From Supermembrane to Matrix String

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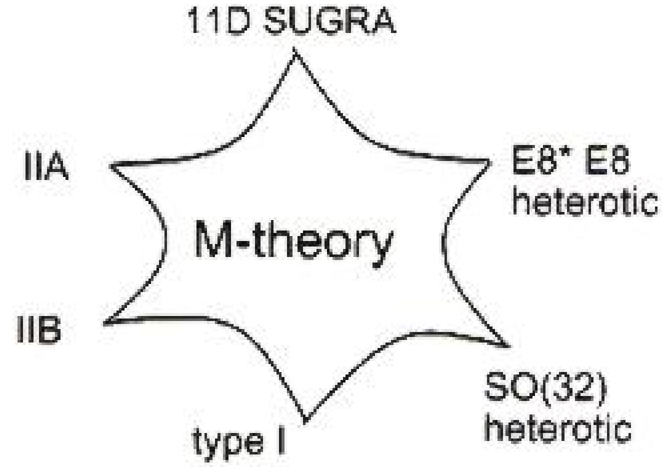
based on

Y. S. and T. Yoneya (U. Tokyo, Komaba), hep-th/0108176.

1. Introduction

Understanding after the discovery of the string dualities: ('95~)



Perturbative superstring theories (5 kinds)
= Various limits of 'M-theory' whose definition is not known.

In one of the limits (the strong coupling limit of 10D type IIA string), theory becomes 11-dimensional.

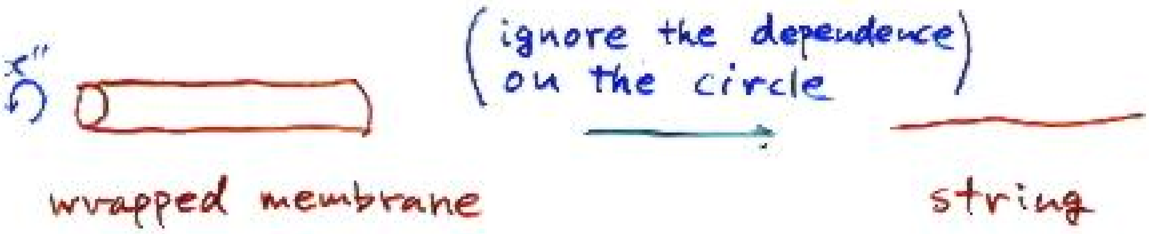
In the low-energy approximation, 11D M-theory is described by 11D SUGRA. However, 'microscopic' description of 11D M-theory is not known.

• What is the fundamental d.o.f. of (11D) M-theory?

It is naturally expected that supermembrane plays crucial role in the 11D M-theory.

- Supermembrane can be consistently coupled to 11D SUGRA. (Bergshoeff, Sezgin and Townsend, 1987)

- Upon double dimensional reduction (simultaneous dim. red. of spacetime and worldvolume), reduces to 10D type IIA superstring. (Duff, Howe, Inami and Stelle, 1987)



double dim. red.

However, there are difficulties for considering the quantum supermembrane as a fundamental theory.

- non-linear interacting 3D theory which is non-renormalizable.

Proposals for the definition of M-theory

Matrix theory (Banks, Fischler, Shenker and Susskind, 1996): (0+1)D U(N) SYM

$$S = \int dt \, \mathrm{Tr}\Big(\frac{1}{2 g_s \ell_s} D_t X_i D_t X_i + i\theta D_t \theta + \frac{1}{4 g_s \ell_s^5} [X_i, X_j]^2 - \frac{1}{\ell_s^2} \theta \Gamma_i [\theta, X_i] \Big)$$

Proposed as the definition of 11D M-theory in the light-cone frame.

- Known as matrix-regularization of the super-membrane in the light-cone gauge
- Reinterpreted as the theory of N D0-branes

String d.o.f. is not directly visible

Relation to (11D or even 10D) SUGRA is not evident

11D physics $\leftarrow$ large N limit, difficult to analyze.

