle_m \to 0,$$
$$\langle 2\mathcal{O}_{1111} - \mathcal{O}_{0202} + \mathcal{O}_{0022} \rangle_m \to 0.$$

$\to$ eigenvalue decomposition (let me skip details)

$\to$ $\langle \rho_1^A \rangle = O(m^2) \quad \langle \rho_2^A \rangle = O(m^2)$

Higher orders…

No additional constraint on $\langle \rho_i^A \rangle$ upto N=6.

For zero-modes, we obtain (instanton effects are suppressed !)

$$\lim_{V\to\infty} \left\langle \frac{N_{R+L}^A}{V} \right\rangle = O(m^N) = 0,$$

$$\lim_{V\to\infty} \left\langle \frac{Q(A)^2}{V} \right\rangle = O(m^N) = 0.$$

Summary of the constraints

$$\lim_{m\to 0} \langle \rho^A(\lambda) \rangle_m = \lim_{m\to 0} \langle \rho_3^A \rangle_m \frac{|\lambda|^3}{3!} + O(\lambda^4)$$

$\langle \rho_3^A \rangle_m \neq 0$ even for "free" theory.

Fate of $U(1)_A$ anomaly

Surprisingly, the constraint

$$\lim_{m\to 0}\langle \rho^A(\lambda)\rangle_m = \lim_{m\to 0}\langle \rho_3^A\rangle_m \frac{|\lambda|^3}{3!} + O(\lambda^4)$$

is strong enough to prove

$$\langle \delta^0 O_{n_1,n_2,n_3,n_4}\rangle = 0 \quad \text{for any} \quad N\ !$$

Namely, [Z_4 sub group of] $U(1)_A$ anomaly is invisible.

(at least for these set of operators)

See Aoki, HF, & Taniguchi, arXiv:1209.2061 for details.

order of chiral phase transition

1. Pisarski & Wilczek 1984
   SU(2)→SU(2)xSU(2) 2nd order, O(4)
   SU(2)→SU(2)xSU(2)xU(1) → 1st order
2. Pelissetto & Vicari 2013 w/ 5 loop effective theory analysis : 2nd order but not O(4)
3. Nakayama & Ohtuski 2014 w/ conformal bootstrap : 2nd order FP exists (not O(4))

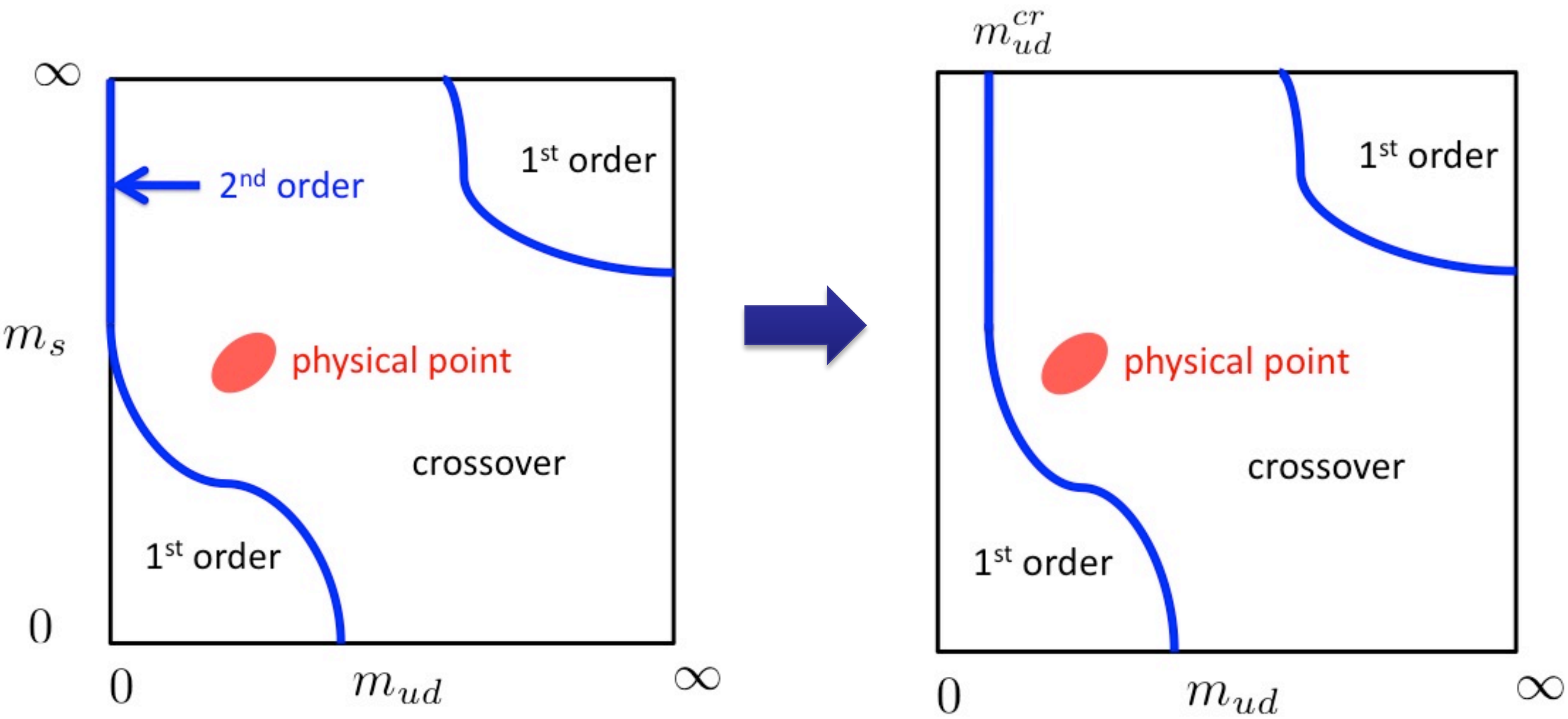
2016 : sub G of U(1) → cannot be full U(1).

