

Exactly Solvable Matrix Models with Spontaneous Breakdown of $SO(D)$ Symmetry

— a mechanism for
dynamical generation of 4d space-time
in nonperturbative string theory —

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hep-th/0108070.

0. Introduction

superstring theory

a natural (promising) candidate
for a unified theory including gravity

- A Big Puzzle :
consistency (unitarity, Lorentz invariance)
→ 10d space-time
- A Possible Scenario :
our 4d space time is realized as a brane
in 10d space time
- A Direct Approach to this issue :
IIB matrix model
(Ishibashi-Kawai-Kitazawa-Tsuchiya '96)
nonperturbative definition of type IIB
superstring theory (conjecture)

A_μ ($\mu = 1, \dots, 10$)

$N \times N$ hermitian matrix

interpreted as dynamical space-time

$$Z = \int dA e^{-S_b[A]} Z_f[A]$$

complex

- MC sim with $|Z_f[A]|$
→ no SSB of $SO(10)$.
- phase of $Z_f[A]$ favours lower-dim config
Saddle-point analysis → $3 \leq d \leq 8$
- a concrete example of matrix models
in which SSB of $SO(D)$ is driven by
the phase of fermion det.

cf.) Random Matrix Model for
finite density QCD.

useful toy model for testing methods
to study IIB m.m.

- "d = 4" from IIB matrix model!
J.N.-Sugino hep-th/0111102

a new type of "mean-field" approach
originally proposed for BFSS Matrix Theory
Kabat-Lifschytz ('00), ...

Convergent series expansion.

tested up to 10-th order in ϕ^4 toy model

- Apply this new method
to exactly solvable matrix model
JN.-Okubo-Sugino in prep.

- A new Monte Carlo technique
to study complex-action system
Anagnostopoulos - J.N. hep-th/0108041

Plan of the talk

§ 0. Introduction

§ 1. Def. of IIB matrix model.

§ 2. A mechanism for 4d space-time
SSB of $SO(10)$ driven by
the phase of $Z_f[A]$.

§ 3. Exactly solvable matrix models

- Random Matrix Theory
- new models with SSB of $SO(D)$.

§ 4. $d=4$ from IIB matrix model.

a new type of "mean-field" approach

§ 5. testing the new method
with exactly solvable m.m.

→ reveals its power

§ 6. Summary

nonperturbative formulations of
superstring/M theories (conjectures)

Banks-Fishler-Susskind-Shenker ('96)

Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)

IIB matrix model (\iff type IIB superstring)

$$\begin{cases} A_\mu \ (\mu = 1, \dots, 10) & \text{bosonic} \\ \psi_\alpha \ (\alpha = 1, \dots, 16) & \text{fermionic} \end{cases}$$

$N \times N$ hermitian matrices

$$\begin{aligned} Z &= \int dA \, d\psi \, e^{-S_b - S_f} \\ S_b &= -\frac{1}{4g^2} \text{tr} ([A_\mu, A_\nu]^2) , \\ S_f &= -\frac{1}{2g^2} (\tilde{\Gamma}_\mu)_{\alpha\beta} \text{tr} (\psi_\alpha [A_\mu, \psi_\beta]) . \quad (1) \end{aligned}$$

A_μ : dynamical space-time
manifest SO(10) inv. and $N=2$ SUSY

g : scale parameter

$N \rightarrow \infty \iff$ cont. lim. in lattice gauge theory

$\tilde{\Gamma}_\mu = C \Gamma_\mu$ (Γ_μ : Weyl-projected γ matrices)

2. A mechanism for 4d space-time

- moment of inertia tensor

$$T_{\mu\nu} = \frac{1}{N} \text{tr}(A_\mu A_\nu) \quad (3)$$

10 × 10 real sym. matrix

eigenvalues λ_i ($i = 1, \dots, 10$)

$$\lambda_1 \geq \dots \geq \lambda_{10} \geq 0$$

- configuration $\{A_\mu\}$ such as

$$\begin{cases} \lambda_1, \dots, \lambda_d & : \text{large} \\ \lambda_{d+1}, \dots, \lambda_{10} & : \text{small} \end{cases}$$

represents d -dimensional space time.

- If $d = 4$ configs. dominate the integral $\int dA$

→ **dynamical generation of 4d space time**
(implies SSB of $SO(10)$ inv.)