Matrix string theory (Dijkgraaf, Verlinde and Verlinde, 1997): (1+1)D U(N) SYM

$$S = \int d\sigma^2 \mathrm{Tr}\Big(\frac{1}{4}(F_{\alpha\beta})^2 + \frac{1}{2}(D_\alpha X^i)^2 - \frac{1}{4g_s^2}[X^i, X^j]^2 + i\overline{\Psi}\Gamma^\alpha D_\alpha \Psi + \frac{1}{g_s}\overline{\Psi}\Gamma^i[X^i, \Psi]\Big)$$

Proposed as the non-perturbative definition of 10D type IIA string theory.

- 'Heuristic derivation': Obtained from Matrix theory by a sequence of string dualities.

$$\underset{(\mathrm{IIA})}{\mathrm{D0}} \xrightarrow{T} \underset{(\mathrm{IIB})}{\mathrm{D1}} \xrightarrow{S} \underset{(\mathrm{IIB})}{\mathrm{F1}} \xrightarrow{T} \underset{(\mathrm{IIA})}{\mathrm{F1}}$$

- Interpretation of the diagonal elements: light-cone Green-Schwarz type IIA string

- Argued that in the $g_s \to 0$ limit, effectively described by the $S^N \mathrm{R}^8$ CFT of diagonal elements. ('IR reduction')

It would describe 10 D SUGRA consistently

How 11-th dimension appear is not clear.

Our work:

Gave a direct correspondence between supermembrane wrapped around a circle and matrix string theory

- not relying on string dualities

- provide a hint for the large N behavior of matrix string theory

- clarify the 11D interpretation of matrix string theory

→ Starting point for analyzing 11D dynamics from matrix string theory

In addition, attempt at

Quantum theory of wrapped supermembrane
('Quantum double dimensional reduction':
integration of KK modes on the membrane)

Plan of the talk

1. Introduction

2. Light-cone supermembrane wrapped around a circle

3. Correspondence between wrapped supermembrane and matrix string

4. Attempt at quantum study of the wrapped supermembrane

5. Concluding remarks

2. Light-cone supermembrane wrapped around a circle

• Supermembrane action in the light-cone gauge (de Wit, Hoppe and Nicolai, 1988)

$$A = \frac{1}{\ell_M^3} \int d\tau \int_0^{2\pi L} d\sigma \int_0^{2\pi L} d\rho\ \mathcal{L},$$

$$\begin{aligned}(2\pi)^2 \mathcal{L} &= \frac{1}{2}(D_0 X^a)^2 + i\bar{\psi}\gamma_- D_0 \psi - \frac{1}{4}(\{X^a, X^b\})^2 \\ &\quad + i\bar{\psi}\gamma_- \Gamma_a \{X^a, \psi\}, \qquad (a = 1, \ldots, 9)\end{aligned}$$

where τ is proportional to the light-cone time

$$\ell_M^3 P^+ \tau / (2\pi L)^2 = X^+$$

and $\gamma_+ \psi = 0$

• Derivatives in spatial coordinate enter through Poisson bracket(PB)

$$\{A, B\} = (2\pi)^2 (\partial_\sigma A \partial_\rho B - \partial_\rho A \partial_\sigma B)$$

• Area preserving diffeo. (residual sym. in the l.-c. gauge)

$$\delta X^i = \{\Lambda(\sigma, \rho), X^i\}, \cdots$$

is gauged by introducing gauge field A_0

$$D_0 X^a = \partial_\tau X^a - \{A_0, X^a\}, \cdots$$

$$\delta A_0 = \partial_0 \Lambda(\tau, \sigma, \rho) + \{\Lambda(\tau, \sigma, \rho), A_0\}$$

Consider a membrane wrapped around a compact direction $X^9 \equiv Y$. (M-theory direction)

$$\text{radius of } Y = L = g_s \ell_s$$

• We choose ρ along Y-direction and set

$$Y(\tau, \sigma, \rho) = \rho + \hat{Y}(\tau, \sigma, \rho)$$

$Y(\rho + 2\pi L) = Y(\rho) + 2\pi L$

where $\hat{Y}$ is periodic in ρ

$$\underline{\hat{Y} = \sum_n \hat{Y}_n(\tau, \sigma) \mathrm{e}^{in\rho/L}.}$$

Also for other fields (X^i, Ψ, A), $(i = 1, \ldots, 8)$

$$X^i(\tau, \sigma, \rho) = x^i(\tau, \sigma) + \hat{X}^i(\tau, \sigma, \rho), \ldots$$
$$\underline{\hat{X}^i = \sum \hat{X}^i_n(\tau, \sigma) \mathrm{e}^{in\rho/L}}$$

• If we simply drop ρ-dependent fields ($\hat{X}^i$, $\hat{Y}$) action reduces to that of type IIA string.