Our result suggests 1st order transition

$$\lim_{V\to\infty} \frac{\langle Q^2\rangle}{V} = O(m^N) = 0, \quad \text{for any } N.$$

1. m=0 point is not special.
2. $\lim_{V\to\infty} \frac{\langle Q^2\rangle}{V}$ =0 for $m<m_{cr}$
3. Phase transition occurs even for finite m.
4. Finite m has no symmetry -> likely to be 1st order.

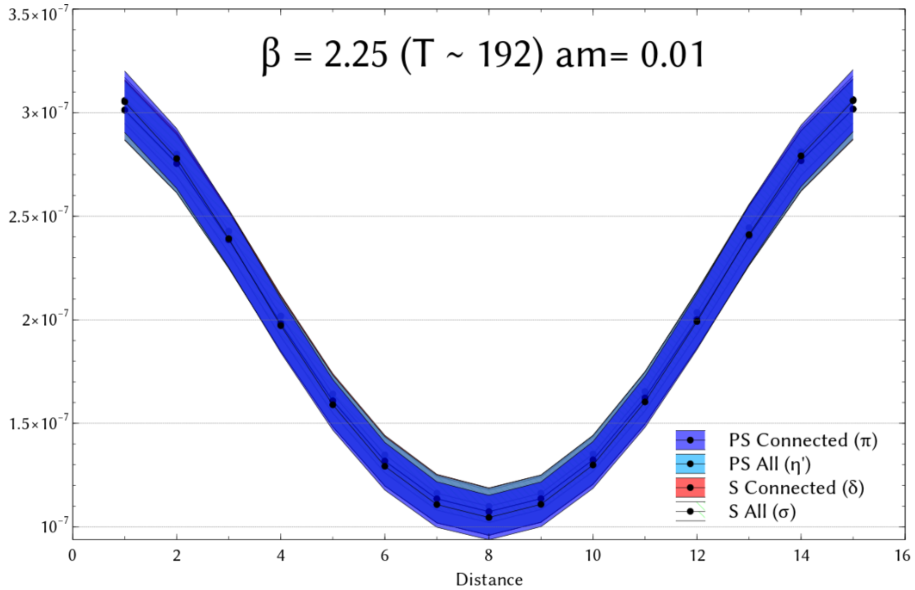
Conventional phase diagram may need a change.

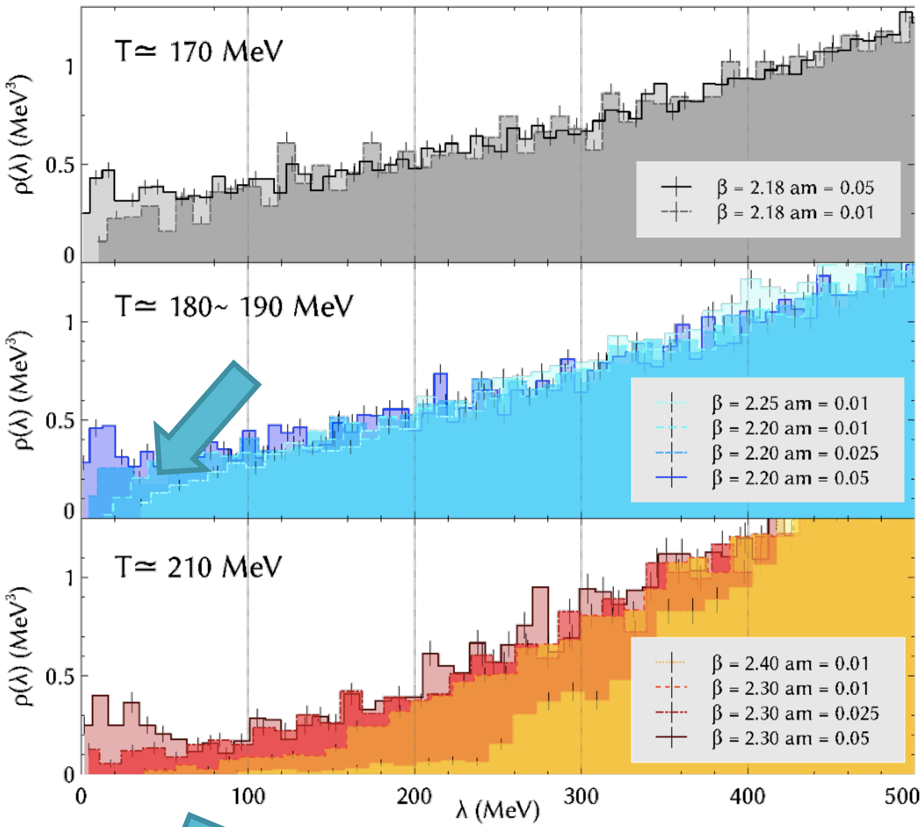


4. Numerical evidences by JLQCD collaboration [2013-2015]

Our previous work w/ overlap fermions [JLQCD 2013]

Strong suppression of $U(1)_A$ anomaly.





But our volume was small (L~1.8fm) and topology was fixed.

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JLQCD project 2013 -

Computers @KEK: SR16000 (55 TFLOPS) + BG/Q (1.2 PFLOPS)

Lattice cut-off : 1.6GeV-2.6GeV

Lattice size : 16^3 x8, 32^3x8 (L～4 fm) ,
32^3x12 (T= 190-330MeV)

Fermion : (Mobius) DomainWall

Quark mass : 1.5-30 MeV [Simulated w/ Iroiro++ G. Cossu et al.]

Topology fluctuates.