– **graviton propagation** only in 4d

demonstrated à la Randall-Sundrum

(Ishibashi-Iso-Kawai-Kitazawa '00)

(2.1) reduced SYM integrals

zero-volume lim. of D -dim. $\mathcal{N} = 1$ SYM theory
($D = 3, 4, 6, 10$)

$$\begin{aligned} Z &= \int dA d\psi e^{-S_b - S_f} \\ &= \int dA e^{-S_b} Z_f(A) \end{aligned} \quad (4)$$

finite for $\forall N, D \geq 4$ (Austing-Wheater '01)

The fermion integral :

$$Z_f(A) = \begin{cases} \text{Det } \mathcal{M} & (D = 4, 6) \\ \text{Pf } \mathcal{M} & (D = 10) \end{cases},$$

where

$$\mathcal{M}_{a\alpha, b\beta} \equiv \text{Tr} \left(t^a (\Gamma_\mu)_{\alpha\beta} [A_\mu, t^b] \right),$$

$(t^a : \text{generators of } \text{SU}(N).)$

size of the matrix \mathcal{M} : $2^{D/2-1} (N^2 - 1)$.

The phase of $Z_f(A)$:

$$Z_f(A) = |Z_f(A)| e^{i\Gamma} \quad (5)$$

$\Gamma = 0$ for $D = 4$,

$\Gamma \neq 0$ in general for $D = 6, 10$.

(2.2) the role of the phase Γ

- **no SSB of $SO(D)$ inv.** occurs for :
 - 1) bosonic model (no fermionic matrices)
(Hotta-J.N.-Tsuchiya '98)
 - 2) $D = 4$ SUSY model ($\Gamma \equiv 0$)
(Ambjørn-Anagnos...-Bietenholz-Hofheinz-J.N. '01)
 - 3) $D = 6, 10$ SUSY model omitting Γ
(Ambjørn-Anagnos...-Bietenholz-Hotta-J.N. '00)

⇒ The phase Γ must play a crucial role!

- effects of Γ in $D = 6, 10$ SUSY model
saddle-point analysis (J.N-Vernizzi, '00)

When $\{A_\mu\}$ is a d -dim. config.

$$\frac{\partial^n}{\partial A_{\mu_1}^{a_1} \partial A_{\mu_2}^{a_2} \cdots \partial A_{\mu_n}^{a_n}} \Gamma = 0 \quad \forall a_i, \mu_i \quad (6)$$

for $n \leq (D - d - 1)$.

⇒ SSB of $SO(D)$ occurs !

$3 \leq d \leq (D - 2)$ allowed

(2.3) exactly solvable matrix model

(J.N. hep-th/0108070)

$$Z = \int dA d\psi d\bar{\psi} e^{-S_b - S_f}$$

$$S_b = \frac{N}{2} \sum_{\mu=1}^4 (m_\mu)^2 \text{tr}(A_\mu)^2 ,$$

$$S_f = -(\Gamma_\mu)_{\alpha\beta} (\bar{\psi}_\alpha A_\mu \psi_\beta) . \quad (7)$$

ψ_α^f ($\alpha = 1, 2$) : N -dim. vector

$f = 1, \dots, N_f$, Veneziano limit $r \equiv N_f/N$.

fermion integral $Z_f[A]$: complex.

Solvable at $N \rightarrow \infty$ using RMT technique.

$m_\mu \rightarrow 1$ lim. keeping the order $m_1 < \dots < m_4$.

$$N^{-1} \langle \text{tr}(A_i)^2 \rangle = 1 + r \quad (i = 1, 2, 3)$$

$$N^{-1} \langle \text{tr}(A_4)^2 \rangle = 1 - r \quad (8)$$

3-dim. space-time is generated dynamically.

The phase of $Z_f[A]$ crucial for SSB.