However, quantum mechanically, there is no justification for dropping the d.o.f.

($\rightarrow$ last part of the talk)

3. Correspondence between wrapped membrane and matrix string

Preliminaries

- To relate the membrane to matrix string, first note that by the substitution $Y = \rho + \hat{Y}$, $\hat{Y}$ can be seen as 'A_σ'.

$$\{X^i, Y\} = \partial_\sigma X^i - \{\hat{Y}, X^i\} \equiv D_\sigma X^i$$
$$D_0 Y = \partial_0 \hat{Y} - \partial_\sigma A - \{A, \hat{Y}\} \equiv F_{0,\sigma}$$

The action takes the form of 2D gauge theory.

$$A = (2\pi)^2 L/\ell_M^3 \int d\tau \int_0^{2\pi} d\sigma \int_0^{2\pi} d\rho$$
$$\Big[\frac{1}{2}F_{0,\sigma}^2 + \frac{1}{2}(D_0 X^i)^2 - \frac{1}{2}(D_\sigma X^i)^2 - \frac{1}{4L^2}\{X^i, X^j\}^2$$
$$+ i\psi^T D_0 \psi - i\psi^T \Gamma_9 D_\sigma \psi + i\frac{1}{L}\psi^T \Gamma_i \{X^i, \psi\}\Big]$$

where we have rescaled $\rho \to L\rho$, $\sigma \to L\sigma$, $\tau \to \frac{L}{(2\pi)^2}\tau$

- $1/L$ plays the role of gauge coupling.

$\star$ What we are going to do is different from 'ordinary' matrix regularization (which would give a 1D theory)

Identification of the variables

Variables of wrapped supermembrane:

$$X^i(\tau,\sigma,\rho) = x^i(\tau,\sigma) + \sum \tilde{X}^i_n(\tau,\sigma)\mathrm{e}^{in\rho}\ , \ \cdots$$

where $0 \le \sigma \le 2\pi$.

Variables of matrix string theory: $N \times N$ matrices

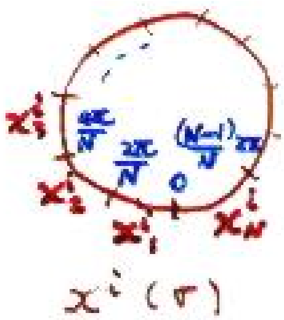
$$X^i_{k\ell}(\tau,\theta)\ ,\ \cdots$$

where $0 \le \theta \le 2\pi$. We denote $x^i_k(\tau,\theta) \equiv X^i_{kk}(\tau,\theta)$

• We assume that ρ-independent mode of the membrane $x^i(\sigma)$ corresponds to diagonal elements of matrix string .

Decompose σ into N segments and identify

$$x^i(\sigma) \Leftrightarrow x^i_k(\theta)$$



with $\quad \sigma = \dfrac{2(k-1)}{N}\pi + \dfrac{\theta}{N}$

where we have imposed the 'long string b.c.' for matrix string:

$$x^i_k(\theta+2\pi) = x^i_{k+1}(\theta), \quad (k=1,2,\ldots,N-1),$$
$$x^i_N(\theta+2\pi) = x^i_1(\theta).$$

• Further assume the correspondence between off-diag. elements and KK modes.