Hitachi SR16000


IBM Blue Gene/Q

(Mobius) domain-wall fermions

=Approximation of

$$D^{overlap} = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \mathrm{sgn}(H)$$

[Kaplan, Furman & Shamir, Borici, Chiu, Brower et al., Edwards and Heller…]

Kernel = scaled Shamir Kernel:

$$2H_T = \gamma_5 \frac{2D_W}{2+D_W}$$

Sgn function = Tanh:

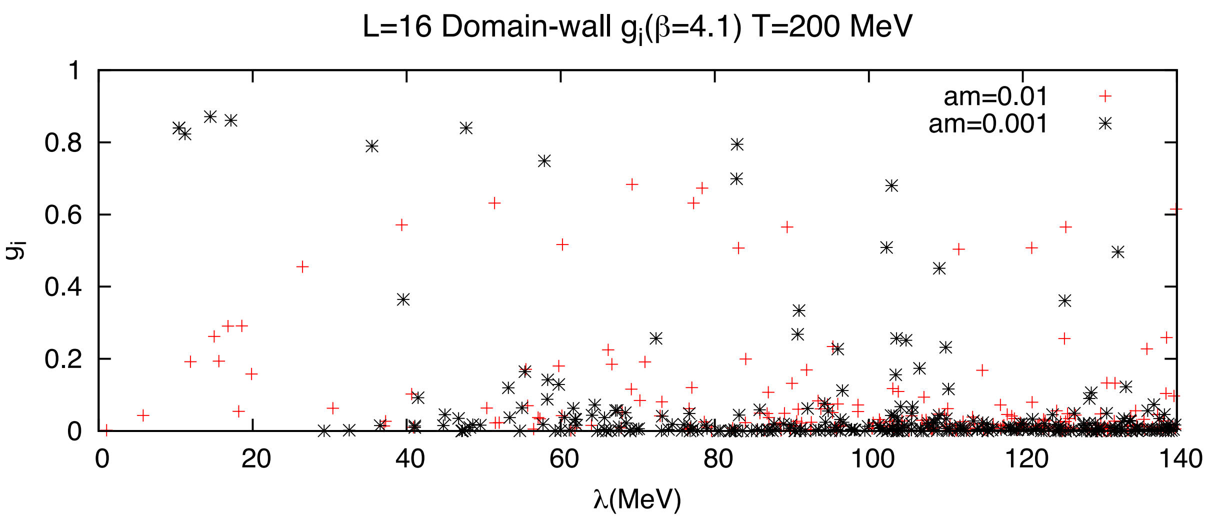
$$\mathrm{sgn}_{\tanh}(2H_T) = \frac{(1+2H_T)^{L_s} - (1-2H_T)^{L_s}}{(1+2H_T)^{L_s} + (1-2H_T)^{L_s}} = \tanh(L_s \tanh^{-1}(2H_T))$$

Ls = O(10)

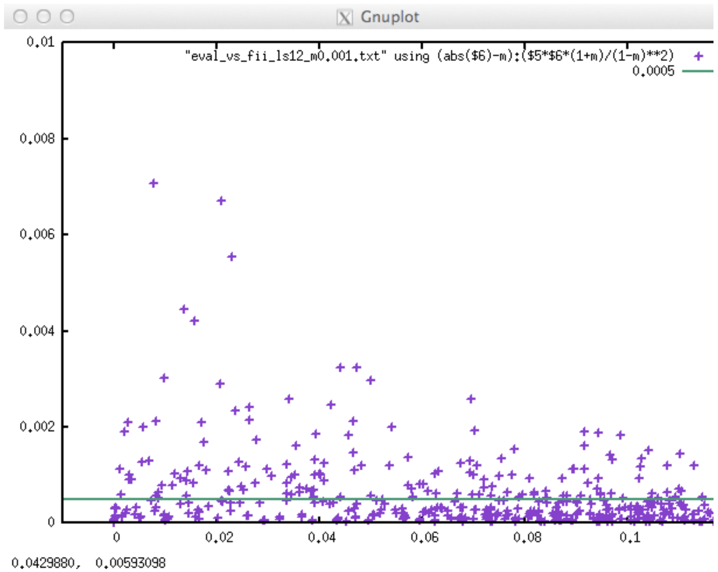
→ m_{res}<1MeV, Chiral symmetry ~ 10^{-3}

However, Mobius DW is NOT good enough at T>Tc

For low-modes at T>Tc , violation of Ginsparg-Wilson relation ~ 90 % !



Cf. T=0 case (quenched QCD)



Overlap/Domain-wall reweighting

$$\langle O \rangle_{\mathrm{overlap}} = \frac{\langle O R(A) \rangle_{\mathrm{Domain-wall}}}{\langle R(A) \rangle_{\mathrm{Domain-wall}}}$$

$$R(A) = \frac{\mathrm{Det}\, D_{\mathrm{ov}}^2(m)}{\mathrm{Det}\, D_{\mathrm{DW}}^2(m)} \frac{\mathrm{Det}\, D_{\mathrm{DW}}^2(1/2a)}{\mathrm{Det}\, D_{\mathrm{ov}}^2(1/2a)}.$$

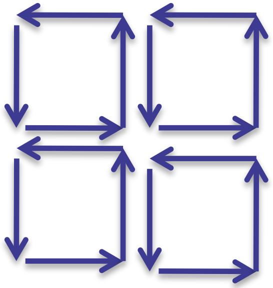
$$D_{ov} = \underbrace{\frac{1}{2} \sum_{\lambda_i < \lambda_{th}} (1 + \gamma_5 \mathrm{sgn} \lambda_i) \, |\lambda_i\rangle\langle\lambda_i|}_{\text{Exact low modes}} + \underbrace{D_{DW}^{4D} \left(1 - \sum_{\lambda_i < \lambda_{th}} |\lambda_i\rangle\langle\lambda_i|\right)}_{\text{High modes}},$$

We also use its index for topological charge.

* For coarsest lattice ($\beta = 4.10$) we use lowest 100 mode approximation of R(A).