c.f.) If we replace $Z_f[A]$ by $|Z_f[A]|$,

$$N^{-1} \langle \text{tr}(A_\mu)^2 \rangle = 1 + \frac{r}{2} . \quad (9)$$

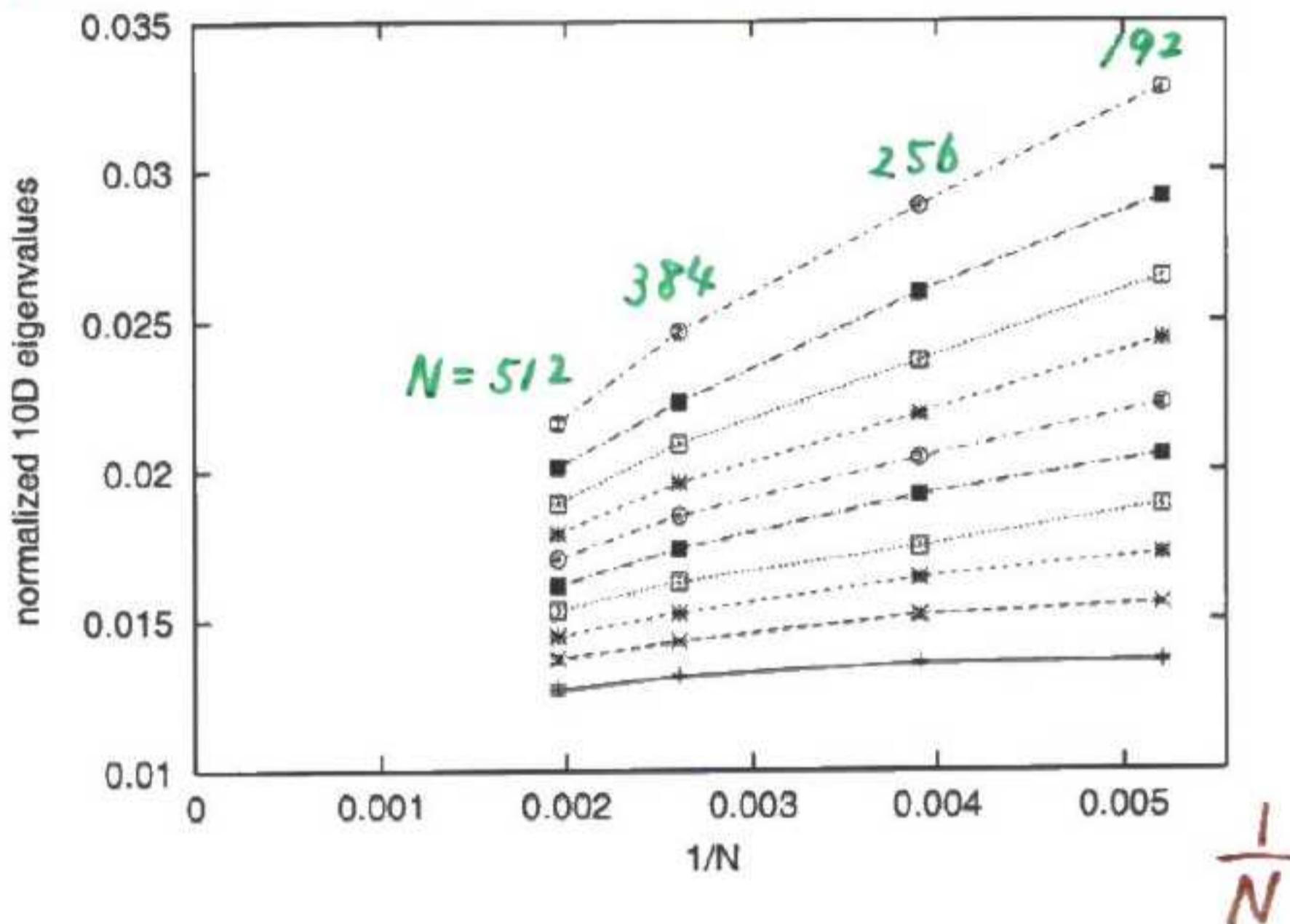
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▲ leading order in r . ($O(r)$)
 Other terms can be calculated
 easily.

eigenval's of $T_{\mu\nu} = \frac{1}{N} \text{tr}(A_\mu A_\nu)$

(moment of inertia)

$$\frac{\langle \lambda_i \rangle}{g \cdot N^{1/2}}$$



no SSB (Γ^{μ} omitted)

Ambjørn - Anagnostopoulos
- Bietenholz - Hotta - J.N. ('00)

Random Matrix Theory for finite density QCD.

$$Z_{\text{QCD}} = \int dA_\mu e^{-S_g[A]} \det D[A]$$

$$D[A] = \gamma^\mu (\partial_\mu + i A_\mu) + m + \mu \gamma_4$$

$$= \begin{pmatrix} m & iW + \mu \\ iW^* + \mu & m \end{pmatrix}$$

$$W = \sigma_i (\partial_i + i A_i) - i (\partial_\infty + i A_\infty)$$

$$\gamma^i = \begin{pmatrix} & i\sigma_i \\ -i\sigma_i & \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$$