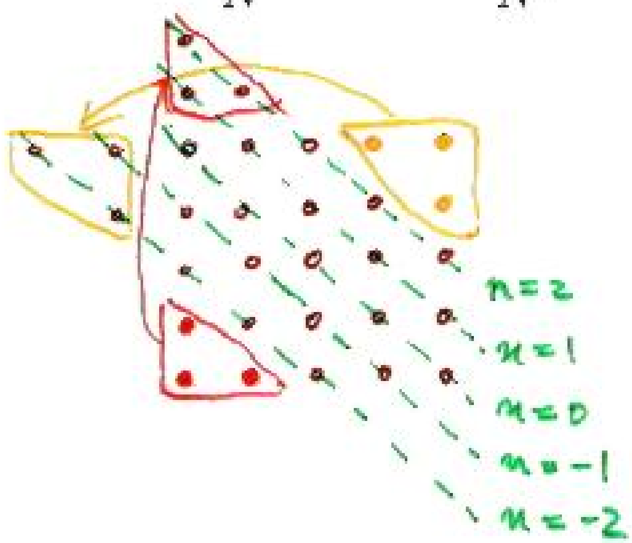
$$X^i_n(\sigma) \Leftrightarrow X^i_{k\ell}(\theta)$$

with (for N=odd; similarly for N=even)

$$\sigma = \frac{(k+\ell-N-2)\pi}{N} + \frac{\theta}{N}, \quad n = k-\ell-N, \ k-\ell \geq \frac{N+1}{2}$$

$$\sigma = \frac{(k+\ell-2)\pi}{N} + \frac{\theta}{N}, \quad n = k-\ell, \quad |k-\ell| \leq \frac{N-1}{2}$$

$$\sigma = \frac{(k+\ell-N-2)\pi}{N} + \frac{\theta}{N}, \quad n = k-\ell+N, \ k-\ell \leq -\frac{N-1}{2}$$



Essentially,
KK momentum
⇔ (row)-(column)

b.c. for the matrix string fields:

$$M(\theta+2\pi) = SM(\theta)S^\dagger, \quad S_{k\ell} = \begin{pmatrix} & & 1 \\ 1 & & \\ & \ddots & \\ & 1 & \end{pmatrix}$$

We also require all the fields of matrix string theory are periodic under $\sigma \to \sigma + \pi$.

• KK-momentum $|n|$ is cut off at $(N-1)/2$. (i.e. ρ is discretized)

Mapping of the products

• With the above identification, we can see that P.B of membrane variables corresponds to matrix commutator (to the leading order in the large N limit)

$$\{X^i, X^j\}_n(\sigma) \Leftrightarrow i(\frac{2\pi}{N})^{-1}[X^i, X^j]_{k\ell}(\theta) \quad (n=k-\ell)$$

∵) e.g. commutator of a diagonal and an arbitrary matrix:

$$\begin{aligned}
&[x^i(\theta), X^j(\theta)]_{k\ell} = (x^i_k(\theta) - x^i_\ell(\theta))X^j_{k\ell}(\theta) \\
&= \frac{2(k-\ell)\pi}{N}\partial_\sigma x^i(\sigma_{k\ell})X^j_{k-\ell}(\sigma_{k\ell}) + O(1/N^2) \\
&= -i(\frac{2\pi}{N})\{x^i, X^j\}_{k-\ell}(\sigma_{k\ell}) + O(1/N^2)
\end{aligned}$$

where $\sigma_{k\ell} = \frac{k+\ell-2}{N}\pi + \frac{\theta}{N}$ (when $k-\ell < N/2$)

• Similarly, we obtain a map between general product of membrane variables and matrix string variables.

$$\begin{aligned}
&\frac{1}{N}\int d\theta \, \mathrm{Tr}\Big(M^{(1)}(\theta)M^{(2)}(\theta)\cdots M^{(\ell)}(\theta)\Big) \\
&= \frac{1}{2\pi}\int d\rho \int d\sigma \exp\Big[-i\frac{\pi}{N}\sum_{\ell\geq i>j\geq 1}(\partial_{\sigma_j}\partial_{\rho_i} - \partial_{\rho_j}\partial_{\sigma_i})\Big] \\
&M^{(1)}(\sigma_1,\rho_1)\cdots M^{(\ell)}(\sigma_\ell,\rho_\ell)\Big|_{\sigma_i=\sigma,\rho_i=\rho}.
\end{aligned}$$