Topological charge density

$$F^{lat}_{\mu\nu} =$$



$$q(x) = \frac{1}{32\pi^2} \mathrm{Tr} \epsilon^{\mu\nu\rho\sigma} F^{lat}_{\mu\nu} F^{lat}_{\rho\sigma}$$

We use $\displaystyle\sum_x q(x) = Q + O(a^2)$

after Wilson flow cooling at $\sqrt{8t} \sim 0.5\mathrm{fm}.$

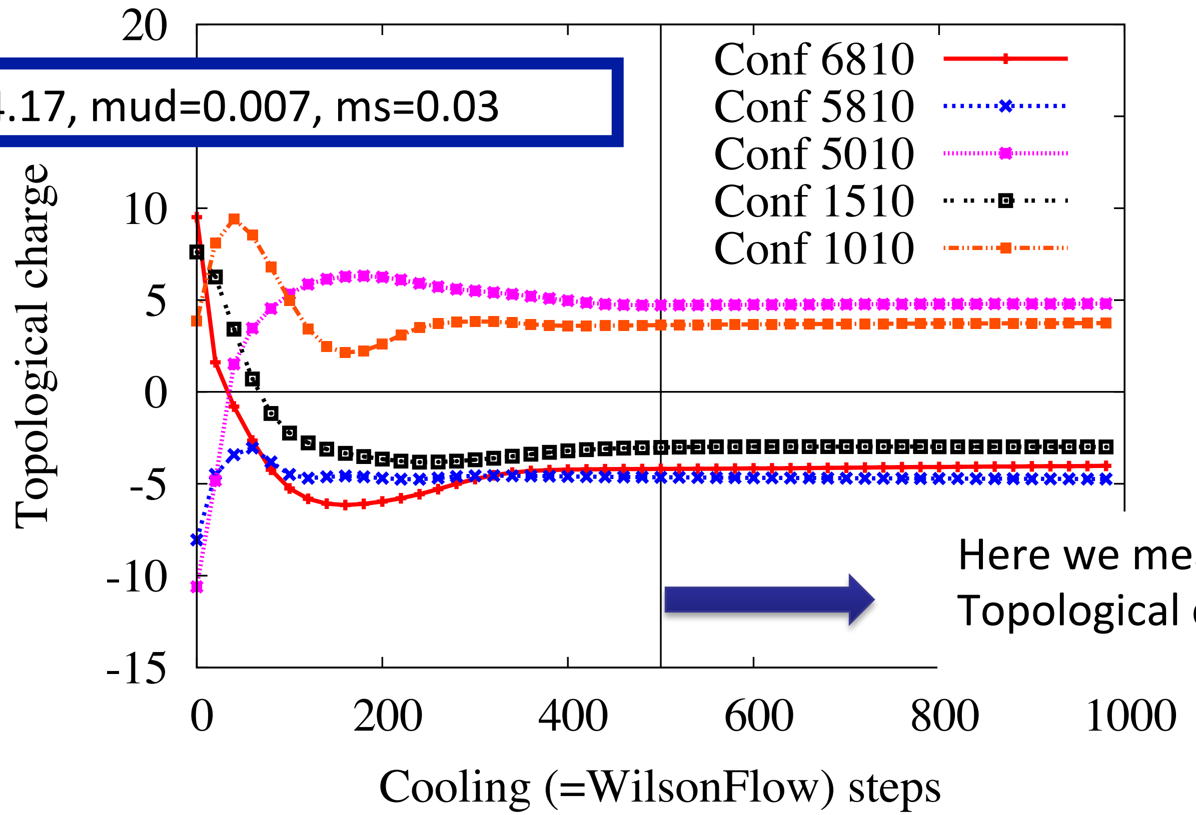
[Luscher 2010]

Wilson flow cooling

After Wilson flow at $\sqrt{8t} \sim 0.5\text{fm}$

topological charge does not change.

Beta=4.17, mud=0.007, ms=0.03



Here we measure Topological charge

Good agreement with Dirac index

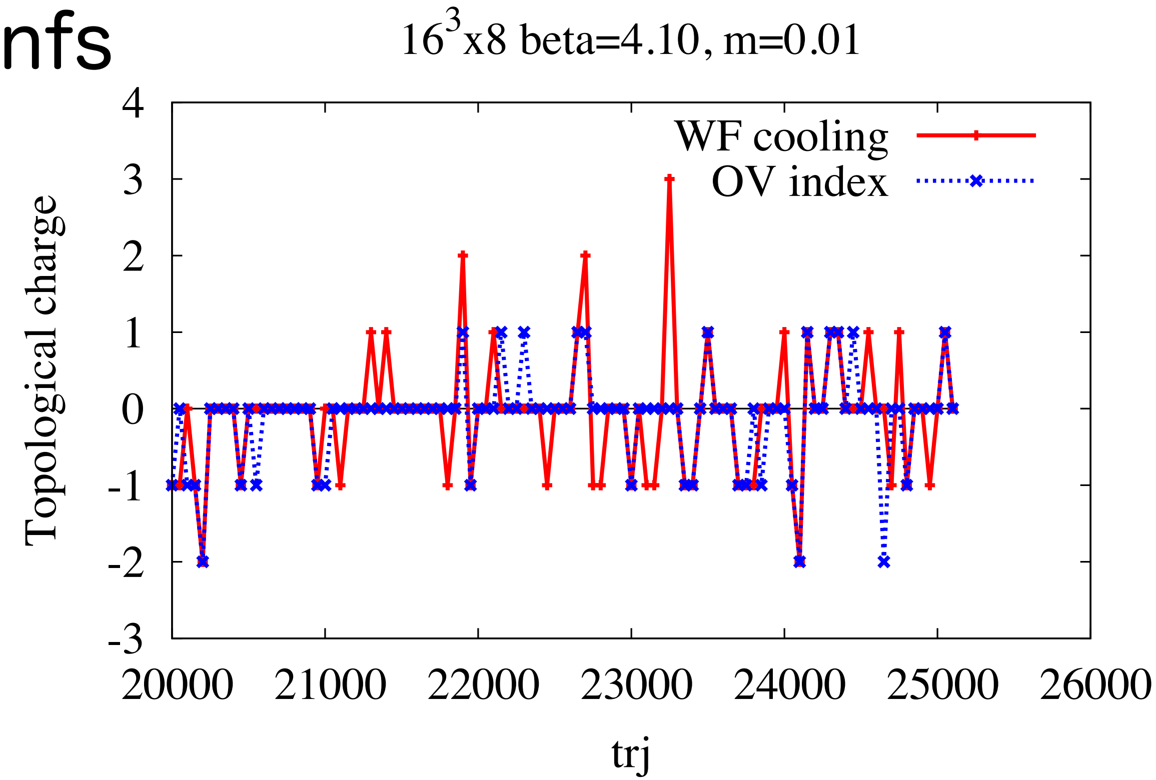
Index theorem : $Q = n_+ - n_-$

$n_\pm$: # of L/R zero-modes of overlap Dirac op.