Assuming that the structure of Dirac op.
is important, (Stephanov '96)

$$Z_{\text{RMT}} = \int dW dW^* e^{-N \text{Tr} W^* W} \det D(W)$$

$$D(W) = \begin{pmatrix} m & iW + \mu \\ iW^* + \mu & m \end{pmatrix}.$$

- exactly solvable (even for finite N).
- the effects of the phase of $\det D(W)$

RMT \Rightarrow powerful tool to explore
low-energy dynamics of QCD.
 χ SB, finite T, θ -term, ...

$$Z = \int dW dW^+ d\Psi d\bar{\Psi} e^{-N \text{Tr} W^+ W + \bar{\Psi} D \Psi}$$

$$\Psi = \begin{pmatrix} \Psi \\ \chi \end{pmatrix}, \quad \bar{\Psi} = (\bar{\Psi}_i, \bar{\chi}_j)$$

$$\bar{\Psi} D \Psi = \bar{\Psi}(iW + \mu) \chi + \bar{\chi}(iW^+ + \mu) \Psi + m(\bar{\Psi}\Psi + \bar{\chi}\chi)$$

integrate over W

$$(W_{ij}^* - \frac{i}{N} \bar{\Psi}_i \chi_j)(W_{ij} - \frac{i}{N} \bar{\chi}_j \Psi_i) - \frac{1}{N} \bar{\Psi}_i \Psi_i \bar{\chi}_j \chi_j$$

$$\bar{\Psi}_i \Psi_i \bar{\chi}_j \chi_j = \frac{1}{4} [(\bar{\Psi}_i \Psi_i + \bar{\chi}_j \chi_j)^2 - (\bar{\Psi}_i \Psi_i - \bar{\chi}_j \chi_j)^2]$$

$$\exp(-AQ^2) \sim \int d\sigma \exp\left(-\frac{\sigma^2}{4A} - iQ\sigma\right).$$

$$e^{\frac{1}{4} \bar{\Psi}_i \Psi_i \bar{\chi}_j \chi_j}$$

$$= \int d\sigma_S \exp(-N\sigma_S^2 + (\bar{\Psi}_i \Psi_i + \bar{\chi}_i \chi_i) \sigma_S)$$

$$\cdot \int d\sigma_A \exp(-N\sigma_A^2 + i(\bar{\Psi}_i \Psi_i - \bar{\chi}_i \chi_i) \sigma_A)$$

$$\sigma = \sigma_S + i\sigma_A$$

integrate over fermions

$$\sum_{i=1}^N (\bar{\Psi}_i \bar{\chi}_i) \underbrace{\begin{pmatrix} \sigma + m & \mu \\ \mu & \sigma^* + m \end{pmatrix}}_{M(\sigma)} \begin{pmatrix} \Psi_i \\ \chi_i \end{pmatrix}$$

$$\Rightarrow |\det M(\sigma)|^N$$

Effective theory of σ

$$Z = \int d\sigma d\sigma^* e^{-N\{\sigma^*\sigma - \ln \det M(\sigma)\}}$$

$$\det M(\sigma) = (\sigma + m)(\sigma^* + m) - \mu^2$$

$N \rightarrow \infty \Rightarrow$ saddle-point eq.

$$\begin{cases} -\sigma + \frac{\sigma + m}{(\sigma^* + m)(\sigma + m) - \mu^2} = 0 \\ (\sigma \leftrightarrow \sigma^*) \end{cases}$$

For $m=0$, two solutions

$$1) |\sigma| = \sqrt{1 + \mu^2} \rightarrow Z \sim e^{-N(1 + \mu^2)} \quad \mu < \mu_c$$