Matrix string action

• Using the above mapping ($\{\ ,\ \} \to [\ ,\ ]$ and $\int d\sigma d\rho \to \int d\theta \mathrm{Tr}$), supermembrane action is rewritten as the matrix string action

$$A = \frac{(2\pi)^2 L}{\ell_M^3} \int d\tau \frac{2\pi}{N} \int_0^{2\pi} d\theta\, \mathrm{Tr}\Big(\frac{1}{2}F_{0,\theta}^2 + \frac{1}{2}(D_0 X^i)^2 - \frac{1}{2}N^2 (D_\theta X^i)^2$$
$$+\frac{1}{4L^2}(\frac{N}{2\pi})^2 [X^i, X^j]^2 + i\psi^T D_0 \psi - Ni\,\psi^T \Gamma_9 D_\theta \psi - \frac{1}{L}\frac{N}{2\pi}\psi^T \Gamma_i [X^i, \psi]\Big)$$

where

$$D_\theta X^i = \partial_\theta X^i - i\frac{1}{2\pi L}[Y, X^i],$$

$$D_0 X^i = \partial_\tau X^i - i\frac{N}{2\pi L}[A, X^i],$$

By performing the redefinition

$$\tau \to \tau/N, \quad L \to L/2\pi, \quad \psi \to \sqrt{N}\psi,$$

the N dependence is eliminated and the action is reduced to the standard matrix-string theory action.

Matrix string in a curved b.g. (example)

Important aspect of our correspondence:
It should serve as a principle for constructing matrix string theory in curved b.g. (starting from supermembrane in general b.g.)

Simple example of exact (not a weak-field expansion) matrix string action on a curved (RR) b.g.

Kaluza-Klein Melvin background

10D spacetime obtained from flat 11D by a compactification with non-trivial topology:

$$(r, y, \varphi) \simeq (r, y + 2\pi Lm, \varphi \underline{+ 2\pi qLm} + 2\pi n)$$

where $y \equiv x^9$ and $x^7 + ix^8 = re^{i\varphi}$.

Define the coordinates which are single-valued in the y-direction.

$$x^7_{flat} + ix^8_{flat} = e^{iqy}(x^7 + ix^8), \; \psi_{flat} = e^{-\frac{q}{2}\Gamma_{78}y}\psi$$

$\rightarrow$ Dimensional reduction along y

$$ds^2_{10} = f(r)[-dt^2 + dx_1^2 + \cdots + dx_6^2 + dr^2 + r^2 f^{-2}(r)d\varphi^2 + dx^2_{10}]$$

$$e^{\phi} = f^{3/2}(r), \quad A_\varphi = qr^2 f^{-2}(r)$$
$$f(r) = (1 + q^2r^2)^{1/2}$$

Rewrite the light-cone membrane action in the 'single-valued' variables. Simply replace the derivative ∂_α with the following 'covariant derivatives'

$$\nabla_\alpha X^m = \partial_\alpha X^m + q\partial_\alpha Y \epsilon^{mn} X^n,$$
$$\nabla_\alpha X^i = \partial_\alpha X^i,\ \nabla_\alpha Y = \partial_\alpha Y, \dots$$

where x^i $(i = 1, \dots, 6)$: 'trivial' transverse directions.

Applying the membrane-matrix string mapping, we obtain matrix string action

$$A = A^0 + A^1 + A^2$$

where A^0: same as the flat space action,

$$\begin{aligned} A^1 &= q\int d\tau d\sigma d\rho\, \epsilon^{mn}\Big[-D_0 Y D_0 X^m X^n + \{X^i, Y\}\{X^i, X^m\}X^n \\ &\quad + \{X^p, Y\}\{X^p, X^m\}X^n - i\psi^T \Gamma_m X^n \{Y, \psi\} \\ &\quad - \frac{i}{4}\psi^T \Gamma_{mn}\psi D_0 Y - \frac{i}{4}\psi^T \Gamma_i \Gamma_{mn}\psi \{X^i, Y\}\Big], \end{aligned}$$

$$\begin{aligned} A^2 &= q^2 \int d\tau d\sigma d\rho \Big[\frac{1}{2}(D_0 Y)^2 (X^m)^2 - \frac{1}{2}\{X^i, Y\}^2 (X^m)^2 \\ &\quad - \frac{1}{2}(X^m\{X^m, Y\})^2\Big]. \end{aligned}$$

- Stability of the b.g., etc. from this formalism. (future study)

4. Attempt at the quantum study of wrapped supermembrane

• Supermembrane wrapped around a circle is expected to reduce to superstring in the $L \to 0$ limit. (double dimensional reduction)

To justify double dim. red. quantum mechanically, we must integrate out KK modes.