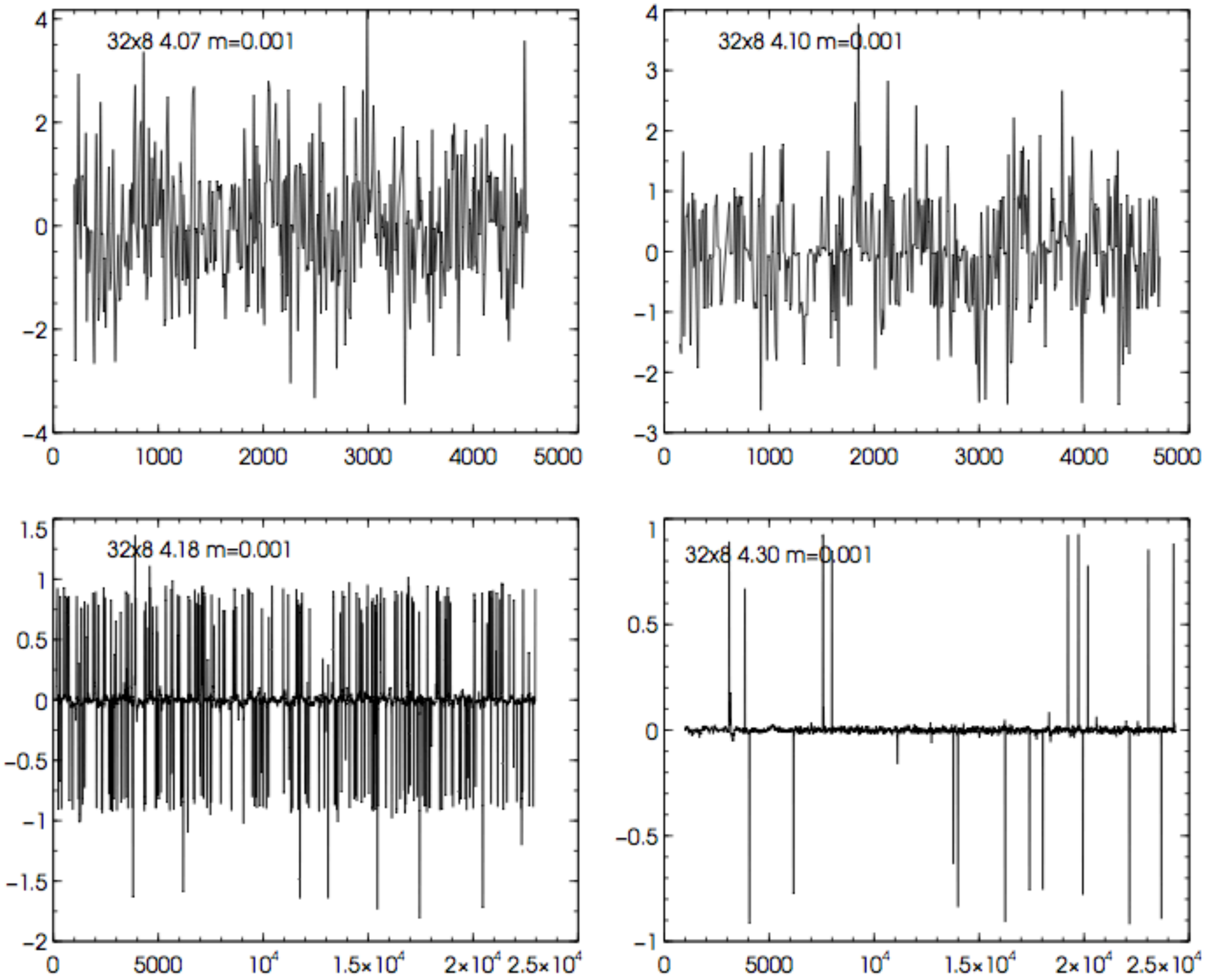
on non-cooled confs

Agreement

~ 80-90%.