$$2) |\sigma| = 0 \rightarrow Z \sim \mu^{2N} \quad \mu > \mu_c$$

critical point $\mu_c = 0.527\dots$

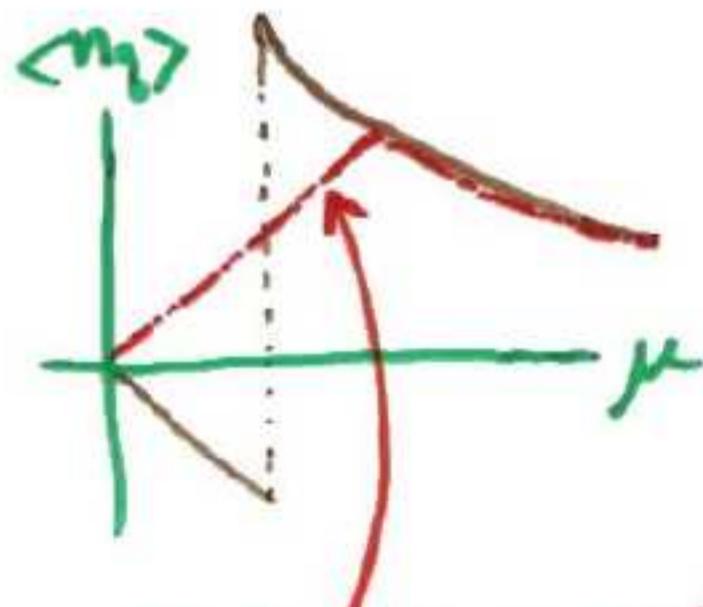
$$-(1 + \mu_c^2) = \ln \mu_c^2$$

quark number density

$$\langle n_q \rangle = \frac{1}{2N} \frac{\partial}{\partial \mu} \ln Z$$

$$= \frac{1}{2N} \left\langle \text{tr} \frac{1}{g_s D} \right\rangle$$

$$= \begin{cases} -\mu & (\mu < \mu_c) \\ \frac{1}{\mu} & (\mu > \mu_c) \end{cases}$$



result for a model with $|\det D(w)|$.

phase of $\det D(w)$ crucial.

Apply the same method
to our $SO(D)$ sym. matrix model

integrate over A_μ

→ four-Fermi action

$$S_{\text{Fermi}} = \frac{i}{N} \sum_{\alpha\beta,\gamma\delta} (\bar{\Psi}_\alpha^f \Psi_\beta^g)(\bar{\Psi}_\gamma^g \Psi_\delta^f)$$

$$\Sigma_{\alpha\beta,\gamma\delta} = \sum_\mu \frac{1}{2m_\mu} (\Gamma_\mu)_{\alpha\delta} (\Gamma_\mu)_{\gamma\beta}$$

introduce σ -variables

$$\sigma_{\alpha\beta}^{fg} \quad f, g = 1, \dots, N_f \\ u, v = 1, \dots, p$$

$$p = 2^{\frac{D}{2}-1}$$

$$Z = \int d\sigma d\sigma^\dagger e^{-N \text{Tr} \sigma^\dagger \sigma} \left\{ \det M(\sigma) \right\}^N \\ p^{N_f} \times p^{N_f}$$

For finite N_f ,

$N \rightarrow \infty \Rightarrow$ saddle-point dominates

$N \rightarrow \infty$ with $r = \frac{N_f}{N}$ fixed (Veneziano limit)

$$\sigma_{\alpha\beta}^{fg} = \sigma_{\alpha\beta} \delta_{fg} + \xi_{\alpha\beta}^{fg}$$

fluctuations

fluctuations can still be neglected at $O(r)$

Explicit results for $D=4$.

$$W(\sigma) = NN_f \left\{ (\sigma_{\text{exp}})^2 - \ln \det M(\sigma) \right\}$$

$$M(\sigma) = \begin{pmatrix} a+ib & ic+d \\ ic-d & a-(b) \end{pmatrix}$$

$$a = \sqrt{P_4} \sigma_{11} \quad b = \sqrt{P_3} \sigma_{22}$$

$$c = \sqrt{P_1} \sigma_{12} \quad d = \sqrt{P_2} \sigma_{21}$$

$$P_\mu = \sum_\nu (-1)^{\delta_{\mu\nu}} \frac{1}{m_\nu}$$

saddle-point eq.