• Naively, KK modes are expected to become infinitely massive and decouple in the $L \to 0$ limit.

$$\left(\partial_\rho \phi^* \partial_\rho \phi \to (n^2/L^2)\phi^2 \quad \text{for } \phi \sim \mathrm{e}^{in\rho/L} \right)$$

However, in our case, decoupling is not straightforward. Instead of mass term, we have interaction term

$$\frac{1}{4}\{X^i, X^j\}^2 = \frac{1}{2}(\partial_\sigma x^i)^2(\partial_\rho \tilde{X}^j)^2 - \frac{1}{2}(\partial_\sigma x^i \partial_\rho \tilde{X}^i)^2 + \cdots$$

• We obtain effective action for string ($x^i(\tau,\sigma)$, $\psi(\tau,\sigma)$) in the $L \to 0$ limit as an expansion in L (strong coupling expansion).

Correction to the free string action in powers of $L = g_s \ell_s = \mathrm{e}^{\phi} \ell_s$ is not expected
→ Confirmed up to order L^2

Backgrounds: ρ-independent modes (x^i, ψ)

Fluctuations: ρ-dependent modes (X^i, Ψ, A, Y, C, $\overline{C}$)

- Adopt the background field gauge

$$\mathcal{L}_{g.f.} = -\frac{1}{2}(\partial_0 A - \{a, A\} + \{x^i, X^i\} + \{\rho, Y\})^2$$

and introduce ghosts C and $\overline{C}$.

- The part of the action which contain no spinor fields (ψ, Ψ) are

$$\mathcal{L}_B = \mathcal{L}_{B,\mathrm{bg}} + \mathcal{L}_{B,0} + L^1\mathcal{L}_{B,1} + L^2\mathcal{L}_{B,2},$$

$$\mathcal{L}_{B,\mathrm{bg}} = \frac{1}{2}(\partial_0 x^i)^2 + \frac{1}{2}(\partial_\sigma x^i)^2 + \frac{1}{2}(\partial_\sigma a)^2,$$

$$\begin{aligned}\mathcal{L}_{B,0} \;=\;& \frac{1}{2}\big((\partial_\sigma a)^2 + (\partial_\sigma x^i)^2\big)(\partial_\rho X^j)^2 + \frac{1}{2}\big((\partial_\sigma a)^2 + (\partial_\sigma x^i)^2\big)(\partial_\rho Y)^2 \\ &+\frac{1}{2}\big((\partial_\sigma a)^2 + (\partial_\sigma x^i)^2\big)(\partial_\rho A)^2 - i\big((\partial_\sigma a)^2 + (\partial_\sigma x^i)^2\big)\partial_\rho\bar{C}\partial_\rho C,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{B,1} \;=\;& -2\partial_0 x^i\{A, X^i\} - 2\partial_\sigma a\{Y, A\} - 2\partial_\sigma x^i\{Y, X^i\} \\ &-\partial_\sigma a\partial_0 A\partial_\rho A - \partial_\sigma a\partial_0 Y\partial_\rho Y - \partial_\sigma a\partial_0 X^i\partial_\rho X^i \\ &-\partial_\sigma a\partial_\rho Y\{Y, A\} - \partial_\sigma a\partial_\rho X^i\{X^i, A\} \\ &-\partial_\sigma x^i\partial_\rho A\{A, X^i\} - \partial_\sigma x^i\partial_\rho Y\{Y, X^i\} - \partial_\sigma x^i\partial_\rho X^j\{X^j, X^i\} \\ &+i\partial_\sigma a\partial_0\overline{C}\partial_\rho C + i\partial_\sigma a\partial_\rho\overline{C}\partial_0 C \\ &+i\partial_\sigma a\partial_\rho\overline{C}\{C, A\} + i\partial_\sigma x^i\partial_\rho\overline{C}\{C, X^i\},\end{aligned}$$