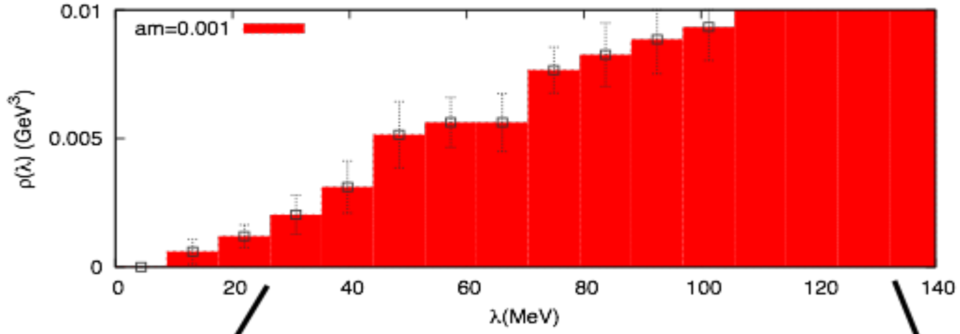
Topology fluctuates for $\beta < 4.30$



Dirac spectrum

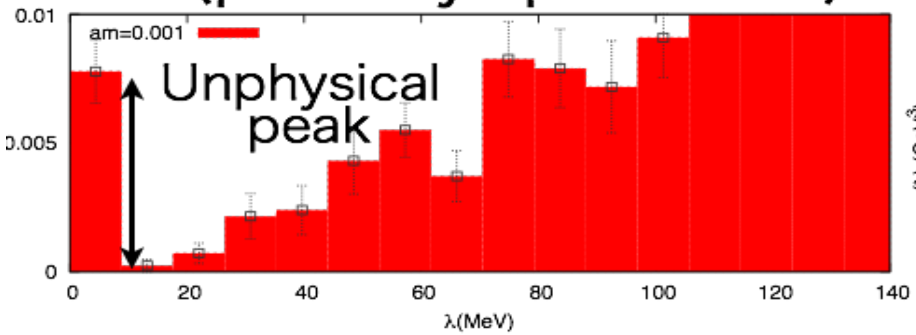
T= 203 MeV for L=2fm,T=1.13 Tc

Domain-wall

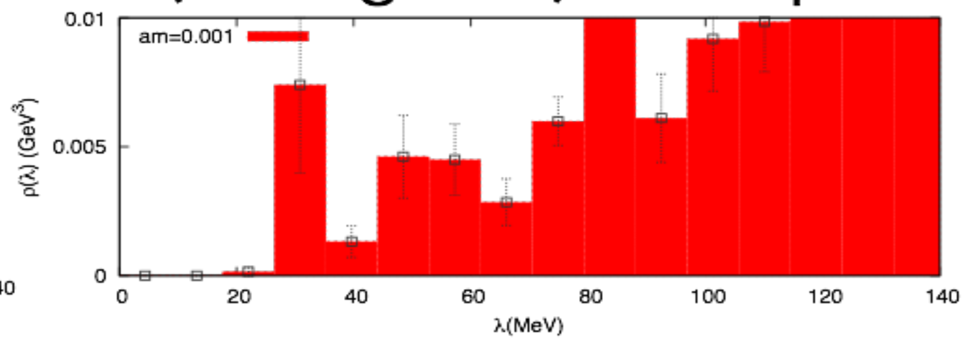


Cf. Dick et al. (2015)

Overlap on domain-wall sea (partially quenched)



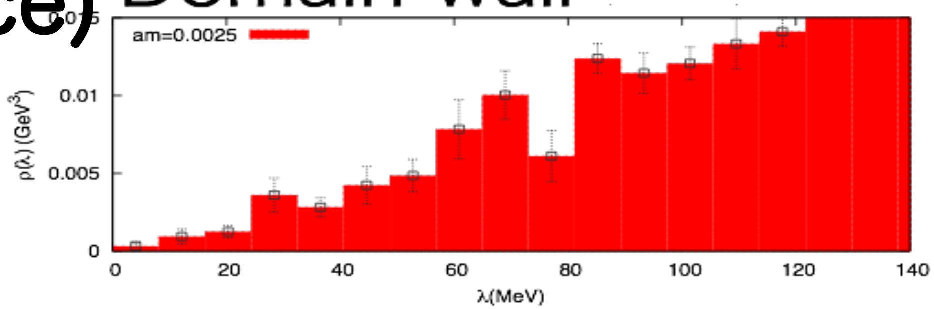
(reweighted)Overlap



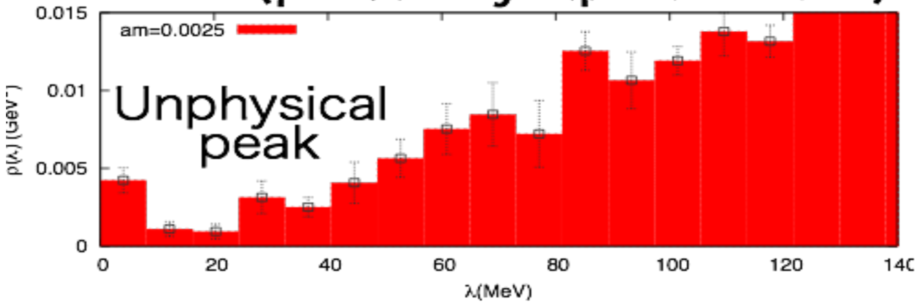
Dirac spectrum

T= 190 MeV for L=3fm,T=1.05 Tc
(fine lattice)

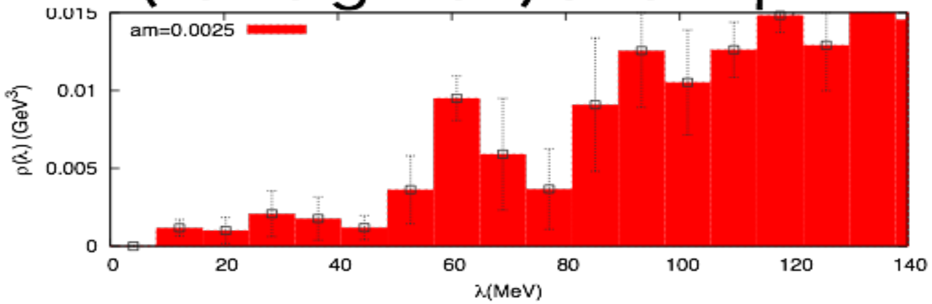
Domain-wall



Overlap on domain-wall sea (partially quenched)