$$\begin{cases} \sigma_{11} = \Delta^{-1} P_4 \sigma_{11} \\ \sigma_{12} = \Delta^{-1} P_1 \sigma_{12} \\ \sigma_{21} = \Delta^{-1} P_2 \sigma_{21} \\ \sigma_{22} = \Delta^{-1} P_3 \sigma_{22} \end{cases}$$

$$\Delta = a^2 + b^2 + c^2 + d$$

four solutions

$$(\sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22})$$

- ① $(1, 0, 0, 0) \Rightarrow W \sim NN_f(1 - \ln P_4)$
- ② $(0, 1, 0, 0) \Rightarrow W \sim NN_f(1 - \ln P_1)$
- ③ $(0, 0, 1, 0) \Rightarrow W \sim NN_f(1 - \ln P_2)$
- ④ $(0, 0, 0, 1) \Rightarrow W \sim NN_f(1 - \ln P_3)$

$$m_1 < m_2 < m_3 < m_4 \Rightarrow P_1 < P_2 < P_3 < P_4$$

① gives the dominant saddle point.

$$Z \sim \frac{1}{(\pi m_\mu)^{N^2/2}} \exp\left[N N_f \ln\left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3} - \dots\right)\right]$$

$$\begin{aligned}\langle \lambda_\mu \rangle &= \left\langle \frac{1}{N} \text{tr}(A_\mu)^2 \right\rangle \\ &= -\frac{2}{N^2} \frac{\partial}{\partial m_\mu} \ln Z \Big|_{m_\mu=1} \\ &= \begin{cases} 1+r & (\mu=1, 2, 3) \\ 1-r & (\mu=4). \end{cases}\end{aligned}$$

$SU(4) \xrightarrow{SSB} SU(3)$

associated with $\langle \bar{\Psi}_\alpha^f \Psi_\alpha^f \rangle \neq 0$.

- fermion determinant = $(\det \mathcal{D})^{N_f}$
 $\mathcal{D} = \Gamma_\mu A_\mu$ $pN \times pN$ matrix

$\det \mathcal{D}$ is complex in general

$$\det \mathcal{D}[A^\dagger] = (\det \mathcal{D}[A])^*$$

$\det \mathcal{D}$ is real if $\sum_\mu n_\mu A_\mu = 0$.

- If we replace it by $|\det \mathcal{D}|^{N_f}$.
 $\rightarrow \langle \lambda_\mu \rangle = 1 + \frac{1}{2}r$

The phase is crucial for SSB.

§4. $d=4$ from IIB matrix model

J.N. - Sugino hep-th/0111102

A new type of "mean-field" approach

$$\text{e.g.) } Z = \int d\phi e^{-S} \quad S = \phi^4$$

$$= \int d\phi e^{-(S-S_0)} e^{-S_0} \quad S_0 = \frac{1}{2}M^2\phi^2$$

$$= \sum_{k=0}^{\infty} \int d\phi \frac{(-1)^k}{k!} (S-S_0)^k e^{-S_0}$$

$$F \equiv -\ln Z = \sum_{k=0}^{\infty} F_k$$

$$\begin{cases} F_0 = -\ln Z_0 \\ F_k = -\frac{(-1)^k}{k!} \langle (S-S_0)^k \rangle_{0,c} \quad k \geq 1 \end{cases}$$

$$\langle \Theta \rangle = \langle \Theta \rangle_0 + \sum_{k=1}^{\infty} \frac{(-1)^k}{k!} \langle \Theta (S-S_0)^k \rangle_{0,c}$$

$$(\text{e.g. } \Theta = \phi^2)$$

If we truncate the series at $k=n$,
the results depend on M .

determine M by $\frac{\partial}{\partial M} \left(\sum_{k=0}^n F_k \right) = 0$

results up to $n=10 \Rightarrow$ converge to exact results

Study SSB of $SO(D)$ sym in matrix models!

§4.1 bosonic model

$$Z = \int dA_\mu e^{-S}, \quad S = -\frac{1}{4} N \text{tr}[A_\mu, A_\nu]$$

$$S_0 = \sum_{\mu=1}^D \frac{N}{v_\mu} \text{tr}(A_\mu)^2.$$

"gap eq." at order 1.

$$\frac{\partial}{\partial v_\mu} (F_0 + F_1) = 0$$

$$\Rightarrow -\frac{1}{2v_\mu} + \frac{1}{4} \sum_{\nu \neq \mu} v_\nu = 0.$$

$$\Rightarrow v_1 = \dots = v_D = \sqrt{\frac{2}{D-1}} \quad (\text{no SSB})$$

$$\left\langle \frac{1}{N} \text{tr}(A_\mu)^2 \right\rangle = \sqrt{\frac{2}{D-1}} \quad (\forall \mu = 1, \dots, D)$$

note:

- bosonic model allows a systematic $1/D$ expansion.
Hotta - J.M. - Tsuchiya ('98)
- the above (order 1) result becomes exact at $D \rightarrow \infty$.
- absence of SSB for $D \geq 3$
all orders in $1/D$ expansion
MC simulation at $D = 3, 4, 6, 8, \dots, 20$

(4.2) supersymmetric case

J.N. - Sugino
hep-th/0111102

- SUSY Gaussian action
 - general SUSY models (Kabat-Lifschytz '00)
 - BFSS Matrix Theory at strong coupling consistent with supergravity dual nonperturbative BH thermodynamics.
 - reduce YM integrals (Sugino '01).
 $\langle W \rangle, \langle P \rangle$ for $D = 4$
 qualitative agreement with MC results.

For $D = 4$, no SSB (\simeq MC results)

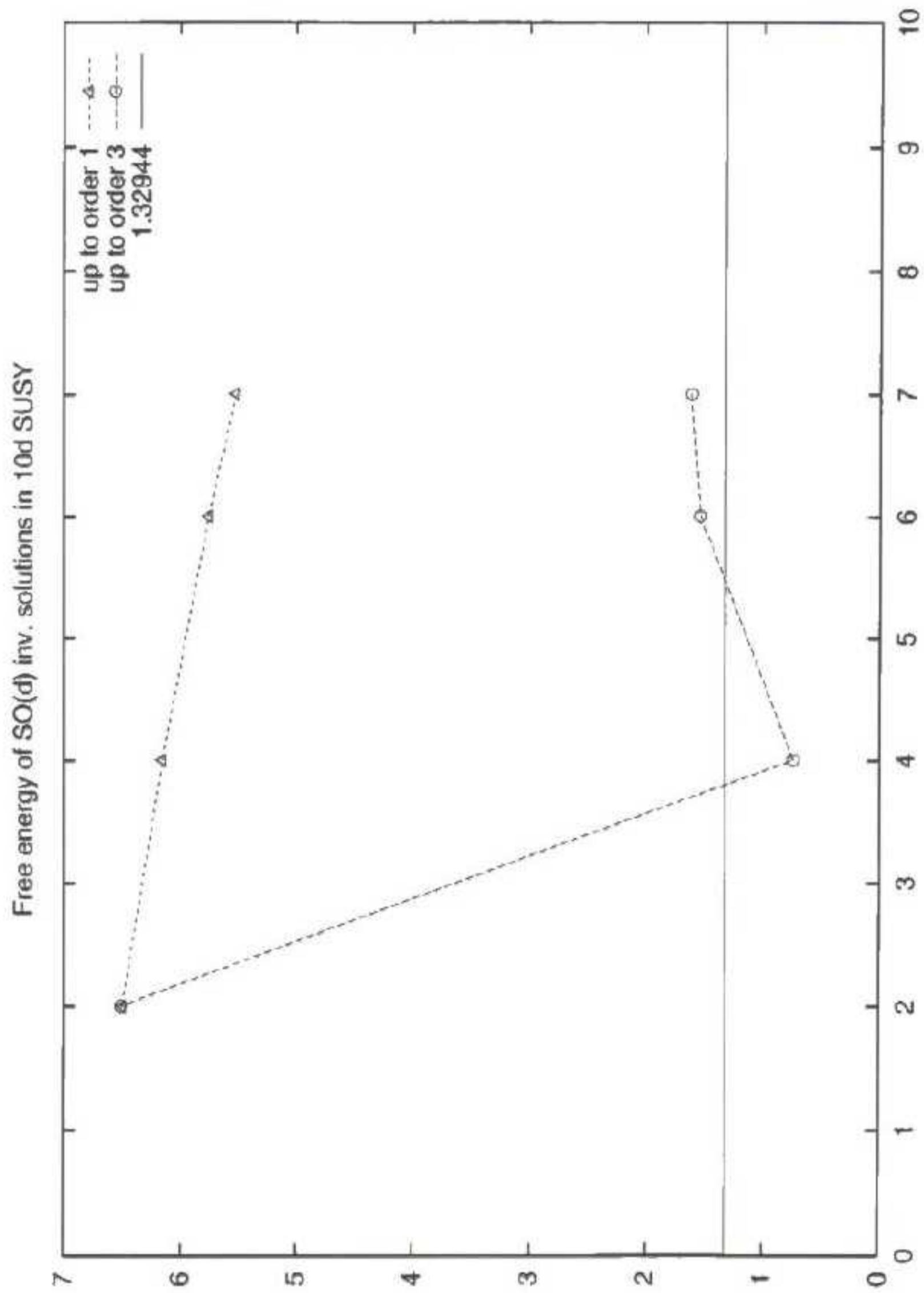
For $D = 6, 10$, SO(4) inv. solution exists !

$$v_1 = \dots = v_4 = V$$

$$v_5 = \dots = v_D = v < V .$$

Higher order corrections (up to $n = 3$)
 space-time further collapses.