$$
\begin{aligned}
\mathcal{L}_{B,2} &= \frac{1}{2}(\partial_0 A)^2 + \frac{1}{2}(\partial_0 X^i)^2 + \frac{1}{2}(\partial_0 Y)^2 + \frac{1}{2}(\partial_\sigma A)^2 + \frac{1}{2}(\partial_\sigma X^i)^2 \\
&+\frac{1}{2}(\partial_\sigma Y)^2 + \partial_\sigma A\{A, Y\} + \partial_\sigma X^i\{X^i, Y\} + \partial_0 X^i\{X^i, A\} \\
&+\partial_0 Y\{Y, A\} + \frac{1}{2}\{A, X^i\}^2 + \frac{1}{2}\{A, Y\}^2 + \frac{1}{2}\{X^i, Y\}^2 + \frac{1}{4}\{X^i, X^j\} \\
&-i\partial_0\overline{C}\partial_0 C - i\partial_\sigma\overline{C}\partial_\sigma C + i\partial_0\overline{C}\{A, C\} + i\partial_\sigma\overline{C}\{Y, C\}.
\end{aligned}
$$

- The part containing spinor fields are

$$\mathcal{L}_F = \mathcal{L}_{F,\mathrm{bg}} + \mathcal{L}_{F,0} + L^{1/2}\mathcal{L}_{F,1/2} + L^1\mathcal{L}_{F,1},$$

$$\mathcal{L}_{F,\mathrm{bg}} = \psi^T\partial_0\psi + i\psi^T\Gamma_9\partial_\sigma\psi,$$

$$\mathcal{L}_{F,0} = -\Psi^T\partial_\sigma a\partial_\rho\Psi - \Psi^T\Gamma_i\partial_\sigma x^i\partial_\rho\Psi,$$

$$\mathcal{L}_{F,1/2} = 2\Psi^T\partial_\rho A\partial_\sigma\psi + 2i\Psi^T\Gamma_9\partial_\rho Y\partial_\sigma\psi + 2i\Psi^T\Gamma_i\partial_\rho X^i\partial_\sigma\psi,$$

$$
\begin{aligned}
\mathcal{L}_{F,1} &= \Psi^T\partial_0\Psi + \Psi^T\Gamma_9\partial_\sigma\Psi \\
&-\Psi^T\{A, \Psi\} - \Psi^T\Gamma_9\{Y, \Psi\} - \Psi^T\Gamma_i\{X^i, \Psi\}.
\end{aligned}
$$

• Leading term in L which we treat as free action for the fluctuations (X^i, A, Y, Ψ $\overline{C}$ and C)

$$\mathcal{L}_0 = \frac{1}{2}\Big((\partial_\sigma a)^2 + (\partial_\sigma x^i)^2\Big)(\partial_\rho X^j)^2 + \cdots$$
$$-\Psi^T(\partial_\sigma a + \Gamma_i \partial_\sigma x^i)\partial_\rho \Psi$$

does not contain derivatives (∂_τ, ∂_σ).

Propagators are proportiaonal to the delta function $(\xi = \tau, \sigma)$

$$\langle X^i_{-n}(\xi) X^j_n(\xi')\rangle = \frac{\delta^{ij}}{n^2} G(\xi, \xi'),$$
$$\langle \Psi_{-n}(\xi)\Psi_n(\xi')\rangle = \frac{-i}{2n}(\partial_\sigma a - i\Gamma_i \partial_\sigma x^i) G(\xi, \xi'),$$

$$G(\xi, \xi') = \frac{1}{(\partial_\sigma a)^2 + (\partial_\sigma x^i)^2}\delta^{(2)}(\xi - \xi').$$

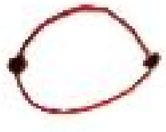
• UV divergences $\delta(0)$ upon loop integration. Definite regularization is needed for a rigorous treatment.

We treat the divergence formally as $\delta(0)$ and see whether its coefficient vanish.

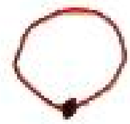
⋆ We are considering general configurations of strings (not restricted to static, BPS etc.)

- L^0 contribution to the effective action (one-loop determinant of the propagator) vanish due to the matching of the bosonic and fermionic d.o.f. (8 (=10-2) bosons: $X^i, Y, A, C, \overline{C}$; 8 (=16/2) fermions: Ψ)

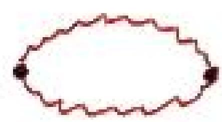
- Next non-trivial order is one-loop, L^2.