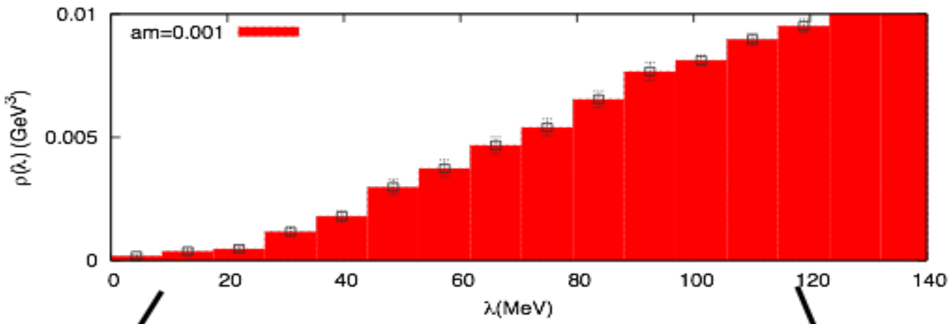
(reweighted)Overlap



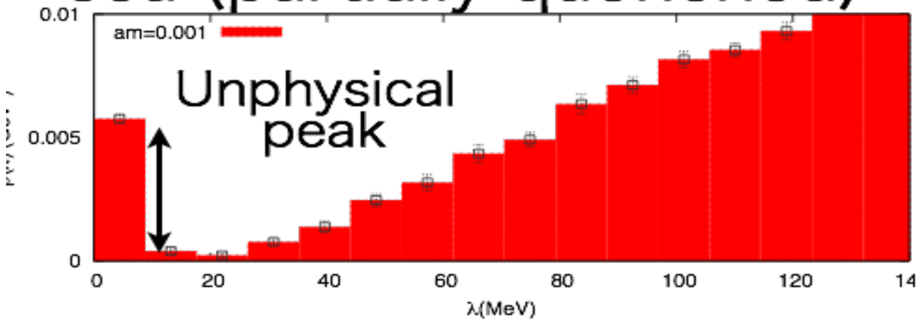
Dirac spectrum

T= 202 MeV for L=4fm, T=1.1 Tc

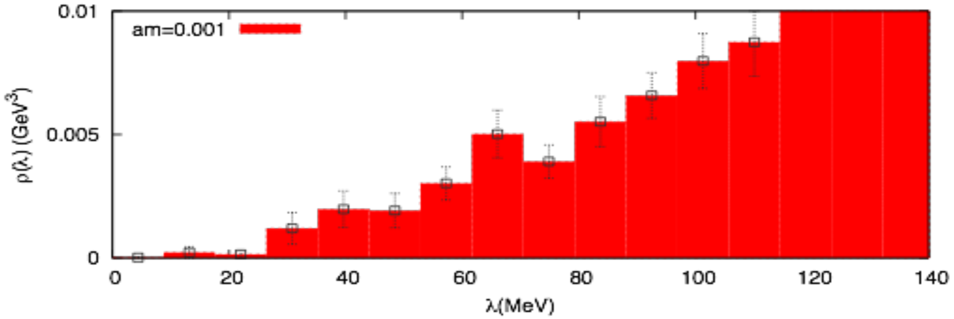
Domain-wall



Overlap on domain-wall sea (partially quenched)



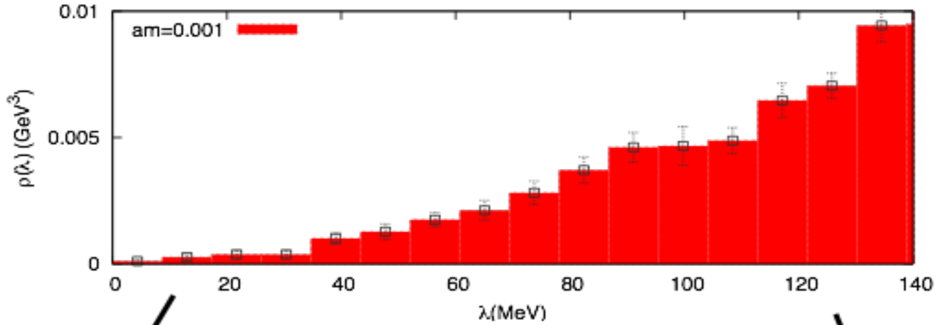
(reweighted)Overlap



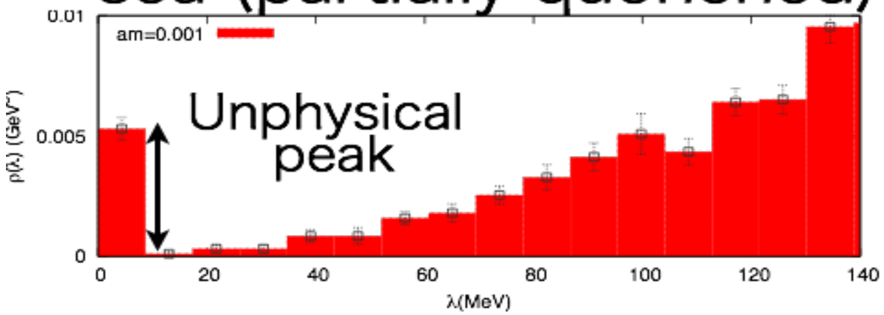
Dirac spectrum

T= 217 MeV for L=4fm, T=1.2 Tc

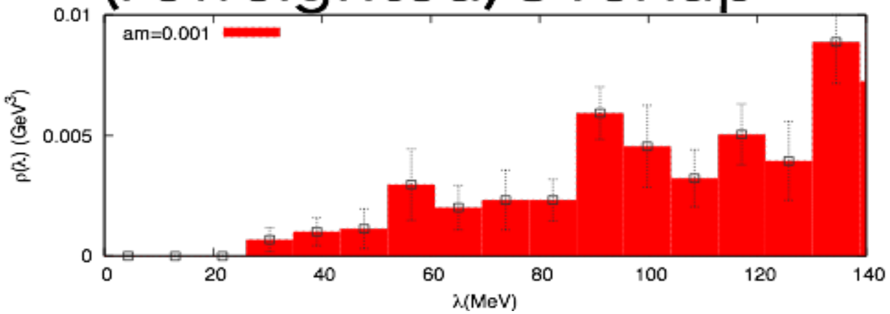
Domain-wall



Overlap on domain-wall sea (partially quenched)



