- Kodaira - Spencer as the string field theory BCOV §5.3

The string field theory of topological B model reduces to a field theory on the target space.

(the path integral localizes on the space of constant maps.)

We can write down the (effective) action directly so that the equation of motion reproduces the K-S eq. (in Tian's form)

We can argue that this action agree with what we would obtain from the general prescription of constructing the action of string field theory [cf Witten hep-th 9207094]

Topological action on Calabi-Yau 3 fold X

$^{\exists}\Omega$: holomorphic $(3,0)$-form on X

$$S = \frac{1}{2}\int_X \Omega \wedge \alpha \qquad \alpha : (0,3)\text{-form on } X$$

Example 1 (Holomorphic Chern-Simons theory)

Open string field theory of topological B-model

B : $\mathfrak{g}$-valued $(0,1)$ form on X

$$\alpha = \mathrm{Tr}\left(B \wedge \bar{\partial} B + \frac{2}{3} B \wedge B \wedge B \right)$$

Eq. of motion $\quad \bar{\partial} B + B \wedge B = F_B^{(0,2)} = 0$ //

Example 2 (BCOV theory : KS gravity)

Closed string field theory of topological B-model

$A \in PV^{1,1}(X) \qquad \partial A = 0 \qquad \partial : PV^{i,j}(X) \longrightarrow PV^{i-1,j}(X)$

(holom. vector field) valued (0,1)-form

$$\beta = A \wedge \partial^{-1} \bar{\partial} A + \frac{1}{3} A \wedge A \wedge A \in PV^{3,3}(X)$$

$$\alpha = \beta^{\vee} = (\Omega \cdot \beta) \qquad (0,3)\text{-form}$$

Eq. of motion

$$\partial^{-1} \bar{\partial} A + \frac{1}{2} (A \wedge A) = 0$$

$$\bar{\partial} A + \frac{1}{2} \partial (A \wedge A) = 0$$

KS eq. in Tian's form.

<u>Remark</u> The action of BCOV theory may be regarded as the holomorphic Chern-Simon action where the gauge group (of the open string field theory) is replaced with Ω-preserving diffeomorphisms of X. (Infinitesimal diffeo $\leftrightarrow$ holom. vector fields)

Here an invariant Killing form on the algebra of holom. vector fields is given by

$$\mathrm{Tr}\, A B = \int_X \Omega \wedge A^\vee \partial^{-1} B^\vee \qquad A, B \in PV^{1,1}(X)$$

Target space interpretation of SUSY generators in the large volume limit

A-model

$$\mathcal{O}_\omega = \omega_{I_1 \cdots I_p \bar{J}_1 \cdots \bar{J}_q} \chi^{I_1} \cdots \chi^{I_p} \bar{\chi}^{J_1} \cdots \bar{\chi}^{J_q}$$

$$[\bar{Q}_+, \phi^{\bar{I}}] = \chi^{\bar{I}} , \quad [Q_-, \phi^{I}] = \chi^{I}$$

$$\bar{Q}_+ \leftrightarrow \bar{\partial} , \quad \bar{Q}_- \leftrightarrow \partial , \quad \Rightarrow \quad \bar{Q}_- \leftrightarrow \bar{\partial}^\dagger , \quad \bar{Q}_+ \leftrightarrow \partial^\dagger$$

$$Q_A = \partial + \bar{\partial} = d \qquad G_{zz} = \partial^\dagger , \quad G_{\bar{z}\bar{z}} = \bar{\partial}^\dagger$$

B-model

$$\mathcal{O}_F = F_{\bar{I}_1 \cdots \bar{I}_p}{}^{J_1 \cdots J_q} \bar{\eta}^{I_1} \cdots \bar{\eta}^{I_p} \theta_{J_1} \cdots \theta_{J_q}$$

$$[\bar{Q}_\pm , \bar{\phi}^{\bar{I}}] = \frac{1}{2} (\bar{\eta}^{\bar{I}} \pm g^{J\bar{I}} \theta_J)$$

target space metric

$$\bar{Q}_\pm \leftrightarrow \frac{1}{2} (\bar{\partial} \pm \partial^\dagger) \qquad G_{\substack{zz \\ \bar{z}\bar{z}}} = \frac{1}{2} (\bar{\partial}^\dagger \pm \partial)$$

$$Q_B = \bar{Q}_+ + \bar{Q}_- = \bar{\partial} \qquad G_{zz} - G_{\bar{z}\bar{z}} = \partial$$

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Comparison with the string field theory

See also Witten hep-th/9207094
in Floer memorial volume.

The B-model is indep. of the scaling of the volume of the target space.

In the large volume limit, only the "boundary" of the moduli space of Riemann surfaces contribute (gives a finite action).

In this limit $\bar{\partial} \sim Q = \bar{Q}_+ + \bar{Q}_-$ (BRST of B-model)

$\partial \sim G_{zz} - G_{\bar{z}\bar{z}} \Rightarrow \bar{b}_0$ (in bosonic string)

$\uparrow$ anti-ghost.

The constraint $\partial A = 0$ corresponds $b_0 \Psi = 0$ in string field theory.

$\uparrow$ string fields

The physical? string field A should have ghost number $(1, 1)$

To write down the kinetic term of closed string theory,

we need $c_0^- = c_0 - \bar{c}_0$ which satisfies $\{c_0^-, b_0^-\} = 1$

We cannot have c_0^- in general, but on the massive sector

we can define $c_0^- = \partial^{-1} = \dfrac{\partial^\dagger}{\Delta}$

the kinetic term of the closed string field theory

$$\frac{1}{2}(\Psi, c_0^- Q \Psi) \sim \frac{1}{2}\int A' \frac{1}{\partial}\bar{\partial} A'$$

reproduces the kinetic term of the BCOV theory

The gauge fixing condition $\bar{\partial}^\dagger A = 0 \longleftrightarrow b_0^+ \Psi = 0$

"the Siegal gauge"

$\longrightarrow$ closed string field propagator $\dfrac{b_0^+ b_0^-}{L_0 + \bar{L}_0} \longleftrightarrow \dfrac{\bar{\partial}^\dagger \partial}{\Delta} \sim \dfrac{1}{2}\dfrac{\partial}{\bar{\partial}}$

The action of BCOV theory is nothing but the closed string field theory action up to cubic terms.

In the usual closed string field theory, it is necessary to introduce higher string vertices (due to the absence of a cell decomposition of the moduli space of Riemann surfaces in accord with perturbation.) The higher string vertices comes from the internal domain of the moduli space. In the case of (type B) topological string theory, we can take the large volume limit and consequently, it has contribution only from the boundary of the moduli space. The higher order vertices are absent in BCOV theory.