$\langle \mathcal{L}_{B,1} \mathcal{L}_{B,1} \rangle^{(one-loop)}$



$\langle \mathcal{L}_{B,2} \rangle^{(one-loop)}$



$\langle \mathcal{L}_{F,1} \mathcal{L}_{F,1} \rangle^{(one-loop)}$

$$\langle \mathcal{L}_{B,1}\mathcal{L}_{B,1}\rangle^{(one-loop)} = -4\int d^2\xi \int d^2\xi' \sum_{n\neq 0}\frac{1}{n^2}$$
$$\left[\partial_0 x^i \partial'_0 \partial'_\sigma x^i \partial_\sigma G(\xi,\xi')G(\xi,\xi') + \partial_\sigma x^i \partial'_\sigma \partial'_\sigma x^i \partial_\sigma G(\xi,\xi')G(\xi,\xi')\right],$$

$$\langle \mathcal{L}_{B,2} \rangle^{(one-loop)} = -4 \int d^2\xi \sum_{n\neq 0} \frac{1}{n^2} \lim_{\xi \to \xi'} \left[\partial_0 \partial'_0 G(\xi,\xi') + \partial_\sigma \partial'_\sigma G(\xi,\xi') \right]$$

We rewrite this term by inserting the delta function

$$\lim_{\xi \to \xi'} 1 = \int d^2\xi' \delta(\xi - \xi') = \int d^2\xi' \partial_\sigma x^i \partial'_\sigma x^i G(\xi,\xi')$$

as

$$\langle \mathcal{L}_{B,2} \rangle^{(one-loop)} = -4 \int d^2\xi \int d^2\xi' \sum_{n\neq 0} \frac{1}{n^2} \partial_\sigma x^i \partial'_\sigma x^i \left[\partial_0 \partial'_0 G(\xi,\xi') G(\xi,\xi') \right.$$
$$\left. + \partial_\sigma \partial'_\sigma G(\xi,\xi') G(\xi,\xi') \right].$$

$$\langle \mathcal{L}_{F,1} \mathcal{L}_{F,1} \rangle^{(one-loop)} = 4 \int d^2\xi \int d^2\xi' \sum_{n\neq 0} \frac{1}{n^2}$$
$$\left[\partial_\sigma x^i \partial'_\sigma x^i G(\xi,\xi') \partial_0 \partial'_0 G(\xi,\xi') + \partial_\sigma x^i \partial'_\sigma x^i G(\xi,\xi') \partial_\sigma \partial'_\sigma G(\xi,\xi') \right.$$
$$\left. + \partial_\sigma x^i \partial'_0 \partial'_\sigma x^i G(\xi,\xi') \partial_0 G(\xi,\xi') + \partial_\sigma x^i \partial'_\sigma \partial'_\sigma x^i G(\xi,\xi') \partial_\sigma G(\xi,\xi') \right].$$

The sum of the one-loop L^2 contribution vanish

$$\langle \mathcal{L}_{B,1} \mathcal{L}_{B,1} \rangle^{(one-loop)} + \langle \mathcal{L}_{B,2} \rangle^{(one-loop)} + \langle \mathcal{L}_{F,1} \mathcal{L}_{F,1} \rangle^{(one-loop)} = 0.$$

- In our treatment, higher-order terms are ambiguous and we do not have definite answer.

- The problem of 'quantum double dimensional reduction' is essentially equivalent to that of the Infra-red reduction in matrix string theory.

5. Concluding remarks

Summary

Established a direct correspondence between light-cone supermembrane and matrix string.

- Explicit mapping of the variables not relying on string-duality arguments.
- Clarified the 11D-interpretation of the off-diagonal elements of matrix string.
- Should serve as a principle for constructing matrix string in curved b.g. (Example: Matrix string action on Kaluza-Klein Melvin b.g.)

Analyzed the effective action for strings by integrating out KK modes on the membrane

- Gave (some) justification to the double dimensional reduction quantum mechanically.

Future problems

- 11D Supergravity from supermembrane or matrix string

- String interaction from membrane picture

- Further (rigorous) analysis of quantum double dimensional reduction

- Matrix string on Kaluza-Klein Melvin b.g.; (Stability of the b.g., fate of type 0 tachyon)

- Covariant matrix string theory?