$\sum_{k=0}^n F_k \rightarrow$ (conjectured) exact result
 (Green-Gutperle, Krauth-Nicolai-Staudacher,
 Moore-Nekrasov-Shatashvili)



§5. testing the new method with exactly solvable m. m.

$$S_0 = \sum_{\mu=1}^P \frac{N}{U_\mu} \text{tr}(A_\mu)^2 + N \sum_{\alpha, \beta=1}^P A_{\alpha\beta} \bar{\Psi}_{\alpha i}^\dagger \Psi_{\beta i}^f$$

$$D=4 \quad A = \sum_{\mu=1}^4 U_\mu \Gamma_\mu$$

order 1

$$F = \underbrace{-2 \ln 2 + (1 - \ln 2)r - 2r^2 + \dots}_{\text{SO(3)}}$$

$$\quad \quad \quad - \frac{3}{2} r^2 + \dots \quad \text{SO(2)}$$

$$\quad \quad \quad - \frac{4}{3} r^2 + \dots \quad \text{SO(1)}$$

$$\quad \quad \quad - \frac{5}{4} r^2 + \dots \quad \text{SO(0)}$$

agrees with
exact result
up to $O(r)$

SO(3) inv. solution gives smallest F
at small r.

$$\left\{ \begin{array}{l} \langle \lambda_i \rangle = \underbrace{1+r - 2r^2 + \dots}_{(i=1,2,3)} \\ \langle \lambda_4 \rangle = \underbrace{1-r + 6r^2 + \dots} \end{array} \right.$$

order 3

$$F = \underbrace{-2 \ln 2 + (1 - \ln 2)r - \frac{7}{3} r^2 - 5.22 r^{5/3} + \dots}_{\text{SO(3)}} \quad \text{SO(3)}$$

$$\quad \quad \quad - 2.64 r^{5/3} + \dots \quad \text{SO(2)}$$

$$\left\{ \begin{array}{l} \langle \lambda_i \rangle = \underbrace{1+r - 2.85 r^{5/3} + \dots}_{(i=1,2,3)} \\ \langle \lambda_4 \rangle = \underbrace{1-r + 8.54 r^{5/3} + \dots} \end{array} \right.$$

Stability of the solutions

SO(3) inv. solutions

3 parameters

$$\begin{cases} X_1 \equiv u_1 = u_2 = u_3 \\ X_2 \equiv u_4 \\ X_3 \equiv u_4 \end{cases}$$

$$(u_1 = u_2 = u_3 = 0)$$

Hessian

$$H_{ij} = \frac{\partial^2 F}{\partial X_i \partial X_j}$$

3 x 3 real sym mat

eigenvalues $(\alpha_1, \alpha_2, \alpha_3)$

order 1.

$$\begin{cases} \alpha_1 = 0.375 - 0.75r + 3r^2 + \dots \\ \alpha_2 = 0.125 + 0.25r - r^2 + \dots \\ \alpha_3 = -0.25r + r^2 + \dots \end{cases}$$

order 3

$$\begin{cases} \alpha_1 = -2.75r + 1.55r^{4/3} + \dots \\ \alpha_2 = 1.04 r^{4/3} + \dots \\ \alpha_3 = 0.148 r^{4/3} + \dots \end{cases}$$

Hessian becomes smaller ($r \sim 0$)
as we go to higher order.

§ 6. Summary

- Dynamical origin of space-time dim.
in nonperturbative string theory.
4 dim from IIB matrix model.
- a mechanism for SSB of $SU(10)$.
phase of fermion det.
- a concrete example of matrix models
which realize this mechanism.
- test new methods
 - a new type of "mean-field"
approach
 - MC technique for complex action

Future prospects :

- further developments
of these methods
- various applications
- other dynamical properties of IIB m.m.
e.g.) gauge group of S.M.