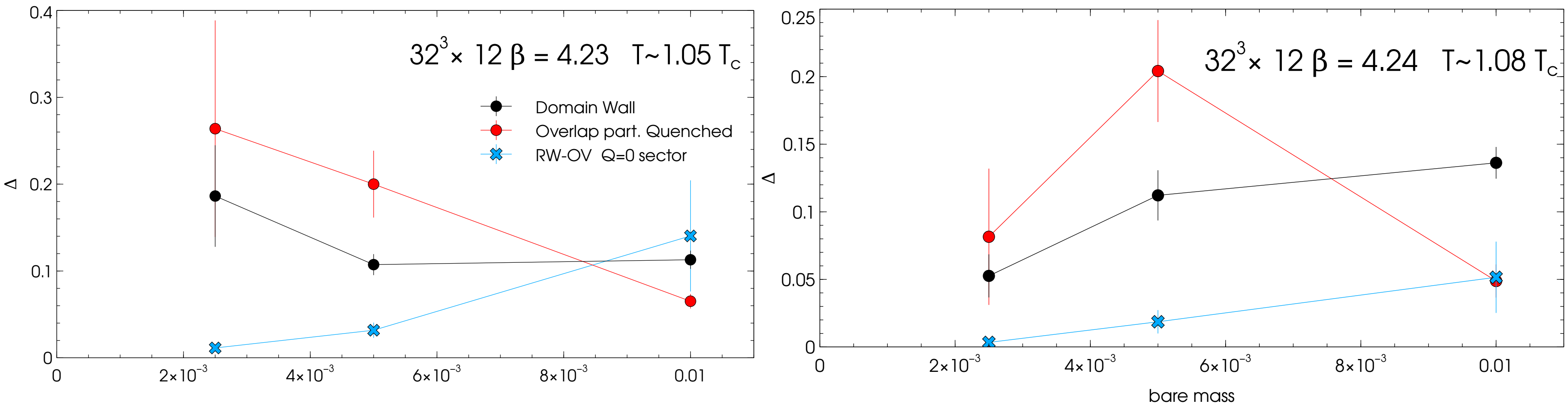
(reweighted)Overlap



U(1)A susceptability

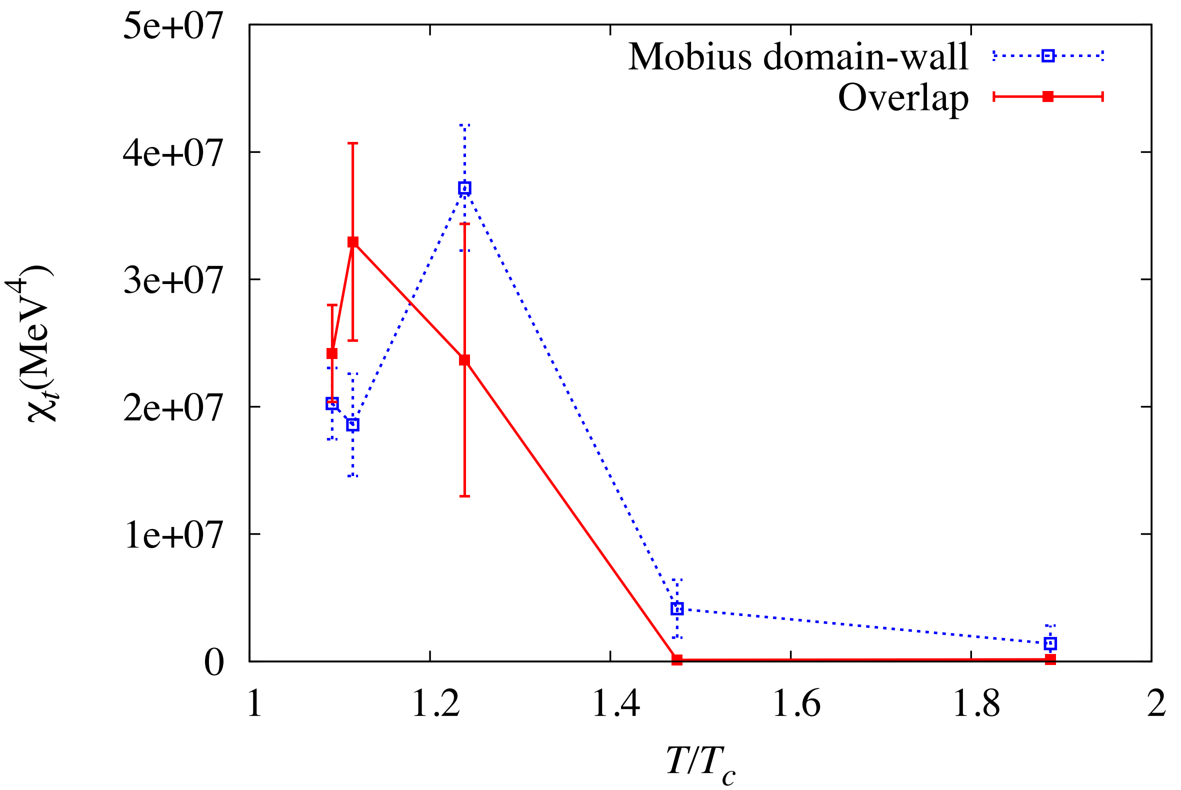
$$\Delta = \int d^4x[\langle\pi(x)\pi(0)\rangle - \langle\delta(x)\delta(0)\rangle]$$

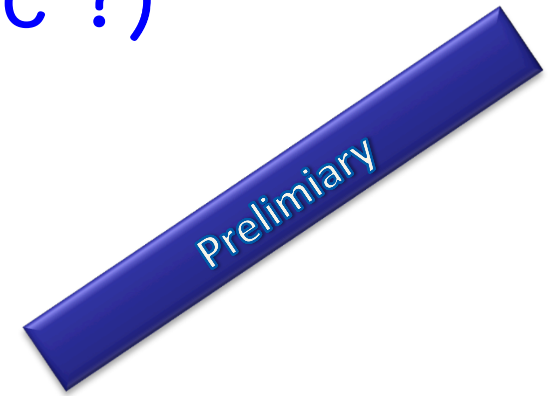
Only overlap converges to zero.



Topological susceptibility

We find strong suppression
(1st order transition at T~1.4Tc ?)





Summary

1. (effective or accidental or part of) axial U(1) symmetry restoration is interesting.
2. Dirac spectrum strongly connects SU(2)xSU(2) and U(1).
3. Chiral symmetry on the lattice is essential. Even Mobius DW is not good enough.
4. Our (reweighted) overlap fermion results suggest strong suppression of instanton effects and $U(1)_A$ anomaly (1^{st} order transition ?).

Summary

For chiral symmetry in QCD,

Spontaneous breaking [Nambu 1961]	⟷ Related !	Anomaly (explicit breaking) [Adler 1969, Bell,Jackiw 1969]

→ Standard 2nd order chiral phase transition with O(4) universality may be different.